LDL Decomposition-based FPGA Real-time Implementation of DOA Estimation

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***Abstract*—An FPGA implementation and real-time experimental verification of proposed direction of arrival (DOA) estimation algorithm employing LDL factorization are presented in this paper. The proposed algorithm is implemented on a Xilinx FPGA using LabVIEW software and its real-time experimental verification is performed using National Instruments (NI) PXI platform. The proposed method has several advantages over well-known methods which are based on either eigen value decomposition (EVD) or singular value decomposition (SVD). It provides faster execution since LDL factorization requires $O(n^3/6)$ number of operations whereas EVD requires $O(n^3)$. Results from Matlab simulations and real-time experiments demonstrate the effectiveness of the proposed method. Successful FPGA compilation reports show low resource usage and faster computation time for LDL-based method compared with QR-based implementations. Performance comparison is done in terms of estimation accuracy, FPGA processing time and resource utilization.**

Keywords*—*FPGAs, LDL factorization, NI PXI platform, pipelined architecture

I. INTRODUCTION

DOA estimation has become a very important research topic owing to its many applications in both civilian and military areas. There are numerous practical applications of DOA estimation in areas such as beam forming ‘smart’ adaptive antenna arrays, source localization in radar and sonar systems, channel estimation and equalization, multiple-input-multiple-output (MIMO) systems, and echo and interference cancellation [1-2]. A detailed study of literature in these areas shows that most of the research work has focused primarily on numerical simulations of DOA estimation algorithms to establish their effectiveness and accuracy [3-8]. However, for practical implementation, the efficiency and accuracy of these algorithms must be established on real hardware, and experimentally validated.

An important consideration in the hardware implementation of DOA estimation algorithms is the computational complexity of the algorithm besides its speed and estimation accuracy. Existing DOA techniques [3-7] based on either EVD or SVD separate noise and signal subspaces for inferring angles of arrival of incident signals. A drawback of these techniques is that they are not suitable for hardware implementation due to their compute intensive nature. Techniques based on QR, LU, and LDL decomposition on the other hand are significantly less complex as they do not require either EVD or SVD.

Another important consideration for hardware implementation is the choice of hardware platform and its suitability for real-time implementation of DOA estimation algorithms in terms of estimation time, memory requirements, scalability, cost and development time. A hardware implementation (using NI PXI platform and LabVIEW software) of novel DOA estimation methods based on QR decomposition has been reported in [9-10]. However, the algorithms execute on a PC-based processor running on a Windows OS; a real-time implementation on a device such as an FPGA was not studied.

Recent works on FPGA real-time implementation of DOA estimation algorithms based on QR decomposition can be found in [11] and that based on LU decomposition in [12]. The QR-based algorithm in [11] is reported to have outperformed existing hardware implementations reported in the literature of unitary-MUSIC [13-14], MUSIC [15] and ESPRIT [16] algorithms. The proposed LU-based implementations in [12] have superior performance compared with the QR-based algorithms in [11].

In the work presented in this paper, we employed LDL decomposition, which is a close variant of Cholesky decomposition. LDL decomposition factors matrix $A = LDL^T$ as a product of a lower triangular matrix with 1's on the diagonal, a diagonal matrix, and the transpose of the lower triangular matrix. LDL has significantly less complexity compared to QR. LDL factorization requires $O(N^3/6)$ flops while QR requires $O(N^3)$ flops. Lower complexity reduces memory requirements and processing time making LDL preferable over other methods for hardware implementation. A multiple target tracking algorithm based on LDL decomposition is presented in [17] which is reported to be computationally efficient compared with extended Kalman filter (EKF) based algorithms.

Xilinx Virtex-5 FPGA [18] was chosen for the hardware implementation of the proposed DOA estimation algorithm based on LDL factorization. LabVIEW FPGA with high throughput modules [19] was used for FPGA programming. Performance of the proposed algorithm has been measured in terms of estimation accuracy, processing time, and resource utilization and has been compared with QR decomposition-based DOA estimation methods (QR-Q, QR-R). The proposed DOA estimation algorithm has been experimentally validated through real-time testing on a hardware prototype built using NI PXI platform [20]. LDL-based prosed method is shown to be superior to QR-based methods in all performance parameters.

This paper is organized as follows: Section II presents the system model and the proposed method based on LDL decomposition; section III describes the FPGA hardware implementation of the proposed DOA estimation algorithm; section IV discusses the FPGA resources utilization and processing time for the proposed method; Section V presents results from Matlab simulation; Section VI presents the experimental results from the real-time DOA estimation; and finally, conclusions are presented in section VII.

II. SYTEM MODEL

The system model shown in Fig. 7 consists of a uniform linear array (ULA) composed of four omni-directional antennas (*M=4*) placed 16 cm apart which is equivalent of having the wavelength of 900 MHz. Up to two sources ($K=1$ and $K=2$) are considered in the far-field region of the ULA for real-time testing using NI PXI platform. The two RF sources are assumed to be located at angles θ_1 *and* θ_2 from the ULA. At any time instant ***t***, the snapshot of the signal received at the ULA can be expressed as:

$$x_m(t)=\sum_{i=1}^{K} s_i(t)\, e^{-j(2\pi/\lambda)dm\cos\theta_i}+n_m(t);\left(m=1,2,\cdots,4\right)\ and\ K=1,2 \qquad (1)$$

where $s_i\left(t\right)$ is the signal from the *i*-th incident source, λ is the wavelength, $\left(d=\lambda/2\right)$ the spacing distance of ULA, and $n_m\left(t\right)$ is the noise at the *m*-th element.

The received data can be expressed as:

$$X\left(t\right)=A\left(\theta\right)S(t)+N\left(t\right), \qquad (2)$$

where $A(\theta)$ is the $(M\times K)$ array response matrix given as:

$$\mathrm{A}(\theta)=\left[\mathbf{a}(\theta_1)\quad \mathbf{a}(\theta_2)\quad \ldots\quad \mathbf{a}(\theta_K)\right], \qquad (3)$$

where $a(\theta_i)$ for $i=1,2,\ldots,K$ is the corresponding array response vector.

$$a\left(\theta_K\right)=\left[1\quad \cdots\quad u_K^M\right]^T, where\ \ u_k=\exp\left(-j2\pi\, d\cos\left(\theta_k\right)/\lambda\right) \qquad (4)$$

where S(t) is the vector of received signals given by:

$$S(t)=\left[s_1(t)\quad s_2(t)\quad \ldots\quad s_K(t)\right]^T, \qquad (5)$$

and

$$N(t)=\left[n_1\left(t\right)\quad \cdots\quad n_M\left(t\right)\right], \qquad (6)$$

is the $\left(M\times 1\right)$ additive white Gaussian noise (AWGN) vector. Here and in the following sections, the superscripts * and *T* denote the conjugate and transpose operations, respectively.

2.1 PROPOSED DOA ESTIMATION USING LDL METHOD

In the proposed method, LDL decomposition is used for estimating either the lower or upper triangular matrix L, which can be used for determining the DOA angle estimates of multiple RF sources incident on the ULA. The signal space contained in the lower triangular matrix L is used for extracting the DOA information while the direction matrix is obtained using the least squares (LS) approach. The following subsections present detailed information about the proposed method.

A step-by-step procedure for the proposed method is presented in detail below for estimating up to two sources ($K=2$) employing a ULA consisting of four antenna elements ($M=4$.

Step 1*:* First, factorize data matrix R by applying LDL^H factorization, as shown below:

$$R=\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14}\\ r_{21} & r_{22} & r_{23} & r_{24}\\ r_{31} & r_{32} & r_{33} & r_{34}\\ r_{41} & r_{42} & r_{43} & r_{44}\end{bmatrix}$$

$$LDL^H(R)=\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0\\ l_{21} & 1 & 0 & 0\\ l_{31} & l_{32} & 1 & 0\\ l_{41} & l_{42} & l_{43} & 1\end{bmatrix}}_{L}\underbrace{\begin{bmatrix} D_{11} & 0 & 0 & 0\\ 0 & D_{22} & 0 & 0\\ 0 & 0 & D_{33} & 0\\ 0 & 0 & 0 & D_{44}\end{bmatrix}}_{D}\underbrace{\begin{bmatrix} 1 & l_{21}^* & l_{31}^* & l_{41}^*\\ 0 & 1 & l_{32}^* & l_{42}^*\\ 0 & 0 & 1 & l_{43}^*\\ 0 & 0 & 0 & 1\end{bmatrix}}_{L^H} \qquad (7)$$

Step 2: Next, since we consider two sources, only the first two columns of $\boldsymbol{L}$ are extracted which span the same signal space as the columns of the steering vectors in $\boldsymbol{A}\left(\theta\right)$. The signal space for the two sources $\boldsymbol{L_s}\in\mathbb{C}^{M\times 2}$ is obtained as follows:

$$L_s=\begin{bmatrix} 1 & 0\\ l_{21} & 1\\ l_{31} & l_{32}\\ l_{41} & l_{42}\end{bmatrix} \qquad (8)$$

The DOA angles estimates can be obtained from the matrix L_s of size$\left(M\times 2\right)$. The following formulae are used for finding the entries of $L\ and\ D$:

$$D_j=r_{jj}-\sum_{k=1}^{j-1} L_{jk}L_{jk}^{*}D_k,$$
$$L_{ij}=\frac{1}{D_j}\left(r_{ij}-\sum_{k=1}^{j-1} D_k L_{jk}L_{jk}^{*}\right)\ ;\text{for } i\succ j. \qquad (9)$$

Step 3*:* Next, the L_s data matrix is partitioned into two sub-matrices of size (3×2) as follows:

$$L_{s1}=L_s(1:3,1:2),$$
$$L_{s2}=L_s(2:4,1:2) \qquad (10)$$

Since the range $\Re[l_s] = \Re[A]$, there must exist a unique matrix T, such that:

$$L_s = \begin{bmatrix} l_{s1} \\ l_{s2} \end{bmatrix} = \begin{bmatrix} A_1(\theta)T \\ A_1(\theta)\Phi T \end{bmatrix}, \tag{11}$$

where $\mathrm{A}_1(\theta) = [\mathbf{a_1}(\theta_1) \quad \mathbf{a_1}(\theta_2)]$ is the array response matrix of size (3×2), $a_1(\theta_1) = \begin{bmatrix} 1 & \cdots & u_1^3 \end{bmatrix}^T$, and Φ is a diagonal matrix of size (2×2) containing information about the DOA angle estimates of the incident sources.

$$\Phi = diag\begin{bmatrix} e^{-\frac{j2\pi d\cos(\theta_1)}{\lambda}} & \cdots & e^{-\frac{j2\pi d\cos(\theta_2)}{\lambda}} \end{bmatrix}$$

It can be easily seen that: $\Re[l_{s1}] = \Re[l_{s2}] = \Re[A_1]$, since l_{s1} *and* l_{s2} span the same signal space. This leads to both spaces being related by a nonsingular transform Λ as follows:

$$l_{s2} = l_{s1}\Lambda \tag{12}$$

Since $\boldsymbol{A}$ is a full rank matrix for uncorrelated sources, equation (12) can be expressed as:

$$\Lambda = T^{-1}\Phi\ T \tag{13}$$

Finding the eigenvalues of the matrix Λ which are the diagonal elements of Φ, will lead to obtaining the DOA angle estimates for the incident sources.

Equation (12) can be solved using the LS approach (LS-ESPRIT) which minimizes the difference between l_{s2} and $l_{s1}\Lambda$.

$$\begin{aligned} \Lambda &= \arg\ \min_{(\Lambda)} \| l_{s2} - l_{s1}\Lambda \|_F^2 \\ &= \arg\min_{(\Lambda)} tr\{[l_{s2} - l_{s1}\Lambda]^H [l_{s2} - l_{s1}\Lambda]\} \end{aligned} \tag{14}$$

The LS solution of (14) can be found as:

$$\Lambda = \left[l_{s1}^H l_{s1}\right]^{-1} l_{s1} l_{s2} \tag{15}$$

Step 4: Next, the eigenvalues Γ_k of the matrix Λ in (15) are computed.

Step 5: Finally, the DOA angle estimates of multiple incident sources are computed using the following expression:

$$\theta_K = \cos^{-1}\left(\frac{angle((\Gamma_K)_\Lambda)}{2\pi d}\right); K = 1,2 \tag{16}$$

where Γ_K is the kth eigenvalue.

III. FPGA IMPLEMENTATION OF PROPOSED DOA ESTIMATION ALGORITHM

The proposed algorithm has been implemented in hardware on NI PXIe-7965R FlexRIO FPGA module [22] using LabVIEW FPGA modules and LabVIEW software [21]. The FlexRIO features Xilinx Virtex-5 SXT FPGA [18] with 512 MB of onboard RAM and 40 MHz of onboard base clock.

FPGA implementation of the proposed algorithm was carried out using LabVIEW FPGA modules with high throughput mathematical functions. Fixed-point data type with a data size of 16/8 was selected (word length/integer length in bits).

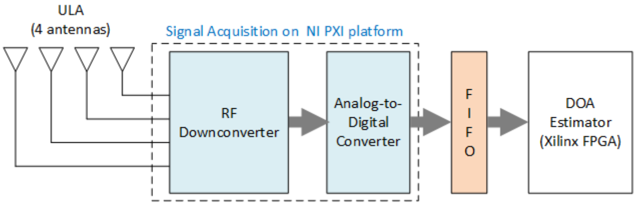

Fig. 1. Hardware implementation model

Fig. 1 shows the hardware implementation model for the real-time DOA estimation using the proposed LDL algorithm. Signals received from the ULA are first down-converted, digitized, and then stored in a FIFO (first-in first-out queue). Signal acquisition and digitization steps are executed on the host (PC) while the proposed DOA estimation algorithm is executed on the FPGA target. The FIFO is used for transferring signal data from the host PC to the FPGA using direct memory access.

The proposed DOA estimation algorithm, as shown in Fig. 2, is implemented on the FPGA using a pipelined architecture to achieve high throughput. The major operations of the algorithm are represented by the different stages of the pipeline.

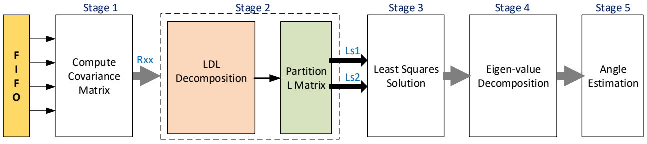

Fig. 2. Pipelined execution of DOA estimation using LDL Decomposition

Stage 1: First, the signal data received from the four-antenna array of the ULA is retrieved from the FIFO and is used to estimate the covariance matrix $\boldsymbol{R}_{xx}$. The covariance matrix is estimated from N snapshots as follows:

$$\hat{\boldsymbol{R}}_{xx} = E\left[\mathbf{x}(t)\mathbf{x}(t)^H\right] = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}(t)\mathbf{x}(t)^H \tag{17}$$

where $x(t)$ is the column vector from the ith antenna element.

In LabVIEW FPGA, (17) is implemented using multiply and accumulate operations.

Stage 2: In this stage, the LDL decomposition is performed as given by (7).

Since up to two incident sources ($K = 2$) are considered, only the first two columns of $\boldsymbol{L}$ matrix are required to be computed. The matrix $\boldsymbol{L}$ is then partitioned into two sub-matrices as given by (10). This step is implemented on the FPGA using LabVIEW FPGA modules as shown in Fig. 3 below.

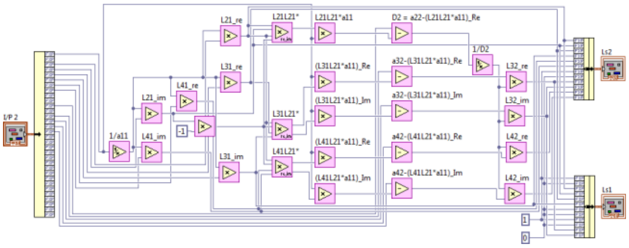

Fig. 3. Generation of Ls matrix and its partitioning in LabVIEW FPGA

Stage 3: In this stage, the LS solution of $\Lambda = \left[\boldsymbol{l}_{s1}^{H}\boldsymbol{l}_{s1}\right]^{-1}\boldsymbol{l}_{s1}\boldsymbol{l}_{s2}$ in (15) is implemented. The LabVIEW FPGA implementation of the LS solution requires a matrix inversion operation and several complex-number multiplication operations.

Stage 4: In the fourth stage, the eigenvalues of matrix $\boldsymbol{\Lambda}$ (given in (15)) are obtained by performing eigenvalue decomposition (EVD). The eigenvalues, for a given matrix $\boldsymbol{A}$, can be calculated (using algebraic method) as $determinant(A - \lambda I) = 0$.

This stage may present a bottleneck in the pipeline owing to the computationally intensive operations such as complex square root and division operations required in computing the EVD.

Stage 5: In this final stage of the pipeline, the DOA angle estimates are computed according to (16). A look-up table (LUT) is used to store pre-computed values of cos^{-1}() for faster angle estimation.

IV. FPGA PROCESSING TIME AND RESOURCE UTILIZATION

The proposed algorithm has been implemented in hardware on Xilinx Virtex-5 SXT FPGA [18] using LabVIEW FPGA modules and LabVIEW software [21].

LabVIEW codes called VIs (Virtual Instruments) were developed to implement the proposed DOA algorithm employing LDL factorization as well as QR factorization (the latter for comparison). A successful compilation of the VIs produces a report on the FPGA processing time required (in MHz) and resources consumed.

Table I below shows FPGA resources consumption in the implementation of DOA estimation algorithm employing QR-Q, QR-R, and LDL. It can be observed clearly that the proposed LDL-based DOA estimation method consumes the least amount of resources while QR-R on the other hand consumes the highest amount.

TABLE I. FPGA RESOURCES CONSUMED FOR DOA ESTIMATION

Resource	**QR-Q**	**QR-R**	**LDL**
Total Slices	9555	10846	8707
Slice Registers	18778	22840	17363
Slice LUTs	24820	30568	22255
Block RAMs	10	10	10
DSP48s	270	418	234

Fig. 4 shows FPGA resources consumption as a percentage of device utilization for data size of 16/8 for DOA angles estimation employing LDL, QR-Q, and QR-R based methods. Total computation time in terms of clock cycles for DOA estimation has also been calculated. The number of clock cycles taken by each stage of the FGPA execution pipeline to calculate the DOA angles is shown in Table II. The number of clock cycles was calculated based on the longest propagation path. We observe in Fig. 5 that the proposed LDL-based method is faster compared with QR-R and QR-Q. The FPGA is expected to take 4.525 µs for DOA angles estimation using LDL (with the onboard clock of 40 MHz). This is the fastest time among the three methods.

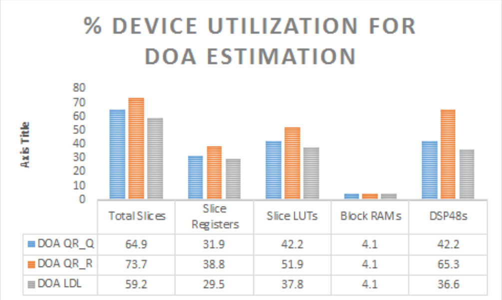

Fig. 4. Percentage Device utilization for DOA estimation

TABLE II. CLOCK CYCLES FOR DOA ESTIMATION USING QR AND LDL

#	**Pipeline Stage**	**QR-Q**	**QR-R**	**LDL**
1	Covariance Matrix computation	3	3	3
2	Matrix Decomposition (4x4)	59	75	44
3	Least square solution	28	28	28
4	Eigen value decomposition (EVD)	82	82	82
5	Angle Estimation	24	24	24
	Total clock cycles	196	212	181

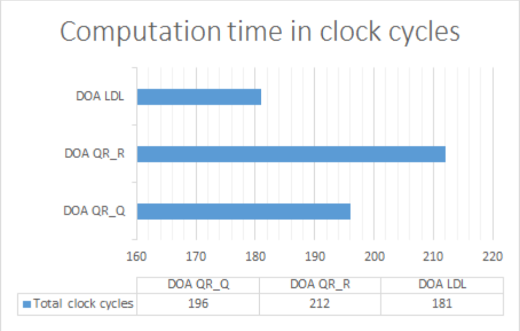

Fig. 5. Computation time in clock cycles for DOA

From these results, it can be concluded that overall LDL outperforms QR-based methods in terms of processing time as well as resources utilization.

V. MATLAB SIMULATION RESULTS

Matlab simulations were conducted for verification of the proposed LDL-based DOA estimation algorithm. Two non-coherent sources ($K=2$) and four antenna elements ($M=2$) are considered. The inter element spacing between the antenna elements is taken as half the wavelength of the incoming signal. Monte-Carlo trials are considered and the performance of the proposed algorithm is measured in terms of root mean square error (RMSE) which is defined as follows:

$$RMSE = \frac{1}{K}\sum_{k=1}^{K}\sqrt{E\left[(\hat{\theta}_k - \theta_k)^2\right]} \qquad (18)$$

where k represents the source index and $E\left[Q\right]$ represents the expectation value of a random variable Q.

Two uncorrelated sources are considered with the DOA angles at 70° and 85° from the ULA reference. The combined RMSE values versus SNR for the two sources is shown in Fig. 6 for the proposed LDL-based method and QR-Q and QR-R methods. The SNR is set from 0 to 30 dB, 500 snapshots are used, and 300 Monte-Carlo trials are conducted. It can be observed in Fig. 6 that the LDL-based method has very good estimation accuracy which is comparable with that of QR-R.

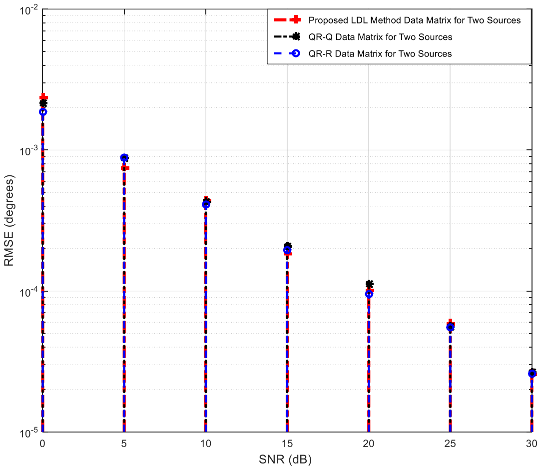

Fig. 6. Combined RMSE values for the proposed LDL and QR-Q, QR-R methods for two sources lying at 70° and 85° from the array reference

VI. REAL-TIME EXPERIMENTAL VALIDATION

Real-time experimental validation of the proposed DOA estimation algorithm was carried out on the NI PXI platform comprising arbitrary waveform generators, data acquisition modules, RF downconverters and up-converters, digitizers, local oscillators, and FlexRIO with Xilinx Virtex-5 FPGA module.

A. Experimental Setup

The experimental setup as shown in Fig. 7 consists of two transmitters (seen in the foreground) and a uniform linear array with four antenna elements deployed at the receiver (seen in the background). The inter-element spacing between the receiver antennas is 16 cm corresponding to half wavelength (λ/2).

The transmitter unit which acts as a source lying in a far field region of the receiver is implemented using an arbitrary waveform generator (AWG) module (NI PXI-5421), an up-converter module (NI PXIe-5652), and RF amplifier module (NI PXI-5691).

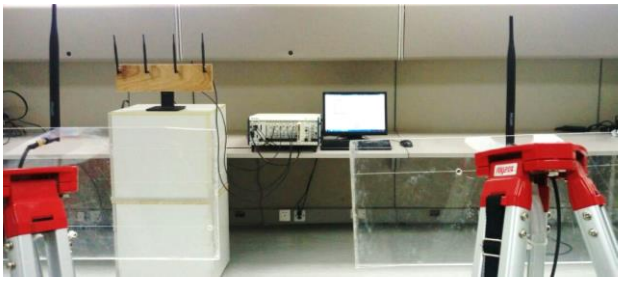

Fig. 7. Experimental setup showing two transmitters and a 4-element antenna array and PXI system

The receiver units are composed of an RF downconverter (PXIe-5601) and a high-speed digitizer (PXIe-5622), with each unit connected to an antenna in the four element ULA. The NI PXI platform houses four such receiver units, a local oscillator, and FlexRIO FPGA module. All the receiver units share the same clock signal generated by the local oscillator (LO).

B. Procedure for Real-time DOA Estimation

The step-by-step procedure listed below was followed for conducting real-time experimental validation of the proposed DOA estimation algorithm.

Step 1: LabVIEW FPGA codes for the proposed DOA angle estimation algorithm are compiled.

Step 2: RF transmitter and receiver units are set up and successful signal reception is checked.

Step 3: DOA FPGA LabVIEW code is executed to run on the NI PXI platform and the front panel is configured for real-time data acquisition.

Step 4: Co-phase synchronization is performed to calibrate the phase differences of all RF receiver channels.

Step 5: LabVIEW FPGA code for DOA angle estimation is executed on the target FPGA and the estimated angles are recorded.

C. Real-Time DOA Angle Estimation Results

Real-time experimental validation of the DOA estimation algorithm in the case of two sources placed at arbitrary angles was performed with 20 trials (10 iterations with 1000 snapshots in each trial). The calculated mean values of DOA estimates are shown in Table III.

TABLE III. MEAN DOA ANGLE ESTIMATES OF TWO SOURCES OF 20 SUCCESSFUL TRIALS FROM REAL-TIME EXPERIMENTS

Actual location: Two sources	Real-time DOA Estimation		
	Proposed LDL	**QR-Q**	**QR-R**
(70°, 120°)	(69.52°, 119.82°)	(68.71°, 119.10°)	(69.63°, 120.22°)
(75°, 110°)	(75.45°, 109.63°)	(74.12°, 109.23°)	(75.34°, 109.64°)
(60°, 100°)	(60.57°, 100.55°)	(58.94°, 101.11°)	(60.46°, 100.62°)
(50°, 90°)	(49.61°, 90.48°)	(49.08°, 90.87°)	(50.21°, 90.52°)

Fig. 8 shows a snapshot of the real-time DOA estimates obtained using LDL and QR-Q based methods for two RF sources placed at arbitrary angles of 70° and 120°, respectively, from the ULA.

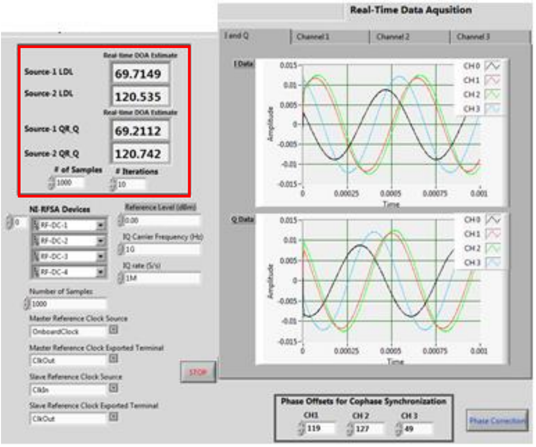
Real-time DOA Estimate
Source-1 LDL 69.7149
Source-2 LDL 120.535
Real-time DOA Estimate
Source-1 QR_Q 69.2112
Source-2 QR_Q 120.742
# of Samples 1000
Iterations 10
NI-RFSA Devices 0: RF-DC-1, RF-DC-2, RF-DC-3, RF-DC-4
Reference Level (dBm) 0.00
IQ Carrier Frequency (Hz) 10
IQ rate (S/s) 1M
Number of Samples 1000
Master Reference Clock Source OnboardClock
Master Reference Clock Exported Terminal ClkOut
Slave Reference Clock Source ClkIn
Slave Reference Clock Exported Terminal ClkOut
Phase Offsets for Cophase Synchronization
CH1 119
CH2 127
CH3 49

Fig. 8. Real-time DOA angle estimates for two sources located at 70° & 120° for LDL and QR-Q methods.

It can be observed from the results shown that both LDL and QR-R offer higher accuracy in DOA angles estimation compared with QR-Q. While the estimation accuracy of both QR-R and LDL is comparable, LDL-based algorithm is better overall since it executes faster and consumes fewer resources.

VII. CONCLUSIONS

In this paper, we presented our proposed DOA estimation algorithm based on LDL decomposition and its hardware implementation on an FPGA. The performance of this algorithm was evaluated through Matlab simulations as well as real-time experiments on NI PXI platform. The performance of the LDL-based method compared with QR-based DOA estimation algorithms was found to be superior for hardware implementation in terms of computational complexity, estimation accuracy, FPGA processing time and resources utilization.

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