Three epistemic paralogisms, one logic of utterances

Fabien Schang

Abstract

Assuming that a paralogism is an unintentionally invalid reasoning, we give an exemplification by means of three epistemic "paradoxes", namely: Fitch's Paradox, Moore's Paradox, and Zemach's Paradox. A common symptom lies on the basis of these three paralogisms, which concerns the satisfaction-conditions of a discourse about truth. From an anti-realist understanding of the paradoxes, we propose an alternative semantic framework in which the meaning of sentences relies upon a question-answer game. Then we set forth some features of this many-valued framework, before applying it to the three preceding paradoxes and displaying its general prospect.

1. Logical paradoxes: one trouble, two treatments

A *logical paradox* is a thought defect that one strives to correct; it occurs within a reasoning, when the premises are taken to be unanimously true but lead to a conclusion which is not taken to be so. The expected ending of a paradox is its resolution: the first point is to understand how the conclusion has been introduced and, then, to determine the way in which such an undesired result should be deleted.

A similar form of defecting thought is to be distinguished from the paradox, namely: the *paralogism*, where the reasoning includes a premise that is unanimously true in appearance but turns out to be false from a certain point of view. The conclusion thus relies upon an initial misunderstanding, so that the problem is solved by its not being so any more. A famous case of paralogism is the naturalistic fallacy, in which the difficulty stems from mistaking a value judgment for a factual judgment. In the same vein, we will turn our attention to another kind of paralogism: an *epistemic paralogism*, where the trouble stems from mistaking a knowledge statement for a sentence about knowledge.

Some kinds of paralogism may be more difficult to eradicate, because they require a preliminary reflection about which premise should be rejected. In other words, the borderline between paradoxes and paralogisms may be muddled in case of a disagreement about the origin of the trouble. The Liar Paradox is a typical case in point: the origin of the trouble and the suggested treatments are not uniquely identified within the community of logicians, because their assessment of the reasoning needn't rely upon one and the same norm of rationality. If so, the disagreement is about the nature of the suggested "solution"; Graham Priest's solution of dialetheism is a radical sort of assessment where the initial problem is not even considered as such.

By supporting philosophy with the help of logical analysis, *philosophical logic* corresponds to the set of logical systems developed in order to deal with paradoxes and give a better understanding of the philosophical concepts that generated the trouble. Although a paradox is commonly viewed as a defect of the reasoning, there is a sensible difference between the *rectification* of such a reasoning and its *clarification*: the former method cancels the defect by submitting thought to the norms of logic; the latter takes the defect into account by researching the mechanisms of thought that led to the undesired conclusion. To rectify the reasoning is to make use of logic as a normative model for thought. To clarify the reasoning is to see logic as a descriptive instrument, i.e. a way among several ones to describe the different steps of reasoning that lead to the blamed conclusion. The current trend pays scant attention of the rectifying stance; rather, it counts on the descriptive role of logic in the process of explanation. Rightly or wrongly, depending upon the light thrown by this description.

Let us consider three examples of epistemic paradoxes which, it seems to us, are paralogisms including one and the same false premise from a certain point of view. False, in the sense that its introduction is unacceptable from the perspective of the logic assumed by its *speaker*.

2. Epistemic paradoxes: three symptoms, one explanation

The traditional definition of knowledge is taken to be granted, but it may lead to a paradoxical situation if one takes into account the context in which this concept is introduced. Following the definition from Plato's *Theaetetus*, a given agent x knows something (expressed by a sentence p) if and only if: (1) x believes that p, (2) x has a justification for p, and (3) p is true. A reformulation of Plato's definition leads to the Principle of Factivity for knowledge: $\mathrm{K}p \rightarrow p$, which means that the knowledge of p entails the truth of p. Such a principle seems to be taken for granted, because of the very meaning of the concept of knowledge; however, its application to some contexts of discourse may be troublesome if one takes into account the way in which an agent is entitled to be said to know something.

Let us consider the case of anti-realism. This epistemological theory takes the opposite position of the Platonician definition of knowledge, insofar as it reverses the explanatory roles between the concepts of knowledge and truth: it is not truth that occurs there as a necessary condition to the acquisition of knowledge; to the contrary, it is knowledge that helps to characterize the truth of a sentence in terms of knowability.

Now logic seems to establish that such a theory, legitimate as it may be, leads to a paradox that threatens its soundness: Fitch's Paradox. Starting from the anti-realist Principle of Knowability, according to which any sentence is true only if it is possible for an agent to have a proof for its truth, the logical analysis introduces an operator of possibility in order to express this principle as follows: $p \rightarrow \Diamond \mathrm{K}p$. Such an assumption entails an unacceptable conclusion through a set of premises and inference rules that seem to hold beyond doubt.

We begin by substituting in $p \rightarrow \Diamond \mathrm{K}p$ the complex formula $(p \wedge {\sim}\mathrm{K}p)$ to the initial sentence p. It follows that the presumed truth of the premise

$p \rightarrow \Diamond p$ implies that of its direct consequence $(p \wedge \sim Kp) \rightarrow \Diamond K(p \wedge \sim Kp)$. One of the admitted inference rules in the demonstration of the paradox is the closure of knowledge upon conjunction, that is: $K(A \wedge B) \rightarrow (KA \wedge KB)$; the latter implies in turn the presumed truth of $(p \wedge \sim Kp) \rightarrow \Diamond(Kp \wedge K\sim Kp)$. And given that the Principle of Factivity seems taken for granted, its application leads to the consequence $(p \wedge \sim Kp) \rightarrow \Diamond(Kp \wedge \sim Kp)$ which is not admissible: it is not possible to know and not to know that p is true at once; but this is exactly what is meant by the above consequent. An application of the rule of *reductio ad absurdum* implies in turn that the falsehood of this consequent generates the falsehood of its antecedent, i.e. the falsehood of $(p \wedge \sim Kp)$. Now if this initial substitution instance is logically false, its negation is logically true and one deduces from it the true statement that every true sentence is known: $p \rightarrow Kp$. Does this whole demonstration really mean that the anti-realist Principle of Knowability is logically indefensible; or does the trouble actually stem from either one of the inference rules that have just been applied? To the question which of these inference rules is responsible of the undesired conclusion, a good number of different solutions has been propounded within the abundant literature devoted to the resolution of Fitch's Paradox and its resolution.[1]

The point is now to identify the origin of the problem in the derivation: the adoption of the substitution instance $(p \wedge \sim Kp)$. Instead of blaming either of the inference rules, our following assessment suspects this paralogism to introduce a sentence that is incompatible with the point of view of the anti-realist: how can such an agent claim a sentence to be true if (s)he is not entitled to know it to be so? This is what is implied by the substitution instance, however. If it is so, then the trouble does not come from the Principle of Knowability but lies in the sentences liable to be uttered by the anti-realist.

This is not the whole story, since the dissolution of Fitch's "paradox" is not so much relevant so long as it does not find any application beyond this particular problem. And it does, or it seems so: this alleged paradox appears to be symptomatic of a more general problem about the making of a truth-claim. Fitch (1963) states this point in the form of a theorem:

Theorem 1. If a is a truth class which is closed with respect to conjunction elimination, then the proposition, $[p \wedge \sim(\alpha p)]$, which asserts that p is true but not a

[1] A case in point is Wansing (2002). Most of the solutions consist in rejecting either of the inference rules in order to block the undesired conclusion, and the choice of the rejected rule may depend upon the logical system assumed by the arguer: intuitionist logic, paraconsistent logic, relevance logic, etc. We do not follow this way of clarifying the paradox, here: we suspect the problem to come from the meaning of one particular sentence that is admitted by any of these logical systems.

member of α (where p is any proposition), is itself necessarily not a member of α.[2]

The two following "paradoxes" clearly exemplify this established rule for the discourse about truth, where α applies to the concept of knowledge and is also extended to the concept of belief. Admittedly, the modal operator of belief does not belong to this truth class mentioned by Fitch, in the sense that Fitch's paradox calls for the Principle of Factivity in order to obtain the paradoxical conclusion. Nevertheless, Moore's Paradox will show that knowledge and belief are equally symptomatic of a more general problem concerning the discourse about truth, rather than truth as such. The problem is related to the *veracity* of a knowledge statement, and not to the factivity of any stated knowledge.

The first example is Moore's Paradox, which is as famous in epistemic modal logic as the preceding case. Georges Edward Moore argued that these two kinds of formulas are absurd but noncontradictory: "It is raining, but I don't believe it", and "It is not raining, but I believe it". It is true that these two formulas are not self-contradictory in epistemic modal logic, assuming that the expression of truth is symbolized by the occurrence of a sentential variable p. The first formula takes the logical form $p \wedge \sim\mathrm{B}p$, and the second has the form $\sim p \wedge \mathrm{B}p$ that is not any more objectionable from a semantic point of view. Yet they are from a pragmatic point of view, as notably claimed by Hintikka (1962): the trouble does not come from *that* which these sentences talk about but, rather, from whoever talks about these, i.e. from their utterance by a speaker. Hence the "semi-performative" character of Moore's Paradox, according to which uttering a declarative sentence about a state of affairs commits the speaker in telling the truth about it[3]. Let us call by "Sincerity Clause" this pragmatic principle inherited from Austin, to the effect that a declarative discourse proceeds as an act of assertion whose satisfaction requires the speaker to believe the truth of what (s)he says as a precondition. Therefore, a treatment of this problem means a transition from the semantic level of the relations between language and the world to the pragmatic level of the relations between language and its users, in accordance to Charles Morris' tripartition between the syntactic, semantic, and

[2]Fitch (1963), p. 138. The truth class corresponds to the set of sentences that satisfy the Principle of Factivity: $\alpha p \rightarrow p$.

[3]Hintikka (1962) thus argues: "Given the fallacious aspect of the introspective arguments, it is important to realize that none of the conditions or rules that we adopted thus far is based on these. The arguments we gave for them concerned all the circumstances in which a set of *explicitly* made statements can be reasonably *said* to be defensible. No reference has been made to what can be known by inspecting on one's mind." (p. 55, my italics). For a detailed review of Moore's Paradox and his illocutionary reading, see Schang (2007), especially section 2.2.4.2.2.

pragmatic features of language. Hintikka encoded such a transition into his formal language: in doxastic modal terms, a formalization of the problem consists in going from the self-consistent (logically defensible) level of the sentences $p \wedge \sim \mathrm{B}p$ and $\sim p \wedge \mathrm{B}p$ to the self-contradictory (logically indefensible) level of their statements "$p \wedge \sim \mathrm{B}p$" and "$\sim p \wedge \mathrm{B}p$"; a formal translation of the Sincerity Clause can be performed by inserting the initial sentences into the scope of a further doxastic operator B: $\mathrm{B}(p \wedge \sim \mathrm{B}p)$ and $\mathrm{B}(\sim p \wedge \mathrm{B}p)$. The modal logical properties of belief entail that the speaker both believes and does not believe that p is true:

$$\mathrm{B}(p \wedge \sim \mathrm{B}p) \leftrightarrow (\mathrm{B}p \wedge \mathrm{B}{\sim}\mathrm{B}p) \leftrightarrow (\mathrm{B}p \wedge \sim \mathrm{B}p),$$

and

$$\mathrm{B}(\sim p \wedge \mathrm{B}p) \leftrightarrow (\mathrm{B}{\sim}p \wedge \mathrm{BB}p) \leftrightarrow (\mathrm{B}{\sim}p \wedge \mathrm{B}p) \leftrightarrow (\sim \mathrm{B}p \wedge \mathrm{B}p).$$

Therefore, the intermediary procedure of formalization helps to show that the initial absurdity of the sentences comes now with a contradictoriness of their statement. Furthermore, the pragmatic explanation in terms of speech acts avoids any muddling detour through psychological arguments of introspection or transparency of the mental states in order to account for the occurrence of the paradox: the Moorean speaker believes that (s)he believes that it is raining (or not) just because (s)he just said it and cannot thereby retract what (s)he just affirmed.

The second example corroborates the preceding in the form of an apparent logical circularity that equally brings out the origin the problem. Zemach (1969) depicted it as a pragmatic paradox that has to do with the third criterion of the Platonician knowledge. Returning to the point of view of the speaker, the latter is entitled to say that p is true only if (s)he is in position to acknowledge that: (s)he believes that p is true; (s)he has a proof for p; p is true. Now saying that p is true requires us to have the proof of it as a precondition and, therefore, to know it. Therefore, the criterion for the statement of a truth proceeds in such a way that knowledge becomes a necessary condition to the affirmation of truth. For

> once I discovered that the [third] condition is met, i.e. that p is the case, I know that p: it is impossible for me to establish the fact that p without coming to know that p.[4]

The author notes in the same time that this pragmatic paradox does not entail any logical circularity in the definition of knowledge:

> I don't claim that the admitted definition of knowledge is logically circular. But I claim that it is pragmatically circular, i.e. necessarily it becomes self-defeatingly circular, in its application to itself. In other words, its relation to the above case of

[4] Zemach (1969), p. 284.

circularity is similar to the relation of "p, but I don't believe that p" or "I cannot make statements in English" to "p and not p".[5]

Formally speaking, this means that $\mathrm{K}p \rightarrow p$ is still valid but not its converse $p \rightarrow \mathrm{K}p$, because the latter does not represent a proper formalization of the condition under which a truth can be stated. A more appropriate modal version of this condition would be something in the form $\mathrm{K}(\mathrm{K}p \rightarrow p)$, thus implying the validity of $\mathrm{K}p \rightarrow \mathrm{KK}p$ by distributivity of the operator K and confirming the view of Hintikka (1962) according to which the Theorem of Epistemic Introspection is closely related to the same logic of statements as the three paradoxes considered in this paper.

The apparent flavor of a paradox is caused here by a twofold semantic and pragmatic understanding of the problem. On the one hand, knowledge presupposes truth: p is known provided that p is true. On the other hand, any discourse about truth presupposes that one does have knowledge: p can be said to be true only if p is known to be true. Albeit apparent only, such a paralogism supports the difficulty that is expressed by Fitch's Theorem 1 and instantiated by Moore's statements. That is: declarative sentences do not share the same logic as their statements, and Fitch's Paradox shows to what extent a careless use of formalism is more likely to muddle the situation than to throw some light upon the initial trouble.

In order to emphasize the role of utterance and its resulting statements in the interpretation of our daily reasonings, we propose in the following a formal language from an anti-realist perspective of truth: focused on the speaker's attitudes, on the one hand; based on a non-referential semantics, on the other hand.

3. Question-Answer Semantics

If the assessment of a reasoning should take into account the context in which the latter is performed, then the impersonal point of view that is expressed by the Platonician definition of knowledge should be superseded by a more relative point of view. Rebuschi & Lihoreau (2009) set forth a contextualist semantics in this sense: insisting upon the influence of context in the assignment of knowledge to an agent, both authors argue that

> the question is to know which context we deal with. Two large conceptions are opposing to each other: the one, contextualism, according to which the knowledge of an agent is dependent upon the context of assignment; the other, subjectivism,

[5] Zemach (1969), p. 283. A further connection is made in this quotation between Moore's and Zemach's Paradoxes.

which considers that the knowledge of the agent is dependent upon his (her) own context, i.e. his (her) own epistemic standards. In both cases we have to do with relativist conceptions of knowledge.[6]

For this purpose, we suggest a *subjectivist* approach of knowledge assignment: an agent can be said to know the truth of a sentence only if (sh)he has a sufficient ground to justify it and deny the converse (i.e. that (s)he does not know that p or, worse, that (s)he knows p to be false). Such an approach is likely to assimilate belief with knowledge: if the "objective" truth of p is not taken into account, then knowledge is made synonymous with certainty, or strong belief. But such an apparent disadvantage may be qualified in two respects: knowledge may be presented as a common belief shared by a set of speakers; the following semantic framework purports to have its own advantages in turn.

By a *Question-Answer Semantics* (thereafter: **QAS**), we mean a semantic framework in which the meaning of a sentence is determined by a set of questions about this sentence and its corresponding sentences. Our subjectivist approach entails that the meaning is fixed by the speaker: the sentence makes sense through a speech act and gives a sense to the whole reasoning in which it occurs. Roughly speaking, this semantic framework is composed of a statement-forming operator **Q**, a logical matrix and a valuation function **A**; let us consider a set of resulting logics, namely: the logics of acceptance and rejection ($\mathbf{AR}_V$). For every sentence p, **Q** applies to p to form a declarative statement; the aim of such a speech act is telling the truth, in accordance to what Searle & Vanderveken (1985) called by *assertive* acts. $\mathbf{Q}(p) = \langle \mathbf{q}_1(p),...,\mathbf{q}_n(p)\rangle$ includes at least $n = 2$ ordered questions about a sentence, whereby $\mathbf{q}_1(p)$: "Is p true?" and $\mathbf{q}_2(p)$: "Is p false?". The question is of a metalinguistic order: it implicitly addresses the speaker, who implicitly answers to it by performing his (her) speech act. Assuming that the sense of the sentence is fixed by the set of these questions, its reference is denoted by the set of the corresponding answers and results in a logical value in **QAS**: "yes" and "no" are the basic answers, to be symbolized by 1 and 0. But other possible sorts of answers can be devised in a more probabilistic approach, when "maybe" is inserted between "yes" and "no". As a matter of fact, the number of logical values is $V = m^n$, where n symbolizes the number of questions and m the number of the sorts of answers that can be given to each question.

This semantics is algebraic, given that is establishes an ordering relation between the logical values in order to characterize the logical operations. It is non-referential, in the sense that a logical value is not a usual *truth*-value but

[6]Rebuschi & Lihoreau (2009), p. 55.

stands for a truth-claim that accounts for the preceding distinction between truth and truth-claim. If $m = n = 2$, we thus obtain a system $\mathbf{AR}_4$ with 4 logical values expressing the specific "degree of strength" of a declarative attitude: $\mathbf{A}(p) = \langle 1, 0\rangle$ for certainty, $\mathbf{A}(p) = \langle 1, 1\rangle$ for conjecture, $\mathbf{A}(p) = \langle 0, 0\rangle$ for doubt, and $\mathbf{A}(p) = \langle 0, 1\rangle$ for negative certainty. The number n of the questions can thus be modified to give another characterization of the truth-claims. For example, let $\mathbf{Q}(p) = \langle \mathbf{q}_1(p),\mathbf{q}_2(p),\mathbf{q}_3(p)\rangle$ with $\mathbf{q}_1(p)$: "Is there every reason to hold p as true?", $\mathbf{q}_2(p)$: "Is there some (but not all) reason to hold p as true?", and $\mathbf{q}_3(p)$: "Is there no reason to hold p as true?". We thus obtain an extended logic $\mathbf{AR}_8$ with eight logical values, thus equating truth-claims with degrees of belief whose optimal elements correspond to cases of subjective "knowledge".

The notion of a degree of strength is inherited from Searle's speech act theory; but contrary to its subsequent expression in illocutionary logic, our semantic framework is not a Kripkean model with possible worlds; rather, it is a many-valued model. **QAS** also shares with dialogic the purpose to reject the traditional distinction between semantics and pragmatics by rejecting the referential or truth-conditional view of meaning; rather, it intends to determine the meaning of sentences by means of a question-answer game, whether this game takes place between two agents or I and Nature.

Returning to the three preceding paradoxes, $\mathbf{AR}_4$ accounts for the cases in which a trouble does arise or does not. Fitch's Theorem 1 expounds an objectionable logical form, because one and the same a occurs both in the statement act and its sentential content; in such a case, the semantic framework of $\mathbf{AR}_4$ shows that the premise from which Fitch's Paradox emerged is incoherent[7] because it both affirms and denies the same attitude of certainty uttered by the speaker: $\mathbf{A}(p) = \langle 1, 0\rangle$, and $\mathbf{A}(p) \neq \langle 1, 0\rangle$. However, the trouble does not arise if the degree of strength of the statement act is smaller than that of the attitude expressed in the sentential content. Thus, the following variant of Moore's Paradox is not objectionable at all: "p, but I don't know whether p", given that the speaker expresses a belief synonymous with conjecture and thereby concedes that (s)he does not have a proof for p. Hintikka mentioned such a variant in his epistemic logic: $\mathrm{B}(p \wedge \sim\mathrm{K}p)$.

[7]Incoherent, and not inconsistent. An agent cannot give two different answers to one and the same question in $\mathbf{AR}_4$, which characterizes the property of incoherence. Nevertheless, (s)he may give the same answer to two different questions: (s)he may admit the truth and the falsehood of one and the same sentence at once, so long as (s)he is not asserting anything about it and is merely conjecturing. The case of conjecture can be inconsistent in $\mathbf{AR}_4$, since the logical value $\langle 1, 1\rangle$ means that the speaker believes in a weaker sense of the word that both p and $\sim p$. About the difference between incoherence and inconsistency, see (2009b), especially section 4.

Our account of this formula in $\mathbf{AR}_4$ consists in arguing that the speaker merely believes p ($\mathbf{A}(p) = \langle 1, 1 \rangle$) without being certain of it ($\mathbf{A}(p) \neq \langle 1, 0 \rangle$), where the concepts of knowledge and belief don't occur as modal operators but correspond to logical values that characterize various degrees of strength. Consequently, the logical values refer to degrees of belief, and the degrees of belief express degrees of strength in the declarative acts (or statements).

4. Conclusion: the answer is in the question

We assumed that a logical paradox is relevant provided that it helps to emphasize a general feature in our ways of reasoning; if so, then philosophical logics are worthwhile if they catch our attention to the problem before any solution to it. Any narrow and hasty search for a solution to the paradox yields nothing but dissolution of the problem, and its philosophical import is irrelevant; by contrast, our treatment of the epistemic paradoxes attempted to emphasize the role of statements acts within a many-valued semantics.

This adoption of many-valuedness purports to satisfy further explanatory needs. On the one hand, **QAS** restores the Principle of Bivalence with some respect: the choice of two answers among "yes" and "no", within a semantics where the two basic values are not the referential properties of truth and falsehood but, rather, the pragmatic acts of affirmation and denial. On the other hand, **QAS** results from a more general reflection where the three epistemic paradoxes occur as mere particular symptoms: our anti-realist semantics of speech acts resulted in a question-answer game whose application goes beyond the restricted case of epistemic modalities. The underlying theory of meaning for this game has been presented as an extension of the Fregean distinction between sense and reference, without truth-values and with alternative logical values. The distinction between several degrees of belief in a discourse of truth has found some applications in the realm of Indian logics[8]; the composite nature of these logical values also led to an original algebraic semantics for oppositions, where opposite-forming operators are defined recursively and replace the Tarskian notions of consequence and truth by those of opposition and negation[9].

An alternative process of logical analysis has been suggested by means of many-valuedness, and we attempted to show at the same time that the real

[8]See Schang (2009a) and (2011), where two non-classical logics are reconstructed in the line of **QAS**: the Jain theory of seven-fold predication, and Nāgārjuna's Principle of Four-Cornered Negation.

[9]See Schang (2010a), where the valuations don't refer to epistemic attitudes because they rely upon some other sorts of questions $\mathbf{Q}$ than those occurring in $\mathbf{AR}_V$.

import of a logical paradox does not lie so much in the ways of solving it than in the reasons of its occurrence.

References

Fitch (1963): "A logical analysis of some value concepts", *The Journal of Symbolic Logic* **28**, 135-142

Hintikka (1962): *Knowledge and Belief. An Introduction to the Logic of the Two Notions.* Cornell Univ. Press, New York

Rebuschi M. & Lihoreau F. (2009): "Sur la formalisation du contexte en épistémologie", in Prépublications de la MSH Lorraine **5**, 48-61

Schang, F. (2007): *Philosophie des modalités épistémiques (La logique assertorique revisitée).* PhD dissertation, Université Nancy 2

Schang, F. (2009a): "A plea for epistemic truth: Jaina logic from a many-valued perspective", in *Logic in Religious Discourse* (A. Schumann (ed.), Frankfurt & Paris, Ontos Verlag, 54-83

Schang, F. (2009b): "Relative charity", *Revista Brasileira de Filosofia* **233**, 159-172

Schang, F. (2011): "Two Indian dialectical logics: *saptabhaṅgī* and *catuṣkoṭi*", *Journal of the Indian Council of Philosophical Research* **27**, 45-75.

Schang, F. (2012): "Questions and Answers about Oppositions", in *New Perspectives on the Square of Opposition*, Béziau, J.-Y. Payette, G. (eds), Peter Lang, Bern (2011), 289-319

Searle, J. (1969): *Speech Acts*, Cambridge Univ. Press

Searle, J. & Vanderveken, D. (1985): *Foundations of Illocutionary Logic*, Cambridge Univ. Press

Wansing, H. (2002): "Diamonds are a philosopher's best friend: The Knowability Paradox and Modal Epistemic Relevance Logic", *Journal of Philosophical Logic* **31**, 591-612.

Zemach, E. (1969): "The pragmatic paradox of knowledge"