Depicting negation in diagrammatic logic: legacy and prospects

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Abstract

Here are considered the conditions under which the method of diagrams is liable to include non-classical logics, among which the spatial representation of non-bivalent negation. This will be done with two intended purposes, namely: a review of the main concepts involved in the definition of logical negation; an explanation of the epistemological obstacles against the introduction of non-classical negations within diagrammatic logic.

1. From dichotomy to bivalence

In the primary diagrams suggested in the logic of classes, each predication of the form *S is P* was presented by a circle or closed curve symbolizing the class P and in which an element S occurred; in other words, *S is P* is a true proposition if and only if the individual value occurs *in* the region purported to figure the class of individuals which satisfy the property P. A translation of classes in terms of propositions or predications is thus possible, provided that the class P is considered as a predicate term attached to a subject term S. As to the set of individuals which don't satisfy the property P, they must occur *outside* the closed curve that determines the set of P's.

One first epistemological obstacle to non-classical logics is the lack of any universe of discourse in the first suggested diagrams, as shown by the distinction below.

Figure 1

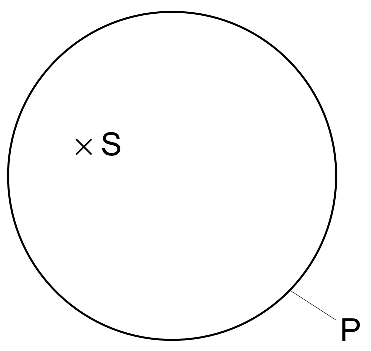
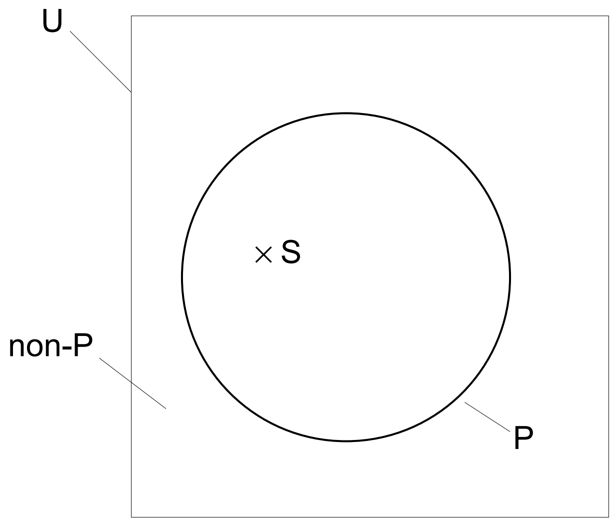


Figure 2



In Figure 1, the set of individuals that are not P (i.e. the non-P's) are not located in a specific space since only the closed curve is taken into account; whereas in Figure 2, all the non-P's correspond to the region located outside the closed curve and are within the *universe of discourse* U. We call by *dichotomy* the opposition between the whole individuals located within (the class) P and the whole remaining ones located outside (the class) P. The opposition between P and non-P thus characterizes logical negation as a dichotomy; it is presented in diagrammatic logic as a spatial contrast *in* vs. *out*.

As to the semantic notion of *bivalence*, it extends dichotomy in splitting the universe of discourse into two classes of truth-values. If S is located in the region of P's, then the proposition *S is P* is true; if S is not located in the region of P's, then *S is P* is false. Now since S cannot be both in and outside P, the truth of the proposition *S is P* entails the falsehood of its negation *S is not P*, and the falsehood of the proposition *S is P* entails the truth of its negation *S is not P*. Consequently, every proposition does have a truth-value within the universe of discourse: either it is true, or it is false; no proposition can be both in and outside the closed curve, so that if a proposition is true then it is not false and if it is false then it is not true. Another way to state these two properties of bivalence is to say that a diagram is *complete* and *consistent*.

It seems difficult to challenge bivalence in logical diagrams, assuming that it implies for any element S in U both its occurrence and non-ubiquity. Nevertheless, the logic of diagrams gave way to the introduction of non-classical constants in its modern variants, including non-classical negation. How to embed the latter without questioning both preceding requirements of completeness and consistency?

Our answer is: by internalizing truth-values within the logic of diagrams, and by making a distinction between two readings of negation. Our final claim is that non-classical negation is less revolutionary than it might appear first, insofar as it doesn't really challenge the general property of dichotomy.

2. From exclusion negation to choice negation

By an *internalization*, we mean the process that consists in introducing a metalanguage notion within the object-language. While truth-values belong to metalanguage in the logic of diagrams, given that they don't occur in the logical space U, some non-classical logics make use of the following process in order to depict non-classical constants, namely: to see the general proposition of the form *S is P* as an element in a class of truth-value (a "semantic class", say); e.g. [*S is P: p*] *is true*, with [*S is P: p*] for *S* and *true* for *P*. We thus obtain the subsequent illustrations of non-classical logic, where the Venn scheme is internalized in the semantic level of truth-values.

In Figure 3, a diagram is generally presented as a range of possible relations between two distinct classes and the combinations of which are counted as $2^2 = 4$; by analogy with the classes of propositions, the classes of truth-values equally propose four possible relations between the class T of true propositions and the class F of false propositions.

Figure 3

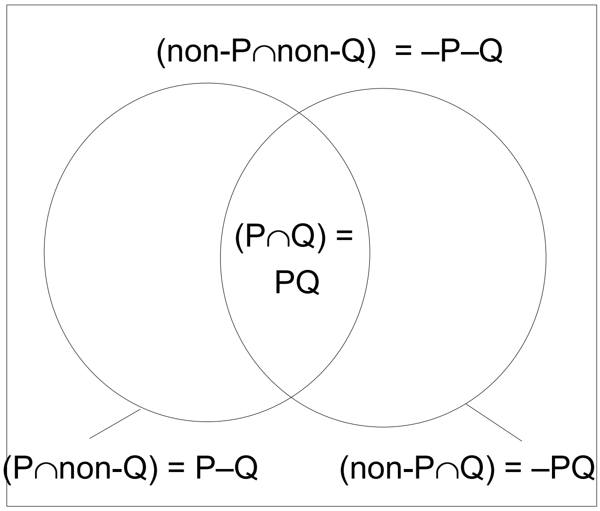
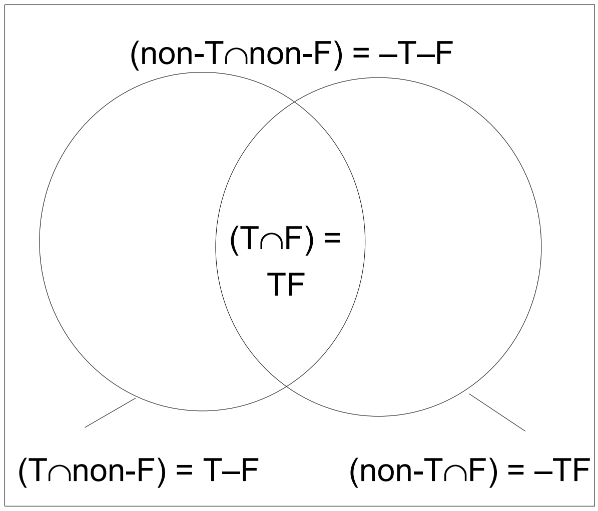


Figure 4



Any proposition can thus be considered as: true and non-false, false and non-true, true and false, non-true and non-false. Unlike diagrams in classical logic, Figure 4 displays a non-bivalent space in which a proposition can be neither true nor false or both true and false. In this sense, the space U' of non-classical logics is not complete but *paracomplete*; it is not consistent, either, but *paraconsistant*. A sample of paracomplete three-valued logics are Kleene's intuitionistic logic in [3] or Łukasiewicz's logic of indetermination in [4], in which some propositions are neither true nor false indécidable; a case of paraconsistent logic is Priest's dialetheist logic in [6], in which a proposition can be paradoxal and, therefore, both true and false.

However, paraconsistency doesn't mean that a proposition may be both *in* and *outside* one and the same class: if it is both true and false, then it is located within the class TF that intersects T and F. Paracompleteness doesn't mean any more that a proposition may be absent from the universe of discourse, given that propositions that are neither true nor false *are* located in U'. Therefore, the introduction of non-classical diagrams does not entail the revision of dichotomy as such: if a proposition is not true, for instance, then it is located in the non-true (in –T) and, thus, outside the true (i.e. outside T); although it may be both true and false, such a case does not constitute any more an infringement to dichotomy than for an element S to be both in the class P and the class Q. In other words, the change caused by non-classical logics may generate some plausible confusion between the concepts of *bivalence* and *dichotomy*: the former states that any true proposition is non-false and any false proposition is non-true, hence an obvious connection between such a formulation and dichotomy. However, dichotomy merely requires the distinction between an arbitrary class and any other one that is not itself: P and non-P, true and non-true, false and non-false, and so on.

In order to bring out the distinction between mere *difference* and *incompatibility*, it can be said that while the class T of *strictly* true propositions and the class F of *strictly* false propositions are dichotomous in classical logic and even in a number of non-classical logics, T's and F's are no more dichotomous in non-classical logics since TF is a new class in U' including both true and false propositions. Consequently, this means that a number of non-classical negations are not dichotomous whenever they don't always require true propositions to be outside the false region and false propositions to be outside the true region.

Borrowing a terminology from Terence Parsons in [4], the difference is made here between negation as a *choice operation* and negation as an *exclusion operation*. While exclusion negation generally consists in projecting any proposition from a class (whether semantic or not) into another class, choice negation turns a proposition into another proposition but doesn't necessarily projects it from a semantic class to another one. Łukasiewicz's or Priest's logics are an illustration of it, since the (choice) negation of a proposition that is neither strictly true nor strictly false (say, indeterminate: –T–F = I) results in an unchanged truth-value when the negation is said to be *normal*: v(p) = v(~p) = I, so that *p* and non-*p* do have the same truth-value. Such a non-classical negation does not exhibit any more the property of exclusion that characterizes dichotomy. Let us note that, while not every logical negation acts as an exclusion operation, some non-classical negations still do. These are said to be *non-normal*, e.g. Post'c cyclic negation: for an ordered series of {1, ...,i, ... *n*} truth-values, v(~P) = i+1 whenever v(P) = i, so the truth-values of P and not-P are never the same. How to characterize logical negation intensionally, if not every negation is to be depicted as a exclusion operation?

As Brady put it in [1], we can consider an intensional property of logical negation that is more general than dichotomy and does justice to both classical and non-classical negations, that is: negation as a *mirror-image* concept with an axis of symmetry. If one depicts a range of three truth-values with T and F as opposite sides and I in the middle, then the negation of I results in I just as the mirror-image of any point located in an axis of symmetry results in this point itself.

Thus, not every logical negation is an exclusion property when applied to propositions with a non-classical value; nevertheless, the distinction between classical and non-classical remains as a dichotomy in itself between {T,F} and not-{T,F}.

The overall situation may be summarized as follows:

- exclusion negation '–' is an *intensional* property: it rejects a proposition –(*p)* outside the semantic class of *p*, but without determining the class in which –(*p*) should be located; and conversely, choice negation '~' is *extensional* since it determines a specific semantic class for ~(*p*) without rejecting it necessarily outside the semantic class of *p*. Priest's paraconsistent negation, for example, locates the negation of a strictly true proposition in the strictly false region F, the negation of a strictly false proposition in the strictly true region T, and the negation of a paradoxical proposition (i.e. both true and false) in the paradoxical region TF.

- the assimilation of logical negation to dichotomy is due to the behavior of choice negation within classical, bivalent logic: given that only two strictly separate semantic classes occur in it (i.e. T and F), classical negation behaves exactly like an exclusion negation and the distinction between choice '–' and exclusio '~' is thus made impossible extensionaliy speaking, contrary to the case with most of non-classical logics and especially the three-valued ones.

- being *inconsistent* does not mean the same as being *contradictory* if, by contradictory, we mean the possibility to be both X and non-X whatever the syntactic category of X may be (whether a class of propositions or a class of truth-values). Paraconsistent logic does not include both true and non-true propositions or false and not-false propositions: T–T and F–F cannot occur in U'. Such a non-classical (choice) negation doesn't infringe dichotomy at all: it seems to do so only for whoever goes on to think of it in a classical universe of discourse U without including the more complex universe U'. But to accept TF in U is an impossible thing to do, so that the confusion between paraconsistency and true contradictions is nothing more than a confusion between the logical spaces U and U' as different frameworks for thinking about truth-values.

Conclusion: non-classical negations are harmless for dichotomy as such

In sum, the introduction of non-classical logics into diagrams entails neither a significant revolution within the logical space, nor the obligation to modify our representation of such a space so as to allow the ubiquity of propositions both in and outside one and the same class. The process of internalization merely helps to make propositions *relatively* true and false and to go beyond the strictly separate relation between T and F within a bivalent framework; now if such a process if refused, then it appears as literally impossible to think of negation from a non-classical perspective, given the spatial relation in-out as a undebatable statement of dichotomy. In a nutshell: either logical negations are dichotomous or not, as choice operations; but they don't infringe to dichotomy in a logical space at all, because such an infringement would assume a substantial change in the very geometrical properties of the logical space. Nothing similar occurs in paraconsistent or paracomplete logics in U', anyway.

One task remains to be accomplished in non-classical diagrammatic logic once the internalization is accepted, namely: assuming that not every non-classical logic is a exclusion operation and, thus, an operation of *complementation*, how to represent the noncomplementary laws of projection between *p* and not-*p* for paraconsistent and paracomplete negations in U', and which of these projections should replace the dichotomous relation in-out that marks classical negation? Such a task will be accomplished in a later paper about the specific case of da Costa & Béziau's overclassical logic (see [3]), a case of paraconsistent negation that rejects the laws of non-contradiction and excluded middle while maintaining De Morgan's laws and double negation.

References

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