

Two Indian dialectical logics: *saptabhaṅgī* and *catuṣkoṭī**

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Abstract. A rational interpretation is proposed for two ancient Indian logics: the Jaina *saptabhaṅgī*, and the Mādhyamika *catuṣkoṭī*. It is argued that the irrationality currently imputed to these logics relies upon some philosophical preconceptions inherited from Aristotelian metaphysics. This misunderstanding can be corrected in two steps: by recalling their assumptions about truth; by reconstructing their ensuing theory of judgment within a common conceptual framework.

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1 Two logics?

A note on Indian “logics” is in order, to begin with. By a logic, it is ordinarily meant a specific set of consequence relations between a set of premises Γ and a conclusion B such that, for every formula $A \in \Gamma$, if A is true then so is B . Formally: if $v(A) = T$ then $v(B) = T$, where v is a valuation function from a set of formulas to a set of truth-values. But such a modern definition of logic as a set of rules for truth preservation cannot be properly applied to ancient logics, including those from India. Rather, ancient and medieval logics include epistemology in the scope of the formal discipline: how to assess the content of a judgment isn’t separable in Aristotle’s *Organon* or the Port-Royal Logic, for instance, and Indian logics are not an exception.

The epistemological import of Indian logics largely accounts for their peculiar content; the metaphysical assumptions that underlie these Indian schools of philosophy also results in specific theories of truth, and the main aim of the present paper will be to give a formal presentation of the ways to produce a judgment or predication such as “ S is P ” or “ S is not P ” (where S is the subject-term and P the predicate-term). As a matter of rule, Indian logics are about judgments and not about the sentences expressing them; we will restrict our attention to two such cases: the Jaina *saptabhaṅgī*; and the *catuṣkoṭī* from the Buddhist school of Mādhyamaka (literally, “Middle Way”).

As a general rule, the logics emerging from the Jaina and Mādhyamika schools include both a theory of knowledge (about how to come to know something) and a complementary theory of judgment (about how to express this something known). Concerning the theory of knowledge, the *nayavāda* is a Jaina theory (*vāda*) of standpoints (*nayas*) that includes seven kinds of justification for the truth of a sentence.¹ Furthermore, a set of seven (*sapta*) distinct judgments (*bhaṅgī*) can be made about a given topic. There is no causal relation between the number of standpoints and judgments, however. After all, the Greek skeptic Agrippa proposed five kinds of justification while

¹The seven kinds of justification (*nayas*) include metaphysical, physical and grammatical features. These are the following: *naigama-naya* (non-distinguished standpoint); *saṃgraha-naya* (collective standpoint); *vyavahāra-naya* (particular standpoint); *ṛju-sūtra-naya* (momentary viewpoint); *śabda-naya* (synonym viewpoint); *samabhirūḍha-naya* (etymological viewpoint); and, finally, *evaṃbhūta-naya* (momentary etymological viewpoint). For instance, “the existence of an entity such as a pot, depends upon its being a particular substance (an earth-substance), upon its being located in a particular space, upon its being in a particular time, and also upon its having some particular (say, dark) feature. With respect to a water-substance, it would be non-existent, and the same with respect of another spatial location, another time (when and where it was non-existent), and another (say, red) feature. It seems to me that the indexicality of the determinants of existence is being emphasized here.” ([12], p. 132).

sticking to an Aristotelian or bivalent view of judgments: either S is P or S is not P, period. Rather, the number of the Jaina judgments is due to their endorsement of a metaphysical pluralism according to which reality is many-faceted and cannot be restricted to a unique predication. As to the Mādhyamika school and its founder Nāgārjuna (\simeq 100 C.E.), they did not present a competing theory of knowledge but advanced four (*catus*) main sorts of stances (*koṭi*) for any subject-matter.

As noted in [15], “logic is not metaphysically neutral”, and the difference between the Jaina seven and Nāgārjuna’s four judgments is due to their rival views of truth. Ganeri advances (in [6], p. 268) a relevant distinction between three semantic views of truth-assignment, namely: doctrinalism, skepticism, and pluralism. According to the doctrinalist view, “it is always possible, in principle, to discover which of two inconsistent sentences is true, and which is false.” This doctrine is related to Aristotle’s two-valued logic, where only two judgments can be made about any subject-matter (S is P, S is not P) and only one of which comes to be accepted as true while the other is to be false. Bivalence is the logical cornerstone of such a doctrine and entails that every judgment is either a truth- or a falsity-claim, i.e. a statement. Skepticism and relativism challenge this binary view in opposite directions. According to skepticism, “the existence both of a reason to assert and a reason to reject a sentence itself constitutes a reason to deny that we can justifiably either assert or deny the sentence”, so that some sentences can be taken to be neither true nor false. Conversely, the pluralistic watchword is “to find some way conditionally to assent to each of the sentences, by recognizing that the justification of a sentence is internal to a standpoint”; in this sense, one and the same sentence can be taken to be both true and false depending upon the condition under which its content is assessed.

We take these three doctrines of truth-assignment to be the crucial path for a better understanding of Indian logics. While these have been dismissed by Western thinkers, as having “irrational” or “unintelligible” outlook², we suspect this uncharitable preconception to stem from a narrow reading of bivalence that takes Frege’s modern logic as a standard for any meaningful judgment. If so, the next sections insist upon the discursive and non-standard form of judgments in Jaina and Mādhyamika logics: it is still possible to preserve bivalence within these Indian theories and, thus, to preserve their intelligibility, but only if such a bivalence is not defined in Fregean terms and reformulated as a question-answer game between speakers.

²“Manifoldness in this context is understood to include mutually contradictory properties. Hence on the face of it, it seems to be a direct challenge to the law of contradiction. However, this seeming challenge should not be construed as an invitation to jump into the ocean of irrationality and unintelligibility” ([12], pp. 129-30).

2 Two opposite logics?

An intriguing feature of Jaina and Mādhyamika logics concerns their attitude towards inference: the relativist doctrine of truth seems to entail a fully inconsistent logic, whereas the skeptic doctrine of truth would entail a fully incomplete logic. This means that, for any sentences A and B, B seems to be inferred from every premise A in Jaina logic (say, J): $A \not\vdash_J B$ (for every B); whereas no sentence B would be inferred from A in Nāgārjuna’s logic (say, N): $A \not\vdash_N B$. Parsons described in [14] these cases in terms of ultimate eclecticism and complete nihilism, respectively³.

Is Jaina logic a formal system of eclecticism, and Nāgārjuna’s logic a system for nihilism? This is not so, at least for one simple reason: nihilism assumes that the premise A is accepted as true, while the coming exposition of Nāgārjuna’s Principle of Four-Cornered Negation amounts to a denial of every sentence including A. As to the Jaina logic, the role of standpoints means that not every conclusion B can be inferred from A irrespective of the context in which A and B are assessed. This entails that not everything can be inferred from every given context, and Priest recalls this fact in [15] to make his own dialetheist reading of Jaina logic immune from triviality. We will return to this modern translation in Section 5.

Two Sanskrit notions will be introduced now, in order to throw some light upon the Jaina and Mādhyamika ways of doing logic. The first concept is *anekāntavāda*: this term means non one-sidedness and characterizes the Jaina conditional view of truth, according to which the truth of a sentence is never one-sided (*ekānta*) but always depends upon the context in which it is assessed. The second concept is *prasajya pratishedha* (see [5],[11],[13]); Mohanta mentions this concept in [13] as a non-relational negation which somehow corresponds to the contemporary denegation or illocutionary negation⁴.

³See [14], p. 141. Roughly speaking, eclecticism refers to the view that sentences of two different theories can be accepted consistently within a third embracing theory: $T_1 \models p$, $T_2 \models q$, $T_3 \models p$ and $T_3 \models q$. This is not the point of Jainism. As to nihilism, it refers to the belief that nothing is true. This is not the point of Mādhyamaka, either. The difference between such nihilists and the latter could be made clearer by the difference between atheism (negative assertion about the existence of God) and agnosticism (mere denial about the existence of God).

⁴Illocutionary negation (denial, or denegation) has been defined by John Searle in [19]. Let the speech act $F(p) =$ “I promise that I will come”, where F is the act of promise and p the sentential content “I will come”; then its locutionary negation $F(\sim p)$ is “I promise that I will not come”, while its illocutionary negation $\sim(Fp)$ is “I do not promise that I will come”. Denial has been ordinarily rendered as a reversed turnstile \nmid , in reference to Frege’s turnstile of assertion, while Keiff views it in [11] as a merely failed assertion $\not\vdash$. In both cases, denial occurs as an operator; in **QAS**, however, denial is an operand (a logical value: the no-answer $\mathbf{a}_i = 0$).

In contrast to the Jaina conditions for truth-assignment, the Mādhyamikas defended the view that being dependent upon anything else is a sufficient ground for denying a corresponding predication: S cannot be said to be P or not to be P whenever S is not self-originated and is caused by another substance than itself. This refers to the two-truths doctrine and its distinction between *absolute* truth (*paramārtha-satya*) and *conventional* truth (*saṃvṛti-satya*) in the Mādhyamika’s *sūnyavāda* (doctrine of emptiness); we will see how this doctrine leads to Ganeri’s previous distinction between the pluralist and skeptic conditions for truth-assignment. While the Jains favor a contextual theory of affirmation, Nāgārjuna endorses a peculiar use of denial which is to be rigorously distinguished from negative assertion and departs from falsity-assignment. Thus, saying that S is not P results in an ambiguous judgment between affirming that the sentence “S is P” is false and denying that “S is not-P” is true. From an Aristotelian or doctrinalist approach, affirming S not to be P and denying S to be P are synonymous with each other; from a Mādhyamika or skeptic approach, however, P may be denied to be true of S without being affirmed to be false of S. Such a confusion amounts to a harmful confusion between two sorts of Indian negations (*pratisedha*), namely: the previous *prasajya pratisedha* and *paryudāsa pratiṣedha*, which is a relational (see [13]) or locutionary negation used by the later Navya school.

To sum up, Jaina and Mādhyamika logicians do oppose each other with respect to their underlying criterion for truth-assignment. Given two opposite sentences “S is P” and “S is not P”, how to decide on the truth of either? The main difference between Jainas and Mādhyamikas lies in their answer to this question. Thus, Matilal claims (in [12], p. 129) that “the difference between Buddhism and Jainism in this respect lies in the fact that the former avoids by *rejecting* the extremes altogether, while the latter does it by *accepting* both with qualifications and also by reconciling them.” It is worthwhile to note that these opposite modes of truth-assignment also foreshadow the contemporary opposition between semantic realism and anti-realism: [22] and [23] notice that the Jains countenance a correspondence theory of truth, whereas Siderits’ comparison (in [21]) between Nāgārjuna’s denials and Dummett’s anti-realist semantics entails that Nāgārjuna’s conception of truth doesn’t transcend recognitional capacity by a given agent.

Before approaching this last problem about the relations between judgments, let us consider the way to describe their various admitted judgments within a clear and uniform formal semantics.

3 Two many-valued logics?

One of the primary aims of the paper is to insist upon the dialectical nature of Indian logics, i.e. their presentation in terms of speech-acts within an argumentative framework of questions and answers. To put it in other words, each truth- or falsity-assignment proceeds by means of an intermediary act of affirmation and denial. Importantly, we take the asymmetry between the pairs true-false and affirmation-denial to be the key for a better understanding of Indian logics. A number of logical techniques have been proposed in the literature to catch the dialectical or discursive feature of Indian logics: relational or possible-world semantics ([15]), dialogics ([8],[11]), and algebraic or many-valued semantics ([6],[15],[18],[20])⁵.

In order to give a more fine-grained description of Jaina and Mādhyamika logics, we resort here to many-valuedness. Roughly speaking, the various ways of making a judgment require the introduction of alternative logical values beyond the doctrinalist values of truth and falsity. In the case of Jaina philosophy, no judgment uniquely claims plain truth or falsity because of its underlying one-many correspondence theory of truth: a given sentence partly describes a fact following the perspective from which its content may be described.⁶ In the case of Nāgārjuna's Principle of Four-Cornered Negation, it will be shown that the assumption of bivalence cannot make sense of the four negative stances together (see section 5). At the same time, the metaphysical pluralism of the Jains does not entail that new truth-values

⁵Gokhale rejected the many-valued interpretation of Jain logic because, according to him, a difference is to be made between epistemological and logical values. Thus: "The middle value designated by the term *avaktavyam* is therefore better understood as the epistemic middle rather as the logical middle. It is closer to the middle truth-value called 'undeterminable' of Kleene's three-valued system than to the Łukasiewiczian third truth-value called 'indeterminate'. (...) As a result we can say that *avaktavya* is not the third truth-value in the logical sense of the term, because it does not arise out of the violation of the laws of logic such as non-contradiction and excluded middle" ([7], p. 75). This objection assumes that every logical value should have an ontological import, but our purely algebraic viewpoint of logic does not require this and Belnap's four-valued system is an instance where all the logical values have an epistemological import.

⁶Sylvan noted that "Jainism apparently entailed a correspondence theory of truth" (p. 62), so that the Jain values have an ontological import that differs from Belnap's four values in FDE: a sentence *is* true and false (in some respects), rather than *told* true and *told* false. The difference between Jain and Aristotelian logic relies upon their underlying ontology: the latter takes a true sentence to correspond to a fact, while the former reject such a one-one correspondence between sentences of a language and states of affairs of the world. Thus Tripathi argued in [23] that "Jainism is a realistic system. It not only holds that reality is pluralistic, but also that reality is many-faced (*anantadharmātmakam vastu*)." ([21], p. 187) The Wittgensteinian *Bildtheorie* should be strictly kept apart from the Jain view of reality, consequently.

should be devised in addition to the Aristotelian framework of bivalence. Rather, these alternative logical values are various *combinations* of truth and falsity inside the initial set of values T (for true) and F (for false).

In particular, the Jaina theory of sevenfold predication (*saptabhaṅgī*) reminds one of Belnap's system of generalized truth-values and Shramko & Wansing's extension from 2 to n truth-values (see [3],[20]). Taking $2 = \{T, F\}$ as a basic set and its two elements of truth and falsity, an extension from 2 to 4 results from its powerset $\wp(2)$, that is the set of the subsets of 2. Thus $4 = \{\{T\}, \{F\}, \{T, F\}, \emptyset\}$, and Belnap symbolized the new combinations of truth-values as $\{T, F\} = B$ (for "both true and false") and $\emptyset = N$ (for "neither true nor false") in its four-valued logic FDE (First Degree Entailment). The same process can be applied indefinitely, leading to a set of $\wp(n)$ elements for any n -valued logic (where $n \geq 1$). Another such generalized set is $\wp(3)$, with $n = 3$ basic elements T, F and $\{T, F\}$. One of these generalized sets is $8 = \{\{T\}, \{F\}, \{B\}, \{\{T\}, \{F\}\}, \{\{T\}, \{B\}\}, \{\{F\}, \{B\}\}, \{\{T\}, \{F\}, \{B\}\}, \emptyset\}$. We will see that the latter set can be made very similar to the Jaina semantics, even though the odd number of the seven Jaina judgments may surprise at a first blush. Moreover, Bahm rightly noted in [2] that Indian logics are not just formal combinations of truth-values but require a more comprehensive reading of their original texts.

For this purpose, we propose now a conceptual framework to grasp the rationale of Indian logics: a Question-Answer Semantics (**QAS**) that encompasses Belnap's generalizations and helps to account for the Mādhyamika's dialectical logic of Four-Cornered Negation.

DEFINITION 1. A question-answer semantics is a model **QAS** = $\langle \mathfrak{M}, \mathbf{A} \rangle$ upon a sentential language \mathcal{L} and its set of logical connectives \odot . It includes a logical matrix $\mathfrak{M} = \langle \mathbf{Q}, V, D \rangle$, with:

- a function $\mathbf{Q}(\alpha) = \langle \mathbf{q}_1(\alpha), \dots, \mathbf{q}_n(\alpha) \rangle$ that turns any sentence α of \mathcal{L} into a specific speech-act (the sense of which is given by appropriate questions about it);
- a set V of logical values (where $Card(V) = m^n$);
- a subset of designated values $D \subseteq V$.

It also includes a valuation function \mathbf{A} , such that the logical value $\mathbf{A}(\alpha) = \langle \mathbf{a}_1(\alpha), \dots, \mathbf{a}_n(\alpha) \rangle$ of V that characterizes a statement by giving an ordered set of m sorts of answers to each question \mathbf{q}_i in $\mathbf{Q}(\alpha) = \langle \mathbf{q}_1(\alpha), \dots, \mathbf{q}_n(\alpha) \rangle$. This semantic framework results in a variety of logics $\mathcal{L} = \langle \mathcal{L}, \models_{\mathcal{M}} \rangle$ that include an entailment relation in a model $\models_{\mathcal{M}}$ such that, for every set of premises Γ and every conclusion α in \mathcal{L} , if $\mathbf{A}(\Gamma) \subseteq D$ then $\mathbf{A}(\alpha) \subseteq D$: $\Gamma \models_{\mathcal{M}} \alpha$.

A crucial difference with the more familiar logics is the meaning of the

semantics values in **QAS**: each element $\{\mathbf{a}_1(\alpha), \dots, \mathbf{a}_n(\alpha)\}$ of $\mathbf{A}(\alpha)$ is a basic answer $\mathbf{a}_i(\alpha)$ (where $1 \geq i \geq n$) with the symbol 1 for *affirmations* (yes-answers) and the symbol 0 for *denials* (no-answers). Let us call by the general heading of “logical value” every such ordered set of answers, rather than the customary “truth-values”: these values are a combination of yes-no answers to corresponding questions, whereas not every question is to be asked about the truth-value of a sentence in **QAS**.

Once the formal structure is set out for any question-answer game, let us have a closer look at our two Indian logics at hand while attempting to reconstruct their argumentative games.

4 Jaina’s theory of seven-fold predication

It has been previously claimed that not everything can be derived from every premise from a Jaina perspective: meaningfulness presupposes that a restricted set of sentences can be accepted on the basis of certain premises in a given language, while the remaining sentences of the language should not be accepted. But the question is how the Jaina predications do make sense in a consistent set of statements. In particular, the Jaina theory of seven-fold predication (*saptabhaṅgī*) has been viewed as a challenge to Aristotle’s logic.

According to Aristotle, the Principle of Non-Contradiction (PNC) is a universal law of thought that cannot be violated without committing its opponent into plain nonsense. It is stated in [1] as follows:

“It is impossible for the same thing to belong and not to belong at the same time to the same thing and in the same respect.” (Book IV, 1005b19-20)

An instant reflection suffices to see that the Jains did not oppose to this principle as it stands: their semantic pluralism relies upon a doctrine of conditioned, relative or partial truth (*syādvāda*). The Jaina philosopher Vādiveda Sūri (1086-1169 C.E.) displayed the following set of seven predications and witnessed the crucial role of *syād* (“arguably”, or “in some respect”) in every corresponding statement, where every predication expresses a conditioned judgment about a sentence⁷:

⁷The *saptabhaṅgī* clearly departs from the Fregean logic of propositions, where a sentence expresses a thought and refers to a unique truth-value. To the contrary, the seven arguments of *nayavāda* assume that the meaning of a sentence is context-dependent and doesn’t refer to some eternal entity as the True. Thus Matilal: “Realists or believers in bivalence (as Michael Dummett has put it) would rather have the proposition free from ambiguities due to the indexical elements - an eternal sentence (of the kind W. V. Quine talked about) or a Thought or *Gedanke* (of the Fregean kind) - such that it would have a value, truth or falsity - eternally fixed (...) We may assume that a proposition has an eternally fixed truth-value, but it is not absolutely clear to us what kind of a proposition that

- (1) *syād asty eva*: arguably, it (some object) exists.
- (2) *syān nāsty eva*: arguably, it does not exist.
- (3) *syād asty eva syān nāsty eva*: arguably, it exists; arguably, it does not exist.
- (4) *syād asty eva syād avaktavyam eva*: arguably, it exists; arguably, it is non-assertible.
- (5) *syād asty eva syād avaktavyam eva*: arguably, it exists; arguably, it is non-assertible.
- (6) *syān nāsty eva syād avaktavyam eva*: arguably, it does not exist; arguably, it is non-assertible.
- (7) *syād asty eva syān nāsty eva syād avaktavyam eva*: arguably, it exists; arguably, it does not exist; arguably, it is non-assertible.

Each of these predications is a combination of three basic semantic predicates (*mūlabhaṅgas*)⁸, namely: assertion, or *truth-claim*; denial, or *falsity-claim*⁹; and a third sort of judgment that Jains called by non-assertibility (*avaktavya*). Before discussing the meaning of this third predicate #, it follows from their combinations that the three basic statements are very similar to the set $\mathbf{3} = \{T, F, \#\}$ and its eight combined subsets in $\wp(\mathbf{3}) = \mathbf{8} = \{\{T\}, \{F\}, \{\#\}, \{T, F\}, \{T, \#\}, \{F, \#\}, \{T, F, \#\}, \emptyset\}$. The logical structure of **QAS** brings out the two main features of this sevenfold predication, where each component is to be rendered in terms of corresponding questions and answers.

DEFINITION 2. A Jaina predication expresses an ordered answer $\mathbf{A}(\alpha) = \langle \mathbf{a}_1(\alpha), \mathbf{a}_2(\alpha), \mathbf{a}_3(\alpha) \rangle$ to $n = 3$ basic questions $\mathbf{Q}(\alpha) = \langle \mathbf{q}_1(\alpha), \mathbf{q}_2(\alpha), \mathbf{q}_3(\alpha) \rangle$, such that \mathbf{q}_1 : “Is α asserted?”, \mathbf{q}_2 : “Is α negated?”, and \mathbf{q}_3 : “Is α non-assertible?”. There are $m = 2$ kinds of exclusive answers $\mathbf{a}_i(\alpha) \mapsto \{0, 1\}$ to each ordered question \mathbf{q}_i , where 0 is a denial “no” and 1 is an affirmation “yes”. This yields the following list of $m^n = 2^3 = 8$ predications and their counterparts in a Belnap-typed set $\mathbf{8}$:

would be. For it remains open to us to discover some hidden, unsuspected determinants that would force us to withdraw our assent to it.” ([12], p. 136)

⁸A judgment proceeds as a statement in which a semantic value is predicated of the sentence. Gokhale claims for this higher-order level of discourse: “A *syāt*-statement, in so far as it is a statement about a sense of a sentence, is a metalinguistic statement and not an object-linguistic one.” ([7], p. 80).

⁹Jain “denial” corresponds to the relational negation of the realists (*paryudāsa pratishedha*), by contrast to the Mādhyamika non-relational negation (*prasajya pratishedha*). Accordingly, the “denial” of the second *mūlabhaṅgi* (2) amounts to an act of negative assertion or falsity-claim and stands for a commitment of the speaker about how the world is not, whereas every disciple of Mādhyamaka typically endorses an attitude of non-commitment.

$$\begin{array}{ll}
(1) = \langle 1, 0, 0 \rangle \text{ for } \{\text{T}\} & (2) = \langle 0, 1, 0 \rangle \text{ for } \{\text{F}\} \\
(3) = \langle 1, 1, 0 \rangle \text{ for } \{\{\text{T}\}, \{\text{F}\}\} & (4) = \langle 0, 0, 1 \rangle \text{ for } \{\#\} \\
(5) = \langle 1, 0, 1 \rangle \text{ for } \{\{\text{T}\}, \{\#\}\} & (6) = \langle 0, 1, 1 \rangle \text{ for } \{\{\text{F}\}, \{\#\}\} \\
(7) = \langle 1, 1, 1 \rangle \text{ for } \{\{\text{T}\}, \{\text{F}\}, \{\#\}\} & (8) = \langle 0, 0, 0 \rangle \text{ for } \emptyset
\end{array}$$

Each of the seven Jaina statements is an expression of single yes-answers ($\mathbf{a}_i = 1$) among three possible ones, while the remaining no-answers ($\mathbf{a}_i = 0$) are left silent by the affirmative nature of Jaina philosophy. The first two statements (1) and (2) mean that *every* standpoint is such that it makes a given sentence true or false, respectively. (3) means that there are standpoints for asserting the truth and the falsity of the sentence, while noting that a standpoint does not make this sentence both true and false at once. The *internal* consistency of the standpoints is stated in terms of *successive* assertion and denial. (4) is the troublesome statement that the sentence is non-assertible: although this semantic predicate seems to entail merely that a given sentence cannot be asserted (made true), this should leave place for strong denial (falsity-claim); but such a translation would collapse (4) into (2), all the more that this third *mūlabhaṅgi* is translated as a case of *simultaneous* assertion and denial. How can one and the same sentence be non-assertible and asserted at once? We return to this point in the next paragraph. The three remaining predications are combinations of the four preceding ones: (5) and (6) mean that there are standpoints that make the sentence true and non-assertible, or false and non-assertible. (7) is a combination of the three basic predications such that the available standpoints make the sentence true, false, and non-assertible.

The ultimate subset (8) doesn't appear in the list of the Jaina predications, however; hence the odd number of $8-1 = 7$ elements. A combinatorial account for this odd number of predications can be given as follows: there is an infinite number of particular arguments for any predication, and all of these are classified among a set of seven general standpoints in the *nayavāda*. Now since any two different kinds of standpoints may result in one and the same statement of the *syādvāda*, it follows from it that every sentence is made (or claimed to be) either true, false or non-assertible by a variable set of related standpoints. Therefore, there is always at least one standpoint $\mathbf{a}_i(\alpha) = 1$ for any sentence α . This entails that no sentence α can be an exception to these three basic judgments $\langle \mathbf{a}_1(\alpha), \mathbf{a}_2(\alpha), \mathbf{a}_3(\alpha) \rangle$, and the answer $\mathbf{A}(\alpha) = \langle 0, 0, 0 \rangle$ is made an impossible case.

As rightly noted by Priest¹⁰, no contemporary counterpart has been de-

¹⁰“What are the semantic values of such compound sentences? Such a question is not one that Jaina logicians thought to ask themselves, as far as I know. So we are on our own here.” ([15], p. 268).

vised for the so-called “Jaina logic”: the Jains have not defined any closed formal language with a set of constants (connectives) and a closed set of consequences. However, we can develop a plausible Jain logic within **QAS**.

DEFINITION 3. Jain logic is a model $\mathbf{J}_7 = \langle \mathfrak{M}, A \rangle$ upon a sentential language \mathcal{L} and its set of logical connectives $\odot = \{\sim, \wedge, \vee, \rightarrow\}$. It includes a logical matrix $\mathfrak{M} = \langle \mathbf{Q}; \mathbf{7}; D \rangle$, with:

- a function $\mathbf{Q}(\alpha) = \langle \mathbf{q}_1(\alpha), \mathbf{q}_2(\alpha), \mathbf{q}_3(\alpha) \rangle$;
- a set $\mathbf{7}$ of logical values;
- a subset of designated values $D \subseteq \mathbf{7}$.

The cardinality of D and the different matrices for \odot cannot be uniquely determined without solving an intermediary problem: the meaning of the “non-assertible” *avaktavya* in \mathbf{q}_3 , by contrast to the two “assertible” *vaktavya* (*asti*, *nasti*) that constitute expressible predications in \mathbf{q}_1 and \mathbf{q}_2 . Each ordered answer is a logical value from our many-valued perspective, and the meaning of the semantic predicate “non-assertible” is crucial to determine whether a positive answer to $\mathbf{q}_3(\alpha)$ results in a designated or non-designated value¹¹. For if $\mathbf{A}(\alpha) = (4) = \langle 0, 0, 1 \rangle$, then $\mathbf{a}_1(\alpha) = \mathbf{a}_2(\alpha) = 0$ and $\mathbf{a}_3(\alpha) = 1$. Assuming with Priest that a semantic value is designated if it expresses truth, then a non-assertible sentence should be asserted to be at least true in order to be designated. Is it so?

There are three main interpretations of *avaktavya*: (4.1) neither true nor false, (4.2) both true and false, (4.3) none (taking to be granted that not two of these can be accepted without extending the set of semantic predicates from 8–1 to 16–1 = 15 elements)¹². Given the crucial role of the number 7, only one of these three possibilities is to be accepted as *the third mūlabhanḡi*.

¹¹An alternative way consists in characterizing logical consequence in terms of an ordering relation between the elements of V , such that $p \models_{\mathbf{J}_7} q$ if and only if $\mathbf{A}(p) \leq \mathbf{A}(q)$. See [3],[22] about this process. An algebraic presentation for Jain logic is also given in [20],[22] and results in a bi-and- a-half-lattice (a product of two Belnap’s bi-lattices) with no lower bound \emptyset ($\langle 0, 0, 0 \rangle$, in \mathbf{J}_7). But given that nothing seems to justify a specific hierarchy between the seven logical values, we stick to the view of logical consequence as preserving the designated value.

¹²Priest mentions the possibility of four-valued facets or *mūlabhanḡi* and a subsequent 15-valued logic in [15], in such a way that a sentence could be said to be either asserted or denied, or both, or neither. Some other extensions of the basic predications have been entertained in [2] for Jain logic, assuming it to be a positive counterpart of the *catuṣkoṭi*; these yield an extension from 4- to 8- and 12-valued logics, where a given standpoint is “more asserted (or not)” than another. But such a probabilistic extension misleadingly takes the doctrine of relative truth for a logic of partial truth-values. Gokhale argues against this reading, because “*nayavāda*, as has generally been held, gives us a class of ‘partial truths’, whereas *syādvāda* gives us a class of whole truths (or the whole truth).” ([7], p. 74). In other words, each sentence is plainly true (or not) from each given standpoint.

The first interpretation is defended by [6], [7], and [9]; the second is urged by [4], [12], and [15]. [15] and [18] admit both interpretations, while the third interpretation is supported by [2] and [23].

Those who advocate (4.1) usually claim that the Jains always sustained internal consistency or non contradiction as an unquestionable meta-principle (*paribhāsā*); this amounts to reject any case of simultaneous assertion and denial from the same standpoint. Ganeri advanced in [6] a *reductio* argument against the inconsistent interpretation, to the effect that admitting a simultaneous assertion and denial would reduce the logical values (5) and (6) to (4). This collapsing argument is rejected in [18]], insofar as it omits to take the difference between the standpoints \mathbf{a}_1 and \mathbf{a}_2 into account¹³. As a further argument for (4.3), Tripathi claimed that the incomplete interpretation (4.1) cannot square with the *affirmative basis* of the Jaina predications¹⁴. The latter means that any sentence can be made true from at least one standpoint, so that no sentence can be said to be neither true nor false. Assuming that “affirmative basis” essentially refers to an act of assertion (the second predication is a negative assertion), this implies that every Jaina predication asserts something about a sentence and cannot amount to a pure denial without assertive counterpart¹⁵.

Conversely, Priest quotes some sources in support of (4.2) and takes them

¹³Ganeri’s argument (see [6], p. 272) proceeds as follows: if *avaktavyam* means (4.2): $\{T, F\}$, then the fifth and sixth predicates yield (5.2): $\{T, \{T, F\}\}$ and (6.2): $\{F, \{T, F\}\}$, respectively; now (5.2) and (6.2) are “logically equivalent” with $\{T, F\}$, given the trivially twofold occurrence of T and F. Hence the adoption of (4.2) entails that (5) and (6) conflate into (4), and the *sevenfold* predication is done. Ganeri’s mistake is due to his set-theoretical equation between sets and subsets of elements in V : “this argument seems to rely upon a conflation of two distinct standpoints: to state that p is asserted from one standpoint and both asserted and denied from another standpoint doesn’t entail that p is merely asserted and denied, unless the crucial *syād* is suddenly removed from the meaning of a statement. But it could not be so, and Ganeri unduly commits the following simplification: $p \wedge (p \wedge \sim p) = (p \wedge \sim p)$.” ([18], pp. 63-4)

¹⁴“To say that a thing neither exists (*asti*) nor does not exist (*nāsti*) is sheer skepticism, and the Jaina would never accept it as a *bhāṅga* (predicate), and as one of the *mūlabhāṅgas* (primary predicates) at that. (...) What is worse, the interpretation of the *avaktavya* as “neither” would make it indistinguishable from the fourth *koṭi* (alternative viewpoint) of the Mādhyamika *catuṣkoṭi*, as also from the *anirvanacanīya* (indescribable as either being or not-being) of the Vedānta.” ([21], pp. 187-8). The argument is unconvincing, however, given that the Mādhyamikas deny the “neither ... nor”- position and don’t affirm it (see Section 5); no confusion should arise from (4.1), accordingly.

¹⁵It could be objected to the view of a pure denial that any first-order denial implicitly contains a second-order assertion. Such an objection suggests that (4.3) includes a second-order affirmative basis (something like “arguably, I assert that I don’t assert anything about p ”); see Section 6 about this.

to mean a plausible admission of internal inconsistency¹⁶. The present paper does not purport to have the final word, but to note two main properties of \mathbf{J}_7 that are established in [18]¹⁷. On the one hand, the essential occurrence of standpoints gives rise to a *quasi-value-functional* set of logical matrices for \mathbf{J}_7 where the logical value of a complex sentence is partly determined by the value of its components¹⁸. On the other hand, the incomplete or inconsistent interpretation of *avaktavya* makes \mathbf{J}_7 quasi-equivalent to two famous many-valued systems: Kleene’s 3-valued logic \mathbf{K}_3 or Priest’s 3-valued Logic of Paradox \mathbf{LP} , respectively. This can be stated by the two following theorems:

THEOREM 1. \mathbf{J}_7 is a *paranormal* logic that is either paraconsistent or paracomplete. That is: for some sentences α, β of \mathcal{L} , either $\alpha, \sim\alpha \not\models \beta$ or $\not\models \alpha$ does not entail $\models \sim\alpha$. \mathbf{J}_7 is paracomplete and quasi-equivalent with \mathbf{K}_3 if and only if (4) is interpreted incompletely, and \mathbf{J}_7 is paraconsistent is quasi-equivalent with Priest’s 3-valued logic \mathbf{LP} if and only if (4) is interpreted inconsistently.

THEOREM 2. The matrices for the connectives \odot of \mathbf{J}_7 are invariant, irrespective of the interpretation of (4). For every connective $\bullet \in \odot$, $\mathbf{A}(\alpha \bullet \beta)_{icm} = \mathbf{A}(\alpha \bullet \beta)_{ics}$ for every value of α and β including the incomplete (icm) or inconsistent (ics) reading of $\#$.

Apart from these technical results, it remains that no definite interpre-

¹⁶Priest adduces his usual argument for dialetheism, according to which some (but not every) contradictions are true: “What should seem to be meant by two things being contradictory here is that they cannot obtain together. If [(4)] is *both true and false*, then $[p]$ and $[\sim p]$ are precisely not contradictories in *this* sense.” ([14], pp. 271-2). Does this mean that a difference should be made between possibly true and impossibly true contradictions? A plea for possibly true contradictions has been made in [16], arguing that (4.1) could mean that some standpoint affords an evidence both for and against the truth of p . But the latter explanation does not seem to match with the definite value of a sentence in each standpoint, according to Gokhale (see note 12 above). This is why the third interpretation (4.3) will be favored in the following.

¹⁷A quantified epistemic interpretation of the standpoints has been suggested in [17]: each standpoint stands for a single belief within a community of agents, so that each Jain statement about α is translated as $\exists x B_x(\alpha)$ and reminds us of Jaśkowski’s discussive logic \mathbf{D}_2 . Such a translation helps to explain the *paraconsistent* behavior of the Jains: a set of inconsistent standpoints does not entail the truth of everything. Nevertheless, it doesn’t account for Jain *realism* (see note 6 above): a standpoint is not the mere epistemic expression of a belief or opinion, but the genuinely ontological expression of a facet of reality.

¹⁸Quasi-truth-functionality is due to the relative truth of standpoints. Two any sentences α and ψ can be true from two different standpoints; but there may be no standpoint from which α and ψ should obtain at once, according to the existential translation of a standpoint in [18]: $v(\exists x B_x(\alpha)) = \mathbf{T}$ and $v(\exists x B_x(\psi)) = \mathbf{T}$ don’t entail $v(\exists x B_x(\alpha \wedge \psi)) = \mathbf{T}$, but $v(\exists x B_x(\alpha \wedge \psi)) = \mathbf{T}$ or \mathbf{F} . On the origins of quasi-truth-functionality, see [17].

tation of *avaktavya* occurs in the literature and thus leaves the Jaina set of logical consequences indeterminate. The next point is to see whether a meaningful interpretation can be given to the third interpretation (4.3): what can be meant by *avaktavya*, if it is neither “both asserted and denied” nor “neither asserted nor denied”? For even though such an alternative reading prevents Jaina logic from reducing to what Matilal called a mere “facile relativism”¹⁹, a formal approach hardly makes obvious any statement beyond being either true, or false, or both true and false, or neither true nor false.

For one thing, Bahm takes it (in [2]) to mean something like an *incomplete* thought: a sentence is non-assertible whenever no property P can be completely predicated of S. But this is the essential feature of *anekāntavāda*, the partial truth for every standpoint of the Jaina *nayavāda*: the cornerstone of their pluralist metaphysics is that reality is an indefinite collection of incomplete perspectives. Assertion and denial are not categorical or one-sided speech-acts, therefore, and the essential incompleteness of any *syād* is likely to undermine Bahm’s explanation.

A more insightful reading seems to emerge in [23], where non-assertibility is synonymous with *non-distinction*: a sentence is non-assertible whenever its object S cannot be said to be properly P or not P. The difference is thus made with the interpretation (4.2), in the sense that S is said to be both P and not-P by including both opposite properties from one contradictory standpoint. But again, Tripathi claims in [23] that the Jains fully subscribed to the law of non-contradiction and would have refused any *self*-contradictory statement²⁰. A plausible account of being indistinguishable refers to the Hegelian view of an internal or inclusive contradiction without exclusive opposition between its terms. In support of this awkward view of contradiction, it is worthwhile to note that most of the Jaina or Mādhyamika sentences are about such metaphysical subjects as ātman, Brahman and their being existent. One may be hesitant about the logical form of an expression like “ātman

¹⁹“It also amounts to a view which announces that all predicates are *relative* to a point of view; no predicates can be *absolutely* true of a thing of a thing or an object in the sense that it can be applied unconditionally at all times under any circumstances. Jainas in this way becomes identified with a sort of facile relativism.” ([12], p. 133). Again, the crucial role of standpoints clearly points out that the Jain logic is not a real challenge to PNC.

²⁰“No system of philosophy can afford to accept self-contradiction as valid, because if self-contradiction is accepted as valid without any qualifications, then there remains no weapon for criticism, anything which is said will have to be accepted, because even self-contradictories is valid. It is certain that the Jaina does not take leave of logic and consistency; he does criticize others by pointing out self-contradiction. Every system of philosophy has its contradictory which is regarded as false. This is why when a system has to accept a synthesis of contradictories as valid, it has to invent one device or another which at least seems to take off the edge from the contradictories.” ([21], p. 188).

is self-existent”, where existence occurs as a predicate; but a more charitable reading would be to the effect that the subject-term S is elliptically said to exist or to be as falling under a certain property P. Consequently, *avaktavya* might mean that S is not any more P than non-P. But which sort of S could be so indistinguishable as not only to cover both P and all its complementary properties, but also to cancel any distinction between these properties? Tripathi mentions as a “non-expressible” sentence that which can be thought but cannot be expressed (for want of a distinguishable set of properties)²¹. Such a subject should be kept silent, according to the Wittgensteinian stance that the limits of language are the limits of thought. (But our former reference to Hegel should give rise to a non-Wittgensteinian relationship between language and the world.) While noting that Hegel’s philosophy supported a transcendental idealism and clearly differs from the Jaina realism, a common point between Jainism and the Buddhist trend of Mādhyamikas seems to be their common rejection of *logical atomism*: reality is not a whole whose parts would be objects and their properties, or at least not for some extra-natural entities that transcend the empirical level of illusory data (*prātibhāsika*). This plausible account of (4.3) will be pursued in the next section, because it might make sense of Nāgārjuna’s radical skepticism.

To conclude our discussion of Jaina logic, Priest uses in [15] an analogy with the cube to make sense of complete truth: every facet of reality is a side of a cube, and reality is the collection of every such facet. But Jaina cubism is such that the indefinite number of facets turns the cube into a polygon even more complex than Descartes’ chiliagon. Just as Picasso wanted to catch a conceptual reality by pooling different perspectives of a character together in one and the same profile, the Jaina philosophy relies upon a plurality of standpoints to grasp the essence of reality. A logical translation of this view is given in [4]: plain truth amounts to a complete knowledge (*pramāṇa*) whose expression in a complete judgment consists in the addition of the seven sorts of predication. Is this a right way to describe the transition from partial to complete truth²²?

²¹Bahm’s account must be distinguished from Meinong’s famous example of a “round square”, which has frequently been mentioned as a case of impossible object and a challenge to PNC. A round square is an object that can be expressed (described) but cannot be thought (imagined, or conceived mentally). To the contrary, the third interpretation of *avaktavya* refers to something that can be thought but cannot be expressed. Is there such a subject S that can fulfill this requirement? A Wittgensteinian reader would answer negatively to this question, assuming that “whereof one cannot speak, thereof one must be silent”.

²²The following definition of plain truth is given in [4]: “An object X can be viewed from any one of the seven standpoints. However, since the totality of all these seven possibilities comprises the *pramāṇa-saptabhaṅgī* (complete judgment of the phenomenal world in terms

An alternative account would be to state that a subject is completely described when absolutely every particular standpoint is listed, rather than just the seven kinds of argument from the *nayavāda*. Such an exhaustive completion is impossible, given the infinite sort of standpoints that constitute the proper description of any object.

A natural translation of (4) within \mathbf{J}_7 might be taken to be the twofold answer “yes and no” to the third basic question: $\mathbf{a}_3(\alpha) = \{1, 0\}$. But it is not so, given that this third question is positively answered if the corresponding sentence is inexpressible. No yes-no answer occurs in the Jaina question-answer game, consequently: two different questions can result in the same answer or not, but no single question can be answered oppositely by “yes” and “no” at once²³. This is the gist of self-contradiction, and even the third basic predicate of inexpressibility does not state it because non-distinction does not mean an internal coexistence of opposite properties. These cannot coexist, by definition.

Whatever the final word may be about (4), we argue two things about complete truth: it does not mean for a given sentence either to be assigned a designated value (this is partial truth) or to be uniquely asserted and, therefore, be given the logical value (1) in \mathbf{J}_7 ²⁴; partial truth is a sufficient

of seven possibilities), the disjunction, denoted by \wedge , of these seven predications should lead to a tautology.” ([4], p. 186). In algebraic terms, the Jains would thus assimilate one-sided truth with logical tautology and define the latter as the union of the seven elements of V . That is: $\top = ((1) \cup (2) \cup (3) \cup (4) \cup (5) \cup (6) \cup (7))$. This definition of tautology clearly differs from that of Priest’s in [15] or \mathbf{J}_7 in [18]: a sentence is a tautology if it is designated from every standpoint. But this is a definition of tautology in the *conventional* sense of truth, by contrast to the aforementioned absolute sense of truth that uniquely leads to a *pramāṇa*. One could wonder another thing, with respect to this definition of one-sided tautology: does it correspond to the union of the seven kinds of standpoints or, rather, should it collect the indefinitely many particular standpoints that are included in each of these seven kinds?

²³Three levels of inconsistency can be graded within the framework of **QAS**: *light* inconsistency, or inconsistency from two different standpoints: $\{\{T\}, \{F\}\}$, i.e. $\mathbf{a}_i(\alpha) = \mathbf{a}_j(\sim\alpha) = 1$ (where $i \neq j$); *mild* inconsistency, or inconsistency from one and the same standpoint: $\{\{T, F\}\}$, i.e. $\mathbf{a}_i(\alpha) = \mathbf{a}_i(\sim\alpha) = 1$; and *strong* inconsistency, or inconsistency in one and the same answer: $\{\{T, \sim T\}\}$, i.e. $\mathbf{a}_i(\alpha) = \mathbf{a}_i(\alpha) = \{1, 0\}$. The Jain *anekāntavāda* embodies a logic of light inconsistency; Priest’s Logic of Paradox **LP** argues for a mild inconsistency that corresponds to the inconsistent interpretation (4.1) of *avaktavyam*; but no counterpart seems to occur for the strong inconsistency of self-contradiction, going beyond the so-called “impossible” values of [20]. Indeed, strong inconsistency consists of non-empty subsets including an element and its complement. Such a case is impossible even in a combinatorial approach of semantic values, insofar as Priest’s value $\{T, F\}$ assumes that T and F are not complementary to each other.

²⁴Returning to the comparison with Jaśkowski’s Discussive logic \mathbf{D}_2 , the Polish logician rendered each standpoint by the modality of possibility, \diamond . Accordingly, any sentence α

condition of truth-assignment for the Jains, while the skeptic Mādhyamikas take complete truth to be a necessary condition for truth-assignment. Let us now consider this skeptic logic within a question-answer game of **QAS**.

5 Nāgārjuna’s Principle of Four-Fold Negation

Nāgārjuna’s radical skepticism is summarized in his *Mūlamadhyamaka-kārikā*, where the first verse includes four sentences (or lemmas) that are equally denied by means of stances (*drṣṭis*, or *koṭi*) and result in the the so-called Principle of Four-Cornered Negation (thereafter: 4CN) or Tetralemma (*catuṣkoṭi*). Thus:

- (a) Does a thing or being come out itself? No.
- (b) Does a thing or being come out the other? No.
- (c) Does it come out of both itself and the other? No.
- (d) Does it come out of neither? No.

How can Nāgārjuna consistently deny all the four questions at once? While noting that their content refers to the Mādhyamika’s doctrine of emptiness (*sūnyavāda*), a problem arises about the meaning of negation in the four aforementioned answers. A tentative formalization of (a)-(d) yields the following, where a is a predication of the form “S is P” (with S for “thing” and P for “coming out itself”) and \sim is classical negation:

- (a’) Not (S is P) = $\sim(\alpha)$
- (b’) Not (S is not P) = $\sim(\sim\alpha)$
- (c’) Not (S is P and S is not P) = $\sim(\alpha \wedge \sim\alpha)$
- (d’) Not (neither S is P nor S is not P) = $\sim(\sim(\alpha \vee \sim\alpha))$

Assuming that negation is the relational *paryudāsa pratiṣedha*, the set of four negative statements is clearly inconsistent: (b’) is equivalent with the affirmation α (by double negation), and this is patently contradictory with its negation in (a’). Even more than for the Jains, it is commonly acknowledged that the Mādhyamikas unexceptionably subscribed to PNC and cannot then accept both (a’) and (b’). Furthermore, (d’) occurs as a denial of the denial of the Principle of Excluded Middle (PEM), according to which every sentence or its negation is true. But it clearly appears that the double denial arising

that is uniquely asserted (such that $v(\alpha) = (1)$) is logically necessary because it is cannot be but asserted, and it is not possible for it to be denied or taken to be non-assertible. Thus $v(\alpha) = (1)$ means the same as α . This modal interpretation squares with the idea of one-sidedness; however, the Jain view of *pramāṇa* still goes beyond such a logical necessity (see note 22 above).

in (d') does not amount to an affirmation of PEM, since (a') and (b') already reject the affirmation of both α and $\sim\alpha$.

A way to avoid the contradiction (a')-(b') has been urged by Horn (in [10]), who claimed that the negation of every sentential content should be rendered as a *predicate-term* negation rather than a *predicate* negation²⁵. The distinction between predicate-term and predicate negation cannot be expressed in a modern or Fregean logic, where predicate-terms and predicates are collapsed into a unique function. By using term logic, (b') should be read as "S is not-P", the contrary opposite of (a'). The conjunction (a')-(b') results in a stronger relation of incompatibles, and Horn is right to say that two contraries can be consistently negated without entailing any self-contradiction. In this respect, an application of *intuitionistic* negation ($\neg\alpha$ for "S is not-P") should fill the bill and be preferred to the classical negation ($\sim\alpha$ for "S is not P"): $\sim(\sim\alpha)$ becomes $\sim(\neg\alpha)$, and the latter cannot be reduced to α by the law of double negation.

Does this mean that intuitionistic logic should be seen as a proper logic for 4CN? It is not, given that the last statement (d') leads to another contradiction. For since one of de Morgan's laws states that $(\sim\alpha \wedge \sim(\neg\alpha))$ is equivalent to $\sim(\alpha \vee \neg\alpha)$, how to claim with (a')-(b') that S is neither P nor not-P: $\sim(\alpha \vee \neg\alpha)$ while denying it at the same time with (d'): $\sim(\sim(\alpha \vee \neg\alpha)) \leftrightarrow (\alpha \vee \neg\alpha)$?

The whole result turns 4CN into a case for radical skepticism: not only does the speaker Nāgārjuna ignore whether S is P or not, but he goes on denying that he does ignore it. This troublesome stance has been noted by Raju²⁶ and accounts for the difference between Buddhism and nihilism, as currently urged by a number of commentators: nihilism is the affirmation that nothing is real or can be known to be so; whereas Buddhism argues for

²⁵Horn claims that "crucially, no distinction between contradictory and contrary negation was regularly made within classical Indian logic." ([10], p. 80) However, the contrary or contradictory feature of a negation crucially depends upon the nature of the subject in a predication: are the subjects of a Jain predication sometimes universal, sometimes particular? No definite answer seems to be available to disentangle the meaning of 4CN; it is only the later school of Navya-Nyāya that will deal with such equivocation cases. See in this respect J. Ganeri: "Towards a formal regimentation of the Navya-Nyāya technical language" (parts I,II), in *Logic, Navya-Nyāya and Applications* (Homage to Bimal Krishna Matilal), M.K. Chakraborti and Löwe, B. and Mitra M.N. and Sarukkai S (eds.), College Publications, London, 2008, pp. 105-121.

²⁶The alleged founder of 4CN, Sañjaya (\simeq 6th century B.C.), would have influenced the Greek philosopher Pyrrho in his radical skepticism; Raju states this point by claiming that Pyrrho "maintained that 'I am not only not certain of the knowledge of any object, but also not certain that I am not certain of such a knowledge' " ([16], p. 695). It is worthwhile to note that the Greek principle of indifference *ou mallon* (not any more than) strikingly parallels 4CN.

a mere denial without any positive counterpart. The positive basis of each Jaina statement included a case of negative assertion, as witnessed by the predication (2); but no such assertion arises in 4CN, where negation is pure denial. Before answering to whether there can be a negation without any positive counterpart, we suspect the core difficulty with 4CN to lie in the meaning of its wide scope negation (the answer “No”): it is used to produce a denial, and this no-answer should find a proper treatment within the formal framework of **QAS**.

Unlike the Jaina statements, and following the connection established between Mādhyamika skepticism and anti-realism, we assume that each *koṭi* deals with the impossibility of knowledge: the human failure to catch any absolute truth (*paramārthasatya*) about reality is a sufficient reason to deny any justifiable belief and thus any truth-assignment, according to Nāgārjuna’s *sūnyavāda*. If so, we introduce a four-valued logic of acceptance and rejection for 4CN.

DEFINITION 4. A logic of acceptance and rejection is a model $\mathbf{AR}_4 = \langle \mathfrak{M}, \mathbf{A} \rangle$ upon a sentential language \mathcal{L} and its set of logical connectives $\odot = \{\sim, \wedge, \vee, \rightarrow\}$. It includes a logical matrix $\mathfrak{M} = \langle \mathbf{Q}; 4; D \rangle$, with :

- a function $\mathbf{Q}(\alpha) = \langle \mathbf{q}_1(\alpha), \mathbf{q}_2(\alpha) \rangle$;
- a set $\mathbf{4}$ of logical values;
- a subset of designated values $D \subseteq \mathbf{4}$, where $D = \{ \langle 1, 0 \rangle, \langle 1, 1 \rangle \}$.

$\mathbf{Q}(\alpha)$ is an ordered set of $n = 2$ questions about the sentence α , with \mathbf{q}_1 : “is a justifiably be true?” and \mathbf{q}_2 : “is a justifiably false?”²⁷, and $n = 2$ sorts of answers such that $\mathbf{a}(\alpha) \mapsto \{0, 1\}$. It results in a set V of $m^n = 2^2 = 4$ logical values, each standing for an explicit belief-attitude in $\mathbf{4} = \{ \langle 1, 0 \rangle, \langle 1, 1 \rangle, \langle 0, 0 \rangle, \langle 0, 1 \rangle \}$. The difference with \mathbf{J}_7 is that no third question \mathbf{q}_3 occurs here: *avaktavya* is not a Mādhyamika concept, so that only two basic semantic predicates or *muladr̥ṣṭis* are required in 4CN. At the same time, \mathbf{AR}_4 is a general logic of statements that could include the Jaina stances as well: the Jaina value $\langle \mathbf{a}_1(\alpha), \mathbf{a}_2(\alpha), \mathbf{a}_3(\alpha) \rangle$ can be equated with the value $\mathbf{Q}(\alpha) = \langle \mathbf{q}_1(\alpha), \mathbf{q}_2(\alpha) \rangle$ of \mathbf{AR}_4 by canceling the third *bhaṅga* $\mathbf{a}_3(\alpha)$. Then $\langle 1, 0 \rangle = \{ \langle 1, 0, 1 \rangle, \langle 1, 0, 0 \rangle \}$, $\langle 1, 1 \rangle = \{ \langle 1, 1, 1 \rangle, \langle 1, 1, 0 \rangle \}$, and $\langle 0, 1 \rangle = \{ \langle 0, 1, 1 \rangle, \langle 0, 1, 0 \rangle \}$. A relevant exception concerns the third value $\langle 0, 0 \rangle = \{ \langle 0, 0, 1 \rangle, \langle 0, 0, 0 \rangle \}$, which includes the eighth forbidden value $\langle 0, 0, 0 \rangle$ in \mathbf{J}_7 . This forbidden value is our key to a better understanding of Nāgārjuna’s four stances, with the following definition of negation and its distinction with the speech-act of denial.

²⁷The second question “Is α justifiably false?” is equivalent with “Is $\sim\alpha$ justifiably true?”. This results in the following equation for negation in \mathbf{AR}_4 : $\mathbf{a}_1(\sim\alpha) = \mathbf{a}_2(\alpha)$, and conversely.

DEFINITION 5. For every sentence α such that $\mathbf{A}(\alpha) = \langle \mathbf{a}_1(\alpha), \mathbf{a}_2(\alpha) \rangle$:
 $\mathbf{A}(\sim\alpha) = \langle \mathbf{a}_2(\alpha), \mathbf{a}_1(\alpha) \rangle$.

The import of **QAS** is to bring an algebraic distinction between logical negation and denial: contrary to the usual perplexing presentation of 4CN, denial should not be rendered as a connective that is part of the sentential content α ; rather, a denial is a no-answer that does not stand for a function but its resulting value. Correspondingly, a proper formalization of 4CN is suggested in the following style:

- (a'') $\mathbf{a}_1(\alpha) = 0$
- (b'') $\mathbf{a}_1(\sim\alpha) = 0$
- (c'') $\mathbf{a}_1(\alpha \wedge \sim\alpha) = 0$
- (d'') $\mathbf{a}_1(\sim((\alpha \vee \sim\alpha))) = 0$

Only one valuation of \mathbf{AR}_4 accounts for the consistency of (a'')-(d''), namely: $\mathbf{A}(\alpha) = \langle 0, 0 \rangle$, the forbidden value of Jaina logic. Following the definition of conjunction and disjunction in \mathbf{AR}_4 ²⁸, (a'') and (b'') entail that $\mathbf{a}_2(\alpha) = \mathbf{a}_1(\alpha \wedge \sim\alpha) = \mathbf{a}_1(\sim(\alpha \vee \sim\alpha)) = \mathbf{a}_2(\alpha \vee \sim\alpha) = 0$.

Once again, the usual perplexity caused by Nāgārjuna's stance is due to a confusion between the relational and non-relational reading of negation. The former negation (*paryudāsa pratiśedha*) is not an answer about whether the sentence α is true or false, given that it occurs within its sentential content in the whole expression $\sim\alpha$; most importantly, it assumes bivalence and entails that $\sim\alpha$ is false whenever α is true (and conversely). Therefore, no sentence can be given a "gappy" value (neither true nor false) with such a relational use of negation. Furthermore, introducing the intuitionistic negation \neg for this purpose is not the solution either: that α is said to be neither true nor false cannot explain again why this gappy solution is insufficient to account for the fourth stance (d'). This leads to the conclusion that Nāgārjuna's denial should be strictly distinguished from assertive negation and be equated with the "absolutely no"-answer $\langle 0, 0 \rangle$.

Our point about logical values actually holds for every negation, in the sense that there is no functional difference between classical and intuitionistic negation \mathbf{AR}_4 . For the difference between the two negations does not lie in the definition of their mapping from \mathcal{L} to V but, rather, in the domain of values they range over. Given that classical negation assumes a one-one correspondence theory of truth, this entails that a sentence cannot be said

²⁸A complete description of the semantics for \mathbf{AR}_4 is not required in the context of 4CN, but it includes maximal and minimal functions (*max, min*) upon the values of V , given a total ordering function $<$ between these elements proceeds as follows: $\langle 0, 1 \rangle < \langle 0, 0 \rangle < \langle 1, 1 \rangle < \langle 1, 0 \rangle$. Hence the following definition of the connectives of conjunction and disjunction: $v(\alpha \wedge \psi) = \min(\alpha, \psi)$, and $v(\alpha \vee \psi) = \max(\alpha, \psi)$.

to be either both true and false or neither true nor false; hence a restriction of the range from $V = 4$ to $V = 2 = \{\langle 1, 0 \rangle, \langle 0, 1 \rangle\}$. As to the intuitionistic theory of truth as justifiable truth, no sentence can be said to be true unless the justification is definite and this stringent view of justification implies another restriction from $V = 4$ to $V = 3 = \{\langle 1, 0 \rangle, \langle 0, 0 \rangle, \langle 0, 1 \rangle\}$. The Jaina case embodies a paraconsistent variant, where a sentence can be said to be both true and false but excludes the possibility that it be none; hence a corresponding restriction from $V = 4$ to $V = 3 = \{\langle 1, 0 \rangle, \langle 1, 1 \rangle, \langle 0, 1 \rangle\}$. The relative truth of *nayavāda* also accounted for the combination of such basic answers into new logical values in \mathbf{J}_7 , unlike the non-relative, absolute or one-sided view of truth in the Mādhyamika school.

But that is not the whole story of 4CN. Recalling a former quotation by Raju, two problems remain to be solved. Firstly: does Nāgārjuna deny absolutely everything, including his own denials? And secondly: is the *catuṣkoṭi* a mere reversal of the *saptabhaṅgī*, i.e. the transformation of a common set of positive statements into negative statements?

6 Two contrary logics?

Let us note about the first question that a distinction can be made between two generic forms of skepticism, a moderate and a radical one. The former is closer to what the Buddhists meant by nihilism and wanted to be strictly distinguished from; it means that nothing can be known about reality, but one least thing to be known is precisely that nothing mundane can be known. In contrast to this, the radical version goes on denying any denial about our knowledge about reality: ignorance is not asserted but doubted itself. Whether or not such a distinction relates to the Greek schools of the New Academy (Arcesilas, Carneades) and Pyrrhonism (Pyrrho, Timon of Phlius) does not really matter in what follows. Rather, the point is whether Nāgārjuna endorsed radical skepticism and what his rejection consisted in. In the light of **QAS**, the complete denial of 4CN means that only no-answers are given to preceding questions.

As to the second question, Bahm replies in [2] that the two Indian logics cannot merely seen as mutual contraries: Jaina logic cannot be reduced to a Principle of Four-Cornered Affirmation. **QAS** already brought this point out by the cardinality of the sets of logical values, given the essential occurrence of a third question (about *avaktavya*) in \mathbf{J}_7 . Nevertheless, there is a reason to claim that these philosophical schools are really opposite to each other in some respect. The *catuṣkoṭi* can be taken to be a reversal of *saptabhaṅgī* only if the sentential content of a denial or an affirmation is of the first order,

i.e. stands for a declarative sentence about reality; but the same cannot be safely said for higher-order questions about the answerer's attitudes²⁹.

Let us exemplify this symmetrical behavior by means of two Socratic dialogues, where an initial question about the atomic sentence p is accompanied with a sequence of oratory questions (the questioner expects to have a given answer) and answers. The answerer to a common questioner (the doctrinalist Aristotle) is a Jaina speaker (Vādiveda Sūri) and a Mādhyamika speaker (Nāgārjuna), respectively. It clearly appears that the resulting dialogues are radically opposed to each other, and we bring this out by formalizing them in terms of **QAS**.

DIALOGUE 1: ARISTOTLE VS. VĀDIVEDA SŪRI

1. **Q:** Do you accept p ?
[$\mathbf{a}_1(p) = 1$?]
2. **A:** Yes, I accept p .
[$\mathbf{a}_1(p) = 1$]
3. **Q:** Therefore you reject $\sim p$?
[$\mathbf{a}_2(p) = 0$?]
4. **A:** No, I do not reject $\sim p$.
[$\mathbf{a}_2(p) \neq 0$]
5. **Q:** Does it mean that you also accept $\sim p$?
[$\mathbf{a}_2(p) = 1$?]
6. **A:** Yes, I also accept $\sim p$.
[$\mathbf{a}_2(p) = 1$]
7. **Q:** Therefore you accept p and $\sim p$?
[$\mathbf{a}_1(p \wedge \sim p) = 1$?]
8. **A:** Yes, I accept both.
[$\mathbf{a}_1(p \wedge \sim p) = 1$]
9. **Q:** Therefore you reject $\sim(p \wedge \sim p)$?
[$\mathbf{a}_2(p \wedge \sim p) = 0$?]
10. **A:** No, I don't reject $\sim(p \wedge \sim p)$.
[$\mathbf{a}_2(p \wedge \sim p) \neq 0$]
11. **Q:** Does it mean that you also accept $\sim(p \wedge \sim p)$?
[$\mathbf{a}_2(p \wedge \sim p) = 1$?]

²⁹The order of attitudes and their statements can be reformulated in terms of iterated modalities: the statement " α " is an affirmation and correlated belief about α , $B(\alpha)$; the statement "I affirm that α " is an affirmation and correlated belief about the affirmation and correlated belief about α , $B(B\alpha)$; and so on for any n -ordered statement as a sequence of n beliefs: $B^n(\alpha)$. The difference between **AR**₄ and modal logic is that iterated attitudes are not rendered as modal operators but as logical values in the former semantics. See note 31 below.

12. **A:** Yes, I also accept $\sim(p \wedge \sim p)$.
 $[\mathbf{a}_2(p \wedge \sim p) = 1]$
13. **Q:** Therefore you reject $\sim((p \wedge \sim p) \wedge \sim(p \wedge \sim p))$?
 $[\mathbf{a}_2(((p \wedge \sim p) \wedge \sim(p \wedge \sim p))) = 0 ?]$
14. **A:** No, I don't reject $\sim((p \wedge \sim p) \wedge \sim(p \wedge \sim p))$.
 $[\mathbf{a}_2(((p \wedge \sim p) \wedge \sim(p \wedge \sim p))) \neq 0]$
15. **Q:** Therefore you also accept $\sim((p \wedge \sim p) \wedge \sim(p \wedge \sim p))$?
 $[\mathbf{a}_1(\sim((p \wedge \sim p) \wedge \sim(p \wedge \sim p))) = 1?]$
16. **A:** Yes, I also accept $\sim((p \wedge \sim p) \wedge \sim(p \wedge \sim p))$
 $[\mathbf{a}_1(\sim((p \wedge \sim p) \wedge \sim(p \wedge \sim p))) = 1]$
- ...

It emerges from this abortive maieutic that the doctrinalist questioner fails to make the answerer his own reason: the whole answers are perfectly rational albeit inconsistent, in the light of \mathbf{AR}_4 and its non-classical logical values that are exclusively positive or negative³⁰.

THEOREM 3. For every sentence α (including p , $\sim p$, $p \wedge \sim p$, $\sim(p \wedge \sim p)$, and so on), the answer of the Jaina in \mathbf{AR}_4 is $\mathbf{A}(\alpha) = \langle 1, 1 \rangle$.

Proof: Let us assume that $\mathbf{a}_1(p \wedge \sim p) = 1$; then $\mathbf{a}_1(p) = \mathbf{a}_1(\sim p) = \mathbf{a}_2(p) = 1$. And if $\mathbf{a}_1(\sim(p \wedge \sim p)) = 1$ then $\mathbf{a}_2(p \wedge \sim p) = 1$, i.e. $\mathbf{a}_2(p) = 1$ or $\mathbf{a}_1(\sim p) = 1$. Hence for every α , $\mathbf{a}_1(\alpha) = \mathbf{a}_2(\alpha) = 1$. Hence $\mathbf{A}(\alpha) = \langle \mathbf{a}_1(\alpha), \mathbf{a}_2(\alpha) \rangle = \langle 1, 1 \rangle$.

Let us now apply the same process to a *dual* dialogue between the dogmatist questioner Aristotle and his skeptic answerer. This yields the exact reversal of the preceding dialogue, given that each question about whether a given sentence is *accepted* becomes a question about whether it is *rejected*.

DIALOGUE 2: ARISTOTLE VS. NĀGĀRJUNA

1. **Q:** Do you reject p ?
 $[\mathbf{a}_1(p) = 0?]$
2. **A:** Yes, I reject p .

³⁰The semantics for \mathbf{AR}_4 can be said to be bivalent in this respect: for every answer given to question \mathbf{q}_i about the sentence α , the corresponding answer is either positive ($\mathbf{a}_i(\alpha) = 1$) or negative ($\mathbf{a}_i(\alpha) = 0$). *Tertium non datur*. Concerning any positive and negative answer to one and the same question, it has been argued earlier (see note 23) that it is equally impossible in the pluralist approach of the Jains. Hence the ensuing difference between two grades of inconsistency in \mathbf{AR}_4 : a given answer $\mathbf{A}(\alpha)$ is externally *inconsistent* if and only if $\mathbf{a}_1(\alpha) \neq \mathbf{a}_2(\alpha)$; it is internally inconsistent or *incoherent* if and only if, for any answer x in $\{0,1\}$, $\mathbf{a}_i(\alpha) = x$ and $\mathbf{a}_i(\alpha) \neq x$. Accordingly, there is a crucial difference between sentential inconsistency and non-sentential inconsistency (incoherence): two sentences α and $\sim\alpha$ can be mutually inconsistent while the answers $\mathbf{A}(\alpha)$ and $\mathbf{A}(\sim\alpha)$ about them are internally consistent (coherent).

- [$\mathbf{a}_1(p) = 0$]
3. **Q**: Therefore you accept $\sim p$?
 [$\mathbf{a}_2(p) = 1$?]
4. **A**: No, I do not accept $\sim p$.
 [$\mathbf{a}_2(p) \neq 1$]
5. **Q**: Does it mean that you also reject $\sim p$?
 [$\mathbf{a}_2(p) = 0$?]
6. **A**: Yes, I also reject $\sim p$.
 [$\mathbf{a}_2(p) = 0$]
7. **Q**: Does it mean that you reject both p and $\sim p$?
 [$\mathbf{a}_1(p \vee \sim p) = 0$?]
8. Yes, I reject both p and $\sim p$.
 [$\mathbf{a}_1(p \vee \sim p) = 0$]
9. **Q**: Therefore you accept $\sim(p \vee \sim p)$?
 [$\mathbf{a}_2(p \vee \sim p) = 1$?]
10. **A**: No, I do not accept $\sim(p \vee \sim p)$.
 [$\mathbf{a}_2(p \vee \sim p) \neq 1$]
11. Does it mean that you reject both $(p \vee \sim p)$ and $\sim(p \vee \sim p)$?
 [$\mathbf{a}_1(((p \vee \sim p) \vee \sim(p \vee \sim p))) = 0$?]
12. **A**: Yes, I reject both $(p \vee \sim p)$ and $\sim(p \vee \sim p)$.
 [$\mathbf{a}_1(((p \vee \sim p) \vee \sim(p \vee \sim p))) = 0$]
13. **Q**: Therefore you accept $\sim((p \vee \sim p) \vee \sim(p \vee \sim p))$?
 [$\mathbf{a}_2(((p \vee \sim p) \vee \sim(p \vee \sim p))) = 1$?]
14. **A**: No, I don't accept $\sim((p \vee \sim p) \vee \sim(p \vee \sim p))$.
 [$\mathbf{a}_2(((p \vee \sim p) \vee \sim(p \vee \sim p))) \neq 1$]
15. **Q**: Therefore you also reject $\sim((p \vee \sim p) \vee \sim(p \vee \sim p))$?
 [$\mathbf{a}_2(((p \vee \sim p) \vee \sim(p \vee \sim p))) = 0$?]
16. **A**: Yes, I also reject $\sim((p \vee \sim p) \vee \sim(p \vee \sim p))$
 [$\mathbf{a}_2(((p \vee \sim p) \vee \sim(p \vee \sim p))) = 0$]
- ...

Again, the doctrinalist questioner failed to make the answerer his reason: the whole is rational albeit incomplete, so long as the answerer refuses to commit in the truth of any sentence.

THEOREM 4. For every sentence α (including p , $\sim p$, $p \vee \sim p$, $\sim(p \vee \sim p)$, and so on), the answer of the Mādhyamika in \mathbf{AR}_4 is $\mathbf{A}(\alpha) = \langle 0, 0 \rangle$.

Proof: if $\mathbf{a}_1(p \vee \sim p) = 0$ then $\mathbf{a}_1(p) = \mathbf{a}_1(\sim p) = \mathbf{a}_2(p) = 0$. And if $\mathbf{a}_1(\sim(p \vee \sim p)) = 0$ then $\mathbf{a}_2(p \vee \sim p) = 0$, i.e. $\mathbf{a}_2(p) = 0$ or $\mathbf{a}_1(\sim p) = 0$. Hence for every α , $\mathbf{a}_1(\alpha) = \mathbf{a}_2(\alpha) = 0$. Hence $\mathbf{A}(\alpha) = \langle \mathbf{a}_1(\alpha), \mathbf{a}_2(\alpha) \rangle = \langle 0, 0 \rangle$.

Just as the Jains refuse exclusive acts of positive assertion and contend themselves with inconsistent affirmations, the Mādhyamikas refuse exclusive acts of negative assertion and contend themselves with incomplete denials.

A parallel can be made here with da Costa paraconsistent logics C_1 - C_n : these are non-truth-functional systems where contradictions are variably affirmed or denied according to the structural complexity of the contradictory sentences (p and $\sim p$, in C_0 ; $(p \wedge \sim p)$ and $\sim(p \wedge \sim p)$, in C_1 ; and so on). By the same way, a set of dual paraconsistent logics C'_1 - C'_n can be devised for the dialectical process of 4CN and states that alternatives are variably affirmed or denied according to the structural complexity of the alternative sentences: (p or $\sim p$, in C'_0 ; $(p \vee \sim p)$ or $\sim(p \vee \sim p)$, in C'_1 ; and so on). But the parallel stops here, because the preceding dialogues have shown that the structural complexity of a sentence does not change the attitude of the answerer. In this respect, the Jains and Mādhyamikas are likely to be considered as two contrary attitudes or judgments in the common logic of statements **AR**₄: the former affirm everything whereas the latter deny everything.

Returning to a preceding objection, it remains to consider to what extent such radical speakers can be said to affirm “everything” (doxastic eclecticism) or deny “everything” (doxastic nihilism) in their dialectical games³¹. While the concerned texts mention dialectical games about first-order statements only, it hardly makes sense to contend that Nāgārjuna would have denied his own denials with respect to first-order statements.

Let us make a semantic ascent and consider the second-order statement α' : “I don’t affirm that α (is true)”. A no-answer to the question $\mathbf{q}_1(\alpha')$: “is α' justifiably true?” would mean that the answerer denies to have denied (the truth of) α , while a yes-answer would entail that he affirms to have denied α (as he did). The same objection can be made to a universally affirmative stance in the Jains. Likewise, the Jain would hardly give an affirmative answer to α' without refusing the truth to α and thereby violating his policy of non-one-sidedness³². Actually, the preceding dialogues have already made clear that the Jain did deny three times (steps 4, 10 and 14)

³¹Nāgārjuna’s following stance is the key to his allegedly radical skepticism: “If I had a thesis, I would be wrong. But I have no thesis. Therefore there is nothing wrong with me.” (“To keep one away from the vain discussions”, Number 29). What is the content of the thesis at hand? It is likely to be a first-order thesis, i.e. a statement about any given state of affairs. Whether Nāgārjuna would have also claimed to have no thesis about his own attitudes remains unclear, however.

³²This leads to the reintroduction of the law of double negation in the form of an illocutionary law of *double denial*: the denial of $\sim\alpha$ needn’t entail the affirmation of α , given that $\mathbf{a}_1(\alpha) = 0$ needn’t entail that $\mathbf{a}_2(\alpha) = 1$ (compare with $\mathbf{A}(\alpha) = \langle 0, 0 \rangle$); on the other hand, the denial of the denial of α entails the affirmation of α , given that $\mathbf{a}_1(\alpha) \neq 0$ does entail that $\mathbf{a}_1(\alpha) = 1$.

while the Mādhyamika did affirm five times (steps 2, 6, 8, 12, and 16).

If so, the radically opposed attitudes of the Jainas and Mādhyamikas should find their own limits with the sort of sentences to be questioned: denying and affirming are about the nature of reality, rather than about one's own mental states. Such a limit of dialectic might be what Aristotle had in mind, when he attempted to show the attitude of Heracliteus with respect to the PNC is self-defeating. But he failed to make his point with his elenctic strategy, locating the trouble in the propositions (affirming α and affirming not- α) rather than his opponent's propositional attitudes (affirming α and not affirming α). Admittedly, these Indian logics were much more concerned with metaphysical topics and soteriological ends than having the final word in every yes-no answer game.

7 Conclusion

We have proposed a reconstruction of the Jaina and Mādhyamika logics by means of a question-answer semantics. The result of such an enterprise is a rational reading of these Indian schools through modern logical glasses, including the logical tool of many-valuedness that presented skepticism and pluralism as radically opposed to each other and separated by a middle view of judgment that is Aristotle's bivalent way of doctrinalism. Many-valuedness accounts for the seven judgments of Jaina *saptabhaṅgī*, while a more general logic of attitudes displays Jaina and Nāgārjuna's stances within a four-valued semantics that characterizes both Mādhyamika skepticism (the value $\langle 0, 0 \rangle$) and Jaina pluralism (the value $\langle 1, 1 \rangle$).

Above all, the main import of **QAS** is to pay attention to the dialectical role of questions and answers in the Indian approach of logic: just as the Megarics emphasized the dialogical nature of philosophical investigation in contrast to the Aristotelian monological view of truth and falsity as transcendental values, we want to keep in mind that the Indian logicians introduced their statements in the form of answers to speculative questions. Jaina metaphysical pluralism also made sense of their inconsistent judgments, while the skeptic flavor of Nāgārjuna's philosophy explains his systematic denial to any question about the nature of reality.

Last, but not least: one of the most intriguing case studies has concerned the meaning of *avaktavya* (non-assertibility), the third basic judgment of Jaina logic. This predicate should not be confused with common self-contradiction, where a sentence and its negation are said to be both true at once and in the same respect. The commentators frequently claimed that the Jainas subscribed to PNC in their various reasonings: so non-assertibility

refers to another, milder view of contradiction than coexistence of incompatible properties in the same subject. Rather, we support Tripathi's interpretation of *avaktavya* in the sense of non-distinction: the Jaina third judgment might mean that some objects (S) cannot be predicated by any property, that is, neither of one of them (P) or any of their complementaries (not-P). Rather than a plea for self-contradiction, *avaktavya* seems to argue for the impossibility to predicate anything of some such "absolute subjects" as *ātman* or Brahman because these would stand beyond any set of definite properties. Such a tentative explanation would match with the Hegelian alternative process of *Aufhebung* (or "sursumption"), in contrast to the predicative process of subsumption that systematically describes a subject S as falling under a given set of properties P³³.

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³³Here is an alleged description of the Brahman by himself: "This whole universe is filled by me in immaterial form; all beings are in me, but I am in them. Yet those born are not within me. Behold my kingly rule: my self sustains all beings, is not in them but creates them. Just as the mighty wind everlastingly occupies the space above us and moves throughout it, so do all created beings occupy me." (*Bhagavad-Gītā*: Chapter 9, verses 4-7). This seems to match with our description of S as an ultimate class.

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