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Dewey Hodges' Research in Structural Dynamics, Aeroelasticity and Composites: A Personal Perspective

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The contributions of Dewey Hodges within the specialized areas of structural dynamics, aeroelasticity and composites are highlighted in this paper. Dewey Hodges has published 215 journal and 170 conference papers covering a wide range of topics in aerospace structures. His research and academic career spans nearly five decades and it is difficult, if not impossible to give a detailed commentary on his colossal achievement in the confines of a single paper. It is widely acknowledged that Dewey Hodges' research record is no less than incredible. He has not only given an exceptional account of himself, but has also demonstrated his extraordinary versatility in research. The author of this paper has known him both personally and professionally for nearly three decades and with great humility he acknowledges the enormous benefit he has received from his association with him. Given the huge volume of work Dewey Hodges has produced and the enormity of the task of commenting on it, the author has understandably been highly selective in choosing which contributions to discuss. A sample of selected papers with Dewey Hodges' original and ground-breaking contributions is thus highlighted and commented on.

I. Introduction

The author while as a PhD student at Cranfield University came across for the first time in 1978, the publication of Dewey Hodges' classic 1974 NASA report [1] coauthored by Earl Dowell. As a young researcher, he had limited understanding of Lagrangian dynamics and Hamiltonian mechanics at the time, but he was beginning to expand his interest in classical mechanics by exploring possible alternatives to Newtonian mechanics. This paper by Hodges and Dowell [1] created an instant spark in him, acted as a catalyst to stimulate his interest and provided him with the vital

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clue needed at that time to acquire the necessary knowledge of how to derive the governing differential equations of motion of complex dynamical systems using both Hamilton's principle and Newton's second law. In the NASA report [1], Hodges used Hamilton's principle while Dowell for his part used Newton's second law and they both arrived at the same differential equations for the elastic bending and torsion of a twisted nonuniform rotor blade. The two methods were complementary in that the confirmatory results from the two parallel investigations gave assurance concerning the accuracy of the governing differential equations. It was fantastic. This outstanding and inspiring report has had profound influence on the author and his research career ever since. It was a great opportunity for the author at the time to reconstruct the derivations of Hodges and Dowell [1] after expending considerable efforts. Since then the author has been following Dewey Hodges' research with great interest, but it was not until 1989 when he met him for the first time in one of the SDM conferences in the USA. He made his acquaintance and has continued to maintain his association with him, having met him on numerous occasions at international conferences. Later he visited him in the Daniel Guggenheim School of Aerospace Engineering at the Georgia Institute of Technology a few times. Having known him both personally and professionally, the author gives here an overview of Dewey Hodges' work from a selective range of his research [1-70].

Dewey Hodges' illustrious research career began in the early 1970s and his early preoccupation in research was focused mainly on the free vibration, response and aeroelastic stability analysis of helicopter rotor blades [3-5]. In this endeavor, he was strongly influenced by his association with Robert Ormiston and David Peters [6]. He continued his research in these areas in the 1980s and one of the key aspects of his research during this period was the development of nonlinear large deformation theories for twisted and curved beams [8-11] as applicable to helicopter blades. The principal novelty in this research arises from his innovative formulation of nonlinear beam kinematics [12-13]. Side by side to his research in rotorcraft aeroelasticity, he carried out extensive research on the intrinsic formulation for curved and twisted composite beams [20, 34] in the 1990s which represents one of the significant flagships of his research. During this period, he turned his attention to composite beam modelling [18, 19] and took strong interest in the variational-asymptotic method (VAM) to establish cross-sectional properties of composite beams accurately [28, 29]. His friendship and collaboration with Victor Berdichevsky were instrumental to inspire and stimulate his interest in VAM [36]. He used the cross-sectional properties derived through VAM to investigate the free vibration behavior of composite beams [22] and he drew many significant conclusions. During this period, Dewey Hodges initiated a

programme of research associated with Kanés's equation and multibody dynamics [47, 52, 54], optimal control problems in engineering [21, 26, 30, 38] and the trapeze effect in finite element cross-sectional analysis in composite beams [37, 63]. Since 2000, Dewey Hodges turned his attention to aeroelasticity of aircraft with high aspect ratio wings, both metallic and composites [39, 41]. This was alongside his on-going research interests in geometrically exact intrinsic theory for the curved and/or twisted beams [70] and rotary wing aeroelastics [49]. Although the advancement of nonlinear beam dynamics is an area in which Dewey Hodges excels, he nevertheless made remarkable contributions in plate [65, 68] and shell [57, 69] theories over the years. His recent research which accounts for the effects of damage in structures demonstrates his versatility and has culminated in admirable research papers [64, 67]. Ever since Dewey Hodges started his research career in the 1970s, his work has not been far from potential industrial applications. To this end he has been continually associated with software developments in some form or other, providing tools to the design engineers. Foremost amongst these are: GRASP (General Rotorcraft Aeronautical Stability Program [11, 14, 17]), VABS (Variational Asymptotical Beam Sectional Analysis [35, 46, 59]), RCAS (Rotorcraft Comprehensive Analysis System [49]) and NATASHA (Nonlinear Aeroelastic Trim and Stability of HALE aircraft [58]).

The current paper is now set out using the above background of Dewey Hodges as a prelude. In the subsequent text, some details of his contributions in (i) structural dynamics, (ii) aeroelasticity and (iii) composites which have commonality with the author's research interests are elucidated. Because of the interdisciplinary nature of Dewey Hodges' work, there is inevitably some overlap between these three areas of research. Thus, some of his papers reviewed under the above three broad categories have been referred to more than once. The main emphasis here is given on the developments of his wide range of innovative beam theories, bearing in mind that he has also made large contributions to the literature using plate and shell theories which have not been given coverage in this paper. Furthermore, his significant contributions to multi-body dynamics and control theory have also not been included in this review.

II. Contributions of Dewey Hodges

The three chosen areas of Dewey Hodges' research are now described in some detail referring to a thoughtfully selected assortment of his publications.

A. Structural Dynamics

The work of Dewey Hodges in structural dynamics is without doubt elaborate, extensive and diverse. However, attention here is mainly confined to the dynamical behavior of beams made of both isotropic and anisotropic materials. Any details concerning the dynamical behavior of helicopter rotor blades are also omitted although the beam idealization for such problems is directly relevant. In his work on beam structures, Dewey Hodges always felt the need to establish their mass and stiffness properties accurately and then examine their static behavior before carrying out the subsequent dynamic analysis. In one of his earlier papers [2] he made both theoretical and experimental investigations to examine the nonlinear bending-torsion behavior of a cantilever beam. Following the publication of his classic paper which describes the derivation of the nonlinear equations of motion for the elastic bending and torsion of twisted non-uniform rotor blades [1], he further studied the nonlinear deformation geometry of Euler-Bernoulli beams in a general context [6]. Static stiffness and deflection analysis as well as free vibration analysis of uniform and nonuniform rotating and non-rotating beams have been researched by Dewey Hodges from the very early days of his research career. With the advent of advanced fibre reinforced composite materials, he extended his earlier work on metallic beams [7, 15, 16] to composite beams [18, 19, 22, 24, 25] in later years. Paper [15] is an important contribution which captures the nonlinear effects in the static and dynamic behavior of beams and rotor blades. The paper identified some errors in previous publications by deriving correct equations, including all nonlinear terms up to the third degree. Paper [16] gives both theoretical and experimental results by conducting a thorough nonlinear investigation of a cantilever beam. Although the focus of this paper was on static analysis and deflection prediction, some free vibration results were also included and good agreement between the theory and the experiment was detected. The computer program GRASP [14] which was developed through the initiatives of Dewey Hodges was used when obtaining the theoretical results. Paper [22] is a very interesting paper in that it deals with both stiffness and mass representations of composite beams as well their free vibration analysis. The paper demonstrates the effects of stiffness calculation on the free vibration characteristics of composite beams by performing several comparative studies. A breakthrough came when Dewey Hodges captured the physical behavior of beam dynamics by developing a novel way of calculating the dispersion curves for both isotropic and anisotropic beams in [36]. Various non-classical

effects and their relative importance have been elucidated in this paper by some unusual and unorthodox mode shapes. Paper [48] is basically an authoritative theoretical paper which breaks new ground by developing a geometrically exact intrinsic theory to investigate the dynamic behavior of curved and twisted anisotropic beams. The formulation given in this paper is sufficiently general to capture non-linear rigid-body dynamics and non-linear static analysis as degenerate cases. Dewey Hodges has published numerous papers on the structural dynamics of helicopter rotor blades which are not discussed here, but interested readers are referred to his publications in this area using the website www.dhodges.gatech.edu/ which provides sufficient details.

B. Aeroelasticity

Dewey Hodges' research in aeroelasticity covers both helicopters and fixed wing aircraft, but the former is not a specialization of the present author and hence is not discussed here, but the readers may find Dewey Hodges' survey paper [49] informative and enlightening. Attention here is focused on fixed wing aeroelasticity and some selective papers of Dewey Hodges are reviewed next. In the majority of the aeroelasticity work on fixed wing aircraft, the structural nonlinearity is generally ignored, and paper [39] is an original contribution which deals with nonlinear aeroelastic analysis of complete aircraft. Geometric stiffening of the wing arising from structural nonlinearity and dynamic stall arising from aerodynamic nonlinearity were both included in the analysis. When predicting the aeroelastic instability of the aircraft, Limit Cycle Oscillation (LCO) was observed due to the changes in mode shapes arising from the geometric stiffening of the wing. This work was further extended [41] to analyze the aeroelastic behavior of High-Altitude, Long Endurance aircraft (the so-called HALE aircraft). This is a most interesting paper which shows the effect of the angle of attack on the flutter speed and flutter frequency of the HALE aircraft. The paper clearly demonstrates that the short-period and phugoid modes are significantly affected by wing flexibility of the HALE aircraft. The research described in [39] provided further stimulus to investigate the LCO phenomenon in a high aspect ratio aircraft wing using nonlinear analysis [42]. It was noted that the change in flutter speed with respect to the loaded or trimmed state was somewhat severe, leading to a significant conclusion that linear analysis may lead to over-prediction of the flutter speed which can be dangerous. Published literature on the effect of engine thrust on the flutter behavior of aircraft wings is scarce, and in this respect paper [44] is a striking contribution which showed that for lower values of the bending to torsional rigidity ratio (typically less than 5), the flutter speed increases for a certain value of the thrust, but for a higher value of this ratio, particularly above 10, the flutter speed always decreases. The paper captures very well the frequency coalescence phenomenon generally observed in classical bending-torsion

flutter. Dewey Hodges continued his research on aeroelasticity of high aspect ratio wings using nonlinear theories. This is evident from many of his creative ideas published in the literature [50, 53]. It is worth noting that paper [55] is a groundbreaking contribution which assimilates flight dynamics of highly flexible aircraft using geometrically exact, intrinsic beam elements in the structural idealisation, and two-dimensional, large angle-of-attack, unsteady aerodynamics in the aerodynamic idealisation. The influence of significant aircraft parameters such as wing flexibility, horizontal/vertical tail aerodynamics was investigated in this paper. Alongside to this research on the aeroelasticity of high aspect ratio wings, Dewey Hodges was mindful of the importance of validation exercises when solving new and complex engineering problems. To reinstate the power and strength of his methods, he illuminated his scholarship in paper [58] where he validated his theories and results for aeroelastic trim and stability of highly flexible aircraft using different computer programs based on different theories. It is well recognized that the aeroelastic behavior of an aircraft can be affected by the location of the engine(s) on the wing. In this context, Dewey Hodges made pioneering contributions [61, 62] when investigating the effects of engine locations on the aeroelastic trim and stability of flying wing aircraft. A comprehensive set of results were reported in these papers. Paper [66] investigates the effects of inertial and constitutive properties on the body-freedom flutter of a flying wing. The results were obtained using both linear and nonlinear aeroelastic analyses and were compared and contrasted. Dewey Hodges has published many more papers on aeroelasticity which are not discussed in this paper and interested readers are again referred to his website for further insights into the subject.

C. Composites

To the best of the author's knowledge, Dewey Hodges started taking a vigorous interest in composite materials and structures in the 1980s and his review on composite rotor blade modelling which appeared some years later [18] makes very interesting reading. The different modelling techniques used by different investigators and their relative advantages and disadvantages were discussed in this paper. Paper [19] is a noteworthy contribution which epitomizes non-classical effects in the behavior of thin-walled composite beams. Of considerable importance is the inclusion of the bending-shear coupling and torsional warping and the paper provides considerable insights into the non-classical behavior of composite beams. Although the paper focuses on the static behavior of composite beams of closed cross-section, it paves the foundation for future dynamic analysis of composite beams ensuring that correct stiffness properties are necessarily used. A follow-up paper using a unified approach to predict the deflected shape of a composite beam was published [23] and the predictions from the unified analysis agreed very well with published

experimental results for both nonlinear static and linearized dynamic behavior about the equilibrium position. Paper [24] advances the work of paper [23] in a major way and provides further insights into the cross-sectional properties of both isotropic and anisotropic beams. A different, but related work which deals with a simplified strain energy function for geometrically nonlinear composite beams using an asymptotically exact methodology, based on geometrically nonlinear, three-dimensional elasticity was published [25]. This is probably one of the earliest papers of Dewey Hodges which applied the Variational-Asymptotic Method (VAM) and subsequently this work had far reaching consequences in his later research. The attractiveness of this elegant method is the fact that it allows determination of the governing equations for an isotropic or anisotropic beam with the inclusion of the three-dimensional relationships necessary to predict the displacements, stresses and strains throughout the beam structure. In essence the problem is split into a two-dimensional linear cross-sectional analysis and a one-dimensional nonlinear beam analysis. The outcome is that the results are asymptotically correct. VAM was later applied to predict the stiffness properties of curved and twisted composite beams [29, 31-33]. Based on VAM, the computer program VABS was developed [35]. The program VABS gives accurate cross-sectional properties of arbitrary beam cross-sections allowing for anisotropic and non-homogeneous material properties of the beam. It is now commercially available and integrated within MSC/NASTRAN. VABS has been extensively validated [40, 43, 45, 46, 51, 59] and it was also used within the context of composite rotor blade design [56]. An up-to-date version of VABS was later published [60]. It is now well recognised that VABS is a powerful tool to compute cross-sectional properties of beams with complex geometries and made of anisotropic materials. The development of VABS is probably one of the single most practical applications of the variational-asymptotic method, particularly useful in the design of composite helicopter blades and aircraft wings. From an aeroelastic tailoring point of view, the emergence of VABS is tremendously significant. Active research on further developments of VABS is continuing and its full potential in aeronautical design has yet to be realised.

III. Conclusions

The illustrious career of Dewey Hodges spanning around five decades clearly depicts an academic and research life full of energy and activity. The author of this paper has given only a glimpse of his contributions in research by discussing a highly selective sample of his publications. Dewey Hodges has made gigantic contributions to the scientific and engineering community of the world by carrying out extensive research in wide-ranging topics. Foremost amongst these topics are (i) dynamic stability of helicopter blades, (ii) applications of Hamiltonian mechanics to derive governing differential equations for complex dynamical systems, (iii) static, dynamic and buckling analysis of structures using linear and nonlinear theories, (iv) free vibration analysis of rotating structures, (v) developments of refined beam, plate and shell theories using variational-asymptotic and other methods, (vi) application of symbolic computation in structural engineering research, (vii) multi-body dynamics, (viii) optimal control theory, (ix) aeroelasticity of composite wings, (x) damage analysis and (xi) software development for computer aided structural analysis and design. His research has been truly interdisciplinary in nature, integrating many of the above topics. His publications of 215 journal papers and 170 conference papers give a lucid account of the new knowledge that he has generated during his renowned career. Suffice to say that an academic and research life has been and continued to be well-lived by a formidable and distinguished scholar. It is evident from his publications that Dewey Hodges' search for knowledge has been unrelenting and uncompromising. The application of the mathematical theory of elasticity coupled with the utilization of variational methods in elastodynamics in a rigorous manner has been the flagship of his exemplary research. He has been a role model for generations of future academicians and researchers. It has been a great honor for the author to have known Dewey Hodges for so many years (nearly three decades). Writing this paper has indeed been a privilege and a fitting tribute to a friend and colleague who has come before us as an indomitable researcher and has always given his very best.

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