University of Bath

Examining the Solution Bias of Construction Kits

Dr. Gopsill

February 2018

Dear DESIGN 2018 Programme Committee,

This document details the amendments made to the manuscript entitled "Examining the Bias of Construction Kits". We would like to thank the reviewers for their constructive comments. We feel that the ammendments have further strengthed the contribution of this paper and will spark interesting debate and discussion at DESIGN 2018. Table 1 highlights the key amendments that have been made based on the comments of the three reviewers. In addition, attached is a complete track changes report that evidences the thorough proof read that has been performed to enhance the manuscript and make it suitable for publication in the proceedings of the international DESIGN conference.

Kind regards,

Dr. Gopsill

Table 1: Key amendments to manuscript

Review	Comment	Amendment
1	It is not really clear what some of the previous research shown	An extended discussion concerning previous research have been included in the amended manuscript. (Pages 1 & 2)
1	Then the sentence "To further democratise the process, Bennett et al. (2017) and Mathias et al. (2017) have investigated how to provide the constraints in an increasingly immersive manner. This has been through the embedding of rules within the bricks of construction kits. One of the most commonly used tools in prototyping." The sentence is not really complete "One of the most commonly used tools in prototyping?" What is the most commonly used tool bricks, what types of bricks?	A thorough proof read and clarification of these statements has been made in the amended manuscript. (Page 2)
1	Then in the sentence describing the possible numbers of combinations of the $(B_{(2,4)}(6))$ it could be of interest that you mean six bricks	This has now been clarified within the manuscript: "The analysis revealed that the original value of 103.0×10^6 contiguous combinations of six 2×4 bricks $(B_{(2,4)}(6))$ as reported by LEGO™" as well as the addition of a figure to describe w and h with respect to a brick (Figure 1b). (Page 2)
1	You state "A Gini coefficient of 0 indicates that the number of pathways ('wealth') is equally distributed across the morphologically unique combination ('population'). This would indicate that no bias exists within the construction kit. In contrast, $G = 1 - 1/N$ would indicate a highly biased construction kit with one combination having all the pathways ('wealth'). And I am sorry because I cant follow your argumentation here, and in the equation, you use the capital N, is this the number of people or number of lego pieces?	This section has been re-written to focus solely on the application of the Gini coefficient in terms of combinations and bricks, and not how it has been applied to other domains. Also, an additional figure has been provided to demonstrate how the Gini coefficient is derived from the Lorenz curve (Figure 5). (Pages 6 & 7)

Review	Comment	Amendment
1	Also can you clarify what the pathways is in the simple lego examples with two bricks, in table 2 the minimum pathway for two bricks is 2 and the maximum is 14 and at the same time in the section under fig 4 it states "By colouring the bricks, it can be seen that some combinations have 8 pathways, whilst others have only 1" So what is the $P_{\max}$ and $P_{\min}$ in Table 2?	Figure 4 has been updated to include a breakdown of the combinations and the number of pathways to them (Figure 4c). In addition, $P_{\max}$ and $P_{\min}$ have been defined clearly within the text. (Pages 6 & 8)
1	I have some more questions regarding the Gini coefficient what I can see in several publications is that the Gini coefficient should be in the range of 0 to 1? If we go to the Table 2 and Figure 5 what is listed in the table as G? In the Table, the Gini coefficient is listed as a number 472 for 2 bricks and 1.6×10^6 for 3 bricks?	The authors apologise for the confusion in the table and the nomenclature has since been updated to be in line with established practice. (Page 7)
1	In the discussion, I try to understand "research is required on the metrology of construction kits in order to determine a bricks position with respect to the other bricks in the combination" can you please clarify!	This has been further clarified with respect to how it will support research in determining the human factors involved when constructing with a construction kit. (Page 9)
2	How is bias defined?	This has been clarified in Section 3 with a more detailed description of the metrics and how they relate to bias within a construction kit. (Pages 6 & 7)
2	How is bias measured?	Again, this is clarified through the use of the Gini coefficient as the measure for bias. (Pages 6 & 7)
2	How to avoid bias?	This is discussed in the discussion section (Section 4) of the manuscript where it is highlighted that this is future work and is currently being performed by the researchers of this manuscript. (Page 9)
2	How do the results of this paper may influence design choices in future?	Again, this is discussed in the discussion section (Section 4) of the manuscript where it has been highlighted that this paper has identified the need to understand how prototyping kits influence the design choices of desingers in order to provide a framework for appropriate application. (Page 9)
2	What is the significance of this paper?	This has been clarified in the conclusions and in the introduction and that is to determine whether biases do exist within construction kits and to what level they exist. If proven, there lies opportunities to better understand this phenemomen. From the conclusions: "However, it is posited that how the problem is represented by the construction kit has the potential to introduce an unknown bias where there is statistically more chance for certain solutions to be generated than others. This has been proven by this paper through its scoping study into the construction combinations of n, 2 wide, 4 high LEGO™bricks." (Pages 9 & 10)
2	The paper has some stylistic problems: ill-structured/unfinished sentences, wrong punctuation, typos (e.g 'number of' repeated in the same sentence, trivial mistakes such as 'how additional rules affects' (sic), etc.). It needs very careful proofreading.	The paper has been thoroughly proof-read and checked for the final camera-ready version. Full details of all the corrections have been provided in the supporting track changes document.
3	However, it is not yet convincing whether such an approach can be applied to other construction kits.	The researchers have clarified that the focus on currently on brick-style construction kits where such a process can be applied to. It is out of the scope of this conference article to provide a general solution for the combinatorics of all construction kits.

EXAMINING THE BIAS OF CONSTRUCTION KITS

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Abstract

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1. Introduction

Prototyping is a fundamental design activity with studies showing that the method by which designers prototype along with the constraints set during prototyping sessions can affect design fixation and solution generation (Menold, Jablokow, and Simpson 2017). Youmans (2011) highlights that prototyping can reduce the cognitive load on designers by constraining and providing an easily accessible method of interacting with the design space. Youmans (2011) also concluded that prototyping reduces design exploration and increases likelihood of design fixation. These findings have also been observed by Viswanathan and Linsey (2012) whose study revealed a potential correlation between the complexity of the design problem being represented by the prototyping tools and the level of exploration within the design space.

Prototyping has also been used extensively in design processes where wider stakeholder engagement is desired. Coughlan, Suri, and Canales (2007) have shown the capability of prototyping to democratise the design process by providing a common platform to present the constraints of a design problem. This reduces the time spent communicating, discussing and defining the problem, and more time solving and generating potential solutions. To further democratise the process, Bennett et al. (2017) and Mathias et al. (2017) have investigated how to provide constraints in an increasingly immersive manner (Figure 1a). This has been through the embedding of constraints within the bricks of brick-style construction kits (e.g. LEGO™). By embedding the rules within the bricks, the environment is able to track and react in real-time with regards to the validity of the proposed solution.
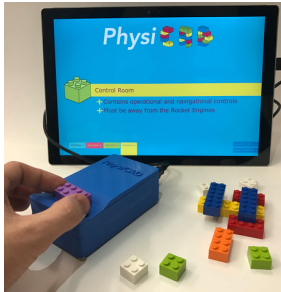
(a) Embedding within bricks (From: Mathias et al. 2017)
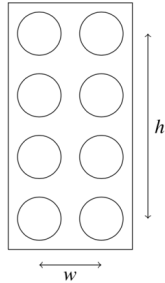
(b) Brick characteristics

Figure 1: Prototyping with construction kits

Construction kits also enable individuals to explore and present designs to the same level of granularity and accuracy by providing ready-made constructs that can be combined together. This also removes the skill barrier one might have when using techniques, such as sketching, where the quality of the output can hinder the communication of the potential design (Craft and Cairns, 2009). Construction kits have been particularly successful in city, town, office and manufacturing facility configuration design tasks as well as developing problem solving skills during a child's development (Hadhrawi and Larson 2016; Eisenberg et al. 2002).

With descriptive studies revealing the profound effect prototyping tools have on the representation and exploration of the design space, further prescriptive studies into the development of the underlying theory of a construction kits suitability for design problems can now be performed. This can now be performed with the advent of high-performance computing, which enables the entire design space to be computed.

For example, mathematicians have examined the number of contiguous combinations of 2 wide by 4 high studded LEGO™ bricks (Eilers 2016) (Figure 1b). The analysis revealed that the original value of 103.0×10^6 contiguous combinations of six 2×4 bricks ($B_{(2,4)}(6)$) as reported by LEGO™ was a factor of 9 out and that the true value is nearer 915.1×10^6. Although this informs us on the complete number of contiguous combinations of $B_{(2,4)}(6)$ bricks that can be created, it does not inform us on the number of pathways one could take to form these combinations and whether there are multiple pathways to achieve a particular combination. In addition, questions can also be asked as to whether construction kits contain an inherent combination bias with some combinations having a greater number of pathways than others. This may lead designers to favour particular solutions and not fully explore the design space. To investigate the number of pathways to combinations and answer the question of whether bias exists within current construction kits, this paper presents the results from a scoping study that has characterised the design space represented by $B_{(2,4)}(n)$ LEGO™ bricks. This has been achieved by solving the combinatorial problem of constructing a breadth-first search algorithm that mimics the behaviour of constructing a combination from $B_{(2,4)}(n)$ LEGO™ bricks. By modelling the problem through the perspective of a designer, features such as the number of morphologically unique combinations and number of pathways to each combination can be derived. The analysis of the distribution of the pathways amongst the combinations can provide insights into the potential bias within the construction kit.

To continue, Section 2 defines and presents the breadth-first search algorithm that is used to solve the combinatorial problem of $B_{(2,4)}(n)$ LEGO™ bricks. This leads into Section 2, which defines the metrics used to determine the bias within construction kits. Section 3 presents the results from a study that has analysed the distribution of pathways across the potential combinations for $B_{(2,4)}(n)$. The role of constraints and their effects on any potential bias within the construction kit has been explored through the consideration of the addition of a No Rotation Constraint (NRC) case. Section 4 then discusses the
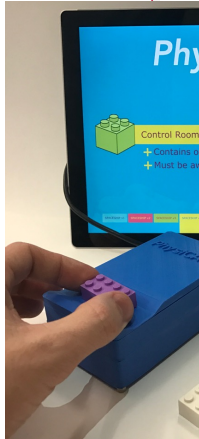

Figure 1: Embedding rules within the bricks

potential bias in construction kits, how one can potentially control the bias, and future work this is developing a theory for the appropriate design and application of construction kits in design scenarios. The paper then concludes by highlighting the key findings from the scoping study.

2. The combinatorics of $B_{(2,4)}(\text{n})$ LEGO™ kits

Before one can explore the potential bias within $B_{(2,4)}(\text{n})$ LEGO™ kits, it is important to be clear and precise in the definition of the combinatorics problem that is being solved. Therefore, this section defines the combinatorial problem as well as the approach that has been taken to solve the problem. The approach is a breadth-first search algorithm that has been designed to mimic the construction of a combination by a designer as well as enabling the inception of further constraints into the process of forming a combination.

2.1. Defining the combinatorics problem

A combination is to be made up of a set of bricks where a brick (b) is defined by fixed dimensions w and h denoting the number of studs along the bricks width and height, respectively. All bricks are of the same depth (z) and $w \leq h$. Therefore, a single brick can be denoted by $b_{(w,h)}$.

The bricks' top and bottom surfaces must be in parallel with the XY-plane with two sides parallel to the x-axis (i.e. the bricks can only be in 90° orientations). There is also the physical constraint of the bricks not being able to occupy the same space as one another. By meeting these constraints for n bricks, a combination is formed. The set of combinations that meet all the constraints for a set of same type bricks is denoted as $B_{(w,h)}(n)$.

In solving the problem, there will be combinations that are rotationally symmetric. This has been explored by Eilers (2016). In this paper, the concept has been extended to identify morphologically equivalent combinations. I.e. combinations that are symmetric through reflection. It is argued that morphologically equivalent combinations are derivations of a single design. A feature that is often seen across Engineering Design with examples such as, left/right-handed versions of products and left/right-hand drive vehicles.

Having defined the combinatorial problem, the paper continues by presenting the approach taken to solve the problem where consideration has been made to provide the ability to add rules to the construction kit.

2.2. Solving the combinatorial problem

The generalised process for solving the combinatorial of construction kits is described by Gopsill (2018, In Review). Therefore, this section describes the application of the process to solve for $B_{(w,h)}(n)$ LEGO™ kits and is illustrated in Figure 2.

Step 1. defines the initial brick in space, which is defined as an object featuring (x, y, z, w, h, θ) where:

x is the x-coordinate of the leftmost line of studs;
y is the y-coordinate of the uppermost line of studs;
z is the layer that the brick has been placed upon;
w is the stud width;
h is the stud height; and,
θ is the brick rotation.

Thus, the first brick in this problem is represented by (0,0,0,2,4,0).

START

Step 1. Initialise Model $B_{(2,4)}(i) = B_{2,4}(1)$

Loop 1. FOR each combination in $B_{(2,4)}(i)$

Step 2. Determine potential positions for b_{new}

Loop 2. FOR Each potential position

Step 3. Check for clashes

IF clash detected — Yes → Disregard combination

No

Step 4. Check for morphological equivalence

IF equivalent to existing combination

No → Append combination to $B_{(2,4)}(i + 1)$

Yes → Increment number of pathways to existing combination in $B_{(2,4)}(i + 1)$

No — **IF** end of **Loop 2** — Yes

No — **IF** end of **Loop 1** — Yes
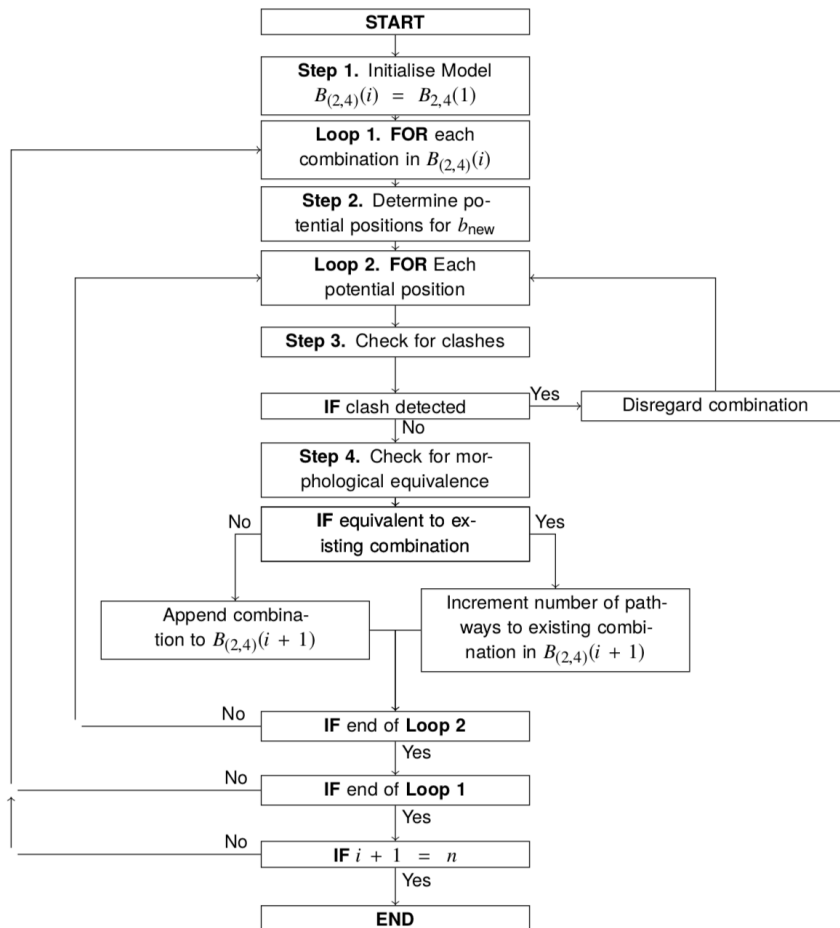
No — **IF** $i + 1 = n$ — Yes

END

Figure 2: The process applied to solve the $B_{2,4}(n)$ combinatorial problem

With the first brick defined, the process can start to add bricks to the first combination. Running through each combination in the set of combinations $B_{(2,4)}(i)$ (**Loop 1.**) the process calculates all the possible positions that it could be placed by looking at all the bricks in the existing combination and creating an array of all the potential x, y positions for b_{new} (**Step 2.**). These positions are calculated using the ranges derived in Table 1.

Following the creation of an array of the potential positions for b_{new}, the process then checks whether the brick would intersect with any of the existing bricks in the combination.

Table 1: Determining the potential

	$b_{\text{prev}}, \alpha = 0$
b_{new} $\alpha = 0$	$x_{\text{prev}} - w_{\text{new}} + 1 \leq x_{\text{new}} \leq x_{\text{prev}} + w_{\text{prev}}$ $y_{\text{prev}} - h_{\text{new}} + 1 \leq y_{\text{new}} \leq y_{\text{prev}} + h_{\text{prev}}$
b_{new} $\alpha = 90°$	$x_{\text{prev}} - h_{\text{new}} + 1 \leq x_{\text{new}} \leq x_{\text{prev}} + w_{\text{prev}}$ $y_{\text{prev}} - w_{\text{new}} + 1 \leq y_{\text{new}} \leq y_{\text{prev}} + h_{\text{prev}}$

Table 1: Determining the potential brick positions for b_{new}

	$b_{\text{prev}}, \theta = 0$	$b_{\text{prev}}, \theta = 90°$
b_{new} $\theta = 0$	$x_{\text{prev}} - w_{\text{new}} + 1 \leq x_{\text{new}} \leq x_{\text{prev}} + w_{\text{prev}}$ $y_{\text{prev}} - h_{\text{new}} + 1 \leq y_{\text{new}} \leq y_{\text{prev}} + h_{\text{prev}}$	$x_{\text{prev}} - w_{\text{new}} + 1 \leq x_{\text{new}} \leq x_{\text{prev}} + h_{\text{prev}}$ $y_{\text{prev}} - h_{\text{new}} + 1 \leq y_{\text{new}} \leq y_{\text{prev}} + w_{\text{prev}}$
b_{new} $\theta = 90°$	$x_{\text{prev}} - h_{\text{new}} + 1 \leq x_{\text{new}} \leq x_{\text{prev}} + w_{\text{prev}}$ $y_{\text{prev}} - w_{\text{new}} + 1 \leq y_{\text{new}} \leq y_{\text{prev}} + h_{\text{prev}}$	$x_{\text{prev}} - h_{\text{new}} + 1 \leq x_{\text{new}} \leq x_{\text{prev}} + h_{\text{prev}}$ $y_{\text{prev}} - w_{\text{new}} + 1 \leq y_{\text{new}} \leq y_{\text{prev}} + w_{\text{prev}}$

2.2.1. Detecting intersections (Step 3.)

To detect whether any of the bricks intersect, the process uses a matrix representation for the layer that b_{new} is being placed upon. The matrix is initialised as a zero array (Figure 3a) and the stud positions of the bricks that already exist on this layer as well as b_{new} are added to the matrix. Once all the bricks have been added to the layer, intersects are identified if the matrix contains any elements that are > 1 (Figure 3b). This indicates that the studs of multiple bricks are occupying the same space. If an intersection is detected then the combination is disregarded.
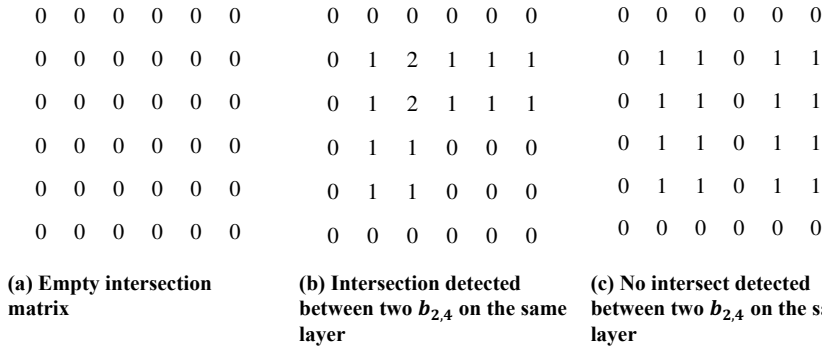
```
0 0 0 0 0 0     0 0 0 0 0 0     0 0 0 0 0 0
0 0 0 0 0 0     0 1 2 1 1 1     0 1 1 0 1 1
0 0 0 0 0 0     0 1 2 1 1 1     0 1 1 0 1 1
0 0 0 0 0 0     0 1 1 0 0 0     0 1 1 0 1 1
0 0 0 0 0 0     0 1 1 0 0 0     0 1 1 0 1 1
0 0 0 0 0 0     0 0 0 0 0 0     0 0 0 0 0 0
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(a) Empty intersection matrix **(b) Intersection detected between two $b_{2,4}$ on the same layer** **(c) No intersect detected between two $b_{2,4}$ on the same layer**

Figure 3: Detecting clashes between bricks

This is performed for each position of b_{new} and at the end of this loop, the process has generated all the possible combinations for $B_{(2,4)}(i+1)$.

2.2.2. Detecting morphologically equivalent combinations (Step 4.)

Having processed all the possible combinations of adding b_{new}, the process then identifies the number of morphologically unique combinations and in doing so, determines the number of pathways one can take to achieve a particular combination. This is achieved by calculating the following features for each combination:

- The sum pair-wise distances between the brick centres.
- The Δ in rotated to non-rotated bricks.
- The number of available studs.

For combinations to be morphologically equivalent, all these features have to equate. Figure 4 illustrates this for $B_{(2,4)}(2)$, where the 2nd brick can only be added to the top of the 1st brick. **Step 2.** and **3.** detects that there are 46 possible combinations for the bricks to be arranged. After checking for morphological equivalence, this is reduced to 14 morphologically unique combinations.

(a) 46 Combinations of $B_{2,4}(2)$

(b) 14 Morphologically unique combinations and their equivalent combinations of $B_{2,4}(2)$

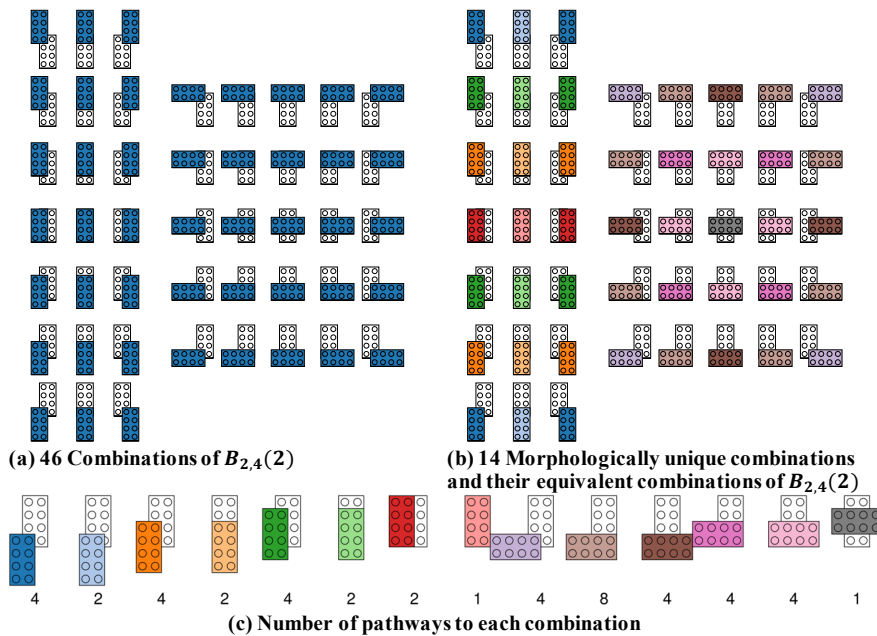**(c) Number of pathways to each combination**

Figure 4: The combinations of $B_{2,4}(2)$

The process also provides the number of pathways one can take to form these combinations. This is illustrated in Figure 4b where the bricks with the same colour have been detected as morphologically equivalent. Figure 4c shows that some combinations have 8 pathways, whilst others have only 1. Hence, some of the combinations are statistically more likely to be generated by a designer and this is being driven by the nature of the construction kit. This initial result provides an indication that a bias to particular solutions is present in the construction kits designers use for prototyping and evidences the need for this study.

Once the morphologically unique combinations have been determined. The process repeats for the next brick and continues through this loop until the combinations for $B_{(2,4)}(n)$ have been generated.

3. Examining the bias within $B_{(2,4)}(n)$ bricks

To explore the potential bias, the paper uses the process described in Section 2.2 to solve the combinatorial problem for $B_{(2,4)}(n)$ bricks. From this, metrics on the distribution of the number of pathways amongst the morphologically unique combinations can be generated and will provide an indication of the potential bias within the construction kit. In addition, an investigation into the potential role additional constraints have on changing the bias within a construction kit is also examined. This has been achieved by preventing the rotation of the bricks during construction.

3.1. Analysing the distribution of pathways to combinations

To investigate the potential bias within a construction kit, an analysis of the distribution of the number of pathways (P) to the number of morphologically unique combinations (M) can be calculated. To achieve this, one can look at the Mean $\left(\frac{P}{M}\right)$ and the Gini coefficient (G) that can be derived from the Lorenz curve of the distribution of pathways amongst the combinations of $B_{(2,4)}(n)$ (Figure 5).
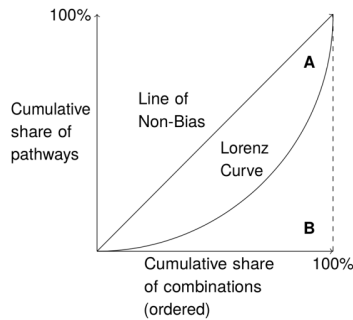
Figure 5: Lorenz curve of the distribution of pathways to combinations

Where the Gini coefficient is given by:

$$G = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j|}{2 \sum_{i=1}^{n} \sum_{j=1}^{n} x_j}$$

Where x_i is the wealth of person i, x_j is the wealth of person j and n is the number of people. In terms of the Lorenz curve in Figure 5, this is the ratio of the area under the Lorenz curve (**B**) and the area for a non-biased kit (**A+B**).

A Gini coefficient of 0 indicates that the number of pathways is equally distributed across the morphologically unique combination. This would indicate that no bias exists within the construction kit. In contrast, a highly biased construction kit is where one combination has all the pathways. For the case of analysing construction kits, additional adaptations to the Gini coefficient have to be made due to the following features:

One Pathway Minimum: All combinations must have at least one pathway otherwise the combination would not exist in the population.
Discrete Allocation Pathways: cannot be split beyond an integer level. Therefore, the remainder of the modulus has to be split as evenly as possible.

$$A_{biased} = \frac{qM^2 + r}{2}$$
$$A_{non-biased} = \frac{M^2 + (P - M)}{2}$$

Where M is the number of morphologically unique combinations, P is the number of pathways to the combinations, and q & r are the quotient and remainder of $\frac{P}{M}$ respectively. These values can then be used to normalise against the area (**B**) calculated for $B_{(2,4)}(n)$ to form the Gini coefficient.

$$G = 1 - \frac{B - A_{biased}}{A_{biased} - A_{non-biased}}$$

3.2. $B_{(2,4)}(n)$ bricks

To analyse the bias within $B_{(2,4)}(n)$, the process was deployed on the High-Performance Computing (HPC) facility at the University of Bath. The system contains 3,072 cores, 18TB main memory and 300TB storage, and has a peak performance of 57Tflops. The maximum job size reported in this paper was of 64 cores running for a period of 6 hours.

Table 2 presents the results for both the no additional constraint (NAC) and no rotation constraint (NRC) scenarios. Focusing on the NAC scenario, it can be seen that n has a large factor in the number of morphologically unique combinations and pathways one can create and use. Although this is a sensible conclusion as you're increasing the degrees of freedom within the construction kit, the rate at which it increases appears to be at an exponential rate. Looking at the mean, it can be seen that this is also increasing, which reveals that the number of pathways is increasing at a higher rate than the number of combinations. Taking the perspective of a designer, this would give the illusion that the number of potential solutions is much greater than there actually is. Looking at P_{min} and P_{max}, which are the minimum and maximum number of pathways to a combination within the set, it can be seen that P_{min} remains fairly consistent and low with some combinations having very few pathways to them. In comparison, P_{max} increases at a similar rate to the number of pathways and combinations, and shows that there are some combinations that are much more likely to be attained by the designer. This is further confirmed by G, which indicates there is indeed a bias within the LEGO™ construction kit and as n increases, G increases revealing that the bias builds within the kit.

Table 2: Number of pathways and morphologically equivalent combinations of $B_{2,4}(n)$

$B_{w,h}(n)$	B	M	$\frac{B}{M}$	P_{min}	P_{max}	$A_{non\text{-}biased}$	A_{biased}	$\mathbf{B}$	G
NAC									
$B_{2,4}(2)$	92	14	6.6	2	16	620	137	472	0.31
$B_{2,4}(3)$	12.3×10^3	429	28.7	4	104	2.6×10^6	98.0×10^3	1.6×10^6	0.39
$B_{2,4}(4)$	2.2×10^6	33.3×10^3	66.6	4	832	36.9×10^9	557.2×10^6	21.4×10^9	0.42
$B_{2,4}(5)$	497.8×10^6	3.0×10^6	166.2	4	14.9×10^3	744.8×10^{12}	4.5×10^{12}	370.8×10^{12}	0.51
NRC									
$B_{2,4}(2)$	42	8	5.3	2	8	162.0	49.0	131.0	0.27
$B_{2,4}(3)$	2.5×10^3	139	17.9	4	32	172.5×10^3	10.8×10^3	126.0×10^3	0.29
$B_{2,4}(4)$	198.5×10^3	4.6×10^3	43.5	4	448	449.7×10^6	10.5×10^6	300.6×10^6	0.34
$B_{2,4}(5)$	19.7×10^6	193.4×10^3	101.6	4	2.0×10^3	1.9×10^{12}	18.7×10^9	1.1×10^{12}	0.42

The addition of the no rotation constraint reveals a reduction in the number of pathways and morphological unique combinations for equivalent n bricks. However, the rate of increase does climb considerably and follows a similar trajectory to the NAC case (Figure 6a). Looking at the Gini coefficient, the addition of the fixed rotation constraint does increase the equality of the system for equivalent n bricks but the increase in bias as n increases is still observed. Figure 6b highlights that the addition of the no rotation constraint provides a translation to the Gini coefficient line.
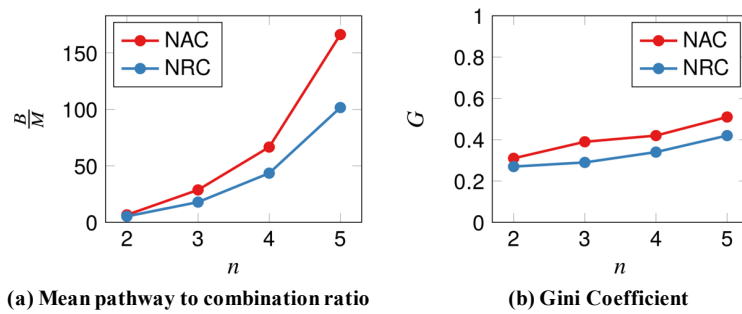

(a) Mean pathway to combination ratio

(b) Gini Coefficient

Figure 6: Characteristics of bias within $B_{2,4}(n)$ construction kits

4. Discussion and future work

These initial results do reveal that biases exist in the LEGO™ construction kit that has been used in many design scenarios around the world (Ringwood, Monaghan, and Maloco 2005; Bethke Wendell and Rogers 2013; Brereton and McGarry 2000). It has been observed that the bias increases as the number of bricks increases. It is also interesting to see the exponential rate at which the design space increases as bricks are added to the combination. Further, the discovery of the number of pathways increases at a higher rate than the number of combinations suggests that there may be a perception of more potential solutions than actual solutions. This 'perceived design freedom' within a construction kit may have an impact on the cognitive loading on a designer and their ability to conceptualise potential designs before they realise them with the kit.

This poses further questions with respect to the role that construction kits have had on the design of products and what would have been if kits featured different biases. For example, does bias:

- effect design fixation?
- change how much of the design space is explored?
- lead to different solutions?

And, how much of an effect has popular construction kits had on shaping designs across society and the way society construct and solve problems? These results show that further research is required in the analysis of construction kits to fully answer these questions.

In addition, there are the human factors involved in using these construction kits. This is an area that the author is actively working upon. It would be interesting to see whether the distribution of the final designs from designers and/or individuals across society correlates with the statistical distribution generated by the combinatorial algorithm. An initial hypothesis is that individuals may be more inclined to use two or more studs when constraining a brick as one stud provides rotational freedom to a brick. One can also go further to analyse aspects of co-creation around construction kits and begin to unravel how it supports and/or hinders design space exploration.

To further support these studies, research is required on the metrology of construction kits in order to determine a bricks position with respect to the other bricks in the combination. Achieving this would enable researchers to capture the sequence of operations a designer performs to construct a design. This information could then be directly correlated with the outputs from the combinatorial algorithm. Further, the combinatorial algorithm needs to be developed further to account for combinations that may be physically possible but would be impossible for an individual to build due to the occlusion of the existing bricks in the structure. Through this, a theory could be developed, which would enable the design of construction kits that can further support future design activities.

5. Conclusion

Prototyping is a fundamental activity across design with many descriptive studies investigating its ability to support design. One of the most common types of tool used during prototyping sessions are construction kits, such as LEGO™. These kits have a number of bricks that designers are able to construct designs from. As long as the constraints are met, a valid design has been achieved. Descriptive studies have concluded that construction kits have been instrumental in enabling wider stakeholder engagement and enable designers to focus on particular elements of the design problem.

However, it is posited that how the problem is represented by the construction kit has the potential to introduce an unknown bias where there is statistically more chance for certain solutions to be generated than others. This has been proven by this paper through its scoping study into the construction combinations of n, 2 wide, 4 high LEGO™ bricks. Through solving the combinatorial problem, the paper has revealed that biases do exist within $B_{(2,4)}(n)$ construction kits. The addition of constraints reduces the design space and decreases the bias for the same n, however the increase in bias still remains as n increases. Even-though a bias is present, this may be hidden from a designer due to the exponential growth rate of potential combinations as one adds more bricks to the system. Thus, further studies are required on the human factors involved in constructing a combination using a kit to evaluate this.

These findings also pose further questions into the bias of other construction kits and how one might want to control bias within a construction kit for given design scenarios. In addition, as these kits are used by infants to develop problem solving skills, these results reveal that society has been continually designing with systems that contain bias towards particular solutions. If one were to build a new construction kit that altered this bias, what societal changes and/or effects would it have on the development of problem solving cognitive abilities? These questions are now able to be explored with computing power made available through HPC facilities.

6. Acknowledgements

7. References

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