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Scalar-top masses from SUSY loops with 125 GeV m_h and precise M_W, m_t

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ABSTRACT

We constrain the masses of scalar-tops (stop) by analyzing the new precision Tevatron measurement of the W-boson mass and the LHC/Tevatron indications of a Higgs boson of mass 125.5 ± 1 GeV. Our study adopts Natural SUSY with low fine-tuning, which has multi-TeV first- and second-generation squarks and a light Higgsino mixing parameter $\mu = 150$ GeV. An effective Lagrangian calculation is made of m_h to 3 loops using the H3m program with weak scale SUSY parameters obtained from RGE evolution from the GUT scale in the Natural SUSY scenario. The SUSY radiative corrections to the Higgs mass imply maximal off-diagonal elements of the stop mass matrix and a mass splitting of the two stops larger than 400 GeV.

Supersymmetry (SUSY) is a theoretically attractive extension of the Standard Model (SM) that may explain the hierarchy of the weak scale and the Planck scale. Of the SUSY particles, the lighter scalar-top squark may have a sub-TeV mass and be detectable by LHC experiments. Existence of a light top squark is particularly suggested by the Natural SUSY model [1–21], that has less fine-tuning. The first- and second-generation squarks have multi-TeV masses to mitigate unwanted flavor changing neutral currents (FCNC) and large CP violation. For a third-generation scalar GUT-scale mass $m_0(3) < 1$ TeV, $m_{\tilde{t}_1}$ is less than 400 GeV from the running of the RGE equations [17].

A light top squark can give a significant radiative contribution to the W-boson mass. The precision of M_W has been improved by recent Tevatron measurements; $M_W = 80{,}387 \pm 12(\text{stat.}) \pm 15(\text{syst.})$ MeV by the CDF Collaboration [22] and $M_W = 80{,}367 \pm 13(\text{stat.}) \pm 22(\text{syst.})$ MeV by the D0 Collaboration [23]. Including these measurements, the world average M_W is shifted downward from [24] $M_W^{\text{exp}} = 80{,}399 \pm 26$ MeV to $80{,}385 \pm 15$ MeV. The SM prediction [25,26] of M_W at 2-loop order is

$$M_W^{\text{SM}} = 80{,}361 \pm 7 \text{ MeV} \tag{1}$$

where we have used the numerical formula of Ref. [27] with central values of parameters [28]. The uncertainties of the SM prediction of M_W resulting from the uncertainties of these input parameters are summarized in Table 1.

Table 1
Uncertainty of the SM M_W prediction from the uncertainties of the parameters. Beside these errors, there is another uncertainty due to missing higher order corrections, which is estimated as about 4 MeV [27].

	δM_W
$\delta m_h = 1.0$ GeV	-0.5 MeV
$\delta m_t = 1$ GeV	6.0 MeV
$\delta M_Z = 2.1$ MeV	2.6 MeV
$\delta(\Delta\alpha_{\text{had}}^{(5)}) = 0.6 \times 10^{-4}$	-1.1 MeV
$\delta\alpha_s(M_Z) = 0.0007$	-0.4 MeV

The LHC experiments have reported indications of a Higgs boson at mass $125.3 \pm 0.4_{\text{stat}} \pm 0.5_{\text{syst}}$ GeV in CMS data [29] and at $126.0 \pm 0.4_{\text{stat}} \pm 0.4_{\text{syst}}$ GeV in ATLAS data [30]. Accordingly, we assume a Higgs boson mass of 125.5 ± 1 GeV in our study. Then, the difference of the experimental and SM values of M_W is

$$M_W^{\text{exp}} - M_W^{\text{SM}} = 24 \pm 15 \text{ MeV}. \tag{2}$$

As can be seen in Table 1, the largest source uncertainty in M_W^{SM} (of 6.0 MeV) is from the uncertainty $\delta m_t = 1$ GeV in the top mass measurement. It is significantly smaller than the experimental uncertainty in M_W^{exp} (of 15 MeV), given in Eq. (2).

The contributions of SUSY particles to the 1-loop calculation of M_W [31] along with the W self-energy at the 2-loop level [32] can account for the 1.6σ deviation of the experimental value from the SM prediction [31]. Conversely, the M_W measurement gives a constraint on the squark masses of the third generation, $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, and $m_{\tilde{b}_L}$. We assume no mixing in sbottom sector since that

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off-diagonal element is proportional to m_b; $m_{\tilde{b}_R}$ is irrelevant to δM_W.

The dominant SUSY radiative corrections to m_h are due to loops of $\tilde{t}_1$ and $\tilde{t}_2$. Implications of a 125 GeV Higgs boson for supersymmetric models are investigated in Ref. [33]. If m_h is confirmed with the value of the present Higgs boson signal ~ 125.5 GeV, the values of $m_{\tilde{t}_1}, m_{\tilde{t}_2}$ and the top-squark mixing angle $\theta_{\tilde{t}}$ can be constrained from the measured m_h. We investigate how a Higgs mass $m_h = 125.5 \pm 1.0$ GeV and the new experimental value of M_W constrain the third-generation SUSY scalar-top masses.

1. Constraint from M_W

The M_W prediction is obtained by calculating the muon lifetime [25,26,31]. The SUSY correction Δr to the Fermi constant G_μ is

$$\frac{G_\mu}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1 + \Delta r) \tag{3}$$

where $s_W = \sin\theta_W$ and θ_W is weak mixing angle which is defined by the experimental values of W/Z pole mass $M_{W/Z}$ as

$$c_W^2 \equiv \cos^2\theta_W = \frac{M_W^2}{M_Z^2}. \tag{4}$$

Δr is calculated [31] in the MSSM, and the corresponding M_W prediction is obtained by iterative solution of the equation

$$M_W^2 = M_Z^2 \times \left\{ \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2} G_\mu M_Z^2}\left[1 + \Delta r(M_W, M_Z, m_t, \ldots)\right]} \right\}. \tag{5}$$

Then, the correction to M_W^2 at 1-loop level is

$$\delta M_W^2 = -M_Z^2 \frac{c_W^2 s_W^2}{c_W^2 - s_W^2} \Delta r. \tag{6}$$

Δr is given by [25,26]

$$\Delta r = \frac{c_W^2}{s_W^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) + \Delta\alpha + (\Delta r)_{\text{rem.}}. \tag{7}$$

The first term on the left-hand side is the on-shell self-energy correction to gauge boson masses; $\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} = -\frac{\Sigma^Z(M_Z^2)}{M_Z^2} + \frac{\Sigma^W(M_W^2)}{M_W^2}$. $\Delta\alpha$ is the radiative correction to the fine structure constant α. The remainder term $(\Delta r)_{\text{rem.}}$ includes vertex corrections and box diagrams at 1-loop level which give subleading contributions compared with the first term of Eq. (7) [31].

The main contribution to δM_W is the on-shell gauge boson self-energy, which is well approximated [32,34] with its value at zero momenta as

$$\Delta r \simeq -\frac{c_W^2}{s_W^2}\left(\frac{\Sigma^Z(0)}{M_Z^2} - \frac{\Sigma^W(0)}{M_W^2}\right) = -\frac{c_W^2}{s_W^2}\Delta\rho \tag{8}$$

where $\Delta\rho$ is the deviation of the ρ parameter due to new physics in the EW precision measurements. It is related to the T parameter [35] by

$$\Delta\rho \simeq \alpha(M_Z) T. \tag{9}$$

The squark, slepton, and neutralino/chargino loops contribute to $\Delta\rho$ at 1-loop level, which we denote as $\Delta\rho_0$. The neutralino/chargino contributions are small [36], and the slepton contributions are suppressed relative to squark contributions by color, and thus the squark contributions are dominant. It is well known [37] that the weak $SU(2)_L$ isospin violation from SUSY doublet masses gives non-zero contributions to δM_W. The scalar-top sector is expected to have a large L–R mixing since the off-diagonal elements of the top-squark mass matrix are proportional to m_t. Finally, δM_W is given by [32,34]

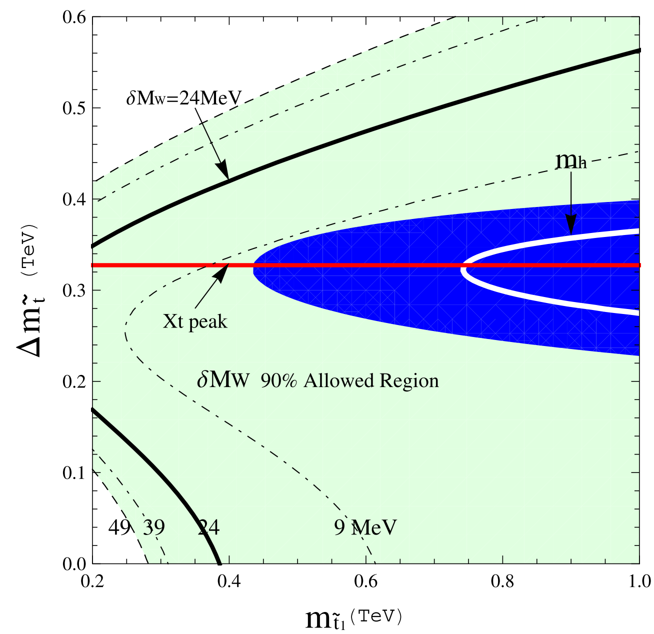

Fig. 1. Allowed regions in the $(m_{\tilde{t}_1}, \Delta m_{\tilde{t}})$ plane for $\theta_{\tilde{t}} = \frac{\pi}{4}$; $\Delta m_{\tilde{t}} = (m_{\tilde{t}_2} - m_{\tilde{t}_1})$. Black (red) solid lines are $\delta M_W = 24$ MeV (maximum m_h with $X_{t\,\text{peak}} = -\sqrt{6} M_{\text{susy}}$). The blue (dark-shaded) region is $m_h = 123.5$ to 127.5 GeV and the white line represents its central value $m_h = 125.5$ GeV. The green (medium-shaded) region is allowed by δM_W at 90% CL, and the dot-dashed lines represent its 1σ deviation, $\delta M_W = 24 \pm 15$ MeV.

$$\delta M_W \simeq \frac{M_W}{2}\frac{c_W^2}{c_W^2 - s_W^2}\Delta\rho_0,$$

$$\Delta\rho_0 = \frac{3 G_F}{8\sqrt{2}\pi^2}\left[-s_{\tilde{t}}^2 c_{\tilde{t}}^2 F_0\left(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2\right) + c_{\tilde{t}}^2 F_0\left(m_{\tilde{t}_1}^2, m_{\tilde{b}_L}^2\right) + s_{\tilde{t}}^2 F_0\left(m_{\tilde{t}_2}^2, m_{\tilde{b}_L}^2\right)\right] \tag{10}$$

where $F_0(a,b) \equiv a + b - \frac{2ab}{a-b}\ln\frac{a}{b}$, $s_{\tilde{t}} = \sin\theta_{\tilde{t}}$, $c_{\tilde{t}} = \cos\theta_{\tilde{t}}$, and $\theta_{\tilde{t}}$ is the top-squark mixing angle. The 2-loop gluon/gluino exchange effects, $\Delta\rho^{\text{SUSY}}_{1,\text{gluon/gluino}}$, are neglected since they are subleading compared with the 1-loop $\Delta\rho$ for $M_{\text{susy}} \gtrsim 300$ GeV [34]. The prediction of M_W in SUSY is then $M_W = M_W^{\text{SM}} + \delta M_W$. From Eq. (10) the δM_W of Eq. (2) corresponds to

$$\Delta\rho = (4.2 \pm 2.7) \times 10^{-4}, \qquad T = 0.054 \pm 0.034. \tag{11}$$

The uncertainty is substantially reduced from that of the previous global electroweak precision analyses: $\Delta\rho = (3.67 \pm 8.82) \times 10^{-4}$ [38], $T = 0.03 \pm 0.11$ [39].

By using Eq. (10) with (2), we can determine the allowed region in the $m_{\tilde{t}_1}, \Delta m_{\tilde{t}}$ plane for a given value of $\theta_{\tilde{t}}$. Here $\Delta m_{\tilde{t}} = (m_{\tilde{t}_2} - m_{\tilde{t}_1})$. The case $\theta_{\tilde{t}} = \frac{\pi}{4}$ is shown in Fig. 1. Note that X_t and $\theta_{\tilde{t}}$ are independent because the soft-SUSY parameters in the diagonal elements are different.

We also note that $m_{\tilde{b}_L}$ in Eq. (10) is given by $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, and $\theta_{\tilde{t}}$

$$m_{\tilde{b}_L}^2 = m_{\tilde{t}_1}^2 \cos^2\theta_{\tilde{t}} + m_{\tilde{t}_2}^2 \sin^2\theta_{\tilde{t}} - m_t^2 + m_b^2 - M_W^2 \cos 2\beta. \tag{12}$$

Eq. (12) is symmetric under the exchange

$$m_{\tilde{t}_1} \leftrightarrow m_{\tilde{t}_2}, \qquad c_{\tilde{t}} \leftrightarrow s_{\tilde{t}}, \quad \text{i.e.}\ \theta_{\tilde{t}} \to \pi/2 - \theta_{\tilde{t}}. \tag{13}$$

2. Constraint from m_{h^0}

The mass of the Higgs boson in the MSSM receives substantial radiative corrections to the tree level result. The scalar-top sector gives the dominant contribution, for which $\Delta m_h^2 \propto m_t^4/v^2$. Tremendous efforts [40–71] have been expended to calculate m_h with sufficient accuracy to compare with LHC measurements, and the Higgs mass has been calculated through the 3-loop level, $\alpha_t\alpha_s^2$, for the leading $(m_t)^4$ corrections [64,65] and partially at 4-loop level [59]. The dominant contributions arise from supersymmetric loops involving the top squarks, along with gluon and gluino exchanges.

There are several different approaches that have been used in the theoretical evaluation of m_h: perturbative calculation of the Higgs self-energy diagrams to (i) 2-loop and (ii) 3-loop orders, (iii) effective field theory (EFT) methods based on second derivatives of an effective Higgs potential, (iv) effective potential method based on RGE evolution from the GUT scale, and (v) the effective Lagrangian method. We succinctly summarize the five methodologies:

(i) The FeynHiggs package [58] calculates m_h diagrammatically in 2-loop order in the on-shell (OS) renormalization scheme.

(ii) A MATHEMATICA program, H3m [65], does the 3-loop calculation; it is interfaced with the 2-loop FeynHiggs program for m_h predictions. A numerical 3-loop accuracy on m_h has been estimated to be < 1 GeV. However, its expansion in mass-squared ratios does not apply in some parameter regions relevant to Natural SUSY.

(iii) In the EFT 2-loop leading-log approximation [48,43,67], m_h^2 is calculated in the limit of stop matrix elements $M_L = M_R$ [67–71], and $M_L \gg M_R$ [50].

The m_h^2 formula in the general case with $M_L \neq M_R$, is given in the large m_A limit by [67]

$$\begin{aligned} m^2_{h,\mathrm{EFT2}}&(m_{\tilde{t}_1}, m_{\tilde{t}_2}, x_t) \\ &= M_Z^2 c_{2\beta}^2 + \frac{3\bar{m}_t^4}{2\pi^2 v^2}\left[\frac{1}{2}\tilde{X}_t + t + \frac{1}{16\pi^2}\left(\frac{3\bar{m}_t^2}{v^2} - 32\pi\alpha_s(\bar{m}_t)\right)\right. \\ &\quad \times \left.\left(\tilde{X}_t t_{\max} + \frac{t_{\max}^2 + t_{\min}^2}{2} + (2t - t_{\max} - t_{\min})t_{\max}\right)\right], \\ t &\equiv \ln\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{\bar{m}_t^2}, \quad t_{\max} \equiv \ln\frac{M_{\max}^2}{\bar{m}_t^2}, \quad t_{\min} \equiv \ln\frac{M_{\min}^2}{\bar{m}_t^2}, \end{aligned} \tag{14}$$

where $v \equiv 1/\sqrt{\sqrt{2}G_F} \simeq 246$ GeV and the contribution from the sbottom sector can be omitted so long as $\tan\beta$ is not close to its upper bound of ~ 60.

In the above equation, $\tilde{X}_t$ is related with the stop-mixing parameter $X_t = A_t - \mu\cot\beta$ by

$$\tilde{X}_t \equiv 2|X_t|^2 \frac{\ln(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} + |X_t|^4 \frac{2 - \frac{m_{\tilde{t}_2}^2 + m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}\ln(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2}. \tag{15}$$

In Eqs. (14) and (15) the X_t is a quantity regularized with the renormalization scale $\mu = M_{\mathrm{susy}}$ in the $\overline{\mathrm{MS}}$ scheme, while the running top-quark mass $\bar{m}_t$ is evaluated at $\mu = \bar{m}_t$ itself in the $\overline{\mathrm{MS}}$ scheme. $\bar{m}_t(\mu)$ was calculated in $\overline{\mathrm{DR}}$ scheme by Ref. [72] and in $O(\alpha_s^4)$ [73,74]. Its value in the $\overline{\mathrm{MS}}$ scheme is $\bar{m}_t = 163.71 \pm 0.95$ GeV [39] which corresponds to the on-shell top-quark mass $M_t = 173.4 \pm 1.0$ GeV.

The $\tilde{X}_t$ in Eq. (15) is well approximated as

$$\tilde{X}_t = 2x_t^2 - \frac{x_t^4}{6}, \quad x_t \equiv \frac{X_t}{M_{\mathrm{susy}}} \tag{16}$$

with the choice of SUSY-breaking scale

$$M_{\mathrm{susy}} = \frac{m_{\tilde{t}_1} + m_{\tilde{t}_2}}{2}. \tag{17}$$

The $m^2_{h,\mathrm{EFT2}}$ of Eq. (14) has its maximum at $|x_t| = |(x_t)_{\max}| = \sqrt{6}$ or $|X_t| = |(X_t)_{\max}| = \sqrt{6}M_{\mathrm{susy}}$, for which $\tilde{X}_t = 6$. It is also a common feature of the analytic EFT formula at 1- and 2-loop levels [67–70]. A region $|X_t| \gtrsim \sqrt{6}M_{\mathrm{susy}}$ is theoretically not allowed from considerations of false vacuum of charge and color symmetry breaking [75–78].

$M_{\max,\min}$ are related to the stop squared-mass matrix $M_{\tilde{t}}^2$ in on-shell (OS) renormalization scheme as

$$\begin{aligned} M_{\tilde{t}}^2 &\equiv \begin{pmatrix} M_L^2 & M_t X_t^{\mathrm{OS}} \\ M_t X_t^{\mathrm{OS}} & M_R^2 \end{pmatrix} \\ &= \begin{bmatrix} m_{\tilde{t}_1}^2 c_{\tilde{t}}^2 + m_{\tilde{t}_2}^2 s_{\tilde{t}}^2 & -(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)c_{\tilde{t}}s_{\tilde{t}} \\ -(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)c_{\tilde{t}}s_{\tilde{t}} & m_{\tilde{t}_1}^2 s_{\tilde{t}}^2 + m_{\tilde{t}_2}^2 c_{\tilde{t}}^2 \end{bmatrix} \end{aligned} \tag{18}$$

$$\begin{aligned} \left(M^{\mathrm{OS}}\right)^2_{\max,\min} &\equiv \max,\min\{M_L^2, M_R^2\} = \frac{m_{\tilde{t}_2}^2 + m_{\tilde{t}_1}^2}{2} \\ &\quad \pm \sqrt{\left(\frac{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}{2}\right)^2 - \left(M_t X_t^{\mathrm{OS}}\right)^2}. \end{aligned} \tag{19}$$

Our sign convention of X_t agrees with that used in Ref. [70]. X_t^{OS} is the on-shell stop mass matrix parameter. The relation between $M_{\mathrm{susy}}^{\mathrm{OS}}$ and X_t^{OS} in OS scheme and those in $\overline{\mathrm{MS}}$ scheme are given in [70], see also [57]. Here we treat $M_{\max,\min}^{\mathrm{OS}}$ as being equal to $M_{\max,\min}$ in Eq. (14) since the difference is small (less than 4%) for $M_{\mathrm{susy}} > 1$ TeV.

In Eq. (17), the r.h.s. is given by the on-shell stop masses and thus, more precisely Eq. (17) is $M_{\mathrm{susy}}^{\mathrm{OS}}$. Here we regard $M_{\mathrm{susy}}^{\mathrm{OS}}$ as being equal to M_{susy} in $\overline{\mathrm{MS}}$ scheme since the difference is small.

On the other hand, X_t affects a relatively large difference between $\overline{\mathrm{DR}}$ and OS schemes. Numerically, we define the ratio

$$\kappa = (X_t)_{\max}/\left(X_t^{\mathrm{OS}}\right)_{\max} \tag{20}$$

which is about 1.2 from the formula relating $\overline{\mathrm{MS}}$ and OS schemes given[1] in Carena et al. [70]. Coincidentally, $\kappa \approx \sqrt{6}/2.0$. We choose this form because the factor $\sqrt{6}$ matches the x_t value in the $\overline{\mathrm{MS}}$ scheme giving maximum $\tilde{X}_t$ of Eq. (16) which leads to maximum $m^2_{h,\mathrm{EFT2}}$ of Eq. (14). The 2.0 in the denominator is given as a numerical value of the ratio $(X_t^{\mathrm{OS}})_{\max}/M_{\mathrm{susy}}$ in Ref. [70]. We have also checked the ratio (20) by using ISAJET 7.83 [80]: ISAJET adopts the $\overline{\mathrm{DR}}$ scheme and $\overline{\mathrm{DR}} \simeq \overline{\mathrm{MS}}$ and converts to OS stop masses using [57]. ISAJET outputs of $X_t^{\overline{\mathrm{DR}}}$ and on-shell stop masses are numerically consistent with the relation $(X_t^{\overline{\mathrm{DR}}})_{\max}/(X_t^{\mathrm{OS}})_{\max} = \sqrt{6}/2.0$. (See, also, the caption of Fig. 4.) We apply this relation (20) in the region close to "maximal mixing", $|X_t|/M_{\mathrm{susy}} \sim \sqrt{6}$:

$$X_t = \kappa X_t^{\mathrm{OS}}, \qquad \kappa = \sqrt{6}/2.0. \tag{21}$$

The EFT method is not gauge-fixing invariant [59]. Nonetheless, it is found to give a good approximation when compared to

[1] $X_t^{\overline{\mathrm{MS}}} = X_t^{\mathrm{OS}} + \frac{\alpha_s}{3\pi}M_{\mathrm{susy}}\left[8 - \frac{X_t^2}{M_{\mathrm{susy}}^2} + \frac{4X_t}{M_{\mathrm{susy}}} + \frac{3X_t}{M_{\mathrm{susy}}}\ln\frac{M_{\mathrm{susy}}^2}{\bar{m}_t^2}\right]$ in 1-loop level [70] where the renormalization prescription is not specified in $O(\alpha_s)$ term.

other methods. The formula (14) with (15) give larger m_h values by about 1 GeV than the results of H3m with the inputs of the Natural SUSY benchmark points, as will be commented on below.

The m_h^2 formula obtained from the 2-loop diagrammatic approach (i) can be matched to the EFT formula above by adjusting the renormalization prescription [70], except for additional non-logarithmic terms in the diagrammatic formula that give asymmetric heights of the peak m_h at $X_t > 0$ and $X_t < 0$. The latter contributions arise from SUSY threshold effects that are not taken into account in the RGE running down from the SUSY-breaking scale that includes logarithms of $M_{\rm susy}/\bar{m}_t$.

(iv) In the unification approach, RGEs are evolved from the GUT coupling unification scale [72], where the first- and second-generation scalars in Natural SUSY have an $m_0 \sim 10$ TeV mass and the third-generation scalars have $m_0 \sim 1$ TeV [17,79]. The Higgs potential at the SUSY-breaking scale $M_{\rm susy}$ is based on 1-loop MSSM radiative corrections that are RGE improved. With the choice of $M_{\rm susy} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, the most important 2-loop effects [66] are included in the effective potential. The RGE evolution is implemented with the ISASUSY package [80,81], with a scan over GUT-scale parameters.

(v) In the effective Lagrangian approach, the gauge couplings, the Yukawa couplings, and the soft-SUSY terms are also RGE evolved to the weak scale from high scale boundary values, where the gauge couplings unify. The ISASUSY program for this RGE evolution incorporates SUSY threshold effects [80,81]. The weak scale parameters so obtained are taken as input to the diagrammatic calculation at 2-loop order by the FeynHiggs [58] or 3-loop order by the H3m [65]. It has been argued [59] that this method may provide the most accurate evaluation of the leading and next-to-leading contributions to m_h in 3-loop order in the approximation of large QCD and top-quark Yukawa couplings.

We adopt the latter approach in the framework of Natural SUSY using ISASUSY [80,81], with a scan over GUT-scale input parameters. We have also converted the sign convention of X_t in ISASUSY in order to match ours. We then evaluate m_h using the H3m program with the ISASUSY input for the SUSY parameters at the weak scale. Specifically, we adopt the benchmark line NS3 of Ref. [17] that has a Higgsino mass term $\mu = 150$ GeV and other Natural SUSY benchmark points RNS1 and RNS2 of Ref. [18].[2] The NS3 gives $m_h = 123.5$ GeV that is consistent with the LHC experimental value. There is a strong preference for $A_t(M_{\rm susy}) > 0$ and $\tan\beta > 10$ in Natural SUSY [17]. Since μ is small in Natural SUSY, X_t is approximately A_t for $A_t \sim$ TeV. We should note that variations of the masses of the first and second generations and gauginos from the NS3 inputs have little effect on m_h since they are heavy in Natural SUSY scenario.

The m_h effective Lagrangian result with the NS3 input parameters can be numerically represented by the formula

$$m_h^2 = m_{h,B}^2(x_t) \equiv M_Z^2 c_{2\beta_B}^2 + \frac{3\bar{m}_t^4}{2\pi^2 v^2}\left[c_0 + (c_1 + c_2 x_t)\tilde{X}_t\right],$$
$$\tilde{X}_t \equiv 2x_t^2\left(1 - \frac{x_t^2}{12}\right), \quad x_t \equiv \frac{X_t}{M_{\rm susy,B}} \qquad (22)$$

where the subscript B means the NS3 benchmark point: $c_{2\beta_B} = \cos 2\beta_B$ is calculated from $\tan\beta_B = 19.4$. $M_{\rm susy,B}$ is the SUSY-breaking scale corresponding to $(m_{\tilde{t}_1,B}, m_{\tilde{t}_2,B}) = (812.5, 1623.2)$ GeV; $M_{\rm susy,B} = (812.5 + 1623.2)/2 = 1212.9$ GeV. The coefficients

$$(c_0, c_1, c_2) = (2.661, 0.2874, 0.01717)$$

have been determined by a least-squares fit with some weighting of the maximal m_h region.

[2] The SOFTSUSY [82], SPheno [83,84] and SuSpect [85] codes use the same algorithm as ISAJET [80,81] and employ similar threshold transitions matching the MSSM to the SM. The four codes produce mass spectrum in the mSUGRA model that are in close agreement. The ISAJET [80,81] code provides the NUHM2 model of our interest.

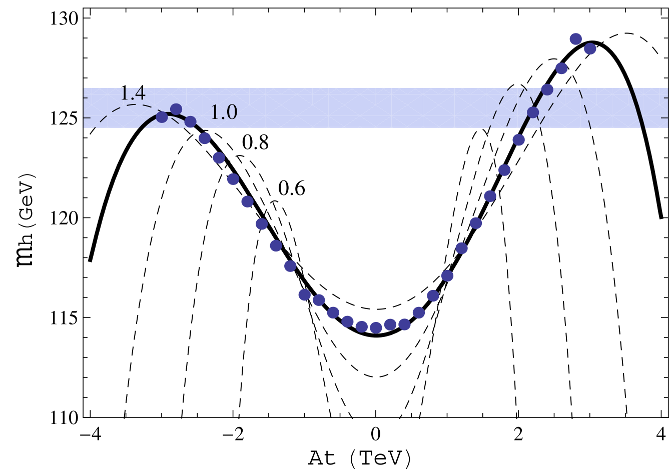

Fig. 2. $A_t(M_{\rm susy})$ dependence of m_h in 3-loop calculation by H3m with the effective Lagrangian method. (Solid circles) The input parameters are a Natural SUSY benchmark line (NS3): $(m_{\tilde{t}_1,B}, m_{\tilde{t}_2,B}) = (812.5, 1623.2)$ GeV which corresponds to $M_{\rm susy} = 1212.9$ GeV. It is obtained by varying the third-generation scalar mass m_0 [17] at the unification scale: The solid line is the formula, Eq. (22), that is designed to numerically reproduce the effective Lagrangian result. The dashed lines are obtained from the formula (23) with inputs $(m_{\tilde{t}_1}, m_{\tilde{t}_2}) = (m_{\tilde{t}_1,B} + \delta m, m_{\tilde{t}_2,B} + \delta m)$ with various δm values corresponding to $M_{\rm susy}(= \frac{m_{\tilde{t}_1}+m_{\tilde{t}_2}}{2}) = 0.6, 0.8, 1.0, 1.4$ TeV. $m_h = 125.5 \pm 1$ GeV is shown by blue band. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

We use $m_{h,B}$ of Eq. (22) as our benchmark at a given value of x_t. m_h values with different $m_{\tilde{t}_{1,2}}$ and $M_{\rm susy} = \frac{m_{\tilde{t}_1}+m_{\tilde{t}_2}}{2}$ inputs are considered to be given with sufficient accuracy by shifting from $m_{h,B}$ with a common value of x_t through 2-loop analytic formula (14).

$$m_h^2(m_{\tilde{t}_1}, m_{\tilde{t}_2}, x_t, \tan\beta)$$
$$= m_{h,B}^2(x_t) + \left[m_{h,\rm EFT2}^2(m_{\tilde{t}_1}, m_{\tilde{t}_2}, x_t, \tan\beta) - m_{h,\rm EFT2}^2(m_{\tilde{t}_1,B}, m_{\tilde{t}_2,B}, x_t, \tan\beta_B)\right]. \qquad (23)$$

In order to estimate the intrinsic uncertainty, we also consider the other Natural SUSY benchmark points, RNS1 and RNS2 [17], where m_h is estimated by using ISAJET 7.83.

	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$M_{\rm susy}$	$A_t^{\rm OS}$	$\tan\beta$	m_h (ISAJET)	m_h (Eq. (23))
RNS1	1416	3425	2420	3764	10	123.7	124.1
RNS2	1843	4921	3382	5054	8.55	125.0	123.4

(24)

Here the masses and the $A_t^{\rm OS}$ are given in units of GeV. The predictions from Eq. (23) are given in the final column. Our formula (23) is made by using a special input of NS3 benchmark point with $M_{\rm susy} \simeq 1.2$ TeV, but it can be applied to wide range of cases with fairly good accuracy. The theoretical error of Eq. (23) is conservatively considered to be 2 GeV in whole range of parameters in Natural SUSY scenario.

In order to see the $M_{\rm susy}$ dependence of m_h, we shift the $m_{\tilde{t}_{1,2}}$ from the NS3 benchmark values commonly with δm. The results are shown by dashed lines in Fig. 2, which suggests the necessity of the maximal mixing condition when $X_t \simeq \sqrt{6}M_{\rm susy}$ [86,87]. The

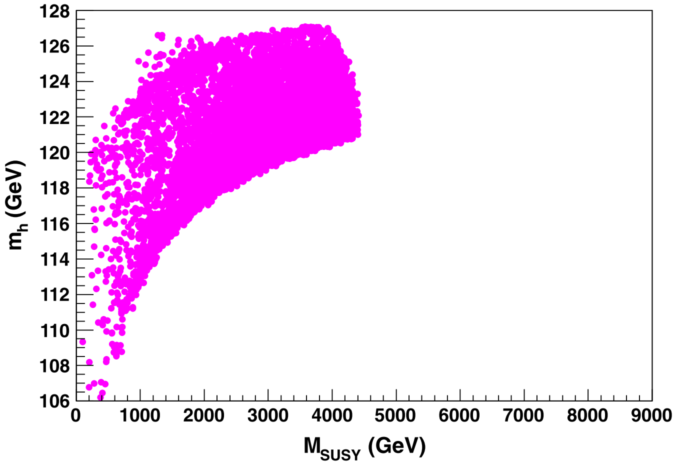

Fig. 3. $M_{\rm susy}$ dependence of m_h in Natural SUSY points following Ref. [17]. The points are obtained from a scan over GUT-scale parameters: the common scalar mass of the first two generations $m_0(1,2)$: 5–50 TeV, the third-generation squark mass $m_0(3)$: 0–5 TeV, the common gaugino mass $m_{1/2}$: 0–5 TeV, $-4 < A_t/m_0(3) < 4$, m_A: 0.15–2 TeV, $\tan\beta$: 1–60. See Ref. [17].

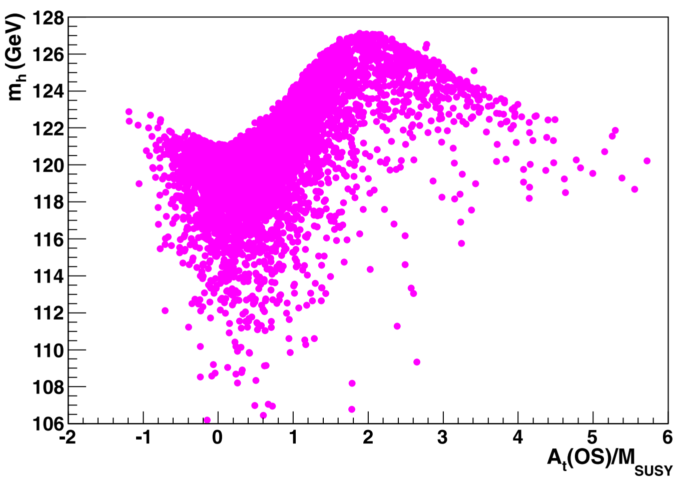

Fig. 4. $A_t(M_{\rm susy})/M_{\rm susy}$ dependence of m_h in Natural SUSY scan points. $X_t = -A_t - \mu\cot\beta \simeq -A_t$ since μ is small, 150 GeV. The maximum of m_h is not obtained at $A_t(M_{\rm susy})/M_{\rm susy} = -\sqrt{6}$ but at about -2, which is due to the difference of renormalization prescription of ISASUSY program, on-shell (OS) renormalization, and the EFT approach using the $\overline{\rm MS}$ scheme. See Ref. [67].

peak value of m_h gradually increases with $\sim \ln M_{\rm susy}$. The Higgs mass constraint $m_h > 124.5$ GeV requires a SUSY-breaking scale $M_{\rm susy} \gtrsim 0.6$ TeV.

The $M_{\rm susy}$ dependence of m_h in Natural SUSY points following Ref. [17] is shown in Fig. 3. The points indicate a $\ln M_{\rm susy}$ dependence, and in order to explain $m_h > 124.5$ GeV, it is indeed plausible that $M_{\rm susy} > 1$ TeV.

The maximal mixing condition $|X_t^{\rm OS}| \simeq 2M_s$, which corresponds to $|X_t| \simeq \sqrt{6}M_s$ in the $\overline{\rm DR}$ or $\overline{\rm MS}$ scheme, can be obtained [88,75] by RGE running from the SUSY-GUT scale, as illustrated for Natural SUSY in Fig. 4; note that $A_t < 0$ is almost absent. The generated points are mainly in the region $0 < A_t < 2$; however, although improbable from the scan, the maximal mixing $X_t = \sqrt{6}M_{\rm susy}$ is possible in Natural SUSY.

By taking $m_h = 125.5 \pm 2.$ GeV as a constraint to Eq. (23), we can determine the allowed region in $(m_{\tilde t_1}, m_{\tilde t_2})$ plane for a given value of $\theta_{\tilde t}$. Here we allow a somewhat large uncertainty of m_h, 2 GeV, because of the theoretical uncertainty of our formula (23). The Higgs mass constraint severely constrains the top-squark sector parameters, especially in that $\Delta m_{\tilde t} (\equiv m_{\tilde t_2} - m_{\tilde t_1})$ has a lower limit. From an ISAJET scan over GUT-scale parameters, we obtain the $\theta_{\tilde t}$ dependence of $\Delta m_{\tilde t}$ in Fig. 5. Almost all data points have large $\theta_{\tilde t}$, $1.3 < \theta_{\tilde t} < \frac{\pi}{2}$, which means $\tilde t_1 \simeq \tilde t_R$. $\Delta m_{\tilde t}$ decreases as $\theta_{\tilde t}$ decreases from $\frac{\pi}{2}$. Actually $\theta_{\tilde t}$ has a lower limit of 1.1 and we find that the on-shell stop mass difference is bounded by

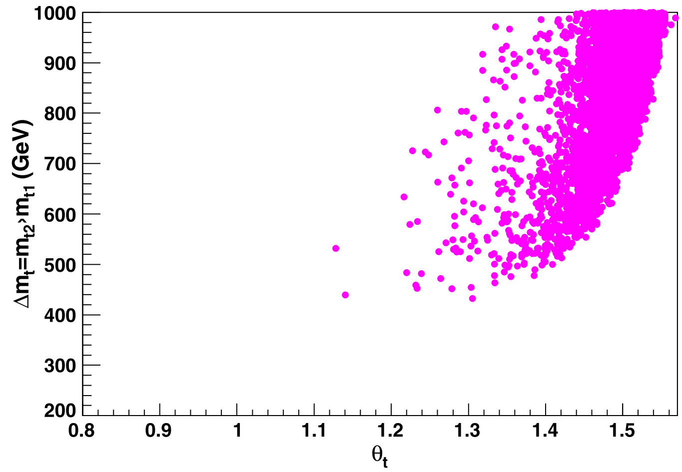

Fig. 5. $\theta_{\tilde t}$ dependence of stop mass difference $\Delta m_{\tilde t} = m_{\tilde t_2} - m_{\tilde t_1}$.

$$\Delta m_{\tilde t} \geqslant 400 \text{ GeV}. \qquad (25)$$

3. Concluding remarks

We have studied the implications for the scalar-top sector of the recent Tevatron M_W measurements and the LHC and Tevatron indications of a 125 GeV Higgs boson. We utilized the H3m package to evaluate m_h through 3 loops in an effective Lagrangian approach with RGE evolution from the GUT scale. Natural SUSY was assumed, for which the third-generation scalar quarks are much lighter than the multi-TeV masses of squarks of the first two generations and the Higgsino mixing parameter μ is small, 150 GeV. A maximal Higgs mass is attained that is close to the LHC experimental indications. The condition for maximal Higgs mass is an off-diagonal value of the stop-mixing matrix $X_t = \sqrt{6}M_{\rm susy}$ in the $\overline{\rm DR}$ renormalization scheme, which requires an on-shell soft-SUSY parameter at the weak scale of $A_t(M_{\rm susy}) \approx 2$ TeV. The minimum value of the mass splitting of two top-squark states was found to be 400 GeV. As can be seen in Fig. 1, the allowed region from the m_h constraint (blue region) satisfies the M_W constraint at 90% Confidence Level, independent of the value of $\theta_{\tilde t}$. For $\theta_{\tilde t} = \frac{\pi}{4}$ a top squark with sub-TeV mass is somewhat favored by the M_W data; $m_{\tilde t_1} < 500$ GeV is possible for almost all values $\theta_{\tilde t}$ when $\tilde t_1 \simeq \tilde t_R$. Precise experimental determination of m_h at the LHC will tighten the restrictions on the top-squark masses. The detection of the scalar-top states at the LHC would establish the SUSY theoretical underpinning of electroweak symmetry breaking.

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