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A Simplified Equation for Modeling Sediment Transport Capacity

S. C. Finkner
Isco, Inc. (Lincoln, Neb.)

M. A. Nearing
University of Georgia, mark.nearing@ars.usda.gov

G. R. Foster
University of Minnesota - Crookston

John E. Gilley
University of Nebraska-Lincoln, john.gilley@ars.usda.gov

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ABSTRACT

Sediment transport capacity for shallow overland flow was represented as a quadratic function of downslope distance using the assumption of a linear increase in overland flow discharge with downslope distance and an approximation to the Yalin equation for sediment transport capacity. The simplified equation for sediment transport applies to complex topography having uniform soil and management characteristics. The simplified equation accurately approximated the Yalin equation when calibrated using the average of the hydraulic shear stresses at the end of a constant slope reference profile and the end of the actual profile. The simplified equation is useful in deriving closed-form solutions to the governing erosion equations for steady state conditions and reduces the computational time when numerical solutions are required.

INTRODUCTION

Many models for soil erosion by water on upland areas dynamically route sediment by solving the continuity equation for sediment transport (Bennett, 1974). The solution of this equation is generally accomplished using numerical methods in association with a hydrologic model which provides the required hydrologic inputs. These dynamic erosion models provide estimates of total sediment discharge from an area, and predict sediment movement within an area by considering the processes of soil detachment, transport, and deposition.

Equations used in many of these erosion models depend on the sediment transport capacity of the overland flow. Transport capacity is compared to the sediment load to determine whether detachment or deposition is occurring. In addition, the rate of detachment or deposition may be assumed to be proportional to the difference between the transport capacity and the sediment load (Foster, 1982). Therefore, a relationship which accurately predicts the sediment transport capacity at all points within an upland area is essential for sediment routing.

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The authors are: S. C. FINKNER, Mechanical Design Engineer, Isco, Inc., Lincoln, NE; M. A. NEARING, Assistant Professor, Agricultural Engineering Department, University of Georgia, Athens; G. R. FOSTER, Professor and Head, Agricultural Engineering Department, University of Minnesota, St. Paul; J. E. GILLEY, Agricultural Engineer, USDA-Agricultural Research Service, University of Nebraska, Lincoln.

Several generalized formulas have been developed for computing sediment transport capacity. Many of the equations were developed for stream flows, and were later applied to shallow overland and channel flows. However, Alonso et al. (1981), who evaluated nine transport formulas, concluded that the Yalin equation (Yalin, 1963) provided reliable estimates of transport capacity for shallow overland and channel flow. Foster and Meyer (1972b) also concluded that the Yalin equation was most appropriate for shallow flows associated with upland erosion. The Yalin equation is defined as

$$\frac{T_c}{(SG)\, d\, \rho_w^{1/2}\, \tau_s^{1/2}} = 0.635\, \delta\, [1 - \frac{1}{\beta} \ln(1+\beta)] \quad \ldots [1]$$

$$\beta = 2.45\, (SG)^{-0.4}\, (Y_{cr})^{0.5}\, \delta \quad \ldots [2]$$

$$\delta = \frac{Y}{Y_{cr}} - 1 \;(\text{when } Y < Y_{cr},\ \delta = 0) \quad \ldots [3]$$

$$Y = \frac{\tau_s/\rho_w}{(SG - 1)\, g\, d} \quad \ldots [4]$$

where

T_c = sediment transport capacity ($ML^{-1}T^{-1}$),
SG = particle specific gravity (unitless),
ρ_w = mass density of water (ML^{-3}),
d = particle diameter (L),
Y = dimensionless shear stress,
Y_{cr} = dimensionless critical shear from Shields Diagram,
g = acceleration of gravity (LT^{-2}),
τ_s = shear stress acting to detach soil ($ML^{-1}T^{-2}$),
β and δ = dimensionless parameters as defined in equations [2] and [3], respectively.

The Yalin equation can be modified to consider mixtures consisting of particles of varying size and density (Foster, 1982).

The Yalin equation (as modified by Foster, 1982) computes transport capacity as a function of flow hydraulics and sediment diameter and density. The equation applies to any point on the landscape provided estimates of hydraulic and sediment properties are available. However, the continuity equation for sediment transport must be solved numerically when using the Yalin equation in its original form (equations [1] through

[4]). The USDA-Water Erosion Prediction Project (Foster, 1987), which utilizes the Yalin equation, required computer code which operated very rapidly. One approach to meet this requirement was to simplify the Yalin equation in order to develop closed-form solutions to the governing differential equations. Closed-form solutions reduce the number of computions and alleviate instabilities associated with numerical solutions.

Objective

The objective of this study was to develop a simplified equation for computing sediment transport capacity that could be used in the Water Erosion Prediction Project (WEPP), as well as other erosion models. The simplified relationship was developed as an approximation to the Yalin equation. The simplified equation was considered to be satisfactory if the maximum difference between transport capacities predicted by the simplified and Yalin equations was no greater than 10% of the maximum transport capacity when the hydraulic shear stress exceeded 1 pascal at some point along the profile.

DEVELOPMENT OF A FUNCTIONAL RELATIONSHIP FOR SEDIMENT TRANSPORT CAPACITY

Julien and Simons (1985) identified slope (S), unit discharge rate (q), rainfall intensity, and shear stress acting to detach soil (τ_s) as the dominant geometric and flow variables for determining sediment transport capacity. Of these variables, S, q, and τ_s were considered to exhibit spatial variability. Shear stress acting on the soil is a derived quantity which incorporates the effects of both S and q. Therefore, τ_s would appear to be the single most significant parameter for a simplified equation for sediment transport capacity. As indicated by Foster and Meyer (1972a), when τ_s was much greater than the critical shear stress for transport of detached particles, the Yalin equation reduced to

$$T_c = K_t \, \tau_s^{3/2} \quad \text{[5]}$$

where T_c is the sediment transport capacity ($ML^{-1}T^{-1}$) and K_t is the transport coefficient. This result is consistent with other transport equations and experimental data (Lu et al., 1989; Simons and Senturk, 1976).

A relationship for T_c as a function of distance along an upland profile was developed using equation [5] by first writing an equation for τ_s as a function of downslope distance, x. Shear stress is given by

$$\tau_s = \gamma \, y_s \, S \quad \text{[6]}$$

where γ is the weight density of water ($ML^{-2}T^{-2}$) and y_s is that portion of the total hydraulic depth which acts to detach soil particles (L).

The value of y_s was determined as (Foster, 1982)

$$y_s = y_t \frac{f_s}{f_t} = \left[\frac{f_t \, q^2}{8 \, g \, S} \right]^{1/3} \frac{f_s}{f_t} \quad \text{[7]}$$

where

y_t = total hydraulic depth (L),
f_s = the total Darcy-Weisbach hydraulic roughness coefficient for smooth, bare soil,
f_t = total hydraulic roughness coefficient,
q = overland flow discharge per unit width (L^2T^{-1}).

Substituting equation [7] into equation [6] yields

$$\tau_s = \frac{\gamma \, f_s}{(8 \, g)^{1/3} \, f_t^{2/3}} \, (q \, S)^{2/3} \quad \text{[8]}$$

If the values of f_s, f_t, γ, and g are assumed constant, then the value of τ_s simply becomes a function of q and S for a particular upland area.

The WEPP model used erosion equations for steady state conditions, in which case discharge may be assumed to vary linearly with downslope distance, x. An expression for q is then

$$q = q_o + \sigma \, x \quad \text{[9]}$$

where q_o is the inflow at the upper end of the profile (L^2T^{-1}) and σ is the rainfall excess per unit area (LT^{-1}).

By substituting this relationship into equation [8], the expression for τ_s was reduced to a function of downslope distance and slope.

At this point, the equations are converted into nondimensional forms to simplify the derivation. Nondimensional equations can also reduce the computational load by simplifying the algebraic expressions and by creating solutions which can be solved "once and for all" with respect to many parameters. A reference profile was defined as a profile with a constant slope which passes through the end points of a complex profile (Foster and Meyer, 1972a), Fig. 1. The slope and conditions at the end of the reference profile were chosen to normalize the erosion equations.

The expression for τ_s may be normalized to the shear stress on the soil at the end of the reference profile, with the result

$$\tau_{s*} = \left[\left(\frac{q_{o*} + x_*}{q_{o*} + 1} \right) S_* \right]^{2/3} \quad \text{[10]}$$

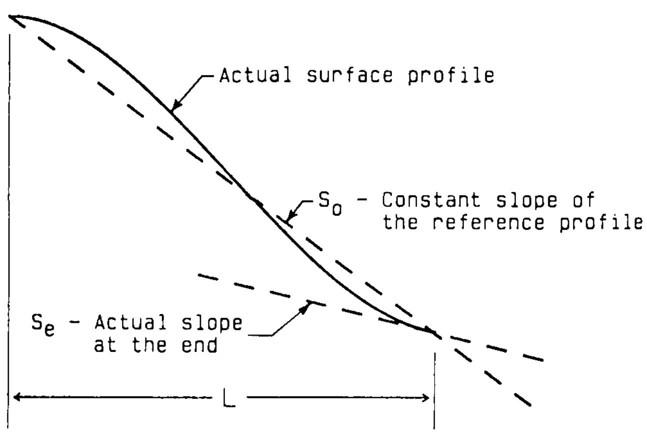

Fig. 1—Representation of a typical upland profile.

where

τ_s* = normalized shear stress,
q_o* = $q_o/\sigma L$,
x_* = x/L,
S_* = S/S_o,
L = profile length,
S_o = gradient of the constant slope reference profile, Fig. 1.

Equation [5] for sediment transport capacity is normalized to the transport capacity at the end of the constant slope profile and becomes

$$T_{c*} = K_{t*}\, \tau_{s*}^{3/2} \quad \text{[11]}$$

The normalized transport coefficient is defined as $K_{t*} = K_t/K_{to}$ where K_{to} is the transport coefficient determined at the end of the reference profile. Procedures for determining K_t and K_{to} will be discussed in detail in the following section.

A complex profile can be represented in a continuous fashion as a series of uniform, convex, and concave slope segments. The normalized slope for each segment can be described as a function of distance by the relationship

$$S_* = a\, x_* + b \quad \text{[12]}$$

where a and b are the slope coefficients which are constant for each slope segment.

Uniform, convex, and concave segments and the degree of curvature can be represented by varying the value of a and b. Finally, using equation [12], the transport capacity may be written as a function of distance as

$$T_{c*} + A\, x^2 + B\, x + C \quad \text{[13]}$$

where

$$A = K_{t*}\, a / (q_{o*} + 1)$$

$$B = K_{t*}\, (a\, q_{o*} + b) / (q_{o*} + 1)$$

$$C = K_{t*}\, q_{o*}\, b / (q_{o*} + 1) \quad \text{[14]}$$

The parameters A, B, and C in equations [13] and [14] would be constant for profile segments of uniform soil characteristics and management practices.

CALIBRATION OF THE SIMPLIFIED EQUATION

If deemed appropriate in form, equation [5] could be used to compute transport capacity for the WEPP model, as well as other upland erosion models. In these models, the value of τ_s would be available from a hydrology model which provides the required hydrologic inputs. The value of K_t, however, would need to be determined independently for each upland area.

No method currently exists for determining K_t directly from available information and experimental data was inadequate to relate K_t to sediment properties. However, estimates are available for the parameters required to determine the transport capacity using the Yalin equation. If a representative shear stress (τ_{so}) could be identified for a particular profile, the value of K_t could be determined as

$$K_t = \frac{T_{co}}{\tau_{so}^{3/2}} \quad \text{[15]}$$

where T_{co} is the transport capacity computed from the Yalin equation using τ_{so}.

In this section, three methods of determining τ_{so}, and thus K_t, are described. These three methods are later evaluated to determine which is most appropriate for approximating the Yalin equation.

Figure 2 shows K_t as a function of τ_s for a typical silt loam soil. At higher values of τ_s, K_t becomes relatively constant. However, K_t rapidly approaches zero when τ_s is small. Therefore, the value of τ_{so} must be carefully selected to be representative of the entire profile, as well as the areas of greatest potential for erosion or deposition.

Conceptually, the end of the profile may be thought of as a "gate" which controls the amount of sediment leaving the profile. Larger discharge rates at the end of a profile may result in much greater sediment detachment rates relative to other locations along the profile when the slope at the end of the profile is comparatively steep. When a profile flattens at the lower end, large sediment loads may be rapidly deposited. Therefore, the conditions at the lower end of a profile could be used to calibrate the simplified equation with the expectation that the simplified equation would provide the highest degree of accuracy at this critical location. Two of the options for calibrating the simplified equation utilized conditions at the end of the profile to determine τ_{so}. The third option considered conditions along the entire profile.

The first calibration option determined τ_{so} based on the discharge at the end of the profile and the slope of the reference profile, S_o. This method was referred to as the Reference Slope Method. The transport coefficient identified by the Reference Slope Method was denoted as K_{to}.

The second method of calibrating the transport

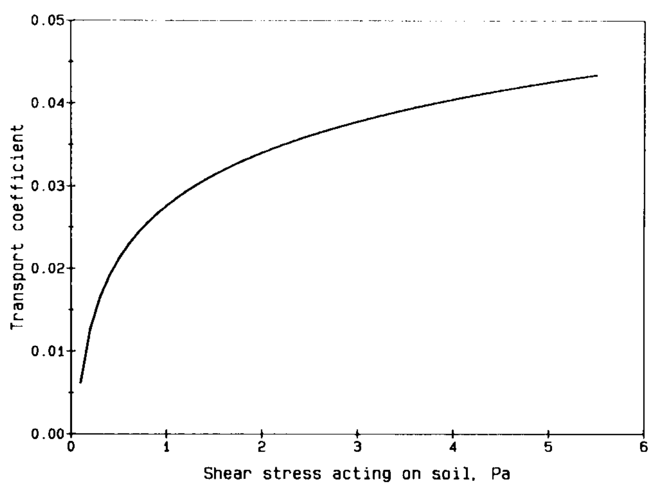

Fig. 2—Values of the transport coefficient (K_t) determined for various levels of soil shear stress.

coefficient was referred to as the Dual Slope Method. This option required computing two hydraulic shear values at the end of the profile. The first hydraulic shear value was based on the slope of the reference profile, S_o, while the second value was based on the actual slope at the end, S_e. The value of τ_{so} was then defined as the average of these two shear stress values. The Dual Slope transport coefficient was denoted as K_{te}.

A third option for calibrating the simplified equation was termed the Average Shear Method. This method yielded the coefficient, K_{ta}, which was based upon the average shear stress along the entire profile. The average shear stress would be representative not only of the slope of the entire profile, but would take into account the combination of slope and discharge along the length of the profile. A relationship describing τ_s as a function of x could be developed using equations [10] and [12]. The average shear stress could then be calculated as

$$\tau_{so} = \frac{\int \tau_s(x)\, dx}{L} \quad \ldots\ldots [16]$$

The value of K_{ta} could then be determined from equation [15] using the average shear stress for τ_{so}.

EVALUATION OF THE SIMPLIFIED TRANSPORT EQUATION

The Reference Slope, Dual Slope, and Average Shear Methods of calibrating the transport coefficient were tested under simulated conditions. Uniform, convex, concave, and s-shaped profile types were simulated. These profiles were chosen to provide a wide range in overland flow hydraulic characteristics. Procedures for calibrating the simplified equation would be thoroughly evaluated using these four profile types.

A uniform profile is characterized by a constant slope. Simulation testing was completed on uniform profiles with slopes of 2, 10, and 20% and lengths of 10, 50, and 100 meters. The slope of the convex profile increased from 2% at the top to 15% at the bottom while the slope of the concave profile decreased from 15% to 2% over the length of the profile. The slope of the s-shaped profile increased from 2% at the top to 15% at the midpoint and then back to 2% at the bottom. The convex, concave, and s-shaped profiles were all 100 m in length. Simulated peak runoff rates for the profiles ranged from 80 to 83 mm per hour.

The Yalin equation was used to compute the sediment transport capacity, T_{cy}, at 100 points spaced evenly along each of the profiles. The simplified equation was also utilized to estimate the transport capacity, T_{cs}, at corresponding points along each profile. Three separate estimates were determined using the simplified equation which corresponded with the three values of the transport coefficient, K_{to}, K_{te}, and K_{ta}. Deviations between T_{cy} and T_{cs} were also identified for each profile and each calibration method. These statistics were then compared to determine whether the simplified equation was an accurate representation of the Yalin equation and which method of calibration provided the best agreement with the Yalin equation.

The sum of the deviations squared are presented in Table 1 for each profile and calibration method. Predicted values of transport capacity were equivalent

TABLE 1. Summary of statistical analysis of three calibration methods for the simplified equation for sediment transport capacity

Profile description		Maximum T_c, kg/m s	Sum of squares, Tc*			Maximum deviation,†%		
			K_{to}	K_{te}	K_{ta}	K_{to}	K_{te}	K_{ta}
2% UNI,+	10 m	0.0006	2.03	2.03	2.66	19.2	19.2	41.2
2% UNI,	50 m	0.0063	0.87	0.87	1.21	12.7	12.7	26.7
2% UNI,	100 m	0.0150	0.60	0.60	0.83	10.5	10.5	21.9
10% UNI,	10m	0.0063	0.86	0.86	1.21	12.7	12.7	26.7
10% UNI,	50 m	0.0483	0.32	0.32	0.43	7.6	7.6	15.6
10% UNI,	100 m	0.1078	0.19	0.19	0.27	6.0	6.0	12.0
20% UNI,	10 m	0.0157	0.58	0.58	0.82	10.4	10.4	21.6
20% UNI,	50 m	0.1082	0.19	0.19	0.27	6.0	6.0	12.0
20% UNI,	100 m	0.2366	0.12	0.12	0.17	4.7	4.7	9.7
CONVEX		0.1707	0.31	0.31	1.32	7.5	4.2	16.8
CONCAVE		0.0404	0.35	0.07	0.04	14.1	8.9	7.1
S-SHAPE		0.0858	0.19	0.14	0.30	7.3	7.4	11.6

\+ UNI = Uniform slope profile

† Maximum deviation between transport capacity values predicted by the simplified and Yalin equations expressed as a percentage of the maximum transport capacity for a profile

for the K_{to} and K_{te} coefficients on uniform profiles as were statistical results. Sums of squares for the K_{to} and K_{te} coefficients were generally lower than those determined using the K_{ta} transport coefficient. For the convex, concave, and s-shape profiles, the K_{te} transport coefficient resulted in sums of squares of the error which were consistently lower than those determined with K_{to}. The K_{ta} coefficient had the best correlation for the concave profile but showed much lower agreement for other profiles. Based on this evaluation, the K_{te} coefficient, determined from the Dual Slope Method of calibration, was selected as the coefficient which provided the best overall correlation with the Yalin equation.

The maximum deviation between T_{cy} and T_{cs} expressed as a percent of the maximum transport capacity is also shown in Table 1. In general, the K_{te} coefficient yielded the smallest maximum deviation of the three calibration coefficients. This result was consistent with the sum of squares test, again indicating that the Dual Slope Method provided the best estimate of results obtained using the Yalin equation.

The criterion established for determining whether the simplified equation provided a suitable estimate of the Yalin equation was that the maximum deviation between T_{cy} and T_{cs} be less than 10% of the maximum transport capacity. However, this criterion was only considered when τ_s exceeded 1 Pa at some point along the profile. This was because the threshold shear stress below which detachment by flow does not occur is generally greater than 1 Pa. For $\tau_s = 1$ Pa, K_t could be identified from Fig. 2 as 0.029 and the corresponding limiting T_c could be computed from equation [5] as 0.029 kg/m s.

Several of the tested profiles had maximum deviations which were greater than 10% of the maximum T_c when the simplified equation was calibrated by the Dual Slope Method. However, a plot of maximum deviation as a function of maximum transport capacity, Fig. 3, demonstrated that maximum deviation was always less than the established criterion for profiles with maximum transport capacities greater than 0.029 kg/m s. Thus,

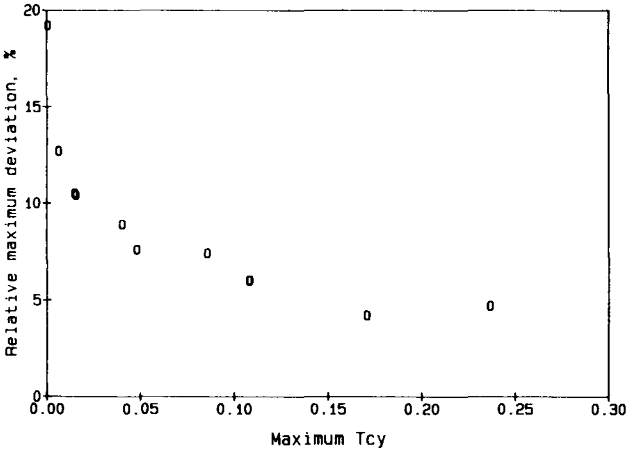

Fig. 3—Relative maximum deviations between the simplified and Yalin equations as determined for profiles of various maximum transport capacities.

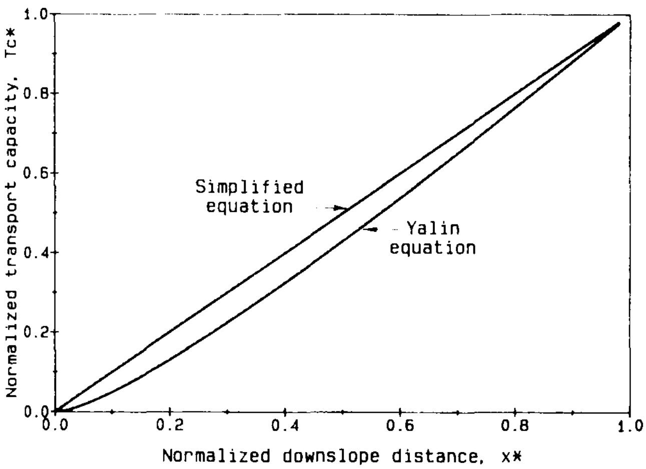

Fig. 4—A comparison of the simplified and Yalin transport capacity functions for profiles of various maximum transport capacities.

the statistical results support the simplified equation calibrated by the Dual Slope Method as a valid approximation to the Yalin equation for the given criterion.

A visual comparison of the results also indicated that the simplified equation calibrated using the Dual Slope Method was an accurate approximation to the Yalin equation. A comparison of transport capacity values predicted by the Yalin and simplified equations is presented in Figs. 4 - 7 for the 10% uniform profile of 50 m length, and the convex, concave, and s-shape profiles, respectively. The simplified equation over estimated the transport capacity for uniform profiles, Fig. 4, due to the lack of a critical shear value in the simplified equation. However, periods of negative as well as positive deviation occurred on the convex and s-shaped profiles, as shown in Figs. 5 and 7, respectively. In all cases, the simplified equation produced results which were in close agreement with the Yalin equation.

When using K_{te} as the calibration coefficient, maximum deviation between the simplified and Yalin equations were less than 10% of the maximum transport capacity, for $\tau_s \geqslant 1$ pascal ($T_c \leqslant 0.029$ kg/m s). A visual comparison of transport capacity values along the test profiles also indicated that the simplified equation provided a suitable approximation to the Yalin equation. The simplified equation calibrated by the Dual Slope Method would therefore be a valid approximation to the Yalin equation for profiles where τ_s exceeds 1 Pa and could be utilized in computer models with resultant savings in computational time.

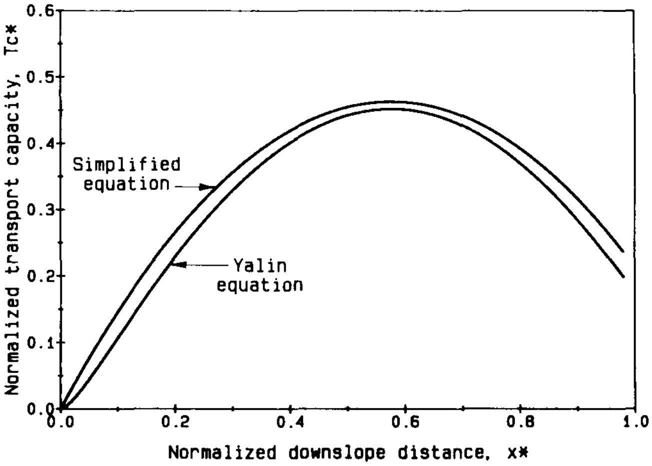

Fig. 6—A comparison of the simplified and Yalin transport capacity functions for a concave profile.

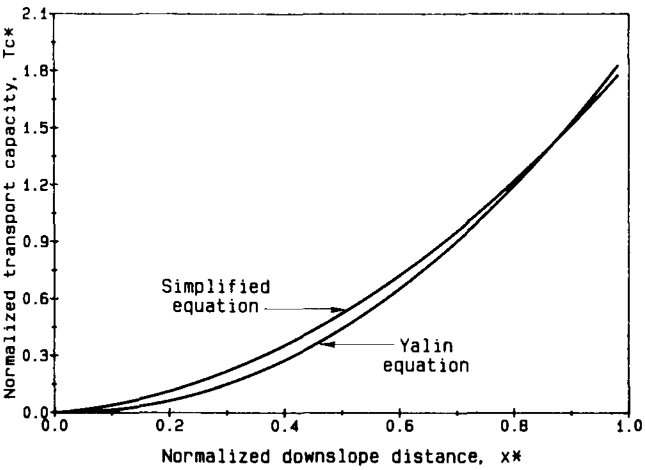

Fig. 5—A comparison of the simplified and Yalin transport capacity functions for a convex profile.

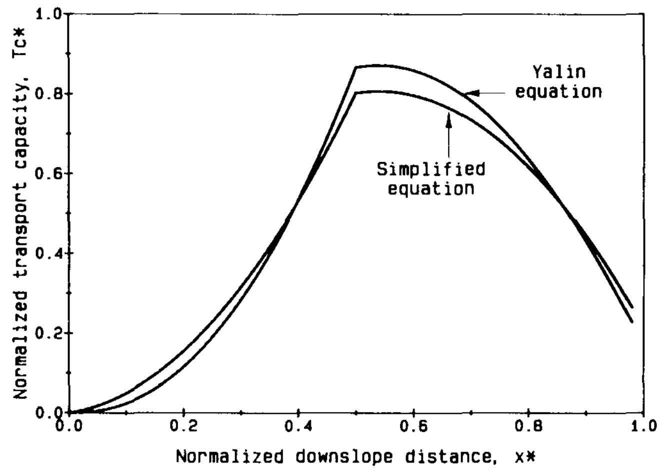

Fig. 7—A comparison of the simplified and Yalin transport capacity functions for an s-shaped profile.

SUMMARY

A simple functional equation was developed which describes sediment transport capacity as a function of distance along a profile. This relationship was based upon the assumption of a linear increase in overland flow discharge with downslope distance and the assumption that transport capacity was proportional to the 3/2 power of the shear stress acting on the soil. The functional form of the simplified equation can be used to derive closed-form solutions to governing erosion equations under steady-state conditions. This simplified relationship is also useful in reducing the amount of computational time required for the numerical solution of the unsteady continuity equation for sediment routing.

Three methods of calibrating a transport coefficient in the simplified equation were evaluated by comparing results of the simplified equation to results obtained using the Yalin equation. The Dual Slope Method of calibration was determined to provide the transport coefficient which resulted in the best agreement. The Dual Slope transport coefficient, K_{te}, was obtained using a representative shear stress which was defined as the average of shear stresses acting on the soil at the end of the constant slope reference profile and at the end of the actual profile. The representative shear stress was used to compute transport capacity based on the Yalin Equation and the transport coefficient was then determined from equation [15]. When the Dual Slope Method of calibrating the simplified equation was used, the maximum deviation between the simplified and Yalin equations was less than 10% of the maximum transport capacity for all profiles where hydraulic shear acting on the soil, τ_s, was greater than one pascal. A visual comparison of the results from the simplified and Yalin equations also indicated close agreement between the two equations.

The simplified equation provides a good approximation to the Yalin equation on profiles where $\tau_s \geqslant 1$ Pa at some point along the profile. However, the simplified equation is not well suited in applications where lower shear stresses and smaller amounts of erosion are of concern. Where applicable, the simplified relationship would be useful in deriving closed-form mathematical solutions to governing erosion equations under steady-state conditions and would reduce the computational time load when numerical solutions are required.

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LIST OF SYMBOLS

a	slope coefficient
A	coefficient in the simplified transport equation
b	slope coefficient
B	coefficient in the simplified transport equation
C	coefficient in the simplified transport equation
d	particle diameter, L
f_s	portion of the total Darcy-Weisbach hydraulic roughness coefficient which may be attributed the bare soil
f_t	total Darcy-Weisbach hydraulic roughness coefficient
g	gravitational acceleration, $L\,T^{-2}$
K_t	transport coefficient in the simplified transport equation, $L^{1/2}\,T^2\,M^{-1/2}$
K_{ta}	transport coefficient based on the average shear stress for the entire profile, $L^{1/2}\,T^2\,M^{-1/2}$
K_{te}	transport coefficient based on the average of the shear stress at the end of the reference profile and the shear stress at the end of the actual profile, $L^{1/2}\,T^2\,M^{-1/2}$
K_{to}	transport coefficient based on the shear stress at the end of the reference profile, $L^{1/2}\,T^2\,M^{-1/2}$
K_{t*}	normalized transport coefficient
L	profile length, L
q	discharge per unit width, $L^2\,T^{-1}$
q_o	inflow at the upper end of a profile, $L^2\,T^{-1}$
q_{o*}	normalized inflow at the upper end of a profile
S	surface slope
S_e	slope at the end of the actual profile
S_o	gradient of the constant slope reference profile
S_*	normalized surface slope
SG	particle specific gravity
T_c	sediment transport capacity, $M\,L^{-1}\,T^{-1}$
T_{co}	sediment transport capacity, computed using the representative shear stress (τ_{so}), $M\,L^{-1}\,T^{-1}$
T_{cs}	sediment transport capacity determined using the simplified equation, $M\,L^{-1}\,T^{-1}$
T_{cy}	sediment transport capacity determined using the Yalin equation, $M\,L^{-1}\,T^{-1}$
T_{c*}	normalized sediment transport capacity
x	downslope distance, L
x_*	normalized downslope distance
y_s	portion of the total hydraulic depth which acts to detach soil, L
y_t	total hydraulic depth, L
Y	dimensionless shear stress
Y_{cr}	dimensionless critical shear parameter from Shields Diagram
β	a parameter in the Yalin equation
δ	dimensionless excess shear in the Yalin equation
γ	weight density of water, $M\,L^{-2}\,T^{-2}$
ρ_w	mass density of water, $M\,L^{-3}$
σ	rainfall excess, LT^{-1}
τ_s	shear stress acting to detach soil particles, $M\,L^{-1}\,T^{-2}$
τ_{so}	shear stress on the soil at the end of the reference profile, $M\,L^{-1}\,T^{-2}$
τ_{s*}	normalized shear stress acting to detach soil particles.