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Letter to the Editor

Static-electric-field behavior in negative ion detachment by an intense, high-frequency laser field

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Abstract

Based upon the exact numerical solution of the complex quasienergy problem for a 3-dimensional short-range potential as well as upon analytical evaluations, we demonstrate for any finite frequency ω that the action of an ultra-intense laser field (with electric vector $F(\omega t)$) on a weakly bound atomic system may be described by the cycle-averaging of results for an instantaneous static electric field of strength $|F(\omega t)|$.

The accurate description of the intensity dependence of the decay rate of a bound level over a broad interval of laser frequencies is one of the challenging problems of strong field laser-atom physics. Existing qualitative results obtained from nonperturbative (in the intensity) analyses of atomic decay rates in a laser field depend significantly on the relation between the laser frequency ω and $\omega_0 = |E_0|/\hbar$ (where E_0 is the binding energy), as well as that between the laser amplitude F (see (2) below) and the characteristic internal atomic field strength, $F_0 = (2m|E_0|^3/|e|\hbar)^{1/2}$. (Below we use the following scaled units: energies, ω and F are measured in units $|E_0|$, ω_0, and F_0, respectively.) For small frequencies, $\omega \ll 1$, and for field strengths $F \geq \omega$ (or equivalently for $\gamma_K < 1$, where $\gamma_K = \omega/F$ is the well- known Keldysh parameter), the tunneling mechanism for the decay is realized, which is valid only for weak (although nonperturbative) fields, $F \ll 1$ (see [1] and the improved analyses in [2]). The tunneling mechanism for the decay has been confirmed by many experiments for frequencies up to $\omega \sim (0.1–0.2)$, particularly for the rare gases [3]. For the case of ground state atomic hydrogen, H(1s), Pont *et al.* [4, 5] performed a low-frequency analysis of the decay rate Γ beyond the Keldysh approach (up to $F \leq 0.2$) using the ω^2 expansion of the complex quasienergy using the basis of quasistationary states of the hydrogen atom in a static electric field (whose magnitude equals that of the instantaneous laser field, see below). For $\omega = 0.134$ ($\lambda = 616$ nm), a comparison of the F dependence of these "static-field-based" results with the exact ones shows a reasonable agreement (which becomes better for stronger F) except for the structure seen in the exact $\Gamma(F)$ which is due to Rydberg levels shifting in and out of resonance as the intensity varies. With increasing F (*e.g.* for $F \geq 0.2$ in the case of H(1s)), over-barrier ionization becomes important. Recently, the over-barrier decay rate Γ in the low-frequency limit, $\omega \ll 1$, has been obtained by Popov [6] using an adiabatic cycle-averaging of the Stark width Γ_{stat} for a strong static electric field. It demonstrates a surprisingly linear dependence of Γ on F (the "intermediate" asymptotic regime [6]),

$$\Gamma \approx k\,(F - F_{cr}) \tag{1}$$

where the fitting parameters k and F_{cr} do not depend on F over a wide fitting interval (*e.g.* $0.6 < F < 2$ for H(1s)) and are smooth functions of the laser ellipticity. For above-threshold frequencies, $\omega > 1$, and in the strongly nonperturbative regime, the concept of quasistationary stabilization of atomic decay rates is conventionally understood to be applicable, in which case $\Gamma(F)$ has a decreasing trend with increasing F (see reviews [7, 8] on the recent status of this problem). However, for a Coulomb potential the existence of a stabilization regime for decay rates in the ultra-strong field limit is still an open question.

The analysis of $\Gamma(F, \omega)$ is simplified for the case of negative ion detachment, for which simple analytic, short-range binding potentials can be utilized. One of them is the 3-dimensional zero-range potential (ZRP) that has been widely used for the description of a weakly bound electron, as for example in the H^- negative ion. The use of a quasistationary quasienergy state (QQES) approach [9] for the ZRP model essentially permits exact predictions of $\Gamma(F, \omega)$ (which is determined by the imaginary part of the complex quasienergy, $\epsilon = \mathrm{Re}\,\epsilon - i\Gamma/2$) for laser intensities extending from the perturbative to the ultrahigh intensity regime and for frequencies extending from the tunneling to the multiphoton regimes. In particular, recently we have demonstrated [10] that, for the ZRP model, the stabilization-like behavior of $\Gamma(F)$ in a high-frequency field only exists for a limited interval of F, up to the closing of the direct photoionization channel caused by the ponderomotive shift. Moreover, for the particular case of circular polarization, the strong field behavior of $\Gamma(F)$ was found to be similar to that for a strong static electric field, both for $\omega < 1$ as well as for the post-stabilization regime at $\omega > 1$.

In this letter we present a global analysis of the dependence of Γ (for the ZRP model) on F, ω, and on the polarization state of a laser field described by the electric vector

$$\boldsymbol{F}(\omega t) = \frac{F}{\sqrt{1+\eta^2}}\{\cos\omega t,\, \eta\sin\omega t,\, 0\}, \qquad -1 \leqslant \eta \leqslant +1. \tag{2}$$

(Instead of the ellipticity, η, it is more convenient in what follows to use the related degree of linear polarization, $l = (1 - \eta^2)/(1 + \eta^2)$.) For details concerning the exact numerical calculations of the complex quasienergy ϵ for the ZRP in the nonperturbative regime, see [10, 11]. The method we employ gives results that are in agreement with those of other authors who employ the ZRP, *e.g.* [12]. Results of exact numerical calculations for $\Gamma(F)$ are presented in Figure 1 for four different values of l and for ω = 1.5, which corresponds to the case of H^- irradiated by a Nd:YAG laser. (These results cover a much greater range of F and l than in [10].) One observes that as F increases, the perturbative regime, in which $\Gamma \sim F^2$, evolves smoothly into a stabilization-like behavior, which breaks up at the closure of the one-photon ionization channel, *i.e.* at $F = F_{th}^{(1)}$. Note that the finite value of Γ at $F = F_{th}^{(1)}$ results from the contributions of partial rates $\Gamma^{(n)}$ for n-photon (above-threshold) detachment with n = 2, 3, 4,..., whose F-dependence (for $n > 2$) is essentially perturbative for $F \sim F_{th}^{(1)}$. It is also seen that the threshold structure of $\Gamma(F)$ at higher thresholds is significantly different from that for n = 1 and depends sensitively on the laser polarization. The frequency dependence of Γ in the interval $0.15 < \omega < 2$ is presented in Figure 2 for l = 0.72 for four different values of F. For weak F, $\Gamma(\omega)$ exhibits the typical perturbative behavior, *i.e.* the step-like increase as ω increases that results from the sequential contributions of the partial

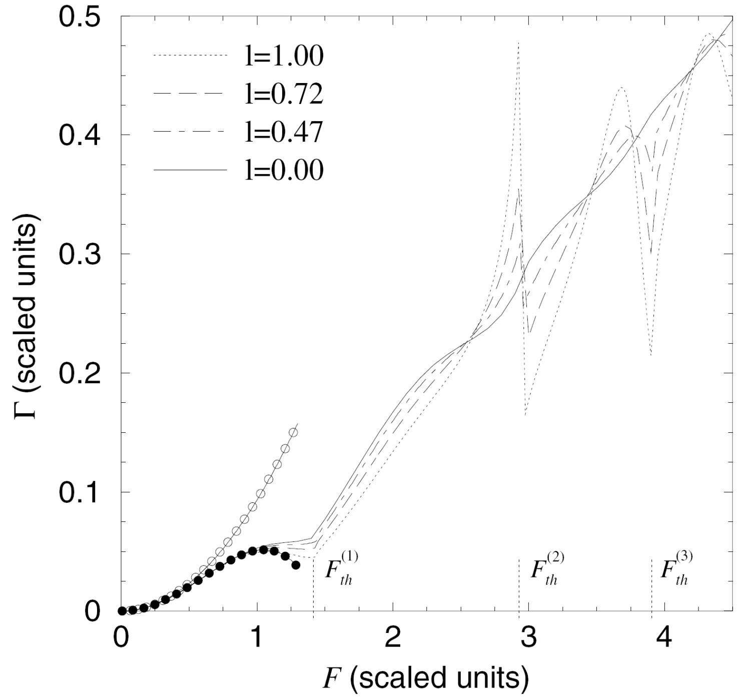

Figure 1. F-dependence of the total detachment rate Γ for ω = 1.5. Full curve, QQES results for four different values of l, as indicated in the figure; open circles, the lowest-order perturbation theory (PT) result for $\Gamma^{(1)} \sim F^2$; full circles, PT result for $\Gamma^{(1)} + \Gamma^{(2)}$ (including terms up to the order of F^4) for l = 0.72.

rates, $\Gamma^{(n)} \sim F^{2n}$, with n becoming smaller as ω increases. As F increases, the stair-step behavior gradually disappears as $\Gamma(\omega)$ nearly becomes insensitive to ω for essentially nonperturbative values of F. This unusual behaviour of $\Gamma(\omega)$ at high F allows us to assume that in the strong field limit the decay mechanism itself becomes essentially independent of the frequency, even in the $\omega > 1$ domain.

To analyze the strong field regime in more detail, instead of the conventional representation for a quasienergy state, $\Psi_\epsilon(r, t) = \Phi_\epsilon(r, t)\exp(-\mathrm{i}\epsilon t)$, we use the following one:

$$\Psi_\epsilon(\boldsymbol{r}, t) = \chi(\boldsymbol{r}, t) \exp\left(-\mathrm{i}\int^{t} \mathcal{E}(t')\,\mathrm{d}t'\right) \tag{3}$$

where $\Psi_\epsilon(r, t)$ is the solution of the Schrödinger equation for a Hamiltonian $H(r, t) = H_{\mathrm{at}}(r) + V(r, t)$, where $H_{\mathrm{at}}(r)$ describes the atom and $V(r, t) = r \cdot F(\omega t)$. The periodic functions $\chi(r, t)$ and $\mathcal{E}(t)$ satisfy the following equation

$$\left(H_{\mathrm{at}}(\boldsymbol{r}) + \boldsymbol{r}\cdot\boldsymbol{F}(\omega t) - \mathcal{E}(t) - \mathrm{i}\frac{\partial}{\partial t}\right)\chi(\boldsymbol{r}, t) = 0. \tag{4}$$

One may easily verify that the quasienergy ϵ is the cycle-average of $\mathcal{E}(t)$,

$$\epsilon = \frac{1}{T}\int_0^T \mathcal{E}(t)\,\mathrm{d}t, \qquad T = 2\pi/\omega. \tag{5}$$

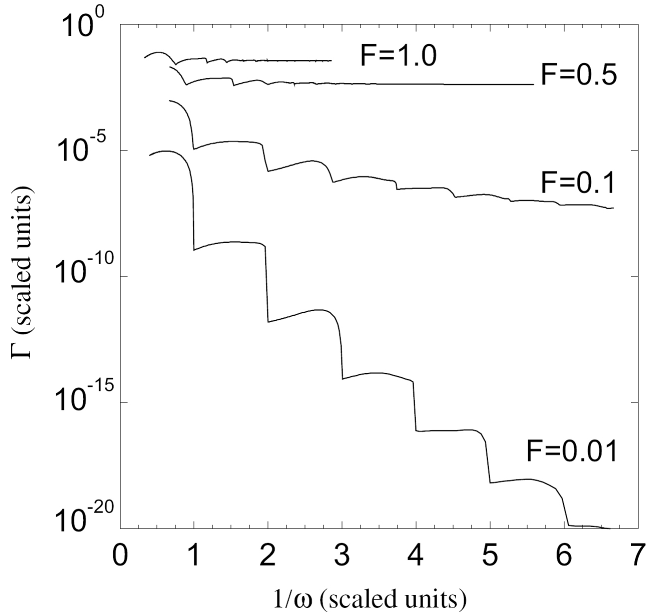

Figure 2. ω-dependence of the total detachment rate Γ for four values of F, as indicated in the figure, and for l =0.72.

Equations (3)–(5) are very general and were used by Langhoff *et al.* [13] in their analyses of so-called "secular terms" in higher orders of perturbation theory (in V), and by Pont *et al.* [5] in the low-frequency analysis of the ionization of H(1s). In [5] the formal development of a perturbation theory in $W = -i\omega\partial/\partial\tau$, where $\tau = \omega t$, is presented for calculations of $\chi(r, t)$ and $\mathcal{E}(t)$ based on the instantaneous state of an atom in a static electric field of strength $\mathcal{F} = |\boldsymbol{F}(\omega t)|$, $\chi^{(0)}(r, t)$, with energy $\mathcal{E}^{(0)}(t)$. In what follows, we employ such an approach to analyze the frequency dependence of ϵ for the ZRP model in the strong field limit. Since we do not assume that ω is small compared to the binding energy $|E_0|$, the key issue is to calculate the next order correction, $\mathcal{E}^{(2)}(t) \sim \omega^2$, to $\mathcal{E}^{(0)}(t)$ in order to estimate the accuracy of the expansion of ϵ in a power series in ω^2, which is generally an asymptotic expansion [5].

The general result for $\mathcal{E}^{(2)}(t)$ is [5],

$$\mathcal{E}^{(2)}(t) = \omega^2 \left\langle \frac{\partial \tilde{\chi}^{(0)}(\boldsymbol{r}, t)}{\partial \tau} \middle| \mathcal{G}'_{\mathcal{E}^{(0)}(t)}(\boldsymbol{r}, \boldsymbol{r}') \middle| \frac{\partial \chi^{(0)}(\boldsymbol{r}', t)}{\partial \tau} \right\rangle \tag{6}$$

where $\mathcal{G}'_{\mathcal{E}(0)(t)}(r, r')$ is the reduced Green function of an atom in a static electric field and $\tilde{\chi}^{(0)}(r, t)$ is the "dual" function, $\tilde{\chi}^{(0)}(r, t) = \chi^{(0)*}(r, -t)$, which is necessary to provide a proper normalization of the quasistationary (resonance) state $\chi^{(0)}(r, t)$ [5, 14]. In the ZRP model (see the review [14] for details), $\mathcal{E}^{(0)}(t)$ at any fixed t can be obtained as the root of the transcendental equation:

$$1 + \pi \mathcal{F}^{1/3} J(\xi) = 0 \tag{7}$$

where $\xi = -\mathcal{E}^{(0)}(t)\, \mathcal{F}^{-2/3}$, $\mathcal{F} \equiv |\boldsymbol{F}(\omega t)| = F\,[(1 + l \cos 2\omega t)/2]^{1/2}$, and $J(\xi)$ is a combination of regular (Ai) and irregular (Bi) Airy functions and their derivatives:

$$J = Ai'(\xi)Bi'(\xi) - \xi Ai(\xi)Bi(\xi) + \mathrm{i}\left[Ai'^2(\xi) - \xi Ai^2(\xi)\right].$$

Using the explicit forms of $\chi_0(r, t)$ and $\mathcal{G}'_{\mathcal{E}(0)(t)}(r, r')$, the matrix element in (6) is calculated analytically (some details regarding the calculation of the integrals that occur can be found in [14]):

$$\mathcal{E}^{(2)}(\tau) = -\omega^2 \left[\left(\frac{\partial \boldsymbol{F}(\tau)}{\partial \tau} \right)^2 I^{(4)} \right] \left(360 \mathcal{F}^{8/3} I \right)^{-1} + \frac{\omega^2}{8\mathcal{F}^2 I} \left(\frac{\partial \boldsymbol{F}(\tau)}{\partial \tau} \cdot \boldsymbol{F}(\tau) \right)^2$$
$$\times \left\{ \frac{I^{(1)}}{I^2} \left(I^{(1)} f(\tau) + \frac{I^{(3)}}{6\mathcal{F}^{4/3}} \right)^2 - \frac{1}{I} \left(I^{(1)} f(\tau) + \frac{I^{(3)}}{6\mathcal{F}^{4/3}} \right) \left(\frac{4}{3} I^{(2)} f(\tau) + \frac{I^{(4)}}{5\mathcal{F}^{4/3}} \right) \right.$$
$$\left. + \frac{1}{3} I^{(3)} f^2(\tau) + \frac{4 I^{(5)}}{45 \mathcal{F}^{4/3}} f(\tau) + \frac{I^{(7)}}{180 \mathcal{F}^{8/3}} \right\} \tag{8}$$

where

$$I(\xi) = -J'(\xi) = Ai(\xi)(Bi(\xi) + \mathrm{i}Ai(\xi)), \qquad I^{(n)} = \frac{\partial^n I(\xi)}{\partial \xi^n}.$$

The function $f(\tau)$ is connected with the derivative of $\mathcal{E}^{(0)}(t)$, which is calculated with the use of (7):

$$\frac{\partial \mathcal{E}^{(0)}}{\partial \tau} = \left(\frac{\partial \boldsymbol{F}(\tau)}{\partial \tau} \cdot \boldsymbol{F}(\tau) \right) f(\tau), \qquad f(\tau) = \frac{1}{3\boldsymbol{F}^2(\tau)} \left[2\mathcal{E}^{(0)} + \left(\frac{\pi I}{\mathcal{F}^{1/3}} \right)^{-1} \right].$$

The result (8) simplifies for the case of a circularly polarized laser field. In this case,

$$\left(\frac{\partial \boldsymbol{F}}{\partial \tau} \right)^2 = \frac{F^2}{2} \quad \text{and} \quad \left(\frac{\partial \boldsymbol{F}}{\partial \tau} \cdot \boldsymbol{F} \right) = -\left(\frac{1}{2} \right) l F^2 \sin(2\tau) = 0.$$

Thus, $\mathcal{E}(t)$ is time-independent and the correction $\epsilon^{(2)}$ is

$$\epsilon^{(2)}_{\mathrm{circ}} = -\frac{\omega^2}{360 \mathcal{F}^{2/3}} \frac{I^{(4)}}{I}. \tag{9}$$

This result coincides with that obtained by an alternative approach in [10], in which the calculations are carried out in a coordinate frame rotating with frequency ω (see also the similar calculations for H(1s) in [4]). In [10] an analytical result for the asymptotic behavior of ϵ_{circ} in ultra-strong fields, $F \gg 1$, has been obtained. In the weak field limit ($F \ll 1$), neglecting exponentially small (tunneling) terms, we obtain the following result for $\epsilon = \epsilon^{(0)} + \epsilon^{(2)}$:

$$\epsilon = -1 - \frac{F^2}{32} \left(1 + \frac{3F^2}{4} \left(1 + \frac{l^2}{2} \right) + \frac{7}{24} \omega^2 \left[1 + \frac{13}{2} F^2 \left(1 + \frac{25}{28} l^2 \right) \right] \right). \tag{10}$$

Note that the Stark-shift in this equation coincides exactly with the first two terms of the . expansion for the known dynamic polarizability and hyperpolarizability of a weakly bound particle in the ZRP model [15]. Thus, for weak fields, the "zero approximation," $\epsilon \simeq \epsilon^{(0)}$, is valid for $\omega \ll 1$ and is equivalent to the standard adiabatic approach. To establish the accuracy of the term $\epsilon^{(0)}$ for the strong field regime, in Figure 3 we present numerical results for real and imaginary parts of the ratio of $\epsilon^{(2)}$ to $\epsilon^{(0)}$ for a number of values of l at fixed $\omega = 1.5$. One observes that with increasing F the two-term approximation, $\epsilon^{(0)} + \epsilon^{(2)}$ (which we call the AA result), is applicable over a wide interval of ω including the above-threshold region, $\omega > 1$.

To check both the relation between the AA results and exact numerical results for ϵ and also the applicability of the ZRP model for real negative ions in a strong laser field, in Table 1 we compare our numerical (QQES) and approximate (AA) results for the detachment of H^- by linearly polarized CO_2 laser radiation (for which $\omega = 0.155$, and the scaled unit of intensity for H^- is 1.494×10^{12} W cm^{-2}) with existing theoretical predictions in [16–19]. The comparison shows the excellent agreement of the exact

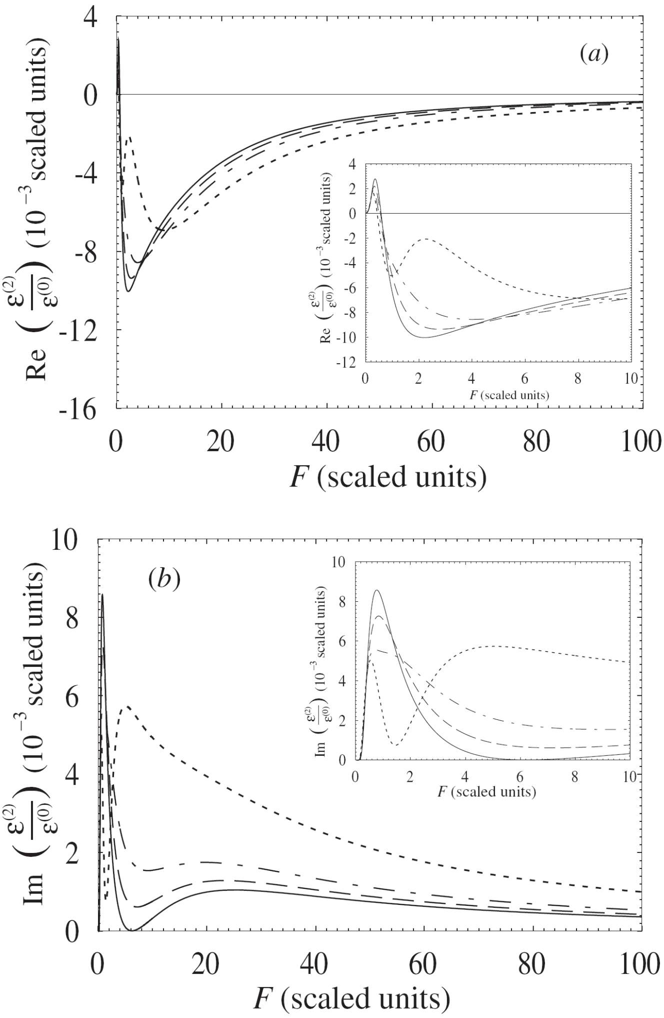

Figure 3. F and l dependences of the real (a) and imaginary (b) parts of the ratio $\epsilon^{(2)}/\epsilon^{(0)}$ for $\omega = 1.5$. Full curve, $l = 0$; long-dashed curve, $l = 0.5$; chain curve, $l = 0.7$; short-dashed curve, $l = 0.9$.

ZRP results with more refined (and time consuming) calculations and also the high accuracy of the AA results for nonperturbative intensities $I \geq 5 \times 10^{10}$ W cm^{-2}, when $F \geq \omega$ (in scaled units).

Comparisons of ϵ_{QQES} and ϵAA as functions of F are presented in Figure 4 for $\omega < 1$ and in Figure 5 for $\omega > 1$. The AA and QQES results for $l = 0$ and $\omega < 1$ are almost indistinguishable: for $\omega = 0.36$ and $F > 0.3$, the difference between ϵ_{QQES} and ϵ_{AA} is less than 3%; for $\omega = 0.56$ and $F > 0.4$, it is less than 2%; and for $\omega = 0.77$ and $F > 0.5$, the difference is less than 4%. Generally, the AA results accurately describe the trends of the position and the width of a quasistationary level but fail to describe the threshold related peculiarities, which are lost by using the ω^2 expansion for the iterative solution of equation (4). These peculiarities are most pronounced for the case of linear polarization and they are exhibited at the points of non-analyticity of the function $\epsilon(F)$, which correspond to the closure of partial detachment channels with increasing F (at $F = F_{th}^{(n)}$). These points are branch points of the type $(\epsilon + U_p + n\omega)^{k+1/2}$ (where U_p is the ponderomotive shift, $U_p = F^2/(2\omega^2)$) and as F increases (and Im ϵ becomes impor-

Table 1. Detachment rates for H^- in the field of a CO_2 laser having linear polarization ((n) ≡ 10^n).

Intensity (W cm^{-2})	Detachment rates (au)					
	[16]	[17]	[18]	[19]	AA	QQES
1.0(10)	(1.04 ± 0.12)(−9)	0.97(−9)	0.91(−9)		0.32(−9)	0.97(−9)
1.12(10)	(2.04 ± 0.11)(−9)			2.7(−9)[a] 2.1(−9)[b]	0.73(−9)	2.28(−9)
2.52(10)	(1.12 ± 0.08)(−7)			1.4(−7)[a] 1.0(−7)[b]	0.88(−7)	1.14(−7)
5.0(10)	(1.81 ± 0.06)(−6)	1.67(−6)	1.76(−6)		1.64(−6)	1.79(−6)
1.0(11)	(1.68 ± 0.03)(−5)	1.61(−5)	1.61(−5)		1.62(−5)	1.66(−5)
1.6(11)	(5.91 ± 0.02)(−5)				5.74(−5)	6.12(−5)
2.0(11)	(9.97 ± 0.01)(−5)				9.75(−5)	9.87(−5)

[a] Floquet calculations with a parametrized one-electron potential.
[b] Faisal–Reiss formulas with a Hylleraas ground state wavefunction.

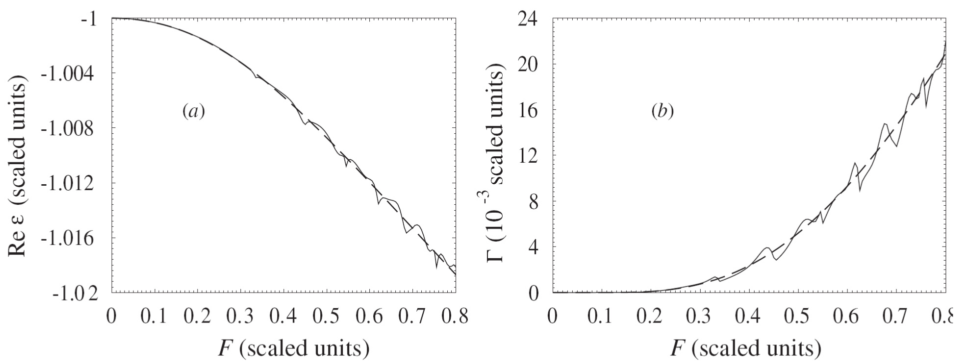

Figure 4. *F*-dependence of the real (a) and imaginary (b) parts of the complex quasienergy (ϵ = Re ϵ − iΓ/2) for ω = 0.36 and l = 1. Full curve, the exact QQES result; dashed curve, the AA result.

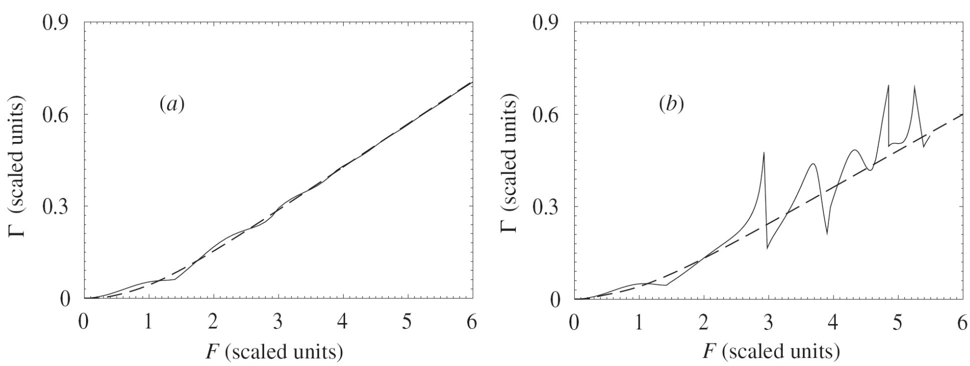

Figure 5. *F*-dependence of Γ for ω = 1.5, and l = 0 **(*a*)** and l = 1 **(*b*)**. Full curve, the exact QQES result; dashed curve, the AA result.

tant) they are shifted to the complex F plane. Thus, in strong fields the peculiarities of $\epsilon(F)$ on the real F axis become smoother. As Figures 4 and 5 demonstrate, in the strong field limit, the behavior of the exact results for $\epsilon(F)$ (when averaged over the threshold peculiarities) show surprisingly close coincidence with the AA results, even in the high-frequency domain, $\omega > 1$. Moreover, over a wide interval of nonperturbative values of F the F-dependence of Γ (averaged over threshold peculiarities) is close to linear, which is similar to the "intermediate" asymptotic (1) found for the hydrogen atom in the low-frequency limit. For instance, at $\omega = \omega_{CO_2}$ (see Table 1), the parameters for this linear dependence are $F_{cr} = 0.86$ and $k = 0.12$ for $l = 1$, and results obtained from formula (1) are in reasonable agreement with the exact ones beginning from $F > 1.5$ (or for $I > 2.25 \times 10^{12}$ W cm^{-2} for H^-). Unlike the adiabatic case ($\omega \ll 1$), for a finite frequency the interval of the applicability of the asymptotic (1) depends on ω: as ω increases, the result (1) becomes applicable at stronger fields. Namely, for $\omega = 1.5$ (when the parameters k and F_{cr} in (1) are $F_{cr} = 0.84$, $k = 0.13$ for $l = 0$, and $F_{cr} = 0.89$, $k = 0.1165$ for $l = 1$) the linear in F regime is realized with an accuracy of about 5% over the interval $2.5 < F < 10$.

In conclusion, the results presented in this letter justify our key conceptual statement, namely, that the decay of a weakly bound atomic system in a strong laser field $\boldsymbol{F}(\omega t)$ with any frequency and polarization state may be described by cycle-averaging the results for an instantaneous static electric field of strength $|\boldsymbol{F}(\omega t)|$.

Acknowledgments

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