1975

Rotational Motion

ROTATIONAL MOTION

INTRODUCTION

There is a motion of a system of masses that is as simple as the motion of a point mass on a straight line. It is the rotation of a rigid body about a fixed axis. For example, we live on a rotating earth, use rotating devices such as a potter's wheel or a phonograph turntable, and test our luck with a spinning roulette wheel. All of these are objects whose motion is described by the time dependence of a single variable, the angle of rotation. We shall study the angular equivalent of uniformly accelerated motion for some rotating objects.

This module also begins the study of rotational dynamics by introducing the dynamical quantities torque and angular momentum for a point mass moving in a plane.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate the vector product of two given vectors (needed for Objectives 1 and 2 of this module)	Vector Multiplication Module
*Describe the motion of a body in uniform circular motion (needed for Objectives 1 and 2 of this module)	Planar Motion
*Mathematically describe the change of linear momentum of a particle or system of particles as a function of time (needed for Objective 3 of this module)	Impulse and Momentum Module
*Apply Newton's second law to the solution of mechanical problems (needed for Objective 3 of this module)	Newton's Laws Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Rotational kinematics - Define angular displacement, velocity, and acceleration for the case of rotation of a rigid body about a fixed axis; for the case of constant angular acceleration, use the relation among these quantities to solve problems in rotational motion.

2. Angular-linear relation - Using the solution of a problem in angular variables, determine the linear displacement, velocity, and acceleration of a point on the rotating body.

3. Rotational dynamics - Define torque and angular momentum and apply them to a point mass moving in a plane. For some specific examples, calculate torque and angular momentum from force and velocity; show in such examples that the time rate of change of angular momentum is equal to the torque.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read the General Comments. Then read Chapter 10; be sure you understand the definition of angle measured in radians as introduced in Section 10.1 and used in Eq. (10.6) and Figure 10.6. Section 10.2 is interesting but not relevant, since we limit this module to rotations about a single fixed axis. The relations of angular kinematics are summarized in Eqs. (10.2), (10.4), and (10.6). Illustrations 10.1 and 10.2 show how these equations are used, as do Problems A and C of this module. Sections 10.5 and 10.6 show how to determine linear accelerations from the angular quantities. This completes Objectives 1 and 2.

Section 11.1 in Chapter 11 introduces the dynamic relation between torque and angular acceleration. As noted in the General Comments, $mr^2\alpha$ is just the time rate of change of angular momentum if r is constant. Section 11.2 reminds you of the vector product; with this in hand, you can then go to Section 12.5 in Chapter 12 where the basic relation $\vec{\tau} = d\vec{J}/dt$ is developed ($\vec{J} = \vec{L} \equiv$ angular momentum). See Problems B and D for applications of their result. Remember that if $\vec{r}$, $\vec{F}$, $\vec{v}$ are all in the same plane, then $\vec{\tau}$ and $\vec{J}(\vec{L})$ are perpendicular to that plane. Work the problem below before taking the Practice Test.

Problem

A 3.00-kg particle is at x = 3.00 m, y = 8.0 m with a velocity of $\vec{V} = (5.0\hat{i} - 6.0\hat{j})$ m/s. It is acted on by a 7.0-N force in the negative x direction. (a) What is the angular momentum of the particle about the origin? (b) What torque about the origin acts on the particle? (c) At what rate is the angular momentum of the particle changing with time? (See Answer below.)

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text*	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems
1	Secs. 10.1 to 10.4	A	Illus. 10.1, 10.2	C	Chap. 10, Probs. 3-5	Chap. 10, Probs. 1, 2, 6
2	Secs. 10.5, 10.6	A		C	Chap. 10, Probs. 7, 8	
3	Secs. 11.1, 11.2, 12.5	B		D	Chap. 11, Probs. 1, 2; Chap. 3, Prob. 16	

*Illus. = Illustration(s).

Answer: (a) $-174\hat{k}$ kg m^2/s. (b) $+56\hat{k}$ N m. (c) $+56\hat{k}$ kg m^2/s^2.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read the General Comments. Then read Chapter 10; be sure you understand the definition of θ in radians, and remember that our relations between linear and angular quantities are true only for θ, ω, α in radian measure. Table 10-1 summmarizes the relations between kinematical variables, but beware of Eqs. (3-14) and (10-4); v and ω are functions of time, as you see in Eqs. (3-12) and (10-3). Example 1 illustrates the use of these equations. Section 10-4 relates the linear velocity and acceleration to the appropriate angular quantities. Figure 10-5 and Example 2 should make these relations clear. Now do Problems A and C, and the assigned problems in Chapter 10. This completes Objectives 1 and 2.

Before starting Chapter 11, you may want to review the definitions of vector (cross) product in Section 2-4 (pp. 19, 20). As you read Sections 11-2 and 11-3, keep in mind that if $\vec{F}$, $\vec{r}$, and $\vec{v}$ all lie in a plane, then the torque $\vec{\tau}$ and angular momentum $\vec{L}$ are always perpendicular to that plane. See Figures 11-1 and 11-3. Example 1 illustrates these concepts in a nontrivial case (same as Problem B). Also work through Problem D and the assigned problems of Chapter 11. Take the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text	
1	Secs. 10-1 to 10-3	A	Chap. 10, Ex. 1	C	Chap. 10, Probs. 9, 10, 13	Chap. 10, Probs. 7, 11, 15, 16
2	Sec. 10-4	A	Chap. 10, Ex. 2	C	Chap. 10, Probs. 1, 4	Chap. 10, Probs. 16, 19, 20
3	Secs. 11-1 to 11-3	B	Chap. 11, Ex. 1	D	Chap. 11, Probs. 1, 3, 7	Chap. 11, Probs. 2, 4-6

*Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read the General Comments. Then read Sections 9-1 through 9-4 in Chapter 9. Be sure you understand the definition of angular measure in radians; the relations found in Section 9-5 are true only for θ measured in radians. The relations between the kinematic variables are derived in Section 9-4. The example in Section 9-4 illustrates their use, as do Problems A and C of this module. The relation between linear accelerations of a point on a rotating body and the angular variables is developed in Section 9-5. This completes Objectives 1 and 2.

Torque is defined in Section 3-1 of Chapter 3, and applied to the rotational dynamics of a point mass m_i in a rotating body in Section 9-6. We shall defer discussion of an extended body and the moment of inertia I until the module Rotational Dynamics. For a point mass, I is mR^2. Angular momentum and torque are discussed for a point mass in Section 9-9. Notice that if $\vec{r}$, $\vec{F}$, and $\vec{v}$ all lie in the same plane, then the torque and angular momentum are parallel to each other and perpendicular to the plane. Problems B and D illustrate these ideas. Work the problems below before taking the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems
1	Secs. 9-1 to 9-4	A	Sec. 9-4, Example	C	9-2 to 9-4	9-1
2	Sec. 9-5	A		C	9-6, 9-9	9-5, 9-7, 9-8
3	Secs. 3-1, 9-6, 9-9	B		D	3-1, 9-42	

Problems

1. A 2.00-kg particle is initially at the location x = 4.0 m and y = 0 m. The particle is subject to a constant force of 6.0 N in the negative y direction. Relative to the point (10.0 m, 0 m): (a) What is the particle's angular momentum, and (b) the torque on the particle, both as functions of time? (c) Show for this particular example that torque equals time rate of change of angular momentum. Notice that this is just like Problem B in the Problem Set.

2. A 3.00-kg particle is at x = 3.00 m, y = 8.0 m with a velocity of $\vec{v} = (5.0\hat{i} - 6.0\hat{j})$ m/s. It is acted on by a 7.0-N force in the negative x direction. (a) What is the angular momentum of the particle? (b) What torque acts on the particle? (c) At what rate is the angular momentum of the particle changing with time?

Answers: 1. (a) $(36t)\hat{k}$ kg m^2/s. (b) $36\hat{k}$ N m.
2. (a) $-174\hat{k}$ kg m^2/s. (b) $+56\hat{k}$ N m. (c) $+56\hat{k}$ kg m^2/s^2.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read the General Comments. Then go to Section 4-5 in Chapter 4 for the definitions of angle and angular speed. Section 12-1 in Chapter 12 discusses linear and angular motion with constant acceleration. Remember that the equations that relate linear and angular quantities (e.g., $s = R\theta$) must have angles in radians. Equations (12-8) and (4-16) relate the components of linear acceleration to the angular variables and radius. This is a good place to go over Problems A and C, and work the problems in Chapters 4 and 12. This completes Objectives 1 and 2.

Sections 11-1 and 11-2 of Chapter 11 give a general discussion of angular velocity and angular momentum. You will probably want to review the definition of the vector (cross) product in Sections 2-6 and 2-7 before trying Sections 11-3 and 11-4. Remember that for the case of a particle moving in a plane, the angular momentum about a point in the same plane is always normal to the plane. The same is true of torque if the force lies in the plane; Figures 11-9 and 11-10 illustrate this point. At this point, study Example 11-1 and Problems B and D. Try the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text*	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems
1	Secs. 4-5, 12-1	A		C	12-1, 12-3	12-4
2	Secs. 4-4 to 4-6, 12-1	A		C	4-23	12-2, 12-6
3	Secs. 11-1 to 11-4	B	Ex. 11-1	D	11-1, 11-3	11-2, 11-4, 11-8, 11-9

*Ex. = Example(s).

GENERAL COMMENTS

For rotation about a fixed axis, each point in a body moves in a circle concentric with the axis of rotation (see Fig. 1). This radius R of the circle is the perpendicular distance of the point from the axis. The linear and angular motion have a simple relationship:

$$s = R\theta, \quad ds/dt = v = R\omega = R\, d\theta/dt, \quad dv/dt = a = R\alpha = R\, d\omega/dt,$$

where s, v, and a are tangential linear distance, velocity, and acceleration, respectively; θ, ω, α are the corresponding angular quantities. The above equations imply that angles θ are measured in radians; other common units for angle are degrees (e.g., 45°) or revolutions (e.g., 10 r). They are related by 2π rad = 360° = 1 r. You may wish to review the relations for linear motion given in Rectilinear Motion. Because of the above relationships between linear and angular variables, one can make the table (constant acceleration):

Linear	Angular
$s = v_0t + (a/2)t^2$	$\theta = \omega_0t + (\alpha/2)t^2$
$ds/dt = v = v_0 + at$	$d\theta/dt = \omega = \omega_0 + \alpha t$
$v^2 = v_0^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

Note that a is tangetial acceleration. A particle moving on a circle of radius R also has a radial acceleration $a_r = -v^2/R = -\omega^2R$. In either the linear or angular case, the first two equations arise by integration of $d^2s/dt^2 = a$ (const) or $d^2\theta/dt^2 = \alpha$(const). The last equation is obtained by eliminating t from the first two; it also follows from conservation of energy.

In order to discuss dynamics, i.e., the relation between forces and acceleration, we start from Newton's second law:

$$\vec{F} = (d/dt)m\vec{v} = d\vec{p}/dt$$

for a point mass m. In discussing rotations, we note that a force $\vec{F}$ is most effective in producing an angular acceleration if it is applied far from the axis of rotation, and is directed perpendicular to both a line from the axis of rotation and the axis of rotation. For example, a door is most easily opened if the knob is as far from the hinge as possible, and the force is perpendicular to the plane of the door.

The mathematical quantity that has those properties is $\vec{r} \times \vec{F}$, which is called <u>torque</u>. The vector $\vec{r}$ is measured from the axis of rotation or from an origin. The vector product $\vec{r} \times \vec{F}$ is largest when $\vec{r}$ and $\vec{F}$ are perpendicular, and the direction of $\vec{r} \times \vec{F}$ is along the axis of the rotation caused by $\vec{F}$. We can introduce torque into the equation of motion of a point mass as follows (see Fig. 2). Start with Newton's second law:

$$\vec{F} = m\vec{a} = m(d\vec{v}/dt).$$

Form the vector product of $\vec{r}$ with each side:

$$\vec{r} \times \vec{F} = \vec{r} \times m(d\vec{v}/dt).$$

Now introduce the angular momentum $\vec{L}$ defined by $\vec{L} = \vec{r} \times m\vec{v}$, and notice that

$$d\vec{L}/dt = (d/dt)(\vec{r} \times m\vec{v}) = (d\vec{r}/dt) \times m\vec{v} + \vec{r} \times m(d\vec{v}/dt), \quad d\vec{L}/dt = \vec{r} \times m(d\vec{v}/dt).$$

The last equation follows from the fact that

$$(d\vec{r}/dt) \times m\vec{v} = \vec{v} \times m\vec{v} = 0.$$

(The vector product of parallel vectors is zero.) We can thus write

$$\vec{r} \times \vec{F} = (d/dt)(\vec{r} \times m\vec{v}) = d\vec{L}/dt, \quad \text{TORQUE} = (d/dt)(\text{ANGULAR MOMENTUM}).$$

For the case of motion and forces in a plane, torque and angular momentum are always perpendicular to the plane; the angular momentum can be expressed simply in terms of angular velocity as follows:

$$\vec{r} = (r \cos\theta)\hat{i} + (r \sin\theta)\hat{j},$$

$$\vec{v} = [(dr/dt)(\cos\theta) - (d\theta/dt)(r \sin\theta)]\hat{i} + [(dr/dt)(\sin\theta) + (d\theta/dt)(r \cos\theta)]\hat{j},$$

$$\begin{aligned}\vec{L} &= \vec{r} \times m\vec{v} \\ &= [(r\cos\theta)\hat{i} + (r\sin\theta)\hat{j}] \times m\{[(dr/dt)(\cos\theta) - (d\theta/dt)(r\sin\theta)]\hat{i} \\ &\qquad + [(dr/dt)(\sin\theta) + (d\theta/dt)(r\cos\theta)]\hat{j}\}.\end{aligned}$$

All terms with dr/dt add to zero because the radial component of $\vec{v}$ is parallel to $\vec{r}$ (see Fig. 3):

$$\vec{L} = [mr^2 \cos^2\theta(d\theta/dt) + mr^2 \sin^2\theta(d\theta/dt)]\hat{k} = (mr^2\omega)\hat{k}.$$

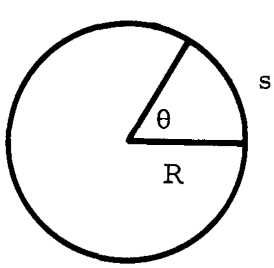

Figure 1

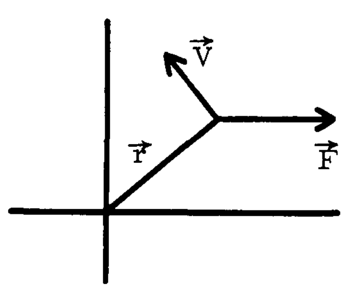

Figure 2

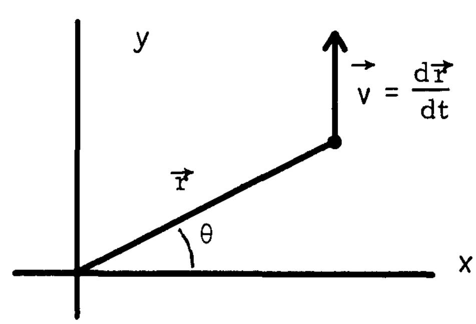

Figure 3

This result is very important and is used in the following module Rotational Dynamics. In the special case that r is constant, we get

$$\tau_z = dL_z/dt = mr^2(d\omega/dt) = mr^2\alpha.$$

This result for a point mass m moving in a circle of radius r is the basis for the treatment of an extended rigid body in Rotational Dynamics. It says that the angular acceleration depends not only on the torque and mass, but also on the distribution of mass (distance from the axis of rotation). The term mr^2 is the rotational inertia, or moment of inertia (I) for a point mass.

PROBLEM SET WITH SOLUTIONS

A(1,2). A phonograph turntable is turning at 3.49 rad/s (33 1/3 r/min) and has a radius of 0.150 m. A friction brake brings it to rest with uniform acceleration in 15.0 s.

(a) What is the angular acceleration of the turntable?

(b) In how many revolutions does it stop?

(c) At ω = 3.00 rad/s while slowing down, what are the radial and tangential accelerations of a point on the turntable rim?

Solution

(a) $\omega = \omega_0 + \alpha t$ has the right variables.

$$\alpha = (\omega - \omega_0)/t = \frac{0 - 3.49\ \text{rad/s}}{15.0\ \text{s}} = -0.233\ \text{rad/s}^2.$$

(b) $\theta = \omega_0 t + (\alpha/2)t^2 = (3.49\ \text{rad/s})(15.0\ \text{s}) + (1/2)(-0.233\ \text{rad/s}^2)(15.0\ \text{s})^2$

$= 26.1\ \text{rad} = 26.1\ \text{rad}\ (1\ \text{r}/2\pi\ \text{rad}) = 4.16\ \text{r}.$

(c) $a_r = -\omega^2 r = -(3.00\ \text{rad/s})^2(0.150\ \text{m}) = -1.35\ \text{m/s}^2.$

$a_t = \alpha r = (-0.233\ \text{rad/s}^2)(0.150\ \text{m}) = -0.035\ \text{m/s}^2.$

B(3). A mass m falls under gravity as shown in Figure 4. The motion is given by $x = s$,

$$y = -(g/2)t^2, \quad v_y = dy/dt = -gt.$$

Calculate the torque and angular momentum about 0 and show that

TORQUE = (d/dt)(ANGULAR MOMENTUM).

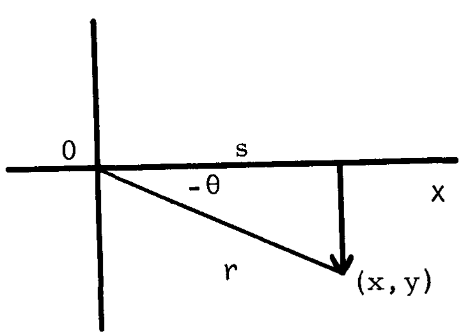

Figure 4

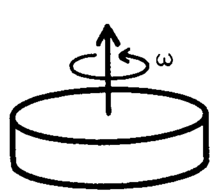

Figure 5

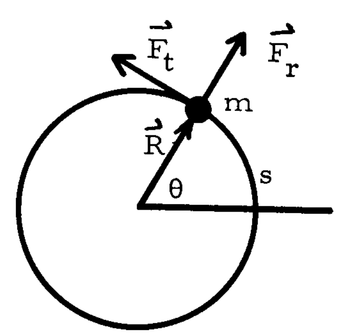

Figure 6

Solution

Since the motion is in a plane all torques and angular momenta are normal to the plane; let $\hat{k}$ be a unit vector out of the paper. Then

$$\vec{\tau} = \vec{r} \times \vec{F} = (s\hat{i} + y\hat{j}) \times (-mg\hat{j}) = -mgs\hat{k},$$
$$\vec{L} = \vec{r} \times \vec{p} = (s\hat{i} + y\hat{j}) \times (-gt\hat{j})m = -mgst\hat{k}.$$

Notice that $\vec{L}$ is just t multiplied by a constant. Differentiate with respect to time:

$$d\vec{L}/dt = (d/dt)(-mgst\hat{k}) = -mgs\hat{k} = \vec{\tau}.$$

Notice that we can write $\vec{L}$ in terms of angular variables. From Figure 4, we see that

$$v_y = s \tan \theta, \qquad v_y = dy/dt = (dy/d\theta)(d\theta/dt) = s\omega/\cos^2 \theta.$$

Substituting, we find

$$\vec{L} = mv_y s\hat{k} = m(s^2/\cos^2\theta)\omega\hat{k} = mR^2\omega\hat{k}.$$

This is a general result mentioned in the General Comments.

Problems

C(1,2). You live on a disk-shaped asteroid of radius 200 m (see Figure 5).

(a) If the radial acceleration at the edge is 9.8 m/s^2, what is the angular velocity?

(b) The acceleration to the above velocity from rest is done with a tangential linear acceleration of 0.50 m/s^2. How long does it take?

D(3). A point mass m moving on a circle of constant radius R is accelerated from rest ($s = v = 0$ at $t = 0$) by a force whose tangential component F_t is constant. See Figure 6.

(a) What is the torque about O.

(b) What is the angular momentum about O at time t?

(c) Show that $\vec{\tau} = d\vec{L}/dt$.

Solutions

C(1,2). (a) $a_r = -\omega^2 R,$

$$\omega = (-a_r/R)^{1/2} = (9.8\ m/s^2/200\ m)^{1/2} = 0.221\ rad/s.$$

(b) $a_t = \alpha R, \qquad \alpha = 0.0025\ rad/s^2, \qquad \omega = \alpha t,$

$$t = \frac{\omega}{\alpha} = \frac{0.221\ rad/s}{0.0025\ rad/s^2} = 89\ s.$$

D(3). (a) Notice that whereas there is usually a radial component of the force it does not contribute to the torque. Only the component perpendicular to R contributes, so $\vec{\tau} = \hat{k}RF_t$.

(b) Use Newton's second law to get a:

$\vec{F}_t = m\vec{a}$, $\quad a = (F_t/m)t$, then $V = at = (F_t/m)t$.

$\vec{L} = \vec{r} \times m\vec{v} = Rm(F_t/m)t\hat{k} = RF_t t\hat{k}$.

(c) As in the last example, notice that $\vec{L}$ is just a constant multiplied by t:

$d\vec{L}/dt = (d/dt)\hat{k}RF_t t = RF_t\hat{k} = \vec{\tau}$.

PRACTICE TEST

1. While waiting to board a helicopter, you notice that the rotor angular velocity went from 300 r/min to 225 r/min with constant acceleration in one minute. The rotor radius is 5.0 m.

   (a) What is the angular acceleration?

   (b) Assuming constant angular acceleration, how long will it take to stop from 225 r/min?

   (c) How many revolutions will it make in this time?

   (d) At 225 r/min, what are the radial and tangential accelerations at the rotor tip?

2. A planet of mass m moves in a circular orbit about the sun at constant speed v. The orbit has radius R, and a constant magnitude force F is directed toward the sun (see Figure 7).

   (a) What is the torque about the sun?

   (b) What is the angular momentum of the planet about the sun?

   (c) Show that $\vec{\tau} = d\vec{L}/dt$.

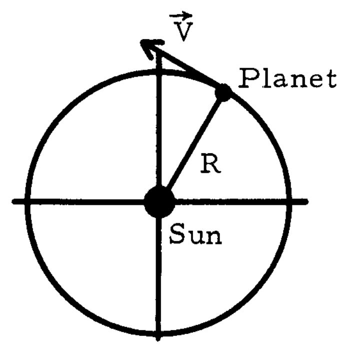

Figure 7

Practice Test Answers

1. (a) $\alpha = -75$ r/min^2,
   $\alpha = -0.131$ rad/s^2.
   (b) $t = 3.00$ min $= 180$ s.
   (c) $\theta = 337.5$ r $= 338$ r.
   (d) $a_t = 0.67$ m/s^2,
   $a_r = 2776$ m/s^2.

2. (a) $\vec{\tau} = 0$, $(\vec{r} \times \vec{F}) = 0$ because $\vec{r}$, $\vec{F}$ are parallel.
   (b) $\vec{L} = mvR\hat{k}$.
   (c) $\vec{\tau} = 0 = d\vec{L}/dt$.

ROTATIONAL MOTION

Mastery Test Form A

Date________________

pass recycle

1 2 3

Name ______________________ Tutor ______________________

1. The spin drier of a washing machine initially turning at 20.0 rad/s slows down uniformly to 10.0 rad/s in 50 revolutions. The drier is a cylinder 0.300 m in radius.

   (a) What is the angular acceleration?

   (b) What is the time required for the 50 revolutions?

   (c) What are the radial and tangential accelerations on the side of the drier as it begins to slow down?

2. A point mass m at $r = x\hat{i} + y\hat{j}$ has a velocity $v = v_x\hat{i} + v_y\hat{j}$ and is accelerated by a force $\vec{F} = F\hat{j}$. If $\vec{\tau}$ and $\vec{L}$ are referred to the origin,

   (a) Calculate the torque $\vec{\tau}$ about the origin.

   (b) Calculate the angular momentum $\vec{L}$ about the origin.

   (c) Show that $\vec{\tau} = d\vec{L}/dt$. Remember that $\vec{v} = d\vec{r}/dt$ and $\vec{F} = m\vec{a}$.

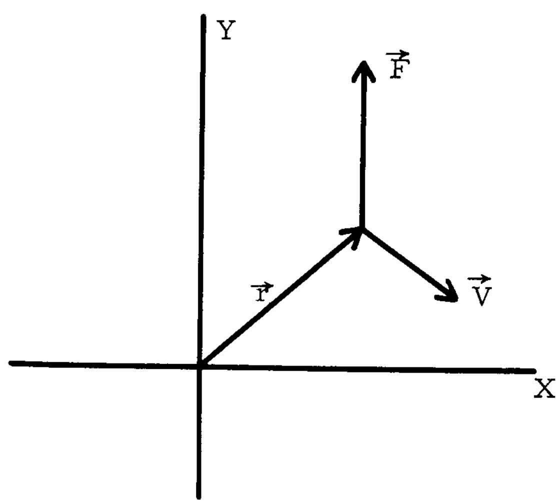

Figure 1

ROTATIONAL MOTION

Mastery Test Form B

Date ______________

pass recycle

1 2 3

Name ______________________ Tutor ______________

1. According to measurements made by cesium clocks, the earth is slowing down at a rate $\alpha = -3 \times 10^{-9}$ r/d yr. If this rate is constant,

   (a) How many days (of current length) will it take to stop? 1 d = 86 400 s.

   (b) How many revolutions (sidereal days) will it make in stopping?

   (c) What is the present radial acceleration of a point on the earth's equator?

   ($R = 6.4 \times 10^6$ m, 1 d = 86 400 s.)

2. A pendulum has a mass m on the end of a massless rod of length R and moves under gravity. Using θ as a coordinate, and the result

   $$|\vec{A} \times \vec{B}| = |\vec{A}| \cdot |\vec{B}| \sin \alpha,$$

   (a) Calculate torque about 0.

   (b) Calculate angular momentum about 0.

   (c) Show that $d^2\theta/dt^2 = -(g/R) \sin \theta$.

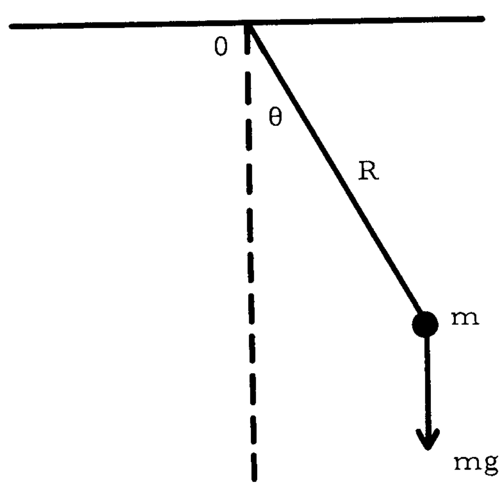

Figure 1

ROTATIONAL MOTION

Mastery Test   Form C

Date ____________

pass   recycle

1   2   3

Name ____________________   Tutor ____________

1. A particle of mass 8.0 kg moves through the point $\vec{r} = (-4.0\hat{i} - 6.0\hat{j})$ m with the velocity $\vec{v} = (6.0\hat{i} + 4.0\hat{j})$ m/s. A force $\vec{F} = (2.00\hat{i} - 3.00\hat{j})$ N acts on the particle.

   (a) What is the torque on the particle with respect to the origin?

   (b) What is the angular momentum of the particle with respect to the origin at this time?

   (c) What is the rate of change of the particle angular momentum?

2. Astronaut training can include work in a centrifuge (rotating cylinder) of 6.0 m radius that spins with an angular velocity $\omega = 2.00$ rad/s.

   (a) What is the radial acceleration at R = 6.0 m?

   (b) The angular velocity is increased to 3.00 rad/s in 15 r. What is the angular acceleration, and how long does it take?

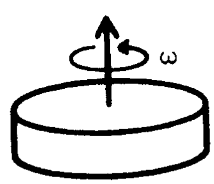

Figure 1

MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
1. Choose correct equation. Convert θ to radians. Correct units and signs.	1. (a) $\alpha = (\omega^2 - \omega_0^2)/2\theta$, $\omega = 10.0$ rad/s, $\omega_0 = 20.0$ rad/s, $\theta = 100\pi$ rad, $\alpha = -0.48$ rad/s^2.
	(b) $t = (\omega - \omega_0)/\alpha = 20.96$ s.
	(c) $a_r = -\omega^2 r = -120$ m/s^2, $a_t = \alpha r = -0.143$ m/s^2.
2. Definitions of $\vec{\tau}$, $\vec{L}$. Evaluation of cross product. Use $V_x = dx/dt$, $F_x = m(dV_x/dt)$.	2. (a) $\vec{\tau} = \vec{r} \times \vec{F} = (x\hat{i} + y\hat{j}) \times F\hat{j}$, $\tau = xF\hat{k}$.
	(b) $\vec{L} = \vec{r} \times m\vec{v} = (x\hat{i} + y\hat{j}) \times m(V_x\hat{i} + V_y\hat{j})$, $\vec{L} = (xV_y - yV_x)\hat{k}\ m$.
	(c) $\frac{d\vec{L}}{dt} = (\frac{dx}{dt}V_y + x\frac{dV_y}{dt} - \frac{dy}{dt}V_x - y\frac{dV_x}{dt})\hat{k}\ m$ $= (x\frac{dV_y}{dt} - y\frac{dV_x}{dt})\hat{k}\ m = \hat{k} \times \vec{F} = \vec{\tau}$.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1. Choose correct equation. Convert to SI units or carry given units through equation.	1. (a) $t = (\omega - \omega_0)/\alpha$, $t = \dfrac{1.00\ \text{r/d}}{3.00 \times 10^{-9}\ \text{r/d yr}}$ $= 3.30 \times 10^{8}\ \text{yr} = 1.20 \times 10^{11}\ \text{d}$.
	(b) $\theta = \omega_0 t + (1/2)\alpha t^2 = 1.00\ \text{r/d} \times 10^{8}\ \text{yr}$ $= \dfrac{-3.00 \times 10^{-9}\ \text{r/d yr}}{2}(3.30 \times 10^{8}\ \text{yr})^2$ $= 6.1 \times 10^{10}\ \text{r}$.
(c) ω must be in radians per second to calculate a_r.	(c) $\omega = \dfrac{2\pi}{86\ 400\ \text{s}} = 7.3 \times 10^{-5}\ \text{rad/s}$, $a_r = -\omega^2 r = -0.034\ \text{m/s}^2$.
2. (a) Definition of $\vec{\tau}$.	2. (a) $\vec{\tau} = \vec{r} \times \vec{F} = mgR \sin\theta$ (into paper).
(b) Definition of $\vec{L}$.	(b) $\vec{L} = \vec{r} \times m\vec{v} = mR^2(d\theta/dt)$ (out of paper).
	(c) $d\vec{L}/dt = \vec{\tau}$, $mR^2(d^2\theta/dt^2) = -mgR \sin\theta$, $d^2\theta/dt^2 = -(g/R)\sin\theta$.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1.(a) Definition of $\vec{\tau}$. Evaluation of vector product. Units.	1.(a) $\vec{\tau} = \vec{r} \times \vec{F} = (-4.0\hat{i} - 6.0\hat{j}) \times (2.00\hat{i} - 3.00\hat{j})$ $= (12.0 + 12.0)\hat{k}$ N m $= 24.0\hat{k}$ N m.
(b) Definition of $\vec{L}$. Vector product. Units.	(b) $\vec{L} = \vec{r} \times m\vec{v} = (-4.0\hat{i} - 6.0\hat{j}) \times 8(6.0\hat{i} + 4.0\hat{j})$ $= 8(-16.0 + 36)\hat{k}$ kg m^2/s $= 160\hat{k}$ kg m^2/s.
	(c) $d\vec{L}/dt = \vec{\tau} = 24.0\hat{k}$ N m.
2.(a) Choose correct equation.	2.(a) $a_r = -\omega^2 r = -(2.00 \text{ rad/s})^2 (6.0 \text{ m})$ $= -24.0 \text{ m/s}^2$.
(b) Choose correct equation. θ is in radians, not revolutions.	(b) $\alpha = (\omega^2 - \omega_0^2)/2\theta$, $\omega_0 = 2.00$ rad/s, $\omega = 3.00$ rad/s, $\theta = 15.0$ r $= 30.0\pi$ rad, $\alpha = 0.0265 \text{ rad/s}^2$, $t = (\omega - \omega_0)/\alpha = 37.7$ s ≈ 38 s.