1975

Magnetic Forces

STUDY GUIDE

MAGNETIC FORCES

INTRODUCTION

It may surprise you to learn that the conversion of electrical energy to mechanical work in electric motors or stereo loud speakers is seldom done by electrostatic forces (Coulomb's law). Magnetic forces associated with moving charges (currents) are the basis of most electromechanical devices. In analogy with the electrostatic case, we introduce an intermediary called the magnetic field. This module considers the forces on currents or moving charges in a magnetic field; the module Ampere's Law will show how magnetic fields are generated by currents.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate vector products (needed for Objectives 1 through 4 of this module)	Vector Multiplication Module
*Define electric charge and field (needed for Objectives 1 and 4 of this module)	Coulomb's Law and the Electric Field Module
*Define current in terms of carrier density and velocity (needed for Objectives 2 through 4 of this module)	Ohm's Law Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Magnetic force - particles - Calculate the force on a moving charged particle in a uniform magnetic field; for the case of $\vec{v}$ perpendicular to $\vec{B}$, find the radius and/or frequency of the resulting circular orbit.

2. Magnetic force - wires - Calculate the force on a current-carrying wire in a uniform magnetic field.

3. Magnetic dipole - Calculate the magnetic moment of a current loop; use this to determine the torque on such a loop in a uniform magnetic field.

4. Hall effect - For problems with balanced electric and magnetic forces (Hall effect, velocity selectors) use the relation $v = E/B$ to relate the fields to parameters such as the Hall field or potential difference, current density, or charge sign and velocity.

GENERAL COMMENTS

Be sure that you are able to find the vector (cross) product of two vectors before reading any further. It is essential to everything in this module. Unlike electric forces and electric fields, magnetic forces are perpendicular to the magnetic field, a relation described mathematically by the cross product.

You will find that a closed circuit carrying a current I experiences a total force (not torque) equal to zero if it is in a uniform magnetic field. An interesting consequence is that the force on a part of such a circuit is independent of the path of the current, and hence is the same as the force on a straight wire connecting the end points.

Notice that although a magnetic dipole (a current-carrying loop) in a magnetic field behaves just like an electric dipole in an electric field, it is a very different object internally. There are no known magnetic charges; the magnetic dipole is a circulating current of electric charge (current loop), which produces the same fields far from the dipole and experiences the same torques as a dipole made from a charge pair.

We also consider the case in which $\vec{v}$, $\vec{B}$, and $\vec{E}$ form a right-handed set of orthogonal vectors like $\hat{i}$, $\hat{j}$, and $\hat{k}$. If $\vec{E} = \vec{B} \times \vec{v}$, then the total force on a charge moving with velocity $\vec{v}$ is zero. This is the basis of velocity filters for charged particles. It also occurs if a current-carrying conductor is placed in a magnetic field. The charge carriers are displaced in the direction $q\vec{v} \times \vec{B}$ until an electric force $q\vec{E}$ caused by the excess charge on one side makes the net transverse force on the charge carrier equal to zero. This field $\vec{E}$ is called the Hall field, and is used to investigate the sign and density of charge carriers in a conductor. One can also use it to measure the magnetic field.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Your text readings are from Chapter 23. Read the General Comments; then go through the textbook in the order given in the Table. As you complete the section for each objective, study the corresponding Problems with Solutions in the Problem Set; then work the Assigned Problems until you have mastered the associated objective. Then take the Practice Test.

An important application of the principles of Objective 1 arises from Illustration 23.5, where there is a calculation of the period of the circular orbit of a particle of mass m in a uniform field $\vec{B}$. The general result is $T = 2\pi m/qB$ independent of the particle energy. This fact is used in the particle accelerator called the cyclotron. Since the period is energy independent, an alternating electric field $E_y = E_0 \sin(2\pi t/T)$ will accelerate the particle. The energy, momentum, and orbit radius all increase as long as the particle remains in both the electric and the magnetic field. This principle is used to accelerate particles to high energies for nuclear research, to study electrons in solids, and to heat ionized gases in thermonuclear research.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
2	Secs. 23.1 to 23.4	B	Illus.[a] 23.1, 23.2, 23.3	E, F	1, 2, 3	4, 16
1	Sec. 23.5	A	Illus. 23.4, 23.5	D	5, 8, 9	10, 11
3	Secs. 23.7, 23.8	C		E	13, 14, 15	
4	Sec. 23.6	D	Illus. 23.6	G	7	

[a]Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

All of your text readings are in Chapter 29. Read the General Comments; then go through the textbook in the order given in the Table below. As you complete the reading for each objective, study the corresponding Problems with Solutions in the Problem Set; then work the Assigned Problems until you have mastered the associated objective. Take the Practice Test and work some Additional Problems if necessary before trying a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 29-1, 29-2, 29-6, 29-7	A	Ex[a]. 1, 4, 5	D	1, 5, 21, 25	4, 6, 23
2	Sec. 29-3	B		E, F	7, 9, 11	8, 10, 12
3	Sec. 29-4	C	Ex. 2	E	13, 15	16
4	Secs. 29-5, 29-8	D		G	19, 41	18, 20, 42

[a]Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Your text readings are from Chapters 30 and 31. Read the General Comments; then go through the textbook in the order given in the Table below. As you complete the section for each objective, study the problems with solutions in the Problem Set, then work the Assigned Problems until you have mastered the associated objective. Try the Practice Test and work some Additional Problems if necessary before taking a Mastery Test. For Objective 3, be sure to note the relation between direction of current and the direction of the magnetic moment for a current loop.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1	Secs. 30-1, 30-2, 30-4, 30-10	A	D	30-1, 30-3, 30-5	30-6, 30-13, 30-19
2	Sec. 31-1	B	E, F	31-1	
3	Sec. 31-3	C	E	31-5	31-4, 31-6
4	Secs. 30-5, 31-2	D	G	30-12, 31-3	31-2

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Your text readings are from Chapter 29. Read the General Comments; then go through the textbook in the order given in the Table below. As you complete the section for each objective, study the corresponding Problems with Solutions in the Problem Set; then work the Assigned Problems until you have mastered the associated objective. Take the Practice Test and work some Additional Problems if necessary before trying a Mastery Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems
1	Secs. 29-1, 29-3, 29-5	A	Ex[a]. 29-2, 29-3	D	29-1, 29-3, 29-11	29-4, 29-8
2	Sec. 29-6	B	Ex. 29-4	E, F	29-16[b]	29-17
3	Secs. 29-7, 29-8	C		E	29-18	29-20, 29-21, 29-23
4	Secs. 29-4, 29-9	F	Ex. 29-5	G	29-9, 29-24	

[a] Ex. = Example(s).

[b] $I = (mg/LB) \tan \theta$.

PROBLEM SET WITH SOLUTIONS

A(1). A particle of mass 10^{-27} kg is moving with speed 10^5 m/s perpendicular to a magnetic field $B = 5.0 \times 10^{-3}$ T* (charge of +e = 1.60×10^{-19} C).

(a) What is the force on the particle?

(b) What is the radius of the circle in which it moves?

(c) What is the period for its motion?

(d) If its speed were doubled, what would its period be?

Solution

(a) Use the magnetic force equation:

$$\vec{F} = q\vec{v} \times \vec{B} = (1.60 \times 10^{-19}\ \text{C})(10^5\ \text{m/s})(5.0 \times 10^{-3}\ \text{T}) = 8.0 \times 10^{-17}\ \text{N}$$

perpendicular to $\vec{v}$, $\vec{B}$.

(b) $\vec{F} = m\vec{a}$. $a = v^2/R$ for circular motion. $qvB = m(v^2/R)$.

$$R = \frac{mv}{qB} = \frac{(10^{-27}\ \text{kg})(10^5\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(5.0 \times 10^{-3}\ \text{T})} = 0.125\ \text{m}.$$

(c) $$T = \frac{2\pi R}{v} = 2\pi\left(\frac{m}{qB}\right) = 2\pi\left(\frac{10^{-27}\ \text{kg}}{(1.60 \times 10^{-19}\ \text{C})(5.0 \times 10^{-3}\ \text{T})}\right) = 7.9 \times 10^{-6}\ \text{s}.$$

(d) From part (c), T is independent of v. No change.

B(s). A square loop of wire, 30.0 cm on each edge, lies in the xy plane with edges parallel to the axes as in Figure 1, and carries a current of 4.0 A. It is in a uniform field $\vec{B} = (0.100\hat{j} + 0.173\hat{k})$ T.

(a) Draw $\vec{B}$ in a $B_xB_yB_z$ coordinate system.

(b) Calculate the force acting on each side of the square wire. Add the forces on the four sides to get the total force.

Solution

(a) See Figure 2. tan (30°) = B_y/B_z = 1/1.73.

(b) See Figure 3. Let L be the length of one side of the loop. Then

$$\vec{F}_1 = IL\hat{j} \times \vec{B} = IL\hat{j} \times (B_y\hat{j} + B_z\hat{k}) = ILB_z\hat{i}, \qquad \vec{F}_3 = -IL\hat{j} \times \vec{B} = -\vec{F}_1,$$

*In SI units, the tesla is used for magnetic flux density. 1 T = 1 Wb/m^2.

$$\vec{F}_2 = -\hat{i}IL \times (B_y\hat{j} + B_z\hat{k}) = -ILB_y\hat{k} + ILB_z\hat{j}, \qquad \vec{F}_4 = \hat{i}IL \times \vec{B} = -\vec{F}_2.$$

Since $\vec{F}_3 = -\vec{F}_1$, $\vec{F}_4 = -\vec{F}_2$,

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{F}_1 + \vec{F}_2 - \vec{F}_1 - \vec{F}_2 = 0,$$

which is always the case in a uniform field.

$$ILB_y = (24\ \text{A})(0.300\ \text{m})(0.100\ \text{T}) = 0.120\ \text{N}, \qquad ILB_z = 0.210\ \text{N}.$$

Figure 1

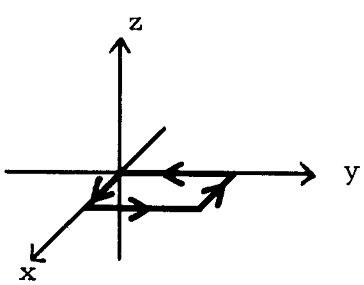


Figure 2

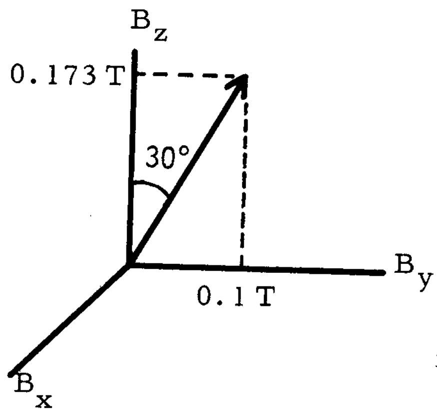
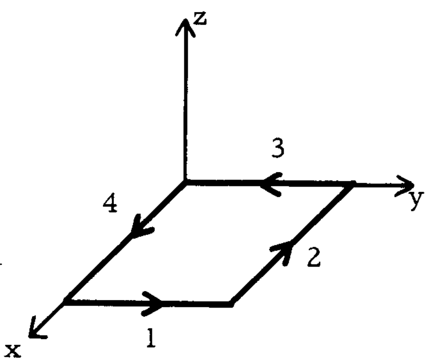


Figure 3



C(3). For the same loop as in Problem B (Figure 1),
(a) What is the vector dipole moment of the loop?
(b) Using the result of part (a), find the vector torque acting on the loop.

Solution

(a) $\vec{\mu} = IA\hat{k}$, where I is the current, and A is the cross-sectional area of the loop. The direction of $\vec{\mu}$ follows a right-hand rule; if fingers follow the current, the thumb picks the correct normal to the loop. Hence

$$\vec{\mu} = [(4.0\ \text{A})(0.300\ \text{m})^2]\hat{k} = 0.36\hat{k}\ \text{A m}^2.$$

(b) $\vec{\tau} = \vec{\mu} \times \vec{B} = \hat{k} \times (B_y\hat{j} + B_z\hat{k}) = -\mu B_y\hat{i} = -0.036\hat{i}$ N m.

D(1, 4). A particle in Figure 4 moving with speed 10^6 m/s along the y axis enters a magnetic field $\vec{B} = B_0\hat{k}$, $B_0 = 0.40$ T, $q = 1.60 \times 10^{-19}$ C, $m = 10^{-27}$ kg.

(a) What is the force on the particle?
(b) What will be the path of the particle if it stays in the field?
(c) What electric field $\vec{E}$ will result in zero net force on the particle?

Solution

(a) $\vec{F} = q\vec{v} \times \vec{B} = (1.60 \times 10^{-19}\ \text{C})(10^6\hat{j}\ \text{m/s})(0.40\hat{k}\ \text{T}) = 6.4 \times 10^{-14}\hat{i}\ \text{N}.$

(b) A circle parallel to the xy plane:

$$R = \frac{mv}{qB} = \frac{(10^{-27}\ \text{kg})(10^6\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.40\ \text{T})} = 1.56 \times 10^{-2}\ \text{m}.$$

(c) $\vec{F} = q(\vec{v} \times \vec{B}) = 0.$

$E = -\vec{v} \times \vec{B} = (-10^6\hat{i}\ \text{m/s})(0.40\hat{k}\ \text{T}) = -4.0 \times 10^5\hat{i}\ \text{V/m}.$

Problems

E(2, 3). A square loop as shown in Figure 5 is pivoted about the z axis and carries a current I = 10.0 A. The loop is in a uniform magnetic field B = 0.50 T parallel to the y axis.
(a) What is the magnitude and direction of force on the side labeled a?
(b) What torque is acting on the loop? (Give magnitude and direction.)
(c) What is the total force on the loop?

Figure 4

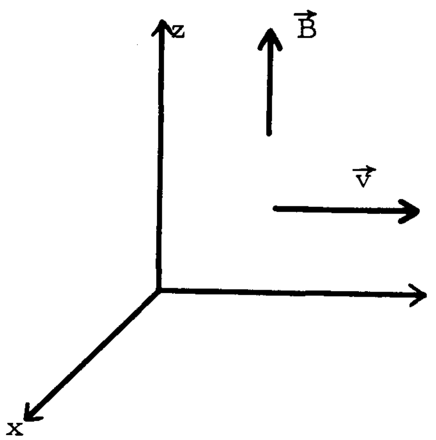
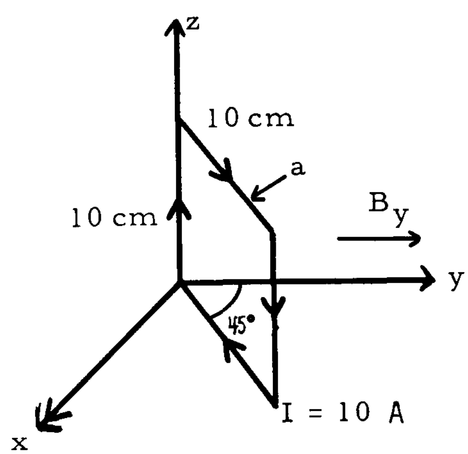


Figure 5



F(2). In Figure 6 a copper rod weighing 2.00 N rests on two horizontal rails 1.00 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction μ is 0.60. What is the smallest vertical magnetic field that would cause the bar to slide, and what is its direction? Remember that the frictional force = μ (normal force).

G(4). The conductor in Figure 7 with square cross section and side a = 0.0250 m carries a current I = 100 A. A magnetic field B_0 = 0.60 T is parallel to the positive z axis. The current is carried by electrons with $q = -1.60 \times 10^{-19}$ C.

(a) What is the direction of the Hall field?

(b) If the density of carriers is 4.0×10^{28} per cubic meter, what is the speed of the electron?

(c) What is the Hall voltage?

Figure 6

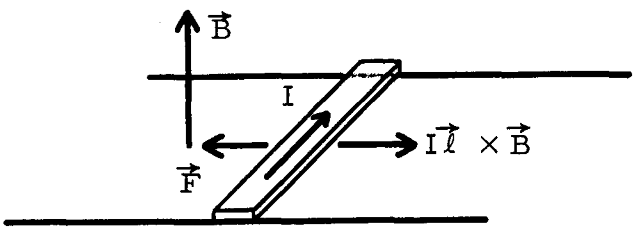
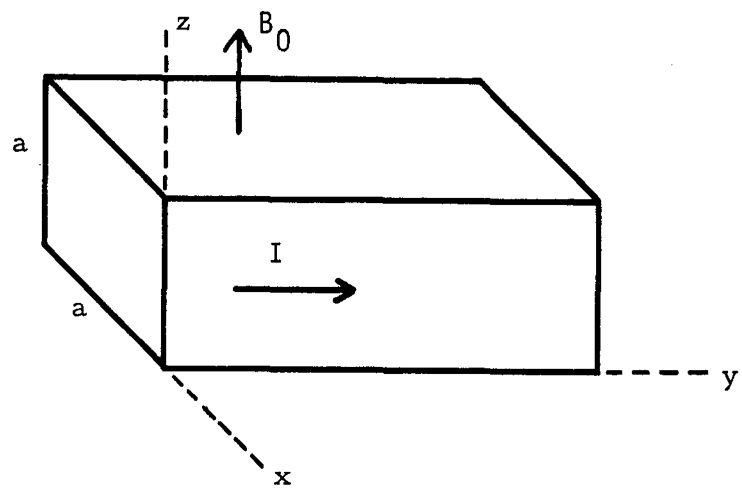


Figure 7



<u>Solutions</u>

E(2, 3). (a) $\vec{F} = I\vec{L} \times \vec{B} = (10.0\ A)(0.100\ m)(\hat{i}/\sqrt{2} + \hat{j}/\sqrt{2})(0.57)\hat{j} = 0.35\hat{k}$ N.

(b) $\vec{\mu} = IA(-\hat{i}/\sqrt{2} + \hat{j}/\sqrt{2}) = (0.100\ A\ m^2)(-\hat{i}/\sqrt{2} + \hat{j}/\sqrt{2})$.

$$\vec{\tau} = \vec{\mu} \times \vec{B} = (0.100\ A\ m^2)(-\hat{i}/\sqrt{2} + \hat{j}/\sqrt{2})(0.57)\hat{j} = -0.035\hat{k}\ N\ m.$$

(c) $\vec{F} = 0$ for closed loop in uniform field.

F(2). B = 0.0240 T up. The rod would move to the right.

G(4). (a) Since carriers have negative charge, they go to the left if the current is to the right. Hall condition is $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})] = 0$.

$\vec{E} = -\vec{v} \times \vec{B}$ points along the positive x axis.

(b) $I = nqvA$,

$$v = \frac{I}{nqA} = \frac{100\ A}{(4.0 \times 10^{28}/m^3)(1.60 \times 10^{-19}\ C)(0.025\ m)^2} = 2.50 \times 10^{-5}\ m/s.$$

(c) $V = Ea$ = Hall voltage,

$E = vB = (2.50 \times 10^{-5}\ \text{m/s})(0.60\ \text{T}) = (1.50 \times 10^{-5}\ \text{V/m})$,

$V = (1.50 \times 10^{-5}\ \text{V/m})(0.0250\ \text{m}) = 3.8 \times 10^{-7}\ \text{V}$.

PRACTICE TEST

1. A four-sided wire loop as shown in Figure 8 (not rectangular) lies in a plane parallel to a uniform magnetic field $\vec{B} = 0.050\hat{i}$ T. A current of $I = 3.00$ A flows clockwise around the loop, as illustrated in the figure.

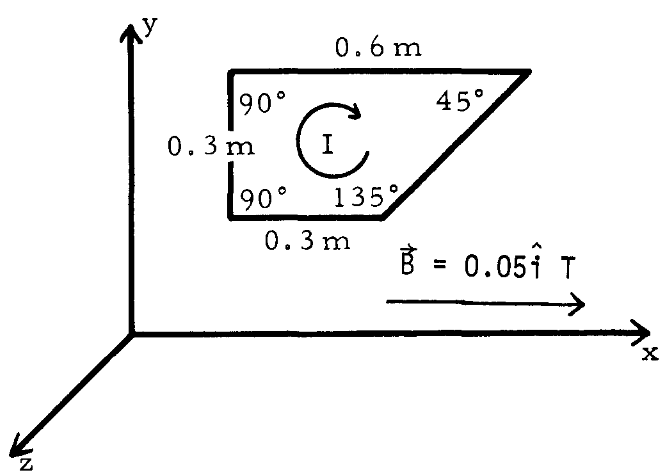
   (a) Calculate the magnitude and direction of the force acting on each straight-line segment of the loop.

   (b) Calculate the resultant force acting on the loop.

   (c) Calculate the magnetic moment of the loop.

   (d) Calculate the torque acting on the loop about any point in the plane of the loop.

Figure 8



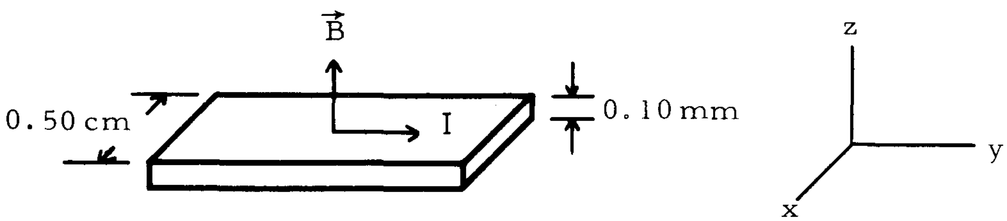
2. The silver wire in Figure 9 is in the form of a ribbon 0.50 cm wide and 0.100 mm thick. A 2.0-A current is passed through the ribbon perpendicular to a 0.80-T magnetic field. The number of charge carriers in silver is 6.0×10^{28} per cubic meter, thus the drift velocity is 4.2×10^{-4} m/s.

   (a) What is the magnetic force on an electron in the wire?

   (b) This force causes a charge accumulation on the sides of the wire until the magnetic force is balanced by an equal electric force. What are the magnitude and direction of the electric field thus produced?

(c) What is the Hall voltage produced across the width of the ribbon?

Figure 9

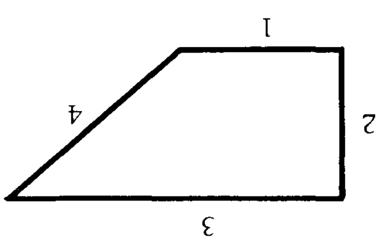


3. A charged particle is accelerated through a potential difference of 5000 V, then enters a magnetic field of magnitude 0.300 T with its velocity perpendicular to the direction of the magnetic field. This particle has a charge-to-mass ratio of $q/m = 4.0 \times 10^6$ C/kg.
(a) Find its speed v.
(b) Use the basic magnetic force law to find the radius of its path.

<u>Practice Test Answers</u>

1. (a) See Figure 10. $\vec{F}_1 = 0$, $\vec{F}_2 = -0.045\hat{k}$ N, $\vec{F}_3 = 0$, $\vec{F}_4 = 0.045\hat{k}$ N.

Figure 10



(b) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$ as always for uniform $\vec{B}$. (c) $\vec{\mu} = -0.41\hat{k}$ A m^2.
(d) $\vec{\tau} = -0.020\hat{j}$ N m.

2. (a) $q\vec{v} \times \vec{B} = -5.3 \times 10^{-23}\hat{i}$ N. (b) $\vec{E} = -3.3 \times 10^{-4}\hat{i}$ V/m.
(c) $V = 1.65 \times 10^{-6}$ V.

3. (a) $v = 2.00 \times 10^5$ m/s. (b) $R = 0.167$ m.

MAGNETIC FORCES

Date ______________________

Mastery Test   Form A

pass   recycle

1   2   3   4

Name ______________________________

Tutor ______________________

1. A gold strip 1.50 cm wide and 0.100 cm thick is placed in a $\vec{B}$ field of 2.00 Wb/m^2 at right angles to the strip. A current of 100 A is set up in the strip along its length. Let $n = 6.0 \times 10^{28}$ per cubic meter and $q = -1.60 \times 10^{-19}$ C.

   (a) Sketch the setup. Include a set of coordinate axes.

   (b) What is the velocity of the charge carriers? Assume they are electrons.

   (c) What is the magnetic force on an electron?

   (d) What Hall potential difference appears across the strip? Indicate the direction of the Hall field in your diagram.

2. A straight wire of length L carrying a current I is situated in a uniform $\vec{B}$ field directed at right angles to the wire segment. Draw a diagram showing the directions of the current in the wire, the $\vec{B}$ field, and the magnetic force acting on the wire. What is the magnitude of the magnetic force? Now imagine that the wire segment is bent into the form of a closed circular loop, the plane of which is oriented at an angle θ to the $\vec{B}$ field, as in Figure 1. What is the net force on the loop? What is the net torque of the loop? $(\sin\theta)(-\hat{i}) + (\cos\theta)\hat{j}$ is the normal to the loop.

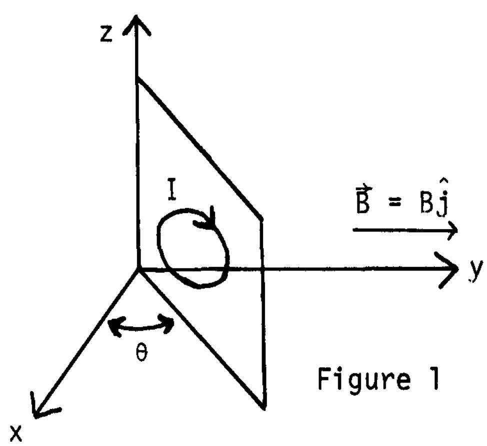


Figure 1

MAGNETIC FORCES — Date ____________

Mastery Test — Form B — pass — recycle 1 2 3 4

Name ______________________ Tutor ____________

1. At the instant a positive ion of charge +e and mass m traveling to the right with a velocity $\vec{v} = v_0\hat{i}$ passes through the origin of a coordinate system as shown in Figure 1, a uniform $\vec{B}$ field directed perpendicular to the xy plane is established and causes the ion to pass through the point x = R, y = R, z = 0.
   (a) Determine the magnitude of the $\vec{B}$ field in terms of e, m, and $\vec{v}$.
   (b) Is the $\vec{B}$ field directed into or out of this exam sheet?
   (c) What is the velocity (magnitude and direction) of the ion at (R, R, 0)?
   (d) How long does it take to make one turn?

   An electric field is introduced to cause a second ion entering the $\vec{B}$ field at the origin of the coordinate system to continue in a straight line with a velocity $\vec{v} = v_0\hat{i}$.
   (e) Indicate the direction of the $\vec{E}$ field.
   (f) Determine the magnitude of the $\vec{E}$ field in terms of the parameters given.

2. In Figure 2 a wire bent into a semicircular arc of radius R in the yz plane carries a current I. A uniform external magnetic field $\vec{B}_0$ is parallel to the x axis. Find the vector force acting on the wire. Remember that closed loops have zero net force.

3. A flat coil of 20 turns, with an area of 20 cm^2 is suspended in a magnetic field of strength B = 0.300 Wb/m^2. When the plane of the coil makes an angle of 45° with the field, the torque has a magnitude of 1.00×10^{-2} N m.
   (a) What is the magnetic moment of the coil?
   (b) What is the current flowing in the coil?

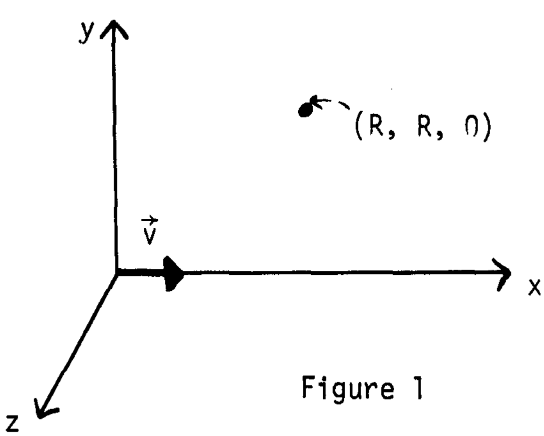


Figure 1

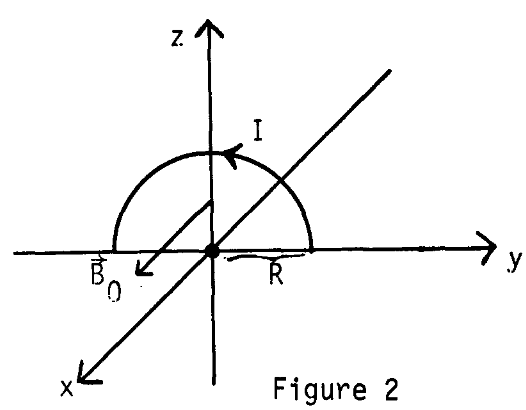


Figure 2

MAGNETIC FORCES

Date ____________

Mastery Test   Form C

pass   recycle
1 2 3 4

Name ______________________

Tutor ____________

1. (a) What is the speed of a proton (charge $q = 1.60 \times 10^{-19}$ C and mass $m = 1.70 \times 10^{-27}$ kg) that moves in a circle of 2.00 m radius in a $\vec{B}$ field of 0.200 Wb/m^2?

   (b) If the speed is doubled, how does the time for one turn change?

   (c) If the velocity of the proton is now along $\hat{i}$ and the magnetic field is along $\hat{k}$, what electric field (direction and magnitude) will cause the velocity to remain constant?

2. The wires of a high-voltage electric power transmission line experience a magnetic force from the Earth's magnetic field. Find the magnetic force per meter of its length if a wire carries a current of 500 A in the local (magnetic) northerly direction. The Earth's field is 2.50×10^{-5} Wb/m^2 and points northward and downward at 70° from the horizontal. Illustrate your solution with a sketch of the problem.

3. You wish to make a square coil out of a 1.00-m-long piece of wire such that it experiences a maximum torque of 0.200 N m when it carries a 4.0-A current and is placed in a uniform 0.84-Wb/m^2 $\vec{B}$ field. How many turns should it have?

MASTERY TEST GRADING KEY - Form A

1. What To Look For: (a) I along strip. B perpendicular to strip. Coordinate axes. (b) Current equation, A in square meters. Direction of v opposite to I (electrons are negative). (c) Lorentz equation. Direction. (d) Force balance or E = vB. Direction of $\vec{E}$ on sketch.

Solution: (a) See Figure 14.

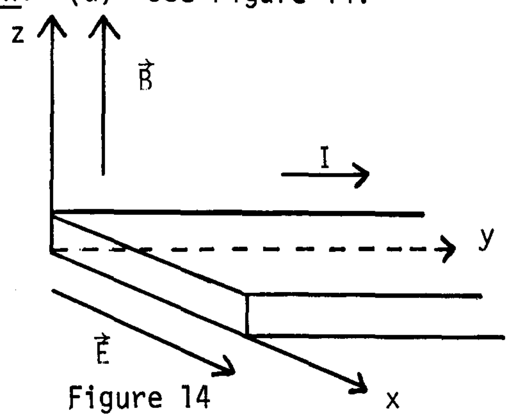


Figure 14

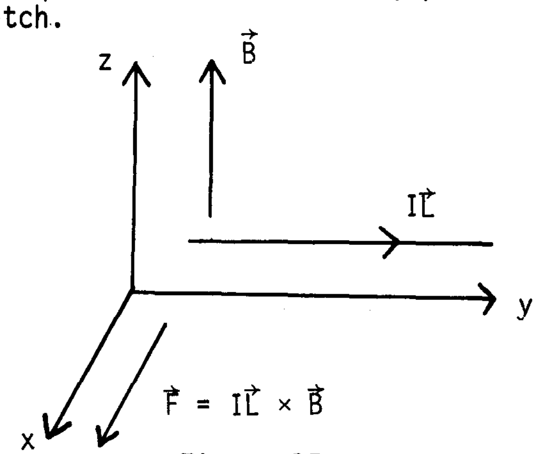


Figure 15

(b) $I = nqvA$, $v = I/nqA$,

$v = 100/(6.0 \times 10^{28})(1.60 \times 10^{-19})(1.50 \times 10^{-5}) = 6.9 \times 10^{-4}$ m/s to left.

(c) $\vec{F} = q\vec{v} \times \vec{B} = -(1.60 \times 10^{-19})(6.9 \times 10^{-4})\hat{j}(2\hat{k}) = 2.20 \times 10^{-22}\hat{i}$ N.

(d) $\vec{F} + q\vec{E}_H = 0$, $\vec{E}_H = -(1/q)\vec{F} = 1.38 \times 10^{-3}$ V/m,

$V_H = dE_H = (0.0150 \text{ m})(1.38 \text{ V/m}) = 2.07 \times 10^{-5}$ V.

2. What To Look For: Force equation, $\vec{B} \cdot I\vec{L} = 0$. Force zero in uniform field. Find area from length. Find magnetic moment. Torque equation.

Solution: See Figure 15. $\vec{F} = 0$ (uniform field).

$L = 2\pi R$, $\mu = IA = I\pi(L/2\pi)^2 = IL^2/4\pi$.

$\vec{\tau} = \vec{\mu} \times \vec{B} = (IL^2/4\pi)[(-\sin\theta)\hat{i} + (\cos\theta)\hat{j}] \times B\hat{j} = -[(IL^2B\sin\theta)/4\pi]\hat{k}$.

some type of method used to show that the radius is R might be helpful — eg.

MASTERY TEST GRADING KEY - Form B

1. What To Look For: (a) Determine radius. $\vec{F} = m\vec{a}$. (b) Get direction of $\vec{B}$ from force. (c) 1/4 turn on circle. (f) Force balance $v_0 = E/B$.

   Solution: See Figure 16. (a) Radius is R,

   $$mv^2/R = evB, \quad B = mv/eR.$$

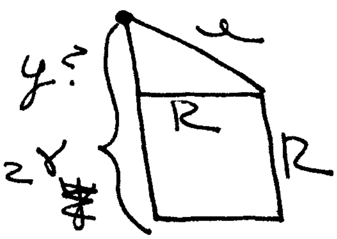
   (b) Force is up, so $\vec{B} = -(mv/eR)\hat{k}$.

   (c) $\vec{v} = v_0\hat{j}$. (d) $T = 2\pi R/v_0$.

   (e) Electric force is down: $\vec{E} = -E_0\hat{j}$.

   (f) $q\vec{E} + q\vec{v} \times \vec{B} = 0$. $E = vB = mv_0^2/eR$.

$r = \ell$
$r^2 = (r-R)^2 + R^2$
$2rR = 2R^2$
$r = R$



2. What To Look For: F = 0 for closed loop. Force equation.

   Solution: See Figure 17. Force on half-loop same as force on line across diameter.

   $$\vec{F} = I\vec{L} \times \vec{B} = I(-2R\hat{j}) \times B_0\hat{i} = 2IRB_0\hat{k}.$$

3. What To Look For: (a) Torque equation. Definition of μ.

   Solution: (a) $\tau = \mu B \sin\theta$,

   $$\mu = \frac{10^{-2}\ \text{N m}}{(0.300\ \text{T})(1/\sqrt{2})} = 0.047\ \text{A m}^2 = NIA.$$

   (b) $I = \mu/NA = (0.047\ \text{A m}^2)/(20 \times 0.00200\ \text{m}^2) = 1.18\ \text{A}$.

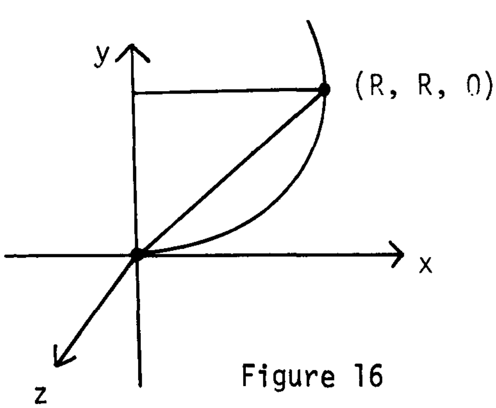


Figure 16

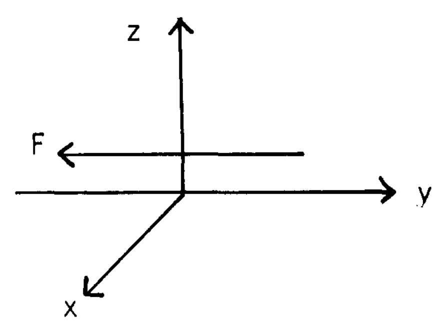


Figure 17

MASTERY TEST GRADING KEY - Form C

1. What To Look For: (a) $F = mA$. (b) T independent of v. (c) Force balance.

   Solution: (a) $mv^2/R = qvB$.

   $v = qBR/m = (1.60 \times 10^{-19})(0.200)(2)/(1.70 \times 10^{-27}) = 3.8 \times 10^7$ m/s.

   (b) No change.

   (c) See Figure 18. $E = vB$ along $\hat{j}$.

   $\vec{E} + \vec{v} \times \vec{B} = 0, \quad E = 0.76 \times 10^7$ V/m.

2. What To Look For: $\vec{F}/\ell = \vec{I} \times \vec{B}$.

   Solution: See Figure 19. $\vec{I} \times \vec{B}$ into paper (West).

   $F/\ell = IB \sin\theta = (500\text{ A})(2.50 \times 10^{-5}\text{ T})(\sin 70°) = 0.0117$ N/m.

3. What To Look For: Definition of μ. Relate area to length. Torque equation.

   Solution: $\mu = NIA, \quad L = N4a$.

   $A = a^2 = (L/4N)^2, \quad \tau_{max} = \mu B = t$.

   $NIAB = t, \quad NI(L/4N)^2B = \tau$,

   $N = L^2BI/\tau 16 = (0.84)(4)/(0.200)(16) = 1.05$ turns.

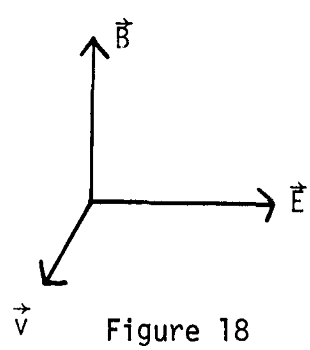


Figure 18

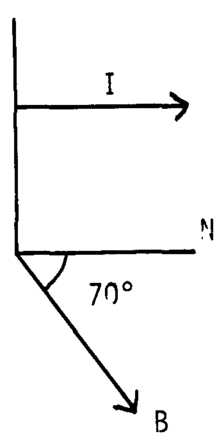


Figure 19