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Kerr-lens mode-locked $\text{Tm}^{3+}:\text{Sc}_2\text{O}_3$ single crystal laser in-band pumped by an Er:Yb fiber MOPA at 1611 nm

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We demonstrate a Kerr-lens mode-locked $\text{Tm}^{3+}:\text{Sc}_2\text{O}_3$ single crystal laser in-band pumped by an $\text{Er}^{3+}:\text{Yb}^{3+}$ fiber MOPA at 1611 nm. Pulses as short as 166 fs with an average output power of 440 mW are obtained. The spectral bandwidth and center wavelength are 29.3 nm and 2124 nm, respectively. At a longer pulse duration of 298 fs we obtain 1 W of average output power. The repetition rate is 95 MHz and the conversion efficiency against the absorbed pump power is as high as 47%. To the best of our knowledge, this is the first Kerr-lens mode-locked Tm^{3+} doped solid state laser. © 2017 Optical Society of America

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Tm^{3+} doped materials are recognized as most promising for highly efficient high power lasers in the 2 μm wavelength range. They have a variety of applications such as material processing (silicon, organic, polymer materials) [1,2], LIDAR [3] and as a pump source for generating coherent light at even longer wavelengths, such as Ho^{3+} lasers around 2.1 μm [4], Cr^{2+} lasers around 2.2-2.9 μm [5], mid-infrared OPOs for wavelengths up to 12 μm [6,7], or mid-infrared supercontinuum generation [8]. Nowadays, mode-locked Tm^{3+} doped solid state lasers were reported based on various kinds of gain media and saturable absorbers e.g. $\text{Tm}^{3+}:\text{Lu}_2\text{O}_3$ with single wall carbon nanotubes [9], $\text{Tm}^{3+}:\text{KYW}$ with semiconductor saturable absorber [10], and $\text{Tm}^{3+}:\text{CLNGG}$ with graphene [11]. Among Tm^{3+} doped materials, Tm^{3+} doped sesquioxides ($\text{Tm}^{3+}:\text{Re}_2\text{O}_3$, $\text{Re}=\text{Sc}$, Lu or Y) are the most attractive gain materials as they possess superior thermal and mechanical properties as well as a broad gain bandwidth above 1980 nm where less water vapor absorption and Tm^{3+} reabsorption exist. In addition, the influence of Stark splitting on Tm^{3+} doped materials is larger than that of Ho^{3+} doped materials so that one of the gain peaks of $\text{Tm}^{3+}:\text{Sc}_2\text{O}_3$ is shifted to a wavelength of 2150 nm (Fig.1a) which is longer than typical peak gain positions of Ho^{3+} doped materials. The Re_2O_3 have high

nonlinear refractive induces (about twice as high as YAG [12]) that increase self-phase modulation effect in mode-locked operation. Although the available crystal size is limited due to the high melting points of $\text{Tm}^{3+}:\text{Re}_2\text{O}_3$ so far and their small absorption cross sections around 800 nm require thick gain media, pulses as short as 175 fs, 218 fs, and 148 fs were obtained by $\text{Ti}^{3+}:\text{sapphire}$ laser pumped mode-locked $\text{Tm}^{3+}:\text{Lu}_2\text{O}_3$ [9], $\text{Tm}^{3+}:\text{Sc}_2\text{O}_3$ [13] and $\text{Tm}^{3+}:\text{LuScO}_3$ [14] lasers, respectively. Promising results have also been obtained with mode-locked $\text{Tm}^{3+}:\text{Lu}_2\text{O}_3$ under direct laser diode (LD) pumping [15]. In prior mode-locked Tm^{3+} solid state lasers, $\text{Ti}^{3+}:\text{Al}_2\text{O}_3$ lasers or LDs emitting at ~ 800 nm were used as pump sources, because thanks to a fortuitous two-for-one cross-relaxation process (${}^3\text{H}_6 \Rightarrow {}^3\text{H}_4 \Rightarrow 2 \times {}^3\text{F}_4$), Tm^{3+} lasers at 2 μm can be highly efficient despite the large energetic difference between the pump and the laser photons (Fig. 1b). This process, however, requires high Tm^{3+} doping levels which may cause thermal problems or detrimental reverse cross relaxation processes at high inversion levels [16]. In addition, the available pump power and efficiency of $\text{Ti}^{3+}:\text{Al}_2\text{O}_3$ pump sources as well as the brightness of direct LD pump sources are strongly restricted so far. Direct in-band pumping from the ground state ${}^3\text{H}_6$ into the upper laser level ${}^3\text{F}_4$ (Fig.1b) at wavelengths around 1.6 μm is another very attractive scheme for Tm^{3+} lasers that would mitigate the restrictions mentioned above and allow for an

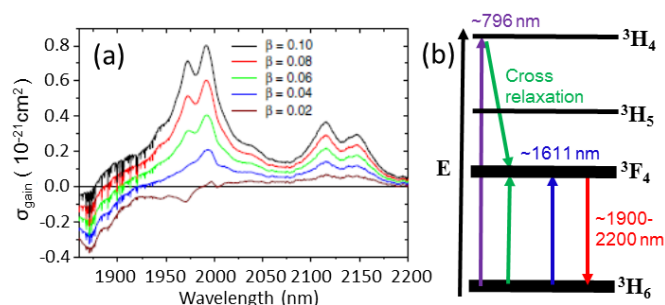


Fig. 1. (a) Gain spectra of $\text{Tm}^{3+}:\text{Sc}_2\text{O}_3$ for a different inversion fraction, β [12, 15]. (b) Energy level of $\text{Tm}^{3+}:\text{Sc}_2\text{O}_3$. Two-for-one cross-relaxation pumping process and in-band pumping process are shown.

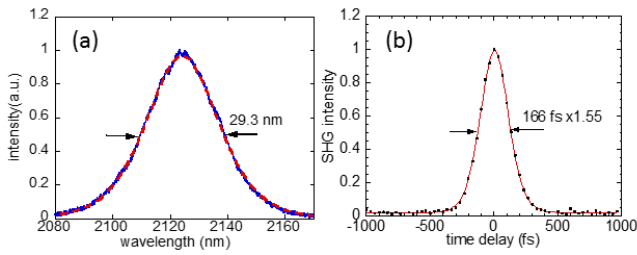


Fig. 4. (a) Measured spectrum (blue solid curve) and its sech^2 fitting (red dashed curve). (b) Autocorrelation trace at the output power of 440 mW. Measured data (black circles) and Sech^2 fitting (red curve).

After the optimization of the cavity for short pulse duration, i.e. alignment of the insertion depth of the prisms, the position of the gain medium along the optical axis, the distance between the folding mirrors as well as the pump power, pulses as short as 166 fs (assuming sech^2 pulses) with a spectral bandwidth of 29.3 nm at a center wavelength of 2124 nm were obtained (Fig. 4a and Fig. 4b). The optical spectrum was measured by an OSA205 (Thorlabs Inc.) and the pulse duration was measured by a home-built autocorrelator with a 300 μm thick type 2 KTP nonlinear crystal. The pulses were close to Fourier limited with a time bandwidth product of 0.32. At the shortest pulse duration the output power was 440 mW at a pump power of 3.15 W. The corresponding pulse energy and peak power were 4.6 nJ and 28 kW, respectively. The intracavity peak power would be as high as 2.8 MW. During the experiment, we observed a red shift of the peak emission wavelength when the laser was aligned for shorter pulse durations (Fig. 5a). The shortest peak wavelength was 2116 nm and it continuously shifted to 2128 nm during the alignment and finally switched to 2139 nm. This switch is explained by the multi-peak gain structure of $\text{Tm}^{3+}:\text{Sc}_2\text{O}_3$ (Fig. 1a). The pulse duration at the sub-peak around 2140 nm was 214 fs (Fig. 5b) at a center wavelength of 2142 nm and a spectral bandwidth of 24.0 nm. The corresponding time bandwidth product was 0.34. In this operation regime, the KLM was very sensitive and CW breakthrough peaks were observed in the spectrum e.g. when increasing the pump power. Under some alignment conditions, we observed a random switching of the center wavelength between the peaks at ~ 2128 nm and ~ 2140 nm or between KLM and CW operation mode, which ultimately limited the available shortest pulse duration. We thus believe that by utilizing optimized mirror coatings and/or improved cavity design with respect to the modulation depth of

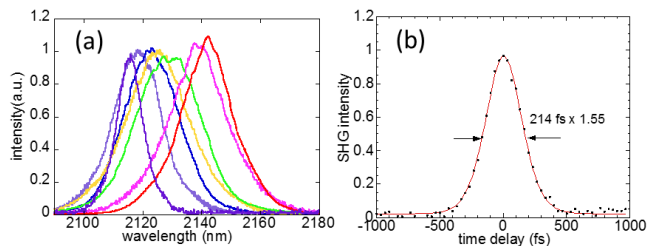


Fig. 5. (a) Measured spectra with different pulse spectral bandwidth. The shift and broadening of the emission peak to longer wavelengths went along with a pulse shortening. The pulse shortening was not observed after the switch of peak emission to ~ 2140 nm. (b) Autocorrelation trace of the center wavelength of 2142 nm.

the KLM, it should be feasible to address the total 3dB gain bandwidth of the double-peak gain structure of $\text{Tm}^{3+}:\text{Sc}_2\text{O}_3$ around 2.1 μm of 50 nm or even more, depending on the inversion level (see Fig. 1a). In this way, sub-100 fs pulse durations should be possible. It should, however, be noted that an estimation of the modulation depth of the current KLM cavity from the increasing of the output power with the onset of KLM (cf. Fig. 3a) is not straight forward. This is because unlike hard aperture KLM based solely on a loss modulation, the soft aperture KLM process is based on an effective gain modulation via changing the mode matching between the laser mode and the pump mode [29].

As previously mentioned, we also performed experiments with the 2.5% OC targeting higher average output powers. Indeed we could increase the average output power to a value as high as 1 W at a pulse duration of 298 fs (Fig. 6a and Fig. 6b), a pump power of 3.7 W, and a somewhat larger tip-to-tip prism separation of 8.5 cm. As expected due to the higher inversion needed to overcome the larger losses in this configuration, the center wavelength of 2115 nm was somewhat shorter than for the best results obtained with the 1% OC. The spectral bandwidth of 15.5 nm yielded a time bandwidth product of 0.31, corresponding (within the error) to Fourier limited sech^2 pulses. The pulse energy and the peak power were 10.5 nJ and 35.2 kW, respectively. It should be noted that the absorption efficiency of the laser crystal was estimated to be only 56% by measuring the residual pump powers. The resulting absorbed pump power of 2.1 W corresponds thus to a conversion efficiency vs. the absorbed pump power as high as 47%. The KLM at the similar output power level was also obtained without the water cooling. The beam profile measured with a scanning beam profiler shown in Fig. 1 (with the 2.5% OC) indicates the expected high beam quality of the KLM laser. In fact, KLM could hardly be achieved with poor beam quality and corresponding larger mode diameters.

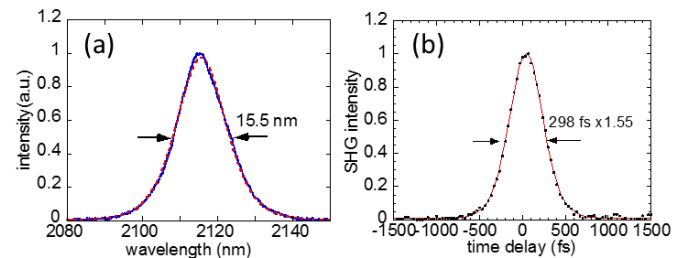


Fig. 6. (a) Measured spectrum (blue solid curve) and its Sech^2 fitting (red dashed curve). (b) Autocorrelation trace of pulses at the output power of 1W and the center wavelength of 2115 nm.

In conclusion, we have demonstrated the first Kerr-lens mode-locked operation of a Tm^{3+} doped solid state laser in the 2 μm wavelength range. We employed an $\text{Er}^{3+}:\text{Yb}^{3+}$ fiber MOPA pump source at 1611 nm for in-band pumping of $\text{Tm}^{3+}:\text{Sc}_2\text{O}_3$. Pulses as short as 166 fs with an average output power of 440 mW were obtained. The center wavelength was 2124 nm and the spectral bandwidth was 29.3 nm. In a configuration optimized for high output power, 1 W was obtained in 298-fs-pulses. This is the first mode-locked operation with a fiber laser in-band pumping scheme for Tm^{3+} and the first watt-level output power mode-locked oscillator in this wavelength range, too. The results were limited by the onset of mode locking instabilities and CW breakthroughs, but neither by the available pump power nor by thermal problems so

far. Thus further increasing of the output power with the similar pulse duration seems feasible by increasing the total GDD of the cavity. Further pulse shortening could also be possible by using a hard aperture inside the cavity. The new in-band pumped short pulse Tm³⁺ solid state laser scheme could also be useful for high power amplifier systems in the 2 μm wavelength range, even though the total efficiency and further power scaling of them is currently limited by the available pump sources. Highly efficient high power operation could be enabled utilizing by more powerful LDs or cascaded Raman laser in-band pump sources in future.

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