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First Iterative Solution of the Thermal Behaviour of Acoustic Cavitation Bubbles in the Uniform Pressure Approximation

Can F Delale[1,2] and **Şenay Pasinlioğlu**[3]
[2]Department of Mechanical Engineering, MEF University, Ayazağa Caddesi No 4, 34980 Maslak, Sarıyer, Istanbul, Turkey
[3]Department of Mathematics, Istanbul Technical University, Büyükdere Caddesi, 34465 Maslak, Sarıyer, Istanbul,Turkey
E-mail: canfuat.delale@mef.edu.tr

Abstract. The thermal behaviour of a spherical gas bubble in a liquid driven by an acoustic pressure is investigated in the uniform pressure approximation by employing an iterative method to solve the energy balance equations between the gas bubble and the surrounding liquid for the temperature distribution and the gas pressure inside the bubble. It is shown that the first iterative solution leads to the first order law of the gas pressure as a polytropic power law of the bubble wall temperature and of the bubble radius, with the polytropic index given as an explicit function of the isentropic exponent of the gas. The resulting first order law of the gas pressure reduces to the classical isothermal and adiabatic laws in the appropriate limits. The first order gas pressure law is then applied to an acoustically driven cavitation bubble by solving the Rayleigh-Plesset equation. Results obtained show that the bubble wall temperature pulsations during collapse and rebound can become a few orders of magnitude higher than the bulk liquid temperature.

1. Introduction

Thermal effects play an important role in the final stage of collapse of inertially controlled bubbles. The pressures and temperatures can reach very high values in applications such as cavitation [1-3], sonochemistry [4] and single-bubble sonoluminescence [5-6]. Various sophisticated models that address the effect of the thermal behaviour of the bubble on the gas pressure inside the bubble in the uniform pressure approximation have been constructed ([7-10] and references therein). Although full numerical computations of these models are available for single bubbles, the complexity of the numerical solutions demonstrates the need of simplified expressions for the gas pressure and temperature. The attempt of modifying the adiabatic approximation for the gas pressure in this direction either by artificially increasing the liquid viscosity or by employing a variable isentropic index was not found to be satisfactory [11]. The aim of this investigation is to derive a simple relation between the gas pressure, the bubble wall temperature, the bubble radius and the isentropic exponent of the gas for use in various applications. For this reason we consider the energy balance between a spherical gas bubble and the surrounding liquid. We investigate the thermal behaviour inside the bubble in the uniform pressure approximation by studying the well-known coupled equations for the gas pressure and temperature, which we attempt to solve iteratively. In particular, in the first iteration we show that the temporal evolution of the gas pressure can be decoupled from that of the gas temperature, resulting in the first order law for the gas pressure as a polytropic power law of the bubble wall temperature and of the bubble

[1] To whom any correspondence should be addressed.

radius, with the polytropic index given explicitly in terms of the isentropic exponent of the gas. The bubble wall temperature is obtained from the Plesset-Zwick solution [12] of the temperature distribution of the liquid side in the thin boundary layer approximation. Results obtained for acoustically driven cavitation bubbles by use of the Rayleigh-Plesset equation of spherical bubble dynamics show that bubble wall temperature pulsations can reach values a few orders of magnitude higher than the bulk liquid temperature during the final stage of bubble collapses.

2. Energy Balance for a Spherical Gas Bubble

In this section we discuss the energy balance between a spherical gas bubble and its surrounding liquid. We focus on the thermal behaviour inside the bubble in the uniform pressure approximation as well as inside the surrounding liquid.

2.1 Thermal Diffusion Through the Gas Bubble in the Uniform Pressure Approximation

The partial differential equation for the temperature field inside a gas bubble in the uniform pressure approximation is given by Prosperetti [13], and in normalized form takes the form [14]

$$\frac{p}{T}\left\{\frac{\partial T}{\partial t}+\frac{D}{p\,R^2}\left[\lambda(T)\frac{\partial T}{\partial y}-y\left(\frac{\partial T}{\partial y}\right)_{y=1^-}\right]\frac{\partial T}{\partial y}\right\}=\frac{(\gamma-1)}{\gamma}\frac{dp}{dt}+\frac{D}{R^2\,y^2}\frac{\partial}{\partial y}\left[y^2\lambda(T)\frac{\partial T}{\partial y}\right] \tag{1}$$

In equation (1) T is the temperature field inside the bubble, normalized with respect to the bulk liquid temperature T_0' ; p is the bubble gas pressure in the uniform pressure approximation, normalized with respect to a reference pressure p_0'; R is the bubble radius, normalized with respect to the initial bubble radius R_0' ; $\lambda(T)$ is the thermal conductivity of the gas, normalized with respect to its value λ_R' at the bubble wall, γ is the isentropic exponent of the gas; y is the radial coordinate, measured from the bubble center and normalized with respect to the instantaneous value of the bubble radius; t is the time, normalized with respect to a characteristic time Θ' and D is the square of the ratio of the penetration length to the initial bubble radius and is given by

$$D=\frac{(\gamma-1)\,\lambda_R'\,T_0'\,\Theta'}{\gamma\,p_0'\,(R_0')^2} \tag{2}$$

The gas pressure in the uniform pressure approximation can then be found by solving the equation

$$\frac{dp}{dt}=\frac{3\,\gamma}{R}\left[\frac{D}{R}\left(\frac{\partial T}{\partial y}\right)_{y=1^-}-p\,\frac{dR}{dt}\right] \tag{3}$$

which requires the solution of equation (1) together with the initial and boundary conditions

$$T(t=0,y)=1\ \ ;\ \ T(t,y=1^-)=T_R(t)\ \ ;\ \ \left(\frac{\partial T}{\partial y}\right)_{y=0}=0 \tag{4}$$

for the evaluation of the bubble wall temperature gradient $c(t)=(\partial T/\partial y)_{y=1^-}$. For this reason we adopt an iterative method of solution by first carrying out an expansion for the temperature field of the gas inside the bubble near the bubble wall as

$$T(t,y)=T_R(t)+\left(\frac{\partial T}{\partial y}\right)_{y=1^-}(y-1^-)+\frac{1}{2}\left(\frac{\partial^2 T}{\partial y^2}\right)_{y=1^-}(y-1^-)^2+O[(y-1^-)^3] \tag{5}$$

together with a similar expansion for the thermal conductivity of the gas. Substitution of the expansions into equation (1) and taking the limit $y=1^-$ leads to the below exact expression for the bubble wall gas pressure gradient

$$c(t)=\left(\frac{\partial T}{\partial y}\right)_{y=1^-}=\frac{R^2}{2D}\left[\frac{p}{T_R}\frac{dT_R}{dt}-\frac{(\gamma-1)}{\gamma}\frac{dp}{dt}\right]-\frac{1}{2}\left(\frac{\partial^2 T}{\partial y^2}\right)_{y=1^-} \tag{6}$$

where the temperature variation of the thermal conductivity $\lambda(T)$ at the bubble wall is neglected (accounting for the derivative of the thermal conductivity with respect to temperature leads to a quadratic equation for c(t)). Equations (1) and (3) together with equation (6) can now be solved iteratively for the gas temperature and gas pressure, utilizing the initial and boundary conditions for the gas temperature and an initial condition for the gas pressure provided that the bubble wall temperature $T_R(t)$ is known (the bubble wall temperature will be obtained from the solution of the thermal diffusion equation of the liquid side taking into account the interface conditions).

2.1.1. First Iterative Solution: The First Order Law of the Gas Pressure during Growth and Collapse
In the first iteration we neglect the effect of the second radial derivative of the gas temperature at the bubble wall in equation (6), i.e. we assume that $(\partial^2 T/\partial y^2)_{y=1^-}=0$. This results in decoupling of equation (3) for the gas pressure from the solution of equation (1) for the temperature field inside the bubble. Substituting the first iterative approximation of $c(t)$ into equation (3), upon exact integration, leads to the first approximation for the law of the gas pressure during the growth and collapse of the bubble in the form

$$p = p_i \left(\frac{\sqrt{T_R}}{R}\right)^{3\kappa} \quad ; \quad \kappa = \frac{2\,\gamma}{(3\,\gamma - 1)} \tag{7}$$

where p_i is the initial gas pressure to be obtained from the initial equilibrium of the bubble and κ is a polytropic index. It can further be demonstrated that the gas pressure law given by equation (7) reduces to the isothermal law whenever γ=1 and T_R=1 and to the adiabatic law when the bubble wall temperature satisfies the isentropic relation $T_R = (p/p_i)^{(\gamma-1)/\gamma}$. For the temperature distribution inside the liquid, we use the Plesset-Zwick solution [12] in the thin boundary layer approximation which provides the normalized bubble wall temperature T_R.

3. Results for Acoustic Cavitation Bubbles and Conclusion
In this section we consider acoustic cavitation (gas-vapor) bubbles where the pressure inside the bubble is taken as the sum of the partial vapor pressure and the partial gas pressure, which in the first iterative approximation in normalized form can be written as

$$p = p_v + p_i \left(\frac{\sqrt{T_R}}{R}\right)^{3\kappa} \quad . \tag{8}$$

Using the interfacial condition for the heat flux at the bubble wall of a vapor-gas bubble and assuming that the thermal conductivity of the liquid is much greater than that of the gas-vapor mixture, the phase change at the bubble wall can be thought to be dominated by the liquid side. In this case the Plesset-Zwick solution for the bubble wall temperature from the liquid side becomes [1, 14]

$$T_R(t) = 1 - \mathcal{B} \int_0^t \frac{R^2(\xi)\ (dR/d\xi)}{\left[\int_\xi^t R^4(\tau)d\tau\right]^{1/2}}\, d\xi \tag{9}$$

where the constant $\mathcal{B}$ is given by

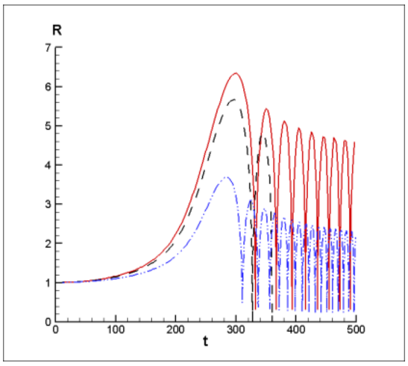
$$\mathcal{B} = \left(\frac{L'}{T_0'\, c_{p\ell}'}\right)\left(\frac{\rho_v'}{\rho_\ell'}\right)\frac{R_0'}{(\pi\,\Theta'\alpha_\ell')^{1/2}} \tag{10}$$

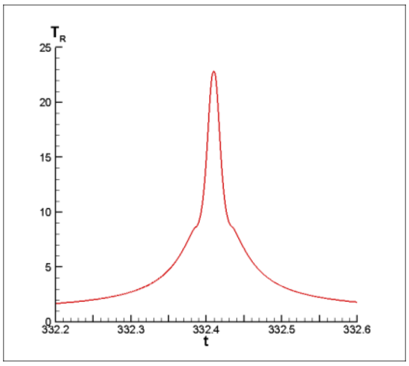
with L' denoting the latent heat of condensation, ρ_v' denoting the saturated vapor density, ρ_ℓ' denoting the liquid density, α_ℓ' denoting the thermal diffusivity of the liquid and $c_{p\ell}'$ denoting the specific heat of the liquid. To close the system of equations, we employ the classical Rayleigh-Plesset equation to describe the radial dynamics of the bubble by implementing the first order gas pressure law for the partial gas pressure inside the bubble. For the driving acoustic pressure we use the equation

$$p_\infty(t) = -0.25\left[1 - \cos\left(\frac{2\pi t}{500}\right)\right] \quad ; \quad 0 < t < 500 \tag{11}$$

where $p_\infty(t)$ is the normalized driving acoustic pressure. For applications we consider the acoustic cavitation of water-vapor/air bubbles in water at T_0'=20ºC with saturated vapor pressure p_v'= 0.0234 bar, surface tension coefficient S' = 0.071 N/m, viscosity of water μ_ℓ' = 10^{-3} kg/m-s, cavitation number σ = 0.492, initial equilibrium bubble radius R_0'=100 μm and a characteristic time Θ' = 10^{-5} s under the normalized driving acoustic pressure given by equation (11). The Rayleigh-Plesset equation incorporating the first order gas pressure law is then integrated using the Runge-Kutta-Fehlberg method together with the solution of equation (9) obtained by Simpson's 3/8-rule of numerical integration**.** Figure 1(a) shows a comparison of the temporal behaviour of the bubble radius using the present first order gas pressure law with thermal damping against the classical isothermal and adiabatic laws for the gas pressure. It is seen that the bubble radius grows to values greater than those of the isothermal case with a noticeable thermal damping effect. Figure 1(b) shows the detailed temporal evolution of the bubble wall temperature during the first collapse period, with bubble wall absolute gas temperature growing twenty three times the absolute bulk liquid temperature.

The present paper presents results for acoustic cavitation bubbles with the inclusion of thermal effects by a reduced order gas pressure law in spherical bubble dynamics. The inclusion of higher iterative approximations for the reduced gas pressure law together with comparison of the results with those of full numerical simulation of the radial PDE system for possible error analysis are left out for future work.





(a) **(b)**

Figure 1. (a) The temporal evolution of the normalized bubble radius driven by the acoustic pressure given by equation (11) for water-vapor/air bubbles in water with initial equilibrium radius R_0'=100 μm, cavitation number $\sigma = 0.492$ using the first order gas pressure law given by equation (8) (red solid line), the isothermal law (black dashed line) and the adiabatic law (blue dotted dashed line), **(b)** detailed temporal evolution of the normalized bubble wall temperature during the first collapse and rebound.

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