



PUBLISHED FOR SISSA BY SPRINGER

RECEIVED: June 9, 2018
ACCEPTED: August 20, 2018
PUBLISHED: August 27, 2018

Combining QED and QCD transverse-momentum resummation for Z boson production at hadron colliders

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ABSTRACT: We consider the transverse-momentum (q_T) distribution of Z bosons produced in hadronic collisions. At small values of q_T , we perform the analytic resummation of the logarithmically enhanced QED contributions up to next-to-leading logarithmic accuracy, including the mixed QCD-QED contributions at leading logarithmic accuracy. Resummed results are consistently matched with the next-to-leading fixed-order results (i.e. $\mathcal{O}(\alpha^2)$) at small, intermediate and large values of q_T . We combine the QED corrections with the known QCD results at next-to-next-to-leading order ($\mathcal{O}(\alpha_S^2)$) and next-to-next-to-leading logarithmic accuracy. We show numerical results at LHC and Tevatron energies, studying the impact of the QED corrections and providing an estimate of the corresponding perturbative uncertainty. Our analytic results for the combined QED and QCD resummation, obtained through an extension of the q_T resummation formalism in QCD, are valid for the production of generic neutral and colourless high-mass systems in hadronic collision.

KEYWORDS: Resummation, Perturbative QCD, Precision QED

ARXIV EPRINT: [1805.11948](https://arxiv.org/abs/1805.11948)

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1 Introduction

The production of vector bosons (W and Z/γ^*), through the Drell-Yan (DY) mechanism [1], is one of the most prominent hard-scattering processes at high-energy hadron colliders, such as the Tevatron and the LHC. In particular, it provides strong tests of the Standard Model (SM) and of QCD, it is important for precise measurements of the mass and the width of the W boson and for the extraction of the electroweak (EW) mixing angle, it gives stringent information on the parton densities functions of the colliding hadrons, and it represents an important background to other processes and to various possible signals from physics beyond the SM.

Owing to the above reasons, it is of primary importance to provide accurate theoretical predictions for vector-boson production cross sections and related kinematical distributions. This task requires detailed computations of the higher-order radiative corrections in QCD [2, 3]–[11] and in the EW theory [12–16]–[27].

Among the various kinematical distributions, the transverse-momentum (q_T) spectrum of the vector boson has a particularly high relevance. A good understanding of the Z boson q_T distribution gives important information on the W boson production mechanism. In turn, an accurate theoretical prediction of the q_T distribution of the W boson, in the intermediate and small q_T region, is crucial for a precise measurement of the W mass [28–30]. In order to fully exploit the high precision of the LHC measurements, corresponding theoretical predictions with increasing accuracy are therefore essential.

In the large- q_T region, where the transverse momentum is of the order of the vector boson mass M , fixed-order perturbative calculations are theoretically justified [31–34]. However the bulk of the vector boson cross section lies in the small- q_T region ($q_T \ll M$) where the reliability of the fixed-order expansion is spoiled by the presence of large logarithmic corrections of the type $\ln(M^2/q_T^2)$ due to the initial state radiation of soft and/or collinear partons.

In order to obtain reliable predictions, the enhanced-logarithmic terms have to be evaluated and systematically resummed to all orders in perturbation theory. The formalism

to resum these large logarithmic corrections in QCD has been developed from the late seventies [35–38]–[49] starting from results settled for QED [50].

While the resummation of the large logarithmic corrections in QCD in the small q_T region is mandatory, the impact of the large logarithmic QED corrections from the initial states is expected to be substantial only at extremely small values of q_T due to the relative smallness of the electromagnetic coupling α with respect to the strong coupling α_S .

Nonetheless all-order resummation of QED emissions can have non-negligible impact on pure QCD resummed results. Moreover since the first-order QCD contribution at finite value of q_T is an $\mathcal{O}(\alpha_S)$ correction relative to the Born-level result, the QED contribution corresponds to an $\mathcal{O}(\alpha/\alpha_S)$ correction to the QCD result. Given the present accuracy of the theoretical predictions [51–53] and the high-precision experimental LHC data, it is therefore important to quantify in a consistent way the impact of QED contributions for the q_T distribution of vector bosons.

In this paper we extend the QCD formalism of refs. [46, 47] in order to combine QCD and QED transverse momentum resummation. In particular we explicitly consider the case of Z production at the Tevatron and the LHC by performing QED and QCD resummation up to next-to-leading logarithmic (NLL) and at next-to-next-to-leading logarithmic (NNLL) accuracy, respectively. We also include the resummation of the mixed QCD-QED corrections at leading logarithmic accuracy. In order to achieve a uniform theoretical accuracy for the entire range of transverse momenta, our resummed calculation has been consistently matched at small and intermediate-large values of q_T with the fixed-order corrections at next-to-leading order (NLO) in QED (i.e. $\mathcal{O}(\alpha^2)$) and next-to-next-to-leading order (NNLO) in QCD (i.e. $\mathcal{O}(\alpha_S^2)$). Owing to the unitarity constraint of the resummation formalism, the integral over q_T of our predictions with QED effects reproduce the corresponding NLO QED corrections to the total cross section, which have been obtained taking the abelian limit of the corresponding QCD result.¹

We show that the QED corrections have a non negligible impact on the q_T spectrum of the Z boson. In particular the inclusion of the NLL+NLO QED corrections reduces the effect of renormalization scale ambiguity of the QED coupling α by consistently including running coupling effects.

The paper is organized as follows. In section 2 we present the combined QED and QCD resummation formalism, which is valid for a generic process involving a colorless and neutral final state (namely, Z/γ^* , ZZ , $\gamma\gamma$, Higgs boson(s), etc.), providing the explicit expression for the universal coefficients necessary to include QED resummation up to NLL accuracy. In section 3 we show illustrative numerical results for Z boson production at Tevatron and LHC energies. In particular, we present combined resummation results up to NLL+NLO in QED and NNLL+NNLO in QCD, quantifying the impact of QED effects with respect to the corresponding NNLL+NNLO QCD results. We then perform a study of the scale dependence of the QED corrections in order to estimate the corresponding perturbative uncertainty. In section 4 we summarize our results.

¹Through a similar abelianization procedure the total cross section for on-shell Z production up to NNLO in QED and in QCD \times QED have been recently obtained in ref. [54].

2 Combined QED and QCD transverse-momentum resummation

In this section we extend the QCD transverse-momentum resummation formalism of refs. [46, 47] in order to consistently include the resummation of QED perturbative logarithmic corrections in the generic case of the production of high-mass systems composed by colourless and neutral particles in hadronic collision. In particular we highlight the differences that arise when considering the inclusion of the QED emissions, by considering the specific case of the hadroproduction of an on-shell Z boson: $h_1 + h_2 \rightarrow F + X$, ($F = Z$).

We start our presentation of combined QCD and QED transverse-momentum resummation by briefly recalling the main features of q_T resummation for QCD [46, 47]. According to the QCD factorization theorem, the transverse-momentum differential cross section for this process can be written as

$$\begin{aligned} & \frac{d\sigma_{h_1 h_2 \rightarrow F}}{dq_T^2}(q_T, M, s) \\ &= \sum_{a_1, a_2} \int_0^1 dx_1 \int_0^1 dx_2 f_{a_1/h_1}(x_1, \mu_F^2) f_{a_2/h_2}(x_2, \mu_F^2) \frac{d\hat{\sigma}_{a_1 a_2 \rightarrow F}}{dq_T^2}(q_T, M, \hat{s}; \mu_F^2), \end{aligned} \quad (2.1)$$

where $f_{a/h}(x, \mu_F^2)$ ($a = q, \bar{q}, g$) are the parton densities of the hadron h at the factorization scale μ_F , $d\hat{\sigma}_{a_1 a_2 \rightarrow F}$ are the partonic cross sections and s ($\hat{s} = x_1 x_2 s$) is the hadronic (partonic) centre-of-mass energy.

The partonic cross sections $d\hat{\sigma}/dq_T^2$ in eq. (2.1) can be decomposed as follows: $d\hat{\sigma} = d\hat{\sigma}^{(\text{res.})} + d\hat{\sigma}^{(\text{fin.})}$, where the ‘resummed’ component ($d\hat{\sigma}^{(\text{res.})}$) contains all the logarithmically enhanced contributions at small q_T proportional to $\delta(q_T^2)$ or to $1/q_T^2 \ln^m(M^2/q_T^2)$ and the ‘finite’ component ($d\hat{\sigma}^{(\text{fin.})}$) is free of such contributions.

The resummation procedure is carried out in the impact-parameter (b) space conjugated to q_T . The resummed component is then obtained by performing the inverse Bessel transformation with respect to b :

$$\frac{d\hat{\sigma}_{a_1 a_2 \rightarrow F}^{(\text{res.})}}{dq_T^2}(q_T, M, \hat{s}; \mu_F) = \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(b q_T) \mathcal{W}_{a_1 a_2}^F(b, M, \hat{s}; \mu_F), \quad (2.2)$$

where $J_0(x)$ is the 0th-order Bessel function. The resummation structure of $\mathcal{W}_{a_1 a_2}^F$ can be organized in an exponential form by considering the Mellin N -moments $\mathcal{W}_{a_1 a_2, N}^F$,² with respect to $z = M^2/\hat{s}$ at fixed M . In the simplified flavour-diagonal case ($a_1 a_2 = c\bar{c}$) it reads:

$$\begin{aligned} \mathcal{W}_N^F(b, M; \mu_F) &= \hat{\sigma}_F^{(0)}(M) \mathcal{H}_N^F(\alpha_S; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2) \\ &\quad \times \exp \left\{ \mathcal{G}_N(\alpha_S, L; M^2/\mu_R^2, M^2/Q^2) \right\}, \end{aligned} \quad (2.3)$$

where $\hat{\sigma}_F^{(0)}$ is the lowest-order partonic cross section, $L = \log(Q^2 b^2/b_0^2 + 1)$ is the logarithmic expansion parameter, Q is the resummation scale [47] and $b_0 = 2e^{-\gamma_E}$ (with $\gamma_E = 0.5772\dots$ the Euler-Mascheroni constant). In eq. (2.3) we explicitly introduced the renormalization scale μ_R and the perturbative QCD coupling $\alpha_S = \alpha_S(\mu_R^2)$.

²The N -moments of a generic function $h(z)$ are defined as $h_N = \int_0^1 dz z^{N-1} h(z)$.

The universal form factor $\exp\{\mathcal{G}_N\}$ includes (and resums to all orders) the large logarithmic terms $\alpha_S^n L^m$ ($1 \leq m \leq 2n$). It can be systematically expanded in powers of α_S as follows:

$$\mathcal{G}_N(\alpha_S, L) = L g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \sum_{n=4}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n-2} g_N^{(n)}(\alpha_S L), \quad (2.4)$$

where the term $L g^{(1)}$ collects the leading logarithmic (LL) contributions, the function $g_N^{(2)}$ controls the NLL contributions, $g_N^{(3)}$ includes the NNLL terms and so forth.

All the perturbative terms that behave as constants in the limit $b \rightarrow \infty$ are included in the process dependent hard-collinear function \mathcal{H}_N^F which has a customary perturbative expansion:

$$\mathcal{H}_N^F(\alpha_S) = 1 + \frac{\alpha_S}{\pi} \mathcal{H}_N^{F(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}_N^{F(2)} + \sum_{n=3}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \mathcal{H}_N^{F(n)}. \quad (2.5)$$

We now discuss how to extend the QCD resummation formalism in order to include and consistently resum the large logarithmic QED corrections of the type $\alpha^n L^m$ ($1 \leq m \leq 2n$). We start from eq. (2.1) and we consider also the inclusion of the photon parton density $f_{\gamma/h}(x, \mu_F^2)$. The eq. (2.3) have then to be replaced by the following generalized expressions:³

$$\begin{aligned} \mathcal{W}_N^F(b, M; \mu_F) &= \hat{\sigma}_F^{(0)}(M) \mathcal{H}_N^F(\alpha_S, \alpha; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2) \\ &\times \exp\left\{\mathcal{G}'_N(\alpha_S, \alpha, L; M^2/\mu_R^2, M^2/Q^2)\right\}, \end{aligned} \quad (2.6)$$

where the form factor $\mathcal{G}'_N(\alpha_S, \alpha, L)$ in eq. (2.6) has a double perturbative expansion in powers of α_S and α

$$\begin{aligned} \mathcal{G}'_N(\alpha_S, \alpha, L) &= \mathcal{G}_N(\alpha_S, L) + L g'^{(1)}(\alpha L) + g_N'^{(2)}(\alpha L) + \sum_{n=3}^{\infty} \left(\frac{\alpha}{\pi}\right)^{n-2} g_N'^{(n)}(\alpha L) \\ &+ g'^{(1,1)}(\alpha_S L, \alpha L) + \sum_{\substack{n,m=1 \\ n+m \neq 2}}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n-1} \left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L, \alpha L), \end{aligned} \quad (2.7)$$

where $\alpha = \alpha(\mu_R)$ is the electromagnetic coupling evaluated at the renormalization scale μ_R . The term $L g'^{(1)}$ collects the LL contributions in QED, the function $g_N'^{(2)}$ controls the NLL QED and so forth, while the terms $g'^{(1,1)}(\alpha_S L, \alpha L)$ and $g_N'^{(n,m)}(\alpha_S L, \alpha L)$ include respectively the leading and subleading mixed QCD-QED corrections. The extension of the eq. (2.5) has an analogous double perturbative expansion in powers of α_S and α :

$$\begin{aligned} \mathcal{H}_N^F(\alpha_S, \alpha) &= \mathcal{H}_N^F(\alpha_S) + \frac{\alpha}{\pi} \mathcal{H}_N^F(1) + \sum_{n=2}^{\infty} \left(\frac{\alpha}{\pi}\right)^n \mathcal{H}_N^F(n) \\ &+ \sum_{n,m=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \left(\frac{\alpha}{\pi}\right)^m \mathcal{H}_N^F(n,m), \end{aligned} \quad (2.8)$$

³In order to simplify the expressions in this section we do not explicitly introduce an additional dependence on perturbative scales in QED (i.e. we fix them to be equal to the corresponding QCD ones).

where the pure QED corrections are controlled by the coefficients $\mathcal{H}_N^{F(n)}$ while the mixed QCD-QED ones are contained in the coefficients $\mathcal{H}_N^{F(n,m)}$.

The LL and NLL functions $g^{(1)}$ and $g_N^{(2)}$ in eq. (2.7) have the same functional form of the corresponding QCD ones

$$g^{(1)}(\alpha L) = \frac{A_q^{(1)}}{\beta_0'} \frac{\lambda' + \ln(1 - \lambda')}{\lambda'} , \quad (2.9)$$

$$g_N^{(2)}(\alpha L) = \frac{\tilde{B}_{q,N}^{(1)}}{\beta_0'} \ln(1 - \lambda') - \frac{A_q^{(2)}}{\beta_0'^2} \left(\frac{\lambda'}{1 - \lambda'} + \ln(1 - \lambda') \right) + \frac{A_q^{(1)} \beta_1'}{\beta_0'^3} \left(\frac{1}{2} \ln^2(1 - \lambda') + \frac{\ln(1 - \lambda')}{1 - \lambda'} + \frac{\lambda'}{1 - \lambda'} \right) , \quad (2.10)$$

while the novel function $g^{(1,1)}(\alpha_S L, \alpha L)$, which controls the mixed QCD-QED correction at leading logarithmic accuracy, reads

$$g^{(1,1)}(\alpha_S L, \alpha L) = \frac{A_q^{(1)} \beta_{0,1}}{\beta_0'^2 \beta_0'} h(\lambda, \lambda') + \frac{A_q^{(1)} \beta_{0,1}'}{\beta_0'^2 \beta_0} h(\lambda', \lambda) , \quad (2.11)$$

with

$$h(\lambda, \lambda') = -\frac{\lambda'}{\lambda - \lambda'} \ln(1 - \lambda) + \ln(1 - \lambda') \left[\frac{\lambda(1 - \lambda')}{(1 - \lambda)(\lambda - \lambda')} + \ln \left(\frac{-\lambda'(1 - \lambda)}{\lambda - \lambda'} \right) \right] - \text{Li}_2 \left(\frac{\lambda}{\lambda - \lambda'} \right) + \text{Li}_2 \left(\frac{\lambda(1 - \lambda')}{\lambda - \lambda'} \right) , \quad (2.12)$$

where $\lambda = \frac{1}{\pi} \beta_0 \alpha_S L$, $\lambda' = \frac{1}{\pi} \beta_0' \alpha L$, and $\beta_0, \beta_0', \beta_1', \beta_{0,1}, \beta_{0,1}'$ are the coefficients of the QCD and QED β functions:

$$\frac{d \ln \alpha_S(\mu^2)}{d \ln \mu^2} = \beta(\alpha_S(\mu^2), \alpha(\mu^2)) = - \sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_S}{\pi} \right)^{n+1} - \sum_{n,m=1=0}^{\infty} \beta_{n,m} \left(\frac{\alpha_S}{\pi} \right)^{n+1} \left(\frac{\alpha}{\pi} \right)^m , \quad (2.13)$$

$$\frac{d \ln \alpha(\mu^2)}{d \ln \mu^2} = \beta'(\alpha(\mu^2), \alpha_S(\mu^2)) = - \sum_{n=0}^{\infty} \beta_n' \left(\frac{\alpha}{\pi} \right)^{n+1} - \sum_{n,m=1=0}^{\infty} \beta_{n,m}' \left(\frac{\alpha}{\pi} \right)^{n+1} \left(\frac{\alpha_S}{\pi} \right)^m . \quad (2.14)$$

The function $g^{(1,1)}$ in eqs. (2.11), (2.12) has been obtained by plain integration, starting from the integral representation (see e.g. eq. (19) of ref. [47]) of the form factor in eq. (2.7). The mixed QCD-QED terms included in eqs. (2.11), (2.12) originate from the QED evolution of the QCD coupling and from the QCD evolution of the QED coupling, respectively in the pure QCD and QED form factors. Indeed in eqs. (2.13), (2.14) we have consistently included the mixed QCD-QED contributions to the running of the QCD and QED couplings through the coefficient $\beta_{n,m}$ and $\beta_{n,m}'$. The explicit expressions of the

coefficients $\beta_0, \beta'_0, \beta'_1, \beta_{0,1}, \beta'_{0,1}$ are:

$$\beta_0 = \frac{1}{12}(11 C_A - 2 n_f), \quad \beta_{0,1} = -\frac{N_q^{(2)}}{8}, \quad (2.15)$$

$$\beta'_0 = -\frac{N^{(2)}}{3}, \quad \beta'_1 = -\frac{N^{(4)}}{4}, \quad \beta'_{0,1} = -\frac{C_F C_A N_q^{(2)}}{4}, \quad (2.16)$$

where we have defined

$$N^{(n)} = N_c \sum_{q=1}^{n_f} e_q^n + \sum_{l=1}^{n_l} e_l^n, \quad (2.17)$$

$$N_q^{(n)} = \sum_{q=1}^{n_f} e_q^n, \quad (2.18)$$

with $N_c = 3$ the number of colours, $C_A = N_c$, $C_F = (N_c^2 - 1)/(2N_c)$, n_f (n_l) the number of quark (lepton) flavours and e_q (e_l) the quark (lepton) electric charges ($e_q = 2/3$ for up-type quarks, $e_q = -1/3$ for down-type quarks, $e_l = 1$ for leptons).

The coefficients $A_q^{(1)}$ and $A_q^{(2)}$, which have been obtained from the corresponding coefficients in QCD [42–44, 55] through an Abelianization algorithm [56, 57], read

$$A_q^{(1)} = e_q^2, \quad A_q^{(2)} = -\frac{5}{9} e_q^2 N^{(2)}, \quad (2.19)$$

while the coefficient $\tilde{B}'_{q,N}{}^{(1)}$ is

$$\tilde{B}'_{q,N}{}^{(1)} = B_q^{(1)} + 2\gamma'_{qq,N}{}^{(1)}, \quad (2.20)$$

with

$$B_q^{(1)} = -\frac{3}{2} e_q^2, \quad (2.21)$$

$$\gamma'_{qq,N}{}^{(1)} = e_q^2 \left(\frac{3}{4} + \frac{1}{2N(N+1)} - \gamma_E - \psi_0(N+1) \right), \quad (2.22)$$

$$\gamma'_{q\gamma,N}{}^{(1)} = \frac{3}{2} e_q^2 \frac{N^2 + N + 2}{N(N+1)(N+2)}, \quad (2.23)$$

where $\psi_0(N)$ is the digamma function and $\gamma'_{ab,N}{}^{(1)}$ are the leading-order (LO) anomalous dimensions in QED.⁴

The LL mixed QCD-QED corrections are included in the $g^{(1,1)}(\alpha_S L, \alpha L)$ function which depends on the coefficients $A_q^{(1)} = C_F$, $A_q^{(2)}$, β_0 , β'_0 , $\beta_{0,1}$, $\beta'_{0,1}$. We note that these logarithmic terms do not contribute to the $\mathcal{O}(\alpha\alpha_S)$ fixed order corrections and their dominant contribution is of the order $\alpha\alpha_S^2 L^3$.

⁴The QED anomalous dimension $\gamma'_{q\gamma,N}{}^{(1)}$ is required at the NLL in the general multiflavour case (see appendix A of ref. [47]).

In order to reach the complete NLL+NLO accuracy for QED, the functions $\mathcal{H}_{a_1 a_2, N}^{\prime F(1)}$ and the finite component of the partonic cross sections $d\hat{\sigma}_{a_1 a_2}^{(\text{fin.})}$ at the first non-trivial order are needed. The hard-collinear functions $\mathcal{H}_{a_1 a_2, N}^{\prime F(1)}$ read

$$\mathcal{H}_{q\bar{q} \leftarrow q\bar{q}, N}^{\prime F(1)} = \frac{e_q^2}{2} \left(\frac{2}{N(N+1)} - 8 + \pi^2 \right), \quad (2.24)$$

$$\mathcal{H}_{q\bar{q} \leftarrow \gamma q, N}^{\prime F(1)} = \mathcal{H}_{q\bar{q} \leftarrow q\gamma, N}^{\prime F(1)} = \frac{3 e_q^2}{(N+1)(N+2)}, \quad (2.25)$$

$$\mathcal{H}_{q\bar{q} \leftarrow \gamma\gamma, N}^{\prime F(1)} = \mathcal{H}_{q\bar{q} \leftarrow qq, N}^{\prime F(1)} = \mathcal{H}_{q\bar{q} \leftarrow \bar{q}\bar{q}, N}^{\prime F(1)} = 0. \quad (2.26)$$

The explicit perturbative scale dependence of the formulae in eqs. (2.9), (2.10), (2.24) is the same as the corresponding one in QCD (see eqs. (22), (23), (46) of ref. [47]).

3 Numerical results

In this section we present selected numerical results, by explicitly considering the transverse-momentum distribution of on-shell Z bosons in hadronic collisions. We start our analysis from the resummed results in QCD at NNLL+NNLO as implemented in the DYqT numerical code [58, 59] and we include the QED corrections up to NLL+NLO.

As for the electroweak couplings, we use the following input parameters: $\alpha(m_Z^2) = 1/127.95$, $\sin^2 \theta_W = 0.23129$ and $m_Z = 91.1876$ GeV [60]. The electromagnetic coupling α is evaluated at 1-loop and 2-loops respectively at LO and NLO in QED. The hadronic cross section is computed using the NNPDF3.1QED parton distribution function (PDF) set at NNLO in QCD [61, 62], which includes the photon PDF as determined within the LUX method [63, 64] as well as the LO QED effects in the evolution of the parton densities. The strong coupling α_S is evaluated at 3-loop order with $\alpha_S(m_Z^2) = 0.118$ within the $\overline{\text{MS}}$ renormalization scheme. We work with $n_l = 3$ charged leptons and $n_f = 5$ flavours of light quarks in the massless approximation.

We set the central value of the renormalization, factorization and resummation scales at $\mu_R = \mu_F = 2Q = m_Z$. We provide an estimate of the perturbative uncertainties of the calculation due to missing higher-order QED terms performing the variation of the renormalization (μ'_R) and resummation (Q') scales associated to the QED contributions. Specifically, the variation of μ'_R and Q' around their central value can be used to estimate the effects of yet uncalculated QED contributions respectively at fixed order and logarithmic accuracy level. We thus perform an independent variation of μ'_R and Q' in the range $m_Z/2 \leq \{\mu'_R, 2Q'\} \leq 2m_Z$ with the constraint $1/2 \leq \{\mu'_R/Q'\} \leq 2$. In principle it would be possible to consider an independent variation also for the factorization scale (μ'_F) related to the QED emissions. However since the QCD and QED factorization scales are assumed to be equal in the evolution of parton densities [62] we keep the factorization scale fixed at the central value $\mu'_F = \mu_F = m_Z$ in our numerical study. The dependence of the resummed QCD predictions from factorization, renormalization and resummation scales has been studied in detail in refs. [58, 59, 65].

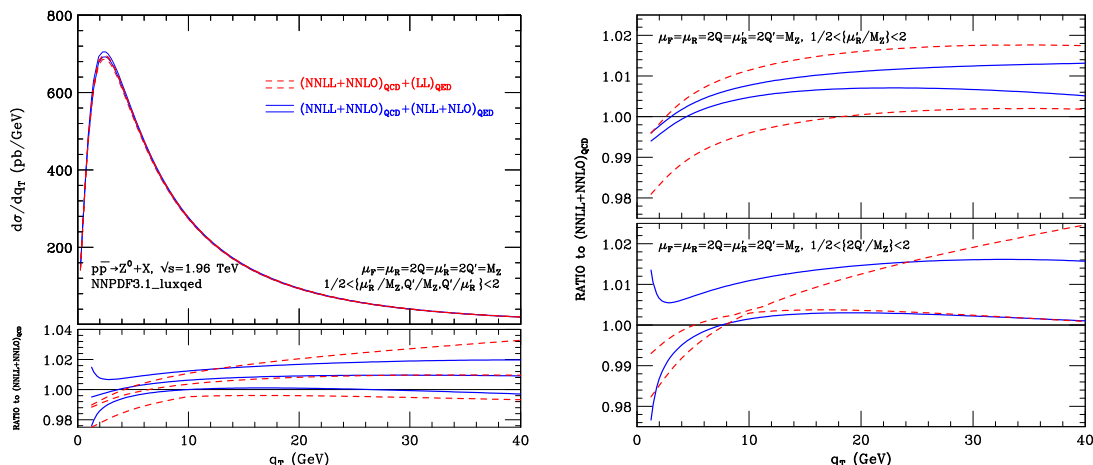


Figure 1. The q_T spectrum of Z boson at the Tevatron ($\sqrt{s} = 1.96$ TeV). Left panel: NNLL+NNLO QCD results are combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands. The bands are obtained by performing the variation of the μ'_R and Q' scales in QED around their central value as described in the text. The lower panel shows the ratio of the QED scale-dependent results with respect to the standard NNLL+NNLO QCD result at central value of the scales. Right panel: upper (lower) panel shows the ratio of the resummation (renormalization) QED scale-dependent results with respect to the standard NNLL+NNLO QCD result at central value of the scales.

We start considering on-shell Z production in $p\bar{p}$ collisions at Tevatron Run II energies ($\sqrt{s} = 1.96$ TeV). In figure 1 (left panel) we present the NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED corrections. The lower panel presents the ratio of our predictions with respect to the standard NNLL+NNLO QCD result at the corresponding central scale.

We observe that the resummation of the QED contributions at LL accuracy for central scale values ($\mu'_R = 2Q' = m_Z$) has the effect to make the q_T spectrum slightly harder. The QED effects at LL are negative in the small q_T region ($q_T \lesssim 6$ GeV) while they are positive in the q_T region $6 \lesssim q_T \lesssim 40$ GeV. The impact of these effects reaches the $\mathcal{O}(1\%)$ level. We recall that, owing to the unitarity constraint of the resummation formalism that we used [47], the LL QED effects for central values of the scales give vanishing contribution to the total (i.e. q_T integrated) cross section: they only affect the *shape* of the q_T distribution by (slightly) *shifting* part of the cross section to higher values of q_T . This physical effect to the q_T distribution is expected and it is due to the contribution to the Z recoil generated by soft and collinear photon emissions to all orders in QED. The NLL+NLO QED effects at central value of the scales $\mu'_R = 2Q' = m_Z$ are of $\mathcal{O}(0.5\%)$ level and they are positive for the entire q_T region considered ($q_T \lesssim 40$ GeV). Thanks to the perturbative unitarity of the resummation formalism, the contribution to the integrated q_T distribution of the NLL+NLO QED result exactly coincides with the NLO QED correction to the total cross section which is equal to +0.3%.

By considering the scale variation band, we observe that the LL QED effects have an uncertainty of around 2% in the small q_T region ($q_T \lesssim 6$ GeV) which increases up to 4% in the intermediate q_T region ($30 \lesssim q_T \lesssim 40$ GeV). The inclusion of the NLL+NLO QED corrections reduces the scale variation band by roughly a factor 2. In order to better illustrate the dependence on the QED scales, in figure 1 (right panel) we separately consider μ'_R and Q' variations. The upper plot corresponds to the variation of μ'_R by a factor 2 around the central value and the lower plot corresponds to an analogous variation of Q' . As expected from the QED running of α , the q_T distribution decreases (increases) by decreasing (increasing) the value of μ'_R . The μ'_R scale dependence is flat and at the level of 2% at LL and it decreases to around 0.5–1% at NLL+NLO. The Q' dependence does not affect the integral of the q_T distribution which is constrained by perturbative unitarity. The resummation scale dependence at LL is about 1% for $q_T \lesssim 5$ GeV, it reduces up to 0.1% for $q_T \sim 10$ GeV and it increases again up to about 2% for $q_T \sim 30$ –40 GeV. The reduction of the Q' dependence in the region $q_T \sim 10$ GeV is due to the crossing (necessary in order to fulfill unitarity) between the predictions with $Q' = m_Z$ and $Q' = m_Z/4$. This accidental cancellation suggests that the Q' variation band at LL might underestimate the perturbative uncertainty of the prediction in the region around $q_T \sim 10$ GeV. In fact at NLL+NLO the Q' dependence for $q_T \gtrsim 4$ GeV is constant and at $\mathcal{O}(1\%)$ level. Below the peak the Q' dependence rapidly increases. We note that in the region $q_T \lesssim 3$ GeV the q_T distribution is steeply falling to zero. We finally observe that at LL the scale variation is mainly driven by μ'_R because the Born-level cross section is proportional to α , while at NLL+NLO the scale uncertainty is dominated by the Q' dependence.

The integral over q_T of the predictions with NLL+NLO QED effects are in agreement (for fixed values of renormalization and factorization scales) with the corresponding NLO QED contributions to the total cross section at the $\mathcal{O}(0.1\%)$ level,⁵ thus checking the numerical accuracy of our implementation.

We now turn to consider on-shell Z production in pp collisions at the LHC energies. In figure 2 (left panel) we present our results for LHC Run I ($\sqrt{s} = 8$ TeV) and in figure 3 (left panel) the results for LHC Run II ($\sqrt{s} = 13$ TeV). The effects of the QED contributions at the LHC are qualitatively similar and quantitatively slightly smaller with respect to the case of the Tevatron.

From the left panels of figures 2 and 3 we observe that the QED effects at LL accuracy, for central value of the scales, give a negative $\mathcal{O}(1\%)$ contribution in the small q_T region ($q_T \lesssim 8$ –10 GeV) and a positive $\mathcal{O}(0.5\%)$ contribution in the q_T region $10 \lesssim q_T \lesssim 40$ GeV. The NLL+NLO QED effects are at the level of $\mathcal{O}(0.5\%)$ and positive for the entire q_T region considered ($q_T \lesssim 40$ GeV). The NLO QED contribution to the total cross section is equal to +0.3%.

The impact on the q_T distributions of the mixed QED-QCD corrections is numerically very small (lower than 0.1% level) both at the Tevatron and at the LHC, and it has not been separately shown in the plots of figures 1–3.

⁵The NLO QED ($\mathcal{O}(\alpha^2)$) corrections to the total cross section and to the finite component $d\hat{\sigma}^{(\text{fin.})}$ of the q_T distribution have been obtained through the Abelianization of the calculations in refs. [2, 3] and in ref. [33] respectively.

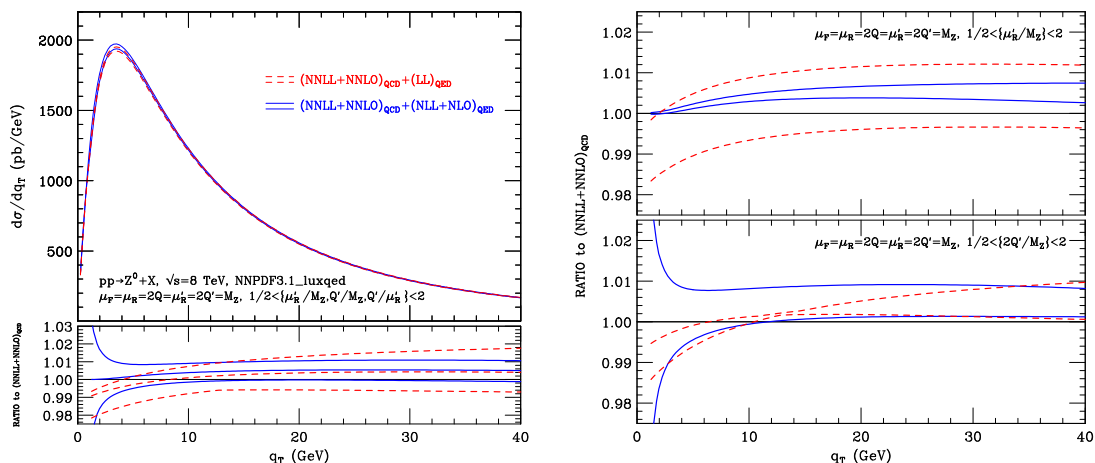


Figure 2. The q_T spectrum of Z boson at the LHC ($\sqrt{s} = 8$ TeV). Left panel: NNLL+NNLO QCD results are combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands. The bands are obtained as in figure 1.

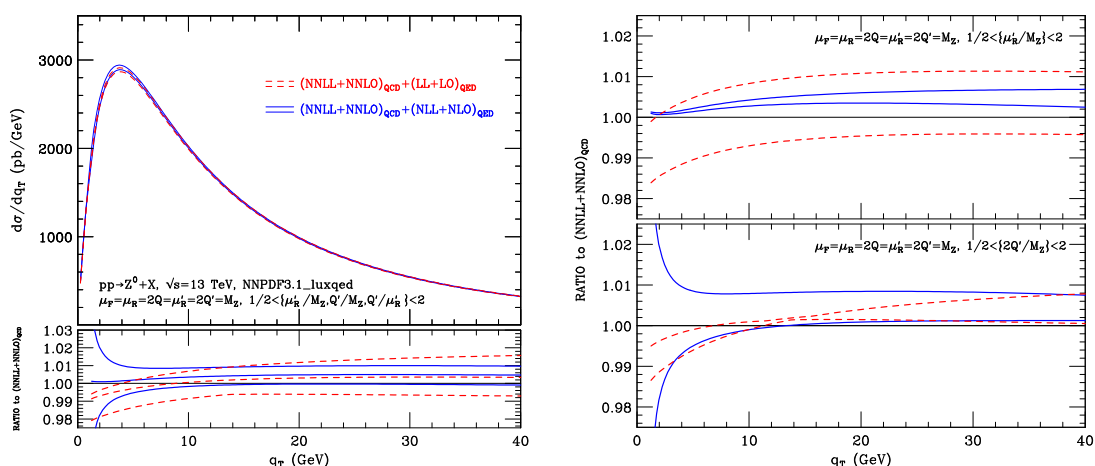


Figure 3. The q_T spectrum of Z boson at the LHC ($\sqrt{s} = 13$ TeV). Left panel: NNLL+NNLO QCD results are combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands. The bands are obtained as in figures 1 and 2.

By considering the perturbative uncertainty shown in the lower-left panels of figures 2 and 3, we observe that the LL QED effects have an uncertainty of around 2% in the small q_T region ($q_T \lesssim 10$ GeV) which increases up to 2.5–3% in the intermediate q_T region ($30 \lesssim q_T \lesssim 40$ GeV). The inclusion of the NLL+NLO QED corrections reduces the scale variation band by roughly a factor 1.5–2. As in the Tevatron case the QED uncertainty is dominated by the renormalization (resummation) scale dependence at LL (NLL+NLO).

4 Summary

In this paper we have extended the QCD transverse-momentum (q_T) resummation formalism of refs. [46, 47] in order to deal with the simultaneous emission of QCD and QED initial state parton radiation. In particular, through the application of the Abelianization algorithm of refs. [56, 57], we have obtained analytic resummed results which are valid for the production of generic neutral and colourless high-mass systems in hadronic collisions.

We have applied our formalism to the transverse-momentum (q_T) distribution of on-shell Z bosons produced at the Tevatron and the LHC. Starting from the known QCD results at next-to-next-to-leading logarithmic accuracy (NNLL) and next-to-next-to-leading order (NNLO) (i.e. $\mathcal{O}(\alpha_S^2)$) [58, 59], we have included the resummation of the logarithmically enhanced QED contributions at small values of q_T up to next-to-leading logarithmic accuracy (NLL) and of the mixed QCD-QED contributions at leading logarithmic accuracy (LL). Our results have been consistently matched with the next-to-leading fixed-order (NLO) results (i.e. $\mathcal{O}(\alpha^2)$) in QED at small, intermediate and large values of q_T .

We have found that the resummation of the QED contributions at leading-logarithmic accuracy has the effect of slightly shifting (roughly $\mathcal{O}(\pm 1\%)$) part of the cross section to higher values of q_T . The NLL+NLO QED effects at the central value of the scales are $\mathcal{O}(+1\%)$ and rather flat for the entire q_T region considered ($q_T \lesssim 40$ GeV).

We have then performed a study of the scale dependence of the QED contribution in order to estimate the corresponding perturbative uncertainty. We have shown that the scale variation band at LL is of $\mathcal{O}(2-4\%)$ and that the inclusion of the NLL+NLO QED results reduces the scale variation band by roughly a factor 1.5-2. The QED uncertainty is dominated by the renormalization and resummation scale dependence at LL and NLL+NLO respectively.

In conclusion we have found that, given the present theoretical and experimental accuracy, the QED resummation effects for on-shell Z boson production are not negligible. In particular the inclusion of the NLL+NLO QED corrections reduces the effect of the renormalization scale ambiguity of the QED coupling α by consistently including running coupling effects.

The inclusion of the QED effects from the leptonic decay of the Z boson and the generalization of our results to the case of charged final states (e.g. W^\pm) are left for future investigations.

Acknowledgments

We gratefully acknowledge Stefano Catani, Daniel de Florian and Alessandro Vicini for useful discussions and comments on the manuscript. G.S. would like to thank also Germán Rodrigo and Félix Driencourt-Mangin for interesting discussions. This research was supported in part by Fondazione Cariplo under the grant number 2015-0761 and by the COST Action CA16201 PARTICLEFACE.

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