The Capacitated Team Orienteering Problem with Incomplete Service

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Abstract

In this paper we study the capacitated version of the Team Orienteering Problem (TOP), that is the Capacitated TOP (CTOP) and the impact of relaxing the assumption that a customer, if served, must be completely served. We prove that the profit collected by the CTOP with Incomplete Service (CTOP-IS) may be as large as twice the profit collected by the CTOP. A computational study is also performed to evaluate the average increase of the profit due to allowing incomplete service. The results show that the increase of the profit strongly depends on the specific instance. On the tested instances the profit increase ranges between 0% and 50%. We complete the computational study with the increase of the profit of the CTOP due to split deliveries, that is multiple visits to the same customer, and to split deliveries combined with incomplete service.

Keywords: Capacitated Team Orienteering Problem, Incomplete Service, Split Deliveries, Exact Algorithms

1 Introduction

In the Team Orienteering Problem (TOP) a set of potential profitable customers is given. The time distance between each pair of customers and each customer and the depot is also given. A fleet of vehicles, with a time limit on the duration of each tour, is available to serve a subset of the potential customers. The TOP consists in finding the customers that maximize the total collected profit, while satisfying the time duration of the tour of each vehicle. The TOP extends to the case of multiple vehicles the Orienteering Problem (OP), the most studied of the traveling salesman problems with profits (see the survey by Feillet et al. [10]).

The TOP is a well studied problem that, to the best of our knowledge, was first presented and heuristically solved by Butt and Cavalier [7]. The problem was originally called Multiple Tour Maximum Collection Problem, whereas the name Team Orienteering Problem was introduced by Chao et al. in [8]. Several heuristics and metaheuristics were proposed for the solution of the TOP (see the papers by Tang and Miller-Hooks [15], by Archetti et al. [4], Ke et al. [11] and by Vansteenwegen et al. [13]). Exact algorithms for the TOP were proposed by Boussier et al. in [6] and Viana et al. [19]. The TOP with time windows has been studied by Montemanni and Gambardella [12], Vansteenwegen et al. [17], Tricoire et al. [16] and by Souffriau et al. [14]. For a survey on the OP and the TOP the reader is referred to [18].

In the capacitated version of the TOP, called Capacitated TOP (CTOP), customers have demands and vehicles are capacitated. The CTOP was introduced in Archetti et al. [5] where applications are described and exact and heuristic algorithms were presented. An improved exact method was presented in Archetti et al. [1]. In the CTOP a customer, if served, must be completely served. This is a restrictive assumption that may lower the profit collected by the vehicles. The possibility of serving a customer only partially depends on the application. There may be cases where a partial service is not allowed for organizational reasons or because the products to be delivered to a customer cannot be split on different vehicles. However, there are situations where a partial service is possible. In these situations one may take advantage of the possibility to partially serve a customer and increase the collected profit thanks to a better exploitation of the vehicle capacity. In this situation, if a customer will be served only partially, the remaining unserved demand may be served at a later time by the same fleet of vehicles or by a different fleet of vehicles, possibly together with the customers that have not been served. In the CTOP with Incomplete Service (CTOP-IS), each customer is served only once, either completely or partially. If a customer is partially served, then the profit collected is assumed to be proportional to the demand served.

Problems related to the CTOP-IS are the Split Delivery CTOP (SDCTOP), where customers have to be completely served but a customer may be served by more than one vehicle, if beneficial, and the SDCTOP with Incomplete Service (SDCTOP-IS) where customers may be served by several vehicles and the service may be incomplete. The SDCTOP and the SDCTOP-IS have been studied in [3] and in [2], respectively. In these papers, worst-case results are provided to show the maximum increase of the profit achievable by allowing split deliveries and by combining split deliveries with incomplete service, with respect to the CTOP. Computational results have also been presented to show the average increase of the profit and how the increase depends on specific characteristics of the instance, namely the ratio between the vehicle capacity and the average demand.

Whereas the profit collected in the CTOP-IS is certainly not less than the profit collected in the CTOP, we analyze the maximum possible gain and show that it may be possible to double the profit collected by the CTOP. Moreover, we solve the CTOP-

IS exactly and compare it with different 'relaxations' of the CTOP studied so far, i.e. the SDCTOP and SDCTOP-IS, previously studied, on different classes of instances. The results show in particular that, whereas the increase of the profit due to incomplete service may be as large as 100%, on the tested instances the profit increase ranges between 0% and 50%.

In Section 2 we define the CTOP-IS. In Section 3 we present properties of the problem and show the maximum profit increase with respect to the CTOP. Sections 4 and 5 are devoted to the description of the solution approach for the CTOP-IS and to the computational results, respectively.

2 The Capacitated Team Orienteering Problem with Incomplete Service

In the TOP a complete undirected graph $G = (V, E)$ is given, where $V = 1, \ldots, n$ is the set of vertices and E is the set of edges. Vertex 1 is the depot and vertex $i = 2, \ldots, n$ represents a potential customer. A nonnegative profit p_i is associated with each vertex i ($p_1 = 0$) and a travel time c_{ij} is associated with each edge $(i, j) \in E$. We assume that the travel times satisfy the triangle inequality. A set of m vehicles is available to visit a subset of the potential customers. Each vehicle starts and ends its tour at vertex 1. The duration of each tour cannot exceed a time limit T_{max}. The profit of any customer i can be collected by one vehicle at most. The TOP consists in maximizing the total collected profit while satisfying the time limit T_{max} for each vehicle.

In the CTOP each customer has a demand d_i and each vehicle has a capacity Q. We assume that $d_i \leq Q$. The objective is to maximize the total collected profit while satisfying the time limit T_{max} on the duration of each tour and the vehicle capacity constraint.

In the CTOP with Incomplete Service (CTOP-IS) a customer is allowed to be partially served. If a customer is partially served, a proportional part of the profit is collected.

We denote by $z(CTOP-IS)$ the value of an optimal solution of the $CTOP-IS$ and, in general, by $z(P)$ the value of an optimal solution of the problem P.

3 Properties and worst-case analysis

In this section we show that the profit collected in the CTOP may be doubled when incomplete (partial) service of the customers is allowed.

We first show a property of the optimal solution of the CTOP-IS that will be useful in the following. The same property has been shown to hold for the SDCTOP-IS in [2]. The proof here follows the same lines and we report it for the sake of completeness.

Theorem 1 *An optimal solution of the CTOP-IS exists where each tour has at most one customer with incomplete service.*

Proof: Let us take an optimal solution of the CTOP-IS and suppose that more than one customer receives incomplete service. Take a tour with two such customers, say i and j, and assume, w.l.o.g., that $p_i/d_i \geq p_j/d_j$. Then, we modify the optimal solution by moving as much as possible of the demand served of the least profitable customer j to the most profitable one i. We decrease the demand served of j to increase the demand served of i by the same quantity. Let us observe the modified solution. If the demand served of j was greater than or equal to the unserved demand of i, the demand of i is now completely satisfied. In this case, the number of customers with incomplete service is reduced by 1. Moreover, the modified solution is not worse than the original, optimal, solution. If, instead, the demand served of j was smaller than the unserved demand of i, then customer i remains partially served but customer j is not served at all by the considered tour. In this case, we modify the solution by removing customer j from the tour. Also in this case the number of partially served customers is reduced by 1 with respect to the original solution. Moreover, the modified solution is not worse than the original, optimal, solution. We repeat this procedure until at most one customer with incomplete service remains in the tour. Then we repeat the procedure on all the other tours.

□

We note that, for the property to hold, the triangle inequality assumption on the time distances is essential.

We now analyze the increase of the profit that can be achieved by allowing incomplete service in the CTOP. In the CTOP-IS each customer is visited by at most one tour, both in the case the customer is completely served and in the case it is only partially served. The proof of the result follows the lines of a similar result that shows the increase of the profit due to the incomplete service for the SDCTOP (see [2]).

Theorem 2

$$\frac{z(CTOP)}{z(CTOP-IS)} \geq \frac{1}{2}$$

and this bound is tight.

Proof: Let us consider an optimal solution of the CTOP-IS where each tour has at most one customer with incomplete service. Theorem 1 guarantees that such a solution exists. We modify this solution to obtain a solution where all customers are completely served, that is a feasible solution of the CTOP. The tours where all customers are completely served remain unchanged.

If a tour exists where a customer has incomplete service we proceed as follows. We consider two different ways to modify this tour. The first one is to remove the customer with incomplete service and keep only the customers with complete service. In this case the profit of the customer with incomplete service is lost. We also create a tour where only the customer with incomplete service is included and completely served. In this case the profits of the customers with complete service are lost. Note that both tours satisfy the capacity and time constraints. In the modified solution, we replace the tour that we have started from with the most profitable of these two modified tours. By construction, the profit of this tour is greater than or equal to half the profit of the tour we have started from. We repeat the procedure on all tours with customers with incomplete service. The procedure guarantees that the modified solution is a feasible solution of the CTOP with profit not lower than half the optimal profit of the CTOP-IS.

To show that the bound is tight, consider the instance depicted in Figure 1 with two customers, each with $d_i = p_i = \frac{Q}{2} + 1$, one vehicle with capacity Q and a large value of T_{max}. The optimal solution of the CTOP serves only one customer, whereas the optimal solution of the CTOP-IS serves $\frac{Q}{2} + 1$ of the demand of one customer and $\frac{Q}{2} - 1$ of the demand of the other customer (see Figure 1). When Q tends to infinity the ratio tends to 1/2.

$\square$

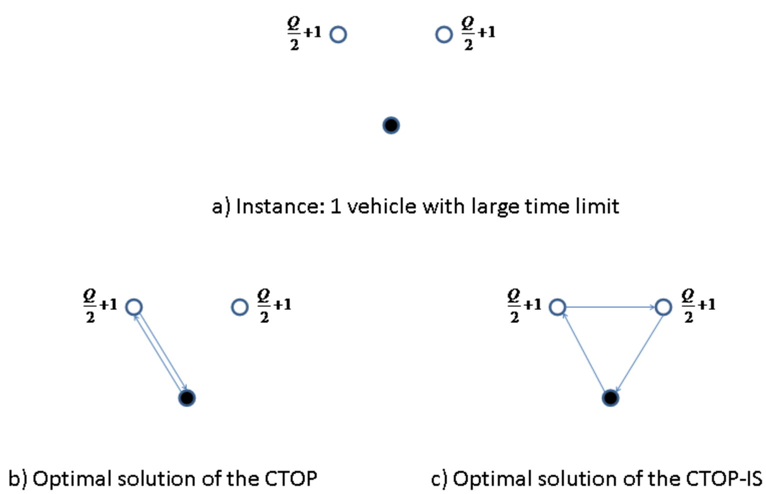

Figure 1: Instance with $z^*(CTOP) = \frac{Q}{2} + 1, z^*(CTOP - IS) = Q$

We summarize in Table 1 the theoretical results known for the studied variants of the CTOP, namely the CTOP-IS, the SDCTOP and the SDCTOP-IS. These variants are all 'relaxations' of the CTOP, where incomplete service or/and multiple visits to a customer are allowed. In each variant the maximum theoretical increase of the profit with respect to the CTOP is 100%. These results imply that either split deliveries or incomplete service may increase the profit by the same maximum amount and that combining the two relaxations does not help in increasing the profit more.

	CTOP-IS	SDCTOP	SDCTOP-IS
CTOP	$\mathbf{\frac{1}{2}}$	$\frac{1}{2}$	$\frac{1}{2}$

Table 1: Comparison of worst-case results for CTOP

One may also wonder what is the benefit of having split deliveries in the CTOP-IS, that is in studying the ratio between $z(CTOP-IS)$ and $z(SDCTOP-IS)$. As obviously $z(CTOP-IS) \geq z(CTOP)$, then from Table 1 $\frac{z(CTOP-IS)}{z(SDCTOP-IS)} \geq \frac{1}{2}$. However, it seems quite unlikely that this bound is tight, i.e., that split deliveries may double the collected profit with respect to the one collected in the CTOP with incomplete service. We have found an instance where split deliveries increase the profit by one third but could not find instances where the profit increases by more. Let us consider three customers. All distances are equal to 1 but the distance between customers 1 and 3 which is 2. Let us take two vehicles with $Q = 2$ and $T_{max} = 3$. Moreover, $d_i = p_i$, $\forall i$, $d_1 = d_3 = 1$, $d_2 = 2$. It can be seen that $z(CTOP-IS) = 3$ while $z(SDCTOP-IS) = 4$. Thus, we conjecture that $\frac{z(CTOP-IS)}{z(SDCTOP-IS)} \geq \frac{3}{4}$.

4 A branch-and-price algorithm

In the CTOP-IS, like in the SDCTOP and in the SDCTOP-IS, decisions have to be taken about the quantities to deliver to each customer. Nevertheless, contrary to what happens in the cited problems, each customer can be visited at most once. The problem can thus be formulated by means of a set-packing model where variables are associated with the feasible vehicle tours. Tours are circuits in the graph G starting and ending at the depot. A tour is feasible if the total travel time and the total quantity delivered do not exceed the given time limit T_{max} and the vehicle capacity Q, respectively. According to Theorem (1), when looking for an optimal solution of the CTOP-IS, it is sufficient to consider the feasible tours where at most one customer is partially served.

We addressed the CTOP-IS by means of a branch-and-price algorithm. Let us first formulate the Master Problem (MP). We define $\mathcal{R}$ as the set of all feasible tours. For each tour $r \in \mathcal{R}$, let p_r be the profit collected by the tour and a_{ir} be a binary coefficient equal to 1 if customer $i \in V \setminus \{1\}$ is visited by the tour. The MP can be formulated as follows:

$$\max \sum_{r\in\mathcal{R}} p_r\lambda_r \tag{1}$$

$$\sum_{r\in\mathcal{R}} a_{ir}\lambda_r \leq 1 \qquad \forall i \in V' = V \setminus \{1\} \tag{2}$$

$$\sum_{r\in\mathcal{R}} \lambda_r \leq m \tag{3}$$

$$\lambda_r \in \{0,1\} \qquad \forall r \in \mathcal{R} \tag{4}$$

where binary variable λ_r, $r \in \mathcal{R}$, is equal to 1 if the corresponding tour is selected in the optimal solution. The objective function (1) aims at maximizing the overall collected profit, constraints (2) guarantee that each customer is visited at most once while (3) establishes that at most m vehicles are used.

The Linear relaxation of the MP (LMP) is solved by means of column generation. The MP, when restricted to a subset of variables (columns), is called Restricted Master Problem (RMP). We initially solve the linear relaxation of a RMP (RLMP), restricted to an initial set of columns. Then, the dual information obtained from the optimal solution of the RLMP is used to find new positive reduced cost columns to be inserted in the RLMP. The procedure is repeated until no such column exists. The optimal solution of the final RLMP is the optimal solution of the LMP. In order to find positive reduced cost columns, a pricing problem is solved which corresponds to a Shortest Path Problem with Resource Constraints (SPPRC), where the resources correspond to tour duration and quantity delivered. As we know that an optimal solution exists where at most one customer is partially served in any tour, an additional resource is used to limit to one the number of partially served customers along a tour. The pricing problem is solved through a labeling algorithm adapted from the one presented in [3] for the SDCTOP.

As the optimal solution of the LMP can be fractional, branching is needed. Branching on the fractional use of arcs is sufficient to guarantee the integrality of the $\boldsymbol{\lambda}$ variables.

Feasible solutions of the CTOP-IS can be obtained by solving the RMP restricted to any subset of columns, that is by running a restricted master heuristic. At each node of the branch-and-bound tree we solve a RMP restricted to a subset of the columns generated by the pricing problem. The RMP is solved through a commercial solver. These solutions give primal bounds and speed-up the solution approach. For more details on the computation of the primal bounds we refer to [1] where a similar approach is described.

5 Computational results

In this section we computationally test the increase of the profit due to incomplete service or/and split deliveries.

The experiments were carried out on an Intel Xeon processor E5520, 2.26 GHz machine with 12 GB of RAM. CPLEX 12.2 was used to solve the linear relaxation of the MPs and the restrcted MPs of the restricted master heuristic. The overall execution time limit for each run was set to 6 hours. The parameters setting is the one proposed in [3]. In particular, the restricted master heuristic is run at the root node of the tree before each exact solution of the pricing problem, whereas, at non-root nodes, it is run after the solution of the LMP. The time limit for each individual run of the restricted master heuristic has been set to 1800 seconds. These settings have been chosen according to the results obtained in [1] and [3], where it is shown that the use of the restricted master heuristic improves the performance of the branch-and-price approach.

The tests were made on the set of instances proposed in [2] with up to 100 customers. They were generated as follows. Starting from 5 benchmark instances for the Vehicle Routing Problem (see [9]), for each of them 11 different instances were defined by randomly generating the demand of the customers from a uniform distribution on the interval $[\alpha Q, \beta Q]$, with the following 11 pairs of values for (α, β): $(0.01, 0.10)$, $(0.10, 0.30)$, $(0.10, 0.50)$, $(0.10, 0.70)$, $(0.10, 0.90)$, $(0.30, 0.50)$, $(0.30, 0.70)$, $(0.30, 0.90)$, $(0.50, 0.70)$, $(0.50, 0.90)$ and $(0.70, 0.90)$.

The comparison between the SDCTOP and the CTOP was reported in [3], while in [2] the SDCTOP was compared with the SDCTOP-IS. Our aim here is to give a global overview of the comparison of the profit collected by the CTOP with the profit collected by all the studied 'relaxations', namely the CTOP-IS, SDCTOP and SDCTOP-IS.

We first present, in Table 2, the computational results concerning the branch-and-price algorithm for the CTOP-IS. The first column contains the name of the instance. The last two numbers appearing in the name are the values of α and β, respectively. The following four columns report instance data. Finally, computational results are reported: '$\overline{z}^*$' is the final upper bound, '$\underline{z}^*$' is the best feasible solution found and 'gap(%)' is the percentage difference between the two. A gap equal to 0 means that the instance is solved to optimality and in the last column we report the computational time needed to solve it. From the table we can see that the branch-and-price algorithm was able to solve 22 instances over 55, 7 of which with 101 nodes.

The approach was not able to solve the root node of the branch-and-bound tree within the time limit in only 4 cases. In all instances not solved to optimality and for which the value of $\overline{z}^*$ is available, the final gap is always lower than 0.3%.

Table 3 is devoted to the comparison of the CTOP with the CTOP-IS, SDCTOP and SDCTOP-IS. Values in columns 6 and 9 are taken from [3] while values in column 11 are taken from [2]. Each entry in these columns represents the best feasible solution found for the corresponding problem instance. A '*' means that the solution value is optimal. In columns '(%)' we report the improvements, in percentage, over the corresponding CTOP solution value. Note that a negative value in the column corresponding to the improvement of the CTOP-IS with respect to the CTOP is due to the fact that the corresponding CTOP-

IS instance was not solved to optimality. It is important to note that, although for many instances the optimal solution is not available, the optimality gaps reported in Table 2 for the CTOP-IS and in [2] and [3] for the SDCTOP-IS and SDCTOP, respectively, guarantee that the value of the solutions reported in Table 3 are very close to optimality. Therefore, the columns '(%)' provide very good estimates of the percentage increase of the profit collected by the CTOP that is achieved by allowing incomplete service, split deliveries or by allowing both.

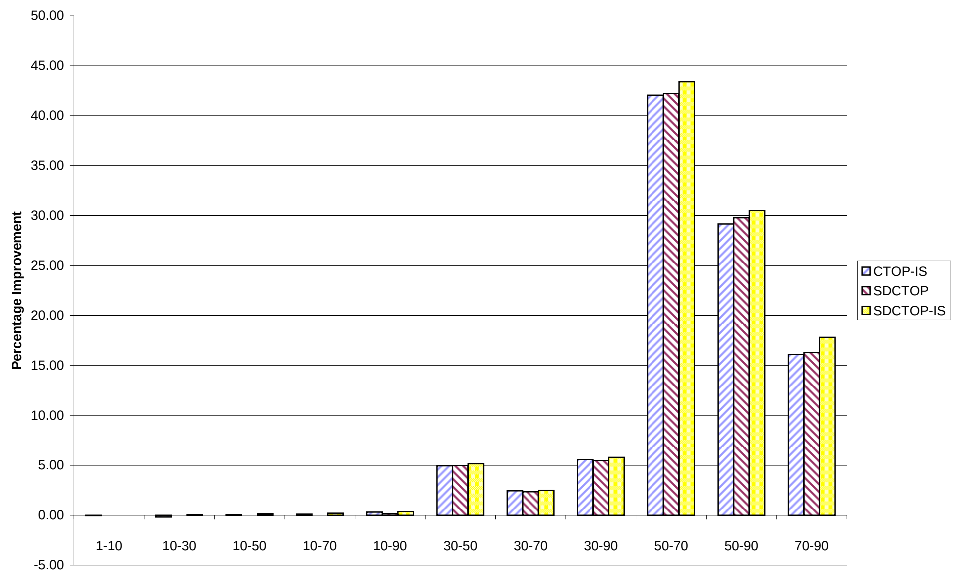

Figure 2. Average percentage improvements with respect to the CTOP

Figure 2 reports the average improvements of the profit achieved by the CTOP-IS, SDCTOP and SDCTOP-IS with respect to the CTOP, classified on the basis of the values of α and β. On the x-axis the tested pairs of 100α and 100β are indicated. The increase of the profit always depends on the specific instance and is on average substantially smaller than the maximum theoretical increase. On the tested instances the profit increase due to incomplete service only ranges between 0% and 50%, whereas the maximum improvement is 100%. Split deliveries and incomplete service show a similar behavior and give the largest improvement when customers demands are high, especially when they are just above half of vehicles capacity. The combination of incomplete service and split deliveries allows a small increase of the profit achieved by either incomplete service or split deliveries.

Table 2: Computational results

Instance					CTOP-IS Branch-and-price			
name	n	m	Q	T_{max}	$\overline{z}^*$	$\underline{z}^*$	gap(%)	
p03_1_10	101	15	200	200	1409.0000	1406.000	0.213	
p03_10_30					1305.1930	1303.000	0.168	
p03_10_50					1117.4490	1116.000	0.130	
p03_10_70					961.3103	961.000	0.032	
p03_10_90					1006.2174	1005.500	0.071	
p03_30_50					927.7140	927.686	0.003	
p03_30_70					811.7869	811.249	0.066	
p03_30_90					752.8447	752.820	0.003	
p03_50_70					-	737.695	-	
p03_50_90					-	636.193	-	
p03_70_90					-	585.880	-	
p06_1_10	51	10	160	200	761.0000	761.000	0.000	(1138")
p06_10_30					758.3333	758.333	0.000	(7911")
p06_10_50					689.2982	689.298	0.000	(14446")
p06_10_70					584.1488	584.060	0.015	
p06_10_90					497.5631	497.531	0.006	
p06_30_50					538.3997	538.350	0.009	
p06_30_70					490.2505	490.250	0.000	(2897")
p06_30_90					434.1991	434.199	0.000	(5177")
p06_50_70					430.0833	430.083	0.000	(6221")
p06_50_90					397.4848	397.485	0.000	(5021")
p06_70_90					337.3898	337.390	0.000	(9701")
p07_1_10	76	20	140	160	1327.0000	1327.000	0.000	(72")
p07_10_30					1327.0000	1327.000	0.000	(302")
p07_10_50					1292.1154	1292.115	0.000	(3798")
p07_10_70					1181.5652	1180.000	0.132	
p07_10_90					1077.7119	1077.699	0.001	
p07_30_50					1138.1391	1138.097	0.004	
p07_30_70					980.7741	980.774	0.000	(1224")
p07_30_90					893.7880	893.788	0.000	(1818")
p07_50_70					878.2732	878.273	0.000	(2982")
p07_50_90					805.2451	805.241	0.000	
p07_70_90					720.2753	720.275	0.000	(3340")
p08_1_10	101	15	200	230	1409.0000	1409.000	0.000	(5536")
p08_10_30					1326.7895	1324.000	0.210	
p08_10_50					1159.4400	1157.000	0.210	
p08_10_70					1046.2935	1045.000	0.124	
p08_10_90					910.5037	910.437	0.007	
p08_30_50					935.7564	935.741	0.002	
p08_30_70					838.3221	838.322	0.000	(10743")
p08_30_90					777.0167	776.979	0.005	
p08_50_70					724.3128	724.313	0.000	(16209")
p08_50_90					692.3763	672.486	2.873	
p08_70_90					587.0910	587.091	0.000	(21590")
p14_1_10	101	10	200	1040	1710.0000	1710.000	0.000	(611")
p14_10_30					1319.7692	1309.000	0.816	
p14_10_50					1041.3793	1040.000	0.132	
p14_10_70					931.8780	930.000	0.202	
p14_10_90					824.9325	824.003	0.113	
p14_30_50					863.1202	863.119	0.000	
p14_30_70					755.1489	755.140	0.001	
p14_30_90					649.3793	649.379	0.000	(4483")
p14_50_70					617.8339	617.834	0.000	(4061")
p14_50_90					569.7406	559.762	1.751	
p14_70_90					-	490.155	-	

Table 3: Improvements with respect to the CTOP

Instance					CTOP	CTOP-IS		SDCTOP		SDCTOP-IS	
name	n	m	Q	T_{max}	$\underline{z}^*$	$\underline{z}^*$	(%)	$\underline{z}^*$	(%)	$\underline{z}^*$	(%)
p03_1_10	101	15	200	200	1409*	1406.000	-0.21	1409*	0	1409.000*	0
p03_10_30					1305*	1303.000	-0.15	1305*	0	1305.193*	0.01
p03_10_50					1117*	1116.000	-0.09	1117*	0	1117.449*	0.04
p03_10_70					961*	961.000	0	961*	0	961.310*	0.03
p03_10_90					1005*	1005.500	0.05	1005*	0	1006.217*	0.12
p03_30_50					892*	927.686	4.00	927	3.92	928.876*	4.13
p03_30_70					807*	811.249	0.53	810	0.37	811.357*	0.54
p03_30_90					704*	752.820	6.93	755*	7.24	755.673*	7.34
p03_50_70					549*	737.695	34.37	739	34.61	741.633*	35.09
p03_50_90					517*	636.193	23.05	643*	24.37	643.794*	24.52
p03_70_90					517*	585.880	13.32	585	13.15	592.800*	14.66
p06_1_10	51	10	160	200	761*	761.000*	0	761*	0	761.000*	0
p06_10_30					757*	758.333*	0.18	757*	0	758.333*	0.18
p06_10_50					687*	689.298*	0.33	687*	0	689.298*	0.33
p06_10_70					581*	584.060	0.53	581*	0	584.162*	0.54
p06_10_90					493*	497.531	0.92	495*	0.41	497.640*	0.94
p06_30_50					504*	538.350	6.82	538*	6.75	539.600*	7.06
p06_30_70					477*	490.250*	2.78	490*	2.73	490.563*	2.84
p06_30_90					409*	434.199*	6.16	432	5.62	435.342*	6.44
p06_50_70					289*	430.083*	48.82	428	48.10	434.511*	50.35
p06_50_90					308*	397.485*	29.05	396	28.57	400.875*	30.15
p06_70_90					289*	337.390*	16.74	335	15.92	342.993*	18.68
p07_1_10	76	20	140	160	1327*	1327.000*	0	1327*	0	1327.000*	0
p07_10_30					1327*	1327.000*	0	1327*	0	1327.000*	0
p07_10_50					1292*	1292.115*	0.01	1292*	0	1292.115*	0.01
p07_10_70					1180*	1180.000	0	1180*	0	1181.565*	0.13
p07_10_90					1075*	1077.699	0.25	1076	0.09	1077.752*	0.26
p07_30_50					1076*	1138.097	5.77	1142*	6.13	1142.215*	6.15
p07_30_70					966*	980.774*	1.53	980*	1.45	981.125*	1.57
p07_30_90					852*	893.788*	4.90	894*	4.93	895.000*	5.05
p07_50_70					631*	878.273*	39.19	884*	40.10	884.350*	40.15
p07_50_90					627*	805.241	28.43	811	29.35	813.650*	29.77
p07_70_90					619*	720.275*	16.36	723	16.80	728.840*	17.74
p08_1_10	101	15	200	230	1409*	1409.000*	0	1409*	0	1409.000*	0
p08_10_30					1326*	1324.000	-0.15	1326*	0	1326.790*	0.06
p08_10_50					1158*	1157.000	-0.09	1158	0	1159.440*	0.12
p08_10_70					1045*	1045.000	0	1045*	0	1046.294*	0.12
p08_10_90					909*	910.437	0.16	910*	0.11	910.509*	0.17
p08_30_50					893*	935.741	4.79	936	4.82	937.786*	5.02
p08_30_70					805*	838.322*	4.14	838	4.10	839.148*	4.24
p08_30_90					750*	776.979	3.60	777*	3.60	777.873*	3.72
p08_50_70					517*	724.313*	40.10	725	40.23	733.288*	41.84
p08_50_90					517*	672.486	30.07	678	31.14	680.646*	31.65
p08_70_90					517*	587.091*	13.56	585	13.15	594.966*	15.08
p14_1_10	101	10	200	1040	1710*	1710.000*	0	1710*	0	1710.000*	0
p14_10_30					1319*	1309.000	-0.76	1319*	0	1319.769*	0.06
p14_10_50					1040*	1040.000	0	1040	0	1041.379*	0.13
p14_10_70					930*	930.000	0	930	0	931.878*	0.20
p14_10_90					822*	824.003	0.24	823	0.12	824.943*	0.36
p14_30_50					835*	863.119	3.37	862	3.23	863.798*	3.45
p14_30_70					732*	755.140	3.16	754	3.01	755.768*	3.25
p14_30_90					611*	649.379*	6.28	647	5.89	650.303*	6.43
p14_50_70					418*	617.834*	47.81	619	48.09	625.227	49.58
p14_50_90					414*	559.762	35.21	561	35.51	564.796	36.42
p14_70_90					407*	490.155	20.43	498	22.36	500.103	22.88

Conclusions

In this paper we studied the impact of allowing incomplete (partial) service in the CTOP. We analyzed the possible gainings both from the worst-case point of view and from the experimental side. We proved that the maximum gaining is equal to 100% the value of the CTOP solution. From the computational point of view, we compared the gainings achieved by allowing incomplete service with the gainings that can be achieved by allowing split deliveries and by simultaneously allowing incomplete service and split deliveries.

From the theoretical point of view, the profit of the CTOP may double by allowing incomplete service or split deliveries or by allowing simultaneously both. The computational results confirm the theoretical analysis, that is that the combination of incomplete service and split deliveries does not significantly increase the profit with respect to the increase achieved by incomplete service or split deliveries only. The computational analysis enlightens the fact that the amount of increase of the profit with respect to the CTOP strongly depends on the instance and in particular on the ratio between the average demand of the customers and the vehicles capacity. The highest profit increase is obtained when the customers demand is above half of the vehicle capacity.

The results of this paper suggest to practitioners that, in routing problems with profits, it is worthwhile to allow incomplete service or split deliveries, especially when the customer demands are large compared to the vehicle capacity.

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