

SELF-CUSTOMIZATION AND PRICE COMPETITION

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Self-customization and price competition

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Abstract

New technologies increasingly provide firms with abilities to design self-customizable products, that can be redeveloped by end-users at their own expenses. The decision to market only a standard product or also a self-customizable one is a strategic one; we analyze this decision in a duopoly with product differentiation. In our model adding a customizable product cannibalizes part of own demand but also allows exploitation of a distinct segment of consumers who attach high value to customizability; it also diverts demand from the rival firm. Firms use second degree price discrimination, attaching a different price to the different products. We find the conditions leading to both firms introducing the self-customizable product, both refraining from it, and to asymmetric equilibria. Our results indicate that self-customization appears in equilibrium; it is profit improving; it can be used by only one or both firms according to the value of the market for customizability. It also leads to lower prices. An increase in consumers' ability to self-customize reduces profits, while a higher cost of self-customization increases profits. Finally a first-mover advantage arises in offering a self-customizable product.

JEL codes: D4; L11; M31

Keywords: self-customization; pricing; duopoly.

1. Introduction

Product customization is an increasingly important phenomenon and one that has generated a consistent body of literature (see Fogliatto et al., 2012, and the references therein). Customization can be of two types: producer-based customization and consumer-based customization. In the first case the producing firm develops a line of products tailored to meet the specific needs of the end-users and bears the full cost of customization. In the second type of customization it is the end-user who bears the cost of changing the product characteristics, but the firm must be able to provide the essential components, the “ingredients” to be added/changed. In between these two extremes customization involves both producer and customer at different degrees; for instance the good is co-designed by producer and customer.

The implications of producer-based customization have been deeply investigated in the literature. For example, Dewan et al. (2003) develop a model of product customization by a monopolistic firm, and show that the monopolist could increase its profits by producing both the standard and the customized product. When considering a duopoly, however, Dewan et al. (2000 and 2003) show that customization might not benefit firms. Syam et al. (2005) consider a situation where the products have several attributes and the firms can choose to customize only some attributes, and they find that in equilibrium only partial customization emerges. Ghose and Huang (2009) consider quality customization by firms, and they show that all firms might get higher profits. Loginova and Wang (2011) consider a two-dimensional model where two competing firms can customize the horizontal attribute of the product, and find that customization becomes more likely when the two firms differ significantly in their vertical attribute.

In order to engage in producer-based customization, the firms need to know correctly the preferences of the consumers in order to provide the products that fit these preferences (Valenzuela et al., 2009). Therefore, in the case of producer-based customization, it is natural to let the firm engage in personalized pricing as in Arora et al. (2008) and Bernhardt et al. (2007).

However, if the firms do not have such precise information, they might let the consumers to customize the products by themselves, that is, they might turn to consumer-based customization (or self-customization) rather than product-based customization: differently from product-based customization, self-customization typically does not imply personalized pricing.

Consumer-based customization has received dramatically less attention than producer-based customization in the theoretical literature. However, self-customization is gaining momentum in several industries (Business Week, Dec. 2002, Wall Street Journal, Oct. 2004). In particular, consumer-based customization appears to be increasingly relevant in those industries where consumers are perceived as co-creators: one such example is the video-game industry (Ondrejka, 2004). In E-retailing it is also possible to design the websites so as to allow the single consumer to arrange the “shop atmosphere” and the searching tools by combining different predetermined features, leading to customization of the design of the shop (Vrechopoulos, 2010). Other examples can be found in the textile industry (Duarte et al., 2017), or in the logistic services (Mammitzsch and Franczyk, 2016). As mentioned by Valenzuela et al. (2009), the Dell Computers website is an example of consumer-based customization. Indeed, the consumers choose the preferred level for each individual attribute that constitute the computer (i.e., the memory, the hard drive size...), and then the computer is assembled. Nike has patented a system that allows consumers to co-design products through computer devices (Franke and Schreier, 2010).

Self-customizable products are also being introduced in the medical devices sector, with specific problems for the associated risks of misuse by patients (Greene, 2016). The current debate on the developments of a third wave of Do-It-Yourself (Fox, 2014) also witnesses the yet undiscovered applications of self-customization in the near future.¹

At the best of our knowledge, the only paper addressing the theoretical implications of consumer-based customization is Gu and Tayi (2015). Gu and Tayi (2015) analyze a monopolist firm producing an indivisible good and choosing whether to offer only a standardized product, only a customizable product, or both at the same time. In their model, the standardized product provides the same utility level to all consumers; the profit maximizing price is then equal to the reservation price of the consumers. By contrast, consumers have different reservation prices for the customizable product, varying over a finite interval with a positive lower bound. The profit maximizing price if the customizable product is sold in isolation is shown to increase with the customizing ability of the consumers and to decrease with their cost of customization. These two parameters therefore account for the profitability of producing only one or the other product. Finally, Gu and Tayi (2015) also provide the conditions under which the monopolist prefers to market both products rather than only one: it is shown that selling both products is profitable when the customization cost and/or the customization efficiency are not too high.

Differently from Gu and Tayi (2015), we consider consumer-based customization in an oligopolistic framework. In particular, we analyze a Hotelling linear city model where at each point there is a proportion of ordinary consumers and a proportion of special (“picky”) consumers whose ideal point

¹ “*Third Wave DIY draws upon the read/write functionality of the Internet, and digitally-driven design/manufacture, to enable ordinary people to invent, design, make, and/or sell goods that they think of themselves*” (Fox, 2014, p.18).

is very far from the perceived location of the firms in the product characteristics space. While ordinary consumers could buy either the standard (non-customizable) product or the customizable product, “picky” consumers do not buy the standard product, since they find it too distant from their ideal type, but would consider the purchase of a customizable product, because self-customization, while implying a sunk cost, allows to reduce the “distance-cost” of the purchased product from the consumer ideal specification. The distance reduction parameter, then, makes the value of self-customization distance-sensitive. Together with the (distance invariant) private sunk cost it determines the relative advantage of buying the self-customizable product. In this context, introducing a self-customizable product has three effects: it cannibalizes part of own demand from type 1 consumers, it addresses a different source of demand (from type-2 consumers); it diverts demand from the rival, an effect that is absent under monopoly.

Our results confirm that the incentives to introduce a customizable version depend upon the market value of customizability. In particular, self-customization is profitable for firms when the segment of special buyers is large enough. However, we also show that self-customization is profitable when the cost of customization for consumers is not too low, and the ability of consumers to self-customize the product is not too high. Indeed, greater ability in self-customization reduces the perceived product differentiation, thus being detrimental for profits. The private cost of customization does not affect the consumer choice between one or the other firm customizable products – when both firms are offering it - and hence does not affect their equilibrium prices; however, it increases the relative attractiveness of the *standard* products with respect to the customizable ones, thus leading to higher equilibrium prices for the standard products, with a positive effect on profits.

We also consider the endogenous choice of the customizability policy by firms. We show that if the segment of special buyers is thin, or if the maximum price that can be extracted from them is low, there is no scope for introducing a customizable product. As the value of the customizable product increases due to a higher mass of special consumers and/or a greater maximum price that can be extracted from them, a firm introduces a customizable product and the other does not. When the value of customizability is high enough both firms introduce customizability. These results are valid both when the firms choose simultaneously the customization policy and when they choose it sequentially.

Furthermore, the effect of introducing a customizable good on the equilibrium price of the standard product is shown to be more complex in the duopoly case than in the monopoly case analyzed by Gu and Tayi (2015). Indeed, in the equilibrium where both firms offer both the customizable and the non-customizable good, the price of the standard product is higher than the price of the customizable good, but lower than in the case where no customizability is possible. Hence all consumers gain from customizability. Also in the asymmetric equilibrium, we find that the price of the standard good is higher than the price of the customizable good.

The rest of the paper proceeds as follows. In Section 2, we introduce the model. In Section 3, we discuss the symmetric cases where both firms offer customizability or no firm offers customizability. In Section 4, we discuss the asymmetric case where only one firm offers a customizable product. In Section 5, we derive the customizability policy emerging in equilibrium. Section 6 concludes. Proofs are in the Appendix.

2. The model

Suppose there are two firms, Firm *A* and Firm *B*, that sell to two types of consumers, type-1 and type-2. Both consumers' types are uniformly distributed along a linear product characteristic space of length one. We assume density one and we denote by $x \in [0, 1]$ the location of each consumer in the product space. At each location there is a quota $\alpha \in (0, 1)$ of type-1 consumers, and a quota $1 - \alpha$ of type-2 consumers (Figure 1).²

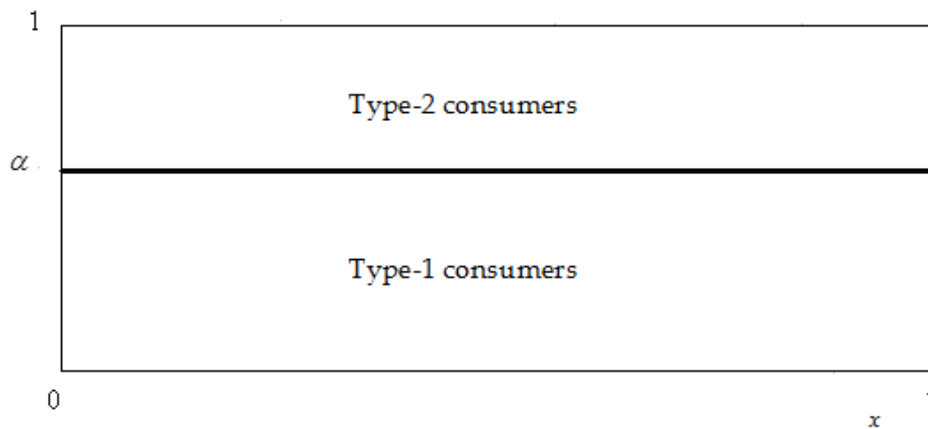


Figure 1: consumers' distribution

Both types can self-customize the customizable product – if it is offered – at a cost c , invariant across types. The two types differ in their perception of the firms' locations. In particular, type-1 consumers consider Firm *A* and Firm *B* to be located at 0 and 1 respectively, whereas type-2 consumers consider Firm *A* and Firm *B* to be located at $-l$ and $l+1$ respectively, where $l > 0$. That is, at each location x , type-1 consumers perceive each firm's product being close to their needs, whereas type-2 consumers ("picky" consumers) perceive each firm's product being far from their needs. Alternatively one may think that "fit uncertainty" for type-2 consumers implies that the expected value of the good is

² We restrict the analysis to the more interesting case where both consumer types exist.

reduced as when the distance is increased by the amount l (see later footnote 5). Firms set the price and decide whether to pursue a customizability strategy or not. Customizability means offering to consumers the possibility to develop the product as customized to their needs in order to increase the utility.

Consider the utility function of a type-1 consumer located at x when he buys a non-customizable or standard product: it is $v - p^A - tx$ if he buys from Firm A , and $v - p^B - t(1-x)$ if he buys from Firm B , where p^A and p^B indicate the price for the standard good and $t > 0$ is the unit “transportation cost”. Next consider the utility function of a type-1 consumer when he buys a customizable product. It is $v - \hat{p}^A - (t - \varepsilon)x - c$ if he buys from Firm A , and $v - \hat{p}^B - (t - \varepsilon)(1-x) - c$ if he buys from Firm B , where \hat{p}^A and \hat{p}^B indicate the price for the customizable good, $\varepsilon \in (0, t)$ measures the effectiveness of customization (indeed, when ε is high, customization dramatically reduces the disutility costs of consumers, while the opposite is true when ε is low), and c is the fixed cost sustained by a consumer that customizes the good.³ It should be noted that p^J and \hat{p}^J , $J = A, B$, constitute a second-degree discriminatory pricing scheme: indeed, differently from producer-based customization which allows personalized pricing, self-customization implies that within a menu of price-product pairs each consumer chooses the preferred one. We assume that c is not too high, namely $c \leq \varepsilon$, and we assume that v is sufficiently high so that the market is covered in any situation. In particular, we assume that $v \geq \underline{v} \equiv (2t - \varepsilon)(2\varepsilon - c)/\varepsilon$.

Now, we consider type-2 consumers. We assume that l is so high that a type-2 consumer buys the good only if he can customize it. This amounts to say that type-2 consumers perceive the good of both firms to be so distant from their needs that they prefer not buying the good unless they can customize it.⁴

³ As in Gu and Tayi (2015), we assume that the customization cost is known ex ante by all consumers.

⁴ For this reason, type-2 consumers can be interpreted as “picky” consumers.

Therefore, the utility function of a type-2 consumer located at x is as follows: $v - \hat{p}^A - (t - \varepsilon)(x + l) - c$ if he buys from Firm A , and $v - \hat{p}^B - (t - \varepsilon)(1 + l - x) - c$ if he buys from Firm B .^{5,6}

Finally, we assume that the marginal costs of production are constant and normalized to zero. Fixed costs of firms are disregarded.

3. Symmetric customizability strategies

In this section we consider the case where the two firms have the same customizability strategy: either both firms offer customizability or no firm offers customizability.

3.1 Both firms offer customizability (CC)

In what follows we assume that both Firm A and Firm B offer both the non-customizable product and the customizable product.

With regard to type-1 consumers, we assume the following market structure, moving from the left to the right of the segment: consumers buying the standard good A , consumers buying the customizable good A , consumers

⁵ Note that assuming that all type-2 consumers never buy the non-customizable good amounts to assume $l > v/t$. As it will be clear later, l plays no role in the analysis, and it is just needed to motivate that type-2 consumers never buy a non-customizable good. Alternatively, one might consider the following expected utility of a type-2 consumer which is uncertain about the fit of the product to his needs before purchase when he buys the non-customizable product of, say, Firm A : $\mu(v - p^A - tx) + (1 - \mu)(v - p^A - \gamma)$, where μ is the probability that the good fits, even if not perfectly, the needs of the consumer, and γ is the loss when there is no match of the product with the needs of consumer x (for example, the consumer must go to the market searching other products). Clearly, if μ is low enough and/or γ is high enough, all type-2 consumers buy only the customizable product. This interpretation yields identical results to the modeling approach used in the text.

⁶ In principle, it is possible to imagine the existence of a third group of consumers, constituted by those consumers that are not able to customize the product, so that they are forced to purchase the standard product. However, since these consumers are only interested in the standard product, the incentives driving the introduction of the customizable product would not be qualitatively affected. Indeed, the substitutability between the two products for type 1 (the ordinary people) is what determines the pressure on the standard product price when a customizable product is introduced, and this would not change much if the third type of consumers is added. Therefore, in order not to complicate the analysis, we do not consider the third type.

buying the customizable good B , and consumers buying the standard good B .⁷ The market structure is illustrated in Figure 2, where the label **A** (**B**) indicates that the consumer is buying from Firm A (B), while the price paid by the consumers is indicated in the brackets. This market structure reflects the idea that consumers for whom the product offered by the firm is close to their ideal point do not need to customize the good. By contrast, those consumers who are distant from the perceived location of the firm in the product characteristic space are the most interested in customizability, since it reduces the distance-related disutility costs. Indeed, customization by the consumer entails a cost equal to c for any consumer, whereas it allows reducing the transportation costs by an amount equal to εx for a consumer buying from Firm A and equal to $\varepsilon(1-x)$ for a consumer buying from Firm B . Therefore, customization is more beneficial for a consumer which is located far from the firm.

On the other hand, type-2 consumers, by assumption, only buy the customizable good. Therefore, the market structure for type-2 consumers is (from the left to the right of the segment): consumers buying the customizable good A and consumers buying the customizable good B .

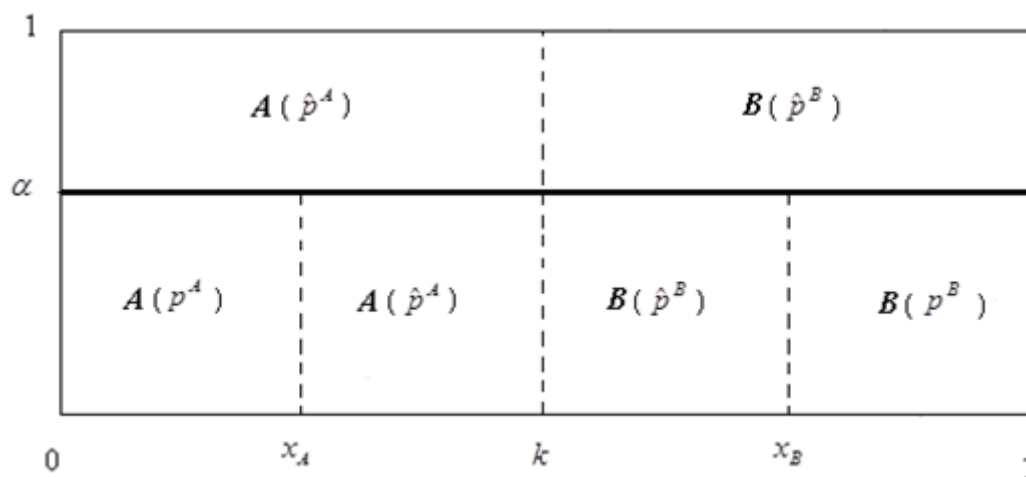


Figure 2: market structure under CC

⁷ We then check that this market structure is confirmed in equilibrium.

The threshold consumers are obtained by equating the appropriate utility functions. Therefore, $p^A + tx = \hat{p}^A + (t - \varepsilon)x + c$ yields $x_A = \frac{c - p^A + \hat{p}^A}{\varepsilon}$, whereas $p^B + t(1 - x) = \hat{p}^B + (t - \varepsilon)(1 - x) + c$ yields $x_B = \frac{\varepsilon - c + p^B - \hat{p}^B}{\varepsilon}$. On the other hand, the consumer which is indifferent between buying the customizable goods does not depend on the type. Indeed, both $\hat{p}^A + (t - \varepsilon)x + c = \hat{p}^B + (t - \varepsilon)(1 - x) + c$ (type-1 consumers) and $\hat{p}^A + (t - \varepsilon)(x + l) + c = \hat{p}^B + (t - \varepsilon)(1 + l - x) + c$ (type-2 consumers) yield $k = \frac{\hat{p}^B - \hat{p}^A + t - \varepsilon}{2(t - \varepsilon)}$.

If the inequalities $1 \geq x_B \geq k \geq x_A \geq 0$ hold, the demand of Firm A for the customizable product is equal to $\hat{D}^A \equiv (1 - \alpha)k + \alpha(k - x_A)$ and that for the non-customizable product is equal to $D^A \equiv \alpha x_A$. The demand of Firm B for the customizable product is equal to $\hat{D}^B \equiv (1 - \alpha)(1 - k) + \alpha(x_B - k)$ and that for the standard product is equal to $D^B \equiv \alpha(1 - x_B)$. Therefore, the profits function of Firm $J = A, B$ is $\pi^J = p^J D^J + \hat{p}^J \hat{D}^J$. By maximizing the profits functions, we get the equilibrium prices:^{8,9}

$$p_{CC}^J * = t - \varepsilon + \frac{c}{2} \quad (1)$$

$$\hat{p}_{CC}^J * = t - \varepsilon \quad (2)$$

⁸ The second-order conditions are always satisfied. Indeed, $\frac{\partial^2 \pi^J}{\partial p^{J^2}} = -\frac{2\alpha}{\varepsilon} < 0$ and

$\frac{\partial^2 \pi^J}{\partial \hat{p}^{J^2}} = \frac{2\alpha(t - \varepsilon) + \varepsilon}{-2t\varepsilon + \varepsilon^2} < 0$, whereas the Hessian is strictly positive: $\frac{2\alpha}{2t\varepsilon - \varepsilon^2} > 0$.

⁹ Note that at the equilibrium prices it is verified that $1 \geq x_B \geq k \geq x_A \geq 0$.

Note that the price of the customizable good is lower than that of the standard good. Indeed, the consumers buying the customizable product are located close to the centre of the segment. It follows that if one firm wants to serve these consumers it has to set a low price, because these consumers are rather far from it and rather close to the rival. Therefore, competition for these consumers is fierce and the equilibrium price is low. This contrasts with the monopolistic case discussed in Gu and Tayi (2015). Indeed, the monopolist can exploit the fact that customizability increases willingness to pay of the consumers. In contrast, such increase cannot be exploited in duopoly, as the two firms compete for the same consumers.

The equilibrium market share of consumers of type-1 buying the standard product (respectively, the customizable one) from Firm A is equal to $c/2\varepsilon$ (respectively, $(\varepsilon - c)/2\varepsilon$). Since the prices are symmetric, the same shares apply to Firm B . Hence, an increase in ε decreases the equilibrium sales of the standard product for both firms and increases those of the customizable good, due to a stronger cannibalization effect of customization. An increase in the customizability cost, c , yields the opposite.

In what follows, we consider the impact of the parameters on the equilibrium prices. Consider parameter ε . When ε increases, the distance-cost for the customizable product decreases. That is, the two customizable goods are less differentiated. This implies that when the efficiency of customization increases, the equilibrium price for the customizable product decreases; since the customizable and the standard products are substitutes, the prices of the standard goods also decrease. Next, consider parameter c . When c goes up, all else being equal, consumers are less attracted by customizability, as it becomes more costly. Therefore, for each firm, the standard product (which is alternative to the customizable product) could be charged with a higher price (see the

conditions determining the threshold consumers x_A and x_B). However, given face-to-face competition between firms for those consumers that buy the customizable products, an increase of c does not affect the threshold consumer k . Therefore, the price for the customizable product is not affected by the customization cost.

Given (1) and (2), the equilibrium profits under CC are:

$$\pi_{CC}^J = \frac{\alpha c^2 + 2\varepsilon(t - \varepsilon)}{4\varepsilon} \quad (3)$$

It should be noted that the equilibrium profits strictly increase with α and c , whereas they strictly decrease with ε . These results have the following explanation. As p_{CC}^J is higher than \hat{p}_{CC}^J , when the number of “picky” consumers decreases (i.e., α goes up), the profits increase. On the other hand, higher differentiation allows higher prices and lower cannibalization. Therefore, when ε decreases, the profits increase. Finally, we know from the above discussion that the price for the standard good increases with c , whereas the price for the customizable good is not affected by c . Consequently, the profits increase with the customization cost.

3.2. Neither firm offers customizability (NN)

In this section we consider the case where both firms offer only the standard product. In this situation, type-2 consumers do not buy. The market is a simple Hotelling linear city with the two firms being located at the extreme points. The indifferent type-1 consumer is obtained by $p^A + tx = p^B + t(1 - x)$, yielding

$g = \frac{p^B - p^A + t}{2t}$. The equilibrium prices and profits are:

$$p_{NN}^J * = t \quad (4)$$

$$\pi_{NN}^J * = \frac{\alpha t}{2} \quad (5)$$

Since customization is not possible, neither c nor ε affect the equilibrium prices and profits, whereas the profits increase with α (i.e. the number of the consumers that purchase). At the same time, it should be noted that the price of the standard good in NN is higher than the corresponding price in CC . Indeed, face-to-face competition between firms for consumers buying the customizable good in CC yields lower price for the customizable good, which, due to strategic substitutability, also reduces the price of the standard good.¹⁰

3.3. Comparison of the two cases

In this section we compare the situation where both firms offer a customizable product together with a non-customizable product, with the situation where both firms offer only the non-customizable product. Let us denote $\tilde{\alpha} \equiv \frac{2\varepsilon(t-\varepsilon)}{2t\varepsilon-c^2}$. We can state the following proposition:

Proposition 1. *When $\alpha \leq (\geq) \tilde{\alpha}$, the profits in the CC case are higher (resp. lower) than the profits in the NN case.*

Proof. Compare $\pi_{CC}^J *$ and $\pi_{NN}^J *$.

Proposition 1 claims that offering customizability will lead to higher profits only if α is sufficiently low. Obviously, customizability leads to higher profits

¹⁰ This result differs from the case of producer-based customizability (see Dewan et al., 2003).

if the proportion of type-2 consumers is higher than a certain threshold, that is, if the market of “picky” consumers is valuable enough. However, this threshold depends upon the other parameters of the model. Indeed, $\tilde{\alpha}$ is strictly increasing in c and strictly decreasing in ε . Hence, an increase in the cost of customizability (up to ε , which is the upper bound for c) tends to enlarge the parameter set where the profits in the *CC* equilibrium are higher than the profits in the *NN* equilibrium. Indeed, as c positively affects the price of the standard good but it does not affect the price of the customizable good due to face-to-face competition between firms for consumers buying the customizable good, an increase in the cost of customizability increases the profits under *CC*, while leaving the profits unaffected in the case *NN*. On the other hand, when considering parameter ε , the profits in the *NN* equilibrium do not depend on ε , whereas the profits in the *CC* equilibrium are negatively affected by the efficiency of customization, as a higher ε implies lower differentiation. Consequently, when ε goes up, the parameter space where the profits in *CC* are larger than the profits in *NN* shrinks. We summarize the above results as follows:

Corollary of Proposition 1. *The higher is c and the lower is ε , the more likely is that the profits in *CC* are higher than the profits in *NN*.*

It is worth comparing our result with Gu and Tayi (2015). Gu and Tayi (2015) find that, in a monopolistic framework, offering both the customizable and the standard product is profitable when the cost of customization is not too high, in contrast with our result. Indeed, a higher c reduces the willingness to pay of the consumers, so the consumers surplus that can be extracted by the monopolist through the customizable product is lower. On the other hand, a

lower ability to customize the product plays a similar role both in the monopolist and in the duopolistic framework, as a too high customization ability destroys the convenience to fall in the *CC* case here and to offer two products in the monopoly (Gu and Tayi, 2015).

We conclude this section with some considerations about the impact of customizability on the consumer surplus. When considering type-2 consumers, they are obviously better off when customization is possible, otherwise they do not purchase. With regard to type-1 consumers, it is easy to show that they are also benefited by the possibility of customization. Indeed, all prices are lower in *CC* than in *NN*. Moreover, type-1 consumers buy the customizable product rather than the non-customizable product only if the former allows greater utility. It follows that the surplus of type-1 consumers is greater in *CC* than in *NN*. Since under some conditions (see Proposition 1) the profits are also higher in *CC* than in *NN*, our analysis shows that symmetric offering of customizability might increase overall welfare relative to no customizability, thus resulting in a win-to-win situation.

4. Asymmetric customizability strategies

In this section we consider the case where only one firm offers customizability (*CN* and *NC*). Since the two firms are symmetric it is sufficient here to analyse the case *CN*. In such situation, type-2 consumers can be served only by Firm *A*, as Firm *B* does not offer the customizable product. When only Firm *A* offers customizability, there are two distinct cases:

- Non-segmented case: some type-1 consumers buy the customizable good of Firm *A*, whereas some others buy the standard product.

- Segmented case: all type-1 consumers buy the standard good of Firm A.

In what follows we derive the equilibrium prices and profits in each of the two cases, and then we briefly discuss the conditions on parameters leading to the different cases.

Let us consider first the non-segmented case. The type-1 consumers buying the standard product are closer to Firm A's location than those buying its customizable product. The consumer who is indifferent between buying from Firm A and Firm B is obtained by $\hat{p}^A + (t - \varepsilon)x + c = p^B + t(1 - x)$ and it is denoted by h , where $h = \frac{p^B - \hat{p}^A + t - c}{2t - \varepsilon}$. This market structure is represented in Figure 3.

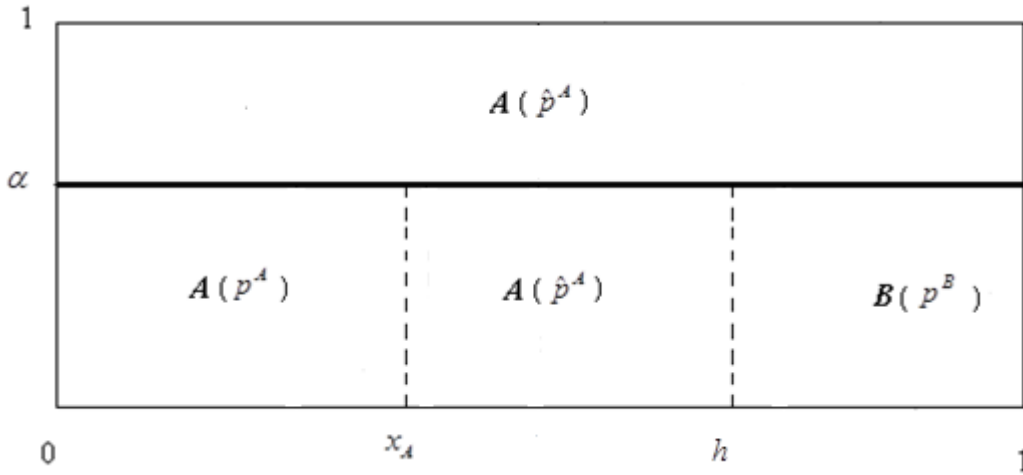


Figure 3: market structure under CN (non-segmented case)

The profit functions of Firm A and Firm B are therefore $\pi^A = \alpha[p^A x_A + \hat{p}^A (h - x_A)] + (1 - \alpha)\hat{p}^A$ and $\pi^B = \alpha p^B (1 - h)$, respectively.

Maximizing the profits function yields the following equilibrium prices:¹¹

¹¹ It should be noted that the second-order condition are satisfied everywhere. We have $\frac{\partial^2 \pi^A}{\partial p^{A^2}} = -\frac{2\alpha}{\varepsilon} < 0$, $\frac{\partial^2 \pi^B}{\partial p^{B^2}} = \frac{2\alpha}{-2t + \varepsilon} < 0$, $\frac{\partial^2 \pi^A}{\partial \hat{p}^{A^2}} = \frac{4t\alpha}{-2t\varepsilon + \varepsilon^2} < 0$ and the Hessian is $\frac{4\alpha^2}{2t\varepsilon - \varepsilon^2} > 0$. The

$$p_{CN}^{A,ns*} = \frac{2t(4-\alpha) + c\alpha - 2\varepsilon(2-\alpha)}{6\alpha} \quad (6)$$

$$\hat{p}_{CN}^{A,ns*} = \frac{t(4-\alpha) - c\alpha - \varepsilon(2-\alpha)}{3\alpha} \quad (7)$$

$$p_{CN}^{B,ns*} = \frac{t(2+\alpha) + c\alpha - \varepsilon(1+\alpha)}{3\alpha} \quad (8)$$

where the superscript “*ns*” refers to the non-segmented case. By using (6)-(8), the equilibrium profits follow:

$$\pi_{CN}^{A,ns*} = \frac{c^2\alpha^2(18t-5\varepsilon) + 4\varepsilon[(2-\alpha)\varepsilon - t(4-\alpha)]\{2c\alpha + [(2-\alpha)\varepsilon - t(4-\alpha)]\}}{36\alpha\varepsilon(2t-\varepsilon)} \quad (9)$$

$$\pi_{CN}^{B,ns*} = \frac{[c\alpha + t(2+\alpha) - (1+\alpha)\varepsilon]^2}{9\alpha(2t-\varepsilon)} \quad (10)$$

It can be observed that, for most of the parameters, the equilibrium profits decrease with ε and α following the impact of these parameters on the prices.¹² On the other hand, $\pi_{CN}^{B,ns*}$ increases with c , whereas $\pi_{CN}^{A,ns*}$ is U-shape in c . Indeed, as the price of Firm *B*'s product increases with c , Firm *B* always benefits by higher costs of customizability. At the opposite, the price of the customizable good of Firm *A* decreases with c , whereas the price of the standard good increases: when c is low the former effect dominates, while the latter effect dominate for high levels of c . Consequently, the profits of Firm *A*

necessary conditions for (6)-(8) to sustain the non-segmented market structure are discussed later in Lemma 1.

¹² When α increases, the share of type-1 consumers over type-2 consumers served through the customizable product by Firm *A* increases, thus increasing the incentive for Firm *A* to set a lower price for this product. Via strategic substitutability of prices, this determines a decrease of all prices. The impact of ε on the prices is the same as in *CC*. In the Appendix, we show that the condition $v \geq \underline{v}$ guarantees that, within the relevant parameter set, the market is covered at the equilibrium prices. Finally, it should be noted that the price of the standard product is higher than the price of the customizable product, as in *CC*.

initially decreases with the cost of customizability, but after a threshold they increase. By looking at the equilibrium value of h , it can also be checked that an increase in ε increases the market share of the firm offering the self-customizable product at the expense of the other firm.

It is interesting to compare the profits of Firm A with the profits of Firm B . We can state the following proposition:

Proposition 2. *In the non-segmented case, when α is low (resp. high), the profits of the firm offering customizability are higher (lower) than the profits of the firm not offering customizability.*

Proof. Compare $\pi_{CN}^{A,ns}$ * and $\pi_{CN}^{B,ns}$ *.

Proposition 2 shows that Firm A , which offers both the customizable and the standard product, might get lower profits than Firm B , which offers only the standard product. This happens when the percentage of type-2 consumers is low enough.¹³ Indeed, when offering the customizable product in the non-segmented case, Firm A must set a price which is sufficiently low in order to sell this product to both type-1 and type-2 consumers. However, if type-2 consumers are few, the profits of Firm A are lower than the profits of the rival, which can set a high price on type-1 consumers for its standard product. The opposite is true if there are many type-2 consumers which can be served only through the customizable product of Firm A .¹⁴

¹³ The relevant threshold of α is: $\alpha^\circ = \frac{2[2c\varepsilon + 2t\varepsilon - 2\varepsilon^2 - \sqrt{\varepsilon[4\varepsilon(c+t-\varepsilon)^2 - 3c^2(2t-\varepsilon)]}]}{3c^2}$.

¹⁴ It can be shown that a higher c reduces the parameter space where the profits of Firm A are larger than the profits of Firm B , whereas the opposite is true for a higher ε . Indeed, c is the cost of customization

Let us consider now the segmented case, where in equilibrium type-1 consumers buy only the standard product, whereas all type-2 consumers only buy the customizable product of Firm A, by assumption. The appropriate market structure is represented in Figure 4.

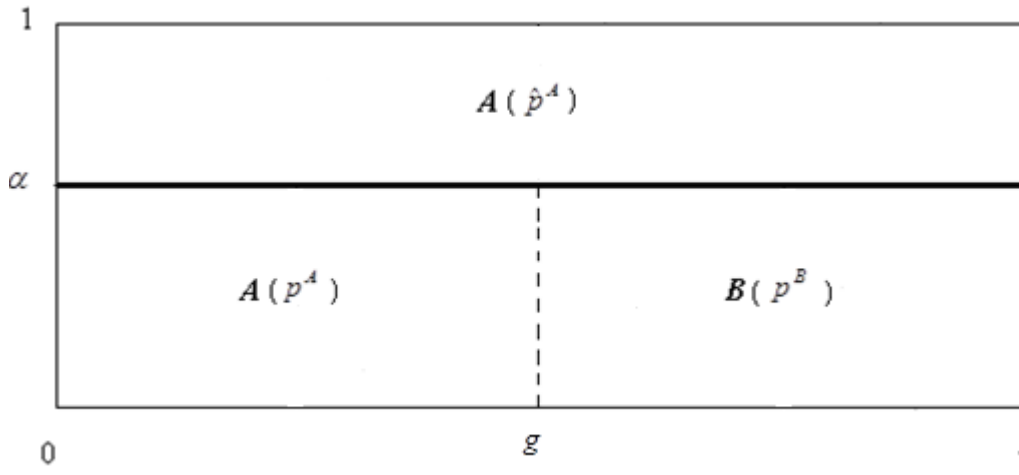


Figure 4: market structure under CN (segmented case)

Since the market is segmented, the equilibrium prices at the type-1 consumers segment are $p_{CN}^{A,s*} = p_{CN}^{B,s*} = t$ (where the superscript "s" refers to the segmented case) as in case NN, and the profits in that segment are simply $\pi_{NN}^A *$ and $\pi_{NN}^B *$.¹⁵ On the other hand, the equilibrium price of Firm A at the type-2 consumers segment is such that the most distant consumer is left with zero surplus when he buys the customizable product of Firm A, that is

and ε measures the efficiency of customization. Therefore, a higher c and a lower ε reduces the profits of the firm offering customizability with respect to the rival.

¹⁵ The necessary conditions for these prices to sustain the segmented market structure are discussed later in Lemma 1.

$\hat{p}_{CN}^{A,s*} = v - c - (t - \varepsilon)$, yielding profits equal to $(1 - \alpha)\hat{p}_{CN}^{A,s*}$.¹⁶ Therefore, the overall equilibrium profits in the segmented case are:

$$\pi_{CN}^{A,s*} = \pi_{NN}^A + (1 - \alpha)[v - c - (t - \varepsilon)] \quad (11)$$

$$\pi_{CN}^{B,s*} = \pi_{NN}^B \quad (12)$$

It is immediate to observe that the profits of Firm A are larger than the profits of Firm B:

Proposition 3. *In the segmented case, the profits of the firm offering customizability are higher than the profits of the firm not offering customizability.*

Proof. Compare $\pi_{CN}^{A,s*}$ and $\pi_{CN}^{B,s*}$.

Indeed, the two firms get the same profits in type-1 consumers market, as both firms offer only standard products. However, Firm A also serves all type-2 consumers through its customizable product, thus gaining additional profits.

Finally, Lemma 1 focuses on the parameters' conditions sustaining each case:

Lemma 1: *The non-segmented case emerges if α is sufficiently high and v is intermediate. The segmented case emerges if v is sufficiently high.*

¹⁶ Condition $v \geq \underline{v}$ guarantees that Firm A wants to serve all type-2 consumers. Indeed, when Firm A serves only a fraction of type-2 consumers (so that $\hat{p}^A > v - c - (t - \varepsilon)$), its profits on this segment are $(1 - \alpha)\hat{p}^A \hat{x}$, where $\hat{x} < 1$ is the most distant consumer which is served at \hat{p}^A , that is: $\hat{x} = \frac{v - c - \hat{p}^A}{t - \varepsilon}$. By comparing $(1 - \alpha)\hat{p}_{cn}^{A,s*}$ with $(1 - \alpha)\hat{p}^A \hat{x}$, it can be seen that the necessary condition for Firm A preferring to serve all type-2 consumers is $v \geq 2(t - \varepsilon)$, which is always satisfied when $v \geq \underline{v}$.

Proof. See the Appendix.

Intuitively, when v is large there is a strong incentive to deviate from the non-segmented case to the segmented case. Vice-versa, when v is quite low,¹⁷ there is a strong incentive to deviate from the segmented case to the non-segmented case. In addition, since the price of the customizable good decreases with α , the percentage of type-1 consumers must be sufficiently high to ensure that some consumers buy the customizable good in the non-segmented case.¹⁸

5. Customizability policy equilibrium

In this section we consider which customizability policy is expected to arise in equilibrium when the two firms are allowed to commit to a customizability policy before setting the prices. First, we consider the case where the two firms commit simultaneously and non-cooperatively to a customizability policy (C or N). Then, we shall consider the case where they commit sequentially to a customizability policy. In both cases, after the customizability policy choice, the firms choose simultaneously and non-cooperatively the price(s) of the product(s) conditioned on the choice in the first stage.

¹⁷ However, v cannot be too low, otherwise the condition $v \geq \underline{v}$ is not respected.

¹⁸ In principle, one should consider both the non-segmented and the segmented case also in *CC*. In what follows, we show that this distinction is redundant in *CC*, as only the non-segmented case emerges. With regard to the non-segmented case, we already have shown that, at the equilibrium prices, the market structure $1 \geq x_B \geq k \geq x_A \geq 0$ is sustained. In *CC*, this implies that there is no profitable deviation to induce the segmented case when the rival is non-segmenting the market. Therefore, the non-segmented case is an equilibrium in *CC*. Next, we show that the segmented case is never an equilibrium in *CC*. If the two firms are segmenting the market, the price of the customizable good (which is purchased only by type-2 consumers) is $t - \varepsilon$, whereas the price of the standard good (which is purchased only by type-1 consumers) is t . The segmented case is sustained only if, given these prices, all type-1 consumers buy the standard good. Consider a consumer located at $1/2$. His utility is $v - 3t/2$ if he buys the standard good, whereas it is $v - 3(t - \varepsilon)/2 - c$ if he buys the customizable good. It is immediate to see that the consumer prefers to buy the customizable good as $c \leq \varepsilon$. Therefore, the segmented case is not sustainable in *CC*.

Consider the case of a simultaneous choice of the customizability policy. The relevant pay-off matrix is given in Table 1:

Table 1: the pay-off matrix

	B	C	N
A			
C		$\pi_{CC}^A *; \pi_{CC}^B *$	$\pi_{CN}^{A,i} *; \pi_{CN}^{B,i} *$
N		$\pi_{NC}^{A,i} *; \pi_{NC}^{B,i} *$	$\pi_{NN}^A *; \pi_{NN}^B *$

where $i = s, ns$. Let us introduce the following threshold:

$$\hat{\alpha} \equiv \frac{4[4t^2\varepsilon - 6t\varepsilon^2 + 2\varepsilon^3 + (2t - \varepsilon)(2c\varepsilon - 3\sqrt{\varepsilon(2t - \varepsilon)(2t\varepsilon - c^2)})]}{c^2(18t - 5\varepsilon) + 8c\varepsilon(t - \varepsilon) - 2\varepsilon(16t^2 - 5t\varepsilon - 2\varepsilon^2)}.$$

We can state the

following proposition:

Proposition 4. *Suppose that the firms choose simultaneously the customizability policy. Suppose $i = ns$: if $\alpha \leq (\geq) \hat{\alpha}$, the equilibria are CN and NC (the equilibrium is NN). Suppose $i = s$: if $\alpha \leq (\geq) \tilde{\alpha}$, the equilibrium is CC (the equilibria are CN and NC).*

Proof. Compare the profits in Table 1.

Let us consider first the case where, if only one firm offers customizability, then type-1 consumers buy both the customizable and the standard product of that firm (non-segmented case). Recall that this requires v to be intermediate and α sufficiently high (Lemma 1). Therefore, if the rival is offering customizability it is optimal for the focal firm not offering customizability, thus

focusing on type-1 consumers. Now suppose that the rival is not offering customizability. In this case, the best-reply of the focal firm depends on the percentage of type-2 consumers. In particular, if there are few type-2 consumers (i.e. $\alpha \geq \hat{\alpha}$), offering customizability is quite costly (see the discussion about Proposition 2), as it requires setting a low price for the customizable good. Therefore, not offering customizability is the dominant strategy and NN is the unique equilibrium. At the opposite, if there are enough type-2 consumers (i.e. $\alpha \leq \hat{\alpha}$), offering customizability when the rival is choosing not customizability is profitable. Therefore, in this case there are two asymmetric equilibria where only one firm offers customizability.

Let us consider now the case where, if only one firm offers customizability, then all type-1 consumers buy the standard good of that firm (segmented case). This case requires that v is high enough. From Proposition 3 it follows that, if the rival is not offering customizability, then the focal firm should offer customizability. Suppose now that the rival is offering customizability. As in the segmented case $\pi_{NC}^{A,s*} = \pi_{NN}^A *$, choosing customizability when the rival offers customizability is profitable when profits in CC are higher than profits in NN . From Section 3, we know that this requires that α is sufficiently low (namely, $\alpha \leq \tilde{\alpha}$). Therefore, if there are many type-2 consumers (i.e. $\alpha \leq \tilde{\alpha}$), customizability is the dominant strategy for both firms, and CC is the unique equilibrium. It is interesting to note that the condition that guarantees that CC is the unique equilibrium (that is, $\alpha \leq \tilde{\alpha}$) is the same condition that guarantees that the profits of the firms when both of them offer a customizable product are larger than the profits when there is no customizability (Proposition 1). This implies that CC emerges in equilibrium when there is no other situation where both firms are better off: that is, firms are not trapped in a prisoner dilemma. On the other hand, if there are few type-2 consumers (i.e. $\alpha \geq \tilde{\alpha}$), then

the focal firm prefers not offering customizability when the rival offers customizability. In this case, there are two asymmetric equilibria.

REMARK: *In all equilibria, a firm introducing the self-customizable product improves its profits by so doing.*

We also consider the impact of c and ε on the relevant thresholds of α . With regard to the segmented case, we showed in Section 3 that $\tilde{\alpha}$ increases in c and decreases in ε : therefore, higher c and lower ε expand the area where both firms choose customizability (CC). On the other hand, with regard to the non-segmented case, we have that $\hat{\alpha}$ decreases in ε , whereas it is U-shape in c . Indeed, we know that, when the rival does not offer customizability, the profits of the firm that offers customizability decrease with ε and are U-shape in c (see Section 4). Therefore, when ε goes up, the area where CN and NC emerge in equilibrium shrinks; when c increases, this area initially shrinks, but then it expands.

It could be noted that, both in the segmented and in the non-segmented case, in the asymmetric equilibrium the firm offering customizability gets greater profits than the rival.¹⁹ Finally, Proposition 4 shows that any customizability policy is possible, depending on the parameter constellation.

In what follows, we consider the case where the customizability policy is chosen sequentially by the two firms before they set simultaneously the prices. A sequential choice of the customization policy might describe those situations where the firms have different capabilities in the adoption of new technologies that allow customizability. We can state the following proposition:

¹⁹ Indeed, this is obvious for the segmented case (see Proposition 3). For the non-segmented case, it can be shown that when an asymmetric equilibrium exists (that is, when $\hat{\alpha} \geq \underline{\alpha}$, where $\underline{\alpha}$ is the necessary lower bound of α for the non-segmented case to occur, see Lemma 1 and the Appendix), then $\alpha^\circ \geq \hat{\alpha}$ holds, which guarantees that the profits of the firm offering customizability are higher than the profits of the firm not offering customizability (see Proposition 2).

Proposition 5. *Suppose that the firms choose sequentially the customizability policy. Suppose $i = ns$: if $\alpha \leq (\geq) \hat{\alpha}$, the leader chooses C and the follower chooses N (both firms choose N). Suppose $i = s$: if $\alpha \leq (\geq) \tilde{\alpha}$, both firms choose C (the leader chooses C and the follower chooses N).*

Proof. See the Appendix.

Proposition 5 reveals that the leader has a first-mover advantage stemming from customizability. Indeed, when an asymmetric equilibrium arises, the firm offering customizability (the leader) obtains greater profits than the firm offering only the standard product (the follower).^{20,21}

6. Conclusions

In this paper, we consider the implications of consumer-based customization (or self-customization) when two firms compete through prices. Consumer-based customization differs from producer-based customization because the former implies that the customization of the product is performed by the end-user rather than by the producer. Consumers are increasingly attracted by the possibility to self-customize the product before purchase (Valenzuela et al., 2009), and firms might pursue the strategy of offering to

²⁰ This kind of first-mover advantage can be found also in the case of producer-based customization (Dewan et al., 2003).

²¹ Differently from Gu and Tayi (2015), we do not consider explicitly the case where only the customizable product is sold by one or both firms. Indeed, it is easy to show that, in the present framework, this strategy is always dominated by the strategy of offering both the standard and the customizable product. Indeed, suppose that Firm A offers only the customizable product. Consider a consumer which is close to point 0. His disutility is equal to $\hat{p}^A + c$, plus negligible transportation costs. Therefore, Firm A can get higher profits by offering a standard good at a price $p^A \in (\hat{p}^A, \hat{p}^A + c)$ in addition to the customizable product.

consumers such possibility. While the literature has discussed the profitability of offering a customizable product together with a standard (non-customizable) product for a monopolist, it is still silent about the implications of self-customization in an oligopolistic framework. To fill this gap, we develop a model where consumers are horizontally heterogeneous and are of two types: the first group of consumers can buy either a standard product or customizable product, whereas the second group of consumers (“picky” consumers) can buy only the customizable product. In such a context, two firms compete through prices.

Our main results are as follows. Self-customization turns out to be profitable for firms when the segment of “picky” is large enough, when the cost of customization for the consumers is not too low, and the ability of consumers to self-customize is not too high. When it is an equilibrium strategy, introducing a self-customizable product is profit increasing; in other words, no firm regrets introducing a self-customizable product in equilibrium. Furthermore, we show that when the firms can commit to the customizability policy before setting the prices, if the segment of “picky” buyers is small, no firm introduces the customizable product. As the value of customizability increases, a firm introduces a customizable product and the other does not (asymmetric equilibrium). Finally, when the value of customizability is high enough, the unique equilibrium consists in both firms introducing customizability. Our results could help explaining the observed variability in customizability strategies across industries.

Our findings also have practical implications for marketers. First, the conditions for the profitability of introducing self-customization strongly depend on the nature of the industry: self-customization might be profitable in a duopoly at those conditions where it is detrimental in a monopoly, and vice-

versa. Second, greater efficiency in customization by consumers might lower the profits, as long as it reduces product differentiation. This highlights the risks connected to those practices aiming to improve the consumers' ability to self-customize the product, as for example by offering free training classes. Lastly, our model emphasizes the relevance of being the first when offering customizability (first-mover advantage).

Our model could be extended for future research. For example, consumers are often characterized by unstable preferences, that is, by preferences that might change across periods (Hoeffler and Ariely, 1999). Re-shaping the present one-period model in a two-period model where the preferences of consumers in the second period are related but not identical to the preferences in the first period might allow to consider the implications of self-customization in a context of varying preferences. Second, the current paper considers only consumer-based customization. However, self-customization is often used together with producer-based customization (Arora et al., 2008). The present model could be extended to allow the firms to choose between self-customization (thus letting the consumers to bear the customization costs) and producer-based customization (thus sustaining the customization costs), or a combination of them (co-design). Lastly, consumers are frequently heterogeneous with regard to their customization capability, depending on their experience, innate skills, and knowledge of their own preferences (Gu and Tayi, 2015). The present model could take account of this additional source of heterogeneity by assuming that at each point x in the horizontal dimension there is a continuum of consumers with different customization capabilities. We leave the exploration of these issues for future research.

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APPENDIX

Proof of Lemma 1. The equilibrium prices calculated for the case $i = ns, s$ sustain the market structure i when two conditions are simultaneously satisfied:

1) given $p_{CN}^{A,i*}, \hat{p}_{CN}^{A,i*}$ and $p_{CN}^{B,i*}$ the market structure assumed for case i is confirmed; 2) given $p_{CN}^{B,i*}$, Firm A has no incentive to deviate in order to induce the other case. Let us consider first the non-segmented case. Condition 1

requires that $1 > h > x_A > 0$ at (6)-(8), which is true if $\alpha \geq \underline{\alpha} \equiv \frac{2\varepsilon(2t - \varepsilon)}{2\varepsilon(5t - 2\varepsilon) - c(6t - \varepsilon)}$.

Condition 2 requires that Firm A has no incentive to deviate in order to induce the segmented case. Note that the profits in the non-segmented case do not depend on v , whereas the profits under the deviation inducing the segmented case are positively affected by v (as the deviation price is $v - c - (t - \varepsilon)$).

Therefore, condition 2 for the non-segmented case is satisfied if v is not too high.²² Furthermore, at the equilibrium prices, the utility of the type-2 consumer located at 1 when $\alpha = \underline{\alpha}$ is zero if $v = \underline{v}$. Therefore, the condition $v \geq \underline{v}$

guarantees that the market is covered at the equilibrium prices. Let us consider now the segmented case. Condition 1 requires that the type-1 consumer located at g prefers buying the standard good of Firm A rather than the customizable good. This occurs if the price of the customizable good is high enough, that is if v is high enough. Condition 2 requires that Firm A has no incentive to deviate in order to induce the non-segmented case. Following the same reasoning as above, the profits in the segmented case are positively affected by v , whereas the profits under the deviation inducing the non-segmented case do not depend

²² At the same time, v cannot be too low, otherwise the condition $v \geq \underline{v}$ is not satisfied. We present a numerical example showing that such parameter set is non-empty. Suppose: $t = 1$, $\varepsilon = 4/5$, and $c = 1/20$, yielding $\underline{v} = 2.32$, and $\underline{\alpha} = 9/10$. Condition 1 is satisfied, as $\underline{\alpha} = 0.37$. Condition 2 requires $v < 2.49$. Therefore, when $v \in (2.32, 2.49)$, the non-segmented case emerges in equilibrium.

on v . Hence, condition 2 for the segmented case is satisfied if v is sufficiently high.²³

Proof of Proposition 5. Suppose Firm $A(B)$ is the leader (follower). Consider $i = ns$. If the leader has chosen C , the follower always chooses N (see Table 1). If the leader has chosen N , the follower chooses C (N) when $\alpha \leq (\geq) \hat{\alpha}$. As $\pi_{CN}^{A,ns*} \leq \pi_{NN}^{A,ns*}$ when $\alpha \geq \hat{\alpha}$, both firms choose N when $\alpha \geq \hat{\alpha}$. On the other hand, as $\pi_{CN}^{A,ns*} \geq \pi_{NC}^{A,ns*}$ when $\alpha \leq \hat{\alpha}$ (Footnote 21), the leader chooses C whereas the follower chooses N when $\alpha \leq \hat{\alpha}$. Consider $i = s$. If the leader has chosen N , the follower always chooses C (see Table 1). If the leader has chosen C , the follower chooses C (N) when $\alpha \leq (\geq) \tilde{\alpha}$. As $\pi_{CC}^{A,s*} \geq \pi_{NC}^{A,s*}$, both firms choose C when $\alpha \leq \tilde{\alpha}$. On the other hand, as $\pi_{CN}^{A,s*} \geq \pi_{NC}^{A,s*}$, the leader chooses C whereas the follower chooses N when $\alpha \geq \tilde{\alpha}$.

²³ When both α and v are low there is no equilibrium. Numerical computations show that multiple equilibria do not arise. Details are available.