Essays on Economic Growth and Health

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Chapter 1

Introduction

Since many years, economists have considered health in their studies following different approaches, theoretical and empirical, both at macro level and at micro level. The aspects that are analyzed throughout these chapters are the impact of longevity - induced by a better health condition - on economic growth, the relationship between health and education in the formation of human capital and the importance of a healthy labor force.

In the last decades, there has been a rapid improvement in individual health conditions that increased the life expectancy (World Bank, 2017). Many economists agree on the positive effect of higher life expectancy on economic growth. Lorentzen et al. (2008) document that a lower risk of death during the first productive years is associated with lower levels of risky behavior, lower fertility, and higher investment in physical capital in African countries. Looking at OECD countries, Gehringer and Prettner (2017) find, empirically and theoretically, that the decrease of mortality positively affects technological progress and productivity growth. Still on the optimistic side, but from a theoretical perspective, higher life expectancy implies higher

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accumulation of human capital and higher accumulation of physical capital. In particular, in the Cervellati and Sunde (2005) model, human capital is a central factor of production and it improves the longevity and productivity of future generations. In turn, individuals decide about human capital acquisition by taking into account both life expectancy and the economic environment. Kalemli-Ozcan et al. (2000) calibrate an OLG framework in continuous time, where individuals make optimal schooling investments according to a probability of death. Mortality decline makes schooling and consumption increase significantly. Boucekkine et al. (2003) build a model based on De la Croix and Licandro (1999) and Boucekkine et al. (2002), where a realistic survival law is embedded into an endogenous growth model with vintage human capital. This framework allows the authors to study how shifts in survival probabilities at different ages affect the investments in human capital and promote growth. As mentioned above, another channel that is explored at the theoretical level is the accumulation of physical capital: i.e. a higher life expectancy creates the incentive to save more: among others Blackburn and Cipriani (2002), Chakraborty (2004) (seminal works on OLG with endogenous survival probability), Azomahou et al. (2009) (age dependent survival probability), Leung and Wang (2010) (savings and health care are complements), Heijdra and Mierau (2012) (focus on the imperfect market annuities market).

On the other hand, there are economists who do not agree with the optimistic side: better health condition of the population might cause faster population growth and therefore a decrease of the growth rate of GDP per capita (Acemoglu and Johnson, 2007), due to the well-known neoclassical capital dilution effect (cf. Solow, 1956; Diamond, 1965). In particular, Acemoglu and Johnson (2007) show empirically that a 1% rise in life expectancy leads to a 1.7-2% increase in the population size but it increases aggregate GDP much less. According to the simulations by Ashraf et al. (2008), the effects of health improvements on income per capita are substantially lower than those that are often quoted by policy-makers, suggesting that proponents of efforts to improve health conditions in developing countries should rely on humanitarian rather than economic reasons. Moreover, exploiting the empirical methodology of the panel Granger-causality, Hartwig (2010) does not support the view that health capital formation fosters long-term economic growth in the OECD area. Applying panel data analysis, Suhrcke and Urban (2010) find that cardiovascular diseases are detrimental to growth just for high income countries and not for low-middle income countries.

Concerning the relationship between education and health: "Much of what we call consumption constitutes investment in human capital. Direct expenditures on education, health, and internal migration to take advantage of better job opportunities are clear examples. [...] Many of them have virtually no schooling, are in poor health, are unskilled, and have little ability to do useful work." (quoted from Schultz, 1961). Especially at the empirical level, there are many examples of how a good health status or the eradication of some particular sicknesses have affected the level of education (Bleakley, 2007; Bleakley and Lange, 2009; Lucas, 2010; Oster et al., 2013).

Moreover, in the literature, the relationship between health in early life and education is highly explored (Conti et al., 2010). Perri (1984), Behrman and Rosenzweig (2004), Currie (2009), among others, show that there is a negative effect of childhood low health status on educational achievements. Healthier children perform better at school and they will have a broader health-related knowledge (Behrman, 2009). According to Case et al. (2005), intergenerational transmission of economic status can potentially take place through the mechanism of health: individuals born into poorer families experience poorer childhood health, lower investments in human capital and poorer health in early adulthood, all of which are associated with lower income in middle age, the years in which they themselves become parents. Despite the empirical literature is wider on the relationship between health and education, there are also some theoretical works. Exploiting the Schumpeterian mechanism,

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Howitt (2005) describes the possible channels of how health can affect human capital. Van Zon and Muysken (2001) present a model of endogenous growth where a good health status is necessary for workers. Specifically Van Zon and Muysken (2001) identify three ways in which health affects intertemporal decision-making: i) health serves as the "conditio sine qua non" to the provision of human capital services; ii) there is competition between the provision of health services and the provision of labor services allocated to the production of output. Besides health competes with the time spent on human capital accumulation; iii) health might generate positive utility on its own. According to Galama et al. (2018), the effect of education on mortality exists in some contexts but not in others, and it seems to depend on different factors. Strulik (2018) presents a model where individuals with a higher return to education choose more education as well as a healthier lifestyle and they spend less on unhealthy consumption than individuals with lower education.

Last but not least, a healthier population usually means also a more productive labor force: Bloom and Canning (2005) and Prettner et al. (2013) show that health is important like education for the workers. In the Kuhn and Prettner (2016) model, health care increases labor participation and at the same time it also diverts labor away from production and R&D. According to Kuhn et al. (2015) a lower morbidity is associated with higher earnings and a lower disutility from labor. In particular, in their theoretical model the authors focus on the decision to retire according to the individuals' health care choices. A very similar set-up, but with different types of health investments is the one by Dalgaard and Strulik (2017). Still on the same approach, Galama et al. (2013) follow the Grossman (1972) model implementing the individual choice of retirement and health care, besides consumption. Cai (2010) estimates a panel data simultaneous equation model to examine the relationship between health and labor force participation in Australia. His findings confirm that health has a positive and significant effect on labor force participation for both males and females: a change from fair to poor health on average reduces the probability of participation by 2.4 percentage points for males and 3.7 percentage points for females. In addition, looking at European countries, García-Gómez (2011) documents that there is a significant causal effect from health on the probability of employment. Indeed, individuals who have a health shock are more likely to quit the job and transit into disability.

This thesis consists of two papers, one co-authored and one single-authored.

The second chapter is a co-authored paper with Professor Alberto Bucci (University of Milan) and Professor Klaus Prettner (University of Hohenheim, Germany): Children's health, human capital accumulation, and R&D-based economic growth. In this joint work, we analyze the role of children's health for human capital accumulation and for long-run economic growth. In particular, we build an R&D-based growth model a la Romer (1990) in which the stock of human capital of the next generation is determined by parental education and health investments. The household side is characterized by parents living for two periods that have to invest an amount of resources in their children's health and education. The results show that, on top of the usual children quality-quantity trade-off, there is also complementarity between the two types of investment: parents who want healthy children want also well-educated children and vice versa. The production side is characterized by five sectors: final goods, intermediate goods, R&D sector, health care and education. Human capital enters all sectors but the intermediate goods sector. We show that higher investment in children's health raises the growth rate of human capital and therefore the growth rate of the central input in the R&D sector. It follows that technological progress will increase, leading to higher economic growth. This type of mechanism based on R&D-based endogenous economic growth explains the positive effect of health on growth that is found for modern economies. Moreover, we find

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that faster population growth implies lower economic growth. Our model offers an additional pathway by which health could exert a positive effect on economic growth besides the neoclassical capital dilution effect and the Ben-Porath mechanism.

The third chapter is a single-authored paper: Different types of Health Expenditures in an overlapping generations framework: living longer or working better? We consider the fact that life expectancy depends on the health status and at the same time being healthy means being more productive at work. Therefore, we combine two strings of the existing literature about the different impacts of health: higher life expectancy and higher worker productivity. As in the second chapter, the household side is characterized by a two periods OLG model, but the second stage of life is uncertain, given a survival probability, depending on the health status. The health status is a linear function of the government health expenditure. Indeed, the government allocates its revenues, coming from a tax on the household income, between two health expenditures: the first type just mentioned above, preventive and curative treatments -hospitals, health care personnel, drugs- and the second type of health expenditure that makes workers more productive. The production side of the economy is characterized by the final good production with a standard *Cobb* Douglas where the inputs are capital and effective labor. The latter one is a linear function of the second health expenditure by the government. The Eurofound¹ reports all the work-related health problems that prevent daily activities at work or directly absence at work. Therefore, the government with ergonomic interventions, better working spaces, financing medical treatments can improve the productivity of workers. As a consequence, the government faces a trade-off allocating the resources to make individuals live longer or to make them work better. We find the optimal combination of the tax rate and of the allocation of the tax revenues between the two health policies, that maximizes the steady state of GDP per worker.

¹European Foundation for the Improvement of Living and Working Conditions

chapter 2

Children's health, human capital accumulation, and

R&D-based economic growth¹

¹This chapter is based on the paper "Children's health, human capital accumulation, and R&Dbased economic growth" by Annarita Baldanzi (University of Milan, University of Pavia), Alberto Bucci (University of Milan) and Klaus Prettner (University of Hohenheim).

Abstract

We analyze the effects of children's health on human capital accumulation and on long-run economic growth. For this purpose we design an R&D-based growth model in which the stock of human capital of the next generation is determined by parental education and health investments. We show that i) there is a complementarity between education and health: if parents want to have better educated children, they also raise health investments and vice versa; ii) parental health investments exert an unambiguously positive effect on long-run economic growth, iii) faster population growth reduces long-run economic growth. These results are consistent with the empirical evidence for modern economies in the twentieth century.

2.1 Introduction

There has been a substantial improvement in childhood health within all industrialized countries over the last decades. According to the World Bank (2016)'s Health Nutrition and Population Statistics, the mortality rate of children under the age of 5 has decreased in the OECD from 63 deaths per 1000 children in 1960 to 7 deaths in 2015. This corresponds to a reduction of the child mortality rate of almost 90% within two generations. Furthermore, over the same time span, the prevalence of certain diseases, such as anemia, has decreased from 24% to around 15% among children. The substantial improvements in the health condition of children are therefore an important driver of the rise in the survival rate to the age of 65, which has increased between 1960 and 2015 from 64% to 83% for men and from 75% to 90% for women.

As far as the relationship between health and national income per head is concerned, there is a strong positive association between these two variables, as reflected in the famous "Preston Curve" (Preston, 1975). However, there are different points of view about the "causality": if some economists claim the positive effects of health on income: Bloom and Canning (2000), Cervellati and Sunde (2005), and Lorentzen et al. (2008), other economists claim the opposite: lower mortality – as induced by a better health condition of the population – might trigger faster population growth and therefore a reduction in the growth rate of income per capita due to the well-known neoclassical capital dilution effect (cf. Solow, 1956; Diamond, 1965).² In their influential work, Acemoglu and Johnson (2007) show that a 1% increase in life expectancy leads to a 1.7-2% increase in the population size but it raises aggregate GDP growth to a lesser extent.³ Consequently, according to their findings, a better

²For the optimistic siade, see also Gallup et al. (1999), Bhargava et al. (2001), and Gehringer and Prettner (2017) for empirical findings and De la Croix and Licandro (1999), Kalemli-Ozcan et al. (2000), Boucekkine et al. (2002), Boucekkine et al. (2003), Lagerlöf (2003), and Bar and Leukhina (2010) for theoretical considerations.

³For the "pessimistic" side see also Ashraf et al. (2008); Hartwig (2010); Suhrcke and Urban (2010)

health condition of the population *reduces* income growth per capita.

Aghion et al. (2011) and Bloom et al. (2014) in turn criticize the findings of Acemoglu and Johnson (2007). Their argument is that the negative effect of higher life expectancy on economic growth might come from the omission of a measure for the initial health condition from the regression specifications. Countries with a lower initial health condition of the population have a larger potential to improve health, but, at the same time, they have a lower economic growth potential. Including initial life expectancy as a proxy for initial health in the regressions, Bloom et al. (2014) show that there is a *causal positive* effect of better health on economic growth. Furthermore, using the same panel data for the period 1940-2000 as Acemoglu and Johnson (2007), Cervellati and Sunde (2011) find that the effect of life expectancy on economic growth might have been negative before the demographic transition when fertility rates stayed constant in the face of decreasing mortality, but that it is unambiguously positive after the onset of the demographic transition when higher life expectancy reduces the fertility rate such that population growth slows down. This implies a positive effect of health on income per capita in a neoclassical-type of growth model because the capital dilution effect is reduced. A complementary effect is that increases in life expectancy raise human capital investments, which also fosters economic growth as shown by Ben-Porath (1967); Cervellati and Sunde (2005, 2013).

The aim of our paper is to contribute to this debate by showing another pathway by which health has the potential to impact on long-run economic growth, especially in modern knowledge-based economies that have already experienced the demographic transition in the past.

Our argument is based on an endogenous growth mechanism where new ideas are created in a research sector by the human capital that a society devotes to R&D.⁴

⁴For endogenous growth models, see Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1995), Kortum (1997), Peretto (1998), Segerström (1998), Young (1998), Howitt (1999), and many others. For frameworks that explicitly model human capital as a result of schooling investments, see, for example, Funke and Strulik (2000), Strulik (2005), Grossmann

The aggregate human capital stock of a country is in turn a compound of the education level and the health condition of the population and there are feedback effects between these two variables (Schultz, 1961; Grossman, 2000; Becker, 2007). In the

tion level and the health condition of the population and there are feedback effects between these two variables (Schultz, 1961; Grossman, 2000; Becker, 2007). In the theoretical literature, there are examples about the interplay between health and education. Howitt (2005) describes six possible channels of how health can affect growth, in particular human capital in a broader sense, in a Schumpeterian model: i) healthier workers are more productive; ii) increases in life expectancy, because of good health, have a positive effect on the steady-state average skill level of the population; iii) health matters for the rate of return to education, indeed healthy children will gain more from a given amount of education; iv) healthy childhood makes a person more creative; v) healthy childhood improve the ability to cope with stress in the adulthood, therefore to adapt to the frequently disruptive and stressful effects of rapid technological change; vi) empirically there is a strong negative correlation between various indicators of population health and measures of income inequality. More related to human capital is the work by Van Zon and Muysken (2001). Lucas (1988) endogenous framework is exploited to describe the trade-off between health and human capital accumulation: an expansion of the health sector promotes growth thanks to the increased health of the population, while a contraction of the health sector frees the resources necessary to increase growth through an increase in human capital accumulation activities. In particular, Van Zon and Muysken (2001) assume that the generation of health services is characterized by decreasing returns and human capital accumulation is modeled with increasing returns. Prettner et al. (2013) show that a fertility decline induces higher education and health investments that are able to compensate for declining fertility under certain circumstances. In Strulik (2018), individuals with a higher return to education choose more education as well as a healthier lifestyle. In particular, a higher return to education makes individuals search more education and have a higher labor income. Since the marginal (2007), Bucci (2008, 2013), Strulik et al. (2013), and Prettner (2014).

return of health expenditure is declining while the marginal damage of unhealthy consumption is increasing, educated and wealthy individuals spend not only more on health but also less on unhealthy consumption than less educated ones. In their empirical work, Galama et al. (2018) show that the effect of education on mortality exists in some contexts but not in others, and it seems to depend on different factors: the gender, the labor market returns to education, the quality of education and whether education affects the quality of individuals peers.

In our model, on the household side, health enters the utility function of parents who choose how much to invest in children's health and in children's education. We show that, if parents want to have better educated children, they also increase health investments in their children. This result is consistent with the empirical findings of Perri (1984), Behrman and Rosenzweig (2004) and Currie (2009), who document a negative effect of childhood ill-health on educational achievements.⁵ In addition, healthier children perform better in school and will themselves have a higher healthrelated knowledge (Behrman, 2009). Overall, in our framework, human capital is used as an input in the production functions of the final goods sector, the R&D sector, the education sector, and the health sector. Given the positive role of health in the creation of human capital, there are more productive resources available for R&D in a healthier economy and this has the potential to lead to faster long-run economic growth (cf. Prettner et al., 2013; Kuhn and Prettner, 2016). Our model therefore characterizes an additional channel by which health could exert a positive effect on economic growth besides the neoclassical capital dilution effect (Cervellati and Sunde, 2011) and the Ben-Porath mechanism (Ben-Porath, 1967; Cervellati and Sunde, 2005, 2013).

The paper is organized as follows. We set up the model in Section 2.2, describe the consumption side, the production side, and the market clearing conditions. Section 2.3 contains the balanced growth path of the economy and the main analytical

 $^{{}^{5}}$ See also Bleakley (2007), Bleakley and Lange (2009), Lucas (2010), and Oster et al. (2013) who document a positive effect of health on human capital.

results. We then proceed to a numerical example to illustrate the transitional dynamics of the system. In Section 2.5.2 we conclude.

2.2 The model

Consider a knowledge-based economy a la Romer (1990) - Jones (1995) with five sectors: final goods production, intermediate goods production, R&D, education, and health. Physical capital and human capital are the two production factors. Physical capital is accumulated according to the savings and investment decisions of households and it is used to produce machines in the intermediate goods sector. Human capital is available in four different forms: as "workers" in the final goods sector for the production of the consumption aggregate, as "teachers" in the education sector for the production of the knowledge and skills of the next generation, as "health care personnel" for the improvement of the health condition of the next generation in the health sector (including also public health projects, for example, improvements in sanitation), and as "scientists" for the production of new blueprints for machines in the R&D sector.

The consumption side of the economy consists of overlapping generations of households who live for two time periods. Households consume, save, and choose the number of children on the one hand, and how much to invest in education and health of each child, on the other hand. The households' expenditures on education are used to hire the teachers to educate the young, while the households' expenditures on health are used to hire the health care personnel to improve the physical well-being of children.⁶

⁶There is a vast literature in which overlapping generation models are employed to endogenize life expectancy (Blackburn and Cipriani, 2002; Chakraborty, 2004; Cervellati and Sunde, 2005; Hashimoto and Tabata, 2005; Bhattacharya and Qiao, 2007; Castelló-Climent and Doménech, 2008; Osang and Sarkar, 2008; De la Croix and Licandro, 2013). Our work abstracts from the survival probability; instead we follow a short-cut formulation in which the health component is one of the determinants of the accumulation of human capital.

2.2.1 Households

We follow Strulik et al. (2013) and Prettner et al. (2013) in assuming that the utility function of households is given by

$$u_{t} = \log (c_{1,t}) + \beta \log (R_{t+1}s_{t}) + \xi \log (n_{t}) + \theta \log (e_{t}) + \zeta \log (f_{t}),$$

where $c_{1,t}$ is first period consumption of the generation born at time t, R_{t+1} is the capital rental rate, s_t denotes savings such that $c_{2,t} = R_{t+1}s_t$ refers to consumption in the second period of life, n_t is the number of children, e_t refers to education investments per child, f_t refers to health investments per child, β is the discount factor, ξ denotes the utility weight of children, θ refers to the utility weight of children's eduction, and ζ is the utility weight of children's health.⁷ For consistency, we employ the parameter restriction $\xi > \theta + \zeta$, which ensures that parents do not want to invest in children's education and health without having children in the first place. In addition, the restriction rules out immediate extinction (i.e., $n_t = 0$). The utility function without the health component of children is frequently used in the literature (cf. Strulik et al., 2013; Prettner et al., 2013; Bloom et al., 2015b) because it operationalizes the "warm-glow motive of giving" as described by Andreoni (1989) and because it is the special case of logarithmic utility of the more general specification employed by Galor and Weil (2000) and Galor (2011). To see this, consider the formulation of Galor and Weil (2000), where parental utility depends positively on the consumption possibilities of children as approximated by their total income $n_t h_{t+1} w_{t+1}$ with w_{t+1} being the wage rate per unit of human capital of the next generation. Computing the logarithm yields $\log(n_t) + \log(h_{t+1}) +$ $\log(w_{t+1})$, where the wage rate per unit of human capital of the next generation is a constant to the parent such that it drops out of the first-order conditions. If

 $^{^7\}mathrm{As}$ in Strulik et al. (2013), each single sex household consists of one parent, to avoid matching problems.

 h_{t+1} is a multiplicative function of education and health, then our corresponding formulation in the utility function as represented by $\xi \log(n_t) + \theta \log(e_t) + \zeta \log(f_t)$ captures all the tradeoffs that parents face when deciding on the number of children and the parental expenditures for children's education and health.

The budget constraint of the household is given by

$$(1 - \psi n_t) h_t w_t = \eta e_t n_t + \kappa n_t f_t + c_{1,t} + s_t,$$

where ψ measures the unit time cost of rearing each child $-\psi n_t$ is the opportunity cost of rising children, η measures the unit cost of the investment in education per child, κ measures the unit cost of the investment in health per child, h_t refers to the human capital level of an adult, which is tantamount to her productivity and is itself a compound determined by the education and health investments of her own parents, and w_t is the wage rate per unit of human capital of the parent generation. Note that, while η and κ are constant, the investment costs in education and health are also determined by the wages of teachers and of health care personnel and therefore depend on the overall stage of economic development.

The result of the optimization problem is given by optimal consumption, savings, fertility, education investments, and health investments as given by

$$c_{1,t} = \frac{h_t w_t}{1 + \beta + \xi},\tag{2.1}$$

$$s_t = \frac{\beta h_t w_t}{1 + \beta + \xi},\tag{2.2}$$

$$n_t = \frac{\xi - \zeta - \theta}{\psi \left(1 + \beta + \xi\right)},\tag{2.3}$$

$$e_t = \frac{\theta \psi h_t w_t}{\eta \left(\xi - \zeta - \theta\right)},\tag{2.4}$$

$$f_t = \frac{\zeta \psi h_t w_t}{\kappa \left(\xi - \zeta - \theta\right)}.$$
(2.5)

At this stage we can state the following intermediate result that is consistent with the empirical findings discussed in the introduction.

Proposition 1.

- i) If households have stronger preferences for a higher number of children, the optimal fertility rate is higher, while optimal consumption, savings, and investments in children's health and education are lower;
- ii) If households have stronger preferences for children's education, parental optimal investments in both education and health are higher, while fertility is lower;
- *iii)* If households have stronger preferences for children's health, parental optimal investments in both education and health are higher, while fertility is lower.

Proof. Part i): By investigating Equations (2.1), (2.2), (2.4), and (2.5) it is straightforward that a higher level of ξ implies lower consumption, savings, children's health, and children's education. To see the effect on fertility, we compute the derivative of (2.3) with respect to ξ :

$$\frac{\partial n_t}{\partial \xi} = \frac{1 + \beta + \zeta + \theta}{\left(1 + \beta + \xi\right)^2 \psi}$$

and observe that the derivative is unambiguously positive.

Part ii): By investigating Equations (2.3) and (2.5), fertility decreases and children's health increases with θ . To see the effect on children's education, we compute the derivative of (2.4) with respect to θ :

$$\frac{\partial e_t}{\partial \theta} = \frac{\left(\xi - \zeta\right)\psi h_t w_t}{\eta \left(\xi - \theta - \zeta\right)^2}.$$

Since we have that $\xi > \theta + \zeta$, the derivative is unambiguously positive.

Part iii): By investigating Equations (2.3) and (2.4), fertility decreases and children's education increases with ζ . To see the effect on children's education, we compute the derivative of (2.5) with respect to ζ :

$$\frac{\partial f_t}{\partial \zeta} = \frac{\left(\xi - \theta\right)\psi h_t w_t}{\kappa \left(\xi - \theta - \zeta\right)^2}.$$

Again, given $\xi > \theta + \zeta$, this derivative is unambiguously positive.

Altogether, we observe that parents who invest more in their children's education also invest more in their children's health and vice versa. At the same time, higher investments in education and health imply that parents have fewer children. This is consistent with the evidence on the relation between health and education (cf. Perri, 1984; Behrman and Rosenzweig, 2004; Currie, 2009; Behrman, 2009; Case et al., 2005) and it is also consistent with the child quality-quantity trade-off as described by Becker and Lewis (1973).⁸

Taking into account Equation (2.3), the evolution of the population size is governed by the difference equation

$$N_{t+1} = n_t N_t = \frac{\xi - \zeta - \theta}{\psi \left(1 + \beta + \xi\right)} N_t \tag{2.6}$$

and the optimal labor force participation rate can be calculated as

$$lpr = 1 - \psi n_t = \frac{1 + \beta + \zeta + \theta}{1 + \beta + \xi}.$$

Naturally, the labor force participation rate is smaller than one because of the time parents spend on rearing children.

2.2.2 Production

The productions side of the economy consists of five sectors: final goods production, intermediate goods production, R&D, education, and health. The description of the first three sectors follows the standard R&D-based growth literature with the only difference being that human capital (as a compound of the number of people, their education level and their health condition) is used instead of raw labor as a factor of

⁸It is noteworthy that ξ has two effects of opposing signs on n_t but the positive one prevails, while there are no multiple effects of opposing signs as regards the impact of θ on e_t and of ζ on f_t .

production. An example of how the health care sector enters the economic growth framework is by Kuhn and Prettner (2016), that model an economy with four sectors: final goods production, intermediate goods production, R&D, and health care. There are two productive factors that can be used in these four sectors: capital and labor. The latter one is in the form of workers. Besides, Schneider and Winkler (2017) model the health sector but in a decentralized overlapping generations economy with a focus on the annuities.

The final goods sector produces a consumption good Y_t with human capital $H_t = h_t N_t$ and machines $x_{t,i}$ as inputs according to the production function

$$Y_t = H_{t,Y}^{1-\alpha} \int_0^A x_{t,i}^{\alpha} \, di,$$
 (2.7)

where A is the technological frontier and $\alpha \in (0, 1)$ denotes the elasticity of output with respect to machines of type *i*. Profit maximization implies

$$w_t = (1 - \alpha) \frac{Y_t}{H_{t,Y}}, \qquad p_{t,i} = \alpha H_{t,Y}^{1-\alpha} x_{t,i}^{\alpha-1}, \qquad (2.8)$$

where $p_{t,i}$ is the price of machines.

The intermediate goods sector is monopolistically competitive as in Dixit and Stiglitz (1977). Firms in the intermediate goods sector have access to the production technology $x_{t,i} = k_{t,i}$, where $k_{t,i}$ denotes physical capital employed by each firm. Operating profits of intermediate goods producers are then given by $\pi_{t,i} = p_{t,i}x_{t,i} - R_t k_{t,i} = \alpha H_{t,Y}^{1-\alpha} k_{t,i}^{\alpha} - R_t k_{t,i}$, such that profit maximization yields the optimal price of a machine as $p_{t,i} = R_t/\alpha$ for all *i*. In this context, $1/\alpha$ is the markup over marginal cost. Due to symmetry with respect to the pricing policy of individual firms, we know that the aggregate capital stock is $K_t = A_t k_t$ such that we can write the aggregate production function as

$$Y_t = (AH_{t,Y})^{1-\alpha} K_t^{\alpha}.$$
 (2.9)

The R&D sector employs scientists $H_{t,A}$ to discover new blueprints A_t according to the production technology

$$A_{t+1} - A_t = \delta A_t^{\phi} H_{t,A}, \qquad (2.10)$$

where δ refers to the productivity of scientists and $\phi < 1$ to the intertemporal spillover effects of technologies that raise the productivity of human capital employed in the research sector (cf. Jones, 1995). R&D firms maximize profits $\pi_{t,A} = p_{t,A} \delta A_t^{\phi} H_{t,A} - w_{t,A} H_{t,A}$, with $p_{t,A}$ being the price of a blueprint that they sell to the intermediate goods producers. From the first-order condition we get

$$w_{t,A} = p_{t,A} \delta A_t^{\phi}, \qquad (2.11)$$

where $w_{t,A}$ refers to the wage rate per unit of human capital of scientists. The interpretation of this equation is straightforward: wages of scientists increase with their productivity as measured by δA_t^{ϕ} and with the price that a research firm can charge for the blueprints that it sells to the intermediate goods producers. In the labor market there is free entry and this assumption allows us to exploit the wages paid in the final goods sector to compute the wages in the other sectors. In particular, the education sector employs teachers with human capital $H_{t,E}$ to produce the knowledge and skills of the next generation.⁹ Employment in the education sector is determined by the equilibrium condition that household expenditures for teachers are equal to the total wage bill of teachers, i.e.,

$$\eta e_t n_t N_t = H_{t,E} w_t \Leftrightarrow H_{t,E} = \frac{\theta H_t}{1 + \beta + \xi}$$

Similarly, the health sector employs health care personnel with human capital $H_{t,F}$

⁹Berk and Weil (2015) underline the problem of older teachers in this context: with the phenomenon of population aging, workers will have older teachers, who might teach outdated knowledge. This observation is very interesting and it could be considered in an extension of our model that allows for this type of the "vintage effect".

to improve the health condition of the next generation. Employment in the health sector is therefore determined by the equilibrium condition that household expenditures for health are equal to the total wage bill of health care personnel, i.e.,

$$\kappa f_t n_t N_t = H_{t,F} w_t \Leftrightarrow H_{t,F} = \frac{\zeta H_t}{1 + \beta + \xi}$$

Individual human capital is a Cobb-Douglas compound of the education level and the health condition such that

$$h_{t+1} = \left(\mu \frac{H_{t,E}}{N_{t+1}}\right)^{\nu} \left(\omega \frac{H_{t,F}}{N_{t+1}}\right)^{1-\nu}$$
(2.12)

where $H_{t,E}/N_{t+1}$ measures the education intensity per child, μ is the productivity in the schooling sector, $H_{t,F}/N_{t+1}$ measures the health care intensity, ω is the productivity in the health care sector, and ν denotes the elasticity of human capital with respect to education.¹⁰

2.2.3 Market clearing

Labor markets are assumed to clear such that $lpr*N_t = L_t = L_{t,Y} + L_{t,A} + L_{t,E} + L_{t,F}$, where L_t is total employment and $L_{t,j}$ for j = Y, A, E, F refers to employment in the four different sectors that use human capital. This implies that $lpr*H_t =$ $H_{t,Y} + H_{t,A} + H_{t,E} + H_{t,F}$ because human capital is embodied. Since there is free movement of labor in the economy, wages in the final goods sector and in the R&D sector will be equal in equilibrium. Inserting (2.8) into (2.11) therefore yields the following equilibrium condition that equates the marginal value product of a worker in the final goods sector and of a scientist in the R&D sector

$$p_{t,A}\delta A_t^{\phi} = (1-\alpha)\frac{Y_t}{H_{t,Y}}.$$
 (2.13)

¹⁰This function will generate $h_{t+1}/h_t > 1$ given sufficiently large productivity in education and health, i.e. μ and ω .

We follow Aghion and Howitt (2005) and assume that patent protection lasts for one generation, which is reasonably in line with the duration of patents in reality (cf. The German Patent and Trade Mark Office, 2016; The United States Patent and Trademark Office, 2012). After a patent expires, the right to sell the blueprint is handed over to the government that consumes the associated proceeds.¹¹ As a consequence, the patent price is given by the one-period profits of the intermediate goods sector, which can be written as

$$\pi_{t,i} = p_{t,A} = (1 - \alpha) \,\alpha k_t^{\alpha} H_{t,Y}^{1-\alpha} = \left(\alpha - \alpha^2\right) \frac{Y_t}{A_t}$$

Plugging this into (2.13) and solving for employment of human capital used in the final goods sector yields $H_{t,Y} = A_t^{1-\phi}/(\alpha\delta)$. Now we can use the relation $H_{t,A} = lpr * H_t - H_{t,Y} - H_{t,E} - H_{t,F}$, which is implied by the labor market clearing condition and the fact that human capital is embodied, to solve for human capital employment in the R&D sector as

$$H_{t,A} = \frac{1+\beta+\zeta+\theta}{1+\beta+\xi}H_t - \frac{A_t^{1-\phi}}{\alpha\delta} - \frac{\theta H_t}{1+\beta+\xi} - \frac{\zeta H_t}{1+\beta+\xi}$$

Since $H_t = h_t N_t$, we obtain¹²

$$H_{t,A} = \frac{(1+\beta)h_t N_t}{1+\beta+\xi} - \frac{A_t^{1-\phi}}{\alpha\delta}.$$
 (2.14)

Plugging the resulting employment level of human capital of scientists into the production function of the R&D sector [Equation (2.10)], yields the following law of motion for blueprints

$$A_{t+1} = \frac{(1+\beta)\,\delta h_t N_t A_t^{\phi}}{1+\beta+\xi} - \frac{(1-\alpha)\,A_t}{\alpha}.$$
(2.15)

¹¹For the long-run balanced growth rate of the economy it would make no difference if the government were allowed to invest part of (or even the total) of these proceeds.

¹²It is noteworthy that $H_{t,A} = max [0, Eq(2.14)].$

We immediately see that, *ceteris paribus*, a higher productivity of scientists (δ), a higher employment level of human capital in the R&D sector [$H_{t,A}$ as defined in Equation (2.14)], and stronger intertemporal knowledge spillovers (ϕ) all lead to a faster accumulation of patents between time t and t + 1.

Capital market clearing requires that total savings $s_t N_t$ are either used for investment in physical capital, K_{t+1} , or for buying newly developed blueprints to establish an intermediate goods producer. Given that the price of a patent is $p_{t,A}$, the value of savings in the form of new patents amounts to $p_{t,A} (A_{t+1} - A_t)$. Thus, the stock of physical capital at time t + 1 is equal to aggregate savings net of savings invested in the shares of intermediate goods producers such that

$$K_{t+1} = s_t N_t - p_{t,A} \left(A_{t+1} - A_t \right) = Y_t - c_{1,t} N_t - c_{2,t-1} \frac{N_t}{n_{t-1}} - G_t, \qquad (2.16)$$

where G_t are governmental expenditures financed by the proceeds of expired patents and the second equality follows from the national accounts identity $Y_t = C_t + K_{t+1} +$ G_t for a closed economy with $C_t = c_{1,t}N_t - c_{2,t-1}\frac{N_t}{n_{t-1}}$ being aggregate consumption. Note that, in this expression, $c_{2,t-1}N_t/n_{t-1}$ refers to total consumption of the generation born at time t - 1, which is in the second phase of its life cycle in year t and is of size N_t/n_{t-1} . Consequently, we have total output net of consumption expenditures by households and the government, i.e., total investment in terms of physical capital, on the right-hand side of Eq. (2.16).

$$K_{t+1} = (1-\alpha) K_t^{\alpha} \left(\frac{A_t^{2-\phi}}{\alpha\delta}\right)^{1-\alpha} - \frac{(1-\alpha) A_t h_t N_t K_t^{\alpha} \left(\frac{A_t^{2-\phi}}{\alpha\delta}\right)^{-\alpha}}{1+\beta+\xi}.$$
 (2.17)

Finally, we solve for the evolution of individual human capital as determined by parental investments in education and health. Plugging $H_{t,E}$ and $H_{t,F}$, which result from the household maximization problem, into the production function of human capital [Equation (2.12)], and also considering Eq. (2.6) yields

$$h_{t+1} = \frac{\left(\theta\mu\right)^{\nu} \left(\zeta\omega\right)^{1-\nu} \psi}{\xi - \zeta - \theta} h_t.$$
(2.18)

Note that, *ceteris paribus*, if parents have stronger preferences towards children's education (higher θ) or if parents have stronger preferences towards children's health (higher ζ), individual human capital accumulation increases. By contrast, if parents prefer having more children (higher ξ), individual human capital accumulation decreases because of the quality-quantity trade-off. The main question that arises regarding aggregate human capital accumulation is whether the increase in individual human capital accumulation due to a stronger preference for children's health and education can overcompensate the associated reduction in the population growth rate.¹³

2.3 Dynamics and long-run equilibrium

We summarize the model dynamics defined by (2.6), (2.15), (2.17), and (2.18) in the following four-dimensional system of difference equations:

$$A_{t+1} = \frac{(1+\beta)\delta h_t N_t A_t^{\phi}}{1+\beta+\xi} - \frac{(1-\alpha)A_t}{\alpha}, \qquad (2.19)$$

$$K_{t+1} = (1-\alpha)K_t^{\alpha} \left(\frac{A_t^{2-\phi}}{\alpha\delta}\right)^{1-\alpha} - \frac{(1-\alpha)A_th_tN_tK_t^{\alpha}\left(\frac{A_t^{2-\phi}}{\alpha\delta}\right)^{-\alpha}}{1+\beta+\xi}, \quad (2.20)$$

$$N_{t+1} = \frac{\xi - \zeta - \theta}{\psi(1 + \beta + \xi)} N_t, \qquad (2.21)$$

$$h_{t+1} = \frac{(\theta\mu)^{\nu}\psi(\zeta\omega)^{1-\nu}}{\xi - \zeta - \theta}h_t.$$
(2.22)

¹³It is noteworthy that a higher value of θ or ζ decreases the growth rate of the population, Eq. (2.6).

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It follows that the variables A, N, and h grow at the following rates:

$$g_A = \frac{(1+\beta)\,\delta h_t N_t A_t^{\phi-1}}{1+\beta+\xi} - \frac{1}{\alpha}, \qquad (2.23)$$

$$g_N = \frac{\xi - \zeta - \theta}{\psi \left(1 + \beta + \xi\right)} - 1, \qquad (2.24)$$

$$g_h = \frac{(\theta\mu)^{\nu} \psi (\zeta\omega)^{1-\nu}}{\xi - \zeta - \theta} - 1.$$
(2.25)

It is obvious from Equation (2.23) that a balanced growth path – along which the growth rate of technology stays constant – has to fulfill

$$\frac{h_t}{h_{t-1}} \frac{N_t}{N_{t-1}} \left(\frac{A_t}{A_{t-1}}\right)^{\phi-1} = 1.$$

From this we can infer the long-run growth rate of technology as

$$g_A^* = \left[(1+g_h) \left(1+g_N \right) \right]^{\frac{1}{1-\phi}} - 1 = \left[\frac{\zeta \left(\theta \mu \right)^{\nu} \omega \left(\zeta \omega \right)^{-\nu}}{1+\beta+\xi} \right]^{\frac{1}{1-\phi}} - 1.$$

From this result and Equation (2.9) we know that the long-run growth rate of per capita GDP that is associated with a constant capital-to-output ratio is given by

$$g_{y}^{*} = \left[(1+g_{h}) \left(1+g_{A}\right) \right] - 1 = \frac{\left(1+\beta+\xi\right)\psi\left[\frac{\zeta(\theta\mu)^{\nu}\omega(\zeta\omega)^{-\nu}}{1+\beta+\xi}\right]^{1+\frac{1}{1-\phi}}}{\xi-\zeta-\theta} - 1, \qquad (2.26)$$

while the growth rates of aggregate GDP and aggregate physical capital are

$$g_Y^* = g_K^* = (1 + g_N) \left(1 + g_h\right) \left(1 + g_A\right) - 1 = \left[\frac{\zeta \left(\theta \mu\right)^{\nu} \omega \left(\zeta \omega\right)^{-\nu}}{1 + \beta + \xi}\right]^{1 + \frac{1}{1 - \phi}} - 1.$$

Next, we state our central results regarding the differential evolution of fertility, education, and health and their corresponding effects on long-run economic growth.

Proposition 2. Shifts in the population growth rate due to changes in preferences towards fertility are accompanied by shifts in the long-run economic growth rate of

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opposing sign.

Proof. The derivative of Equation (2.26) with respect to ξ is

$$\frac{\partial g_y^*}{\partial \xi} = \frac{\left[\zeta + \theta + \xi \left(\phi - 2\right) + \beta \left(\phi - 1\right) + \phi - 1\right] \psi \left[\frac{\zeta(\theta \mu)^{\nu} \omega(\zeta \omega)^{-\nu}}{1 + \beta + \xi}\right]^{1 + \frac{1}{1 - \phi}}}{\left(\zeta + \theta - \xi\right)^2 \left(1 - \phi\right)}.$$

It is noteworthy to remind that $\partial n/\partial \xi > 0$. Recalling that the parameter restriction $\xi > \zeta + \theta$ has to hold to rule out immediate extinction and noting that the term $\phi - 2$ is smaller than -1 because $\phi < 1$, we see that the numerator of this expression is always negative. Since the denominator is always positive, the proof of the proposition is established.

The intuition for this finding is that parents who prefer to have fewer children, reduce fertility. This allows them – for a given income level – to spend more on education and health for each child. In addition, the reduction in fertility allows parents to supply more time on the labor market such that their disposable incomes rise. Part of this additional income is spent on education and health. While the reduction in fertility reduces the growth rate of the aggregate human capital stock, the increase in educational investments and health investments raises growth of aggregate human capital. Since the fall in fertility unleashes additional resources that can be spent on education and health, this effect is so strong that it overcompensates the negative effect of the reduction in fertility. Consequently, aggregate human capital accumulates faster and economic growth increases in case of lower fertility. This is a similar mechanism as in the partial equilibrium framework of Prettner et al. (2013) and Strulik et al. (2013). The implied negative association between fertility and long-run economic growth is consistent with the empirical evidence for modern economies (see, for example, Brander and Dowrick, 1994; Ahituv, 2001; Herzer et al., 2012).

Next, we obtain the following result.

Proposition 3. Shifts in parental education investments due to changes in preferences are accompanied by shifts in the long-run economic growth rate of the same sign.

Proof. Taking the derivative of Equation (2.26) with respect to θ provides

$$\frac{\partial g_y^*}{\partial \theta} = \frac{\left(\beta + \xi + 1\right) \left\{\theta \left[\nu \left(\phi - 2\right) - \phi + 1\right] + \nu \left(\zeta - \xi\right) \left(\phi - 2\right)\right\} \psi \left[\frac{\zeta(\theta \mu)^{\nu} \omega(\zeta \omega)^{-\nu}}{1 + \beta + \xi}\right]^{1 + \frac{1}{1 - \phi}}}{\theta \left(\zeta + \theta - \xi\right)^2 \left(1 - \phi\right)}.$$

It is noteworthy to remind that $\partial e/\partial \theta > 0$. To see that this expression is positive, we note that the denominator is always positive. Furthermore, we inspect the following part of the numerator: $\theta \left[\nu \left(\phi - 2 \right) + 1 \right] + \nu \left(\zeta - \xi \right) \left(\phi - 2 \right) = \theta \nu \left(\phi - 2 \right) - \theta \phi + \theta + \nu \left(\zeta - \xi \right) \left(\phi - 2 \right)$. This is unambiguously positive because i) $\nu \left(\zeta - \xi \right) \left(\phi - 2 \right)$ is positive, ii) $| \theta \nu \left(\phi - 2 \right) | < | \nu \left(\zeta - \xi \right) \left(\phi - 2 \right) |$ since $\xi > \zeta + \theta$, and iii) $-\theta \phi + \theta$ is positive.

The intuition behind this result is that parents who want to have better educated children do not only increase their educational investments but they also reduce fertility $(\partial n/\partial \theta < 0)$ due to the quality-quantity substitution described in Becker and Lewis (1973). This implies in turn that they supply more of their time on the labor market and partly spend the additional income on education and health of their children. The additional investments in the quality of children are larger than the reductions in the investments in their quantity. Consequently, aggregate human capital growth increases, despite the fact that population growth decreases. Due to this increase in the rate of aggregate human capital accumulation, technological progress and economic growth gain momentum.

Finally, we obtain the following result.

Proposition 4. Shifts in parental health investments due to changes in preferences are accompanied by shifts in the long-run economic growth rate of the same sign.

Proof. The derivative of Equation (2.26) with respect to ζ is given by

$$\frac{\partial g_y^*}{\partial \zeta} = \frac{(\beta + \xi + 1) \left\{ \zeta \left[\nu \left(\phi - 2 \right) + 1 \right] + (\nu - 1) \left(\theta - \xi \right) \left(\phi - 2 \right) \right\} \psi \left[\frac{\zeta (\theta \mu)^{\nu} \omega (\zeta \omega)^{-\nu}}{1 + \beta + \xi} \right]^{1 + \frac{1}{1 - \phi}}}{\zeta \left(\zeta + \theta - \xi \right)^2 \left(\phi - 1 \right)}.$$

Again, it is noteworthy to remind that $\partial f/\partial \zeta > 0$. To see that this expression is positive, first note that the denominator is negative. Next, we inspect the following part of the numerator: $\zeta \left[\nu (\phi - 2) + 1\right] + (\nu - 1) (\theta - \xi) (\phi - 2) = \zeta + \zeta \nu (\phi - 2) + (\nu - 1) (\theta - \xi) (\phi - 2)$. This expression is negative because $\xi > \zeta + \theta$, which implies that the derivative is positive.

The intuition behind this result is similar to that of Proposition 3 and it is again rooted in the quality-quantity substitution. Parents who want to have healthier children do not only increase their health investments but they also reduce fertility $(\partial n/\partial \zeta < 0)$. Again, this allows them to work more and spend part of the additional income on education and health of their children. Analogous to the intuition behind the previous result, this leads to faster human capital accumulation, technological progress, and economic growth.

Parameter	Value	Parameter	Value
β	0.6	δ	7
ϕ	0.7	lpha	0.33
ξ	0.85	ζ	0.3
θ	0.4	ψ	0.05
μ	8.68	ω	8.65
ν	0.5		

2.3.1 Numerical illustration

 Table 2.1: Parameter values for simulation

We illustrate the transitional dynamics of the model and the long-run solution by solving the four-dimensional system of difference equations (2.19)-(2.22) for the parameter values displayed in Table 2.1. The discount factor β is computed based on a discount rate ρ that is equal to 2% (Zhuang et al., 2007) and considering that each period lasts for 25 years in our OLG structure. The elasticity of output with respect to physical capital, α , and the knowledge spillover, ϕ , attain the values of 0.33 and 0.7, respectively (Acemoglu, 2009; Jones, 1995; Jones and Williams, 2000; Mankiw et al., 1992). The other parameters are chosen such that we obtain values of the growth rate of per capita GDP and the growth rate of the population along the balanced growth path that are consistent with the US experience averaged over the years 2006-2015 according to the World Bank (2016) data. We consider the growth rates of the population and of GDP per capita from 2006 to 2015 for which we compute the geometric mean. Afterwards, we convert the yearly growth rates into their intergenerational counterparts.¹⁴

Figure 2.1 displays the convergence of the economic growth rate from above to its steady-state level. The dashed line represents the baseline case. We observe that the long-run growth rate of per capita GDP almost reaches the intergenerational growth rate of per capita GDP of the US, which is 14.59%. The growth rate of the population is constant [see Equation (2.24)] and in our simulations we obtain a value of 22.45% which is a reasonable approximation of the US intergenerational population growth rate of 23.26%.

After the fifth period in the simulations, we increase the value of the weight of children's health in the parental utility function (ζ) by 1% in an alternative scenario ($\zeta = 0.303$). We observe that, after the increase in the parameter ζ , the growth rate of GDP per capita is higher. This is exactly what we stated in Proposition 4. The same result can be observed in Figure 2.2, where we plot the levels of technology. After the increase in the utility weight of children's health, technology levels are

¹⁴In particular, ξ , θ , η and ψ are chosen to get the intergenerational growth rate of population. It was the more convenient to stick because of the data and because of the dynamics of population (see Equation 19). The parameters ξ , θ , η are really sensitive because they show up in equations 19 and 20 with opposite sign, one at the numerator and the other one at the denominator. Therefore, we had to pay attention to not get negative numbers.

higher.

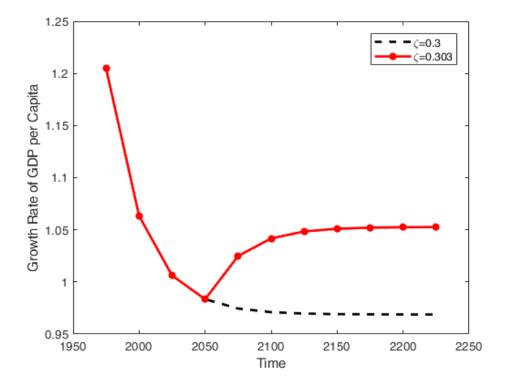


Figure 2.1: Growth rate of GDP per capita. Note that, after the fifth period in the simulations, the weight of health in parental utility (ζ) increases by 1%.

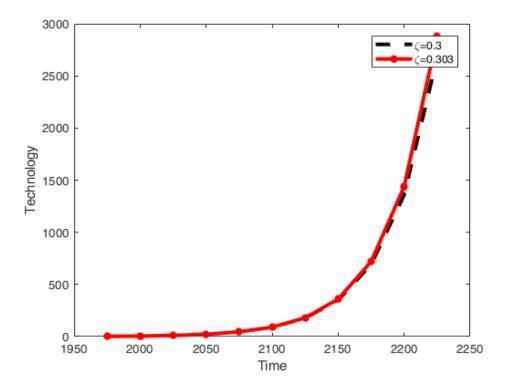


Figure 2.2: Technology level over 15 periods. Note that, after the fifth period in the simulations, the weight of health in parental utility (ζ) increases by 1%.

2.4 Conclusions

We set up a framework of R&D-based economic growth in which the stock of human capital is determined by parental education and health investments. Due to the quality-quantity trade-off, an increase in fertility leads to a reduction in education and health investments to the extent that the growth rate of overall human capital slows down. The converse holds true for falling fertility. Altogether, this generates a pattern in which a lower population growth rate is associated with faster economic growth. This pattern is consistent with the empirical findings for modern economies in the second half of the twentieth century (Brander and Dowrick, 1994; Ahituv, 2001; Herzer et al., 2012). If parents prefer to have better educated children, they do not only increase educational investments but also health investments and if parents put more weight on their children's health they do not only raise health investments but also educational investments. This implies that there is a complementarity between health and education as emphasized in the literature.

In our model, a better health condition of children raises the growth rate of human capital and therefore the growth rate of the central input in the R&D sector. As a consequence, technological progress increases, which in turn raises economic growth. This provides a mechanism based on R&D-based endogenous economic growth to explain the positive effect of health on growth that is found for modern economies (Cervellati and Sunde, 2011). This mechanism is likely to complement the ones that are based on the neoclassical capital dilution effect (Cervellati and Sunde, 2011) and on the Ben-Porath mechanism that a higher life expectancy implies a stronger incentive for education (Ben-Porath, 1967; Cervellati and Sunde, 2005, 2013).

To focus on the most important transmission channels of the effects of children's health on economic growth, we abstracted from some aspects that would be present in a more realistic setting but which would make the model more complicated such that analytical closed-form solutions for the long-run growth rates could not be obtained. For example, i) health might not only be represented by physical well-being but also by longevity, ii) the function by which health and education investments translate into human capital might have a more general form than the currently used Cobb-Douglas specification. While we do not find any reason to believe that generalizations along these lines would render our central results invalid, a consideration of these factors is surely a promising avenue for further research.

2.5 Appendix

2.5.1 Derivation of Equation (2.17)

Equation (2.16) comes from

$$K_{t+1} = s_t N_t - \pi_{i,t} \left(A_{t+1} - A_t \right) + \tau l_t,$$

and

$$K_{t+1} = Y_t - c_{1,t}N_t - c_{2,t-1}N_t / n_{t-1} - (1-\tau)l_t$$

where l_t indicates the proceeds of the patents. We assume that τ is equal to 0, meaning that the government consumes all of the proceeds. We solve by l_t and we obtain the Equation (2.17).

2.5.2 Robustness checks

We suppose that the government does not intervene and the proceeds are spent unproductively on public consumption (Strulik et al., 2013). This will imply that $K_{t+1} = s_t N_t$. Therefore:

$$K_{t+1} = \frac{\left(1-\alpha\right)\beta A_t h_t N_t K_t^{\alpha} \left(\frac{A_t^{2-\phi}}{\alpha\delta}\right)^{-\alpha}}{1+\beta+\xi}.$$

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The other three difference equations do not change:

$$A_{t+1} = \frac{(1+\beta)\delta h_t N_t A_t^{\phi}}{1+\beta+\xi} - \frac{(1-\alpha)A_t}{\alpha},$$

$$N_{t+1} = \frac{\xi-\zeta-\theta}{\psi(1+\beta+\xi)}N_t,$$

$$h_{t+1} = \frac{(\theta\mu)^{\nu}\psi(\zeta\omega)^{1-\nu}}{\xi-\zeta-\theta}h_t.$$

The growth rates at the BGP are still the same.

$$g_A^* = \left[(1+g_h) \left(1+g_N\right) \right]^{\frac{1}{1-\phi}} - 1 = \left[\frac{\zeta \left(\theta\mu\right)^{\nu} \omega \left(\zeta\omega\right)^{-\nu}}{1+\beta+\xi} \right]^{\frac{1}{1-\phi}} - 1.$$
$$g_y^* = \left[(1+g_h) \left(1+g_A\right) \right] - 1 = \frac{\left(1+\beta+\xi\right) \psi \left[\frac{\zeta(\theta\mu)^{\nu} \omega(\zeta\omega)^{-\nu}}{1+\beta+\xi} \right]^{1+\frac{1}{1-\phi}}}{\xi-\zeta-\theta} - 1,$$

$$g_Y^* = g_K^* = (1 + g_N) \left(1 + g_h \right) \left(1 + g_A \right) - 1 = \left[\frac{\zeta \left(\theta \mu \right)^{\nu} \omega \left(\zeta \omega \right)^{-\nu}}{1 + \beta + \xi} \right]^{1 + \frac{1}{1 - \phi}} - 1.$$

Exploiting the iterations from Matlab, we get the final values of g_N , g_h and g_A and we observe that the value of g_K^* is the one reached by the dynamics of the physical capital. The same holds for technology and GDP per capita (See Fig. 2.2 and Fig. 2.3)

$$g_K^* = (1+g_N)(1+g_h)(1+g_A) - 1 = (1+0.2245)(1+0.0005465)(1+0.9678) - 1 = 1.4108,$$

$$g_A^* = [(1+g_h)(1+g_N)]^{\frac{1}{1-\phi}} - 1 = [(1+0.0005465)(1+0.2245)]^{\frac{1}{1-0.7}} - 1 = 0.9678,$$

$$g_y^* = [(1+g_h)(1+g_A)] - 1 = 0.9688.$$

1

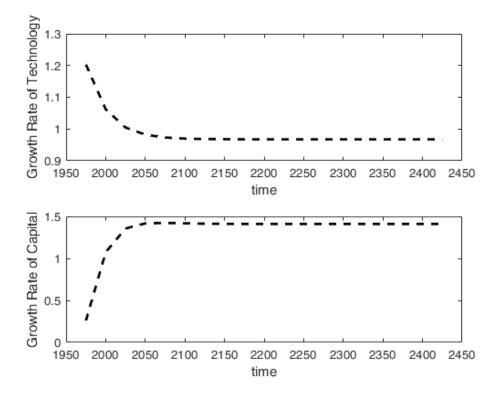


Figure 2.3: Transitional dynamics of technology and physical capital growth rates

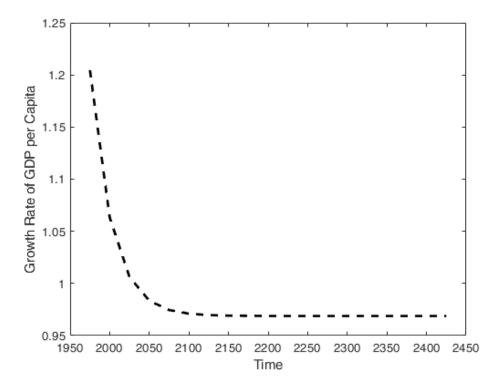


Figure 2.4: Transitional dynamics of the GDP per capita growth rate.

chapter 3

Different Types of Health Expenditures in an OLG framework:

living longer or working better?¹

¹This chapter is based on the paper "Different Types of Health Expenditures in an OLG framework:living longer or working better?" by Annarita Baldanzi (University of Milan, University of Pavia)

Abstract

The health status of individuals affects their longevity and their labor productivity. We include public health expenditures into an overlapping-generations model. The government faces the trade-off investing its revenues either into the health expenditure that makes the labor force productive, or into the health expenditure that increases life expectancy. We show how the government optimally allocates the resources between the two types of health expenditures. The results are remarkable because by using a straightforward structure of the economy, we are able to combine two strings of the existing literature about the impacts of health: higher life expectancy and higher worker productivity. "The demographic challenge of an aging population and the increasing diversity of working life have led to an increased focus on the concept of sustainable work over the life course. This emphasizes the relevance of the quality of a worker's job and their working environment over the entire course of their working life. Working longer implies working better by ensuring work organization and working arrangements that suit individual needs; training and skills development; maintaining health, safety and well-being at the workplace; providing adequate earnings and prospects; and paying attention to working time and worklife balance." (Eurofound, 2017).

3.1 Introduction

Between now and 2030, all countries will experience population aging. During the past six decades, countries have experienced just a small increase in the share of people aged 60 years and older, from 8% to 10%. However, in the next four decades, this share will rise to 22% of the total population, i.e. a jump from 800 million to 2 billion people (Bloom et al., 2015a).² According to World Bank (2017) data, in the European Union in 2016 the percentage of population aged 65 and above was almost 20%.

The increase in the life expectancy is an indirect evidence of the fact that there has been a remarkable improvement in health conditions and health investments among almost all countries in the world over the last decades (see Figure 3.1). There is a wide literature on the positive effects of life expectancy on income. For empirical findings see for example: Bloom and Canning (2000), Lorentzen et al. (2008), and Gehringer and Prettner (2017). From the theoretical perspective: living longer increases the incentives to invest not only in human capital (Cervellati and Sunde, 2005), but also in physical capital as long as it creates a need of saving more for retirement (Azomahou et al., 2009; Heijdra and Mierau, 2012).³ One of the main consequence of population aging is a higher working life expectancy: in Europe many countries have increased the official retirement age and measures to promote higher economic activity among the workers aged 50 and older to avoid substantial financial burdens with the public pension system.⁴

Improving the working conditions is a collective concern, prompted by both hu-

²Population aging can be considered a recent phenomenon, indeed mortality declines have not improved uniformly across age groups over the last two centuries: first we have observed infants and children mortality declines, now mortality declines characterize older ages (Lee, 1994; Wilmoth and Horiuchi, 1999; Cutler et al., 2006).

³For other theoretical works, see also: De la Croix and Licandro (1999), Kalemli-Ozcan et al. (2000), Boucekkine et al. (2002), Boucekkine et al. (2003), Lagerlöf (2003), Echevarría and Iza (2006), Ferreira and de Abreu Pessôa (2007), Jayachandran and Lleras-Muney (2009) and Bar and Leukhina (2010).

⁴Increases in working life expectancy are more pronounced among women than men and differences according to the level of education are substantial (Loichinger and Weber, 2016).

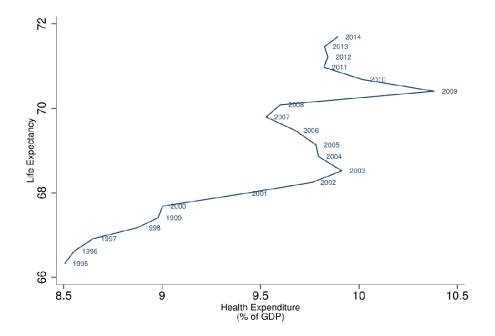


Figure 3.1: World Average of Life Expectancy and Health Expenditure (1995 - 2004) (World Bank, 2018)

manitarian and economic considerations. As pointed out by the Eurofound (2017), since the population works longer, the health, the safety and the well-being at the workplace are even more important. Creating more jobs and jobs of better quality is one of the targets of the EU social policy: a safe and healthy working environment is essential for the quality of work. In general, health status is fundamental for the labor supply: Cai (2010) finds that a change from fair to poor health on average reduces the probability of labor participation for both males and females.⁵ According to Eurostat (2017a), a work-related health problem covers all diseases, disabilities and other physical or mental health problems, apart from accidental injuries, suffered by the person during the last 12 months, and caused or made worse by the work. In the Labor Force Survey (LFS) ad hoc module 2013, people aged 15 to 64 years that work or worked previously are asked whether they suffered from health problems caused or made worse by their job in the past 12 months. In total, 7.9% have a work-related health problem. Table 3.1 shows the most common work-related

⁵See also Campolieti (2002), Cai and Kalb (2006), García-Gómez (2011), Cai et al. (2014).

health problems. In addition, the LFS ad hoc module 2013 assesses the extent in which these health problems limit the ability to carry out normal day activities either at work or outside of work. Among employees with work-related health problems, 25.3% have no limitations, 50% have some limited limitations and 22.3% have considerable limitations in the daily activities.⁶ These types of work-related health problems impact on the workers' productivity who cannot perform their tasks and have to ask for sick leave: among employees aged 35-54 with work-related health problems, 47% report sick leave, while among workers with work-related health problems that are between 55 and 64, 49.8% report sick leave.

Type of work-related health problem	Percentage of employees
Muskolo-skeletal disorders	60.1
Stress, depression, anxiety	15.9
Headache and/or eyestrain	4.8
Cardiovascular disease	4.5
Pulmonary disorders	3.6
Stomach, liver, kidney or digestive problem	1.8
Skin problem	1.2
Hearing problem	1.1
Infectious Disease	1.0
Other not elsewhere mentioned	5.4

Table 3.1: Types of work-related health problems (Eurostat, 2017a)

Nowadays, another most politically debated issue is the high level of health expenditures. It has been estimated that global spending on health care per person will more than double by 2040 (Ahmad Kiadaliri, 2017).

At the European level (EU-28), general government health expenditure has increased relatively smoothly in the period 2002-2015: amounting to 13.7% of total expenditure in 2002, 14.7% of total expenditure in 2009 and 15.2% in 2015. The

 $^{^{6}}$ Venema et al. (2009) underline how the high occurrence of work-related health problems in sectors such as agriculture, mining, manufacturing, is related to the fact that less favorable job characteristics are more prevalent in these types of sectors, such as manual work and atypical working hours.

EU-28 government health expenditure as a share of GDP amounted to 6.2% in 2002 and to 7.2% in 2015. The highest level was in 2009 (7.4% of GDP), at the onset of the economic crisis, because of a decrease in nominal GDP and not because of an unusual increase in government expenditure (Eurostat, 2017b).

The aim of this paper is to contribute to the highly debated issue of health expenditure with a theoretical model, combining two strings of the existing literature about the different impacts of health: higher life expectancy and higher worker productivity. We propose an OLG model in the context of a stationary economy, where the government has to face a trade-off investing into different types of health expenditures:⁷ it can increase the survival probability (preventive and curative treatments) or the labor productivity (assistance for disabled workers, laws for better working spaces, ergonomic interventions to avoid health work related problems listed in Table 3.1). More specifically, the consumption side of the economy consists of households who reach the retirement phase, given a survival probability. The probability of surviving depends on the health stock in the economy, which is determined by how much the government decides to allocate to the first type of health expenditure. The propensity to save is an increasing function of the survival probability, i.e. of the first type of health expenditure of the government. Therefore, the more the government invests to increase the survival probability, the more capital accumulation arises in the economy. The production side of the economy is characterized just by final goods production, described by a Cobb-Douglas, where the inputs are capital and effective labor. The latter input directly depends on the second type of government health expenditure: the more it invests in the second type of health expenditure. the more productive the labor force will be. We find the optimal combination of the tax rate and of the allocation of the tax revenues between the two health policies, that maximizes the steady state of GDP per worker.

⁷The Government has different types of health expenditures: curative care; preventive care; rehabilitative cares; long-term care health; therapeutic appliances and other medical goods (Eurostat, 2017b).

3.2 The model

3.2.1 Households

The consumption side of the economy consists of overlapping generations (Samuelson, 1958; Diamond, 1965) with single-sex individuals who live for three time periods: childhood, adulthood, and retirement. The first period lasts for 20 years and the other two periods last 40 years each.⁸ Therefore, the total years of life are 100. However, adults face a survival probability between adulthood and retirement, that defines the total life expectancy. Children do not make any economic decisions and they fulfill their consumption through parents expenditures. Adults consume, save and work in the first period and they only consume in the second one.⁹ In the middle of adulthood, each adult gives birth to one child such that the latter enters adulthood when adults enter the retirement. Given these assumptions the size of the cohorts will stay constant.¹⁰ We assume, the labor force stays constant: $N_t = \bar{N}, \forall t$. Compared to part of the existing literature about the impact of longevity, we completely abstract from the issue of population growth. Zhang et al. (2001), Doepke (2004), Zhang and Zhang (2005) among others emphasize how an increase in longevity reduces fertility. In our work, we decide to focus just on the impact of longevity, i.e. of health, on savings, because we are interested in the complementary effects of different types of health expenditures affecting longevity and labor productivity in the economy. Adding the issue of fertility might distract from our major intent. The probability of surviving to the second period has the

 $^{^{8}\}mathrm{Also}$ in the theoretical model by Galama et al. (2013), individuals begin working at the age of 20.

⁹In this paper we do not model the concept of working life expectancy. An interesting approach is the one by Fanti and Gori (2011): the authors build an OLG framework where the public health investments affect the labor supply of the old-aged workers. Indeed, according to Eurostat (2017c) 15.9% of EU-28 old-age pensioners continue to work. Of these, 62.8% continue to work mainly for financial reasons, while 37.2% do so mainly for non-financial reasons, e.g. job satisfaction.

 $^{^{10}}$ See Ludwig and Vogel (2010); Baldanzi et al. (2017) for the same structure.

CHAPTER 3.

following specification:¹¹

$$\phi = \frac{\gamma b_t}{1 + b_t},\tag{3.1}$$

where b_t is the health status of an individual depending on the resources invested by the government and γ is the medical technology that positively affects the life expectancy. The survival probability function has the following properties:

- $\phi(b_t) \in (0, 1)$
- $\partial \phi(b_t) / \partial(b_t) > 0$
- $\partial \phi^2(b_t) / \partial b_t^2 < 0$

Our survival probability follows partly the function suggested by Chakraborty (2004). Despite of it, the purpose of this paper is different: in his work, the author focuses on the probability of surviving endogenously determined through public investment in health, but compared to us, there is no need to focus on the role of the government or of different types of health expenditures. Indeed, Chakraborty (2004)'s model relates to developing and underdeveloped countries where high mortality can induce poverty traps because neither savings accumulation nor human capital accumulation occur. Instead in our model, that fits more the developed countries, the longevity issue is just one aspect and the government has to allocate resources among two different types of health expenditures.¹²

The utility function is^{13}

$$u_{t} = \ln(c_{1,t}) + \beta \phi(b_{t}) \ln(c_{2,t}), \qquad (3.2)$$

¹¹Blackburn and Cipriani (2002) argue that changes in life expectancy owe much to changes in public awareness and personal lifestyles brought about unintentionally by changes in levels of education through which human capital is accumulated. Moreover, the survival probability function can be influenced by a number of other factors as well, both internal and external to an agent, such as: private expenditures of income, time and effort (e.g., on medical treatment, hygiene and exercise), government provided services and the quality of the environment (i.e. the extent of public health care, sanitation and pollution).

 $^{^{12}\}mathrm{See}$ Agénor (2012) for different examples of survival probability function.

 $^{^{13}}$ Health does not enter directly the utility function, but through the survival probability. See Van Zon and Muysken (2001) for an example of how health enters the utility through endogenous preferences.

where $c_{1,t}$ is first period consumption of the generation born at time t, $c_{2,t}$ is second period consumption of the generation born at time t and β is the discount factor. Following Zhang and Zhang (2005), the economy is characterized by an actuarially fair annuity market that channels savings in physical capital, K, for production in the next period. With this annuity market, old-age survivors share the savings plus interest left by households who die before reaching old age.¹⁴ The budget constraints of the households are the following:

$$c_{1,t} + s_t = a_t w_t \left(1 - \tau\right),$$
$$c_{2,t} = \left(\frac{R_{t+1}}{\phi(b_t)}\right) s_t.$$

The lifetime budget constraint follows:

$$c_{1,t} + \frac{c_{2,t}\phi(b_t)}{R_{t+1}} = a_t w_t \left(1 - \tau\right), \qquad (3.3)$$

where a_t is the individual labor productivity, w_t is the effective wage rate, τ is the tax rate on households income and $\frac{R}{\phi(b_t)}$ is the rate of return to saving, thanks to the market annuity structure. From the FOCs, we get the Euler Equation:

$$\frac{c_{2,t}}{c_{1,t}} = \beta \left(R_{t+1} \right).$$
(3.4)

The optimal savings are

$$s_t^* = \sigma a_t w_t \left(1 - \tau \right), \tag{3.5}$$

where

$$\sigma = \frac{\phi(b_t)\beta}{1+\phi(b_t)\beta},\tag{3.6}$$

is the propensity to save, which is an increasing function of the survival probability

 $^{^{14}}$ See also Blackburn and Cipriani (2002); Chakraborty (2004); Irmen (2017) among others for the same annuity market structure.

 $(\partial \sigma / \partial \phi (b_t) > 0)$:¹⁵ improvements in longevity incentive to invest more in physical capital. For the same type of reasoning see also: Reinhart (1999); Azomahou et al. (2009); Leung and Wang (2010); Heijdra and Mierau (2012).¹⁶

3.2.2 Production

The production side of the economy is characterized just by final goods production, described by a Cobb-Douglas, where the inputs are capital and effective labor.

$$Y_t = DK_t^{\alpha} \left(a_t \bar{N} \right)^{1-\alpha}, \qquad (3.7)$$

where D is the productivity parameter and α is the elasticity of output with respect to capital. First-order conditions yield the rental rate of capital

$$R_t = \alpha \frac{Y_t}{K_t},\tag{3.8}$$

and the effective wage rate

$$w_t = (1 - \alpha) \frac{Y_t}{\bar{N}a_t}.$$
(3.9)

3.2.3 Government

Following Barro (1990), the government has to balance its budget, i.e. it can neither finance deficits issuing debts nor run surplus accumulating assets.¹⁷ In particular, we assume that government spending is financed contemporaneously by a flat-rate tax, τ , on household income. The government has to allocate its total revenues,

¹⁵Note that the survival probability disappears from the Euler Equation because of the annuity market and it shows up in the optimal savings.

¹⁶In this model, we focus on the savings of young cohorts and not of old cohorts. Actually, studies concerning savings among people aged more than 65 years do not show clear results (Bloom et al., 2015a).

 $^{^{17}}$ In Barro (1990) the taxation is directly on income, instead of on household income. See Aísa and Pueyo (2006) for an example of government debts for health expenditure.

 $\tau \bar{N} a_t w_t$, between two types of health expenditures:

$$h_t^1 + h_t^2 = \tau \bar{N} a_t w_t. \tag{3.10}$$

Therefore substituting Equation (3.9)

$$h_t^1 + h_t^2 = \tau \left(1 - \alpha\right) Y_t, \tag{3.11}$$

where

$$h_t^1 = \lambda \tau \left(1 - \alpha\right) Y_t, \tag{3.12}$$

$$h_t^2 = (1 - \lambda) \tau (1 - \alpha) Y_t, \qquad (3.13)$$

 h_t^1 affects the survival probability, so it represents the investments by the government that make households live longer (preventive and curative treatments). The health status, b_t , introduced in Equation (3.1), is now characterized by

$$b_t = \frac{h_t^1}{Y_t} = \lambda \tau \left(1 - \alpha\right). \tag{3.14}$$

The health stock per person, b_t , is the health expenditure by the government, h_t^1 , as a share of GDP, i.e. the health status depends positively on the share of resources that an economy allocates to the health sector. To see why, consider Equation (3.14) in this way: $b_t = \frac{h_t^1}{Ny_t}$ where $y_t = \frac{Y_t}{N}$ is the GDP per worker. It is straightforward to divide health expenditure by the labor force size \bar{N} . Indeed, if we compare two countries, such as India and Luxembourg, the latter one would have a lower absolute value of health expenditure. However, life expectancy is higher in Luxembourg than in India. Now consider Equation (3.12) rewritten in the following, i.e. before substituting the definition of effective wage:

$$h_t^1 = \lambda \tau \bar{N} a_t w_t$$

Suppose, in the economy the wages per worker, $a_t w_t$, increase, this will result in a higher tax revenue, therefore, for a given λ , in a higher h_t^1 . The latter one is used to pay the health care personnel, doctors and nurses. Since wages increase, also wages in the health care sector increase, therefore although health care expenditure will rise, the amount of health treatments and cures will not be affected resulting in a no-change of health status of individuals. Given the linear relationship between productivity and GDP per capita, it is convenient to divide h_t^1 by aggregate GDP. In the theoretical model by Aísa and Puevo (2006), public health services affect the level of mortality. In particular, they assume that the instantaneous probability of dying is negatively related to public expenditure in health as a percentage of the GDP. A similar approach has been used by Schneider and Winkler (2017) and by Kuhn and Prettner (2016): the authors consider the health expenditure as the share of labor income because the health care sector is labor intensive. Moreover, although the focus is on human capital, Strulik et al. (2013) consider the education expenditure as a share of GDP. Ono and Uchida (2016) refer to pension-to-GDP ratio and public education-to-GDP ratio in their OLG model.¹⁸

Combining Equations (3.6) and (3.14), we can rewrite the saving propensity as:

$$\sigma = \frac{(1-\alpha)\,\lambda\beta\tau\gamma}{1+(1-\alpha)\,\lambda\tau\,(1+\beta\gamma)}.\tag{3.15}$$

The propensity to save is an increasing function of h_t^1 , i.e. of $\lambda \tau$ ($\partial \sigma / \partial \lambda \tau > 0$). Therefore, the more the government invests in h_t^1 , the more capital accumulation arises in the economy.

The second type of health expenditure, h_t^2 , affects the labor productivity

$$a_t = \frac{h_t^2}{Y_t} = (1 - \lambda) \tau (1 - \alpha).$$
(3.16)

¹⁸Also Jones (2002) considers the health expenditure as a share of GDP. Afonso and Furceri (2010) among others relates the growth rate of GDP per capita with general government expenditure variables as percentage of GDP in their regressions.

The labor productivity is the health expenditure by the government h_t^2 as a share of GDP, i.e. the productivity depends positively on the share of resources that an economy allocates to this type of expenditure. Since the resources needed for this type of health expenditure are produced also with physical capital and not just with labor and due to the symmetry of the model, we divide the second health expenditure by aggregate GDP too.¹⁹ The more the government invests in h_t^2 , the less work-related health problems occur, therefore less sick leaves and less limitations in the daily activities of workers: a healthier population usually means also a more productive labor force (Bloom and Canning, 2005; Cai, 2010; Prettner et al., 2013). For the work-related health problems, the government can subsidize ergonomic interventions (better chairs and computer screens, better working spaces), it can pay for doctors and for drugs to solve the specific health problems listed in Table 3.1. Moreover, it can promote laws to encourage firms to create a healthy working environment compatible with a balanced work-life and it can issue environmental laws, especially in the agricultural and manual sectors. Hence, the production function becomes:²⁰

$$Y_t = DK_t^{\alpha} \left[(1 - \alpha) \left(1 - \lambda \right) \tau \right]^{1 - \alpha}.$$
(3.17)

3.2.4 Savings-Investment Balance

The asset market clearing condition requires equality between investment and savings:

$$I_t = s_t. aga{3.18}$$

Physical capital fully depreciates in one period in production.

$$K_{t+1} = s_t. (3.19)$$

¹⁹It is noteworthy that dividing by Y_t implies that a_t is stationary in the steady state and, thereby, output per capita is also stationary in the steady state.

²⁰From now on, to simplify the notation, we normalize $\bar{N} = 1$.

With Equation (3.5)

$$K_{t+1} = \sigma a_t w_t \left(1 - \tau \right),$$

and with Equations (3.9) and (3.17) the accumulation of capital is:

$$K_{t+1} = \sigma \left(1 - \tau\right) \left(1 - \alpha\right) DK_t^{\alpha} \left[\left(1 - \lambda\right) \left(1 - \alpha\right) \tau\right]^{1 - \alpha}.$$

Since $\alpha < 1$, the process is stable. The capital stock at the steady state is:

$$K^{ss} = \left[D\sigma \left(1 - \alpha \right) \left(1 - \tau \right) \right]^{\frac{1}{1 - \alpha}} \tau \left(1 - \lambda \right) \left(1 - \alpha \right), \tag{3.20}$$

with

$$\sigma = \frac{(1-\alpha)\,\lambda\beta\tau\gamma}{1+(1-\alpha)\,\lambda\tau\,(1+\beta\gamma)}$$

3.3 Results

Substituting Equation (3.20) in (3.17), we can derive the steady state of GDP per worker:

$$Y^{ss} = \left[D\left(1-\alpha\right)\right]^{\frac{1}{1-\alpha}} \left[\frac{\left(1-\alpha\right)\lambda\beta\tau\gamma}{1+\left(1-\alpha\right)\lambda\tau\left(1+\beta\gamma\right)}\left(1-\tau\right)\right]^{\frac{\alpha}{1-\alpha}}\tau\left(1-\lambda\right).$$
 (3.21)

Looking at Equation (3.21), we can see the impact of τ and λ : τ has a positive impact through σ (see Equation (3.15)) -higher accumulation of savings- and through $\tau (1 - \lambda)$ -more productive labor force- and a negative impact through $(1 - \tau)^{\frac{\alpha}{1-\alpha}}$, the drag-down effect of tax reducing household income.²¹ We also see the complementarity of the two health expenditures: if $\lambda = 0$, there will not be investments in the first type of health expenditure, h_t^1 , i.e. no preventive and curative treatments

²¹See Pautrel (2012) for an interesting analysis of the two opposing effects of the environmental tax, τ , leading to an inverted-U-shaped relationship between the environmental tax and the steady-state output level.

that make individuals survive and therefore the saving propensity σ will be equal to zero, making no accumulation of capital in the economy, and hence no positive state of the economy; on the other hand, if $\lambda = 1$, the government will not invest in the second type of health expenditure, h_t^2 , i.e. the labor force will not be productive and there will be no output in the economy, so again no positive steady state of the economy. Therefore, both types of health expenditures are necessary: each of them contributes to two different channels, i.e. accumulation of savings and productivity of the labor force. We implement a simple numerical example with the following parameters (see Table 3.2). We manage to compute the GDP per worker of the European Union, exploiting Equation (3.21), i.e. 71939\$.²²

Table 3.2: Parameter values for the numerical example

Parameter	Value	Parameter	Value
au	0.4	λ	0.5
β	0.67	α	0.33
γ	4.5	D	14494.4

We choose $\lambda = 0.5$, assuming that the government would distribute equally the tax revenues among the two types of health expenditures.²³ Following Heijdra and Mierau (2010), the tax rate on labor income is equal to 40%. Considering that in our model the retirement period lasts 40 years and exploiting the survival probability function (Equation 3.1), we choose $\gamma = 4.5$, such that the total life expectancy is 81.13 like in the European Union (World Bank, 2017).²⁴ The discount factor, β , is computed based on a discount rate equal to 1% from Florio (2007) and considering the fact that a period lasts for 40 years in our OLG structure.²⁵ The elasticity of

²²Since we have normalized the size of the labor force to 1, we can consider GDP per worker from the data (total GDP over labor force). Our target is the average from 2007-2016 of the GDP per worker (current US \$) of the European Union.

²³We decide for this value because data about aggregate government expenditures are faraway from our model assumption with just two types of health expenditures.

²⁴The total life expectancy is computed in this way: $20 + 40 + 40 * \frac{(1-\alpha)\gamma\tau\lambda}{(1+(1-\alpha)\tau\lambda)}$.

 $^{^{25}}$ See Zhuang et al. (2007) for a literature review on the measurement of the discount rate and its estimation.

output with respect to physical capital α is taken from Mankiw et al. (1992). We set the productivity parameter D = 14494.4 to get the best fit with data.

3.3.1 Optimal allocation of the resources by the government

In this subsection, we describe the steady state of the economy as a function of the two government policy instruments. The government has to maximize Y^{ss} choosing the optimal tax rate and the optimal allocation of the two health expenditures.²⁶ Given the complexity of the function of Equation (3.21),²⁷ it is not possible to show analytically its concavity (negative definite Hessian matrix), but with numerical simulations, we show that, despite the variations of parameters, the shape of Y^{ss} is still concave and the maximizing combination of τ^* and λ^* is always interior.²⁸ The optimal combination of τ^* ($\partial Y^{ss}/\partial \tau = 0$) and λ^* ($\partial Y^{ss}/\partial \lambda = 0$) to maximize the steady state of the economy is:²⁹

$$\tau^* = \frac{1 - \alpha}{1 - \lambda^* \alpha},\tag{3.22}$$

$$\frac{\alpha}{1-\alpha} \frac{1-\lambda^*}{\left(1+\left(1-\alpha\right)\lambda^*\tau^*\left(1+\beta\gamma\right)\right)} \frac{1}{\lambda^*} = 1.$$
(3.23)

We notice that τ^* and λ^* are correlated one with each other, specifically looking at Equation (3.22) $\partial \tau^* / \partial \lambda^* > 0$. The steady state of the GDP at the optimal value is:

$$Y_{max} = (D(1-\alpha))^{\frac{1}{1-\alpha}} (\sigma^* (1-\tau^*))^{\frac{\alpha}{1-\alpha}} \tau^* (1-\lambda^*).$$
 (3.24)

 $^{^{26} \}rm Park$ and Philippopoulos (2004) assume that the allocation of public expenditure is exogenous and they focus only on the role the tax rate.

²⁷The major problem of the first and second derivatives of Equation (3.21) is how τ and λ interplay in σ .

²⁸We implemented the numerical simulations in Matlab, plotting Equation (3.21) as a function of τ and λ . Each parameter has its own vector with 100 values taken from a normal random distribution with mean and standard deviation according to the literature and to the model restrictions (for example α has mean and deviation standard equal to 0.33 (Mankiw et al., 1992), but all the 100 values of the random vector have to be between 0 and 1).

²⁹A similar result has been reached by Dioikitopoulos (2014) with a different purpose: the author focuses on the allocation of public funds between health and human capital.

In Figure 3.2, we represent Equation (3.24) with respect to the policy instruments τ and λ . We can observe that the steady state of the economy is a concave function and the optimal combination of the two policy instruments is an interior point.

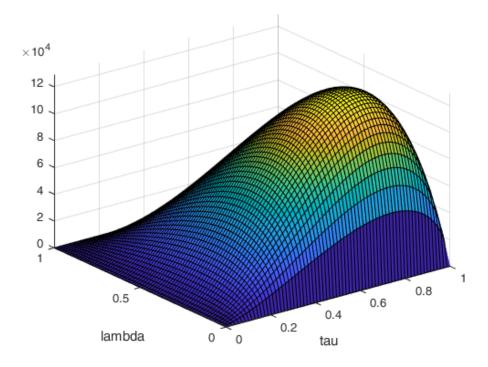


Figure 3.2: Steady state of the economy, $\tau^* = 0.7246$ and $\lambda^* = 0.2464$, $Y^* = 1.2905e + 05$

3.3.2 Sensitivity analysis

In this subsection we conduct the sensitivity analysis for the parameters γ , β and α .

D does not play any significant role with respect to the steady state: when it increases the maximum value of the steady state increases too, without any changes in τ^* and λ^* .³⁰

 γ has the following values: 0.2, 1.5 and 4.5 (see Table 3.3). We obtain respectively

 $^{^{30}\}mathrm{Given}$ the straightforward relationship, we do not report any tables for the sensitivity analysis of D.

the following life expectancies: 61, 67 and 79.5.³¹ $\gamma < 1$ follows Chakraborty (2004) survival probability, indeed we obtain a life expectancy of 61 years old that describes better the conditions of underdeveloped countries. For $\gamma = 4.5$, we obtain a similar value of life expectancy that we have in our numerical example, despite the different τ and λ we choose. We observe that increasing the medical technology makes the optimal value of λ , i.e. the share the government has to invest in medical preventive and curative treatments, decrease: if the medical technology is large to make the life expectancy enough high to have sufficient accumulation of capital than the government has less incentives to invest more resources in the first type of health expenditure.

 β has the following values: 0.30, 0.67 and 0.96. They correspond respectively to the following discount rates: 0.03 (Nordhaus, 1993), 0.01 (Florio, 2007) and 0.001 (Stern, 2007). As in the numerical example, we transform the annual discount rates in intergenerational discount rates considering the periods last 40 years. γ and β have the same role (see Tables 3.3 and 3.4): they increase the maximum value of the steady state of the economy. Both parameters increase the propensity to save $(\partial \sigma / \partial \gamma > 0 \text{ and } \partial \sigma / \partial \beta > 0)$. The rise in the saving rate leads to a higher amount of physical capital investment and therefore in a higher steady state of the economy. α has the following values (see Table 3.5): 0.33 (Mankiw et al., 1992), 0.36 which is the capital share of income in US in 2011 and 0.42 which is the capital share of income in Luxembourg in 2012 (OECD, 2018). If alpha increases, capital becomes relatively more important in the production process. Therefore, the government will try to increase the amount of capital used. Since the stock of capital depends on the amount of savings and the savings in turn depend on the savings rate, the government will try to increase the saving rate. The saving rate depends on the survival probability. Therefore, for the government it is straightforward to invest more in longevity than in productivity. Besides, GDP per worker is higher for higher

³¹The life expectancy has been computed like in the numerical example, but with the optimal values of τ and λ : $\frac{(1-\alpha)\gamma\tau^*\lambda^*}{(1+(1-\alpha)\tau^*\lambda^*)}$.

levels of α and this is consistent with the real data.

Parameters and Variables	Values	Values	Values
α	0.33	0.33	0.33
γ	0.2	1.5	4.5
eta	0.67	0.67	0.67
D	14500	14500	14500
$ au^*$	0.7427	0.7378	0.7299
λ^*	0.2967	0.2785	0.2487
Steady state of the economy	32117	82416	129070

Table 3.3: Sensitivity analysis of γ

Table 3.4: Sensitivity analysis of β

Parameters and Variables	Values	Values	Values
α	0.33	0.33	0.33
γ	4.5	4.5	4.5
eta	0.30	0.67	0.96
D	14500	14500	14500
$ au^*$	0.7362	0.7299	0.7262
λ^*	0.2724	0.2487	0.2346
Steady state of the economy	93625	129070	146730

_

Values	Values	Values
0.33	0.36	0.42
4.5	4.5	4.5
0.67	0.67	0.67
14500	14500	14500
0.7246	0.71014	0.666675
0.2464	0.27536	0.31884
129050	186500	437370
	$\begin{array}{c} 4.5\\ 0.67\\ 14500\\ 0.7246\\ 0.2464\end{array}$	$\begin{array}{cccc} 4.5 & 4.5 \\ 0.67 & 0.67 \\ 14500 & 14500 \\ 0.7246 & 0.71014 \\ 0.2464 & 0.27536 \end{array}$

Table 3.5: Sensitivity analysis of α

3.4 Robustness checks

In this section, we analyze how changing some assumptions of the model impacts on the results. First, we assume a new survival probability function, to see how the saving propensity changes and whether or not the optimal policy instruments change. Second, we change the production function, giving another role to the second type of health expenditure by the government.

The new survival probability function is

$$\phi_{bis} = \gamma b_t^{\xi},\tag{3.25}$$

where γ and ξ belong to (0,1). Substituting the definition of the health status, b_t , the survival probability becomes

$$\phi_{bis} = \gamma \left((1 - \alpha) \,\lambda \tau \right)^{\xi} \,. \tag{3.26}$$

Therefore the propensity to save is

$$\sigma_{bis} = \frac{\beta \gamma \left(\left(1 - \alpha \right) \tau \lambda \right)^{\xi}}{1 + \beta \gamma \left(\left(1 - \alpha \right) \tau \lambda \right)^{\xi}}.$$
(3.27)

The steady state of GDP is still the same

$$Y^{ss} = \left[D\left(1-\alpha\right)\right]^{\frac{1}{1-\alpha}} \left[\frac{\beta\gamma\left(\left(1-\alpha\right)\tau\lambda\right)^{\xi}}{1+\beta\gamma\left(\left(1-\alpha\right)\tau\lambda\right)^{\xi}}\left(1-\tau\right)\right]^{\frac{\alpha}{1-\alpha}}\tau\left(1-\lambda\right).$$
 (3.28)

The steady state of GDP is a concave function of the two optimal policy instruments of the government. The optimal taxation rate does not change, it still depends on the optimal λ :

$$\tau^* = \frac{1 - \alpha}{1 - \alpha \lambda^*}.\tag{3.29}$$

In Figures 3.3, 3.4, 3.5, we show the steady state of the economy with the survival probability function (3.28) by varying the parameter ξ .³² We observe that smaller values of ξ imply smaller values of λ : since the survival probability will increase, the necessary h_t^1 is smaller.

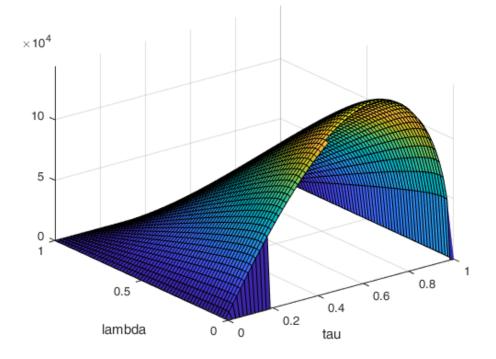


Figure 3.3: Steady state of the economy, $\tau^* = 0.6787$ and $\lambda^* = 0.0387$, $Y^* = 144390$, $\xi = 0.1$

 $^{^{32}}$ The other parameters have the following values: $\alpha = 0.33, \, D = 14500, \, \gamma = 0.5, \, \beta = 0.67$

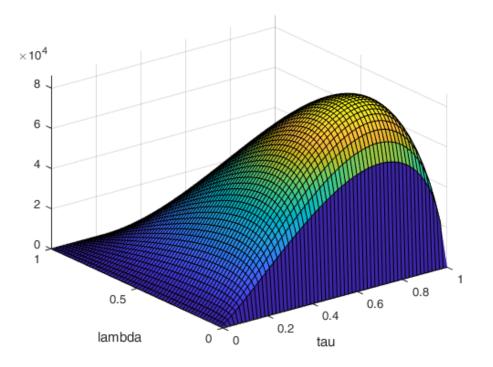


Figure 3.4: Steady state of the economy, $\tau^* = 0.7131$ and $\lambda^* = 0.1831$, $Y^* = 86084$, $\xi = 0.5$

In a second attempt, we change the production function, keeping the original survival probability, Equation (3.1).

$$Y_t = a K_t^{\alpha} \bar{N}^{1-\alpha}, \qquad (3.30)$$

where a is a productivity parameter, which is financed by the government $a = (1 - \alpha) (1 - \lambda) \tau$. In this case, the health expenditure h_t^2 by the government is a positive externality for the whole economy and not just for the labor force. Basically, the health expenditure affect also the use of capital and not just the physical work. Given the new production function, (3.30), we do not compute the effective wage rate, but just the wage rate $(\partial Y_t / \partial \bar{N})$. Despite of it, nothing changes in our analysis, but the steady state of the GDP

$$Y^{ss} = \bar{N} \left(1-\alpha\right)^{\frac{1+\alpha}{1-\alpha}} \left[\frac{\left(1-\alpha\right)\lambda\beta\tau\gamma}{1+\left(1-\alpha\right)\lambda\tau\left(1+\beta\gamma\right)} \left(1-\tau\right)\right]^{\frac{\alpha}{1-\alpha}} \left(\tau\left(1-\lambda\right)\right)^{\frac{1}{1-\alpha}}.$$
 (3.31)

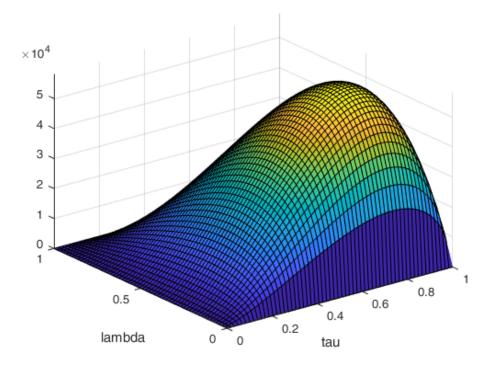


Figure 3.5: Steady state of the economy, $\tau^* = 0.7423$ and $\lambda^* = 0.2950$, $Y^* = 58121$, $\xi = 0.9$

The shape of the function is still concave with respect to the two policy instruments τ and λ (see Figure 3.6).³³ In this case the optimal taxation rate, τ^* , is different compared to (3.22):

$$\tau^* = \frac{1}{1 + \alpha - \alpha \lambda^*}.\tag{3.32}$$

Therefore, we notice that changing the survival probability function changes the saving propensity but not the maximizing taxation rate. Instead, changing the production function, that is giving another role to the health expenditure, we have a different optimal taxation rate. Despite these changes of the assumptions in both the household side and the production side, both health expenditures are necessary at the equilibrium of the economy and the steady state of the GDP is still a concave function of the two policy instruments and the optimal combination of them is always interior.

³³The parameters have the following values: $\alpha = 0.33$, $\bar{N} = 14500$, $\gamma = 4.5$, $\beta = 0.67$.

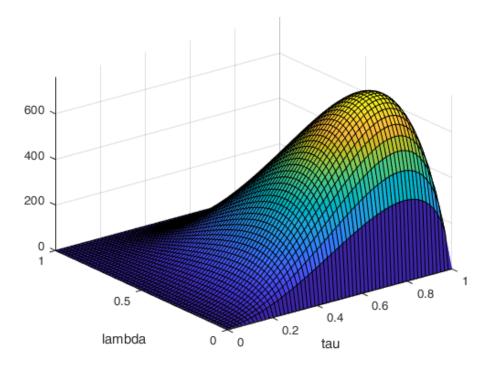


Figure 3.6: Steady state of the economy, $\tau^* = 0.7911$ and $\lambda^* = 0.2000$, $Y^* = 760.8636$

3.5 A comparison with Barro (1990)

Suppose we do not divide the h_t^2 by Y_t . The production function becomes

$$Y_t = DK_t^{\alpha} \left(\left(1 - \lambda \right) \tau \left(1 - \alpha \right) Y_t \right)^{1 - \alpha}.$$
(3.33)

Therefore:

$$Y_t = D^{\frac{1}{\alpha}} K_t \left((1-\lambda) \tau \left(1-\alpha \right) \right)^{\frac{1-\alpha}{\alpha}}.$$
(3.34)

The key assumption is that the production function has constant returns to scale to government spending and capital together, but it has decreasing returns to scale to government spending and capital separately. We end up in an AK model framework like Barro (1990). The associated growth rate of the economy is:

$$g_K = g_Y = (D(1-\alpha))^{\frac{1}{\alpha}} \sigma (1-\tau) (\tau (1-\lambda))^{\frac{1-\alpha}{\alpha}} - 1.$$
 (3.35)

The optimal tax rate is still $\tau^* = (1 - \alpha) / (1 - \lambda^* \alpha)$.

The Barro (1990) model is a breaking point in the literature of public policy and economic growth interaction. In his seminal work, Barro (1990) suggests a simple endogenous growth model from Romer (1986) with government. Public investments, financed through income taxes, are complements to private investments. In the decentralized economy, the maximizing tax rate of the economy is $\tau_{Barro} = 1 - \alpha$.³⁴ In our model, the maximizing tax rate is $\tau^* = \frac{1-\alpha}{1-\lambda^*\alpha}$, which is higher than τ_{Barro} for $\forall \alpha$ and $\forall \lambda$. This happens because compared to Barro (1990), we have two channels of government investments: accumulation of savings and labor productivity. Both of them with diminishing returns increase the output, making it possible to have a higher tax rate, compared to Barro (1990), where the government has just one channel of investment. We will have the same result as Barro (1990) if $\lambda = 0$, i.e. there is no investment in the health expenditure that makes you live longer, having a higher accumulation of capital.

3.6 One policy instrument: λ

To obtain more analytical results, we consider the steady state of GDP per worker as a function of just one policy instrument: λ - the share of the health expenditure, considering τ as a parameter. Moreover, this approach can be a more real world justification: τ is probably more stable than the more specific choice associated with λ . The GDP per worker at the steady state as a function of λ will be

$$Y^{ss} = \left[D\left(1-\alpha\right)\right]^{\frac{1}{1-\alpha}} \left[\frac{\left(1-\alpha\right)\lambda\beta\tau\gamma}{1+\left(1-\alpha\right)\lambda\tau\left(1+\beta\gamma\right)}\left(1-\tau\right)\right]^{\frac{\alpha}{1-\alpha}}\tau\left(1-\lambda\right).$$
(3.36)

³⁴Actually, the maximizing tax rate in Barro (1990) is α , because the production function is $Y_t = K_t^{1-\alpha} G_t^{\alpha}$ where G is the public spending. In our model, it is the opposite: the share of public spending in the production function is $1-\alpha$ (see Equations (3.7) and (3.33)) Therefore, to compare with the Barro (1990) model, we have to assume that the τ_{Barro} is $1-\alpha$.

The optimal value of λ is

$$\lambda^* = \frac{\sqrt{1 + 4\alpha \left(1 - \alpha\right)^2 \tau \left(1 + \beta\gamma\right)} - 1}{2\left(1 - \alpha\right)^2 \tau \left(1 + \beta\gamma\right)}.$$
(3.37)

Also in this case the function is concave with respect to the policy instrument λ .³⁵

3.6.1 Sensitivity analysis

As in Section 3.3.2, we implement a sensitivity analysis. We keep the same values for γ , β , α .

 γ , β , α have the same impact on λ and on the GDP per worker at the steady state like in the case of two policy instruments (see 3.3.2). For τ we choose 0.05, 0.4 and 0.7.³⁶ If τ increases, λ decreases. Since h^2 contributes directly to the increase in income, Y, for the government it is more convenient to invest more into labor productivity, to have a higher income to tax.

Parameters and Variables	Values	Values	Values
α	0.33	0.33	0.33
γ	0.2	1.5	4.5
eta	0.67	0.67	0.67
D	14500	14500	14500
au	0.4	0.4	0.4
λ^*	0.31038	0.29803	0.27534
Steady state of the economy	20029	52384	84595

Table 3.6: Sensitivity analysis of γ

 $^{^{35}\}mathrm{See}$ the Appendix.

³⁶We choose also $\tau = 0.7$, because in the previous sensitivity analysis, we obtained that value as the optimal one.

Parameters and Variables	Values	Values	Values
α	0.33	0.33	0.33
γ	4.5	4.5	4.5
β	0.30	0.67	0.96
D	14500	14500	14500
au	0.4	0.4	0.4
λ^*	0.29364	0.27534	0.26362
Steady state of the economy	59887	84595	97542
U U			

Table 3.7: Sensitivity analysis of β

Table 3.8: Sensitivity analysis of α

Parameters and Variables	Values	Values	Values
α	0.33	0.36	0.9
γ	4.5	4.5	4.5
β	0.67	0.67	0.67
D	14500	14500	14500
au	0.4	0.4	0.4
λ^*	0.2753	0.3005	0.3527
Steady state of the economy	84594	125493	308974

Table 3.9: Sensitivity analysis of τ

Parameters and Variables	Values	Values	Values
α	0.33	0.33	0.33
γ	4.5	4.5	4.5
β	0.67	0.67	0.67
D	14500	14500	14500
au	0.05	0.4	0.7
λ^*	0.32072	0.27534	0.25071
Steady state of the economy	5355.1	84595	128552
Steady state of the economy	0000.1	04090	120002

3.7 Conclusions

In this paper, we combine two different meanings of being healthy: living longer and working better. In addition, we point out the importance of health expenditures, that nowadays are at the center of many political debates (see for example The Economist (2017)). Compared to the existing literature that focuses more on the quantity of life, especially at the macro level, we describe the aspect of the quality of life, in particular the health of the labor force. Indeed nowadays, in developed countries where the life expectancy at 65 has increased dramatically (World Bank, 2017), the concept of the quality of life should be taken into consideration by the economic literature. Since the working life expectancy is increasing, being healthy at work and having a good work-life balance is the first step to tackle the challenges on the European labor market (Eurofound, 2017). We set up an OLG framework, in which health makes households live longer, therefore accumulation of savings occurs, and it makes households work better, hence production in the economy increases. To have positive GDP at the steady state both health expenditures are necessary. We show that the government, given an amount of resources, has a trade-off between two types of health expenditures and we find the optimal tax rate and the optimal allocations of its revenues. Besides, we check that changing the survival probability function and the final goods production function, both health expenditures are still necessary and there exists an optimal combination of the allocation of the government revenues and of the tax rate. As shown in the sensitivity analysis (see Tables 3.3, 3.4 and 3.5), the values of λ , the share of the government revenues invested in the first type of health expenditure, i.e. preventive and curative treatment, are low compared to the share of the government revenues invested in the second type of health expenditure. These results do not only emphasize the importance of the health status at work but they should also encourage future theoretical as well as empirical research, to better understand the interactions between different types of health expenditures and their economic consequences. According to us, the next step should be a vintage growth model where the health status of households changes with the passage of time and impacts differently on the economy. Another issue might be the combination of private and public expenditures for the first type of health (preventive and curative treatment). To focus on the trade-off of the two health expenditures, we abstract from these aspects that will characterize a more realistic setting but which will make the model analytically more complicated. We consider these aspects promising suggestions for future research.

3.8 Appendix

The first derivative of Y^{ss} wrt λ is

$$\frac{\partial Y^{ss}}{\partial \lambda} = \left[D\left(1-\alpha\right) \right]^{\frac{1}{1-\alpha}} \tau \left[\frac{\left(1-\alpha\right)\lambda\beta\tau\gamma}{1+\left(1-\alpha\right)\lambda\tau\left(1+\beta\gamma\right)} \left(1-\tau\right) \right]^{\frac{\alpha}{1-\alpha}} * \left[\left(\frac{\alpha}{1-\alpha}\right) \frac{1-\lambda}{\left(\lambda+\left(1-\alpha\right)\lambda^{2}\tau\left(1+\beta\gamma\right)\right)} - 1 \right].$$

With $\frac{\partial Y^{ss}}{\partial \lambda} = 0$, we obtain

$$(1-\alpha)^2 \lambda^2 \tau \left(1+\beta\gamma\right) + \lambda - \alpha = 0.$$

For the rule of the roots (+ + -), the previous equation has two solutions: one positive and one negative. Since $\lambda \in (0, 1)$, we accept only the positive solution:

$$\lambda^* = \frac{\sqrt{1 + 4\alpha \left(1 - \alpha\right)^2 \tau \left(1 + \beta\gamma\right)} - 1}{2\left(1 - \alpha\right)^2 \tau \left(1 + \beta\gamma\right)}.$$

The second derivative is

$$\frac{\partial^2 Y^{ss}}{\partial \lambda^2} = \left[D\left(1-\alpha\right) \right]^{\frac{1}{1-\alpha}} \tau \left[\frac{\left(1-\alpha\right)\lambda\beta\tau\gamma}{1+\left(1-\alpha\right)\lambda\tau\left(1+\beta\gamma\right)} \left(1-\tau\right) \right]^{\frac{\alpha}{1-\alpha}} *$$

$$*\left[\left(\frac{\alpha}{1-\alpha}\right)^2 \frac{1-\lambda}{\left[\lambda+(1-\alpha)\,\lambda^2\tau\,(1+\beta\gamma)\right]^2} - \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{\left[\lambda+(1-\alpha)\,\lambda^2\tau\,(1+\beta\gamma)\right]} + \right]$$

$$+\frac{\alpha}{1-\alpha}\frac{-\left[\lambda+\left(1-\alpha\right)\lambda^{2}\tau\left(1+\beta\gamma\right)\right]-\left[1+2\left(1-\alpha\right)\lambda\tau\left(1+\beta\gamma\right)\right]\left(1-\lambda\right)}{\left[\lambda+\left(1-\alpha\right)\lambda^{2}\tau\left(1+\beta\gamma\right)\right]^{2}}\right]$$

The first line is positive, hence it does not change the sign of the derivative. In the

squared parenthesis the first term is positive and the other two are negative.

$$\begin{split} & \text{If } \mid \left(\frac{\alpha}{1-\alpha}\right)^2 \frac{1-\lambda}{\left[\lambda+(1-\alpha)\lambda^2\tau(1+\beta\gamma)\right]^2} \mid <\mid -\left(\frac{\alpha}{1-\alpha}\right) \frac{1}{\left[\lambda+(1-\alpha)\lambda^2\tau(1+\beta\gamma)\right]} + \\ & \frac{\alpha}{1-\alpha} \frac{-\left[\lambda+(1-\alpha)\lambda^2\tau(1+\beta\gamma)\right] - \left[1+2(1-\alpha)\lambda\tau(1+\beta\gamma)\right](1-\lambda)}{\left[\lambda+(1-\alpha)\lambda^2\tau(1+\beta\gamma)\right]^2} \mid, \text{ then the derivative is negative and the} \end{split}$$

function is concave.

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