# Accessible Mathematics on Touchscreen Devices: New Opportunities for People with Visual Impairments

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**Abstract**

In recent years educational applications for touchscreen devices (e.g., tablets) become widespread all over the world. While these devices are accessible to people with visual impairments, educational applications to support learning of STEM subjects are often not accessible to visually impaired people due to inaccessible graphics.

This contribution addresses the problem of conveying graphics to visual impaired users. Two approaches are taken into account: audio icons and image sonification. In order to evaluate the applicability of these approaches, we report our experience in the development of two didactic applications for touchscreen devices, specifically designed to support people with visual impairments or blindness while studying STEM subjects: *Math Melodies* and *Audio Functions*. The former is a commercial application to support children in primary school in an inclusive class. It adopts an interaction paradigm based on audio icons. The latter is a prototype application aimed at enabling visually impaired students to explore function diagrams and adopts an image sonification approach.

## 1 Introduction

Starting from the seventies, electronic devices became progressively more accessible to people with disabilities. Currently, screen readers allow blind people or people with severe visual impairments to interact with the OS (e.g., to install and run applications) and with many applications, even those not specifically designed for universal access. For example, blind people can navigate within applications screens, push buttons and access textual information.

While access to textual information is of paramount importance for many activities, for educational purposes there is also the need to access and interact with graphical information, including charts, graphs or drawings. This is particular relevant when mathematics or other STEM subjects are concerned.

Two main approaches have been adopted in the literature to tackle this problem: "image sonification techniques"[1] are aimed to map the graphical representation of the image into sound. With this approach a trained user is expected to recognize the image (or at least some of its properties) by listening to the image sonification. With the "audio icons" approach, the semantics of an image is described through audio, in the form of a textual description, a well-known sound (e.g., a dog's call) or a synthetic sound (e.g., a pure note). A "mixed approach" is also possible, for example by allowing the user to explore an image with a sonification approach and convey some parts of the image with an audio icons approach.

In this contribution we report our experience in the development and evaluation of two applications, one adopting the audio icon approach, the other adopting a mixed approach. *Math Melodies* is an iPad application to support math learning of visually impaired students in primary school. A study, described in this contribution and conducted in the early design

[1]Henceforth "sonification technique" or "sonification"

phase of this app, highlights the need for an approach based on audio icons. *Math Melodies* was first developed as a university research project and then engineered as a commercial product[2]. The other application, *Audio Functions*, allows users to type the equation of a mathematical function and then explore the corresponding chart. Exploration is enabled by a combination of sonification and audio icons.

Note that the two applications have been designed for mobile devices and get advantage from the touchscreen. There are two main advantages that arise from the interaction with the touchscreen. First, proprioception enhances screen exploration, for example providing information that is specific to the current area being explored (i.e., touched). Second, the fact that information is associated to specific locations on the touchscreen allows the user to develop visuospatial skills, that are generally hard to acquire by people with visual impairments.

The following of this paper is organized as follows. Section 2 introduces the background, in terms of assistive tools (both physical and electronic) and research results in the literature. Sections 3 and 4 present *Math Melodies*, and *Audio Functions*, respectively, with focus on the design challenges and evaluation results. Section 5 concludes the paper.

# 2 Background

## 2.1 Physical Tools to Support Learning of STEM subjects

Over the years many different assistive tools have been adopted by visually impaired students to better access graphical information, often used in STEM subjects. Some solutions include tactile drawings, either drawn by pen on plastic sheets or printed, for example by a tactile embosser or on swell paper (see Figure 1(a)).

While these solutions are widespread, they have some drawbacks. First, direct interaction is not possible with printed graphics (i.e., embossed or on swell paper) and it is complicated with plastic sheets, where very good manual skills are required. This is a problem, for example when a student is given a tactile graph of a function and has to draw the symmetric function on the same drawing. The second problem concerns tactile labels (e.g., Braille labels) that often overlap with the drawing itself or with other labels hence posing readability issues. Third, tactile drawings are bulky and heavy. Fourth, printed drawings require specialized hardware, that is often expensive. Finally, when drawings on plastic sheets are adopted, is often necessary that a supervisor hand-draws the image.

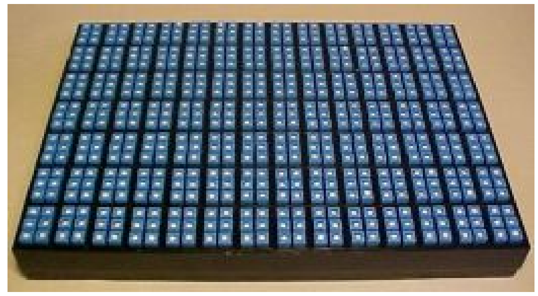
One more tool that is widely used by blind children in doing arithmetic operations is the arithmetic board (Figure 1(b)). This tool enables blind students to arrange digits in a two-dimensional layout to the purpose of solving arithmetic operations. While this solution enables a higher level of interaction, this still requires good manual skills. Also, this device cannot permanently store its content.

## 2.2 State of the Art in Scientific Literature

The problem of enabling sight impaired people to understand graphical information through auditory representations has been widely studied in scientific literature. Sanz et al. [10] and Sarkar et al. [11] present comprehensive surveys of sonification systems used to represent visual scenes and bi-dimensional images.

[2] *Math Melodies* is available for free download from the Apple Store `https://itunes.apple.com/us/app/math-melodies/id713705958?mt=8`

(a) Tactile drawing.



(b) Arithmetic board.

Figure 1: Physical accessibility tools.

We can classify existing sonification approaches into two classes, based on the kind of images they aim to represent: generic images (e.g., frames captured by a camera, pictures, etc.) or simplified graphics (e.g., geometric shapes and function diagrams).

For what concerns the former class (i.e., sonification of generic images), Yeo and Berger [13] created a framework for designing image sonification methods, categorizing various aspects of the sonification process. The authors point out the difference between two methods to organize data for auditory display: *Scanning* and *Probing*. In the *Scanning* method, image data is scheduled to be sonified in a predefined order. Differently, in the *Probing* method, the user can interactively change the portion of the image to be sonified.

Dallas and Erickson [4] propose a scanning technique to convert an image into sound by mapping the vertical position of each pixel to frequency, the horizontal position to time and brightness to loudness. This technique is adopted in the vOICe project [9] that aims at enabling sight impaired persons to explore frames captured through a camera. A conceptually similar solution proposed by Abboud et al. is called *EyeMusic* [1].

Wortwein et al. [12] present a prototype application that combines multiple sonification techniques to explore images on the web. The sight impaired person can choose the most suitable technique for the exploration of an image found while browsing a web page.

In the latter class (i.e., sonification of simplified graphics) Buzzi et al. [3] present a prototype application for recognizing common geometric shapes or topological configurations through sound and vibro-tactile feedback on Android mobile devices.

Many different solutions have been investigated to represent function diagrams through sound. Gardner et al. [5] introduce Audio Graphic Calculator. It is a desktop application that describes a function diagram through a simple sound. Walker et al. [2] introduce Sonification Sandbox. It is a desktop application that describes a function diagram through a synthetic sound. Both of these applications do not provide quantitative information about the function.

Finally, Yoshida [14] investigates a method for exploring images on a touchscreen device through sound. Two sonification modes are designed: local area sonification and distance-to-edge sonification. In the first mode, when the person slides the finger over an edge, a sound representing the line is played. In the second mode, a pulse train signal is used to represent the finger's distance to the closest edge.

All of these techniques present some drawbacks. In particular, as highlighted by S. Maidenbaum, a long and arduous training is necessary to become proficient in exploring generic images

[8]. When simplified graphics are concerned, Gerino at al. have shown that the learning time can be much smaller [6] but that image exploration is still a cognitively demanding challenge even for relatively simple figures, like, polygons.

### 2.3 Available Commercial Software

In recent years, a great deal of commercial applications for touchscreen devices have been developed to support learning from primary school to university. For example, more than $80,000$ educational applications are available for iOS devices[3], many of which are specific for STEM subjects. However, most of these applications are not accessible to students with visual impairments due to the use of inaccessible graphical elements to represent scientific notation, visual interaction paradigms and animations often used to attract the interest of sighted students.

Among the few applications accessible to students with visual impairments, some have been specifically designed addressing their specific needs, while others are accessible thanks to the default system screenreader.

In the former category, there are, for example, Math Robot for iPad [4] and UAbacus for iPad [5]. These applications enable students with visual impairments to practice with basic arithmetic operations in an inclusive class.

In the latter category there is for example iLearn Math for iPad [6], a game for learning arithmetic operations. Some of the exercises are accessible through sound feedback and the magnifier.

A comprehensive list of accessible applications can be found in the ViA app [7] by the Braille Institute.

## 3 Math Melodies

*Math Melodies* is an application for iPad that includes mathematical exercises addressed to children in primary school. The exercises are presented in a narrative context that aims at engaging and motivating the child in practicing maths. This application was developed according to an inclusive design paradigm that enables both sighted, visually impaired and blind students to practice exercises in the same work environment. In the sequel, at first the main design challenges are introduced, then the evaluation results are discussed.

### 3.1 Application design

In the process of designing and developing *Math Melodies*, three main challenges were tackled. First, the application has to present maths exercises that are accessible to both sighted and blind or visually impaired students. Accessibility to sighted students is guaranteed by graphics and “point and tap” gestures. Also, a special interaction paradigm was designed for students with sight impairments. A prototype implementing both a sonification approach and an audio icon approach was first developed with the aim of evaluating which one is more suitable to present maths exercises to young children in primary school. The evaluation (see Section 3.2 highlights that the interaction paradigm based on audio icons is less cognitively demanding and more enjoyable.

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[3] Source: `http://www.apple.com/education/ipad/apps-books-and-more/`

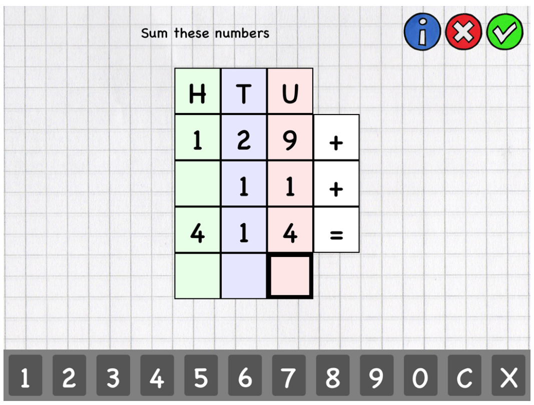
[4] `https://itunes.apple.com/app/math-robot/id704570512`

[5] `https://itunes.apple.com/us/app/uabacus/id688547692?mt=8`

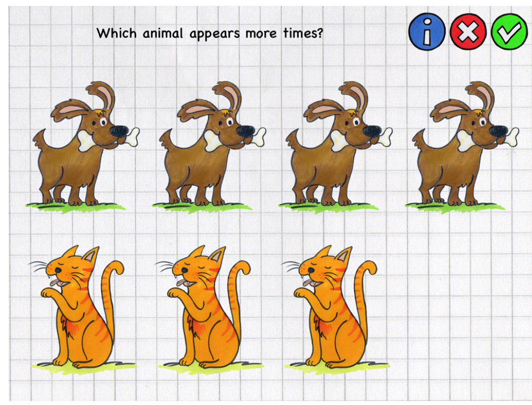
[6] `https://itunes.apple.com/it/app/ilearn-math-lite/id583642155?mt=8`

[7] `https://itunes.apple.com/it/app/via-by-braille-institute/id528499232?mt=8`

According to these results, we designed a set of 17 different types of exercises relying on the audio icons approach. In order to further simplify the interaction model, we decided to organize the objects into a grid layout that, as we observed in our evaluation, helps reducing the time and mental workload required to explore the entire screen. Another choice driven by the need of simplifying the interaction consisted in the definition of two input techniques: a simplified on-screen keyboard to insert the digits only (e.g., for the addition exercises, see Figure 2(a)) and a multiple choice dialog (e.g., to answer an exercise like the one shown in Figure 2(b)).




(a) Addition exercise with simplified keyboard.




(b) Counting exercise.

Figure 2: Two exercises of *Math Melodies*.

The second design challenge concerns taking advantage of reinforcement learning. Children should be stimulated to repeat exercises many times hence correcting their mistakes and learning by reinforcement. To address this challenge, we designed the exercises to have up to 6 difficulty levels. Each exercise is randomly generated according to its type and the difficulty level set. For example, in the "easy" addition exercise the child is asked to add two single-digit numbers, while at an harder level (designed for 3rd grade students) the aim is to add three numbers, each one with up to three digits as in Figure 2(a).

The third design challenge is to motivate children to keep on doing exercises and practicing. To this purpose, the exercises are immersed in a tale, divided into chapters, organized in increasing difficulty levels (two chapters for each grade). Each chapter is further divided into "pages", each one comprising a background image, some text (read by a speech synthesizer) and some audio-icons often associated with an enjoyable sound (see Figure 3). Pages are intertwined with the exercises and there are about 30 exercises in each chapter. Every time each exercise is completed correctly, a short piece of melody is played as a reward for the student. At the end of each chapter, all the pieces of melody are played together to form a full melody.

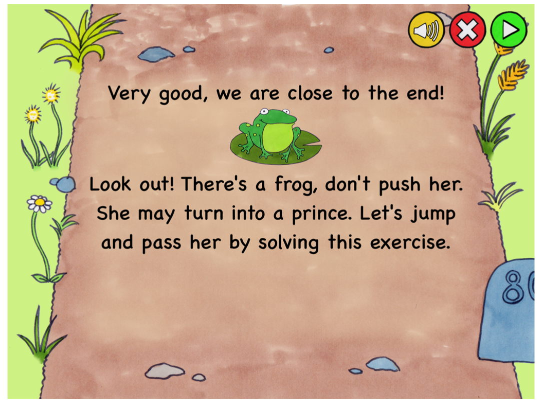
## 3.2 Design methodology and evaluation results

The whole design and development process took benefit from continuous evaluation. Indeed, each design choice was assessed by one of the designers, who is an expert in technologies for education of blind persons. In addition, three main evaluation sessions were organized.

In the first session, four teachers expert in education for blind students were involved[8]. A

[8]Teachers are from the center for the blind people in Brescia, Italy.


(a) A page with a piano audio-icon.




(b) A page with a frog audio-icon.

Figure 3: Two pages of *Math Melodies* story.

prototype implementing exercises based on both sonification and audio icons approaches was presented to the teachers. Then, teachers were asked to assess which approach is more suitable, in particular taking into account a primary school student's needs and abilities. All four teachers agreed that the approach based on audio icons is more suitable for solving maths exercises, while the approach based on sonification is likely to be too demanding for sight impaired children in primary school.

The second session was conducted as a test with three blind children that were asked to use the same prototype adopted for the first session. After a 10 minutes training with the prototype, each child was required to solve eight exercises, four of which adopt the sonification approach, while the others adopt the audio icons approach. Finally students were asked to leave comments on the prototype and all of them reported that the exercises adopting audio icons are easier to understand. Also, two students remarked that exercises with the audio icon approach are also more entertaining.

After the second evaluation session a more advanced prototype was developed that included both the story and the exercises. This prototype was used in the third evaluation session that was conducted with three blind and two sighted students in primary school. The five students were asked to complete all the exercises in the first chapter consisting in counting exercises, sums, etc. All blind children were enthusiast while using the application. Two out of the three blind ones reported that they were entertained and engaged especially by the sounds (e.g. the call of animals and the rewarding melodies. All blind children experienced some difficulties in the early exploration of tables, and needed help by a sighted supervisor. However, after at most 2 minutes of supervised training, all children got familiar with the application and were able to solve the exercises autonomously and, most of the times, providing a correct answer at the first attempt.

The two sighted children enjoyed the application as well. One of the two children initially experienced some difficulties in understanding how to answer. This was partially due to the fact that the child didn't pay much attention to the exercise explanation. After a brief explanation about how to answer, no more help was needed.

*Math Melodies* has been available on the AppStore [9] for about two years. So far, *Math*

[9] `https://itunes.apple.com/us/app/math-melodies/id713705958?mt=8`

*Melodies* has been incrementally improved according to the continuous feedback provided by many users all over the world. In particular, many suggestions of improvement have been received from educators for blind and sight impaired students and were implemented in recent releases. For example, *Math Melodies* has been adapted to use standard Voice Over gestures. This leads to two advantages: first, users that are already acquainted with Voice Over can start using *Math Melodies* with minimum effort; second, users that are not familiar with Voice Over can learn its basic gestures that can then be used to interact with the OS and most of the other applications.

# 4 Audio Functions

*Audio Functions* is an iPad prototype application that enables blind and visually impaired students to explore function diagrams through sound. The students can independently insert the function equation though a specialized keyboard.

## 4.1 Function exploration

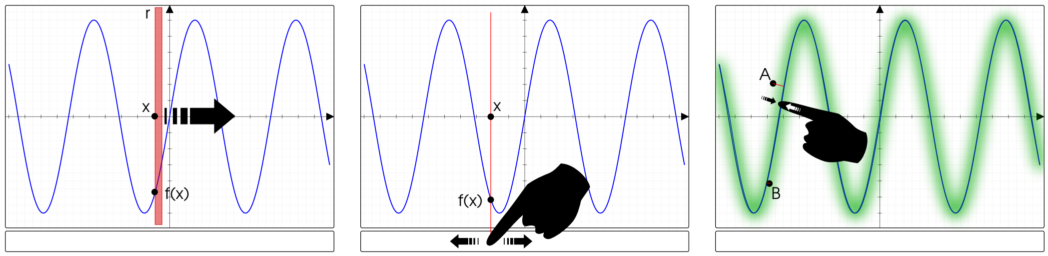
*Audio Functions* allows the students to explore the function diagram with a combination of sonification and audio icons. Sonification is mainly used to convey the function shape while different audio icons are used to convey additional information, including quantitative information (e.g., the pair $\langle x, f(x) \rangle$) and relevant points, like maximum and minimum, for example.

*Audio Functions* adopts three "exploration modes" (see Figure 4) to describe the function shape through sonification. The first one, called "non interactive", is similar to the solution adopted in Audio Graph Calculator and Sonification Sandbox ([5]): by using a "two finger double tap" gesture *Audio Functions* starts/stops a function sonification working as follows. The function domain is divided into small intervals and, for each interval, *Audio Functions* reproduces a sound whose pitch is proportional to the function value in that interval.

With the second exploration mode, called "mono-dimensional interactive" (Figure 4(b)), the student can slide the finger along an horizontal line that represents the abscissa axis. While sliding the finger, *Audio Functions* reproduces a sound whose pitch is proportional to the value of $f(x)$ where $x$ is the point being touched. An advantage of this exploration mode is that, thanks to proprioception, the student is aware of the position of the finger along the abscissa. Therefore, he/she can understand the relation between $x$ and $f(x)$. Furthermore, the student can move forward and backward along the $x$ axis, at the desired speed, hence, for example, focusing on maximum and minimum points.

The third exploration mode is called "bi-dimensional interactive". It enables the student to follow the line of the function diagram with one finger thanks to sonification. When the student's finger is on the function line, *Audio Functions* reproduces a sound with the highest pitch. When the user touches outside the line, the pitch diminishes as the the distance between the finger and the line increases.

In order to provide additional information to the student, *Audio Functions* also provides two forms of audio icons. First, while exploring, *Audio Functions* reproduces some additional sounds at "interesting points", like intersections with the axis, local minimum and maximum and changes in concavity. Second, by double tapping, *Audio Functions* reads details on the current position, including: the values of $x$ and $f(x)$ and the function concavity in that point.



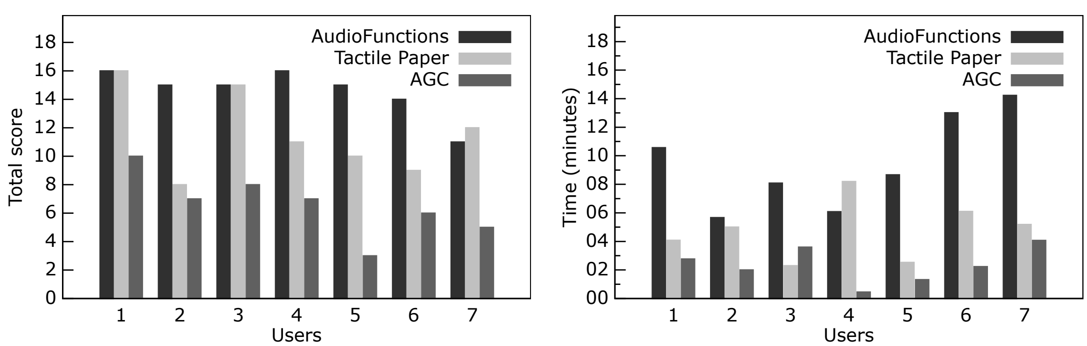

(a) Non interactive (b) Mono-dimensional interactive (c) Bi-dimensional interactive

Figure 4: Function exploration screen and exploration modes

## 4.2 Evaluation methodology and results

The evaluation of *Audio Functions* was conducted to the purpose of assessing how easily and precisely the function diagram is understood. It was conducted with 7 blind students, all with secondary school education in mathematics and all familiar with tactile drawings. Each student received a five minutes training, afterwards he/she was required to explore three different function diagrams, each one with a different tool: *Audio Functions*, Audio Graphic Calculator ([5]) and a tactile drawing. Note that all function diagrams have similar properties and that the tools are used in a random order.

After the exploration, each student was asked to answer 8 questions about the features of the function scored as follows: 0 (totally wrong answer or no answer), 1 (partially correct answer) or 2 (correct answer). Figure 5(a) shows, for each student and technique, the sum of the scores obtained in all the questions. This metric aims at measuring the understanding of the function properties by each user through different techniques. We can observe that every user obtained much better results by using *Audio Functions* with respect to Audio Graphic Calculator. *Audio Functions* also proved to be more effective also compared with tactile drawings that, we recall, all users were acquainted with. Indeed, every user, except user 7, obtained better results with *Audio Functions* than with tactile drawing and for most of the users the results with *Audio Functions* are much better than those with tactile drawings (e.g., users 2, 4, 5 and 6).




(a) Function understanding (b) Total time to answer

Figure 5: Results of the experimental testing.

Figure 5(b) compares the total time required by each user to answer the 8 questions by using each technique. Results show that, by using AGC, users provided answers more quickly (about 2 minutes on average) than with tactile drawings (about 5 minutes on average) and *Audio Functions* (about 9 minutes on average).

# 5 Conclusions and Future Work

This contribution reported our experience in the design of two applications aiming at supporting visually impaired students in learning STEM subjects: *Math Melodies* and *Audio Functions*. Supported by evaluation results obtained with preliminary prototypes, we decided to design the interaction of *Math Melodies* based on audio icons. We can observe that, in the case of *Math Melodies*, there is no need to describe the image appearance; rather, the object semantics as a whole needs to be conveyed. For example, while solving a counting exercise (see Figure 2(b)), the student doesn't need to know how the dog is drawn; as long as the student can distinguish the dog from the cat, the actual aspect of the two icons is irrelevant. Hence, since there are few pre-defined icons, it is easy to define a characterizing sound for each of them.

Vice versa, the design of *Audio Functions* is mainly based on a sonification approach. This is due to the fact that the student is interested in exploring the internal structure of the graphical object (i.e., the function chart). Still, there are some elements that can be pre-defined including the function relevant points and local properties. These elements are indeed represented with audio icons.

There are many research directions that need to be investigated. First, a methodological limitation of many research efforts conducted in the field of assistive technologies is that they rely on supervised empirical evaluations. Since it is often impractical to conduct on-site evaluations with many subjects with disabilities, most of the contributions presented in the last years are evaluated with few subjects only, often in the order of units or tens, at most. Statistically significant results can be hardly obtained. To address this issue, we are currently working on a methodology to conduct unsupervised and remote evaluations, by allowing the tested subject to autonomously use an application and by remotely collecting usage analytics. There are a number of challenges to address with this approach, in particular evaluating the reliability of the collected data. Also, this approach require to tackle a number of engineering issues that need to be addressed in order to publicly release the application. This include, for example, making the application sufficiently stable, localizing it for different languages and cultures, supporting different device hardware and OS versions and maintaining it over time. Even more important, there are two additional design challenges: first, to have a self explaining application and, second, to engage the user in using the application so that a sufficient amount of data can be collected. Some of these issues have been addressed in our previous work [6, 7].

A second research direction is to investigate how to improve interactivity in both *Math Melodies* and *Audio Functions*. For example, as a future work, *Math Melodies* could include exercises that require to move objects and *Audio Functions* could include a functionality to zoom and move in the chart or other ones to draw on a chart and insert annotations. The challenge is to define an interaction technique which provides the user a clear feedback about what it currently being touched inside a graphical representation that is being dynamically altered.

A third research direction consists in exploring how the different sonification techniques proposed in the literature can be combined so that the user can select the most suitable one for a given image. Possibly, the user can switch among different sonification techniques while exploring a single image, for example by first using a scanning sonification technique to quickly grasp a global overview of the image and then using a probing technique to explore the image details. This is partially what we have started investigating in a preliminary work [6].

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