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Letter to the editors

Comments on 'Pleistocene deglaciation and the Earth's rotation: a new analysis', by P. Wu and W. R. Peltier

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The problem dealing with the effects of the Pleistocene ice-age cycles on the Earth's rotational motions is one which has been receiving an increasing amount of attention in the literature (Nakiboglu & Lambeck 1980; Sabadini & Peltier 1981; Peltier 1982; Sabadini, Yuen & Boschi 1982a, b; Yuen, Sabadini & Boschi 1982; Peltier & Wu 1983; Wu & Peltier 1984; Sabadini, Yuen & Boschi 1984). In a recent contribution by Wu & Peltier (1984) it was asserted that the mathematical formation used by Sabadini *et al.* (1982a, b) for calculating the polar speed averaged over glacial cycles is wrong. Additionally, they stated that the net shift of the pole for the last 2 Myr of glacial forcing, obtained by Sabadini *et al.* (1982b), is at least an order of magnitude greater than their own result.

It is our intention here to show that the expression used by Wu & Peltier for calculating the averaged polar velocity is mathematically equivalent to the same equation employed by Sabadini *et al.* (1982a, b). At the same time we also wish to draw attention to the fact that the difference in the net polar speed between the model of Sabadini *et al.* (1982b) and Wu & Peltier is a consequence of increasing layering in the density structure.

It is easy to demonstrate that equations (84) of Wu & Peltier are the same as equation (85) of Sabadini *et al.* (1982a). We begin with equations (67) and (73) of Wu & Peltier. From these two equations one can derive an expression for the product polynomial

$$\prod_{i=1}^{N-1} \lambda_i,$$

where λ_i's are the inverse relaxation times of the rotational spectrum, in terms of the residues of the tidal Love number t_i and the inverse isostatic relaxation times s_i. The total number of modes is denoted by N. Setting s to zero in equation (73) of Wu & Peltier, we find that

$$\prod_{i=1}^{N-1} \lambda_i = \left(\sum_{i=1}^{N} \frac{t_i}{s_i}\right)^{-1} \sum_{j=1}^{N} \frac{t_j}{s_j^2} \prod_{i=1}^{N} s_i. \tag{1}$$

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Substitution of equation (1) into equation (85) of Wu & Peltier yields the following expression for D_2

$$D_2 = l_s \left(\sum_{j=1}^{N} \frac{t_i}{s_j^2}\right)^{-1} \left(\sum_{i=1}^{N} \frac{t_i}{s_i}\right) \tag{2}$$

where l_s is the isostatic relaxation factor associated with the lithosphere (Sabadini & Peltier 1981). The sum involving t_i/s_i can be recast from equation (68) of Wu & Peltier as

$$\sum_{j=1}^{N} \frac{t_j}{s_j} = \frac{\sigma_0}{\sigma_r} k_f = \frac{k_f \sigma_0 A}{\Omega (C - A)} \tag{3}$$

where σ_0 and σ_r are respectively the Chandler wobble frequencies of a homogeneous visco-elastic and a rigid earth, k_f is the tidal fluid Love number, A and C are the principal moments of inertia and the diurnal rotational velocity is given by Ω. By means of equation (5.3.2) of Munk & MacDonald (1960) this expression is simplified to

$$\sum_{j=1}^{N} \frac{t_j}{s_j} = \frac{3 G \sigma_0 A}{a^5 \Omega^3} \tag{4}$$

where G is the universal gravitational constant.

Next we substitute equations (2) and (4) into equation (84) of Wu & Peltier, which is the averaged velocity of polar wandering in the large time regime. We find that

$$\langle \dot{\bar{m}}_j \rangle = \frac{1}{2} \frac{l_s}{\sum_{i=1}^{N} (t_i/s_i^2)} \frac{3G}{a^5 \Omega^2} I_{j_3}^{R} \tag{5}$$

where $I_{j_3}^{R}$ is the inertia perturbation due to the direct effect of the load. This component $I_{j_3}^{R}$ may be recast within the formulation of Sabadini *et al.* (1982a). For $j = 1$, this is given by

$$I_{j^3}^{R} = I_{13} = L_2 l_s \cos \theta_0 \sin \theta_0 \tag{6}$$

where θ_0 is the colatitude of the centre of the ice cap, L_2 is the $l = 2$ component of the surface load, given by equation (20) of Sabadini *et al.* (1982a). The perturbation of inertia I_{13}, given in (6) has been calculated in a coordinate system in which the polar axis goes through the ice cap, whereas $I_{j_3}^{R}$ has been calculated in a geographical coordinate system.

From (3), (5) and (6) it is quite clear that (5), derivable from equation (84) of Wu & Peltier, is, in fact, identical to equation (85) of Sabadini *et al.* (1982a). Our earlier expression for the steady drift velocity is given by

$$\bar{V} = \frac{k_f G_1}{2} \tag{7}$$

where

$$G_1 = \frac{3 G L_2 l_s \cos \theta_0 \sin \theta_0}{\Omega^2 a^3 \sum_{j=1}^{N} (k_i/s_i^2)}. \tag{8}$$

where k_i's are equivalent to t_i's in (5).

We have thus shown that there is no difference in the mathematical formulation between our work (Sabadini *et al.* 1982a) and that of Wu & Peltier, in so far as the averaged polar speed is concerned.

The second point we wish to address concerns the relative magnitudes of the net polar drift accumulated after 2 Myr of periodic glacial forcing, as predicted by the three-layer model of Sabadini *et al.* (1982a, b) and by Wu & Peltier (1984). The basic difference in the predictions of the net polar speed arises from the increasing number of internal density discontinuities in the model, as has already been discussed by Sabadini, Yuen & Boschi (1983). In Fig. 1(a) we display both the three- and four-layer models (Sabadini *et al.* 1982; Yuen *et al.* 1982). The time histories of the averaged velocity, wherein the transient portions of the net drift have also been included (see equations 44–48 of Sabadini *et al.* 1984). The main difference in the two models lies in the presence of a density jump of 0.38 g cm^{-3} at a depth of 670 km in the four-layer model (Yuen *et al.* 1982). The five-layer model of Wu & Peltier contains density jumps of 0.14 g cm^{-3} at a depth of 420 km and 0.27 g cm^{-3} at a depth of 670 km. One observes that initially the averaged velocity is higher for the four-layer model. After about 1 Myr this then decreases to about 0.2 Myr^{-1}, which is about four times smaller than the net drift rate predicted by the three-layer model. It is to be emphasized that in the three-layer model the transient contributions to the long-term drift velocity are very small in comparison to that derived by the four-layer model (Sabadini *et al.* 1984).

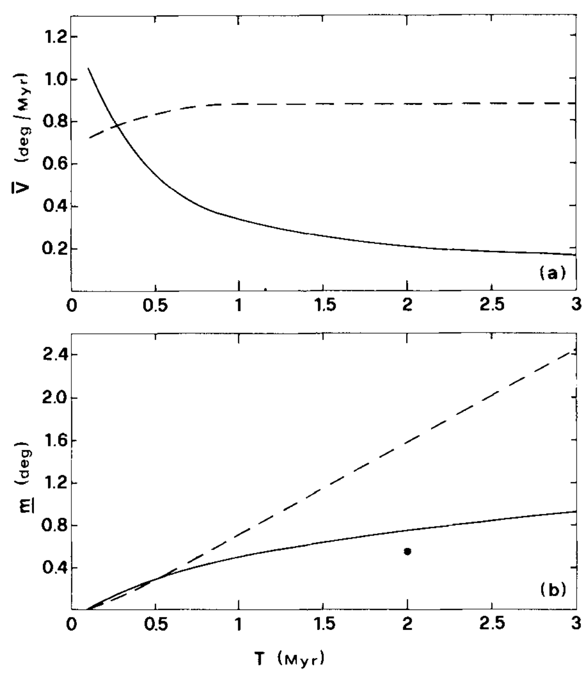

Figure 1. (a) Averaged polar velocity as a function of time for three-layer (dashed curve) and four-layer (solid curve) models. The forcing function is a saw-toothed function with a period of 10^5 yr (Sabadini & Peltier 1981). The amount of equivalent melt water distributed globally is 70 m (Sabadini *et al.* 1982a). The viscosity of the mantle is uniform at 10^{22} P. The physical parameters of the three- and four-layer models are taken from Sabadini *et al.* (1985). In the four-layer model the densities of the lithosphere and upper mantle are the same at 4.18 g cm^{-3}. (b) Net displacement as a function of time for the models displayed in (a). The filled circle is taken from Wu & Peltier's uniform viscosity result (fig. 9 of Wu & Peltier). Other parameters are the same as from (a).

In Fig. 1(b) we plot the total net displacement for the three- and four-layer models and also that derived by Wu & Peltier for a time of 2 Myr (filled circle). As can be observed, the displacement predicted by the three-layer model is only about three times greater than that of Wu & Peltier. The difference between the above value and the 'order of magnitude' quoted by Wu & Peltier is due in part to the lingering presence of the transient terms during the first 2 Myr of continual forcing. On the other hand, the difference in the net displacement between our four-layer model and the five-layer model of Wu & Peltier is only around 30 per cent after 2 Myr.

Finally, we note that the asymptotic drift rate of the five-layer model of Wu & Peltier is about five times smaller than that corresponding to our four-layer model. This reduction, however, may be due to the modelling of the density variation in the vicinity of 400 km depth as a sharp density discontinuity by Wu & Peltier and also to the presence of a non-adiabatic boundary (Fjeldskaar & Cathles 1984). The exact nature of this density change is not as well constrained as the one at 670 km depth, since it may involve both divariant phase transformations consisting of both olivine and a considerable amount of pyroxene (Liu 1979; Sawamoto *et al.* 1984) and also possibly a change in composition (Anderson & Bass 1985). It may then be a transitional type of density change.

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