UNIVERSITÁ DEGLI STUDI DI MILANO

Lombardy Advanced School of Economic Research (LASER)

Department of Economics, Management and Quantitative Methods (DEMM)

Ph.D. COURSE IN ECONOMICS
XXVII cycle

ESSAYS IN HEALTH ECONOMICS AND INDUSTRIAL ORGANIZATION

SECS-P/03

Ph.D. CANDIDATE: Giovanni CREA

SUPERVISOR: PROF. Alberto CAVALIERE

COORDINATOR: PROF. Paolo GARELLA

A.Y. 2013-2014

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Introduction

Industrial organization focuses on imperfectly competitive markets to understand the behavior of firms and the resulting welfare effects. This is a broad definition as most markets are imperfectly competitive and industrial organization research can then focus on a wide variety of topics. Imperfect competition may be due to many reason. Perfect competition in fact requires: a large number of firms and consumers, free entry and exit, marketability of all goods and service including risk, symmetric information with zero search cost. Moreover the list includes no increasing returns, no externalities, and no collusion. Health care markets are a good example for imperfect competition as generally they violate all requirements included in the previous list.

If we focus only on some violation like asymmetric information and no marketability, then health care markets fail in a more clear way than other markets. This justifies the often made claim that the health care market is "different" and implies that any evaluation of its performance must be based on models that explicitly take into account its deviations from the assumption required for perfect competition. The model of perfect competition can still serve as the benchmark of optimal performance, but generally it cannot be used to analyze how health care markets work.

For this reason the common thread of this thesis is to analyze health care markets using the theoretical and empirical tools provided by industrial organization. This thesis is composed by three essays.

In the first one I am going to propose a theoretical framework to analyze product differentiation withconsumers misperception and information disparities. The model is an extension of standard vertical product differentiation (Gabsewictz and Thisse, 1979 and Shaked and Sutton, 1982), where I relax the assumption of perfect information. As I said before asymmetric information is one of the big problem to deal with in health economics. And if products are credence goods, as in case of drugs, many consumers may lack the expertise to ascertain the quality differential with respect to cheaper standard brands, even after purchase. In that case consumers face a risky decision and to the extent they lack information about the true quality differential they may carry out purchase decision according to misperceptions about product quality.

In this paper I extend the analysis of Cavaliere (2005) to include the quality choice by firms, when providing higher quality requires a costly effort, and propose to analyse the case of a duopoly with vertically differentiated products with consumers' misperceptions and information disparities. Consumers are actually split between uninformed and informed consumers. Uninformed consumers are characterized by consumers' misperceptions as they can underestimate or overestimate the quality differential.

As a minimum quality standard is imposed by the Government even uninformed consumers expect that any product sold in the market at least complies with the standard.

As low quality can be said to be verifiable, even uninformed consumers can be confident about low quality products: firms are expected to provide at least the minimum quality standard. This last assumption well fits the case of pharmaceutical products. Actually every developed country has a national institution that enforces and verifies drug's minimum quality standard. The aim of this paper is to shed light on how firm set price and quality when consumers are characterized by asymmetric information and mispercemption obout quality.

We do not analyze information decisions by consumers, which are exogenously given, therefore firms follow a Stackelber behavior vis à vis consumers. However we can analyze quality and price competition between firms for the full range of information disparities, i.e. for any split between informed and uninformed consumers that can affect demand functions. Furthermore we distinguish between the case of optimistic misperceptions (uninformed consumers overestimate the quality differential) and the case of pessimistic consumers (uninformed consumers underestimate the quality differential). Competition between firms is represented by a two stage game, in the first stage the two firms compete in qualities, given the market split between informed and uninformed consumers. In the second stage price competition takes place.

We will show that both price and quality are strictly depend on asymmetric information as expectation and number of informed consumer affect firm's choice. For different quality expectations and share of informed consumer we found market failure. In some cases uninformed consumers are cheated by high quality firm when they purchase high quality product, in other cases, for different information level and expectations, adverse selection arises endogenously in the model.

The second paper consists in a theoretical model where I analyse incentives for cooperative behaviour when heterogeneous health care providers are faced with regulated prices under yardstick competition. Providers are heterogeneous in the degree to which their interests match to those of the regulator.

The basic idea behind yardstick competition is that the price (or price cap) faced by each provider is dependent on the actions of all the other providers (Schleifer, 1985; Laffont and Tirole, 1993). According to Schleifer's rule, the price each provider faces is based on the costs of all other providers in the industry but not its own. This creates strong incentives for cost control.

When there is a large number of providers, this is unlikely to be a problem, mainly because the cost of collusion rises, but even in larger countries, provision might be concentrated among a handful of providers, as is likely for utilities, rail or postal services and for specialist health services, such as bone marrow or lung transplantation.

The innovation with respect to the standard model of yardstick competition is the introduction of heterogeneity in the degree to which the provider's interests correspond to those of the regulator. Because the incentive to collude with other providers will depend on the objectives of the providers, particularly the extent to which their objectives correspond with those of the price-setting regulator. We use "altruism" to describe the behavior of providers whose aims are closely related to those of the regulator and "self-interested" to describe providers whose interests are more divergent from those of the regulator.

If we consider the different ownership types in health services this heterogeneity in "altruism" is evident since we observe full public ownership i.e. altruistic providers and full private hospital i.e. self-interested providers.

This paper aims then to analyse incentives for collusive behaviour when heterogeneous providers are faced with regulated prices under yardstick competition. We analyse the choice of cost when providers do not collude and when they do, and we consider incentives to defect from the collusion agreement

Our results suggest that under the yardstick competition each provider's choice of cooperative cost is decreasing in the degree of the other provider's altruism, so a self-interested provider will operate at a lower cost than an altruistic provider. The prospect of defection serves to moderate the chosen level of operating cost. More general results show that collusion is more stable in homogeneous than in heterogeneous markets.

The third paper is an empirical analysis where I test the hypotheses of physicians' altruism and ex-post moral hazard using a large national panel dataset of drug prescription records from Finland. We estimate the probability that doctors prescribe generic versus branded versions of statins for their patients as a function of the shares of the difference in prices between what patients have to pay out of their pocket and what are covered by insurance.

The role of physicians and insurance in health care markets has been of interest to economists since the seminal contribution of Arrow (1963). Pioneering the economic analysis of physician behavior in the context of health care, Arrow (1963) noticed that doctors may have motives and objectives that differentiate them from purely profit-maximizing agents. The original 'ex-post moral hazard' hypothesis, predicts that health insurance increases the consumption of health care and leads to excessive consumption of services even in a competitive health care market. Ex-post moral hazard has since then been the focus of various empirical and theoretical studies in health economics (see e.g. Feldstein, 1973; Leibowitz, Manning, and Newhouse, 1985; Manning, Newhouse, Duan, Keeler, Leibowitz, and Marquis, 1987; Dranove, 1989; Zweifel and Manning, 2000).

We simultaneously test both altruism and ex-post moral hazard in drug prescription behavior using a large national panel of administrative data from Finland. We first develop a theoretical model on physician decision-making, which, in line with Hellerstein (1998) and Lundin (2000), then use a large national panel dataset with all statin prescriptions in Finland between 2003 and 2010 (n=17 858 829 prescriptions) to test the physicians' altruism and ex-post moral hazard hypotheses, while controlling for a large range of physicians, patients, and drug characteristics. Taking advantage of the panel structure of our national administrative dataset, we directly observe the repeated prescriptions of statins by physicians over time.

We find that although the estimated coefficients associated with ex-post moral hazard and altruism are statistically significantly different from zero, their size is very close to zero and the orders of magnitude is smaller than the effects associated with other key explanatory factors. We also find robust and strong evidence of prescription habit-dependency.

1 Vertical differentiation with consumers misperceptions and information disparities.

1.1 Introduction[1]

In markets where products are vertically differentiated, consumers may be uncertain about the quality differential provided by high quality firms and then consider if this differential is worth the price premium they should pay for products that claim to be ranked as high quality brands. If products are experience goods, ex-post consumption and repeat purchases may provide more precise information to consumers, and firms can establish a reputation for high quality, as shown for example by Shapiro (1982,1983). If products are credence goods, as in case of drugs, chemicals or products sold as green goods, many consumers may lack the expertise to ascertain the quality differential with respect to cheaper standard brands, even after purchase[2].

Actually for products classified as credence goods, consumers may even not know what is the minimum and the maximum quality that a firm can provide. Accordingly it may be difficult for consumers even to assign a probability distribution to the quality choice of the firm. Therefore consumers may carry out purchase decisions according to their perceptions about product quality. In this last case consumers may either overestimate or underestimate the quality differential provided by the seller. For example brand loyalty may imply that consumers are overestimating the quality provided by one brand and turn out to be optimistic about the quality differential actually provided by firms. In this last case firms may profit from imperfect information by charging excessive prices to consumers. But it may also occur that firms claiming to sell higher quality brands face skeptical consumers, which turn out to be pessimistic with respect to the quality choice, thinking that the claimed quality differential cannot be provided by a profit maximizing firms. In both cases competition between vertically differentiated brands will be affected by asymmetric information and consumers misperceptions[3].

[1] Joint work with Alberto Cavaliere, Department of Economics and Management, University of Pavia

[2] Though minimum quality standard exists in this case in order to guarantee a minimum level of safety and reliability to consumers, still some brands may claim they offer a better deal to consumers.

[3] Competition affected by consumers misperception has been seldom analyzed in the economic literature. Spence (1977) in his pioneering work considers a competitive market where firms may fail to provide the adequate level of product safety due to imperfect information about the probability of product failure and proucer liability may be designed accordingly. Liability rules when consumers' misperceptions coexist with market power are analyzed by Polinsky and Rogerson (1982). More recently Iossa and Palumbo (2010) consider consumers that underestimate the risk of product failures

For example in the market for drugs competition betwen generics and branded pharmaceuticals is affected by brand loyalty, as optimistic consumers continue to buy branded drugs being optimistic about their quality, though it is likely to be equivalent to the quality of generic drugs introduced later in the market. On the contrary in the market for green products firms may find it difficult to persuade pessimisitc consumers that are skeptical about the feasibility and propability of selling products with a low environmental impact. As far as the quality choice implies an hidden action for firms, asymmetric information may imply moral hazard. However if no random shock affects quality and given that quality is chosen by firms before purchase, adverse selection is likely to arise. In any case adverse incentives may affect both price competition and product differentiation.

However even in markets where the choice of most consumers is affected by uncertainty about quality, some consumers may be well informed about real quality differentials. Better information may derive either by consumer expertise, costly information gathering activities or better education, leading some consumers to process complex information about quality better than others. For example in the case of drugs firms face both health institutions and educated consumers making informed decisions and uninformed patients affected by misperceptions and brand loyalty. In the case of green products, informed consumers may be able able to distinguish a real environmental commitment from strategic greenwashing, disposing of precise consumer reports provided by associations like Greepeace. Informed consumers may then exert a positive externality on uninformed one and affect the incentive of firms to provide high quality products, as originally shown by Chan and Leland (1982), Cooper and Ross (1984) and Wolinsky (1983), in the framework of perfect competiton and monopolistic competition, or Judd and Riordan (1994) analyzing the case of a new product monopolist. Information disparities between consumers in a model of vertical product differentiation where firstly introduced by Cavaliere (2005), just considering the price competition stage and then neglecting both the quality choice and the cost of quality provision.

In this paper we extend the analysis of Cavaliere (2005) to include the quality choice by firms, when providing higher quality requires a costlier effort. We then analyse the case of a duopoly with vertically differentiated products, asymmetric information about quality, consumers' misperceptions and information disparities. Consumers are split between uninformed and informed consumers. Uninformed consumers are characterized by consumers' misperceptions as they can underestimate or overestimate the quality differential. As a minimum quality standard (MQS) is imposed by the Government, even uninformed consumers expect that any product sold in the market at least complies with the standard. As the MQS can be said to be verifiable, even uninformed consumers can be confident about low quality products: both firms are expected to provide at least the MQS (we show that such an expectation is fullfilled in equilibrium). The firm providing

and show that joint responsibility for breach of contract reduces the incentives of the supplier to mispresent product quality. Consumers mispercetions about product quality have been considered in an oligopolistic setting by Garella and Petrakis(2008) in order to justify minimum quality standard as a solution to market failures due to asymmetric information. Microeconomic foundations for the theory of choice affected by consumers misperceptions have been extensively analyzed by Bordalo Gennaioli and Shleifer (2012) and Gennaioli Shleifer (2009) .

high quality goods claims overcompliance with respect to the MQS and charges higher prices accordingly. But overcompliance is neither obervable by uninformed consumers, nor verifiable. Consumers do not even know what is the extent of the quality choice feasible for the firm. Informed consumers are on the contrary also informed about the quality differential as, beyond the minimum quality standard, they also know the quality provided by the high quality firm.

We also assume that information about quality derives either by higher education or a costly activity, or both. To the extent that both higher education and the willingness to pay for information gathering activities are correlated with income, one particular feature of our model is that uninformed and informed consumers are not randomly distributed in the population of consumers but correlated with the distribution of the willingness to pay for quality[4]. Therefore, by assumption, the higher the willingness to pay for quality the higher the likelyhood that a consumer is informed (such an assumption implies that if a consumer i, with a willingness to pay for quality θ_i is informed any consumer j with a willingness to pay $\theta_j > \theta_i$ will be informed as well)

We do not analyze information decisions by consumers, thar are exogenously given. Therefore firms follow a Stackelberg behavior vis à vis consumers. However we can analyze quality and price competition between firms for the full range of information disparties, i.e. for any split between informed and uninformed consumers that can affect demand functions. Furthermore we distinguish between the case of optimistic misperceptions (uninformed consumers overestimate the quality differential) and the case of pessimistic consumers (uninformed consumers underestimate the quality differential) and will show that such a distinction is built in into the model. Therefore uninformed consumers hold the same misperception, i.e. uninformed consumers may be either optimistic or pessimisitic. Competion between firms is represented by a two stage game, in the first stage the two firms compete in qualities, given the market split between informed and uninformed consumers which affects demand functions. In the second stage price competition takes place.

To the best of our knowledge our model is the first one to analyze the case of pure vertical differentiation with imperfect information, information disparities, and endogenous quality. Previous contribution include Bester (1998) considering a model of horizontal and vertical differentiation where quality is both endogenous and uncertain for consumers and prices can be a quality signal, but information disparities are not analyzed. Garella and Petrakis (2007) consider both information disparities, consumers'misperceptions and endogenous quality but in an oligoplistic setting with imperfect substitutes, according to the Dixit-Spence-Bowley approach. With respect to us they can consider randomly distributed misperceptions but not in a framework of pure vertical differentiation. Other significant contributions mainly concern vertical differentiation with signalling, where quality is exogenously given This strand of literature includes Fluet and Garella (2002), Hetzendorf and Overgaard (2002) and Daughety and Reinganum (2008). Our model does not consider equilibria with signaling, but can provide foundations about the need

[4]As in turn the willingness to pay for quality can be typically correlated with income in vertical differentiation model (cfr. for example Tirole 1989)

for signalling to overcome adverse selection in case of pessimistic beliefs by uninformed consumers. Gabszewicz and Resende (2012) consider price competition in the case of credence goods - as we do - but without considering the issue of quality choice. Moreover they introduce asymmetric information about quality by assuming that consumers do not know which firms sells which quality, building on the previous analysis of Gabszewicz and Grilo (1992). Bonroy and Constantatos (2008) follow this same approach to adress the issue of voluntary versus mandatory labels in credence good markets. Information provision policies are also considered by Brouhle and Khanna (2007) in a duopoly with vertical differentiation and imperfect information about quality. As in our case quality is endogenous in their model, but consumers'heterogeneity depends on their beliefs about the accuracy of information provision, which directly affects consumers utility. Changes in information accuracy affect all consumers in the same way so they cannot consider neither information disparities nor different types of consumers'misperceptions.

The paper is structured as follows. In section 1.2 we present the basic model and consider the analytical distinction netween the case of optimistic and pessimistic consumers affected by misperceptions about the quality differential. In section 1.3 we consider demand functions when uninformed consumers are optimistic. In section 1.4 we introduce the equilibrium analysis. In section 1.5 we carry out equilibrium analysis in case of optimistic consumers. In section 1.6 we consider demand functions and equilibrium analysis in case of pessimistic consumers. Section 1.7 concludes.

1.2 The Basic Model

We consider a market with N consumers. Each consumer buys one unit of the product (we shall assume that the market is completly covered). Consumer preferences can be represented by the following quasi-linear utility function:

$$U = \theta q - P$$

The willingness to pay for quality is represented by θ,which is uniformly distributed between $\underline{\theta}$ and $\bar{\theta}$ with $\bar{\theta} = \underline{\theta} + 1$ and density $f(\theta) = 1$. P is the market price and q represents product quality, which can be low (q_L) or high (q_L)[5]. There is a minimum quality standard q_0, enforceable by the government; thus $q_L \geq q_0$ and q_0 is common knowledge. Consumers have rational expectations about the low quality product, as they expect that $q_L = q_0$ (such an expectations is fullfilled in equilibrium). High quality is perfectly known to the producers but is unknown to the consumer, unless it is informed. Uninformed consumers are uncertain about the quality differential. Due to the existence of a minimum quality standard they can exclude that $q_H < q_0$ but hold consumers'misperceptions about the quality differential which is provided by the firm claiming to sell high quality products. However we assume that each uninformed consumer has the same expectation q_E concerning high quality. As we do not put further restrictions on q_H and q_E, we can distinguish two cases: 1) $q_E > q_H$, i.e. uninformed

[5]The vertical differentiation model with complete information we make reference to is presented by Tirole(1989).

consumers are characterized by **optimistic misperceptions** 2) $q_E < q_H$ i.e. uninformed consumers are characterized by **pessimistic misperceptions**[6].

As to the distinction between informed and uninformed consumers we split the market in two parts, following the distribution of θ. Consumers with a willingness to pay for quality $\theta \geq \theta^*$ are informed and then observe q_H. Consumers characterized by a willingness to pay $\theta < \theta^*$remain uninformed; and make purchase decisions on the basis of an expectation $q_E$concerning high quality products. Therefore, the greater is θ^* and the lower is the portion of informed consumers. In what follows we shall not put any restriction on the value of θ^*except that $\underline{\theta} \leq \theta^* \leq \bar{\theta}$. Therefore demand function will be shaped accordingly. The timing structure of the model can be described in the following way:

1.In the first stage the market is split between uninformed and informed consumers, according to consumers heteogeneity about θ, which is exogenously given

2.In the second stage firms, taking consumers information and expectations about the quality differential as given, choose the quality level

3.In the third stage firms, given their decisions concerning quality, compete in prices.

In the market there are two firms that can produce either a good of quality q_L or a good of quality q_H. Firms are perfectly informed about both product qualities. Let firm one specialize in the production of the good of quality q_L and firm two specialize in the production of quality q_H.We do not consider fixed production cost as we neglect the entry stage and we normalize to zero the variable cost of production. But we suppose that providing higher qualities implies higher efforts. Therefore we consider the cost of quality as αq^2, with $\alpha q_L^2 < \alpha q_L^2$. By considering the cost of quality as the cost of the greater effort of providing high quality goods we can well consider cases where firms should respect a minimum quality standard but can put greater efforts in quality control or any other activity which improves product quality. Low quality goods are sold at price P_L and high quality goods are sold at price P_H. As we assume that the market is covered we suppose that in equilibrium $P_L^* \leq q_L\underline{\theta}$.

In order to define market demand for the low quality and the high quality product we start from the definition of the marginal consumer, who is indifferent between buying from firm one or from firm two. However in this model informed consumers observe the true quality q_H while uninformed consumers just have an expectation about quality: q_E. Both consumers expect that $q_L = q_0$.Thus we are led to define two types of marginal consumer. The first one is the uninformed marginal consumer θ', who is defined by the following equality:

$$\theta q_0 - P_L = \theta q_E - P_H$$

giving

$$\theta' = \frac{P_H - P_L}{q_E - q_0}$$

Let us call $\Delta_E = q_E - q_0$ the expected quality difference perceived by uninformed consumers. Then uninformed consumers, with a willingness to pay $\theta \geq \theta'$ (and $\theta \leq \theta^*$)

[6]For a recent tentative contribution to the microfoundations of optimism and pessimism according to the theory of rational choice one can see (Dillenberger, Postelwaite, Rozen 2013)

choose the high quality product while uninformed consumers with a willingness to pay $\theta \leq \theta'$ (and $\theta \leq \theta^*$) choose the low quality product

The second marginal consumer is the informed one θ'':

$$\theta'' = \frac{P_H - P_L}{q_H - q_0}$$

and let us call $\Delta = q_H - q_0$ the true quality differential, only known to informed consumers. Then informed consumers with a willingness to pay $\theta \geq \theta''$ (and $\theta \geq \theta^*$) choose the high quality product while informed consumers with a willingness to pay $\theta \leq \theta''$(and $\theta \geq \theta^*$) choose the low quality product.

However the definition of demand functions for the low quality and high quality products requires further assumptions on the parameters of the model. For each market splitting between informed and uninformed consumer, i.e. for each location of θ^* with respect to θ' and θ'', market demands can change accordingly, as we shall show in next subsections. Furthermore, when considering the respective locations of the marginal consumers θ' and θ'' across the market, a basic distinction arises into the model, as we are necessary led to consider two main cases. Either $\theta' < \theta''$ or $\theta' > \theta''$. Given P_H, P_L and q_0, the sign of the previous inequality only depends on the relationship between q_E and q_H. Actually either $q_H < q_E$, i.e. uninformed consumers are **optimistic**, or $q_H > q_E$ i.e. uninformed consumers are **pessimistic.** Therefore the distinction between optimistic and pessimistic uniformed consumers is endogenously built-in into the model. In the optimistic case (case A) $\theta' < \theta''$ while in the pessimistic case (case B) $\theta' > \theta''$. Therefore also from the analytical point of view it is necessary to deal separately with these two cases

1.3 Market demands when uninformed consumers are optimistic

The main feature of the optimisitc case is that uninformed consumers overestimate the quality differential as $\Delta_E \geq \Delta$.Due to asymmetric information about the quality choice and consumers'misperceptions $q_E > q_H$.Therefore firms can profit from asymmetric information by charging higher prices and providing lower quality differentials then expected. Then in the optimistic case adverse incentives may potentially lead to consumers cheating.

Equilibrium analysis needs a definition of demand functions. We can define demand functions through the following steps. We start by considering alternative locations for θ^* in the space $[\underline{\theta}, \overline{\theta}]$,given both prices and quality differentials Δ_E and Δ.We shall than be able to restrict our attention to three main cases. For each case we can find restrictions on price domains and expressions for the segments of market demands corresponding to these restrictions.

However further assumptions about Δ_E and Δ need to be introduced to consider the full range of price domains consistent with market segments previously defined.(as in case A1, A2, A3) In the second step we consider in addition the variations in Δ_E and Δ, by looking at the ratio $\frac{\Delta_E}{\Delta} \geq 1$ and obtain restrictions about this ratio consistent with the willingness to pay for quality and the share of informed and uninformed consumers. As a result we shall be able to restrict the definitions of market demands to four alternative cases. In the last step, for each of these four cases we consider price domains and the market segments that define demand functions..

1.3.1 Alternative locations of theta in the space

A.1) $\underline{\theta} \leq \theta' \leq \theta^* \leq \theta'' \leq \bar{\theta}$. (Cf. figure 1.1). From fig.1.1 we see that both the demand for the low quality product (D_L) and the demand for the high quality product (D_H) are given by the sum of the demand of uninformed consumers and of the demand of informed consumers: $D_L = \theta' - \underline{\theta} + \theta'' - \theta^*$; $D_H = \theta^* - \theta' + \bar{\theta} - \theta''$. One can then notice that not only uninformed consumer with a lower willingness to pay buy low quality goods, but also informed consumers with an higher willingness to pay select the low quality product, once they are informed about the quality differential. On the contrary there are consumers - with a compartively lower willingness to pay - that buy high quality goods just because they are uninformed and hold optimisitic misperceptions about the quality differential. Considering the inequalities we can obtain the following restrictions concerning market prices, which will be useful to defining the price domain of demand functions. Concerning D_L we get

$$P_H - \theta^* \Delta_E \leq P_L \leq P_H - \underline{\theta} \Delta_E \tag{1.1}$$
$$P_H - \bar{\theta} \Delta \leq P_L \leq P_H - \theta^* \Delta \tag{1.2}$$

and concerning D_H

$$P_L + \underline{\theta} \Delta_E \leq P_H \leq P_L + \theta^* \Delta_E \tag{1.3}$$
$$P_L + \theta^* \Delta \leq P_H \leq P_L + \Delta \bar{\theta} \tag{1.4}$$

A.2) $\underline{\theta} \leq \theta^* \leq \theta' \leq \theta'' \leq \bar{\theta}$ (Cfr.fig.1.2). The demand for the low quality product is the sum of the demand coming from uninformed consumers, $(\theta^* - \underline{\theta})$ and the demand from informed consumers, $(\theta'' - \theta^*)$: then $D_L = \theta'' - \underline{\theta}$. The demand for the high quality product comes only from informed consumers: $D_H = \bar{\theta} - \theta''$. In this case even consumers with a lower willingness to pay know the true quality and are lead to buy the low quality good. Consumers misperceptions are not affecting neither the demand for the low quality good nor the demand for the high quality good (such a result is ultimately due to the location of θ^*, given that $\theta^* \leq \theta'$) .We can obtain the following restrictions about price domains:

$$P_H - \bar{\theta} \Delta \leq P_L \leq P_H - \theta^* \Delta \tag{1.5}$$
$$P_L + \theta^* \Delta_E \leq P_H \leq P_L + \Delta \bar{\theta} \tag{1.6}$$

A.3) $\underline{\theta} \leq \theta' \leq \theta'' \leq \theta^* \leq \bar{\theta}$. (Cfr. fig. 1.3). The demand for the low quality product comes only from uninformed consumers: $D_L(\theta' - \underline{\theta})$. The demand for the high quality product derives both from uninformed consumers, $(\theta^* - \theta')$,and informed consumers as

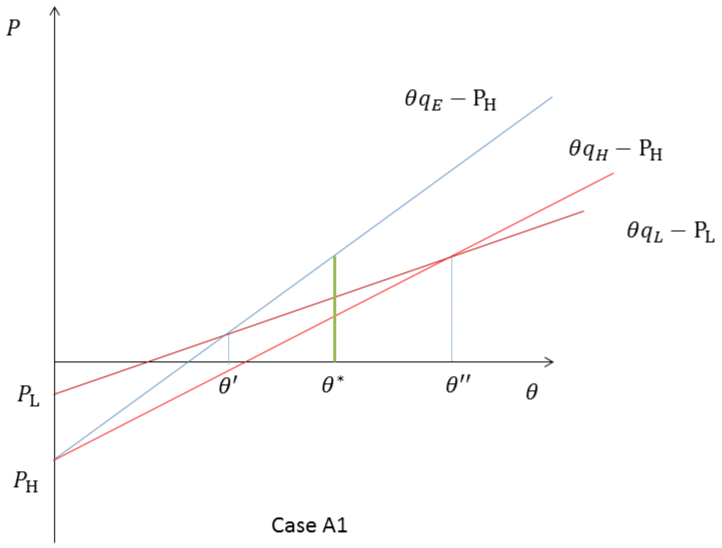

Figure 1.1:

well: $(\bar{\theta} - \theta^*)$: $D_H = \bar{\theta} - \theta'$. Consumers'misperceptions are then affecting both demands, while consumers'information has no impact on demand functions (this is ultimately due to the location of θ^*, given that $\theta'' \leq \theta^*$). The restrictions on price domains arising from case A.3 are the following:

$$P_H - \theta^*\Delta \leq P_L \leq P_H - \underline{\theta}\Delta_E \quad (1.7)$$
$$P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta \quad (1.8)$$

1.3.2 Joint restrictions on quality expectations and the willingness to pay for quality

Considering alternative locations of $\theta^*$to account for the effect of information disparities on market demand is not yet sufficient to define demand functions, as informed consumers observe the real quality differential while the demand of uninformed consumers depend on their misperceptions about high quality products. As the share of informed-uninformed consumers can vary together with both the expected quality differential Δ_E (how much optimistic are optimistic consumers?) and the actual quality provided by the high quality firm Δ , we need to introduce some further restrictions. Furthermore one should also consider that the expected quality differential Δ_E cannot be unbouded. As also the willingness to pay for quality cannot be greater then $\bar{\theta}$,restrictions on Δ_E and Δ depending on on $\underline{\theta}$, $\bar{\theta}$,and θ^* appear to be sensible in the framework of this model.

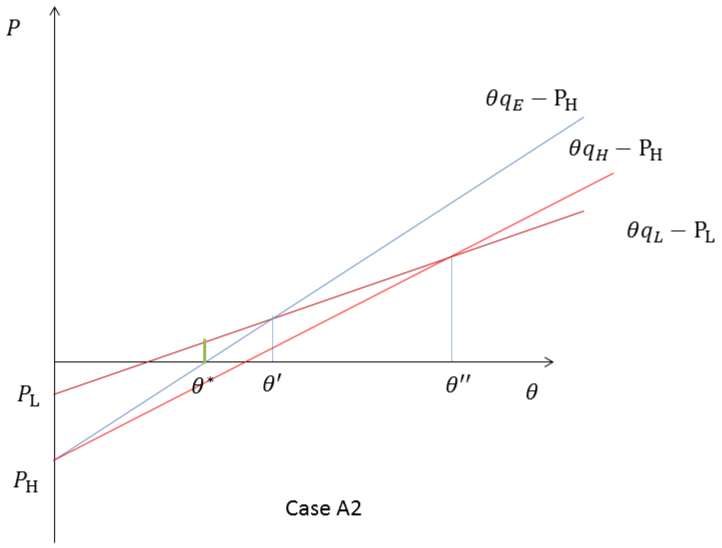

Figure 1.2:

Therefore to fully account for the alternative price orderings that define the price domains of the demand functions we need to introduce altogether restrictions: 1) On the relationship between the quality differerentials expressed by the ratio $\frac{\Delta_E}{\Delta}$;2) on the dimension (value) of the market as measured by $\bar{\theta}$ and $\underline{\theta}$; 3) on the extension of information disparities (i.e $(\theta^* - \underline{\theta})$ and $(\bar{\theta} - \theta^*)$, as measured by the ratios $\frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*}$. By considering alternative orderings of price domains we can obtain the following parameter restrictions that define four alternative couples of demand functions, when uninformed consumers are optimistic:

$$A.a)1 \leq \frac{\Delta_E}{\Delta} \leq Min\left\{\frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}}\right\}; A.b)\frac{\bar{\theta}}{\theta^*} \leq \frac{\Delta_E}{\Delta} \leq \frac{\theta^*}{\underline{\theta}};$$
$$A.c)\frac{\theta^*}{\underline{\theta}} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\theta^*}; A.d)Max\left\{\frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*}\right\} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\underline{\theta}}$$

Considering the ratio $\frac{\Delta_E}{\Delta} \geq 1$, in case A.a the previous restrictions allow for an expected quality differential Δ_E strictly close to the actual one, (i.e $\frac{\Delta_E}{\Delta} \sim 1$). In case A.d we can observe the highest ratio, with "over-optimistic" consumers (i.e $\frac{\Delta_E}{\Delta} \sim \frac{\bar{\theta}}{\underline{\theta}}$). In between these two extremes, we find cases A.b and A.c where the ratio $\frac{\Delta_E}{\Delta}$ can be said to be "intermediate". Furthermore the ratio is bounded in each case through restrictions concerning the share of informed-uninformed consumers (location of θ^*) and the willingness to pay for quality. In cases A. b and A.c the restrictions are such that we

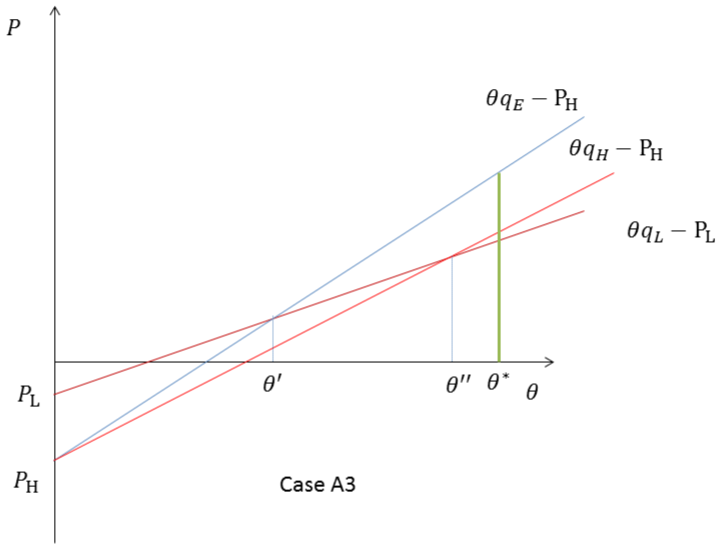

Figure 1.3:

can respectively state that most consumers are uninformed, (as $\theta^* \geq \sqrt{\underline{\theta}\bar{\theta}}$)[7] or most cosnumers are informed (as $\theta^* \leq \sqrt{\underline{\theta}\bar{\theta}}$) [8].

We concentrate our attention on intermediate cases for $\frac{\Delta_E}{\Delta}$. Actually in case (A.a) parameters are such that the model can easily "collpase" to the perfect information case as either the share of informed consumers is very high ($\theta^* \sim \underline{\theta}$) or by chance the expected quality differential by uninformed consumers turn out to be closer to the actual one provided by the high quality firm ($\Delta_E . \sim \Delta$).

1.3.3 Demand Functions in case (A.b)

In order to define the price domains of the demand function we consider the following price ordering for $P_L : P_H - \underline{\theta}\Delta_E \geq P_H - \theta^*\Delta \geq P_H - \bar{\theta}\Delta \geq P_H - \theta^*\Delta_E$, to obtain the demand of the low quality product $D_L(P_L, P_H)$, and the following price ordering for $P_H : P_L + \theta^*\Delta_E \geq P_L + \Delta\bar{\theta} \geq P_L + \theta^*\Delta \geq P_L + \underline{\theta}\Delta_E$ to obtain the demand for the high quality product $D_H(P_L, P_H)$. One can check that the previous price orderings can be

[7]this inequlity say that θ^*must be bigger than geometric mean of minimun willingness to pay $\underline{\theta}$ and the maximum willingness to pay $\bar{\theta}$ for a most precise definition see appendix A1

[8]In case A.a $\sup\left\{1 \leq \frac{\Delta_E}{\Delta} \leq Min\left\{\frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}}\right\}\right\} = Min\left\{\frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}}\right\}$ can be shown to be consistent both with most consumers being uninformed (as $\theta^* \geq \sqrt{\underline{\theta}\left(\underline{\theta}+1\right)}$) and most consumers being (as $\theta^* \leq \sqrt{\underline{\theta}\left(\underline{\theta}+1\right)}$) The same conclusion holds for case A.d, where $\inf\left\{Max\left\{\frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}}\right\} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\underline{\theta}}\right\} = Max\left\{\frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*}\right\}$

reduced to the following condition : $\frac{\bar{\theta}}{\theta^*} \leq \frac{\Delta_E}{\Delta} \leq \frac{\theta^*}{\underline{\theta}}$. The restriction about θ^* ($\frac{\bar{\theta}}{\theta^*} \leq \frac{\theta^*}{\underline{\theta}}$) implies that $\theta^* \geq \sqrt{\underline{\theta}\overline{\theta}}$, i.e the share of informed consumers is smaller with respect to the share of uninformed ones. Given the previous restriction, one can then account for the location of θ^* and define the demand segments for each price domain, by going back to cases A.1, A2 and A.3 above. We start with the demand function for the low quality product.

$$D_L(P_L, P_H) = \begin{cases} \theta^{'} - \underline{\theta} & if \quad P_H - \theta^*\Delta \leq P_L \leq P_H - \underline{\theta}\Delta_E \\ \theta^{'} - \underline{\theta} + \theta^{''} - \theta^* & if \quad P_H - \overline{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta \\ \theta^{'} - \underline{\theta} + \overline{\theta} - \theta^* & if \quad P_H - \theta^*\Delta_E \leq P_L = P_H - \overline{\theta}\Delta \\ \overline{\theta} - \underline{\theta} & if \quad 0 \leq P_L \leq P_H - \theta^*\Delta_E \end{cases}$$

Actually one can check that the price-domain of the first segment of $D_L(P_L, P_H) = \left(\theta^{'} - \underline{\theta}\right)$ is consistent with case A.3. The second segment is consistent with case (A.1), as well as the third segment -$\left(\theta^{'} - \underline{\theta} + \overline{\theta} - \theta^*\right)$.

With the highest price for the low quality good (first price domain), the latter is bought just by uninformed consumer with the lowest willingness to pay and demand is just affected by consumers misperceptions. When P_L decreases we reach the second price domain where also informed consumers characterized by an higher willingess to pay decide to buy the low quality good. Actually it occurs that the reduction in P_L moves $\theta^{''}$towards $\overline{\theta}$ until there is a switch fro $\theta^{''} \leq \theta^*$ to $\theta^{''} \geq \theta^*$ (from A.3 to A.1) implying that a share of informed consumers switch to the low quality good. Their demand is given by $\left(\theta^{''} - \theta^*\right)$, and depends on the location of θ^*. As $\theta^{''}$reaches $\overline{\theta}$ - due to the continuos decrease of P_L- we reach the third segment, where all informed consumers with an higher willingness to pay buy the low quality good, expressing the following demand: $(\overline{\theta} - \theta^*)$. In the third segment the decrease of P_L gradually reduces also the share of uninformed consumer with an intermediate willingness to pay that sticks to the high quality good. As P_L decreases the marginal uninformed consumers $\theta^{'}$ moves towards θ^*, until all uninformed consumers buy the low quality good $\left(\theta^{'} = \theta^*\right)$.Then $D_L(P_L, P_H) = 1$.

The demand for the high quality product $D_H(P_L, P_H)$ then follows (and one can check that it is complementary to $D_L(P_L, P_H)$):

$$D_H(P_L, P_H) = \begin{cases} \theta^* - \theta^{'} & if \quad P_L + \overline{\theta}\Delta = P_H \leq P_L + \theta^*\Delta_E \\ \theta^* - \theta^{'} + \overline{\theta} - \theta^{''} & if \quad P_L + \theta^*\Delta \leq P_H \leq P_L + \overline{\theta}\Delta \\ \overline{\theta} - \theta^{'} & if \quad P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta \\ \overline{\theta} - \underline{\theta} & if \quad 0 \leq P_H \leq P_L + \underline{\theta}\Delta_E \end{cases}$$

Then the first segment - $(\theta^* - \theta^{'})$ - is consistent with case A.1. The second segment of $D_H(P_L, P_H)$ - $(\theta^* - \theta^{'} + \overline{\theta} - \theta^{''})$ - is consistent with case A.1 as well. The third segment of $D_H(P_L, P_H)$ -$\left(\overline{\theta} - \theta^{'}\right)$- is consistent with case A.3, as one can easily check by going back to (6). Looking at the first demand segment one can see that with the highest price for the high quality good only uninformed consumers which overestimate the quality differential are willing to buy it. When P_H decreases and the second price domain is reached then also the demand coming from informed consumers with the highest wilingness to pay $\left(\overline{\theta}-\theta^{''}\right)$ adds to the demand coming from uninformed consumers. Within the second segment, the reduction of P_H implies changes in the location of $\theta^{''}$, moving towards θ^{*}.When $\theta^{''}=\theta^{*}$ the third segment is reached and from now on further reductions of P_H imply a switch from $\theta^{''}\geq\theta^{*}$to $\theta^{''}<\theta^{*}$, which is consistent with case A.3. so that the demand for high quality goods increases further and just the marginal uninformed consumer can affect it (actually $\theta^{''}<\theta^{*}$ implies that $\theta^{''}$can no more affect market demands). As the reduction of P_H also moves $\theta^{'}$ towards $\underline{\theta}$, when P_H is low enough it then happens that $\theta^{'}=\underline{\theta}$. In this last case all consumers buy high quality goods and $D_H\left(P_L,P_H\right)=1$.

Demand functions are then represented in fig 1.4, 1.5 showing their kinked shape which is typical of vertical differentiation models.

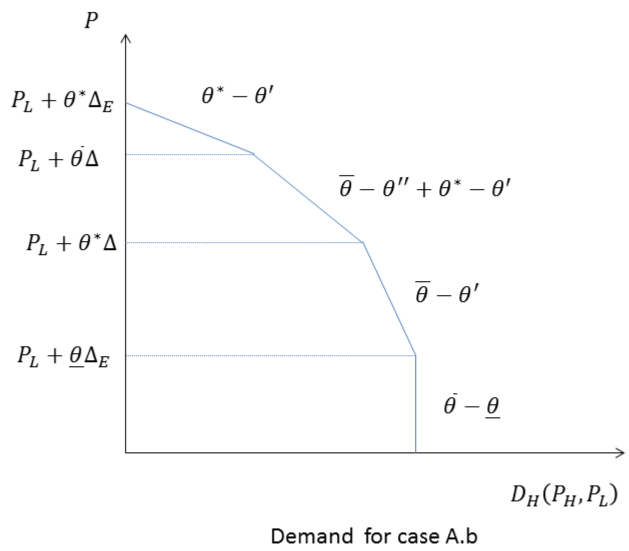

Figure 1.4:

1.3.4 Demand functions in case (A.c)

In this sub case we assume the following price ordering for P_L in order to define the price domain of $D_L\left(P_L,P_H\right):P_H-\theta^{*}\Delta\geq P_H-\underline{\theta}\Delta_E\geq P_H-\theta^{*}\Delta_E\geq P_H-\bar{\theta}\Delta$ and the following price ordering for P_H in order to define $D_H\left(P_L,P_H\right):P_L+\Delta\bar{\theta}\geq P_L+\theta^{*}\Delta_E\geq P_L+\underline{\theta}\Delta_E\geq P_L+\theta^{*}\Delta$. One can check that the previous inequalities reduce to the

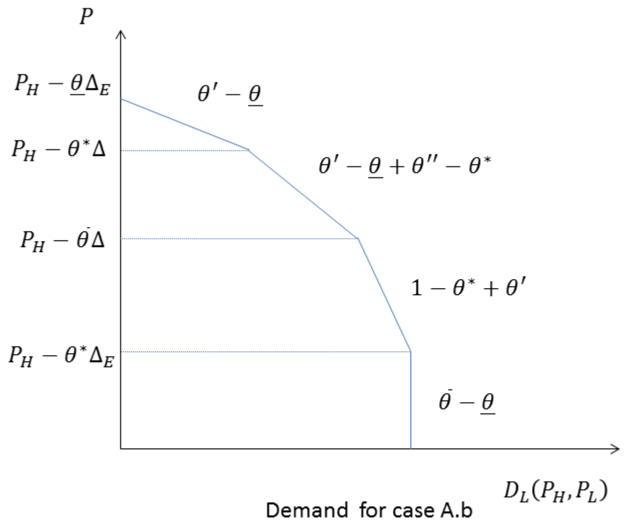

Figure 1.5:

following: $\frac{\theta^*}{\underline{\theta}} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\theta^*}$. The restriction about θ^* ($\frac{\theta^*}{\underline{\theta}} \leq \frac{\bar{\theta}}{\theta^*}$) implies that $\theta^* \leq \sqrt{\underline{\theta}\bar{\theta}}$, i.e the share of informed consumers is larger than the share of uninformed ones. Given the previous restriction, one can then account for the location of θ^*, and define the demand functions segments for each price domain, by going back to cases A.1, A2 and A.3 above, as follows:

$$D_L(P_L, P_H) = \begin{cases} \theta'' - \theta^* & if \quad P_H - \underline{\theta}\Delta_E \leq P_L \leq P_H - \theta^*\Delta \\ \theta'' - \theta^* + \theta' - \underline{\theta} & if \quad P_H - \theta^*\Delta_E \leq P_L \leq P_H - \underline{\theta}\Delta_E \\ \theta'' - \underline{\theta} & if \quad P_H - \bar{\theta}\Delta \leq P_L = P_H - \theta^*\Delta_E \\ \overline{\theta} - \underline{\theta} & if \quad 0 \leq P_L \leq P_H - \bar{\theta}\Delta \end{cases}$$

Actually one can check that the price domain of the first demand segment is consistent with case A.1. The price domain of the second and third segment are consistent with case A.1 as well. We can notice at a glance that in this sub-case, the demand function, turns out being affected mainly by the marginal informed consumer θ'', i.e by the real quality differential provided by the high quality firm, as most consumers are informed.

With the highest price for the low quality good (first price domain) demand comes from informed consumers with an intermediate willingness to pay (first demand segment) and we can notice that the greater is the share of informed consumers (i.e. the lower is θ^*) the greater this piece of demand. It is then interesting to point out that in this case information leads consumers to choose the low quality product even with an high P_L,as the real quality differential is not worth selecting the high quality good. With a decrease of P_L, also a share of uninformed consumers with the lowest willingness to pay $(\theta' - \underline{\theta})$ add to the previous segment, to get $D_L(P_L, P_H) = (\theta'' - \theta^* + \theta' - \underline{\theta})$. As P_L further decreases within the second price domain, the value of the marginal uninformed consumer $\theta^{'}$ increases, i.e. $\theta^{'}$ moves towards θ^* until $\theta^{'} = \theta^*$ and the third price domain is reached. In this last case the marginal uninformed consumer can no more affect the demand function and the latter becomes independent from consumers'misperceptions. Actually the third demand segment will be given by the whole share of uninformed consumers $(\theta^* - \underline{\theta})$ plus the share of informed consumers finding it convenient to buy the low quality good: $(\theta'' - \theta^*)$ .Therefore the third demand segment reduces to $(\theta'' - \underline{\theta})$. As P_L further decreases within the third price domain a parallel decrease of θ''will follow, implying that θ''moves towards $\underline{\theta}$ until $\theta'' = \underline{\theta}$ and then $D_L(P_L, P_H) = 1$.

The demand for the high quality product $D_H(P_L, P_H)$ then follows and one can easily check that it is complementary to $D_L(P_L, P_H)$:

$$D_H(P_L, P_H) = \begin{cases} \overline{\theta} - \theta'' & if \quad P_L + \theta^*\Delta_E \leq P_H \leq P_L + \underline{\theta}\Delta \\ \theta^* - \theta' + \overline{\theta} - \theta'' & if \quad P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta_E \\ \overline{\theta} - \theta'' + \theta^* - \underline{\theta} & if \quad P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta \\ \overline{\theta} - \underline{\theta} & if \quad 0 \leq P_H \leq P_L + \theta^*\Delta \end{cases}$$

One can check that all price domains of $D_H(P_L, P_H)$ are consistent with case A.1.and that all segments of the demand function (but the second one) are affected by the marginal informed consumers $\theta''$and by θ^*. As θ' disapperars from most segments, consumers'misperceptions are not affecting market demands (the only exception being the second price domain). With the highest price for the high quality good, the latter is just purchased by informed consumers with the highest willingness to pay. As P_H decreases and the second price domain is reached, then the demand from uninformed consumers with an intermediate willingnes to pay $(\theta^* - \theta')$ will add to the first demand segment to get $D_H(P_L, P_H) = \left(\theta^* - \theta' + \overline{\theta} - \theta''\right)$ .Within the second price domain the decrease of P_H will lead to a decrease of θ'such that the marginal uninformed consumers will move towards $\underline{\theta}$, and the third price domain is reached when $\theta' = \underline{\theta}$. This implies that all uninformed consumers (included the "poorest" ones) will demand the high quality good. Actully in the third segment $D_H(P_L, P_H) = 1 + \theta^* - \theta''$implying that the demand of the high quality good increases with a decrease of the share of informed consumer and with a decrease of the marginal informed consumer (i.e. when P_H decreases and /or Δ increases- to the extent both changes imply that θ'' is decreasing). When P_H further decreases within the third segment, then $\theta^{''}$ decreases too and moves toward θ^*,until $\theta'' = \theta^*$ and $D_H(P_L, P_H) = 1$.

Demand functions are then represented in fig 1.6, 1.7

1.3.5 Demand functions in case (A.d) and (A.a)

Even if we do not analyze case A.d, we would like to point out that considering the concerned parameter restrictions: $Max\left\{\frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*}\right\} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\underline{\theta}}$ we can find that this case is consistent either with most consumers being informed or most consumers being un-

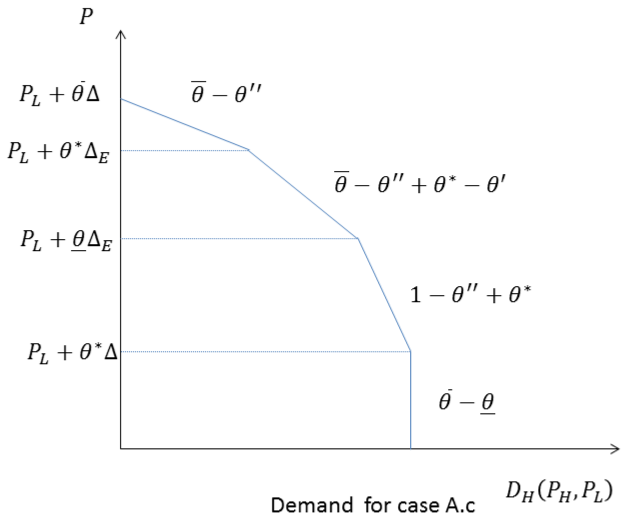

Figure 1.6:

informed (as $Max\left\{\frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*}\right\}$ includes both cases where $\theta^* \leq \sqrt{\bar{\theta}\underline{\theta}}$ and cases where $\theta^* \geq \sqrt{\bar{\theta}\underline{\theta}}$). Actually when checking for the price domains of the demand functions (Appendix A2) one can show they are all consistent with case A.1, as in previous sub-case A.c. Therefore demand functions are expected to be similar to those we have just defined for this sub-case. However what distinguishes sub-case A.d is the fact that the ratio $\frac{\Delta_E}{\Delta}$ is very high, i.e consumers are "over-optimistic" . Due to the fact that Δ is likely to be lower than in other cases with respect to Δ_E, when P_L decreases across price domains then the increase of $\theta^{''}$ is likely to be such that $\theta^{''}$ moves towards and reaches $\bar{\theta}$ before $D_H(P_L, P_H) = 1$, implying of course that it is not necessary to reduce P_L too much to persuade informed consumers with the highest willingness to pay to purchase low quality goods. On the contrary only a decrease of P_H would lead this type of consumers to switch to high quality goods. Therefore when P_H is too high, then high quality good end up being bought just by uninformed consumers with an intermediate willingness to pay[9]. Accounting for this difference one can then show that the segments of the demand functions in case A.d are identical to case A.c, with the exception of the third segment of $D_L(P_L, P_H)$ and, simmetrically, the first segment of $D_H(P_L, P_H)$ where P_L is very low

[9]Likewise, as Δ_E is likely to be very high, when P_H decreases across price domains then the decrease can be such that $\theta^{'}$ moves towards and reaches $\underline{\theta}$ before $D_H(P_L, P_H) = 1$, implying that it is not necessary to reduce P_H to much to persuade uninformed consumers with the lowest willingness to pay to buy high quality goods.

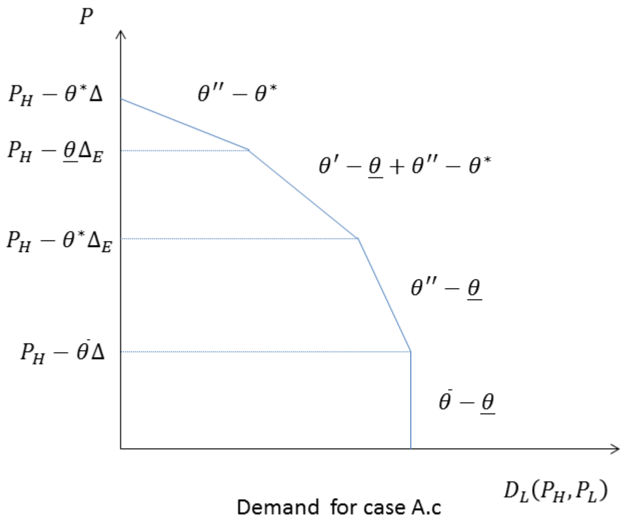

Figure 1.7:

and P_H is very high[10].

Concerning case A.a we can point out that also this case, given the parameter restrictions, is consistent both with most consumers being informed and most consumer being uninformed. Furthermore as $\frac{\Delta_E}{\Delta} \sim 1$,the quality differential are less important then in previous cases in shaping the demand functions. Price changes matter. When P_L is very high and P_H is very low, the low quality good is bough just by uninformed consumers with a lower willingness to pay ($D_L = \theta' - \underline{\theta}$) while the high quality good is bought by uninformed consumers ($\theta^* - \theta'$) and by all informed consumers ($\bar{\theta} - \theta^*$), so that demand functions are just affected by the uninformed marginal consumer θ'(consumers'misperceptions shape demand functions).On the contrary when P_L is very low and P_H is very high,also some informed consumers with an intermediate willingness are lead to buy low quality goods. Actually, given an high P_H a decrease of P_L gradually induces a switch from $\theta' \leq \theta^*$ to $\theta' \geq \theta^*$, such that low quality goods are bought by all uninformed consumers and by a share of informed consumers ($\theta'' - \theta^*$). Demand functions are then dependent on the real quality differential Δ. Actually, given that $\theta' \geq \theta^*$,the marginal uninformed consumer θ'can no more affect demand functions and therefore the model boils down to the full information case. Furthermore in this case θ^* can affect demand functions just when P_H and $P_L$are neither too high nor too low, as it occurs across all sub.cases that we can consider[11].

[10]Appendix A2
[11]Appendix A2

1.4 Equilibrium Analysis

In this section we analyse price and quality competition between the two firms, given expected and real quality differences (Δ_E and Δ), solving the two stage game by backward inuction. In the last stage, firms decide on prices, given qualities chosen in the previous stage and information disparities arising by consumers' decisions in the first stage. Each firm chooses a strategy that is the best reply to the other seller's strategy. Thus let $\Pi_i(P_i, P_j) = P_i D_i(P_i, P_j)\ i, j = L, H$ denote the profit function of firm i., remembering that we have assumed that firm one sells the low quality product and firm two sells the high quality product

Definition: A price (Nash) equilibrium is a pair (P_L^*, P_H^*) such that no firm has an incentive to change its price unilaterally:

$$\Pi_i(P_i^*, P_j^*) \geq \Pi_i(P_i, P_j^*)\ i, j = L, H$$

In the following sub-sections we shall look for a candidate Nash equilibrium in prices both in the optimistic and the pessimistic case. Being demands piecewise linear, for each configuration of the demand function we can find the candidate Nash equilibrium prices, considering each price domain for each demand function. For each sub-case we can moreover obtain the restrictions on the number of informed consumers that result from checking that the candidate equilibrium prices actually belong to the price domains in question[12]. Given equilibrium prices, we then consider the quality choice in the previous stage, to analyzethe degree of product differentiation in equilibrium.

In order to show that the price pairs are indeed a Nash equilibrium we have to check that the last Definition is satisfied.This will be equivalent to checking that the candidate equilibrium prices assure optimisation of the profit functions not only in the price domains considered one at a time, but also in the entire price range characterising each configuration of the demand functions[13].

1.5 Equilibrium Analysis with Optimistic Consumers

1.5.1 Case A.b: Most consumers are uninformed

In this case $\left(\frac{\bar{\theta}}{\theta^*} \leq \frac{\Delta_E}{\Delta} \leq \frac{\theta^*}{\underline{\theta}}\right)$ and $(\theta^* \geq \sqrt{\underline{\theta}\bar{\theta}})$

In order to find the candidate equilibrium price we consider the complementary demand segments of $D_L(P_L, P_H)$ and $D_H(P_L, P_H)$ one at a time:

1.5.1.1 A.b.1

Given the following price domains for $D_L$and D_H

[12] A complete analysis of equilibrium including stability is in Appendix A5

[13] For a similar analytical methodology, see Garella and Martinez-Giralt(1989)

$$P_H - \theta^* \Delta \leq P_L^* \leq P_H - \underline{\theta} \Delta_E$$

$$P_L + \underline{\theta} \Delta_E \leq P_H^* \leq P_L + \theta^* \Delta$$

demand segments lead to the following profit functions:

$$\Pi_L (P_L, P_H) = P_L \left(\theta' - \underline{\theta}\right) - \alpha q_L^2 \qquad \Pi_H (P_L, P_H) = P_H \left(\bar{\theta} - \theta'\right) - \alpha q_H^2 \tag{1.9}$$

We can then obtain tthe following candidate equilibrium prices:

$$P_L^* = \frac{\Delta_E \left(\bar{\theta} - 2\underline{\theta}\right)}{3} \qquad P_H^* = \frac{\Delta_E \left(2\bar{\theta} - \underline{\theta}\right)}{3} \tag{1.10}$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E \left(\bar{\theta} - 2\underline{\theta}\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E \left(2\bar{\theta} - \underline{\theta}\right)^2}{9} - \alpha q_H^2 \tag{1.11}$$

By checking if the candidate equilibrium prices are actually included in the price domains given above, we get a further restictinion on θ^*:

$$\theta^* \geq \frac{\Delta_E \left(2\underline{\theta} + 1\right)}{3\Delta}$$

By considering that across case A.b the following restriction holds $\theta^* \geq \sqrt{\underline{\theta}\bar{\theta}}$,in case A.b.1 θ^* should be even greater if $\frac{\Delta_E}{\Delta} \gtrapprox \frac{3}{2}$ (or lower if $\frac{\Delta_E}{\Delta} \lessapprox \frac{3}{2}$). Still most consumers remain uninformed.

Considering the previous solution for the last stage of the game, we can turn to the quality selection stage, where the degree of product differentiation is found by maximizing equilibrium profits with respect to qualities. Considering the foc we get:

$$\frac{\partial \Pi_L}{\partial q_L} = \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{9} - 2\alpha q_L \leq 0 \tag{1.12}$$

Therefore the low quality firm finds it optimal to keep quality as low as possible. Due to the existence of a MQS q_0, this implies that $q_L^* = q_0$,which is coinsistent with the expectations of any consumer. Concerning the high quality firm, as its profits depend both on the actual level of quality provided (through costs) and on the expected quality (through demand) we consider firstly the impact of the quality increase on costs by maximixation of the profit function to get:

$$\frac{\partial \Pi_H}{\partial q_H} = -2\alpha q_H \tag{1.13}$$

As revenues depend on expected quality, which affects demand but is not under the control of the firm, we can get a restriction on the quality level which is optimal to provide by considering the price domains given above, by checking that the equilibrium prices P_L^* and P_H^* actually belongs to the respective price intervals. We then get the following restriction on q_H:

$$q_H \geq q_0 + \frac{\Delta_E \left(2\underline{\theta} + 1\right)}{3\theta^*} \tag{1.14}$$

By jointly considering the foc and the previous inequality we can obtain a corner solution for $q_H^* = q_0 + \frac{\Delta_E(2\underline{\theta}+1)}{3\theta^*}$. According to this solution we can state that $q_H^* > q_0$ (there is some product differentiation as the quality differential $\Delta$is positive) and morever that $(q_H^* - q_0) < \Delta_E$, (the high quality firm actually provides a lower quality differential than expected by uninformed consumers).

As P_H^* and Π_H^* depends on Δ_E, but the actual quality differential is lower, we can state that adverse incentives lead to consumer cheating in equilibrium, as consumers of high quality products pay an excessive price premium with respect to the actual quality differential. However high quality will not collapse to the MQS. As the price domains are such that all informed consumers buy the high quality good ($\theta^* > \theta'$ and $\theta^* = \theta''$) q_H^* should be consistent with the price that informed consumers are willing to pay. Therefore it is convenient for the high quality firm to provide a quality level sufficiently high to justify the price charged to informed consumers. The latters in this case are characterized by the highest willingness to pay. Furthermore by considering the expression of q_H^* one can easily check that a decrease of informed consumers (shown by an increase of θ^*) leads to a reduction of q_H^* while an increase of informed consumers leads to an increase of q_H^*. Such an effect is independent from equilibrium prices, but depends on the restrictions on q_H^* obtained by checking if P_H^* belongs to the price domain defining case A.c.1 (actually for $q_H < q_H^*$, P_H^* cannot be an equilibrium price as it will not belong to the price domain characterizing A.b.1)).

Therefore even if the share of informed consumers does not directly affect equilibrium prices, such a share indirectly affect the quality differential in equilibrium with a positive externality for uninformed consumers buying high quality goods. An increase of the share of informed consumers leads the high quality firm to provide an higher quality differential. However given that most consumers remain uninformed, the share of informed consumers in never sufficient to affect equilibrium prices, which remain distorted upwards as they depend on the quality differential expected by uninformed consumers. Both firms can profit from imperfect information as they can charge prices and get profits depending on Δ_E,though high quality cannot collapse to the MQS due to the contribution of informed consumer to the demand for high quality products which constrains the high quality firm to provide some product differentiation. Moreover product differentiation increases (and the extent of consumer cheating decreases) with the share of informed consumers.

1.5.1.2 A.b.2

Considering the following price domain:

$$P_H - \bar{\theta}\Delta \leq P_L^* \leq P_H - \theta^*\Delta$$

$$P_L + \theta^* \Delta \leq P_H^* \leq P_L + \bar{\theta}\Delta$$

and the related demand segments, we get the following profit functions:

$$\Pi_L\left(P_L, P_H\right) = P_L\left(\theta' - \underline{\theta} + \theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H\left(P_L, P_H\right) = P_H\left(\bar{\theta} - \theta'' + \theta^* - \theta'\right) - \alpha q_H^2$$

leading to the following equilibrium prices:

$$P_L^* = \frac{\Delta_E \Delta\left(1 - \left(\underline{\theta} + \theta^*\right)\right)}{3\left(\Delta_E + \Delta\right)} \qquad P_H^* = \frac{\Delta_E \Delta\left(\underline{\theta} + \theta^* + 2\right)}{3\left(\Delta_E + \Delta\right)}$$

In this case both the low quality good and the high quality good are bought by uninformed and informed conumers. Then candidate equilibrium prices are affected by all parameters of the model. However one can notice that an increase in the share of informed consumers (lower θ^*) implies an increase of P_L^* and a reduction of P_H^*. While a decrease of this share (higher θ^*) has the opposite effect. Therefore in this case consumer information affects price competition, with opposite effect on firms. More informed consumers provide a price benefit to the low quality firm. Less informed consumers provide a price benefit to the high quality firm. Considering the restriction given by price domains we get a further restriction of θ^* at equilibrium.

$$\theta^* \leq min\left\{\frac{\Delta_E\left(1 + 2\underline{\theta}\right)}{\Delta_E + 3\Delta}, \frac{\Delta_E\left(2 + \underline{\theta}\right) + 3\left(\underline{\theta} + 1\right)\Delta}{2\Delta_E}\right\}$$

By substitution we can find equilibrium profit functions as follows:

$$\Pi_L^* = \frac{\Delta_E \Delta\left(1 - \left(\underline{\theta} + \theta^*\right)\right)^2}{9\left(\Delta_E + \Delta\right)} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E \Delta\left(\underline{\theta} + \theta^* + 2\right)^2}{9\left(\Delta_E + \Delta\right)} - \alpha q_H^2$$

Turning then to the quality selection stage, by profit maximization in qualities we get:

$$\frac{\partial \Pi_L}{\partial q_L} = -\frac{\gamma\left(q_E^2 + q_H^2 + 2q_L^2 - 2q_H q_0 - 2q_E q_L\right)}{\left(9q_E + 9q_H - 18q_L\right)^2} - 2\alpha q_L \qquad and \quad \gamma = \left[1 - \left(\underline{\theta} + \theta^*\right)\right]^2$$

$$\frac{\partial \Pi_L}{\partial q_L} \leq 0 \quad if: q_E \geq q_H \geq q_0\,,\; \gamma \geq 0$$

$$\frac{\partial^2 \Pi_L}{\partial q_L^2} = -\frac{2\gamma\left(q_E - q_H\right)^2}{81\left(q_E + q_H - 2q_0\right)^3} - 2\alpha \leq 0$$

Therefore[14] the low quality firm is lead to produce the minimum quality, i.e $q_L^* = q_0$ as in the previous case.

Concerning the high quality firm we get:

[14]proof appendix A3

$$\frac{\partial \Pi_H}{\partial q_H} = \frac{\varphi (q_E - q_0)^2}{9(q_E + q_H - 2q_0)^2} - 2\alpha q_H \geq 0 \qquad \varphi = (2 + \underline{\theta} + \theta^*)^2$$

$$\frac{\partial^2 \Pi_H}{\partial q_H^2} = -\frac{2\varphi (q_E - q_0)^2}{9(q_E + q_H - 2q_0)^3} - 2\alpha$$

$$q_H^* = -\frac{1}{3*2^{2/3}\alpha} (\Phi)^{1/3} + \frac{-324\alpha^2 q_E^2 + 1296\alpha^2 q_E q_0 - 1296\alpha^2 q_0^2}{(\Phi)^{1/3}} - \frac{2(\alpha q_0 - 2\alpha q_0)}{3\alpha}$$

According to the foc we get $q_H^* > 0$[15]. However when considering the restrictions given by the price domains, we can find that q_H^* is bounded, both upwards and downward.

$$q_0 + \frac{\Delta_E (2\theta^* - 2 - \underline{\theta})}{3\bar{\theta}} \leq q_H^* \leq q_0 + \frac{\Delta_E (1 + 2\underline{\theta} - \theta^*)}{3\theta^*}$$

Actually in this case the upper bound increases if θ^*decreases, but the lower bound decreases for a decrease of θ^*, implying that increasing the share of informed consumers can widen the set of optimal quality levels at equilibrium. However considering also the f.o.c., high quality can be given by an internal solution or a corner solution if $q_H^*$is outside the set of optimal quality.

Considering the corner solution $q_H^* = q_0 + \frac{\Delta_E(1+2\underline{\theta}-\theta^*)}{3\theta^*}$ we can add that there will always be some product differentiation in equilibrium, as $q_H^* > q_0$ just requires that $(1 + 2\underline{\theta}) > \theta^*$a condition which always holds, given the basic model. On the contrary if the corner solution is on the lower bound and $q_H^* = q_0 + \frac{\Delta_E(2\theta^*-2-\underline{\theta})}{3\bar{\theta}}$, to get product differentiation we need $\left(\theta^* > 1 + \frac{\underline{\theta}}{2}\right)$ a condition that not necessarily holds. However by considering the previous inequality one could observe that it can hold the lower the share of informed consumers is (higher θ^*) implying in turrn that the high quality firm can profit from a lower and lower share of informed consumers to provide a lower and lower quality level. Therefore we can state that $\Delta \longrightarrow 0$ as $\theta^* \longrightarrow 1$, implying that we cannot exclude cases where product differentiation is negligible, despite the prices paid by consumers purchasing the high quality good.

To further consider the effect of the share of informed consumers on product differentiation, one could check that $\frac{d(\frac{\Delta_E\left(1+2\underline{\theta}-\theta^*\right)}{3\theta^*})}{d\theta^*} = \frac{-3\Delta_E(1+2\underline{\theta})}{9\theta^{*2}} < 0$. Therefore an increase of θ^*, (i.e, a decrease of the share of informed consumers) reduces the set of optimal quality level q_H^* and leads to less product differentiation. While more informed consumers lead to more product differentiation. By considering again equilibrium prices we point-out that

[15]Let us consider

$$\begin{aligned}\Phi &= -3\varphi^2\alpha (q_E - q_0) + \sqrt{3}\sqrt{8\alpha^5 q_E^3 \varphi (q_E - q_0)^2 - 48\alpha^5 q_E^2 q_0 \varphi (q_E - q_0)^2} + \\ &+ \sqrt{3}\sqrt{-64\alpha^5 q_0^3 \varphi (q_E - q_0)^2 + 96\alpha^5 q_E q_0^2 \varphi (q_E - q_0)^2 + 3\alpha^4 \varphi (q_E - q_0)^4} \\ &+ \sqrt{3}\sqrt{+96\alpha^5 q_E q_0^2 \varphi (q_E - q_0)^2 + 3\alpha^4 \varphi (q_E - q_0)^4} + \left(-4\alpha^3 q_E^3 + 24\alpha^3 q_E^2 q_0 - 48\alpha^3 q_E q_0^3 + 32\alpha^3 q_0^3\right)\end{aligned}$$

and check appendix A3

a larger share of informed consumers also reduces P_H^* and increases P_L^*. Therefore an increasing share of informed consumers benefits also uninformed consumers buying high quality goods, as they are provided higher quality levels at lower prices. The benefits are extended to the low quality firm while low quality consumers are affected by a negative externality as they are provided the same quality level at an higher price.

1.5.1.3 A.b.3

Considering the price domains :

$$P_H - \theta^* \Delta_E \leq P_L^* \leq P_H - \bar{\theta}\Delta$$

$$P_L + \bar{\theta}\Delta \leq P_H^* \leq P_L + \theta^* \Delta_E$$

And the realtive demand segmentswe get the following profit functions

$$\Pi_L (P_L, P_H) = P_L \left((\bar{\theta} - \underline{\theta}) - \theta^* + \theta'\right) - \alpha q_L^2 \qquad \Pi_H (P_L, P_H) = P_H \left((\theta^* - \theta'\right) - \alpha q_H^2$$

leadingto the following candidate equilibrium prices:

$$P_L^* = \frac{\Delta_E (2 - \theta^*)}{3} \qquad P_H^* = \frac{\Delta_E (1 + \theta *)}{3}$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E (2 - \theta^*)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E (1 + \theta^*)^2}{9} - \alpha q_H^2$$

Checking if equilibrium prices belongs to the price domain characterizing A.b.3, we get a further restictinion on θ^*:

$$\theta^* \geq \frac{1}{2} + \frac{3\Delta\bar{\theta}}{2\Delta_E}$$

Considering the quality selection stage we get:

$$\frac{\partial \Pi_L}{\partial q_{\mathrm{L}}} = -\frac{{\theta^*}^2 - 4\theta^* + 4}{9} - 2\alpha q_L \leq 0$$

$$\frac{\partial^2 \Pi_L}{\partial q_L^2} = -2\alpha q_L$$

Still leading to $q_{\mathrm{L}}^* = q_0$. Concerning the high quality firm, by considering profit maximization with respect to quality we get:

$$\frac{\partial \Pi_H}{\partial q_H} = -2\alpha q_H ; \frac{\partial^2 \Pi_H}{\partial q_H^2} = -2\alpha$$

The previous f.o.c. and s.o.c. account for the negative effect of cost on the level of quality. And by considering the restriction on equilibrium prices arising from the price domain we obtain

$$q_H \leq q_0 + \frac{\Delta_E (2\theta^* - 1)}{3\bar{\theta}}$$

Therefore by jointly considering both the f.o.c. and the previous restriction we find that the firm will find it optimal to provide the lowest possible quality: $q_H^* \to q_0$. As in this sub-case (A.b.3) $\theta'' = \bar{\theta}$, considering that across case A.b most consumers are uninformed, then the small share of informed consumers with an high willingness to pay $(\bar{\theta} - \theta^*)$ finds it optimal to purchase low quality goods, together with uninformed consumers with the lowest willingness to pay $(\theta' - \underline{\theta})$. Being $D_H (P_L, P_H) = (\theta' - \theta^*)$, we can notice that only uninformed consumers with an intermediate willingness to pay buy high quality goods. These consumers are cheated in equilibrium, as they pay an higher price with respect to all other consumers to get the same quality q_0.

Therefore there is no real product idfferentiation in equilibrium, as $q_H^* \to q_0$ even if consumers pay a price premium according to the expected quality differential Δ_E. Due both to consumers misperceptions by uninformed consumers and informed consumers buying low quality goods, there is no incentive for the high quality firm to supply a quality level greater than the MQS, while P_L^*,and P_H^* both reflect the expected quality differential Δ_E.

Therefore better information can prevent consumers to be cheated about quality, as they buy low quality goods, but still they pay an higher price with respect to the full information case as price competition is affected by uninformed consumers and their misperceptions. Actually the high share of uninformed consumers exert a negative externality on informed ones, due to equilibrium prices reflecting a quality differential which is not really provided by the high quality firm. Equilibrium prices are then distorted upwards.

However to the extent that P_L^*, and P_H^* also depend on θ^* one can check that an increase of informed consumers (lower θ^*) leads to a decrease of P_H^* and to an increase of P_L^*, that reduces the price difference. Information counterbalances optimistic misperceptions and then reduces price distortions. On the contrary an higher and higher θ^* has just the opposite effect as, by reducing the number of informed cosnumers, leads to an increase of the price difference. By checking equilibrium profits one can see that more information implies further gains for the low quality firm and further losses for the high quality firm.

Therefore informed consumers exert a negative externality on high quality firms and a positive externality on the low quality firm. Acordingly more informed consumers exert a positive externality on uninformed ones by reducing the price they pay to buy "virtual" high quality products. On the contrary more informed consumers cotribute to increase the price of low quality goods (i.e the products bought by them), due to the effect of consumer information on price competition.

1.5.2 Case A.c, Most consumers are informed

In this case $\left(\frac{\theta^*}{\underline{\theta}} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\theta^*}\right)$ and $(\theta^* \leq \sqrt{\underline{\theta}\bar{\theta}})$

1.5.2.1 A.c.1

$$P_H - \underline{\theta}\Delta_E \leq P_L^* \leq P_H - \theta^*\Delta$$

$$P_L + \theta^*\Delta \leq P_H^* \leq P_L + \underline{\theta}\Delta_E$$

By considering demand segments defined by the previous price domain we can get the following profit functions:

$$\Pi_L\left(P_L, P_H\right) = P_L\left(\theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H\left(P_L, P_H\right) = P_H\left((\bar{\theta} - \underline{\theta}) - \theta'' + \theta^*\right) - \alpha q_H^2$$

leading to the following equilibrium prices:

$$P_L^* = \frac{\Delta\left(1(\bar{\theta} - \underline{\theta}) - \theta^*\right)}{3} \qquad P_H^* = \frac{\Delta\left(2(\bar{\theta} - \underline{\theta}) + \theta*\right)}{3}$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta\left(1 - \theta^*\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta\left(2 + \theta^*\right)^2}{9} - \alpha q_H^2$$

By considering that equilibrium prices should belong to the above price domain we get a further restriction on θ^*:

$$\theta^* \leq min\left\{1\,,\ \frac{3\underline{\theta}\Delta_E}{2\Delta} - \frac{\bar{\theta} - \theta}{2}\right\}$$

Considering then the quality selection stage, by profit maximization in qualities we get:

$$\frac{\partial \Pi_L}{\partial q_L} = -\frac{{\theta^*}^2 - 2\theta^* + 1}{9} - 2\alpha q_L \leq 0; \frac{\partial^2 \Pi_L}{\partial q_0^2} = -2\alpha$$

$$\frac{\partial \Pi_H}{\partial q_{\mathrm{H}}} = \frac{{\theta^*}^2 + 4\theta^* + 4}{9} - 2\alpha q_H$$

trough the f.o.c. we get the optimal quality level

$$q_H^* = \frac{{\theta^*}^2 + 4\theta^* + 4}{18\alpha}$$

Actually considering the restrictions given by the price domain we can show that the following condition about expected quality holds in equilibrium:

$$q_E \geq q_0 + \frac{\Delta\left(2\theta^* + 1\right)}{3\underline{\theta}}$$

This lower bound, on expected quality, ensure that uninformed consumers $(\theta^* - \underline{\theta})$ purchase high quality.

High quality goods are also bought by informed consumers with the greatest wllingness to pay $\left(\bar{\theta} - \theta''\right)$. Accordingly the low quality good is bought by informed consumers, with an intermediate willingness to pay:.$(\theta'' - \theta^*)$. As one can check, both demands depend on the real quality differential, then also equilibrium prices will reflect Δ. The greater share of informed consumers then prevents consumers'misperception from affecting equilibrium prices. Competition between the two firms only depends on θ'' and θ^*. The avoided price distortion, due to most consumers being informed, benefits all cosnumers (then informed consumers exert a positive externality on uninformed ones) and reduces firms'profits with respect to the case where prices depend on Δ_E (informed consumers reduce profitability for both firms).

Furthermore we can notice that the higher the share of informed consumers (the lower is θ^*) the higher is P_L^* (as well as Π_L^*) and the lower is P_H^*.(as well as Π_H^*). Actually the lower is θ^* the lower the demand for high quality good arising from (uninformed) consumers with a lower willingness to pay and the higher the demand for low quality goods due to (informed) consumers with an intermediate willingness to pay. Actually the increase in the share of informed consumers leads the high quality firm to retain market shares by reducing prices. As the share of uninformed consumers shrinks, more consumers with a low willingness to pay will switch to low quality goods, i.e. the demand for high quality goods by uninformed consumers shrink, due to a lower θ^*. Then the high quality firm gathers either uninformed cosnumers with a lower and lower willingness to pay and informed consumers with the highest willingness to pay. A reduction of P_H^* when θ^*reduces is then justified as it can help the firm both to keep its market share arising from uninformed consumers and to widen its market share due to informed consumer. Actually a reduction of P_H^* reduces the value of θ'', and can extend the demand segment given by $\left(\bar{\theta} - \theta''\right)$ .One could also state that as the location of θ^* moves towards $\underline{\theta}$, then the optimal strategy is to move the location of θ'' towards θ^*, until $\theta'' = \theta^*$.

The low quality firm that just gathers informed consumers with an intermediate willingness to pay can profit by the exogenous reduction of θ^* but is hurt by the reduction of P_H^* that by reducing θ'' negatively affect $D_L = (\theta'' - \theta^*)$. Actually given that the location of $\theta^*$is exogenously given, and considering that the low quality firm, due to a decrease of θ^* is more and more gathering previously uninformed consumers with an higher willingness to pay, once they switch to low quality they can be charged an higher price. Therefore, it is profitable for the low quality firm to increase its price proportionally to the decrease of θ^*, though an increase of $P_L$also contributes to decrease θ''. Therefore an increase of P_L is optimal until the increase in the mark-up of the firm is greater with respect to the market share lost, according to a decrease of θ''. On the contrary if θ^* increases and the share of informed consumers shrinks P_L proportionally decreases and P_H proportionally increases, implying an increase of θ''. In some cases when the value of the market, as measured by the increase of $\underline{\theta}$, is high and $\theta^* = 1$,the low quality firm could even be excluded from the market and the high quality firm becomes a monopolist.

Given the previous analysis we can then state that both an increase and a decrease of the share of informed consumers affects price competition between the two firms, as the difference in prices widens with the share of informed consumers, but with more informed consumers low quality firms can charge higher prices while with less informed consumers it is the high quality firm that can afford a price increase.

With respect to product differentiation we can state that quality competition is softened by the increase in the share of informed consumers while more vertical differentiation arises if the share of informed consumers shrinks Actually one can check that with more informed consumers a positve quality differential $(q_H - q_0)$ is provided by the high quality firm if $q_H^* = \frac{{\theta^*}^2 + 4\theta^* + 4}{18\alpha} > q_0$. However one can easily check that q_H^* is an increasing function of θ^*. A lower θ^* implies that with an increase of informed consumers it is optimal for the high quality firm to provide a lower level of quality. This occurs because with a lower and lower θ^* the high quality firm gathers more and more consumers with a lower willingness to pay for quality (then it is not worthwhile to increase quality too much, as when consumers become informed they expect a lower quality differential) and moreover by reducing the level of quality the high quality firm can also attract consumers with an intemediate willingness to pay without loosing consumers with the highest willingness to pay. As we saw before, these consumers may be attracted by reducing P_H^* with the effect of reducing θ'', entering then in competition with the low quality firm. However a reduction of q_H reduces Δ as well, implying a greater θ'' in equilibrium,leading in turn to an increase in the demand for low quality goods by informed consumers and a decrease in the demand for high quality goods by these same consumers.

On the contrary an increase of θ^*, by reducing the share of informed consumers leads the high quality firm to provide an high quality differential, to charge an higher price and gain higher profits for the high quality good. On the contrary the low quality firm is lead to reduce prices and obtains lower profits in equilibrium. Less informed consumers imply then more product differentiation and still asymmetric effects on price competition, as P_H^* increases and P_L^* decreases.

1.5.2.2 A.c.2

This sub-case is completely equivalent to case A,b.2 and is analyzed in Appendix A4

1.5.2.3 A.c.3

In this sub-case we consider the following price domains

$$P_H - \bar{\theta}\Delta \leq P_L^* \leq P_H - \theta^* \Delta_E$$

$$P_L + \theta^* \Delta_E \leq P_H^* \leq P_L + \bar{\theta}\Delta$$

and given the related demand segments we get the following profit functions:

$$\Pi_L \left(P_L, P_H\right) = P_L \left(\theta'' - \underline{\theta}\right) - \alpha q_L^2 \qquad \Pi_H \left(P_L, P_H\right) = P_H \left(\bar{\theta} - \theta''\right) - \alpha q_H^2$$

Leading to the following candidate equilibrium prices:

$$P_L^* = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \qquad P_H^* = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3}$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)^2}{9} - \alpha q_H^2$$

By considering the price domain we get the following restictinctions on θ^*:

$$\theta^* \leq \frac{\Delta\left(2\underline{\theta} + 1\right)}{3\Delta_E}$$

Considering then quality competition, by maximization of the respective profit functions we get:

$$\frac{\partial \Pi_L}{\partial q_L} = \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{9} - 2\alpha q_L \leq 0$$

and

$$\frac{\partial \Pi_H}{\partial q_H} = \frac{\bar{\theta}^2 + 4\underline{\theta}^2 - 4\bar{\theta}\underline{\theta}}{9} - 2\alpha q_H = 0$$

considering the f,o,c, we get the following interior solution for q_H :

$$q_H^* = \frac{\left(\bar{\theta} - 2\underline{\theta}\right)^2}{18\alpha}$$

and considering price domains we also get the following restriction concerning q_H^*

$$q_H^* \geq q_0 + \frac{3\Delta_E\theta^*}{1 + 2\underline{\theta}}$$

In this sub-case it is worthwhile to consider the previous restrictions on $\theta^* \leq \frac{\Delta\left(2\underline{\theta}+1\right)}{3\Delta_E}$.As across case A.c most consumers are informed due to $\theta^* \leq \sqrt{\underline{\theta}\bar{\theta}}$, the previous restrictions allow for a even larger number of consumers being informed as $\theta^* \leq \frac{\Delta\left(2\underline{\theta}+1\right)}{3\Delta_E} \leq \sqrt{\underline{\theta}\bar{\theta}}$. Furthermore, being $\theta^* < \theta' < \theta''$, $D_L = (\theta'' - \theta^*) + (\theta^* - \underline{\theta}) = (\theta'' - \underline{\theta})$ and then $D_H = \left(\bar{\theta} - \theta''\right)$, one can see that in this sub-case the model is equivalent to vertical differentiation with perfect information.

1.6 Market demands and Equilibrium Analysis when uninformed consumers are pessimistic

In the pessimistic case uninformed consumers are suspicious and underestimate the quality differential: $\Delta_E \leq \Delta$,as $q_E \leq q_H$ [16].

[16] If quality were exogenous and consumers knew the minimum and the maximum quality and the probability distribution (Akerlof 1970), one could also state that $q_E = E(q_H) = q_L p + q_H(1-p)$,where

In order to define demand functions we can follow the same steps as in the optimistic case. Concerning alternative locations of θ^* in the space $\left[\underline{\theta}, \bar{\theta}\right]$ we are lead to consider three cases that lead us to list the possible demand segments related to each price domain to be considered when defining demand functions:

B.1) $\left(\underline{\theta} \leq \theta'' \leq \theta^* \leq \theta' \leq \bar{\theta}\right)$. This case is represented in fig 1.8 Informed consumers only buy high quality goods, while uninformed consumers only buy low quality goods. Thus information disparities create a separation between the two markets. We get: $D_L\left(P_L, P_H\right) = \theta^* - \bar{\theta}$ and $D_H\left(P_L, P_H\right) = \bar{\theta} - \theta^*$. Therefore market demands are affected only by the weight of informed consumers and result to be perfectly inelastic with respect to prices. Actually in this case neither the marginal uninformed consumers nor the marginal informed consumer can affect market demands, which end up being inelastic to prices. From this point of view one could state that vertical differentiation mitigates adverse selection, as both products can be sold in equilibrium, though neither $D_L\left(P_L, P_H\right)$ nor $D_H\left(P_L, P_H\right)$ turn out to be sensitive to prices and such to completely cover the market, unless all consumers are either informed or uninformed. Information disparities imply as a further effect perfectly inelastic demands. The restrictions on price domains arising from B.1 are given by:

$$
\begin{aligned}
P_H - \theta^*\Delta &\leq P_L \leq P_H - \underline{\theta}\Delta && (1.15)\\
P_H - \bar{\theta}\Delta_E &\leq P_L \leq P_H - \theta^*\Delta_E && (1.16)
\end{aligned}
$$

and by:

$$
\begin{aligned}
P_L + \underline{\theta}\Delta \leq P_H &\leq P_L + \theta^*\Delta && (1.17)\\
P_L + \theta^*\Delta_E \leq P_H &\leq P_L + \bar{\theta}\mathbf{\Delta_E} && (1.18)
\end{aligned}
$$

B.2) $\left(\underline{\theta} \leq \theta'' \leq \theta' \leq \theta^* \leq \bar{\theta}.\right)$ .See fig 1.9 Informed consumers only buy high quality goods while uninformed consumers buy both low quality and high quality goods. Thus we get $D_L\left(P_L, P_H\right) = \theta' - \bar{\theta}$ and $D_H\left(P_L, P_H\right) = \bar{\theta} - \theta^* + \theta^* - \theta' = \bar{\theta} - \theta'$. And demands will be affected only by the uninformed marginal consumer. The restrictions on price domains arising from B.2 are given by:

$$P_H - \theta^*\Delta_E \leq P_L \leq P_H - \underline{\theta}\Delta \qquad (1.19)$$

---

p and $(1-p)$ are their prior probailities about the high quality firm delivering a low or high quality level. As we deal with credence goods no updating of the probaility distribution is possible. However we can remark that when we consider the quality differential, with this approach we would get $\Delta_E = q_L p + q_H(1-p) - q_L = (1-p)(q_H - q_L)$. Then we always obtain $\Delta_E \leq \Delta$,We can then notice that by considering expected quality as above: 1) we would always end up in the case of pessimistic consumers 2) we are in a typical adverse selction framework. Therefore, as far as vertical differentiation with asymmetric information is concerned, pessimisitic consumers beliefs implies adverse selection (and viceversa). However, being in a vertically differentiated duopoly, the effect of adverse selection is expected to be different with respect to the standard case, as both low quality goods and high quality goods can be sold in equilibrium. Therefore product differentiation is expected to mitigate adverse selection

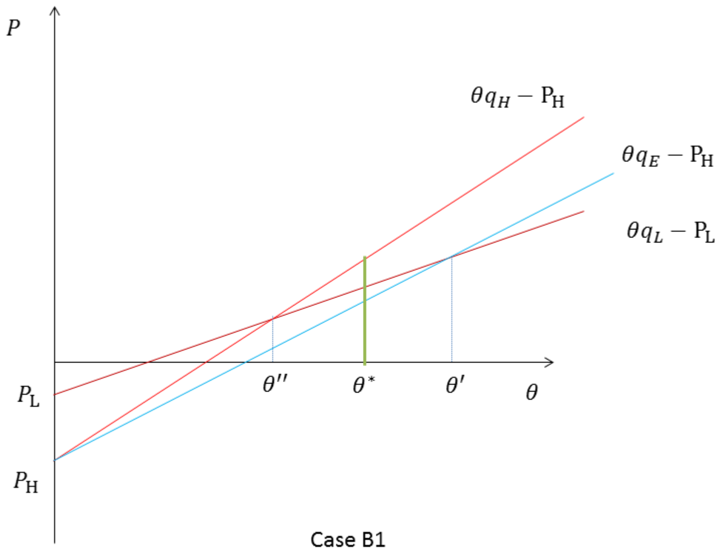

Figure 1.8:

and:

$$P_L + \underline{\theta}\Delta \leq P_H \leq P_L + \theta^*\Delta_E \tag{1.20}$$

B.3)$(\ \underline{\theta} \leq \theta^* \leq \theta'' \leq \theta' \leq \bar{\theta})$See fig. 1.10. In this case uninformed consumers buy low quality goods while informed consumers buy both high quality and low quality goods. Thus we get $D_L(P_L, P_H) = \theta^* - \underline{\theta} + \theta'' - \theta^* = \theta'' - \underline{\theta}$ and $D_H(P_L, P_H) = \bar{\theta} - \theta''$. Therefore in this case market demands are affected by the marginal informed consumer. The restrictions on price domains arising in this case are the following:

$$P_H - \bar{\theta}\Delta_E \leq P_L \leq P_H - \theta^*\Delta \tag{1.21}$$

and

$$P_L + \theta^*\Delta \leq P_H \leq P_L + \bar{\theta}\Delta_E \tag{1.22}$$

As a second step we consider joint restrictions on $\frac{\Delta_E}{\Delta}$, $\frac{\theta^*}{\underline{\theta}}$, $\frac{\bar{\theta}}{\theta^*}$, and as in the case of optimistic consumers we can define market demand functions in four alternative cases,:

$$B.a)\frac{\underline{\theta}}{\bar{\theta}} \leq \frac{\Delta_E}{\Delta} \leq Min\left\{\frac{\theta^*}{\bar{\theta}}, \frac{\underline{\theta}}{\theta^*}\right\}; B.b)\frac{\theta^*}{\bar{\theta}} \leq \frac{\Delta_E}{\Delta} \leq \frac{\underline{\theta}}{\theta^*}; \tag{1.23}$$

$$B.c)\frac{\underline{\theta}}{\theta^*} \leq \frac{\Delta_E}{\Delta} \leq \frac{\theta^*}{\bar{\theta}}; B.d)Max\left\{\frac{\theta^*}{\bar{\theta}}, \frac{\underline{\theta}}{\theta^*}\right\} \leq \frac{\Delta_E}{\Delta} \leq 1 \tag{1.24}$$

Considering the ratio $\frac{\Delta_E}{\Delta} \leq 1$, the previous restrictions allow for the lowest expected quality differential Δ_E in case B.a (over-pessimistic consumers) and the highest ratio

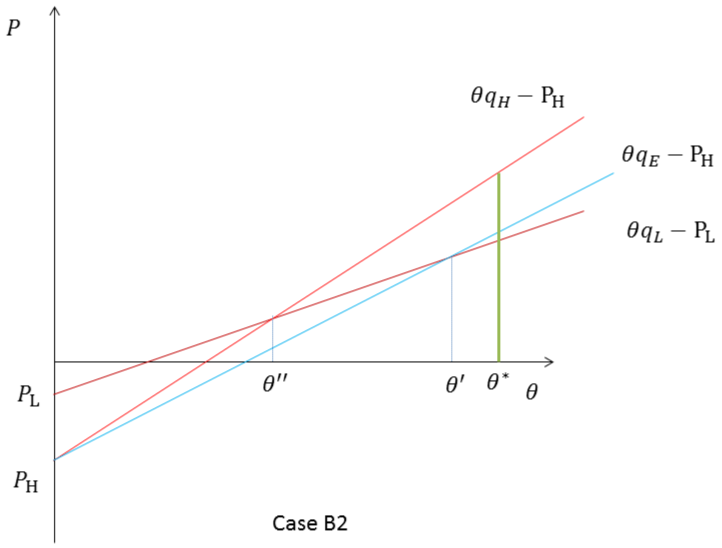

Figure 1.9:

$\frac{\Delta_E}{\Delta}$ in case B.d (consumers are only slightly pessimistic). In between the two extremes, we find intermediate cases like B.b and B.c,Furthermore the ratio is bounded in each case through restrictions on the share of informed-uninformed consumers (location of θ^*) and the value of the willingness to pay (as a proxy of the value of the market under consideration).

In cases B. b and B.c the restrictions are such that we can respectively state that most consumers are informed (as $\theta^* \leq \sqrt{\bar{\theta}\underline{\theta}}$) and most consumers are uninformed, (as $\theta^* \geq \sqrt{\bar{\theta}\underline{\theta}}$). We shall consider in detail the definitions of demand functions and equilibrium prices in case B.d, both because demand functions in the other cases turn out to be combinations of the demand segments already considered in case B.d (thus candidate equilibrium prices turn out to be the same) and also due to the fact that demand functions in the other cases may be either perfectly inelastic to prices or even not defined across some price domain.

1.6.1 Demand functions

Considering then case B.d we assume that the following restrictions hold: $P_H - \underline{\theta}\Delta \geq P_H - \theta^*\Delta_E \geq P_H - \theta^*\Delta \geq P_H - \bar{\theta}\Delta_E$ and $P_L + \bar{\theta}\Delta_E \geq P_L + \theta^*\Delta \geq P_L + \theta^*\Delta_E \geq P_L + \underline{\theta}\Delta$ to get:

$$max\left\{\frac{\theta^*}{\bar{\theta}}, \frac{\underline{\theta}}{\theta^*}\right\} \leq \frac{\Delta_E}{\Delta} \leq 1$$

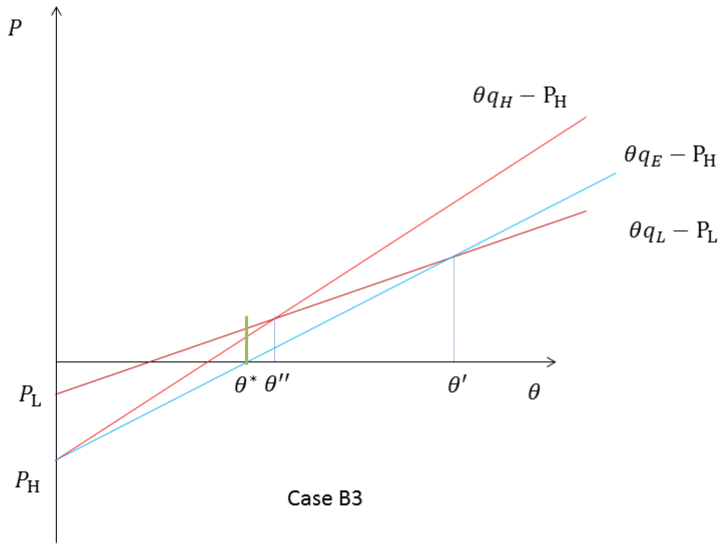

Figure 1.10:

The previous restriction allow for most consumers being either informed or uninformed. By sticking to that restriction, as to demand functions and their price domains we get:

$$D_L\left(P_L,P_H\right)=\begin{cases}\theta'-\underline{\theta} \quad if & P_H-\theta^*\Delta_E\leq P_L\leq P_H-\underline{\theta}\Delta\\ \theta^*-\underline{\theta} \quad if & P_H-\theta^*\Delta\leq P_L\leq P_H-\theta^*\Delta_E\\ \theta''-\underline{\theta} \quad if & P_H-\bar{\theta}\Delta_E\leq P_L\leq P_H-\theta^*\Delta\\ \bar{\theta}-\underline{\theta} \quad if & 0\leq P_L\leq P_H-\bar{\theta}\Delta_E\end{cases}$$

Considering the price domains one at a time we can show that in the first price domain we are in case B.2, as described above. In the second price domain we are in case B.1 and in the third price domain we are in case B.3.

Considering the highest price, the demand for the low quality good is just given by uninformed consumers with the lowest willingness to pay. When prices are lower the uninformed marginal consumer moves towards $\theta^*$and also uninformed consumers with an intermediate willingness to pay switch to low quality goods. As shown also by fig. 1.8 one can check that even for a set of lower and lower prices belonging to the second price domain, $D_L\left(P_L,P_H\right)$ is inelastic to prices . Actually when all uninformed consumer buy low quality goods, informed consumers, who can evaluate the real quality differential, need to pay a very low price (than pessimistic uninformed consumers) to switch to low quality goods. A further decrease of P_L leads some of them to switch, until P_L is so low that the low quality firm can cover the entire market.

As to the demand for high quality goods, it is complementary to $D_L(P_L, P_H)$ and can be expressed as follows:

$$D_H(P_L, P_H) = \begin{cases} \bar{\theta} - \theta'' & : if: \ P_L + \theta^*\Delta \leq P_H \leq P_L + \bar{\theta}\Delta_E \\ \bar{\theta} - \theta^* & : if \ \ P_L + \theta^*\Delta_E \leq P_H \leq P_L + \theta^*\Delta \\ \bar{\theta} - \theta' & : if \ \ P_L + \underline{\theta}\Delta \leq P_H \leq P_L + \theta^*\Delta_E \\ \bar{\theta} - \underline{\theta} & : if \ \ 0 \leq P_H \leq P_L + \underline{\theta}\Delta \end{cases}$$

With the highest prices, the high quality good is demanded just by informed consumers with the highest willingness to pay, as they know the real quality differential. Informed consumers with an intermediate willingness to pay are those buying the low quality good. When P_H decreases even the latter switch to high quality goods until all informed cosnumers buy them. When the second price domain is reached then $D_H(P_L, P_H)$ becomes inelastic to prices as the pessimist beliefs of uninformed consumers are such that a significant decrease of P_H is needed to persuade them to buy the high quality good. Then for further price decreases $D_H(P_L, P_H)$ is such to capture all the uninformed consumers until the entire market is covered by the high quality firm.

Demand functions are then represented in fig 1.11, 1.12

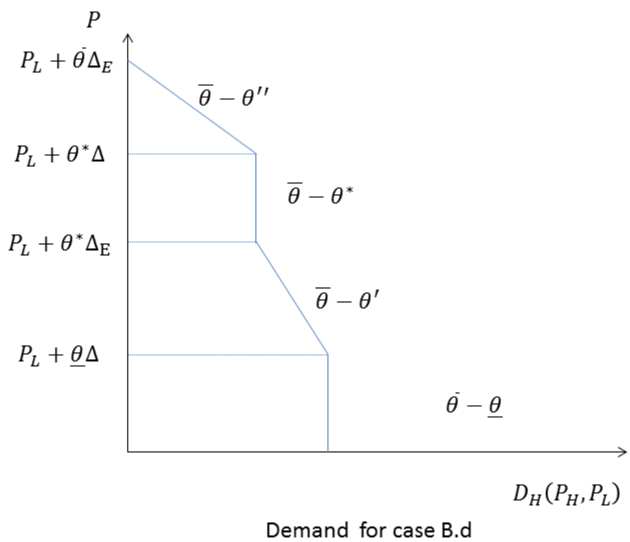

Figure 1.11:

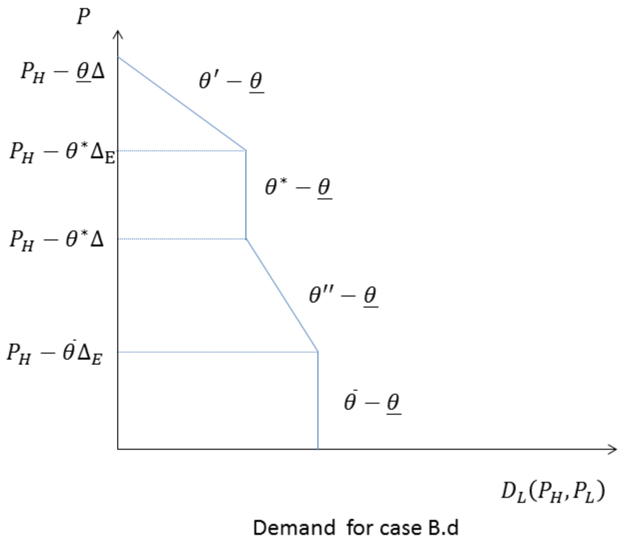

Figure 1.12:

1.6.2 Equilibrium analysis with pessimistic consumers

Still restricting our attention to case B.d, and then considering the demand functions we have just analyzed, we look for candidate equilibrium prices for each price domain. We make reference to the concept of Nash Equilibrium, as analyzed in section 1.4 to discuss the optimistic case.

1.6.2.1 B.d.1

By considering the first price domain, equilibrium prices should be such that:

$$P_H - \theta^* \Delta_E \le P_L^* \le P_H - \underline{\theta}\Delta$$

$$P_L + \underline{\theta}\Delta \le P_H^* \le P_L + \theta^* \Delta_E$$

leading to the following profit functions:

$$\Pi_L\left(P_L, P_H\right) = P_L\left(\theta' - \underline{\theta}\right) - \alpha q_L^2 \qquad \Pi_H\left(P_L, P_H\right) = P_H\left(\bar{\theta} - \theta'\right) - \alpha q_H^2$$

and the following candidate equilibrium prices:

$$P_L^* = \frac{\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \qquad P_H^* = \frac{\Delta_E\left(2\bar{\theta} - \underline{\theta}\right)}{3}$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E \left(\bar{\theta} - 2\underline{\theta}\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E \left(2\bar{\theta} - \underline{\theta}\right)^2}{9} - \alpha q_H^2$$

By furtherly restricting the share of informed consumers we get:

$$\theta^* \geq \frac{2\underline{\theta} + 1}{3}$$

Considering then the quality choice we get:

$$\frac{\partial \Pi_L}{\partial q_L} = \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{9} - 2\alpha q_L \leq 0$$

implying then:

$$q_L = \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{18\alpha} \qquad \leq 0$$

As low quality in equilibrium should be as low as possible, the low quality firm will stick to the MQS. Therefore the low quality firm will supply $q_L = q_0$.Considering then the high quality firm, the foc imply:

$$\frac{\partial \Pi_H}{\partial q_H} = -2\alpha q_L < 0$$

by considering the restriction on equilibrium prices in addition, we get:

$$q_H \leq q_0 + \frac{\Delta_E \left(2\underline{\theta} + 1\right)}{3}$$

Considering that with pessimistic consumers $\Delta \geq \Delta_E$, the previous condition holds for $\Delta = \Delta_E$, implying in turn that $\underline{\theta} = 1$. Therefore pessimistic expectations lead the high quality firm to supply $q_H = q_E$, i.e. the quality expected by uninformed consumers. Actually if the high quality firm decided to supply the MQS (as implied by the foc) informed consumers will switch to the low quality good, therefore it is convenient for this firm to increase q_H above the MQS in order to retain the goodwill of informed consumers, even though the pessimistic expectations constrain the size of the quality differential. Informed consumers in this case still exert a positive externality on uninformed ones as the quality level will not drop to the MQS. Moreover as $q_H = q_E$ equilibrium prices will reflect the quality differential provided by the high quality firm.

1.6.2.2 B.d.2

In this sub case we consider the following restriction on equilibrium prices:

$$P_H - \theta^*\Delta \leq P_L^* \leq P_H - \theta^*\Delta_E$$

$$P_L + \theta^*\Delta_E \leq P_H^* \leq P_L + \theta^*\Delta$$

And given that demand segments are such that $D_L\left(P_L,P_H\right)=\left(\theta^*-\underline{\theta}\right)$ and $D_H\left(P_L,P_H\right)=\left(\bar{\theta}-\theta^*\right)$, the profit functions are given by :

$$\Pi_L(P_L,P_H)=P_L\left(\theta^*-\underline{\theta}\right)-\alpha q_L^2 \qquad \Pi_H(P_L,P_H)=P_H\left(\bar{\theta}-\theta^*\right)-\alpha q_H^2$$

In this sub case, as demand functions are perfectly inelastic to price, we observe that the market is split between informed consumers, buying high quality goods, and uninformed consumers buying low quality goos. Then in this price domain we observe market segmentation according to information disparities. Equilibrium prices should be just consistent with the restriction concerning the price domain. As perfectly inelastic demands imply that the firm can charge a price as high as possible, but included in the price domain, we are lead to consider the following candidate equilibrium prices:

$$\begin{aligned} P_L^* &= P_H-\theta^*\Delta_E \\ P_H^* &= P_L+\theta^*\Delta \end{aligned}$$

Therefore a continuum of price equilibria can exist. If we resort to the condition which ensures that the market is covered one equilibrium is the following: the maximun price that firm L can charge is $P_L^*=\underline{\theta}q_0$, the price of firm H follows from the restriction on the price domain ($P_H^*\leq P_L^*+\theta^*\Delta$), i.e $P_H=\underline{\theta}q^\circ+\theta^*\Delta$.

Equilibrium profits will be given by:

$$\Pi_L^*=\underline{\theta}q_0\left(\theta^*-\underline{\theta}\right)-\alpha {q_0}^2 \qquad \Pi_H^*=\left(\underline{\theta}q_0+\theta^*\Delta\right)\left(\bar{\theta}-\theta^*\right)-\alpha q_H^2$$

No further restrictions on $\theta^*$are necessary in this case, but those already arising from the price domain. In this equilibrium the low quality firm supplies the MQS to uninformed consumers. Concerning the high quality firm we can consider the foc:

$$\frac{\partial \Pi_H}{\partial q_H}=\bar{\theta}\theta^*-{\theta^*}^2-2\alpha q_H=0$$

and the soc:

$$\frac{\partial^2 \Pi_H}{\partial q_H^2}=-2\alpha$$

through the foc we get:

$$q_H=\frac{\bar{\theta}\theta^*-{\theta^*}^2}{2\alpha}>0$$

therefore the high quality firm is lead to increase q_H with the increase of informed consumers (i.e when θ^* decreases).

1.6.2.3 B.d.3

Considering the restriction on equilibrium prices:

$$P_H-\bar{\theta}\Delta_E\leq P_L\leq P_H-\theta^*\Delta$$

$$P_L + \theta^* \Delta \leq P_H \leq P_L + \bar{\theta}\Delta_E$$

and the demand segments resulting from this restriction, we get the following profit functions:

$$\Pi_L\left(P_L, P_H\right) = P_L\left(\theta'' - \underline{\theta}\right) - \alpha q_L^2 \qquad \Pi_H\left(P_L, P_H\right) = P_H\left(\bar{\theta} - \theta''\right) - \alpha q_H^2$$

and obtain the following candidate equilibrium prices:

$$P_L^* = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \qquad P_H^* = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3}$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)^2}{9} - \alpha q_H^2$$

We can then notice that in this case, equilibrium prices and profits boil down to the case of vertical differentiation with perfect information. Therefore prices will reflect the real quality differential provided by the high quality firm. By considering that the candidate equilibrium prices should be included in the price domain given above, we get a further restriction on θ^*:

$$\theta^* \leq \frac{2\underline{\theta} + 1}{3}$$

This restriction then implies that in this sub-case the share of informed consumers is likely to be high, and therefore is consistent with the previous equilibrium results as the share of informed consumers need to be high enough to let prices aand priofits collapse to the perfect information case. Considering then the quality choice we get:

$$\frac{\partial \Pi_L}{\partial q_L} = \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{9} - 2\alpha q_L$$

implying then:

$$q_L = \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{18\alpha} \quad \leq 0$$

As low quality in equilibrium should be as low as possible, the low quality firm will stick to the MQS and then $q_L = q_0$.Concerning the high quality firm, by considering the foc we get

$$\frac{\partial \Pi_H}{\partial q_H} = \frac{\bar{\theta}^2 + 4\underline{\theta}^2 - 4\bar{\theta}\underline{\theta}}{9} - 2\alpha q_H$$

$$q_H^* = \frac{\bar{\theta}^2 + 4\underline{\theta}^2 - 4\bar{\theta}\underline{\theta}}{18\alpha} \geq 0$$

Therefore the high quality firm, in equilibrium is lead to provide a real quality differential $\Delta = q_H^* - q_0$.Therefore in equilibrium, the share of informed cosnumers should be high eneough to let prices reflect the real quality differential as in a standard model of vertical differentiation and product differentiation will be such to relax price competition. Interestingly this result holds even though not all consumers are informed, what matters is the extension of the share of informed consumers in equilibrium.

1.7 Conclusions

In this model we have considered vertical product differentiation in a duopoly with credence goods. According to our assumptions both low quality and high quality firms comply with a MQS and consumers have rational expectations about low quality, as low quality firms actually choose to provide the MQS in equilibrium. High quality remains completely unknown to consumers unless they are informed according to their willingness to pay for quality and their information about product quality. Uninformed consumers make decisions according to their misperceptions about the quality differential. A difference endogenously arises in the model between optimistic and pessimistic uninformed consumers, due to the location of the informed marginal consumers with respect to the uninformed marginal consumer. The shares of informed and uninformed consumers depend on the location of a further marginal consumer. Such a location is exogneously given but in our analysis we consider the definition of demand functions and then sub-game perfect equilibria for any possible split between informed and uninformed consumers in the market.

The difference between optimistic uninformed consumers and pessimistic uninformed consumers arises endogenously into the model and we analyse these two cases sparately from each other. Therefore we cannot consider random distributions of consumers beliefs across the population, but we can analyse markets with vertical differentiation where either optimistic beliefs or pessimistic beeliefs prevail across uninformed consumers. We claim that competition between branded drugs and lately introduced generics can be a good example of a market for a credence good with a MQS, where uninformed consumers are characterized by optimistic beliefs (as in the case of brand loyalty). On the contrary in markets where firms claim to supply green goods to be ranked as high quality products, uninformed consumers may be pessimistic as far as they are skeptical about the feasibility and profitability of selling environemntally friendly product, such that firms overcomply with respect to existing environmental regulations.

With optimistic beliefs by uninformed consumers we find a confirmation of a well known result already found in the economic literature: if the share of informed consumers is sufficiently high then firms have an incentive to provide high quality goods, prices will reflect the quality differential actually provided by firms and informed consumers exert a positive externality on the small share of uninformed consumers. However past result concerned perfect competition and monopolistic competiton, while we consider them in a framework where asymmetric information and information disparities are introduced in a vertical differentiation model. Such a result appears in our analysis when, due to the very high share of informed consumer, the model collapses to vertical differentiation with perfect information. However our model is rich enough to consider, within the case of optimistic uninformed consumers, different equilibria according to the size of the expected quality differential, the extension of the share of informed consumer, and the fact that firms may compete either for informed or for uninformed consumers.

If the expected quality differential is not too large, most consumers are uninformed, and in equilibrium all informed consumers buy the high quality good, then equilibrium prices are distorted upwards (with respect to the full information case) and reflect the expected quality differential. Considering the incentive to product differentiation by the high quality firm we find that though the real quality differential is lower than the expected one, still an increase of informed consumers leads the high quality firm to increase the quality differential in equilibrium (considering that the low quality firm sticks to the MQS). We can further notice in this case that while "rich" and informed consumers stick to the high quality good being aware of the quality differential, less rich uninformed consumers buy the high quality good due to their quality expectation.Therefore uninformed consumers are cheated in equilibrium. Furthermore both equilibrium prices are distorted upwards, as the low quality firm compete in prices with the high quality firm just to achieve a greater share of uninformed consumers, while richer and informed consumers stick to high quality goods, given their high willingness to pay for quality. Therefore product differentiation softens price competition as in the canonical model with perfect information, but uninformed consumers are cheated in equilibrium as they would have chosen the low quality good if they were informed.

Still, if most consumers remain uninformed but their share is even lower than in the previous case we can find a different equilibrium, where on the contrary rich and informed consumers choose the low quality good while "less rich" uninformed consumers buy the high quality good and the poorest one buy the low quality good. In this last case the high quality firm just competes for uninformed consumers, given that all (rich) and informed consumers stick to low quality. In this equilibrium there are no incentives for real quality differentiation from the point of view of the firm claiming to sell high quality good. Actually product differentiation is purely virtual as in practice the high quality firm is lead to provide the MQS as its competitor does. That is why all informed consumers stick to low quality goods, despite their higher willingness to pay. In equilibrium, prices continue to depend on expected quality and are then distorted upward as before. Therefore uninformed consumers buying high quality goods are still cheated in equilibrium. However equilibrium prices also depend on the share of informed consumers, but consumers information affects equilibrium prices asymmetrically as the price of the "virtually" high quality good decreases with consumers'information and the price of the low quality good increases with it. Actually as firms compete for uninformed consumers, the high quality firm is especially hurt by an increase of the share of informed consumers, as competition for uninformed ones becomes fiercer and in order to compete with the low quality firms it should set lower prices On the contrary the low quality firm benefits from consumer information as it can extend its market share due to informed consumers and still sell to uninformed consumers with the lowest willingness to pay, who cannot afford to pay higher prices.

In our opinion this kind of equilibrium can well be adapted to represent the case of competition between branded drugs and generics in markets with a minority of informed consumers. In this last case even consumers with an high willingness to pay may switch to a cheaper generic drug, when introduced in the market, as they know that generics are equivalent or slightly equivalent to branded drugs. On the contrary optimistic misperceptions may lead uninformed consumers with an intermediate willingness to pay to continue to buy branded drug, despite their price. Generics are also bought by consumers with the lowest willingness to pay due to their lower prices. However both prices are distorted upwards, therefore the price of branded drugs may not necessarily decrease after the introduction of generics but changes in consumers information may affect price competition and drive price reductions of branded drugs. Actually information provision policies, widely used in this market, can then be grounded on our results.

If we consider the case with a majority of informed consumers and an even greater expectation about the quality differential (uninformed consumers are more "optimistic" than before), then if the share of informed onsumers is very large we can boil down to the canonical case of vertical differentiation with perfect infiormation: As we said we can then extend the result of the literature concerning information disparities in perfect and monopolistic competition to vertical product differentiation.

In case the share of informed consumers is not so high (though most consumers are informed) we can find another equilibrium where prices can reflect the real quality differential (as with complete information) but also depend on the share of informed consumers. In this equilibrium low quality goods are just bought by informed consumers with an intermediate willingness to pay, while low quality goods are bought by all uninformed cosnumers and by informed consumers with a the greatest willingness to pay. Therefore the high quality firm competes for both uninformed an informed cosnumers, while the low quality firm just sells to informed consumers This equilibrium actually requires a very high expectation about the quality differential provided by the high quality firm, as even consumers with the lowest willingness to pay are lead to buy the high quality product. Furthermore as in equilibrium also a share of informed consumers buys the high quality good, then the high quality firm has an incentive to provide an high quality differential, though lower with respect to the expected one. Furthermore as both equilibrium prices and the equilibrium level of high quality depend on the share of informed consumers, one can show that the higher the share of informed consumers, the lower the level of high quality and the lower the price charged by the high quality firm. Therefore with information disparities, if most consumers are informed there are less incentives for product differentiation and also more price competition resulting in lower prices for the high quality good.

On the contrary the low quality firm can afford to charge higher prices with more informed consumers. Actually if the share of informed consumers increases then we have more competition between firms to achieve informed consumers and this leads the high quality firm to reduce the equilibrium price, as with more informed consumers the share of uninformed consumers buying high quality goods (just on the basis of their expectation) shrinks, increasing consumers adressing to the low quality firm. The latter can profit from consumers switching by charging an higher price. If the profitable strategy for the high quality firm is to reduce both price and the quality level, we can then conclude that increasing consumers information is detrimental to product differentiation and a stimulus for price competition

While considering the pessimistic case, as uninformed consumers are skeptical about the quality differential provided by the high quality firm, the latter faces a different type of adverse incentives, as even though supplying a level of quality higher than expected may be feasible and profitable, the firm will not find it convenient as consumers may not be willing to pay a corresponding price premium to the high quality firm. The market may fail in providing higher quality goods, unless the share of informed consumers is sufficiently high. And actually also in the case of pessimistic consumers if the share of informed consumers is very high the model boils down to the canonical model of vertical differentiation with perfect information.Therefore the high quality firm is lead to supply the quality differential provided with perfect information, equilibrium prices reflect the real quality differential and there is a positive externality for the negligible share of uninformed consumers.

But still restricting our attention to the case where uninformed consumers are not too pessimistic, if the share of informed consumer is lower than in the previous case we can find another equilibrium where prices reflect the expected quality differential and therfore at a first glance they appear to be distorted downward with respect to the canonical case with perfect information. When considering the equilibrium level of product differentiation we can find that the high quality firm finds it optimal to supply exactly the quality differential expected by uninformed consumers.

It is interesting to point out that with pessimistic consumers a third type of equilibrium can arise in case the location of (informed and uninformed) marginal cosnumers is such to lead all uninformed consumers to buy low quality goods and all informed consumers to buy high quality goods. In this case demand functions are perfectly inelastic to prices and there is perfect market segmentation according to consumer information. As a continuum of equilibria may arise in this case we just characterize one of these equilibria by making resort to the assumption that the market is completely covered, which is made across all the paper just for tractability reasons. (the more the number of marginal consumers the more the analysis becomes cumbersome).

In our analysis consumers information and consumers beliefs are exognously given. However we think that it could be possible to consider also a further stage about firm entry and to consider sunk costs as R&D, which may affect the real quality differential, or advertising, that could affect consumers beliefs and the expected quality differential. The latter could then become endogenous to the model. In our framework expenditure in persuasive advertising may then be provided a foundation through the analysis of the optimistic case, while non informative advertising as a signal seem to be consistent with remedies for market failures arising in the pessimistic case. Furthermore if persuasive advertising can modify consumers beliefs, an even richer model could be considered where beliefs become endogenous. Finally as the case of optimistic consumers can be well adapted to deal with competition in the drug market, with slight modifications we can account also for price regulation and information provision to consider competition between generics and branded drugs affected by public policies.

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2 Collusion in regulated pluralistic markets

2.1 Introduction[1]

Many countries have introduced some form of yardstick competition in order to regulate prices in contexts where providers face limited competitive pressure. Examples are the maximum price limits each water company may charge its customers in the UK (Ofwat, 1993); price caps imposed by the Federal Energy Regulatory Commission to hold down the wholesale price of natural gas and electricity in interstate commerce in the US (US Department of Energy, 2002); postal tariffs determined by independent regulators in countries such as Germany, the Netherlands and the UK (NERA, 2004); and prospective payment system (PPS) that have been introduced to pay for health care services in many countries (Schreyögg et al., 2006; Ma, 1994).

The fundamental idea behind yardstick competition is that the price (or price cap) faced by each provider is dependent on the actions of all the other providers (Schleifer, 1985; Laffont and Tirole, 1993). According to Schleifer's rule, the price each provider faces is based on the costs of all other providers in the industry but not its own. This creates strong incentives for cost control: each provider's cost reducing effort will not be detrimental to the price it faces. A potential drawback with yardstick competition is that providers have an incentive to collude on higher costs, first because they can get a higher price for their services and, second, because they can exert less cost reducing effort, thereby benefiting from slack (Wilson, 1989).

In contexts where there is a large number of providers, this is unlikely to be problematic, mainly because the cost of collusion rises (Pope, 1989). But there is greater potential for collusive behaviour in contexts where there is a limited number of providers. This is most obviously a problem for small countries: for instance, Northern Ireland or Iceland have considered introducing PPS arrangements for health services despite there being fewer than five hospitals in each country. But even in larger countries, provision might be concentrated among a handful of providers, as is likely for utilities, rail or postal services and for specialist health services, such as bone marrow or lung transplantation. There is also potential for collusion for services regulated at local authority or municipal level, as is often the case for social care or education.

The incentive to collude with other providers will depend on the objectives of the provider, particularly the extent to which their objectives correspond with those of the

[1] Joint work with: Marisa Miraldo,Imperial College Business School, Imperial College of London; Andrew Street, Centre for Health Economics, University of York and Roberta Longo, Institute of Health Sciences, University of Leeds

price-setting regulator. We use the terms "altruistic" to describe providers that have objectives closely related to those of the regulator and "self-interested" to describe providers whose interests are more divergent from those of the regulator (Rose-Ackerman, 1996; Bozeman, 1984; Rainey et al. 1976). If providers differ in their degree of altruism, they may behave quite differently in response to financial incentives (Aas, 1995). Divergence among providers may arise in situations where greater plurality of provision has been encouraged, as is evident for various services that may have been provided in the past predominantly by public providers. For example, since the 1980s in England and more recently in Denmark, there has been a trend away from public toward private local provision of long-term adult social care, whether this be residential or domiciliary care (Glendinning and Nolan 2009; Fotaki et al 2013). Some countries are encouraging mixed provision of secondary school education, an example being the Academy programme in England whereby academy schools are not regulated by their local authority but directly accountable to the national Department of Education (National Audit Office 2010, Machin and Vernoit 2011). Similarly traditionally public National Health Service systems in England, France, Italy, Portugal and Spain are encouraging more private sector organisations to enter the health care market to provide care to patients funded by the NHS (Oliveira and Pinto, 2002; Aballea et al 2006; Levaggi, 2007; Pollock and Godden 2008).

Greater plurality may accentuate the principal-agent problem, extenuating divergence between regulatory and provider interests. Public providers may have a strong sense of mission, aiming to maximize the well-being of the people they serve (Wilson 1989), just as the regulator would like. But private providers are also accountable to their shareholders, with an interest in profit making. This implies that they have a weaker sense of "public" service mission, and might have objectives that are less closely aligned to those of the regulator (Newhouse, 1970; Hansmann, 1980; Glaeser and Shleifer, 2001).

If objectives are misaligned, providers may be more likely to resist or undermine yardstick competition arrangements. One way of doing this is by not making available the information required by the regulator to set prices or budgets. The National Audit Office notes that the Department of Education has to make "complex annual calculations with certain adjustments" to determine funding but that Academy schools are often late or fail altogether in submitting financial data (NAO 2011). Unlike their NHS counterparts, private treatment centres in England are not required to submit information about their costs to the Department of Health (Mason et al 2008). It is difficult to base prices on costs if cost information is unavailable.

Another way to undermine yardstick competition is by colluding with other providers with regard to the costs that they incur and, by implication, the cost information that they do submit to the regulator. We explore the implications of this strategy in this paper. There are a number of works that have addressed the issue of collusion under yardstick competition (Boardman et al 1986; Tangeras, 2002; Chong and Huet, 2009). However the existing literature assumes homogeneous providers, so does not address the implications of greater plurality of provision. Our paper is close to Potters et. al. (2004) in which the authors present an adapted version of Schleifer's model (Schleifer, 1985) and test it experimentally in order to explore collusive incentives under different yardstick competition schemes.

We extend this work by analysing incentives for collusive behaviour when heterogeneous providers are faced with regulated prices under yardstick competition. We analyse the choice of cost when providers do not collude and when they do so, and we consider incentives to defect from the collusion agreement. Our results show that in markets served by purely altruistic providers there is no collusion on costs while in markets served by purely self-interested providers there is scope for collusion. The paper is organized as follows. Section 2.2 introduces the main assumptions of the model, and considers cooperative behaviour under a yardstick competition model. Section 2.3 presents summarizes the main results of the paper and section 2.4 draws the main conclusions.

2.2 The Model

Consider a market with three types of agent: consumers, providers and a regulatory authority. We consider two providers (with $i = 1, 2$) each with its own population of consumers defined geographically, so that each provider is a local monopolist facing a downward-sloping demand curve with $\hat{p}$ being the price paid by consumers for each unit of service q . In sectors that provide services of public interest, such as for postal services, utilities, or the healthcare sector consumers may face the full or a partially subsidised price. Under yardstick competition, the regulator establishes a payment that gives the providers incentives to reduce costs. In particular each provider faces a regulated price set beforehand equal to the average (say) of the marginal costs of all the other providers in the market except its own (Shleifer, 1985). We assume that costs are observed by the regulator. The regulator sets a cap -$\hat{p}$ - on the price that each provider can charge. Note that this restriction will bind in equilibrium otherwise there would be no need for regulation. The main objectives of a regulation policy are to promote technical efficiency and allocative efficiency by simulating the outcomes of competitive markets (Laffont and Tirole, 1993). When providers enjoy a degree of monopoly power, they can provide a lower volume of output than they would in a competitive situation and, thereby, secure higher prices. This causes welfare loss. Moreover, monopoly firms lack incentives to be cost efficient, thus undermining technical efficiency.

The utility of provider i -U_i - is a function of the regulated price, the marginal costs and the altruism level. We assume that altruistic and self-interested providers are distinguished by the degree to which they are concerned about consumer surplus,

$$CS = \int_{p}^{\infty} q(x)\, d(x) \tag{2.1}$$

This is graphically represented by the area under the demand curve for their services, above their price. Recall that consumer surplus is decreasing in the unit price of the service, so that the greater the degree of altruism, the greater the utility providers derive from lower prices. We further assume that the provider cares about consumer welfare to some proportion α_i with $i = 1, 2$. Without loss of generality we assume that provider 2 is at least as altruistic as provider 1, i.e. $\alpha_1 \leq \alpha_2$. We further assume that providers benefit from slack, i.e. they derive utility from avoiding cost reducing effort (Bradford and Craycraft, 1996; Pope, 1989). The benefit of slack $S(c_i)$ is an increasing function of cost at a decreasing rate (i.e. $S'(c_i) > 0, S''(c_i) < 0$). Thus the utility of each provider is given by the sum of net revenues, the benefit from slack, and the utility the provider derives from increased consumer welfare,

$$U_i\left(\hat{p_i}, c_i; \alpha_i\right) = \left(\hat{p_i} - c_i\right) q\left(\hat{p_i}\right) + S\left(c_i\right) + \alpha \int_{\hat{p_i}}^{\infty} q\left(x\right) d\left(x\right) \tag{2.2}$$

Assumption 1:For $i = 1, 2$ $i \neq -i$ we have: (i) $\partial U_i^2 / \partial c_i^2 < 0$;(ii)$|\partial U_i^2 / \partial c_i^2| > |\partial U_i^2 / \partial c_i \partial c_{-i}|$

Assumption (i) insures that U_i is well behaved and therefore that the second order conditions for a maximum are met (it also ensures that the trace of the Jacobian matrix is negative); (ii) states that the own price effects on marginal utility are of greater magnitude than cross-price effects; (i) and (ii) ensure that the Jacobian determinant is positive that is a sufficient condition for the equilibrium to be stable.

2.2.1 The first best and free price scenario

For comparison purposes we first develop a first best benchmark. Consider a first best scenario by which the regulator can decide on both the price and the cost of each service. In each local market the optimum is then characterized by the pair p_i^*, c_i^* that maximizes social utilitarian welfare $W(.)$ given by the sum of consumer surplus and the provider's utility[2] , i.e.:

$$W(p_i, c_i) = (1 + \alpha_i) \int_{p_i}^{\infty} q(x) d(x) + (p_i - c_i) q(p_i) + S(c_i) \tag{2.3}$$

with $i = 1, 2$. Maximizing welfare with respect to price and cost, the social optimum[3] is then given by the first order conditions (FOC henceforth) with respect to the price,

$$(p_i - c_i) q'(p_i) = \alpha_i q(p_i) \tag{2.4}$$

and with respect to the cost,

$$S'\left(c_i\right) = q(p_i) \tag{2.5}$$

According to (2.4) the optimal price should be such that the marginal net revenues due to an increase in the price equal the change in consumer surplus weighed by the altruistic parameter . Correspondingly (2.5) entails that the provider's marginal benefit from slack should be equal to the effect of increased costs on revenues. From (2.4), the socially optimal price rule can be written as:

[2]Note that consumer surplus shows twice in this utilitarian welfare function because some providers are altruistic. We have assumed a utilitarian welfare function as it is commonly used in the literature. Other functional forms would have an impact on our results but that is out of the scope of our analysis

[3]Social optimum solved in Appendix B1

$$\frac{p_i - c_i}{p_i} = -\frac{\alpha_i}{|\varepsilon_i|} \tag{2.6}$$

With ε_i being the price elasticity. For $0 < \alpha_i \leq 1$ we have a negative mark-up i.e. $p_i^* < c_i^*$while for $\alpha_i = 0$ the mark-up is zero, i.e. $p_i^* = c_i^*$. For the existence of an interior solution the condition $\alpha_i < 1 - (q'(p_i))/(S''(c_i))$ must hold (ensures a negative definite Hessian).

Note that the optimal price differs with the level of altruism, so if $\alpha_1 \leq \alpha_2$ the price for the less altruistic provider is at least the same as the price for the more altruistic provider, i.e. $p_2^* \leq p_1^*$, while by (2.5) the first best cost of the more altruistic provider is lower than the cost of the least altruistic provider, i.e. $c_2^* \leq c_1^*$ (see Appendix B1).

Before proceeding with the analysis it is useful to evaluate a free price scenario.
Maximizing

$$U_i\left(\hat{p_i}, c_i; \alpha_i\right) = \left(p_i - c_i\right) q\left(p_i\right) + S\left(c_i\right) + \alpha_i \int_{p_i}^{\infty} q\left(x\right) d\left(x\right)_i \tag{2.7}$$

With respect to pi and c_i the optimal price p_i^f and cost c_i^f are the solution for the following FOCs: are:

$$\frac{\partial U_i}{\partial p_i} = \left(p_i - c_i\right) q'\left(p_i\right) - \alpha_i q\left(p_i\right) + q\left(p_i\right) = 0 \tag{2.8}$$

$$\frac{\partial U_i}{\partial c_i} = -q\left(p_i\right) + S'\left(c_i\right) = 0 \tag{2.9}$$

Rearranging (2.8)

$$\frac{p_i - c_i}{p_i} = \frac{1 - \alpha_i}{|\epsilon_i|} \tag{2.10}$$

We have a positive mark up, if $\alpha_i < 1$ (i.e. $p_i^f > c_i^f$); zero mark-up for $\alpha_i = 1$ (i.e. $p_i^f = c_i^f$). In the latter when $\alpha_i = 1$ the free price solution is the same as in the first best solution with zero altruism.

Comparing (2.10) with (2.6) $\forall \alpha_i$, $-\alpha_i/|\varepsilon_i| \leq 0$ and $(1 - \alpha_i\ )/|\varepsilon_i| \geq 0$, it follows that $p_i^f > p_i^*$. Moreover given that the mark-up decreases with the altruism level it follows that $p_1^f \geq p_2^f$ for $\alpha_1 \leq \alpha_2$. Furthermore given (2.9) it follows that $c_1^f \geq c_2^f$.

The following proposition summarizes the results.

Proposition 1: In a free price scenario the price and cost of the more altruistic provider are lower than those of the most altruistic provider. Furthermore providers' prices and costs are higher than in the first best.

Proof: Proof in Appendix B2. According to Proposition 1, in the absence of regulation the provider would optimally price higher than the socially optimal price. Therefore, any price cap regulation will bind in equilibrium.

2.2.2 The provider's problem

Non-cooperative solution

We will analyse two types of games. First we start by describing a setting in which providers strategically choose the cost level in a one shot game. In section 2.3 we characterize a repeated game.

In a one-shot non-cooperative game, each provider i maximises its utility by choosing the cost c_i given the price rule to which the regulator will commit. Provider i 's problem is given by,

$$Max_{c_i} \qquad U_i\left(\hat{p_i}, c_i; \alpha_i\right) = \left(\hat{p_i} - c_i\right) q\left(\hat{p_i}\right) + S\left(c_i\right) + \alpha \int_{\hat{p}_i}^{\infty} q\left(x\right) d\left(x\right) \tag{2.11}$$

Since we are considering a two-agent model, the yardstick rule is such that provider i faces a price per service that is equal to the competitor's $(-i)$ marginal cost in providing the same service, i.e. $(\hat{p_i} = c_{-i})$.

The FOC with respect to cost$\partial U_i / \partial c_i$, is given by:

$$\frac{\partial U_i}{\partial c_i} = \left(\frac{\partial \hat{p}_i}{\partial c_i} - 1\right) q\left(\hat{p}_i\right) + \left(\hat{p}_i - c_i\right) q'\left(\hat{p}_i\right) \frac{\partial \hat{p}_i}{\partial c_i} + S'\left(c_i\right) - \alpha_i q\left(\hat{p}_i\right) \frac{\partial \hat{p}_i}{\partial c_i} = 0 \tag{2.12}$$

Proposition 2: Under a non-cooperative the equilibrium is such that providers optimally choose the same level of costs, i.e. $c_1^{nc} = c_2^{nc} = c^{nc}$, the cost does not change with the altruism level. For $\alpha_1 \leq \alpha_2 \, \forall \alpha_i \geq 0$, $c_1^f \geq c_2^f \geq c_1^{nc} = c_2^{nc} \geq c_1^* \geq c_2^*$, while $p_1^f \geq p_2^f \geq p_1^{nc} = p_2^{nc} \geq p_1^* \geq p_2^*$.

Proof: Proof in Appendix B3.

Corollary 1: When providers are purely altruistic i.e. $\alpha_1 = \alpha_2 = 1$ then $p^f = c^f = c^{nc} c = p^{nc} > c^* > p^*$. When providers are purely self-interested i.e.$\alpha_1 = \alpha_2 = 0$, then $p^f > c^f > c^{nc} = p^{nc} = c^* = p^*$.

Proof: Proof in Appendix B3.

The scenario under which providers are purely self-interested ($\alpha_1 = \alpha_2 = 0$) is akin to Scheiffer's (1985) original model and the first best price coincides with the yardstick price the regulator has committed to. It follows that under such regulated price while a provider's cost reduction leads to a reduced price faced by the other provider, it does not adversely affect its own price. This arrangement gives both providers strong incentives to operate at a socially optimal cost level. Take the more altruistic provider ($i = 2$), which affords greater weight to consumer surplus. The price this provider faces depends on the costs of the other provider, implying that the consumer surplus has less influence on its own choice of costs. The opposite rationale holds for the more self-interested provider. These results hold independently of the degree of altruism. Indeed, it is straightforward to show that $dc_i^{nc}/\alpha_i = dc_i^{nc}/\alpha_{-i}$(see Appendix B3).

However, for any other levels of altruism the regulated price is no longer set according to the first best price rule (indeed in the first best $p_i^* < c_i^*, \forall \alpha_i \neq 0$, $i = 1, 2$). Consequently as the yardstick price is a weaker regulatory instrument when compared to p_i^* it follows that providers costs levels will be higher than in the first best, i.e. $c_1^{nc} = c_2^{nc} > c_1^* > c_2^*$.

Cooperative solution

Still on a one shot game, we will now characterize the cooperative solution within which providers maximize their joint utility $U = \sum_i U_i$. The advantage of agreeing on a strategy is that the providers can avoid "competing" against each other in lowering their production costs. Collusion allows providers to limit their cost reducing effort while receiving a higher price for their services. Offsetting these benefits, there are the negative effects resulting from lower demand as well as reduced consumer surplus (which affects utility in proportion α_i). Thus, the final outcome will depend on the balance of these effects. Letting the superscript c indicate the cooperative solution, the following proposition summarises the results in a cooperative scenario.

Proposition 3: In a cooperative scenario, for $\alpha_2 \geq \alpha_1$ and $\alpha_i \in [0,1]$ for $i = 1,2$ the providers' cost strategies are such that $c_2^c \geq c_1^c$, $\forall \alpha_i \in [0,1]$ $i = 1,2$ and $\alpha_1 \leq \alpha_2$. With asymmetric levels of altruism $c_2^c > c_1^c > c^{nc} > c_i^*$ for $i = 1,2$. Providers cost strategies decrease on both levels of altruism (i.e. , $dc_i/\alpha_{-i} < 0$, $dc_i/\alpha_i < 0$).
Proof: Proof in Appendix B4.

Corollary 2: In the case of homogeneous purely altruistic providers, i.e. $\alpha_2 = \alpha_1 = 1$ cost strategies are such that $c_2^c = c_1^c = c^{nc} > c_i^*$. In the case of homogeneous purely self-interested providers, i.e. $\alpha_2 = \alpha_1 = 0$ it follows $c_2^c = c_1^c > c^{nc} > c_i^*$.
Proof: Proof in Appendix B4.

Note that, in a cooperative scenario, providers maximize joint surplus and therefore provider i's decision rule displays provider $-i$'s altruism level. It follows that the providers' optimization problem is symmetric apart from the differences between providers' altruistic levels namely $-\alpha_2 q(c_1)$ and $-\alpha_1 q(c_2)$. This implies that the costs of one provider decrease in relation to the level of altruism displayed by the other.

As in the non-cooperative solution, the more altruistic provider cannot influence the consumer surplus it produces as this depends solely on the cost chosen by the other, less altruistic, provider. It can impact, though, on the other provider's consumer surplus even if this weighs less in the optimal decision rule. The situation under this yardstick regime is akin to the two providers swapping their roles. Indeed, even though provider 2 is more altruistic than provider 1, a situation of pure collusion is such that provider 1 exhibits the strongest cost response in order to reflect the impact of costs on consumer surplus. In this way provider 2 can afford a higher cost. This higher cost will allow provider 1 a higher yardstick price that will counterbalance the decreased benefit from slackness caused by a lower cost.

For a given c_{-i}^c , we note that the cooperative and the non-cooperative best responses of provider i, are given respectively by:

$$\frac{\partial JU}{\partial c_i} = -q\left(c_{-i}\right) + S'\left(c_i\right) + \left(c_i - c_{-i}\right) q'\left(c_i\right) + \left(1 - \alpha_{-i}\right) q\left(c_i\right) \tag{2.13}$$

and

$$\frac{\partial U_i}{\partial c_i} = S'\left(c_i\right) - q\left(c_{-i}\right) = 0 \tag{2.14}$$

These expressions differ in the quantity $\left(c_i - c_{-i}\right) q'\left(c_i\right) + \left(1 - \alpha_{-i}\right) q\left(c_i\right)$. The term $\left(1 - \alpha_{-i}\right) q\left(c_i\right)$ is the net effect that provider i's cost directly has on provider $-i$'s revenues. The term $\left(c_i - c_{-i}\right) q'\left(c_i\right)$ is the effect of a unit of provider i's cost on the joint surplus as determined through the demand function. We note that, with regard to provider 1, the impact is positive because $c_1^c < c_2^c$. Thus we can conclude that, for a given c_2^c , the cooperative strategy of the more self-interested provider 1 is that it will operate at a higher cost than in the non-cooperative scenario. It can be shown (see Appendix B4) that this result holds also for the more altruistic provider 2.

If the market is served by two purely altruistic providers, the costs will be the same as in the non-cooperative scenario

To summarise the results, given that the consumer surplus depends on the regulated price and given that the regulatory scheme sets $\hat{p}_i = c_{-i}$, the maximization of the joint utilities (JU) is such that provider i's choice will affect provider $-i$'s consumer surplus. It follows that provider i makes a decision on costs bearing in mind the altruism level of the other provider.

Defection solution

This cooperative solution can never be sustainable in a one shot game. Indeed, consider provider i. If this provider defects from the cooperative agreement (considering that provider $-i$ plays according to the cooperative strategy), then it will revert to behaving according to the best response function as in (2.14) with the optimal defection cost c_i^d (where the superscript d indicates defection) satisfying:

$$S'\left(c_i^d\right) - q\left(c_{-i}^c\right) = 0 \tag{2.15}$$

Proposition 4: Provider i's defection cost lies between the optimal non-cooperative and the cooperative strategies i.e. $c_i^* \leq c^{nc} \leq c_i^d$. Furthermore $c_2^d \leq c_1^d$ for $\alpha_1 \leq \alpha_2$ and the defection costing strategies are decreasing with both providers altruism level, i.e. $\partial c_i^d / \partial \alpha_i < 0$, $\partial c_i^d / \partial \alpha_{-i} < 0$ for $i = 1, 2, -i = 1, 2, i \neq -i$.

Proof: Proof in Appendix B5.

Intuitively provider i's best response is $c_i^d < c_i^c$, given the choice of the other, and it would still face a higher price and therefore increase its surplus. The provider's decision is based on the maximization of its own utility and the FOC will coincide with the non-cooperative FOC. However the defection level will differ from the non-cooperative level as provider $-i$ is still playing the cooperative solution.

Given the latter, since $q'(.) < 0$ the negative impact of the yardstick price on providers' profit through the cost of providing the service (i.e. $-q(c^c_{-i})$) is higher in absolute value for provider 2 (since in the cooperative scenario the demand for this provider is higher than for provider 1) moreover since$S''(.) < 0$ the cost of reducing the slack is lower for provider 2. Therefore these imply that for provider 2 it pays off more to decrease its marginal cost of providing the service (even if that implies a reduction in the benefit from slack) than for provider 1 and therefore the deviation strategies in equilibrium are such that $c^d_2 \leq c^d_1$.

Given that the cooperative costing strategies decrease with the altruism level, then it follows that the cost of providing the service for each provider also increases with the altruism due to an increase in the demand (i.e. $-q(c^c_{-i})$ is bigger in absolute value for higher levels of altruism). Therefore both providers will need to deviate further from the cooperative agreement in order to compensate for this impact of increased altruism on the cost of providing the service.

To sum up we have shown that in a one shot game deviation from the cooperative agreement is always profitable and, consequently, collusion is never sustainable. Therefore the one shot Nash equilibrium is non-cooperative. This result is consistent with the findings of the existent literature (Tirole, 1988).

2.3 Repeated game: Incentives to collude

Let us consider a repeated game in which the providers can play grim trigger strategies (Friedman, 1971). At the beginning of each period the two providers choose the cost level and act according to the following trigger strategies. If one of them defects in some period t, by choosing a cost level $c^d_i \neq c^c_i$, then in any subsequent period the other provider reverts to play her best response to defection from that point onwards. This is a typical "trigger strategy", whereby if a provider deviates from the cooperative agreement all providers revert to the one shot Nash equilibrium from thereon. Therefore, in deciding whether to stick to the cooperative agreement, a provider compares the stream of profits of cooperating $U^c_i/(1-\delta_i)$ with the stream of profits obtained by deviating i.e. $U^d_i + \delta_i U^{nc}_i/(1-\delta_i)$. It is easy to show that collusion is sustainable for provider i if and only if $\delta_i \geq (U^d_i - U^c_i)/(U^d_i - U^{nc}_i)$ where U^{nc}_i is the equilibrium payoff provider i receives in the non-cooperative scenario, U^c_i is the payoff gained in collusion and U^d_i is the payoff obtained in defection. The outcome depends on the individual discount rate $\delta_i \in [0,1]$, that represents the extent to which each provider considers short term profits more valuable than profits accrued later in time. The higher the rate the lower is each provider's incentive to collude. Therefore it follows that collusion is sustainable for $\delta \geq \delta^*$

$$\delta^* = max\left\{\delta_i, \delta_{-i}\right\}$$

With

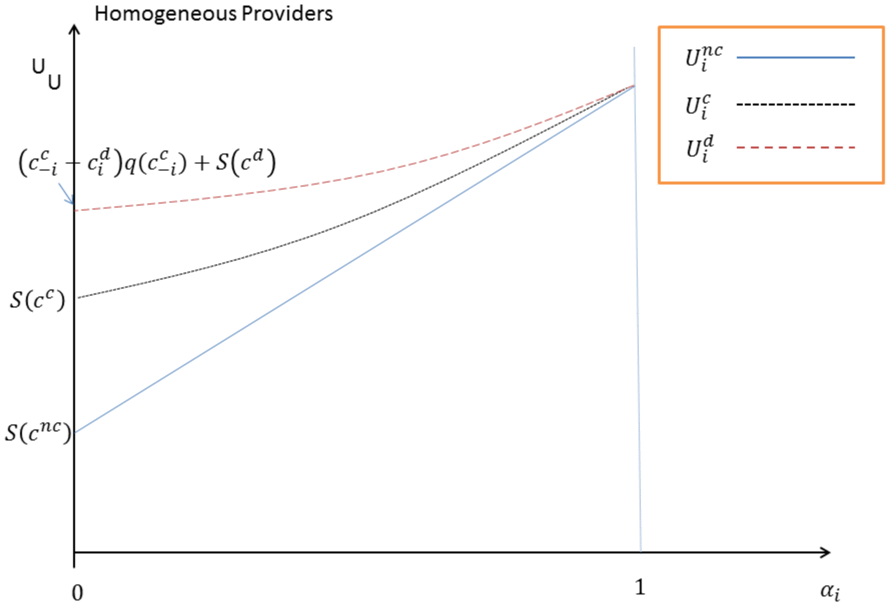

Figure 2.1:

$$\delta_i \geq \frac{(U_i^d - U_i^c)}{(U_i^d - U_i^{nc})} \qquad \delta_{-i} \geq \frac{(U_{-i}^d - U_{-i}^c)}{(U_{-i}^d - U_{-i}^{nc})}$$

When the market is served by two purely altruistic providers, i.e. $\alpha_i \to 1$ it is easy to see that providers have no incentive to collude since $U_i^c \to U_i^{nc}$. In fact providers do not have an incentive to deviate from the non-cooperative cost under joint profit maximization.

If providers are homogeneous and self-interested, it follows that since $\delta_1 = \delta_2 = \delta$ and δ decreases with the altruism level. Collusion in such a market is more likely than in a market served by two purely altruistic providers.

In heterogeneous markets collusion stability will depend on δ^*. Sustainability of collusion in the presence of asymmetric providers depends on the shape of the demand and slack functions. In particular for the more altruistic provider (provider 2) cooperation is profitable only if the benefit from slack is big enough to offset the financial loss and the decrease in consumer surplus that more altruistic firms have to bear in cooperation, i.e. $U_2^{nc} < U_2^c$ holds if and only if:

$$(S(c_2^{nc}) - S(c_2^c)) \leq (c_1^c - c_2^c)\, q(c_1^c) - \alpha_2 \int_{c^{nc}}^{\infty} q(x)\, dx + \alpha_2 \int_{c^c}^{\infty} q(x)\, dx \tag{2.16}$$

If this condition is not verified, if provider 2 were to collude he would sustain a loss

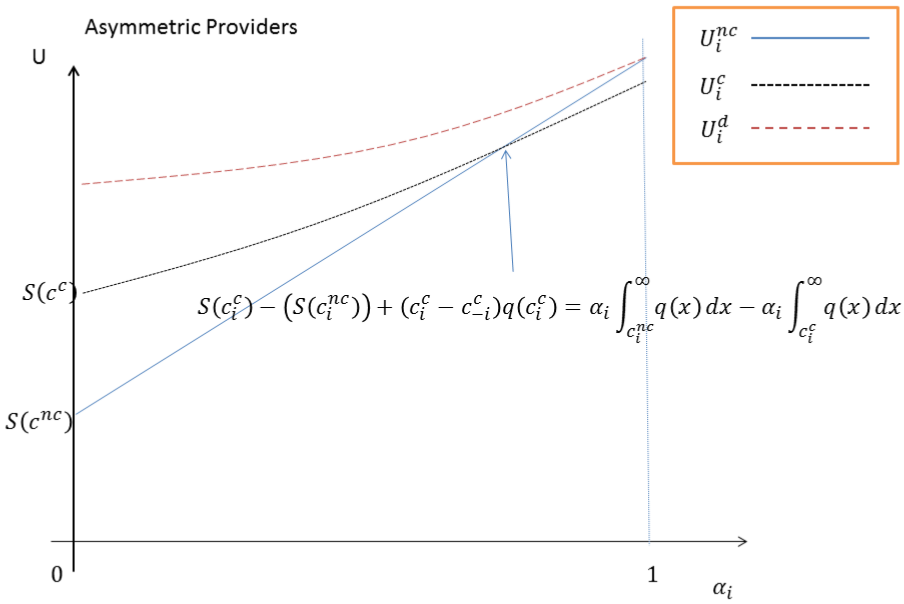

Figure 2.2:

with respect to the non-cooperative strategy in every single period, i.e. $\delta_2 \geq 1$ implying that there would be no collusion (see Figure 2.2).

In order to infer how δ_i changes according to the level of altruism we performed a comparative static analysis to study how provider i's rate changes with their own and the competitors degree of altruism. Proposition 5 summarizes the results.

Proposition 5: A provider's incentive to collude is a decreasing function of its own level of altruism and an increasing function of its competitor's altruism.
Proof: Proof in Appendix B7.

Departing from a homogeneous sector in which providers have the same level of altruism, increasing one provider's level of altruism decreases its incentive to collude in the market, while increasing its competitor level of altruism increases the incentive to collude. Thus in a heterogeneous market, the altruistic provider's incentive to collude[4]is lower than it would be in a market where providers have the same level of altruism. Intuitively it might be that homogeneous providers find it easier and more profitable to collude because of their symmetric objectives.

With regards to the more self-interested provider 1, similarly it can be shown that, as

[4]Note that here we are merely referring to the individual incentive to stick with the cooperative agreement rather than the sustainability of collusion as that will depend on the actions of both providers considered simultaneously. That analysis follows in the paper.

provider 2's degree of altruism increases, provider 1's rate decreases. Thus in a heterogeneous market, the self-interested provider's incentive to collude is higher than it would be in a market where providers have the same level of altruism.

Comparing δ_i across providers it is easy to show that $\delta_2 \geq \delta_1$ for $\alpha_2 \geq \alpha_1$ (see Appendix B7) therefore as long as (2.16) is verified there is scope for collusion in heterogeneous markets and collusion is less (more) likely when the altruism of the more (less) altruistic provider increases (decreases).

Corollary 3: For a given level of α_i, since $\delta_2 \geq \delta_1$ collusion is more likely to be sustained in homogeneous than in heterogeneous markets.
Proof: Proof in Appendix B7.

This result is in line with the existing literature that has shown that asymmetries between providers are an obstacle to collusion (see for e.g. Scherer, 1970; Barla, 2000; Compte and Ray, 2002). Given that in many contexts policy has been to encourage the entry of private providers in traditionally public settings these results suggest that increasing the plurality in service provision renders collusion less likely to occur. Finally, when comparing pure altruistic homogeneous markets with pure self-interested markets, we notice collusion on costs higher than the non-cooperative costs is more likely in the latter.

2.4 Conclusions

A potential drawback with yardstick competition regulation is that it might be susceptible to collusion, because by colluding on higher costs, providers may be able to secure a higher price for their services. We find that the incentive will depend on the degree to which provider objectives correspond to those of the regulator under yardstick competition arrangements.

We generalize the literature analysis by allowing for provider heterogeneity in their degree of altruism. By relaxing the assumption of provider homogeneity we are able to explore a fundamental change in the provision of public services where greater plurality is being encouraged. This is important given the trends toward greater mixed public and private provision in many regulated sectors of the economy, including health care, social care and education services, with governments encouraging more private sector organisations to enter markets traditionally served by public providers.

Our analysis demonstrates that it is important to consider the composition of the market when designing yardstick competition arrangements. We show that in markets served by purely altruistic providers there is no collusion on costs while in markets served by purely self-interested providers there is scope for collusion. We show that collusion is more stable in homogeneous than in heterogeneous markets, i.e. departing from a scenario where providers are homogeneous, we find that a change in the altruism of one provider decreases the stability of collusion in a repeated game. To sum-up, the incentives to collude depend on the extent to which providers share similar objectives. With pluralistic markets being encouraged in many countries and sectors of the economy it is increasingly important that provider heterogeneity is taken into account when designing regulatory policies.

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3 Physicians' altruism and ex-post moral hazard: (weak) evidence from Finnish national prescriptions data

3.1 Introduction[1]

In this article we test the hypotheses of physicians' altruism and ex-post moral hazard using a large national panel dataset of statin prescriptions records from Finland (n=17 858 829). The role of physicians and insurance in health care markets has been of interest to economists since the seminal contribution of Arrow (1963). Pioneering the economic analysis of physician behavior in the context of health care, Arrow (1963) noticed that doctors may have motives and objectives that differentiate them from purely profit-maximizing agents.

Together with Arrow (1963), (1968) developed the original 'ex-post moral hazard' hypothesis, predicting that health insurance increases the consumption of health care and leads to excessive consumption of services in a competitive health care market. Ex-post moral hazard has since been the focus of numerous empirical and theoretical studies in health economics (see e.g. Feldstein, 1973; Leibowitz, Manning, and Newhouse, 1985; Manning, Newhouse, Duan, Keeler, Leibowitz, and Marquis, 1987; Dranove, 1989; Zweifel and Manning, 2000).

In the context of prescription drugs Hellerstein (1998) has further developed and tested the ex-post moral hazard hypothesis by analyzing the decisions of a physician facing a choice between branded and generic versions of a drug. Hellerstein (1998) assumed that both the (indirect) utility of the patient and insurance expenditures enter the utility function of the physician that internalizes a proportion of patients' utility in their own utility functions, together with a proportion of the drug costs covered by the insurance company.

In the theory model Hellerstein (1998) assumed that the branded version of the drug is more expensive than the generic version, and showed that if the physician places a higher weight on the patient's utility than on insurance expenditures, an increase in the insurance coverage decreases (increases) the likelihood of the generic (branded) prescription. An increase in the insurance coverage, in fact, increases insurance expenditures and decreases patient's expenditures, ceteris paribus. As both these variables have a similar

[1] Joint work with: Matteo M. Galizzi London School of Economics, Behavioural Research Lab, LSE Health, and Department of Social Policy; Ismo Linnosmaa Centre for Health and Social Economics, Finnish National Institute for Health and Welfare and Marisa Miraldo Imperial College Business School, Imperial College of London

effect on the physician's utility, higher insurance coverage leads to a lower probability of generic prescribing when the physician values the utility of the patient more than insurance expenditures.

Hellerstein (1998) then empirically tested the ex-post moral hazard hypothesis using data from the 1989 National Ambulatory Medical Care Survey (NAMCS) in the USA. Physicians selected in the NAMCS survey recorded information on a random sample of their patients who visited their offices over a two-week period over the course of a year in 1989. The dataset consisted of 38 384 patient visits to 1233 office-based physicians, for a total of 492 multisource prescription drugs corresponding to 149 different generic compounds for which both branded and generic versions were available. Hellerstein (1998) estimated a random effects probit model for whether physicians prescribed the branded or generic version of the drugs, and, while controlling for the characteristics of the physician, found no evidence of ex-post moral hazard in insurance for multisource prescription drugs in the NAMCS data.

Lundin (2000) further analyzed the effect of health insurance on the probability of physician prescribing a generic or branded version of a drug in Sweden. Making use of prescriptions data (n=6142) on seven different drugs collected from two pharmacies in Tierp (Sweden) in 1992 and 1993, Lundin (2000) estimated a random effects probit model for whether physicians prescribed the branded or generic version of the drugs. Unlike Hellerstein (1998), Lundin (2000) found some support for both the physicians' altruism and ex-post moral hazard hypotheses in the market for prescription drugs in Sweden: higher coverage decreased (increased) the probability of prescribing a generic (branded) version of a drug.

Our research builds on these contributions in that we simultaneously test both altruism and ex-post moral hazard in drug prescription behavior using a large national panel of administrative data from Finland. We first develop a theoretical model on physician decision-making, which, in line with Hellerstein (1998) and Lundin (2000), predicts that the higher is the patient's insurance coverage for pharmaceutical expenditures, the more likely it is that physicians prescribe an expensive branded version of a drug. We then use a large national panel dataset with all statin prescriptions in Finland between 2003 and 2010 (n=17 858 829 prescriptions) to test the physicians' altruism and ex-post moral hazard hypotheses, while controlling for a large range of physician, patient, and drug characteristics.

Our work contributes to the previous literature in three respects. First, we consider the complete national administrative records for statin prescriptions in Finland: both Hellerstein (1998) and Lundin (2000) considered specific samples of prescriptions in the USA and Sweden, respectively, and included very diverse types of drugs in their analyses. Second, we directly control for a broad range of physician, patient, and drug characteristics: neither Hellerstein (1998) nor Lundin (2000) had information on income and other patient characteristics, and we also have direct information on prices for each branded and generic version of the statins in Finland. Third, taking advantage of the panel structure of our national administrative dataset, we directly observe the repeated prescriptions of statins by physicians over time (to the same patient and to the whole sample of patients of the same physician), so that we directly account for multiple observations for both patients and physicians, and we explicitly model habit-dependent prescriptions in our panel estimations, that is important in the context of chronic diseases. To the best of our knowledge, ours is the largest national panel dataset to date (n=17 858 829) on which the hypotheses of ex-post moral hazard, altruism, and prescription habits have been explicitly tested in regard to drug prescription behavior.

Our main findings are the following. We find weak and mixed evidence in support of the hypotheses of physicians' altruism and ex-post moral hazard: although the estimated coefficients associated with ex-post moral hazard and altruism are statistically significantly different from zero (due to the large number of observations), their size is very close to zero and their order of magnitude smaller than the effects associated with other key explanatory factors, such as the class of the prescribed statins and the year of prescriptions. We find, moreover, robust and strong evidence that the physicians' decisions to prescribe branded versions of statins in Finland are habit-dependent: physicians who have prescribed more branded drugs in the past are significantly less likely to switch to generic versions.

The rest of the article is organized as follows: Section 3.2 presents the theoretical framework generating the hypotheses to be tested empirically. Section 3.3 presents the data and Section 3.4 the econometric model. Results and conclusions are discussed in Sections 3.5 and 3.6.

3.2 Theoretical model

Although we use a different setting to test predictions, the theoretical modeling proceeds along the lines presented in Hellerstein (1998). We consider a physician $i = 1, 2 \dots I$ (she) and a patient $j = 1, 2 \dots J$ (he). The physician acts as a double agent for the patient and the health insurer (Blomqvist, 1991; Hellerstein, 1998; Lundin, 2000). The physician has diagnosed the patient and chosen a therapeutic treatment that is effective in curing the patient. The physician now faces a choice between branded and generic versions of the drug, denoted as b and g, respectively.

By consuming the version s in b, g of the drug, the patient j obtains utility:

$$U_{js} = v\left(q_s\right) - \left(1 - r\right) p_s \tag{3.1}$$

where q_s is the quality of the version s; the function $v(q)$ measures the patient's utility of consuming one unit of the drug with quality $q \geq 0$; ps is the price of the version s; and r_j measures the fraction of the price covered by the patient's health insurance. The term $(1 - r_j)p_s$ thus measures the copayment of patient j consuming the version s of the drug. Since the branded and generic versions are bioequivalent, it is assumed throughout all the following analysis that $q_b = q_g$(see also Hellerstein, 1998).

By choosing the version s in b, g of the drug, the physician obtains random utility:

$$V_{ijs} = v_{ijs} + \varepsilon_{ijs} = y_i + \gamma_1 \left[v\left(q_s\right) - \left(1 - r\right) p_s\right] - \gamma_2 r_j p_s + \varepsilon_{ijs} \tag{3.2}$$

where y_i refers to physician's labor income, and the terms $(1 - r_j)p_s$ and $r_j p_s$ measure the copayment of patient j consuming the version s of the drug and the corresponding insurance expenditure, respectively. In the simpler case where patients pay no deductibles, the patient's copayment amounts to her total out-of-pocket expenditure for the version s. In reality, out-of-pocket payments may include both copayments and deductibles. Deductibles, however, do not play any role in physician's choice between generic and branded versions of the drug, since the deductible level is usually lower than the price of the cheapest version. It is assumed throughout the article that the insurance coverage is determined independently of the version of the drug prescribed by the physician.

The parameter γ_1 in the utility function (3.2) measures the physician's altruism towards the welfare of the patient. The parameter γ_2, on the other hand, measures the degree to which the physician takes into account the consequences of her treatment choices on insurance expenditures.

The value of the parameter γ_1 is zero for a selfish physician and increases with the level of altruism. The hypothesis on altruistic physicians is supported empirically if γ_1 is positive and differs statistically from zero. The parameter γ_2 can be either positive, or negative, valued. If $\gamma_2 > 0$, the physician internalizes the consequences of her decisions on insurance expenditures and restrains pharmaceutical consumption. If $\gamma_2 = 0$, the physician ignores possible cost consequences. Finally, if $\gamma_2 < 0$, the physician takes advantage of health insurance as an external means to finance the consumption of pharmaceuticals and increases pharmaceutical consumption.

The random term ε_{ijs} in the physician's utility function captures unobservable factors affecting the physician's choice between the two versions of the drug. Such factors may be the advertising efforts of pharmaceutical companies to promote the sales of their products (Gonul et al., 2001) or the costs of prescribing generic versions of drugs (Hellerstein, 1998). We assume that the random terms ε_{ijs} are identically and independently distributed.

The rational physician i prescribes the generic drug version to patient j, if $V_{ijg} \geq V_{ijb}$. Assuming that the random terms ε_{ijs} are type 1 extreme value distributed, the probability of the physician prescribing the generic version of the drug is given by (see McFadden, 1974):

$$Pr\left(s_{ij}=g\right)=\frac{e^{v_{ijg}}}{e^{v_{ijb}}+e^{v_{ijg}}}=\frac{exp\left\{V_{ijg}\text{-}V_{ijb}\right\}}{1+exp\left\{V_{ijg}\text{-}V_{ijb}\right\}} \tag{3.3}$$

where

$$V_{ijg}\text{-}V_{ijb}=\gamma_1\left(1-r_j\right)\Delta p+\gamma_2 r_j \Delta p \tag{3.4}$$

and $\Delta p \equiv p_b - p_g$ is the price difference between the branded and generic versions of the drug. Given that the price difference Δp is not affected by physician's prescriptions, the effect of the insurance coverage on the probability of prescribing the generic version is given by:

$$\frac{\partial Pr\left(s_{ij}=g\right)}{\partial r_j}=\frac{exp\left\{V_{ijg}\text{-}V_{ijb}\right\}\left(\gamma_1-\gamma_2\right)\Delta p}{\left(1+exp\left\{V_{ijg}\text{-}V_{ijb}\right\}\right)^2} \tag{3.5}$$

It is natural to assume that the branded version of the drug is more expensive than the generic version (see e.g. Hellerstein 1998), which implies that $\Delta p > 0$. Then it follows from the expression (3.5) that an increase in the patient's insurance coverage will decrease (increase) the probability that the physician prescribes the generic (branded) version of the drug if $\gamma_1 \geq \gamma_2$, and the physician gives a higher weight to patient welfare than to insurance expenditures. The following statement summarizes the main prediction of the model:

Prediction 1 Provided that the branded version of a drug is more expensive than the generic version, an increase in the patient's insurance coverage decreases (increases) the probability to prescribe the generic (branded) version of a drug if $\gamma_1 \geq \gamma_2$.

We next test this prediction using national prescriptions data from the Finnish pharmaceutical market.

3.3 Data

Data on pharmaceutical prescriptions was obtained from Kela, the Social Insurance Institution of Finland, which insures all Finnish residents. The original data contain information on all drugs that were prescribed and dispensed in outpatient settings in Finland during each year between 2001 and 2010. The original data include lipid modifying agents (Anatomical Therapeutic Class (ATC) C10), depression drugs (ATC N06A) and two biological drugs (epoetin ATC B03XA01, and ilgrastim ATC L03AA03). In this article we focus on prescriptions for lipid modifying agents ('statins').

Although information about the drug version (either a branded or generic) is available for all prescriptions, data on patients' income are only available from 2003 onwards. Therefore, in this article we consider all the prescriptions of statins in Finland from 2003 to 2010 for which generic substitution was available.

In the period from 2003 to 2010, a total number of 17 858 829 statin prescriptions were issued in Finland[2]. More specifically, in the period 2003 to 2010, six statins were prescribed in Finland: in decreasing order by total number of prescriptions, these were Simvastatin (ATC C10AA01, 56.83% of the prescriptions), Atorvastatin (C10AA05, 22.08% of the prescriptions), Rosuvastatin (C10AA07, 6.92% of the prescriptions), Fluvastatin (C10AA04, 6.24% of the prescriptions), Pravastatin (ATC C10AA03, 4.52% of the prescriptions), and Lovastatin (ATC C10AA02, 3.39% of the prescriptions).

The dataset contains information on the characteristics of dispensed pharmaceutical products, patients, and physicians. We have access to information about the name, strength, form, producer, ATC-class, number of sold packages, and Defined Daily Doses (DDDs) of the pharmaceutical products (in the above-mentioned ATC-classes) prescribed by each physician and dispensed from all Finnish pharmacies each day for the period starting 1st January 2003 to 31st December 2010.

In addition, the dataset contains detailed information about the full price of the prescription and the amount reimbursed by the social insurer. Hence, both insurance coverage and coinsurance could be computed for each prescribed package in our dataset. The data also reveal if the prescribed product was substituted for another product at the pharmacy, so that we are able to precisely disentangle whether the generic substitution occurred by the initiative of physicians or pharmacists.

Observations are defined at the level of an individual prescription, each of which containing the above information about the pharmaceutical product, and linked to data on the patient and the prescribing physician characteristics.

Original data does not contain information on whether the prescribed product is either a branded or generic version of the medicine. The most updated official information linking prescriptions with the versions was obtained from the Finnish Medicine Agency (FIMEA) and directly incorporated into the dataset. In particular, in our empirical analyses we explicitly control for the type of statins by including dummy variables for each of the 7-digit ATC groups (the reference group being Simvastatin, ATC C10AA01, accounting for 56.83% of the prescriptions). Generic substitution in Finland is, in fact, possible only within the same 7-digit ATC class.

The price difference between the branded and generic versions plays a key role in both the theoretical and empirical analyses (see Hellerstein, 1998; Lundin, 2000; and Section 2) and is expected to influence physicians' choices between the two versions of

[2]This is excluding Cerivastatin (C10AA06), for which no generic substitution was available in Finland, and only one branded product was in the market in the period considered here.

drugs. In our dataset, we observe information about the prescribed medicine, including its price, but the data do not provide an indication of alternative products in competition with the prescribed product, nor of their prices. We calculated the price of alternative prescriptions, building on the principles developed by Lundin (2000). Lundin (2000) considered as an alternative price to a generic prescription the price of the branded competitor, and, as an alternative price to a branded prescription, the price of the generic competitor with the largest market share.

We generalize the same principles by allowing prescribing physicians to have only a coarse knowledge of the prices of alternative drugs (Kolassa, 1995). In particular, if the prescribed drug was a generic version, we define the price of the alternative prescription as the average price in the same calendar quarter as the prescription of the branded product within the same 7-digit ATC class, with the same active ingredient, package size, and strength. Similarly, if the prescribed drug was a branded version, we define the price of the alternative prescription as the average price in the market in the same quarter of the prescription of all generic products within the same 7-digit ATC class with the same active ingredient, package size, and strength[3] .

In order to calculate the average price, the price of each alternative product, either generic or branded, was weighted in proportion to the share of the sales of that product over the total sales of generic and branded versions within that 7-digit ATC class in the same quarter of the prescription. If, for a specific quarter, the branded product was no longer in the market (for instance because of the widespread use of generic alternatives), the last known price of the branded version was used as an alternative price. Applying these principles, we were able to compute the price difference (*PriceDiff* henceforth) between the branded and generic versions for each prescribed drug k as $\Delta p_k \equiv p_{bk} - p_{gk}$.

Information about patients in the dataset is quite rich. For each patient in the data, we observe gender (*Gender*), the date of birth (from which we can calculate age, *Age*, and the proportion of patients over 75 years old, *Over75*), the eventual date of death, the taxable income (*Income*), and the type and amount of social security benefits, if present. In addition, we also have information on the reimbursement status of the patients. The latter two variables are used to build illness severity proxies.

In the Finnish social insurance system, health insurance reimbursements can be classified into three classes on the basis of the severity of a patient's illness and the type of medication. Table 3.1 describes reimbursement rates for any pharmaceutical expenditure in excess of the patients' deductibles in each reimbursement class: the variable r_j stands for the reimbursement rate of patient j, while d_j is the deductible of patient j.

All drugs approved by FIMEA are granted the *basic* reimbursement level. To qualify for the *lower special* reimbursement rate, the drug must target the treatment of difficult-to-treat chronic diseases, such as hypertension or chronic obstructive pulmonary disease (COPD). Finally, to qualify for the *higher special* reimbursement rate, the drug must be remedial and target the treatment of severe and life-threatening diseases, such as cancer or diabetes mellitus.

Statin prescriptions are granted either the basic reimbursement or the low special

[3]Note that in each ATC group in our data there is only one branded drug.

Table 3.1: Reimbursement classes in the Finnish health insurance

	reimbursement rate r_j, %	Deductible d_j €
Special reimbursement		
- Higher	100	4.20
- Lower	72	4.20
Basic reimbursement	42	8.41

reimbursement rate, depending on the characteristics of the patient. The reimbursement rate, therefore, can be used as a proxy of patients' health status.

We define a variable called *Severe* as a proxy of the severity of patients' disease based on the reimbursement class they are in. The dummy variable *Severe* takes the value 1 if the dispensed prescription received low special reimbursement, and 0 if the prescription received basic reimbursement. Alternatively, we were able to control for patients' disease severity by using another dummy variable, *Ill*, which is based on information about the health insurance codes (so-called SVA codes) present in the prescription registers. A patient purchasing the prescription was defined as 'severely ill' (the variable *Ill* taking value 1) if the prescription was associated with health insurance SVA codes 203, 211, or 213, the *Ill* variable taking the value 0 otherwise. Although the information contained in the *Severe* and the *Ill* variables coincide for 99.99 percent of the prescriptions[4], we report empirical results using either variable alternatively in the regressions.

The price difference $\Delta p_k = p_{bk}-p_{gk}$, together with the individual rates of reimbursement r_j allow us to define the two key variables that measure the shares of the price difference paid by patients ($PatOOP$) and by the health insurer (*InsExp*). In particular, patient j's share of the price difference is given by $PatOOP = (1 - r_j)\Delta p_k$, while $InsExp = r_j\Delta p_k$ is the corresponding share of the price difference faced by the health insurer. As discussed in Section 2, in our analysis, a physician's altruism is operationally measured by the coefficient of the variable $PatOOP$, while the ex-post moral hazard is measured as the degree of physicians' responsiveness to variables $PatOOP$ and *InsExp*.

Our dataset identifies prescribing physicians on the basis of a code (*sv-code*) that entitles a physician to practice the medical profession as a certified practitioner. Every physician with at least four years of university education can apply to receive such a code in Finland. In addition to the *sv-code*, we observe the date and the field of specialization of each physician in the dataset that we use to build a proxy for experience ($PhysExper$) measured as the number of years from the first specialization degree. Fields of specialization, in particular, were grouped into 17 fields (internal medicine, cardiology, anesthesiology, surgery, neurology, psychiatry, diagnostic imaging, clinical and anatomical pathology, infectious diseases, pharmacology, oncology, gynecology and obstetrics, ears

[4]In particular, 977 Ill prescriptions were not categorized as Severe, while 22 Severe prescriptions were not categorized as Ill.

nose and throat medicine, neonatology and pediatrics, dermatology, geriatric medicine, and others). We also control for the physician's prescribing habits (*PastBranded*), proxied by the portion of branded drugs prescribed by a given physician over the total prescriptions in the previous twelve months, for each prescription of a 7-digit ATC statin with a specific active ingredient, package size, and strength.

Table 3.2 (below) describes the main variables used in the empirical analysis. The dependent variable is the binary variable *Generic*, taking the value 1 if a generic version was prescribed, and 0 otherwise.

Table 3.2: Descriptive statistic

Variable	**Obs**	**Mean**	**Std.Dev**	**Min**	**Max**
Generic	17 858 829	0.6131	0.4870	0	1
PriceDiff	13 350 469	39.0176	29.0871	0	227.2695
PatOOP	13 350 469	20.5571	15.7529	0	288.0607
InsExp	13 350 469	18.4605	160.828	0	163.6355
Gender	17 858 829	0.5078	0.4994	0	1
Age	17 858 829	65.8538	11.1833	0	105
Over75	17 858 829	0.2412	0.4278	0	1
Income	17 858 829	21 038	21 651	0	8 246 447
Severe	17 858 829	0.1941	0.3956	0	1
Ill	17 858 829	0.1942	0.3956	0	1
Simvastatin	17 858 829	0.5683	0.4953	0	1
Atorvastatin	17 858 829	0.2208	0.4148	0	1
Rosuvastatin	17 858 829	0.0692	0.2537	0	1
Fluvastatin	17 858 829	0.0624	0.2419	0	1
Pravastatin	17 858 829	0.0452	0.2078	0	1
Lovastatin	17 858 829	0.0339	0.1812	0	1

Men accounted for 50.8 percent of the patients prescribed statins in our data. The mean age of patients using statins was 65.85 (with a standard deviation of 11.18 years), with about one quarter of the statins prescribed to patients aged over 75 years. The mean taxable income among the patients in the data was 21 038.54 euro (with a standard deviation of 21 651.87 euro). In terms of disease severity, about 19.42 and 19.41 percent of the patients were classified as *Ill* or *Severe*, respectively, in the latter case thus having the right to the lower special reimbursement.

About 61.31 percent of the total statin prescriptions were generic versions. The average price difference between the branded and generic prescriptions was 39.02 euro (with a standard deviation of 29.08 euro). Patients paid on average about 52.69 percent of the price difference (20.56 euro), while the remainder was paid by insurance (18.46 euro).

In addition to the variables described in Table 3.2, in some of the empirical specifications we use some further explanatory variables. In particular, to account for changes in the economic, institutional, and health policy context (e.g. generic substitution and reference pricing were introduced in 2003 and 2009, respectively[5]), we included dummy variables for each year from 2004 to 2010 (the reference year being 2003). Furthermore, we also used two physician-specific variables that are described in the next Section (*PhysExper* and *PastBranded*).

3.4 Econometric model

The dependent variable of interest is the binary variable $y_{ijk} = Generic$, which takes the value 1 if the medical doctor i prescribes the generic version of drug k to patient j, and 0 if the branded version of the drug is prescribed. In the empirical analyses we estimate the probability that the medical doctor i prescribes the generic (versus branded) version of a drug k to patient j, $Prob(y_{ijk} = 1)$[6].

In order to directly test for the altruism and ex-post moral hazard hypotheses, in the baseline empirical specification the probability of prescribing a generic drug is modeled as a function of the price difference paid by the patients (*PatOOP*) and by the health insurance (*InsExp*); the main patient-specific characteristics X_j(*Gender*, taking the value 1 for females; *Age*; and *Income*); and the drug-specific dummy variables D_k (*Atorvastatin, Rosuvastatin, Fluvastatin, Pravastatin, Lovastatin*, five dummies capturing the 7-digit ATC classes of statins, with the reference drug being *Simvastatin*).

Further empirical models include among the explanatory variables different measures for the severity of the patient's illness (either *Ill* or *Severe*, two dummies for the severity of patients' medical conditions); year-specific dummy variables (the reference year being 2003); and physician-specific characteristics Ph_i. The latter are either physician's experience (*PhysExper*) or physician prescribing habits (*PastBranded*) as defined above.

Formally, the most general econometric model thus takes the form:

$$Prob(y_{ijk} = 1) = \Lambda\left(\alpha + \gamma_1(1 - r_j)\Delta p_k + \gamma_2 r_j \Delta p_k + \beta Ph_i + \delta X_j + \phi D_k + \tau Y\right) \quad (3.6)$$

where Λ is the cumulative distribution function of the logistic distribution; r_j is the reimbursement rate of patient j; Δp_k is the price difference between the branded and generic versions of drug k; Ph_i is the variable for physician experience as defined above; X_j refers to a vector of patient-specific variables; D_k is a vector of drug-specific dummies; and Y a vector of year dummies.

[5] The reference pricing (RP) is a pharmaceutical price regulation scheme where the patient (or insurance) is financially responsible for the difference between the price of the purchased drug and a predetermined 'reference price'. See Galizzi, Ghislandi, Hokkanen, Kangasharju, Linnosmaa, Miraldo and Valtonen (2009) and Galizzi, Ghislandi and Miraldo (2011) for reviews of the RP experiences in Finland and internationally, respectively.

[6] We have also replicated the analysis while considering a different dependent variable based on whether the pharmacist dispenses a generic version of the drug to a patient, and found substantially similar results (available upon request).

As discussed in Section 3.2, the parameters γ_1 and γ_2 measure the weights that the physician places on patients' welfare and health insurance expenditures, respectively. The empirical specification (3.6) allows the estimation of the parameters. The results of the empirical analysis give support to the doctors' altruism hypothesis, if $\gamma_1 > 0$, and to the ex-post moral hazard hypothesis if $\gamma_1 > \gamma_2$. The identification of the parameters rests on the assumption that the price difference Δp_k is independent of physicians' prescriptions, which is indeed satisfied given the Finnish institutional context and the principles based on which we have constructed the price differences.

In line with the theoretical model in Section 2, we estimate the logit specification of the empirical model (3.6). To take advantage of the unique longitudinal dimension of our dataset, we estimated panel logit models. In particular, we estimate a set of random-effects (RE) panel logit models, which look at each physician–patient pair $i-j$ over time, and treats the pair-specific effects as unobserved random variables uncorrelated with the regressors:

$$Pr(y_{i-jt} = 1|x_{i-jt}, \alpha_{i-j}) = \Lambda\left(\alpha_{i-j} + x'_{i-jt}\beta\right) \tag{3.7}$$

where $\Lambda(z) = e^z/((1+e^z))$, the vector x_{i-jt} contains the independent variables discussed above, and $\alpha_{i-j} \sim N(0, \sigma^2_\alpha)$. To correct for possible error correlation over time for a given physician–patient pair, we use cluster-robust standard errors at the $i-j$ pair level. The pair-specific effects are integrated out over the joint density function. Since there is no analytical solution to the integral, numerical methods are used in the estimation, such as the adaptive 12-point Gauss-Hermite quadrature.

In our case, the RE panel logit model results in an unbalanced panel structure with the number of time observations for each doctor–patient pair being equal to the number of drug prescriptions in the 2003-2010 period under consideration. On average, the doctor–patient pairs have 10.54 drug prescriptions over that period (with a standard deviation of 10.29), with half of the observations having 6 or more prescriptions, and a quarter having 12 or more, up to a maximum of 150.

In order to obtain a clean panel structure using the calendar time as the temporal dimension, we eliminated the rare cases (a total of 41 976 observations, 0.23% of the total number of observations) where more than one drug was prescribed on the same day within the same physician–patient pair. We have directly tested whether the observations excluded were systematically different from the remainder in terms of observable characteristics, and could not reject the null hypothesis of no significant differences between the two groups.

Notice that the estimation of an RE panel logit model entails the assumption that the physician–patient pair-specific effects are uncorrelated with the regressors. Relaxing this assumption would in principle require the estimation of a fixed-effect (FE) panel logit model, treating the pair-specific effects as unobserved random variables that potentially correlate with the regressors. In our case, however, the FE panel logit model is not a viable option, since jointly estimating the high number of incidental physician–patient fixed effects together with the other model parameters would lead to inconsistent estimations due to very few time points in our short panel.

All estimations were conducted with Stata 13, using the High Performance Computer at Imperial College London.

3.5 Results

We first estimate a baseline RE panel logit model (*Model 1*) where the probability of prescribing a generic drug is a function of the shares of the price differences paid by patients and the health insurance, controlling for age, gender, and income of the patients, and for drug-specific dummy variables. We then add dummy variables into the model to control for patients' severity of disease *(Models 2* and *3*), as well as dummy variables for each year in our dataset (*Model 4*). The same analysis is then replicated by estimating the corresponding RE panel logit models while controlling for either physicians' experience (*Models 5-8*), or physicians' past shares of the prescriptions of the branded versions of the drugs (*Models 9-12*). Further sets of estimations (all available on request) have been replicated in order to test for robustness when considering the sub-samples of the population (e.g. older than 75); generic substitutions by the pharmacists (instead of the physicians); probit models (instead of logit); and sub-sets of years (e.g. excluding years after 2009 when a reference price was introduced). Tables 3.3, 3.4, and 3.5 below display the estimation results of the three main sets of estimations.

As can be seen in Tables 3.3, 3.4, and 3.5, our results seem to provide some support for the physicians' altruism hypothesis in the Finnish statin market. The higher the share of the price difference paid by patients, the higher is the probability that physicians prescribe the generic version of the pharmaceutical (i.e. $\gamma_1 > 0$). From this perspective, our results are thus similar to Lundin's (2000) findings in Sweden for 1993[7].

Given the large number of observations of our prescriptions dataset, it is not too surprising that the estimated effect of the altruism coefficient is statistically significant across the different models. Three further aspects deserve a more developed discussion.

First, the estimated altruism coefficient is very small in magnitude and close to zero. It is true that its size is broadly in line with the effects of patients' socio-demographics, such as income or age, but it should also be noticed that the estimated effect is orders of magnitude smaller than the effects associated with other key explanatory variables.

Second, other factors appear to be the main drivers of generic substitution in Finland. Rather than being based on considerations related to the patients' out-of-pocket payments, the physicians' decision to prescribe the generic or branded version of the statins is mainly associated with the class of the prescribed statins; the timing of the prescriptions; the status of the patients; and the physicians' past experience. In particular, the generic prescription is far less likely when the prescribed statin is not Simvastatin, with particularly remarkable effects for Fluvastatin, Rosuvastatin, and Atorvastatin. Moreover, generic substitution becomes gradually much more likely as time has passed, re-

[7] Our estimates of the parameters γ_1 are larger in magnitude than Lundin's (2000) estimates, suggesting that the effects of doctors' altruism may be larger in Finland in 2003–2010 than in Sweden in 1993. Such differences in the results, however, may be due to different modeling choices or data, or to differences in the institutional context (for instance, the introduction of the generic substitution policy in Finland in 2003).

Table 3.3: Results of the random effects panel logit model

Variables	Model 1	Model 2	Model 3	Model 4
PatOOP	0.0376***	0.0110***	0.0110***	0.0381***
	0.0003	0.0003	0.0003	0.0003
InsExp	0.0080***	0.0359***	0.0359***	-0.0127***
	0.0003	0.0003	0.0003	0.0003
Gender	0.2889***	0.0549***	0.0552***	0.6969***
	0.0120	0.0116	0.0116	0.0117
Age	0.0555***	0.0576***	0.0576***	-0.0999***
	0.0005	0.0005	0.0005	0.0006
Income	0.0317***	0.0273***	0.0273***	-0.0020***
	0.0003	0.0003	0.0003	0.0003
Atorvastatin	-14.6338***	-13.8391***	-13.8379***	-14.0945***
	0.0286	0.0279	0.0279	0.0245
Rosuvastatin	-28.9538***	-23.4818***	-23.4783***	-23.3092***
	0.0476	0.0745	0.0737	0.0619
Fluvastatin	-32.5642***	-31.5209***	-31.5210***	-23.4584***
	0.0404	0.0417	0.0417	0.0882
Pravastatin	-9.7950***	-12.0383***	-12.0363***	-8.3476***
	0.0268	0.0366	0.0375	0.0245
Lovastatin	-3.2090***	-2.8483***	-2.8479***	-0.5434***
	0.0253	0.0236	0.0236	0.0261
Ill		-3.5365***		
		0.0191		
Severe			-3.5412***	
			0.0191	
2004				2.1208***
				0.0150
2005				4.6528***
				0.0164
2006				6.2075***
				0.0171
2007				7.4898***
				0.0171
2008				8.5222***
				0.0192
2009				9.0679***
				0.0210
2010				11.1415***
				0.0257
Constant	5.2378***	5.5576***	5.5583***	8.3670***
N	17 858 829	17 858 829	17 858 829	17 858 829
Log-L	-1960969	-1942662.3	-1942661	-1664209.8
Wald χ 2	1268203.38***	1074915.73***	1075638.49***	605518.14***

Table 3.4: Results of the random effects panel logit model with doctors' experience effects

Variables	Model 5	Model 6	Model 7	Model 8
PatOOP	0.0336***	0.0147***	0.0145***	0.0334***
	0.0004	0.0005	0.0005	0.0004
InsExp	0.0101***	0.0302***	0.0301***	0.0068***
	0.0004	0.0005	0.0005	0.0004
Gender	0.2512***	0.0442**	0.0380**	0.6142***
	0.0185	0.0189	0.0187	0.0173
Age	0.0586***	0.0622***	0.0634***	-0.0893***
	0.0007	0.0008	0.0008	0.0008
Income	0.0209***	0.0170***	0.0172***	-0.0029***
	0.0002	0.0004	0.0003	0.0003
PhysExper	0.2087***	0.1957***	0.1977***	0.0025***
	0.0010	0.0013	0.0013	0.0009
Atorvastatin	-14.3656***	-12.9302***	-12.9925***	-13.0618***
	0.0327	0.0292	0.0286	0.0316
Rosuvastatin	-28.6611***	-26.7189***	-26.8181***	-22.2205***
	0.1876	0.1606	0.1715	0.0780
Fluvastatin	-34.6019***	-31.8933***	-32.0945***	-21.8357***
	0.1103	0.0744	0.0726	0.0639
Pravastatin	-12.3583***	-11.3999***	-11.3979***	-8.1665***
	0.0408	0.0490	0.0415	0.0368
Lovastatin	-3.5421***	-3.2445***	-3.2573***	-0.6222***
	0.0427	0.0411	0.0416	0.0402
Ill		-3.1744***		
		0.0274		
Severe			-3.1744***	
			0.0274	
2004				2.0664***
				0.0212
2005				4.4990***
				0.0230
2006				5.9747***
				0.0238
2007				7.2670***
				0.0249
2008				8.2781***
				0.0263
2009				8.7422***
				0.0276
2010				10.8485***
				0.0321
Constant	3.4693***	3.3262***	3.3245***	6.0195***
N	4 219 510	4 219 510	4 219 510	4 219 510
Log-L	-812241.7	-806476.5	-806006.5	-691989.7
Wald $\chi 2$	395956.15***	416510.38***	437581.45***	366641.07***

Table 3.5: Results of the random effects panel logit model with doctors' habits effects

Variables	Model 9	Model 10	Model 11	Model 12
PatOOP	0.0385***	0.0048***	0.0048***	0.0366***
	0.0003	0.0003	0.0003	0.0003
InsExp	-0.0094***	0.0258***	0.0259***	-0.0123***
	0.0003	0.0003	0.0003	0.0003
Gender	0.5133***	0.3035***	0.3022***	0.6303***
	0.0102	0.0105	0.0105	0.0109
Age	-0.0608***	-0.0477***	-0.0477***	-0.0939***
	0.0005	0.0005	0.0005	0.0005
Income	0.0030***	0.0013***	0.0013***	-0.0022***
	0.0003	0.0003	0.0002	0.0003
PastBranded	-11.0729***	-10.5522***	-10.5550***	-6.0536***
	0.0182	0.0176	0.0177	0.0236
Atorvastatin	-3.0324***	-2.4207***	-2.4249***	-8.2275***
	0.0213	0.0190	0.0191	0.0281
Rosuvastatin	-10.0697***	-9.7872***	-10.2294***	-16.6532***
	0.0935	0.1241	0.1871	0.0522
Fluvastatin	-15.7918***	-14.6833***	-14.7048***	-17.5825***
	0.0506	0.0483	0.0494	0.1443
Pravastatin	-3.7717***	-3.4179***	-3.4219***	-5.5751***
	0.0237	0.0221	0.0222	0.0244
Lovastatin	-1.6221***	-1.3076***	-1.3059***	-0.7553***
	0.0242	0.0239	0.0239	0.0254
Ill		-2.9006***		
		0.0143		
Severe			-2.8992***	
			0.0145	
2004				1.1770***
				0.0153
2005				2.9486***
				0.0169
2006				3.7311***
				0.0183
2007				4.3404***
				0.0198
2008				4.8913***
				0.0213
2009				5.1713***
				0.0224
2010				6.7571***
				0.0254
Constant	13.4153***	12.5973***	12.6016***	10.3093***
N	9 714 779	9 714 779	9 714 779	9 714 779
Log-L	-1628755.7	-1615883.5	-1615826.7	-1575877.8
Wald $\chi2$	907837.72***	804929.79***	789530.22***	761915.17***

flecting general changes in the policy and the institutional context (*Models 4, 8*, and *12*) in Finland. On the other hand, physicians tend to prescribe more branded versions of statins to patients affected by more severe health conditions (*Models 2, 3, 5, 6, 10*, and *11*). Finally, while physicians' experience does not seem to play a major role in generic substitution (*Models 5-8*), physicians' habits do: the larger is the past share of branded versions of the drug, the more likely it is that the physician continues to prescribe branded statins (*Models 9-12*).

Third, in only half of the estimated models (*Models 1, 4, 5, 8, 9*, and *12*) were we able to find evidence that $\gamma_1 > \gamma_2$. The ex-post moral hazard hypothesis that physicians value patients' welfare more than insurance expenditures, in particular, is not supported in the models while controlling for patients' illness severity (*Models 2, 3, 6, 7, 10*, and *11*). Moreover, the sign of the estimated γ_2 coefficient is not robust across different specifications. As observed above for γ_1, this is because, although both the altruism and the moral hazard coefficients appear significantly different from zero due to the very large number of observations, their size is very close to zero, so that slight variations in the empirical specifications can shift their sign from positive into negative (*Models 4, 9*, and *12*).

All in all, while we find remarkable effects of physicians' habits and path-dependence in generic versus branded prescriptions, the evidence on both altruism and ex-post moral hazard is mixed and rather weak in our data.

We replicated the empirical analysis, focusing on patients aged over 75 years only, which yielded substantially analogous results, with the only exception of a consistently negative effect of age: older subjects are less likely to be given generic versions of the drugs. We also replicated the analysis while considering as the dependent variable the generic substitution at the pharmacy level, finding substantially equivalent results. Finally, we also replicated the estimations using probit rather than logit models, and considering only sub-sets of years, for instance, excluding the years after 2009, when a reference price was introduced. The main findings described above are substantially unaltered across all these robustness check estimations. All additional analyses are available upon request.

3.6 Conclusions

We tested the physicians' altruism and moral hazard hypotheses using a large national dataset of drug prescription records from Finland (n=17 858 829). We estimated the probability that physicians prescribe generic versus branded versions of statins drugs for their patients as a function of the shares of the difference in prices that patients have to pay out of their pocket and that are covered by insurance. Given that in Finland physicians bear no financial consequences whatsoever for the type of drug they prescribe, doctors are assumed to behave altruistically if they prescribe generic versions of the drug when patients' out-of-pocket payments are large. Moreover, prescribing of more branded versions of the drugs when insurance reimbursement rates are high provides evidence for physicians' moral hazard.

We have estimated the probability that physicians prescribe the generic version of statins using a set of random effects panel logit models while controlling for a wide range of doctor, patient, and drug characteristics. The results of all our estimations provide mixed and rather weak evidence on the hypotheses of physicians' altruism and ex-post moral hazard in the Finnish statins market: although the estimated coefficients associated with ex-post moral hazard and altruism are, due to the large number of observations, statistically significantly different from zero, their size is very close to zero, and is orders of magnitude smaller than the effects associated with other key explanatory factors, such as the class of the prescribed statins, and the year of prescriptions.

We find, moreover, robust and strong evidence that the physicians' decision to prescribe branded versions of statins in Finland is a self-reinforcing pattern, in the sense that physicians who have prescribed more branded drugs in the past are less likely to switch to generic versions. To the best of our knowledge, ours is the largest national panel dataset to date on which the hypotheses of ex-post moral hazard, altruism, and habit-dependence have been explicitly tested in regard to drug prescription behavior.

More broadly, our results complement and qualify the findings from other streams of the health economics literature that have found some evidence for physicians' altruism, mainly in lab experiments (Hennig-Schmidt, Selten, and Wiesen, 2011; Godager and Wiesen, 2013; Galizzi, Godager, Linnosmaa, Tammi, and Wiesen, 2015). Further research is needed to test whether physicians' altruism depends on the healthcare context (e.g. drug prescription versus treatment choice), the type of drug prescriptions (e.g. repeated versus one-off), or the nature of the diseases (e.g. cardiovascular versus mental health).

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4 Appendix A

4.1 A1

a restiction on informed consumer came from price range in demand function, when we have $\frac{\bar{\theta}}{\theta^*} \leq \frac{\theta^*}{\underline{\theta}}$,

we obtain $\theta^* \geq \sqrt{\bar{\theta}\underline{\theta}}$ this inequlity say that θ^*must be bigger than geometric mean of minimun willingness to pay $\underline{\theta}$ and the maximum willingness to pay $\bar{\theta}$

and considering $\bar{\theta} = \underline{\theta} + 1$

we have the result $\theta^* \geq \sqrt{\underline{\theta}\left(\underline{\theta}+1\right)}$.

Using a laurent expansion for $\theta = \infty$

$\underline{\theta} + \frac{1}{2} - \frac{1}{8\underline{\theta}} + \frac{1}{16\underline{\theta}^2} - \frac{5}{128\underline{\theta}^3} + \mathcal{O}$

so the number of informer consumers is approaching 1/2 of the market size when $\underline{\theta}$ increase.

for opposite case $\frac{\bar{\theta}}{\theta^*} \geq \frac{\theta^*}{\underline{\theta}}$, we obtain: $\theta^* \leq \sqrt{\underline{\theta}\bar{\theta}}$so θ^*must be lower than geometric mean of minimun willingness to pay $\underline{\theta}$ and the maximum willingness to pay $\bar{\theta}$

4.2 A2

Optmist $\theta' \leq \theta''$ or $\Delta_E \geq \Delta$

We assume an effort to increase quality level. the functional forma for effor is the usual found in licterature αq_i^2

$\Delta_E = q_E - q_0$ and $\Delta = q_H - q_L$ or if $q_L = q_0\Delta = q_H - q_0$

A.a

restriction:

$$P_H - \underline{\theta}\Delta_E \geq P_H - \theta^*\Delta \geq P_H - \theta^*\Delta_E \geq P_H - \bar{\theta}\Delta \tag{4.1}$$

$$P_L + \bar{\theta}\Delta \geq P_L + \theta^*\Delta_E \geq P_L + \theta^*\Delta \geq P_L + \underline{\theta}\Delta_E$$

obtain:

$$1 \leq \frac{\Delta_E}{\Delta} \leq min\left\{\frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*}\right\}$$

Given these restrictions we can specify price domains and market demands as follows:

$$D_L\left(P_L, P_H\right) = \begin{cases} \theta' - \underline{\theta} & if\ P_H - \theta^*\Delta \leq P_L \leq P_H - \underline{\theta}\Delta_E \\ \theta' - \underline{\theta} + \theta'' - \theta^* & if\ P_H - \theta^*\Delta_E \leq P_L \leq P_H - \theta^*\Delta \\ \theta'' - \underline{\theta} & if\ P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta_E \\ \bar{\theta} - \underline{\theta} & if\ 0 \leq P_L \leq P_H - \bar{\theta}\Delta \end{cases}$$

$$D_H\left(P_L,P_H\right)=\begin{cases}\bar{\theta}-\theta'' & if\ P_L+\theta^*\Delta_E\leq P_H\leq P_L+\bar{\theta}\Delta\\ \bar{\theta}-\theta''+\theta^*-\theta' & if\ P_L+\theta^*\Delta\leq P_H\leq P_L+\theta^*\Delta_E\\ \bar{\theta}-\theta' & if\ P_L+\underline{\theta}\Delta_E\leq P_H\leq P_L+\theta^*\Delta\\ \bar{\theta}-\underline{\theta} & if\ 0\leq P_H\leq P_L+\underline{\theta}\Delta_E\end{cases}$$

A.a.1

restriction

$$P_H-\theta^*\Delta\leq P_L^*\leq P_H-\underline{\theta}\Delta_E$$

$$P_L+\underline{\theta}\Delta_E\leq P_H^*\leq P_L+\theta^*\Delta$$

profit function:

$$\Pi_L\left(P_L,P_H\right)=P_L\left(\theta'-\underline{\theta}\right)-\alpha q_L^2 \qquad \Pi_H\left(P_L,P_H\right)=P_H\left(\bar{\theta}-\theta'\right)-\alpha q_H^2$$

price:

$$P_L^*=\frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)}{3} \qquad P_H^*=\frac{\Delta_E\left(2\bar{\theta}-\underline{\theta}\right)}{3}$$

Equilibrium profit:

$$\Pi_L^*=\frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)^2}{9}-\alpha q_L^2 \qquad \Pi_H^*=\frac{\Delta_E\left(2\bar{\theta}-\underline{\theta}\right)^2}{9}-\alpha q_H^2$$

restictin on θ^*:

$$\theta^*\geq\frac{\Delta_E\left(2\underline{\theta}+1\right)}{3\Delta}$$

Quality stage:
derivate:

$$\frac{\partial\Pi_L}{\partial q_0}=\frac{4\bar{\theta}\underline{\theta}-\bar{\theta}^2-4\underline{\theta}^2}{9}\leq 0$$

$$\frac{\partial\Pi_L}{\partial q_{\mathrm{L}}}=-2\alpha q_L\leq 0$$

$$\frac{\partial\Pi_H}{\partial q_H}=-2\alpha q_H\leq 0$$

$$\frac{\partial\Pi_H}{\partial q_{\mathrm{E}}}=\frac{\bar{\theta}^2+4\underline{\theta}^2-4\bar{\theta}\underline{\theta}}{9}\geq 0$$

from restriction

$$q_H \geq q_0 + \frac{\Delta_E \left(2\underline{\theta} + 1\right)}{3\theta^*}$$

$$\Pi_L^* = \frac{\left(q_E - q_0\right)\left(\bar{\theta} - 2\underline{\theta}\right)^2}{9} - \alpha q_0^2 \qquad \Pi_H^* = \frac{\left(q_E - q_0\right)\left(2\bar{\theta} - \underline{\theta}\right)^2}{9} - \alpha\left(q_0 + \frac{\Delta_E \left(2\underline{\theta} + 1\right)}{3\theta^*}\right)^2$$

The results are equivalent to case A.b.1 already discussed in chapter 1

A.a.2

Restiction:

$$P_H - \theta^* \Delta_E \leq P_L^* \leq P_H - \theta^* \Delta$$

$$P_L + \theta^* \Delta \leq P_H^* \leq P_L + \theta^* \Delta_E$$

profict function:

$$\Pi_L \left(P_L, P_H\right) = P_L \left(\theta' - \underline{\theta} + \theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H \left(P_L, P_H\right) = P_H \left(\bar{\theta} - \theta'' + \theta^* - \theta'\right) - \alpha q_H^2$$

price:

$$P_L^* = \frac{\Delta_E \Delta \left(1 - \left(\underline{\theta} + \theta^*\right)\right)}{3\left(\Delta_E + \Delta\right)} \qquad P_H^* = \frac{\Delta_E \Delta \left(\underline{\theta} + \theta^* + 2\right)}{3\left(\Delta_E + \Delta\right)}$$

1 came from $\left(\bar{\theta} - \underline{\theta}\right)$so it is the market extention.
restriction on θ^*

$$\frac{\Delta \left(1 + 2\underline{\theta}\right)}{3\Delta_E + \Delta} \leq \theta^* \leq \frac{\Delta_E \left(1 + 2\underline{\theta}\right)}{\Delta_E + 3\Delta}$$

profit funcition:

$$\Pi_L^* = \frac{\Delta_E \Delta \left(1 - \left(\underline{\theta} + \theta^*\right)\right)^2}{9\left(\Delta_E + \Delta\right)} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E \Delta \left(\underline{\theta} + \theta^* + 2\right)^2}{9\left(\Delta_E + \Delta\right)} - \alpha q_H^2$$

quality stage:
for Π_L

$$\frac{\partial \Pi_L}{\partial q_0} = -\frac{\gamma\left(q_E^2 + q_H^2 + 2q_0^2 - 2q_H q_0 - 2q_E q_0\right)}{\left(9q_E + 9q_H - 18q_0\right)^2} - 2\alpha q_L \qquad and \quad \gamma = \left[1 - \left(\underline{\theta} + \theta^*\right)\right]^2$$

$$\frac{\partial \Pi_L}{\partial q_0} \leq 0 \quad if\; q_E \geq q_H \geq q_0\,,\, \gamma \geq 0$$

$$\frac{\partial^2\Pi_L}{\partial q_0^2} = -\frac{2\gamma\left(q_E-q_H\right)^2}{81\left(q_E+q_H-2q_0\right)^3} - 2\alpha \leq 0$$

for Π_H

$$\frac{\partial\Pi_H}{\partial q_H} = \frac{\varphi\left(q_E-q_0\right)^2}{9\left(q_E+q_H-2q_0\right)^2} - 2\alpha q_H \qquad \varphi = \left(2+\underline{\theta}+\theta^*\right)^2$$

$$\frac{\partial^2\Pi_H}{\partial q_H^2} = -\frac{2\varphi\left(q_E-q_0\right)^2}{9\left(q_E+q_H-2q_0\right)^3} - 2\alpha$$

$q_H^* = -\frac{1}{3*2^{2/3}\alpha}\left(\Phi\right)^{1/3} + \frac{-324\alpha^2q_E^2+1296\alpha^2q_Eq_0-1296\alpha^2q_0^2}{\left(\Phi\right)^{1/3}} - \frac{2(\alpha q_0-2\alpha q_0)}{3\alpha}$

$$\begin{aligned}\Phi &= -3\varphi^2\alpha\left(q_E-q_0\right)+\sqrt{3}\sqrt{8\alpha^5q_E^3\varphi\left(q_E-q_0\right)^2-48\alpha^5q_E^2q_0\varphi\left(q_E-q_0\right)^2}+\\ &+\sqrt{3}\sqrt{-64\alpha^5q_0^3\varphi\left(q_E-q_0\right)^2+96\alpha^5q_Eq_0^2\varphi\left(q_E-q_0\right)^2+3\alpha^4\varphi\left(q_E-q_0\right)^4}\\ &+\sqrt{3}\sqrt{+96\alpha^5q_Eq_0^2\varphi\left(q_E-q_0\right)^2+3\alpha^4\varphi\left(q_E-q_0\right)^4}+\left(-4\alpha^3q_E^3+24\alpha^3q_E^2q_0-48\alpha^3q_Eq_0^3+32\alpha^3q_0^3\right)\end{aligned}$$

this is the only rearl solution of $\frac{\partial\Pi_H}{\partial q_H}$, we have to check ih the $q_H^*$is consistent with the follow contrain.

Possibility of different scenarios. or differentiation or convergence to case A.b.3

using restiction we have some bound for q_H:

on right side:

$$q_H \leq q_0 + \frac{\left(q_E-q_0\right)\left(1+2\underline{\theta}\right)-\theta^*\left(q_E-q_0\right)}{3\theta^*}$$

on left side:

$$q_H \leq q_0 + \frac{3\left(q_E-q_0\right)\theta^*}{1+2\underline{\theta}-\theta^*}$$

The results are equivalent to case A.b.2 and A.c.2 already discussed in chapter 1

A.a.3

restriction

$$P_H - \bar{\theta}\Delta \leq P_L^* \leq P_H - \theta^*\Delta_E$$

$$P_L + \theta^*\Delta_E \leq P_H^* \leq P_L + \bar{\theta}\Delta$$

profit function:

$$\Pi_L\left(P_L,P_H\right) = P_L\left(\theta''-\underline{\theta}\right)-\alpha q_L^2 \qquad \Pi_H\left(P_L,P_H\right) = P_H\left(\bar{\theta}-\theta''\right)-\alpha q_H^2$$

price:

$$P_L^* = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \qquad P_H^* = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3}$$

Equilibrium profit:

$$\Pi_L^* = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)^2}{9} - \alpha q_H^2$$

restictin on θ^*:

$$\theta^* \leq \frac{\Delta\left(2\underline{\theta} + 1\right)}{3\Delta_E}$$

Quality stage:
derivate:

$$\frac{\partial \Pi_L}{\partial q_0} = \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{9} - 2\alpha q_L$$

$$q_L = \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{18\alpha} \quad and \quad \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{18\alpha} \leq 0$$

$$\frac{\partial \Pi_H}{\partial q_H} = \frac{\bar{\theta}^2 + 4\underline{\theta}^2 - 4\bar{\theta}\underline{\theta}}{9} - 2\alpha q_H$$

$$q_H^* = \frac{\bar{\theta}^2 + 4\underline{\theta}^2 - 4\bar{\theta}\underline{\theta}}{18\alpha}$$

from restriction

$$q_H \geq q_0 + \frac{3\Delta_E \theta^*}{1 + 2\underline{\theta}}$$

if q_H^*respect the last constrain the profit functions are:

$$\Pi_L^* = \frac{\left(\bar{\theta} - 2\underline{\theta}\right)^3 - q_0\left(\bar{\theta} - 2\underline{\theta}\right)^2}{162\alpha} - \alpha q_0^2 \qquad \Pi_H^* = \frac{\left(2\bar{\theta} - \underline{\theta}\right)^3 - q_0\left(\bar{\theta} - 2\underline{\theta}\right)^2}{162\alpha} - \alpha\left(\frac{\left(\bar{\theta} - 2\underline{\theta}\right)^2}{18\alpha}\right)^2$$

here add comment about the quality range and possible profit or deviation.
The results are equivalent to case A.c.3 already discussed in chapter 1

A.d

restriction:

$$P_H - \theta^*\Delta \geq P_H - \underline{\theta}\Delta_E \geq P_H - \bar{\theta}\Delta \geq P_H - \theta^*\Delta_E \tag{4.2}$$

$$P_L + \theta^*\Delta_E \geq P_L + \bar{\theta}\Delta \geq P_L + \underline{\theta}\Delta_E \geq P_L + \theta^*\Delta$$

from this restriction in this case we need $\underline{\theta} > 0$
obtain:

$$max\left\{\frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}}\right\} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\underline{\theta}}$$

Given these restrictions we can specify price domains and market demands as follows:

$$D_L(P_L, P_H) = \begin{cases} \theta'' - \theta^* & if\ P_H - \underline{\theta}\Delta_E \leq P_L \leq P_H - \theta^*\Delta \\ \theta' - \underline{\theta} + \theta'' - \theta^* & if\ P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \underline{\theta}\Delta_E \\ 1 - \theta^* + \theta' & if\ P_H - \theta^*\Delta_E \leq P_L \leq P_H - \bar{\theta}\Delta \\ \bar{\theta} - \underline{\theta} & 0 \leq P_L \leq P_H - \theta^*\Delta_E \end{cases} \tag{4.3}$$

$$D_H(P_L, P_H) = \begin{cases} \theta^* - \theta' & if\ P_L + \bar{\theta}\Delta \leq P_H \leq P_L + \theta^*\Delta_E \\ \bar{\theta} - \theta'' + \theta^* - \theta' & if\ P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \bar{\theta}\Delta \\ 1 - \theta'' + \theta^* & if\ P_L + \theta * \Delta \leq P_H \leq P_L + \underline{\theta}\Delta_E \\ \bar{\theta} - \underline{\theta} & if\ 0 \leq P_H \leq P_L + \theta * \Delta \end{cases} \tag{4.4}$$

A.d.1

restriction

$$P_H - \underline{\theta}\Delta_E \leq P_L^* \leq P_H - \theta^*\Delta$$

$$P_L + \theta^*\Delta \leq P_H^* \leq P_L + \underline{\theta}\Delta_E$$

profit function:

$$\Pi_L(P_L, P_H) = P_L\left(\theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H(P_L, P_H) = P_H\left((\bar{\theta} - \underline{\theta}) - \theta'' + \theta^*\right) - \alpha q_H^2$$

price:

$$P_L^* = \frac{\Delta\left(1(\bar{\theta} - \underline{\theta}) - \theta^*\right)}{3} \qquad P_H^* = \frac{\Delta\left(2(\bar{\theta} - \underline{\theta}) + \theta *\right)}{3}$$

Equilibrium profit:

$$\Pi_L^* = \frac{\Delta\left(1-\theta^*\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta\left(2+\theta^*\right)^2}{9} - \alpha q_H^2$$

restictin on θ^*:

$$\theta^* \leq min\left\{1\,,\ \frac{3\underline{\theta}\Delta_E}{2\Delta} - \frac{\bar{\theta}-\theta}{2}\right\}$$

Quality stage:
derivate:

$$\frac{\partial\Pi_L}{\partial q_0} = -\frac{{\theta^*}^2 - 2\theta^* + 1}{9} - 2\alpha q_L \leq 0$$

$$\frac{\partial^2\Pi_L}{\partial q_0^2} = -2\alpha$$

$$\frac{\partial\Pi_H}{\partial q_{\mathrm{H}}} = \frac{{\theta^*}^2 + 4\theta^* + 4}{9} - 2\alpha q_H$$

$$q_H = \frac{{\theta^*}^2 + 4\theta^* + 4}{18\alpha}$$

from restriction

$$q_H \leq q_0 + \frac{3\Delta_E\theta}{2\theta^* + 1}$$

The results are equivalent to case A.c.1 already discussed in chapter 1

A.d.2

Restiction:

$$P_H - \bar{\theta}\Delta \leq P_L^* \leq P_H - \underline{\theta}\Delta_E$$

$$P_L + \underline{\theta}\Delta_E \leq P_H^* \leq P_L + \bar{\theta}\Delta$$

profict function:

$$\Pi_L\left(P_L, P_H\right) = P_L\left(\theta' - \underline{\theta} + \theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H\left(P_L, P_H\right) = P_H\left(\bar{\theta} - \theta'' + \theta^* - \theta'\right) - \alpha q_H^2$$

price:

$$P_L^* = \frac{\Delta_E\Delta\left(1-\left(\underline{\theta}+\theta^*\right)\right)}{3\left(\Delta_E+\Delta\right)} \qquad P_H^* = \frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)}{3\left(\Delta_E+\Delta\right)}$$

1 came from $\left(\bar{\theta}-\underline{\theta}\right)$so it is the market extention.

restriction on θ^*

$$\frac{\underline{\theta}-1}{2}+\frac{3\Delta_E\underline{\theta}}{2\Delta}\leq\theta^*\leq\frac{\underline{\theta}+2}{2}+\frac{3\Delta\bar{\theta}}{2\Delta_E}$$

profit funcition:

$$\Pi_L^*=\frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)^2}{9\left(\Delta_E+\Delta\right)}-\alpha q_L^2 \qquad \Pi_H^*=\frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)^2}{9\left(\Delta_E+\Delta\right)}-\alpha q_H^2$$

quality stage:
for Π_L

$$\frac{\partial\Pi_L}{\partial q_0}=-\frac{\gamma\left(q_E^2+q_H^2+2q_0^2-2q_Hq_0-2q_Eq_0\right)}{\left(9q_E+9q_H-18q_0\right)^2}-2\alpha q_L \qquad \mathit{and} \quad \gamma=\left[1-(\underline{\theta}+\theta^*)\right]^2$$

$$\frac{\partial\Pi_L}{\partial q_0}\leq 0 \quad if\; q_E\geq q_H\geq q_0\,,\,\gamma\geq 0$$

$$\frac{\partial^2\Pi_L}{\partial q_0^2}=-\frac{2\gamma\left(q_E-q_H\right)^2}{81\left(q_E+q_H-2q_0\right)^3}-2\alpha\leq 0$$

for Π_H

$$\frac{\partial\Pi_H}{\partial q_H}=\frac{\varphi\left(q_E-q_0\right)^2}{9\left(q_E+q_H-2q_0\right)^2}-2\alpha q_H \qquad \varphi=\left(2+\underline{\theta}+\theta^*\right)^2$$

$$\frac{\partial^2\Pi_H}{\partial q_H^2}=-\frac{2\varphi\left(q_E-q_0\right)^2}{9\left(q_E+q_H-2q_0\right)^3}-2\alpha$$

$q_H^*=-\frac{1}{3*2^{2/3}\alpha}\left(\Phi\right)^{1/3}+\frac{-324\alpha^2q_E^2+1296\alpha^2q_Eq_0-1296\alpha^2q_0^2}{\left(\Phi\right)^{1/3}}-\frac{2(\alpha q_0-2\alpha q_0)}{3\alpha}$

$$\begin{aligned}\Phi &= -3\varphi^2\alpha\left(q_E-q_0\right)+\sqrt{3}\sqrt{8\alpha^5q_E^3\varphi\left(q_E-q_0\right)^2-48\alpha^5q_E^2q_0\varphi\left(q_E-q_0\right)^2}+\\ &+ \sqrt{3}\sqrt{-64\alpha^5q_0^3\varphi\left(q_E-q_0\right)^2+96\alpha^5q_Eq_0^2\varphi\left(q_E-q_0\right)^2+3\alpha^4\varphi\left(q_E-q_0\right)^4}\\ &+ \sqrt{3}\sqrt{+96\alpha^5q_Eq_0^2\varphi\left(q_E-q_0\right)^2+3\alpha^4\varphi\left(q_E-q_0\right)^4}+\left(-4\alpha^3q_E^3+24\alpha^3q_E^2q_0-48\alpha^3q_Eq_0^3+32\alpha^3q_0^3\right)\end{aligned}$$

this is the only rearl solution of $\frac{\partial\Pi_H}{\partial q_H}$, we have to check ih the $q_H^*$is consistent with the follow contrain.
using restiction we have some bound for q_H:
on right side:

$$q_0+\frac{2\Delta_E\theta^*-\Delta_E\left(\underline{\theta}+2\right)}{3\bar{\theta}}\leq q_H\leq q_0-\frac{3\left(q_E-q_0\right)\underline{\theta}}{\underline{\theta}-1-2\theta^*}$$

$\underline{\theta}-1-2\theta^*$is negative becouse $\underline{\theta}\leq\theta^*$by definition.
The results are equivalent to case A.b.2 and A.c.2 already discussed in chapter 1

A.d.3

restriction

$$P_H - \theta^* \Delta_E \leq P_L^* \leq P_H - \bar{\theta}\Delta$$

$$P_L + \bar{\theta}\Delta \leq P_H^* \leq P_L + \theta^* \Delta_E$$

profit function:

$$\Pi_L\left(P_L, P_H\right) = P_L\left(\left(\bar{\theta} - \underline{\theta}\right) - \theta^* + \theta'\right) - \alpha q_L^2 \qquad \Pi_H\left(P_L, P_H\right) = P_H\left(\left(\theta^* - \theta'\right) - \alpha q_H^2\right.$$

price:

$$P_L^* = \frac{\Delta_E\left(2 - \theta^*\right)}{3} \qquad P_H^* = \frac{\Delta_E\left(1 + \theta^*\right)}{3}$$

Equilibrium profit:

$$\Pi_L^* = \frac{\Delta_E\left(2 - \theta^*\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E\left(1 + \theta^*\right)^2}{9} - \alpha q_H^2$$

restictin on θ^*:

$$\theta^* \geq \frac{1}{2} + \frac{3\Delta\bar{\theta}}{2\Delta_E}$$

Quality stage:
derivate:

$$\frac{\partial \Pi_L}{\partial q_0} = -\frac{{\theta^*}^2 - 4\theta^* + 4}{9} - 2\alpha q_L \leq 0$$

$$\frac{\partial^2 \Pi_L}{\partial q_0^2} = -2\alpha$$

$$\frac{\partial \Pi_H}{\partial q_H} = -2\alpha q_L$$

$$\frac{\partial \Pi_E}{\partial q_E} = \frac{{\theta^*}^2 + 2\theta^* + 1}{9} \geq 0$$

from restriction

$$q_H \leq q_0 + \frac{\Delta_E\left(2\theta^* - 1\right)}{3\bar{\theta}}$$

The results are equivalent to case A.b.3 already discussed in chapter 1

4.3 A3

proff of equilibium in case A.b.2 and A.c.2

startin from the follow profit function:

$$\Pi_L\left(P_L,P_H\right)=P_L\left(\theta'-\underline{\theta}+\theta''-\theta^*\right)-\alpha q_L^2 \qquad \Pi_H\left(P_L,P_H\right)=P_H\left(\bar{\theta}-\theta''+\theta^*-\theta'\right)-\alpha q_H^2$$

leading to the following equilibrium prices:

$$P_L^*=\frac{\Delta_E\Delta\left(1-\left(\underline{\theta}+\theta^*\right)\right)}{3\left(\Delta_E+\Delta\right)} \qquad P_H^*=\frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)}{3\left(\Delta_E+\Delta\right)}$$

By substitution we can find equilibrium profit funcitions as follows:

$$\Pi_L^*=\frac{\Delta_E\Delta\left(1-\left(\underline{\theta}+\theta^*\right)\right)^2}{9\left(\Delta_E+\Delta\right)}-\alpha q_L^2 \qquad \Pi_H^*=\frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)^2}{9\left(\Delta_E+\Delta\right)}-\alpha q_H^2$$

Turning then to the quality selction stage, by profit maximization in qualities we get:

$$\frac{\partial\Pi_L}{\partial q_L}=-\frac{\gamma\left(q_E^2+q_H^2+2q_L^2-2q_Hq_0-2q_Eq_L\right)}{\left(9q_E+9q_H-18q_L\right)^2}-2\alpha q_L \qquad and \quad \gamma=\left[1-\left(\underline{\theta}+\theta^*\right)\right]^2 \tag{4.5}$$

$$\frac{\partial\Pi_L}{\partial q_L}\leq 0 \quad if: q_E\geq q_H\geq q_0\,,\ \gamma\geq 0$$

$$\frac{\partial^2\Pi_L}{\partial q_L^2}=-\frac{2\gamma\left(q_E-q_H\right)^2}{81\left(q_E+q_H-2q_0\right)^3}-2\alpha\leq 0 \tag{4.6}$$

solving equation (4.5) for $q_0$we found 3 solution one real solution and two solution in the domain of complex number, so we discart non real solution.

now we have to undestand if real solution is positive or negative. First of all aur derivativative (eq 4.5) is negative for $q_0=0$ and also the lim for $q_0\to\infty$is $-\infty$. Using the second derivative equation (4.6) we can say that the first derivative is alwais decreasing so under that condition the only real solution must be a negative solution.

Thereforethe low quality firm is lead to produce the minimum quality, i.e $q_L^*=q^\circ$ as corner solution

.Concerning the high quality firm we get:

$$\frac{\partial\Pi_H}{\partial q_H}=\frac{\varphi\left(q_E-q_0\right)^2}{9\left(q_E+q_H-2q_0\right)^2}-2\alpha q_H\geq 0 \qquad \varphi=\left(2+\underline{\theta}+\theta^*\right)^2 \tag{4.7}$$

$$\frac{\partial^2\Pi_H}{\partial q_H^2}=-\frac{2\varphi\left(q_E-q_0\right)^2}{9\left(q_E+q_H-2q_0\right)^3}-2\alpha \tag{4.8}$$

from equation (4.7) we obtain three solution uno real solution and two complex solution, discarting the complex one we obtain the follow real solution:

$$q_H^* = -\frac{1}{3*2^{2/3}\alpha}\left(\Phi\right)^{1/3} + \frac{-324\alpha^2 q_E^2+1296\alpha^2 q_E q_0-1296\alpha^2 q_0^2}{\left(\Phi\right)^{1/3}} - \frac{2(\alpha q_0-2\alpha q_0)}{3\alpha}$$

$$\begin{aligned}
\Phi &= -3\varphi^2\alpha\left(q_E-q_0\right)+\sqrt{3}\sqrt{8\alpha^5 q_E^3\varphi\left(q_E-q_0\right)^2-48\alpha^5 q_E^2 q_0\varphi\left(q_E-q_0\right)^2}+\\
&+ \sqrt{3}\sqrt{-64\alpha^5 q_0^3\varphi\left(q_E-q_0\right)^2+96\alpha^5 q_E q_0^2\varphi\left(q_E-q_0\right)^2+3\alpha^4\varphi\left(q_E-q_0\right)^4}\\
&+ \sqrt{3}\sqrt{+96\alpha^5 q_E q_0^2\varphi\left(q_E-q_0\right)^2+3\alpha^4\varphi\left(q_E-q_0\right)^4}+\left(-4\alpha^3 q_E^3+24\alpha^3 q_E^2 q_0-48\alpha^3 q_E q_0^3+32\alpha^3 q_0^3\right)
\end{aligned}$$

now we have to prove that the above solution is positive, the proo is the follow:

the first derivati eq(4.7) evalued in $q_H = 0$ is positve $\frac{\partial \Pi_H}{\partial q_H}|_{q_H=0} > 0$ and the the linit of $\frac{\partial \Pi_H}{\partial q_H}$ for $q_H \to \infty$is $-\infty$from these result and the second derivative always $\frac{\partial^2 \Pi_H}{\partial q_H^2} < 0$ we can say that the only real solutio is unique and positive.

as before we have rescriction on $q_H$so we have to check if those restriction are respected for each case, otherwise we have a corner solution.

4.4 A4

this case is completely equivalent to case A,b.2 and is analyzed in Appendix II

This sub-case is defined by the following price domains:

$$P_H - \theta^* \Delta_E \leq P_L^* \leq P_H - \underline{\theta} \Delta_E$$

$$P_L + \underline{\theta} \Delta_E \leq P_H^* \leq P_L + \theta^* \Delta_E$$

Considering the related demand segments we get the following profit functions:

$$\Pi_L (P_L, P_H) = P_L \left(\theta' - \underline{\theta} + \theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H (P_L, P_H) = P_H \left(\bar{\theta} - \theta'' + \theta^* - \theta'\right) - \alpha q_H^2$$

price:

$$P_L^* = \frac{\Delta_E \Delta \left(1 - (\underline{\theta} + \theta^*)\right)}{3 (\Delta_E + \Delta)} \qquad P_H^* = \frac{\Delta_E \Delta (\underline{\theta} + \theta^* + 2)}{3 (\Delta_E + \Delta)}$$

By checking that equilibrium prices belong to the price domains we get a further restriction on θ^*

$$\theta^* \geq max \left\{ \frac{\Delta (1 + 2\underline{\theta})}{3\Delta_E + \Delta}, \frac{\underline{\theta} (3\Delta_E + \Delta)}{2\Delta} - \frac{\Delta}{2\Delta} \right\}$$

profit funcition:

$$\Pi_L^* = \frac{\Delta_E \Delta \left(1 - (\underline{\theta} + \theta^*)\right)^2}{9 (\Delta_E + \Delta)} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E \Delta (\underline{\theta} + \theta^* + 2)^2}{9 (\Delta_E + \Delta)} - \alpha q_H^2$$

quality stage:
for Π_L

$$\frac{\partial \Pi_L}{\partial q_0} = -\frac{\gamma \left(q_E^2 + q_H^2 + 2q_0^2 - 2q_H q_0 - 2q_E q_0\right)}{(9q_E + 9q_H - 18q_0)^2} - 2\alpha q_L \qquad and \quad \gamma = [1 - (\underline{\theta} + \theta^*)]^2$$

$$\frac{\partial \Pi_L}{\partial q_L} \leq 0 \quad if : q_E \geq q_H \geq q_0 \, , \, \gamma \geq 0$$

$$\frac{\partial^2 \Pi_L}{\partial q_L^2} = -\frac{2\gamma (q_E - q_H)^2}{81 (q_E + q_H - 2q_0)^3} - 2\alpha \leq 0$$

for Π_H

$$\frac{\partial \Pi_H}{\partial q_H} = \frac{\varphi (q_E - q_0)^2}{9 (q_E + q_H - 2q_0)^2} - \alpha q_H^2 \qquad \varphi = (2 + \underline{\theta} + \theta^*)^2$$

$$\frac{\partial^2 \Pi_H}{\partial q_H^2} = -\frac{2\varphi \left(q_E - q_0\right)^2}{9 \left(q_E + q_H - 2q_0\right)^3} - 2\alpha$$

Still considering the price domains together with equilibrium prices we can find lower and upper bounds for q_H:

$$q_0 + \frac{3 \left(q_E - q_0\right) \underline{\theta}}{1 + 2\theta^* - \underline{\theta}} \leq q_H \leq q_0 \frac{3 \left(q_E - q_0\right) \theta^*}{1 + 2\underline{\theta} - \theta^*}$$

4.5 A5

Stability Condition

Case A (optimist)

A.a.1

A.a.1 vs A.a.2

A.a.1 vs A.a.2 for L $\Pi_L^{A.a.1}(P_L^*, P_H^*) \geq \Pi_L^{A.a.2}(P_L', P_H^*)$

$\frac{\Delta_E(\bar{\theta}-2\underline{\theta})^2}{9} \geq P_L'(\theta' - \underline{\theta} + \theta'' - \theta^*)$

if $P_H^* = \frac{\Delta_E(2\bar{\theta}-\underline{\theta})}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta_E}{6} * \frac{-2\underline{\theta} + 2\Delta + \underline{\theta}\Delta_E + 2\Delta_E - 3\theta^*\Delta}{\Delta + \Delta_E}$$

or we can rewrite

$$P_L^d = \left(\frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3} - \theta^*\Delta + \frac{\Delta_{\mathrm{E}}\left(2\bar{\theta} - \underline{\theta}\right)}{3} - \underline{\theta}\Delta\right)\frac{1}{2}\frac{\Delta_E}{\Delta + \Delta_E}$$

comment is like an average weight price
on this case restiction on θ^*we have:

$$\left[\left(\frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3} + \frac{\Delta_E\left(2\bar{\theta} - \underline{\theta}\right)}{3} - \underline{\theta}\Delta\right)\frac{1}{2}\frac{\Delta_E}{\Delta + \Delta_E} - \frac{\Delta_E\left(2\bar{\theta} - \underline{\theta}\right)}{3}\right]\frac{1}{\Delta_E}\left(-\frac{2\Delta_E + 2\Delta}{2\Delta_E + \Delta}\right) \leq \theta^*$$

$$\theta^* \leq \left[\left(\frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3} + \frac{\Delta_E\left(2\bar{\theta} - \underline{\theta}\right)}{3} - \underline{\theta}\Delta\right)\frac{1}{2}\frac{\Delta_E}{\Delta + \Delta_E} - \frac{\Delta_E\left(2\bar{\theta} - \underline{\theta}\right)}{3}\right]\frac{1}{\Delta}\left(-\frac{2\Delta_E + 2\Delta}{\Delta_E + 2\Delta}\right)$$

and profit:

$$\Pi_L^d = \frac{\Delta_E\left(-2\underline{\theta}\Delta + \underline{\theta}\Delta_E - 3\Delta\theta^* + 2\Delta + 2\Delta_E\right)\left(-4\underline{\theta}\Delta + \underline{\theta}\Delta_E + 2\underline{\theta} - 3\theta^*\Delta + 2\Delta + 2\Delta_E\right)}{36\left(\Delta + \Delta_E\right)} - \alpha q_L^2$$

A.a.1 vs A.a.2 for H $\Pi_H^{A.a.1}(P_L^*, P_H^*) \geq \Pi_H^{A.a.2}(P_L^*, P_H'')$

$\frac{\Delta_E(2\bar{\theta}-\underline{\theta})^2}{9} \geq P_H''\left(\bar{\theta} - \theta'' + \theta^* - \theta'\right)$

if $P_L^* = \frac{\Delta_E(\bar{\theta}-2\underline{\theta})}{3}$from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E}{6}\frac{\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right) + \Delta\left(\bar{\theta} - 2\underline{\theta}\right) + 3\Delta\bar{\theta} + 3\Delta\theta^*}{\Delta_E + \Delta}$$

or rearranged

$$P_H^d = \left(\frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \theta^*\Delta + \frac{\Delta_{\mathrm{E}}\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \bar{\theta}\Delta \right) \frac{1}{2} \frac{\Delta_E}{\Delta + \Delta_E}$$

on this case restiction on θ^*we have:

$$\theta^* \leq \left[\left(\frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \frac{\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \bar{\theta}\Delta \right) \frac{1}{2} \frac{\Delta_E}{\Delta + \Delta_E} - \frac{\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \right] \frac{2\Delta + 2\Delta_E}{2\Delta^2 + \Delta\Delta_E}$$

$$\theta^* \geq \left[\left(\frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \frac{\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \bar{\theta}\Delta \right) \frac{1}{2} \frac{1}{\Delta + \Delta_E} - \frac{\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \right] \frac{2\Delta + 2\Delta_E}{\Delta + \Delta\Delta_E}$$

profit:

$$\Pi_H^d = \frac{\Delta_E\left(-2\underline{\theta}\Delta + \underline{\theta}\Delta_E - 3\Delta\theta^* + 4\Delta + 2\Delta\right)\left(4\underline{\theta}\Delta^2 + 7\underline{\theta}\Delta\Delta_E - 4\underline{\theta}\Delta - \underline{\theta}\Delta_E + 3\Delta^2\theta^* + 2\Delta^2 + 6\theta^*\Delta\Delta_E + \right.}{36\left(\Delta + \Delta_E\right)^2}$$

A.a.1 vs A.a.3

A.a.1 vs A.a.3 for L $\Pi_L^{A.a.1}\left(P_L^*, P_H^*\right) \geq \Pi_L^{A.a.3}\left(P_L', P_H^*\right)$
$\frac{\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right)^2}{9} \geq P_L''\left(\theta'' - \underline{\theta}\right)$
if $P_H^* = \frac{\Delta_E\left(2\bar{\theta} - \underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta_E\left(2\bar{\theta} - \underline{\theta}\right)}{6} - \frac{\underline{\theta}\Delta}{2}$$

or in an other form $P_L^d = \frac{\Delta_E\underline{\theta}}{6} + \frac{\Delta_E}{3} - \frac{\underline{\theta}\Delta}{2}$
on this case restiction on θ^*we have:

$$\theta^* \leq \frac{\left(2\bar{\theta} - \underline{\theta}\right)}{6} + \frac{\underline{\theta}\Delta}{2\Delta_E}$$

and profit:

$$\Pi_L^d = \frac{\left[\Delta_E\left(2\bar{\theta} - \underline{\theta}\right) - 3\underline{\theta}\Delta\right]^2}{36\Delta} - \alpha q_L^2$$

A.a.1 vs A.a.3 for H $\Pi_H^{A.a.1}(P_L^*, P_H^*) \geq \Pi_H^{A.a.3}(P_L^*, P_H'')$

$\frac{\Delta_E\left(2\bar{\theta}-\underline{\theta}\right)^2}{9} \geq P_H''\left(\bar{\theta}-\theta''\right)$

if $P_L^* = \frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)}{3}$from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)}{6} + \frac{\bar{\theta}\Delta}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \leq -\frac{\left(\bar{\theta}-2\underline{\theta}\right)}{6} + \frac{\bar{\theta}\Delta}{2\Delta_E}$$

profits is:

$$\Pi_H^d = \frac{\left[\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)+3\underline{\theta}\Delta\right]^2}{36\Delta} - \alpha q_H^2$$

A.a.2

A.a.2 vs A.a.1

A.a.2 vs A.a.1 for L $\Pi_L^{A.a.2}(P_L^*, P_H^*) \geq \Pi_L^{A.a.1}\left(P_L^d, P_H^*\right)$

$\frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)^2}{9(\Delta_E+\Delta)} \geq P_L^d\left(\theta'-\underline{\theta}\right)$

if $P_H^* = \frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta\Delta_{\mathrm{E}}\left(2+\underline{\theta}+\theta^*\right)}{6\left(\Delta+\Delta_{\mathrm{E}}\right)} - \frac{\underline{\theta}\Delta_E}{2}$$

or after some manipulation $P_L^d = \frac{\Delta_E}{6}\frac{2\Delta-2\underline{\theta}\Delta+\Delta\theta^*-3\underline{\theta}\Delta_E}{\Delta+\Delta_E}$
on this case restiction on θ^*we have:

$$\theta^* \geq \frac{2\Delta\Delta_{\mathrm{E}}\left(2+\underline{\theta}\right)+\underline{\theta}\Delta\left[6\left(\Delta+\Delta_E\right)\right]}{2\Delta\left[6\left(\Delta+\Delta_E\right)+1\right]}$$

$$\theta^* \geq \frac{3\underline{\theta}\left(\Delta+\Delta_E\right)}{\Delta} - \underline{\theta} - 2$$

and profits is:

$$\Pi_L^d = \frac{\left[\Delta\Delta_{\mathrm{E}}\left(2+\underline{\theta}+\theta^*\right)-3\underline{\theta}\Delta_E\left(\Delta+\Delta_E\right)\right]^2}{36\left(\Delta+\Delta_{\mathrm{E}}\right)^2\Delta_{\mathrm{E}}} - \alpha q_L^2$$

A.a.2 vs A.a.1 for H $\Pi_H^{A.a.2}(P_L^*, P_H^*) \geq \Pi_H^{A.a.1}\left(P_L^*, P_H^d\right)$

$\frac{\Delta_E \Delta\left(\underline{\theta}+\theta^*+2\right)^2}{9(\Delta_E+\Delta)} \geq P_H^d\left(\bar{\theta}-\theta'\right)$

if $P_L^* = \frac{\Delta_E \Delta\left(1-(\underline{\theta}+\theta^*)\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E \Delta\left(1-(\underline{\theta}+\theta^*)\right)}{6\left(\Delta_E+\Delta\right)} + \frac{\bar{\theta}\Delta_E}{2}$$

or after some manipulation $P_H^d = \frac{\Delta_E}{6}\frac{2\underline{\theta}\Delta+4\Delta-\theta^*\Delta+3\underline{\theta}\Delta_E+3\Delta_E}{\Delta_E+\Delta}$
on this case restiction on θ^*we have:

$$\theta^* \geq \left[-\frac{\Delta\Delta_E\left(1-(\theta^*+\underline{\theta})\right)}{6\left(\Delta+\Delta_E\right)} + \frac{\bar{\theta}\Delta_E}{2}\right] * \frac{1}{\Delta}$$

and profit:

$$\Pi_H^d = \frac{\left[\Delta\Delta_E\left(1-(\theta^*+\underline{\theta})\right) + 3\bar{\theta}\Delta_E\left(\Delta+\Delta_E\right)\right]^2}{36\left(\Delta+\Delta_E\right)\Delta_E} - \alpha q_H^2$$

A.a.2 vs A.a.3

A.a.2 vs A.a.3 for L $\Pi_L^{A.a.2}(P_L^*, P_H^*) \geq \Pi_L^{A.a.3}\left(P_L^d, P_H^*\right)$

$\frac{\Delta_E \Delta\left(1-(\underline{\theta}+\theta^*)\right)^2}{9(\Delta_E+\Delta)} \geq P_L^d\left(\theta''-\underline{\theta}\right)$

if $P_H^* = \frac{\Delta\Delta_E\left(\underline{\theta}+\theta^*+2\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta\Delta_\mathrm{E}\left(2+\underline{\theta}+\theta^*\right)}{6\left(\Delta+\Delta_\mathrm{E}\right)} - \frac{\underline{\theta}\Delta}{2}$$

or after some manipulation $P_L^d = \frac{\Delta}{6}\frac{2\Delta_E-2\underline{\theta}\Delta_E+\Delta_E\theta^*-3\underline{\theta}\Delta}{\Delta+\Delta_E}$
on this case restiction on θ^*we have:

$$\theta^* \leq \frac{6\left(\Delta_E+\Delta\right)}{\Delta} - \underline{\theta} - 2$$

$$\theta^* \leq \left(\frac{3\underline{\theta}\Delta+2\underline{\theta}\Delta_E-2\Delta_E}{\Delta_E}\right)\left(\frac{\Delta}{\Delta+6\left(\Delta+\Delta_E\right)}\right)$$

profits:

$$\Pi_L^d = \frac{\left[\Delta\Delta_\mathrm{E}\left(2+\underline{\theta}+\theta^*\right) - 3\underline{\theta}\Delta\left(\Delta+\Delta_E\right)\right]^2}{36\left(\Delta+\Delta_\mathrm{E}\right)^2\Delta} - \alpha q_L^2$$

A.a.2 vs A.a.3 for H $\Pi_H^{A.a.2}(P_L^*, P_H^*) \geq \Pi_H^{A.a.3}\left(P_L^*, P_H^d\right)$

$\frac{\Delta_E \Delta\left(\underline{\theta}+\theta^*+2\right)^2}{9(\Delta_E+\Delta)} \geq P_H^d\left(\bar{\theta}-\theta''\right)$

if $P_L^* = \frac{\Delta_E \Delta\left(1-(\underline{\theta}+\theta^*)\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E \Delta\left(1-(\underline{\theta}+\theta^*)\right)}{6\left(\Delta_E+\Delta\right)} + \frac{\bar{\theta}\Delta}{2}$$

or after some manipulation $P_H^d = \frac{\Delta}{6}\frac{2\underline{\theta}\Delta_E+4\Delta_E-\theta^*\Delta_E+3\underline{\theta}\Delta+3\Delta}{\Delta_E+\Delta}$

on this case restiction on θ^*we have:

$$\theta^* \leq \left[-\frac{\Delta\Delta_E\left(1-(\theta^*+\underline{\theta})\right)}{6\left(\Delta+\Delta_E\right)} + \frac{\bar{\theta}\Delta}{2}\right] * \frac{1}{\Delta_E}$$

profits:

$$\Pi_H^d = \frac{\left[\Delta\Delta_{\mathrm{E}}\left(1-(\underline{\theta}+\theta^*)\right)+3\bar{\theta}\Delta\left(\Delta+\Delta_E\right)\right]^2}{36\left(\Delta+\Delta_{\mathrm{E}}\right)^2\Delta} - \alpha q_H^2$$

A.a.3

A.a.3 vs A.a.1

A.a.3 vs A.a.1 for L $\Pi_L^{A.a.3}(P_L^*, P_H^*) \geq \Pi_L^{A.a.1}\left(P_L^d, P_H^*\right)$

$\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)^2}{9} \geq P_L'\left(\theta'-\underline{\theta}\right)$

if $P_H^* = \frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{6} - \frac{\underline{\theta}\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \geq \frac{\left(2\bar{\theta}-\underline{\theta}\right)}{6} + \frac{\underline{\theta}\Delta_E}{2\Delta}$$

and profits is:

$$\Pi_L^d = \frac{\left[\Delta\left(2\bar{\theta}-\underline{\theta}\right)-3\underline{\theta}\Delta_E\right]^2}{36\Delta_E} - \alpha q_L^2$$

A.a.3 vs A.a.1 for H $\Pi_H^{A.a.3}(P_L^*, P_H^*) \geq \Pi_H^{A.a.1}\left(P_L^*, P_H^d\right)$

$\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)^2}{9} \geq P_H^d\left(\bar{\theta}-\theta'\right)$

if $P_L^* = \frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_H^d = \frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{6} + \frac{\bar{\theta}\Delta_E}{2}$$

or after some manipulation $P_H^d =$
on this case restiction on θ^*we have:

$$\theta^* \leq \frac{-\Delta\left(\bar{\theta}-2\underline{\theta}\right)+3\bar{\theta}\Delta_E}{6\Delta_E}$$

and profit:

$$\Pi_H^d = \frac{\left[\Delta\left(\bar{\theta}-2\underline{\theta}\right)+3\bar{\theta}\Delta_E\right]^2}{36\Delta_E} - \alpha q_H^2$$

A.a.3 vs A.a.2

A.a.3 vs A.a.2 for L $\Pi_L^{A.a.3}\left(P_L^*,P_H^*\right) \geq \Pi_L^{A.a.2}\left(P_L^d,P_H^*\right)$
$\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)^2}{9} \geq P_L'\left(\theta'-\underline{\theta}+\theta''-\theta^*\right)$
if $P_H^* = \frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_L^d = \left(\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \theta^*\Delta_E + \frac{\Delta_{\mathrm{E}}\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \underline{\theta}\Delta_E\right)\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}$$

on this case restiction on θ^*we have:

$$\left[\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \left(\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3} + \frac{\Delta_{\mathrm{E}}\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \underline{\theta}\Delta_E\right)\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}\right]\frac{1}{\Delta_E}\left(\frac{2\Delta_E+2\Delta}{2\Delta_E+\Delta}\right) \leq \theta^*$$

$$\theta^* \leq \left[\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \left(\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3} + \frac{\Delta_{\mathrm{E}}\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \underline{\theta}\Delta_E\right)\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}\right]\frac{1}{\Delta}\left(\frac{2\Delta_E+2\Delta}{2\Delta+\Delta_E}\right)$$

profits:

$$\Pi_L^d = \frac{\left(2\bar{\theta}-\underline{\theta}\right)^2\Delta^2\left(\Delta+\Delta_E\right)^2 + 6\left(2\bar{\theta}-\underline{\theta}\right)\Delta_E\left(\Delta+\Delta_E\right)^2 + 9\Delta_E\left(\theta^*+\underline{\theta}\right)\left(\Delta^2\left(-\left(\Delta_E\left(\theta^*+\underline{\theta}\right)+4\right)\right)-6\Delta\right.}{36\Delta_E\left(\Delta+\Delta_E\right)^2}$$

A.a.3 vs A.a.2 for H $\Pi_H^{A.a.3}\left(P_L^*,P_H^*\right) \geq \Pi_H^{A.a.2}\left(P_L^*,P_H^d\right)$
$\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)^2}{9} \geq P_H''\left(\bar{\theta}-\theta''+\theta^*-\theta'\right)$
if $P_L^* = \frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_H^d = \left(\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3} + \theta^*\Delta_E + \frac{\Delta_{\mathrm{E}}\left(\bar{\theta}-2\underline{\theta}\right)}{3} + \bar{\theta}\Delta_E\right)\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}$$

or after some manipulation $P_H^d = \frac{\Delta}{6}\frac{3\bar{\theta}\Delta_E+\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)+3\theta^*\Delta_{\mathrm{E}}+\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{\left(\Delta_E+\Delta\right)}$

on this case restiction on θ^*we have:

$$\theta^* \leq \left[\left(\frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \frac{\Delta_{\mathrm{E}}\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \bar{\theta}\Delta_E \right) \frac{1}{2} \frac{\Delta}{\Delta + \Delta_E} - \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \right] \left(\frac{2\Delta + 2\Delta_E}{2\Delta^2 + \Delta\Delta_E} \right)$$

$$\left[\left(\frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \frac{\Delta_{\mathrm{E}}\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \bar{\theta}\Delta_E \right) \frac{1}{2} \frac{\Delta}{\Delta + \Delta_E} - \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \right] \left(\frac{2\Delta + 2\Delta_E}{2\Delta_E^2 + \Delta\Delta_E} \right) \leq \theta^*$$

profits:

$$\Pi_H^d = \frac{\Delta\left[\left(\bar{\theta} - 2\underline{\theta}\right)\left(\Delta + \Delta_{\mathrm{E}}\right) + 3\Delta_E\left(\theta^* + \bar{\theta}\right)\right]^2}{36\left(\Delta + \Delta_E\right)} - \alpha q_H^2$$

A.b.1

A.b.1 vs A.b.2

A.b.1 vs A.b.2 for L $\Pi_L^{A.b.1}\left(P_L^*, P_H^*\right) \geq \Pi_L^{A.b.2}\left(P_L', P_H^*\right)$
$\frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)^2}{9} \geq P_L'\left(\theta' - \underline{\theta} + \theta'' - \theta^*\right)$
if $P_H^* = \frac{\Delta_E\left(2\bar{\theta}-\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta_E}{6} * \frac{-2\underline{\theta} + 2\Delta + \underline{\theta}\Delta_E + 2\Delta_E - 3\theta^*\Delta}{\Delta + \Delta_E}$$

or we can rewrite

$$P_L^d = \left(\frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3} - \theta^*\Delta + \frac{\Delta_{\mathrm{E}}\left(2\bar{\theta} - \underline{\theta}\right)}{3} - \underline{\theta}\Delta \right) \frac{1}{2} \frac{\Delta_E}{\Delta + \Delta_E}$$

comment is like an average weight price
on this case restiction on θ^*we have:
$\gamma = \frac{1}{2}\frac{\Delta_E}{\Delta+\Delta_E}$

$$\theta^* \leq \frac{-\Delta_E\underline{\theta} - 2\Delta_E + 3\underline{\theta}\Delta + 3\Delta + 4\Delta\gamma - \Delta\underline{\theta}\gamma}{3\Delta\gamma}$$

$$\theta^* \leq \frac{2\Delta\gamma\left(\underline{\theta} - 1\right) + \Delta_E\underline{\theta}\left(1 - \gamma\right) + 2\Delta_E\left(1 - \gamma\right)}{3\Delta\left(1 - \gamma\right)}$$

and profit:

$$\Pi_L^d = \frac{\Delta_E\left(-2\underline{\theta}\Delta + \underline{\theta}\Delta_E - 3\Delta\theta^* + 2\Delta + 2\Delta_E\right)\left(-4\underline{\theta}\Delta + \underline{\theta}\Delta_E + 2\underline{\theta} - 3\theta^*\Delta + 2\Delta + 2\Delta_E\right)}{36\left(\Delta + \Delta_E\right)} - \alpha q_L^2$$

A.b.1 vs A.b.2 for H $\Pi_H^{A.b.1}(P_L^*, P_H^*) \geq \Pi_H^{A.b.2}(P_L^*, P_H'')$

$\frac{\Delta_E\left(2\bar{\theta}-\underline{\theta}\right)^2}{9} \geq P_H''\left(\bar{\theta}-\theta''+\theta^*-\theta'\right)$

if $P_L^* = \frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)}{3}$from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E}{6}\frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)+\Delta\left(\bar{\theta}-2\underline{\theta}\right)+3\Delta\bar{\theta}+3\Delta\theta^*}{\Delta_E+\Delta}$$

or rearranged

$$P_H^d = \left(\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3}+\theta^*\Delta+\frac{\Delta_{\mathrm{E}}\left(\bar{\theta}-2\underline{\theta}\right)}{3}+\bar{\theta}\Delta\right)\frac{1}{2}\frac{\Delta_E}{\Delta+\Delta_E}$$

on this case restiction on θ^*we have:

$\gamma = \frac{1}{2}\frac{\Delta_E}{\Delta+\Delta_E}$

$$\theta^* \leq \frac{\Delta_E\underline{\theta}\left(1-\gamma\right)-\Delta_E\left(1-\gamma\right)+4\Delta\gamma+2\underline{\theta}\Delta\gamma}{3\Delta\left(1-\gamma\right)}$$

$$\theta^* \leq \frac{\Delta_E\underline{\theta}\left(1-\gamma\right)-\Delta_E\left(1-\gamma\right)-4\Delta\gamma-2\underline{\theta}\Delta\gamma+3\underline{\theta}\Delta+3\Delta}{3\Delta\gamma}$$

profit:

$$\Pi_H^d = \frac{\Delta_E\left(-2\underline{\theta}\Delta+\underline{\theta}\Delta_E-3\Delta\theta^*+4\Delta+2\Delta\right)\left(4\underline{\theta}\Delta^2+7\underline{\theta}\Delta\Delta_E-4\underline{\theta}\Delta-\underline{\theta}\Delta_E+3\Delta^2\theta^*+2\Delta^2+6\theta^*\Delta\Delta_E+\right.}{36\left(\Delta+\Delta_E\right)^2}$$

A.b.1 vs A.b.3

A.b.1 vs A.b.3 for L $\Pi_L^{A.b.1}(P_L^*, P_H^*) \geq \Pi_L^{A.b.3}(P_L', P_H^*)$

$\frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)^2}{9} \geq P_L''\left(1-\theta^*+\theta'\right)$

if $P_H^* = \frac{\Delta_E\left(2\bar{\theta}-\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta_E\left(2\bar{\theta}-\underline{\theta}\right)}{6}+\frac{\Delta_E}{2}-\frac{\theta^*\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \geq max\left\{\frac{\Delta_E\left(1-\underline{\theta}\right)}{3\Delta_E}, \frac{6\Delta\left(\underline{\theta}+1\right)-\underline{\theta}\Delta_E}{3\Delta_E}\right\}$$

and profit:

$$\Pi_L^d = \frac{\Delta_E\left(\underline{\theta}-3\theta^*+5\right)^2}{36}-\alpha q_L^2$$

A.b.1 vs A.b.3 for H $\Pi_H^{A.b.1}(P_L^*, P_H^*) \geq \Pi_H^{A.b.3}(P_L^*, P_H'')$

$\frac{\Delta_E(2\bar{\theta}-\underline{\theta})^2}{9} \geq P_H''(\theta^* - \theta')$

if $P_L^* = \frac{\Delta_E(\bar{\theta}-2\underline{\theta})}{3}$from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right)}{6} + \frac{\theta^*\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\frac{1-\underline{\theta}}{9} \leq \theta^* \leq \frac{1-\underline{\theta}}{3} - \frac{6\Delta(\underline{\theta}+1)}{3\Delta_E}$$

profits is:

$$\Pi_H^d = \frac{\Delta_E(3\theta^* + 1 - \underline{\theta})^2}{36} - \alpha q_H^2$$

A.b.2

A.b.2 vs A.b.1

A.b.2 vs A.b.1 for L $\Pi_L^{A.b.2}(P_L^*, P_H^*) \geq \Pi_L^{A.b.1}\left(P_L^d, P_H^*\right)$

$\frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)^2}{9(\Delta_E+\Delta)} \geq P_L^d(\theta' - \underline{\theta})$

if $P_H^* = \frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta\Delta_{\mathrm{E}}(2 + \underline{\theta} + \theta^*)}{6(\Delta + \Delta_{\mathrm{E}})} - \frac{\underline{\theta}\Delta_E}{2}$$

or after some manipulation $P_L^d = \frac{\Delta_E}{6}\frac{2\Delta - 2\underline{\theta}\Delta + \Delta\theta^* - 3\underline{\theta}\Delta_E}{\Delta+\Delta_E}$

on this case restiction on θ^*we have:

$$\theta^* \geq max\left\{\frac{2\Delta\Delta_E + 4\Delta\Delta_E\underline{\theta} + 3\underline{\theta}\Delta_E^2}{5\Delta\Delta_E + 6\Delta^2}, \frac{2\underline{\theta}\Delta + 3\underline{\theta}\Delta_E - 2\Delta}{\Delta}\right\}$$

and profits is:

$$\Pi_L^d = \frac{\left[\Delta\Delta_{\mathrm{E}}(2 + \underline{\theta} + \theta^*) - 3\underline{\theta}\Delta_E(\Delta + \Delta_E)\right]^2}{36(\Delta + \Delta_{\mathrm{E}})^2\Delta_{\mathrm{E}}} - \alpha q_L^2$$

A.b.2 vs A.b.1 for H $\Pi_H^{A.b.2}(P_L^*, P_H^*) \geq \Pi_H^{A.b.1}\left(P_L^*, P_H^d\right)$

$\frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)^2}{9(\Delta_E+\Delta)} \geq P_H^d\left(\bar{\theta} - \theta'\right)$

if $P_L^* = \frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E\Delta(1 - (\underline{\theta} + \theta^*))}{6(\Delta_E + \Delta)} + \frac{\bar{\theta}\Delta_E}{2}$$

or after some manipulation $P_H^d = \frac{\Delta_E}{6}\frac{2\underline{\theta}\Delta+4\Delta-\theta^*\Delta+3\underline{\theta}\Delta_E+3\Delta_E}{\Delta_E+\Delta}$
on this case restiction on θ^*we have:

$$\theta^* \geq max\left\{\frac{3\Delta_E\left(\underline{\theta}-1\right)+2\Delta\left(\underline{\theta}-1\right)}{\Delta}, \frac{3\underline{\theta}\Delta_E^2+4\underline{\theta}\Delta\Delta_E+3\Delta_E^2+2\Delta\Delta_E}{5\Delta\Delta_E-6\Delta^2}\right\}$$

and profit:

$$\Pi_H^d = \frac{\left[\Delta\Delta_E\left(1-\left(\theta^*+\underline{\theta}\right)\right)+3\bar{\theta}\Delta_E\left(\Delta+\Delta_E\right)\right]^2}{36\left(\Delta+\Delta_E\right)\Delta_E} - \alpha q_H^2$$

A.b.2 vs A.b.3

A.b.2 vs A.b.3 for L $\Pi_L^{A.b.2}\left(P_L^*, P_H^*\right) \geq \Pi_L^{A.b.3}\left(P_L^d, P_H^*\right)$
$\frac{\Delta_E\Delta\left(1-\left(\underline{\theta}+\theta^*\right)\right)^2}{9\left(\Delta_E+\Delta\right)} \geq P_L^d\left(1-\theta^*+\theta'\right)$
if $P_H^* = \frac{\Delta\Delta_E\left(\underline{\theta}+\theta^*+2\right)}{3\left(\Delta_E+\Delta\right)}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta\Delta_{\mathrm{E}}\left(2+\underline{\theta}+\theta^*\right)}{6\left(\Delta+\Delta_{\mathrm{E}}\right)} - \frac{\theta^*\Delta_E}{2} + \frac{\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \geq max\left\{\frac{\Delta\left(\underline{\theta}-1\right)-3\Delta_E}{2\Delta+3\Delta_E}, \frac{7\Delta\Delta_E+5\underline{\theta}\Delta\Delta_E+3\Delta_E^2+6\Delta^2\left(\underline{\theta}+1\right)}{3\Delta_E^2+4\Delta\Delta_E}\right\}$$

profits:

$$\Pi_L^d = \left[\frac{\Delta\Delta_{\mathrm{E}}\left(2+\underline{\theta}+\theta^*\right)}{6\left(\Delta+\Delta_{\mathrm{E}}\right)} - \frac{\theta^*\Delta_E}{2} + \frac{\Delta_E}{2}\right]^2 - \alpha q_L^2$$

A.b.2 vs A.b.3 for H $\Pi_H^{A.b.2}\left(P_L^*, P_H^*\right) \geq \Pi_H^{A.b.3}\left(P_L^*, P_H^d\right)$
$\frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)^2}{9\left(\Delta_E+\Delta\right)} \geq P_H^d\left(\theta^*-\theta'\right)$
if $P_L^* = \frac{\Delta_E\Delta\left(1-\left(\underline{\theta}+\theta^*\right)\right)}{3\left(\Delta_E+\Delta\right)}$ from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E\Delta\left(1-\left(\underline{\theta}+\theta^*\right)\right)}{6\left(\Delta_E+\Delta\right)} + \frac{\theta^*\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \geq max\left\{\frac{7\Delta\Delta_E+\underline{5\theta}\Delta\Delta_E+6\Delta^2\left(\underline{\theta}+1\right)}{4\Delta\Delta_E+3\Delta_E^2}, \frac{\Delta\left(\underline{\theta}-1\right)}{2\Delta+3\Delta_E}\right\}$$

profits:

$$\Pi_H^d = \left[\frac{\Delta_E\Delta\left(1-\left(\underline{\theta}+\theta^*\right)\right)}{6\left(\Delta_E+\Delta\right)} + \frac{\theta^*\Delta_E}{2}\right]^2 - \alpha q_H^2$$

A.b.3

A.b.3 vs A.b.1

A.b.3 vs A.b.1 for L $\Pi_L^{A.b.3}(P_L^*, P_H^*) \geq \Pi_L^{A.b.1}\left(P_L^d, P_H^*\right)$
$\frac{\Delta_E(2-\theta^*)^2}{9} \geq P_L'(\theta' - \underline{\theta})$
if $P_H^* = \frac{\Delta_E(1+\theta^*)}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta_E(1+\theta^*)}{6} - \frac{\underline{\theta}\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \geq max\left\{\frac{\Delta_E + 3\Delta_E\underline{\theta}}{6\Delta - \Delta_E}, 1 - 3\underline{\theta}\right\}$$

and profits is:

$$\Pi_L^d = \frac{\Delta_E(3\underline{\theta} - \theta^* - 1)^2}{36} - \alpha q_L^2$$

A.b.3 vs A.b.1 for H $\Pi_H^{A.b.3}(P_L^*, P_H^*) \geq \Pi_H^{A.b.1}\left(P_L^*, P_H^d\right)$
$\frac{\Delta_E(1+\theta^*)^2}{9} \geq P_H^d\left(\bar{\theta} - \theta'\right)$
if $P_L^* = \frac{\Delta_E(2-\theta^*)}{3}$ from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E(2-\theta^*)}{6} + \frac{\bar{\theta}\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \geq max\left\{3\underline{\theta} - 1, \frac{2\underline{\theta}\Delta_E + \Delta_E}{6\Delta - \Delta_E}\right\}$$

and profit:

$$\Pi_H^d = \frac{\Delta_E\left(2 - \theta^* + 3\bar{\theta}\right)^2}{36} - \alpha q_H^2$$

A.b.3 vs A.b.2

A.b.3 vs A.b.2 for L $\Pi_L^{A.b.3}(P_L^*, P_H^*) \geq \Pi_L^{A.b.2}\left(P_L^d, P_H^*\right)$
$\frac{\Delta_E(2-\theta^*)^2}{9} \geq P_L'(\theta' - \underline{\theta} + \theta'' - \theta^*)$
if $P_H^* = \frac{\Delta(1+\theta^*)}{3}$ from optitimization proces we optain:

$$P_L^d = \left(\frac{\Delta(1+\theta^*)}{3} - \theta^*\Delta + \frac{\Delta_\text{E}(1+\theta^*)}{3} - \underline{\theta}\Delta\right)\frac{1}{2}\frac{\Delta_E}{\Delta + \Delta_E}$$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{3\underline{\theta}\left(1-\gamma\right)+2\Delta+\gamma\left(\Delta+\Delta_E\right)}{\Delta+2\Delta\gamma-\Delta_E\gamma},\frac{\Delta\left(1-\gamma\right)-\Delta_E\gamma+3\underline{\theta}\gamma\Delta}{2\Delta\left(1-\gamma\right)+\Delta_E\gamma}\right\}$$

$\gamma = \frac{1}{2}\frac{\Delta_E}{\Delta+\Delta_E}$
profits:

$$\Pi_L^d = \frac{\Delta\left(-3\underline{\theta}\Delta-2\Delta\theta^*+\Delta+\Delta\Delta_E+\Delta_E\right)\left(-6\underline{\theta}\Delta\Delta_E+3\underline{\theta}\Delta-6\Delta\Delta_E\theta^*+\Delta\Delta_E+4\Delta\Delta_E+\Delta+\Delta_E\right)}{36\Delta_E\left(\Delta+\Delta_E\right)}-\alpha q$$

A.b.3 vs A.b.2 for H $\Pi_H^{A.b.3}\left(P_L^*,P_H^*\right) \geq \Pi_H^{A.b.2}\left(P_L^*,P_H^d\right)$
$\frac{\Delta_E(1+\theta^*)^2}{9} \geq P_H''\left(\bar{\theta}-\theta''+\theta^*-\theta'\right)$
if $P_L^* = \frac{\Delta_E(2-\theta^*)}{3}$ from optitimization proces we optain:

$$P_H^d = \left(\frac{\Delta\left(2-\theta^*\right)}{3}+\theta^*\Delta+\frac{\Delta_\mathrm{E}\left(2-\theta^*\right)}{3}+\bar{\theta}\Delta\right)\frac{1}{2}\frac{\Delta_E}{\Delta+\Delta_E}$$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{2\Delta_E\left(\gamma-1\right)+3\underline{\theta}\Delta\gamma+5\Delta\gamma}{3\Delta+\Delta_E\left(\gamma-1\right)-2\Delta\gamma},\frac{2\Delta_E\left(1-\gamma\right)+3\underline{\theta}\Delta\left(1-\gamma\right)+3\Delta\left(1-\gamma\right)-2\Delta\gamma}{2\Delta\gamma+\Delta_E\left(1-\gamma\right)}\right\}$$

profits:

$$\Pi_H^d = -\frac{\Delta\left(3\underline{\theta}\Delta+4\Delta\theta^*-6\underline{\theta}\Delta_E+\Delta-5\Delta_E\Delta-8\Delta_E\right)\left(3\underline{\theta}\Delta+2\Delta\theta^*+5\Delta-\Delta_E\theta^*+2\Delta_E\right)}{36\left(\Delta+\Delta_E\right)\Delta_E}-\alpha q_H^2$$

A.c.1

A.c.1 vs A.c.2

A.c.1 vs A.c.2 for L $\Pi_L^{A.c.1}\left(P_L^*,P_H^*\right) \geq \Pi_L^{A.c.2}\left(P_L',P_H^*\right)$
$\frac{\Delta(1-\theta^*)^2}{9} \geq P_L'\left(\theta'-\underline{\theta}+\theta''-\theta^*\right)$
if $P_H^* = \frac{\Delta(2+\theta^*)}{3}$ from optitimization proces we optain:

$$P_L^d = \left[\frac{\Delta\left(2+\theta^*\right)}{3}-\underline{\theta}\Delta_E+\frac{\Delta_E\left(2+\theta^*\right)}{3}-\theta^*\Delta_E\right]\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}$$

comment is like an average weight price
on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{-2\Delta\left(1-\gamma\right)-3\underline{\theta}\Delta_E\gamma-2\Delta_E\gamma}{\Delta\left(1-\gamma\right)-3\Delta_E\left(1-\gamma\right)-\Delta_E\gamma},\frac{2\Delta\left(1-\gamma\right)-3\underline{\theta}\Delta_E\left(1-\gamma\right)-2\Delta_E\gamma}{\Delta\left(1-\gamma\right)-2\Delta_E\gamma}\right\}$$

and profit:

$$\Pi_L^d = \frac{\Delta\left(\left(2+\theta^*\right)\left(\Delta+\Delta_E\right)-3\Delta_E\left(\underline{\theta}+\theta^*\right)\right)^2}{36\Delta_E\left(\Delta+\Delta_E\right)} - \alpha q_L^2$$

A.c.1 vs A.c.2 for H $\Pi_H^{A.c.1}\left(P_L^*, P_H^*\right) \geq \Pi_H^{A.c.2}\left(P_L^*, P_H''\right)$

$\frac{\Delta(2+\theta^*)^2}{9} \geq P_H''\left(\bar{\theta}-\theta''+\theta^*-\theta'\right)$

if $P_L^* = \frac{\Delta(1-\theta^*)}{3}$from optitimization proces we optain:

$$P_H^d = \left[\frac{\Delta\left(1-\theta^*\right)}{3} + \bar{\theta}\Delta_E + \frac{\Delta_E\left(1-\theta^*\right)}{3} + \theta^*\Delta_E\right]\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}$$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{4\Delta_E\gamma - 3\underline{\theta}\Delta_E\left(1-\gamma\right)-\Delta\left(1-\gamma\right)}{-2\Delta_E\gamma-\Delta\left(1-\gamma\right)}, \frac{\Delta\left(1-\gamma\right)-3\underline{\theta}\Delta_E\gamma-2\Delta_E\gamma}{2\Delta_E\gamma-3\Delta_E-\Delta}\right\}$$

profit:

$$\Pi_H^d = \frac{\Delta\left(\left(1-\theta^*\right)\left(\Delta+\Delta_E\right)-3\Delta_E\left(\underline{\theta}+\theta^*\right)\right)^2}{36\Delta_E\left(\Delta+\Delta_E\right)} - \alpha q_H^2$$

A.c.1 vs A.c.3

A.c.1 vs A.c.3 for L $\Pi_L^{A.c.1}\left(P_L^*, P_H^*\right) \geq \Pi_L^{A.c.3}\left(P_L', P_H^*\right)$

$\frac{\Delta(1-\theta^*)^2}{9} \geq P_L''\left(\theta''-\underline{\theta}\right)$

if $P_H^* = \frac{\Delta(2+\theta^*)}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta\left(2+\theta^*\right)}{6} - \frac{\underline{\theta}\Delta}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{4+3\underline{\theta}, \frac{2\Delta+3\underline{\theta}\Delta}{6\Delta_E-\Delta}\right\}$$

and profit:

$$\Pi_L^d = \frac{\Delta\left(2+\theta^*-3\underline{\theta}\right)^2}{36} - \alpha q_L^2$$

A.c.1 vs A.c.3 for H $\Pi_H^{A.c.1}(P_L^*, P_H^*) \geq \Pi_H^{A.c.3}(P_L^*, P_H'')$
$\frac{\Delta(2+\theta^*)^2}{9} \geq P_H''\left(\bar{\theta} - \theta''\right)$
if $P_L^* = \frac{\Delta(1-\theta^*)}{3}$from optitimization proces we optain:

$$P_H^d = \frac{\Delta(1-\theta^*)}{6} + \frac{\bar{\theta}\Delta}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{2\Delta + 3\underline{\theta}\Delta}{6\Delta - \Delta_E}, 6 + 3\underline{\theta}\right\}$$

profits is:

$$\Pi_H^d = \frac{\Delta\left(1 - \theta^* + 3\bar{\theta}\right)^2}{36} - \alpha q_H^2$$

A.c.2

A.c.2 vs A.c.1

A.c.2 vs A.c.1 for L $\Pi_L^{A.c.2}(P_L^*, P_H^*) \geq \Pi_L^{A.c.1}\left(P_L^d, P_H^*\right)$
$\frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)^2}{9(\Delta_E+\Delta)} \geq P_L^d(\theta'' - \theta^*)$
if $P_H^* = \frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta\Delta_{\mathrm{E}}(2 + \underline{\theta} + \theta^*)}{6(\Delta + \Delta_{\mathrm{E}})} - \frac{\theta^*\Delta}{2}$$

or after some manipulation $P_L^d = \frac{\Delta_E}{6}\frac{2\Delta - 2\underline{\theta}\Delta + \Delta\theta^* - 3\underline{\theta}\Delta_E}{\Delta + \Delta_E}$
on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{5\underline{\theta}\Delta\Delta_E - 2\Delta\Delta_E + 6\underline{\theta}\Delta_E^2}{4\Delta\Delta_E + 3\Delta^2}, \frac{2\Delta_E(\underline{\theta}+1)}{6\Delta_E + 3\Delta}\right\}$$

and profits is:

$$\Pi_L^d = \frac{\Delta(\underline{\theta}\Delta_E - 3\theta^*\Delta - 2\theta\Delta_E + 2\Delta_E)^2}{36(\Delta + \Delta_E)} - \alpha q_L^2$$

A.c.2 vs A.c.1 for H $\Pi_H^{A.c.2}(P_L^*, P_H^*) \geq \Pi_H^{A.c.1}\left(P_L^*, P_H^d\right)$
$\frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)^2}{9(\Delta_E+\Delta)} \geq P_H^d(1 - \theta'' + \theta^*)$
if $P_L^* = \frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E\Delta(1 - (\underline{\theta} + \theta^*))}{6(\Delta_E + \Delta)} + \frac{\theta^*\Delta}{2} + \frac{\Delta}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{2\Delta_E+\underline{\theta}\Delta_E+3\Delta}{2\Delta_E+3\Delta}, \frac{5\underline{\theta}\Delta\Delta_E+6\underline{\theta}\Delta_E^2-2\Delta\Delta_E-3\Delta^2}{4\Delta\Delta_E+3\Delta^2}\right\}$$

and profit:

$$\Pi_H^d=\left[\frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)}{6\left(\Delta_E+\Delta\right)}+\frac{\theta^*\Delta}{2}+\frac{\Delta}{2}\right]^2-\alpha q_H^2$$

A.c.2 vs A.c.3

A.c.2 vs A.c.3 for L $\Pi_L^{A.c.2}\left(P_L^*,P_H^*\right)\geq\Pi_L^{A.c.3}\left(P_L^d,P_H^*\right)$

$\frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)^2}{9(\Delta_E+\Delta)}\geq P_L^d\left(\theta''-\underline{\theta}\right)$

if $P_H^*=\frac{\Delta\Delta_E\left(\underline{\theta}+\theta^*+2\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_L^d=\frac{\Delta\Delta_{\mathrm{E}}\left(2+\underline{\theta}+\theta^*\right)}{6\left(\Delta+\Delta_{\mathrm{E}}\right)}-\frac{\underline{\theta}\Delta}{2}$$

or after some manipulation $P_L^d=\frac{\Delta}{6}\frac{2\Delta_E-2\underline{\theta}\Delta_E+\Delta_E\theta^*-3\underline{\theta}\Delta}{\Delta+\Delta_E}$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\left(\frac{3\underline{\theta}\Delta+2\underline{\theta}\Delta_E-2\Delta_E}{\Delta_E}\right)\left(\frac{\Delta}{\Delta+6\left(\Delta+\Delta_E\right)}\right), \frac{6\left(\Delta_E+\Delta\right)}{\Delta}-\underline{\theta}-2\right\}$$

profits:

$$\Pi_L^d=\frac{\left[\Delta\Delta_{\mathrm{E}}\left(2+\underline{\theta}+\theta^*\right)-3\underline{\theta}\Delta\left(\Delta+\Delta_E\right)\right]^2}{36\left(\Delta+\Delta_{\mathrm{E}}\right)^2\Delta}-\alpha q_L^2$$

A.c.2 vs A.c.3 for H $\Pi_H^{A.c.2}\left(P_L^*,P_H^*\right)\geq\Pi_H^{A.c.3}\left(P_L^*,P_H^d\right)$

$\frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)^2}{9(\Delta_E+\Delta)}\geq P_H^d\left(\bar{\theta}-\theta''\right)$

if $P_L^*=\frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_H^d=\frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)}{6\left(\Delta_E+\Delta\right)}+\frac{\bar{\theta}\Delta}{2}$$

or after some manipulation $P_H^d=\frac{\Delta}{6}\frac{2\underline{\theta}\Delta_E+4\Delta_E-\theta^*\Delta_E+3\underline{\theta}\Delta+3\Delta}{\Delta_E+\Delta}$

on this case restiction on θ^*we have:

$$\theta^* \leq \left[-\frac{\Delta\Delta_E\left(1-(\theta^*+\underline{\theta})\right)}{6\left(\Delta+\Delta_E\right)}+\frac{\bar{\theta}\Delta}{2}\right]*\frac{1}{\Delta_E}$$

profits:

$$\Pi_L^d=\frac{\left[\Delta\Delta_{\mathrm{E}}\left(2+\underline{\theta}+\theta^*\right)+3\underline{\theta}\Delta\left(\Delta+\Delta_E\right)\right]^2}{36\left(\Delta+\Delta_{\mathrm{E}}\right)^2\Delta}-\alpha q_H^2$$

A.c.3

A.c.3 vs A.c.1

A.c.3 vs A.c.1 for L $\Pi_L^{A.c.3}\left(P_L^*,P_H^*\right)\geq\Pi_L^{A.c.1}\left(P_L^d,P_H^*\right)$

$\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)^2}{9}\geq P_L'\left(\theta''-\theta^*\right)$

if $P_H^*=\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_L^d=\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{6}-\frac{\theta^*\Delta}{2}$$

on this case restiction on θ^*we have:

$$\theta^*\leq min\left\{\frac{6\underline{\theta}\Delta_E-\underline{\theta}\Delta-2\Delta}{3\Delta},\frac{2+\underline{\theta}}{3}\right\}$$

and profits is:

$$\Pi_L^d=\frac{\Delta\left(\underline{\theta}+2-3\theta^*\right)^2}{36}-\alpha q_L^2$$

A.c.3 vs A.c.1 for H $\Pi_H^{A.c.3}\left(P_L^*,P_H^*\right)\geq\Pi_H^{A.c.1}\left(P_L^*,P_H^d\right)$

$\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)^2}{9}\geq P_H^d\left(1-\theta''+\theta^*\right)$

if $P_L^*=\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_H^d=\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{6}+\frac{\theta^*\Delta}{2}+\frac{\Delta}{2}$$

or after some manipulation $P_H^d=$

on this case restiction on θ^*we have:

$$\theta^*\leq min\left\{\frac{2+\underline{\theta}}{3},\frac{6\underline{\theta}\Delta_E-2\Delta-\underline{\theta}\Delta}{3\Delta}\right\}$$

and profit:

$$\Pi_H^d=\frac{\Delta\left(4-\underline{\theta}+3\theta^*\right)^2}{36}-\alpha q_H^2$$

A.c.3 vs A.c.2

A.c.3 vs A.c.2 for L $\Pi_L^{A.c.3}\left(P_L^*, P_H^*\right) \geq \Pi_L^{A.c.2}\left(P_L^d, P_H^*\right)$

$\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)^2}{9} \geq P_L'\left(\theta' - \underline{\theta} + \theta'' - \theta^*\right)$

if $P_H^* = \frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_L^d = \left(\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \theta^*\Delta_E + \frac{\Delta_{\mathrm{E}}\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \underline{\theta}\Delta_E\right)\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}$$

on this case restiction on θ^*we have:

$$\left[\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \left(\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3} + \frac{\Delta_{\mathrm{E}}\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \underline{\theta}\Delta_E\right)\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}\right]\frac{1}{\Delta_E}\left(\frac{2\Delta_E+2\Delta}{2\Delta_E+\Delta}\right) \leq \theta^*$$

$$\theta^* \leq \left[\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \left(\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3} + \frac{\Delta_{\mathrm{E}}\left(2\bar{\theta}-\underline{\theta}\right)}{3} - \underline{\theta}\Delta_E\right)\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}\right]\frac{1}{\Delta}\left(\frac{2\Delta_E+2\Delta}{2\Delta+\Delta_E}\right)$$

profits:

$$\Pi_L^d = \frac{\left(2\bar{\theta}-\underline{\theta}\right)^2\Delta^2\left(\Delta+\Delta_E\right)^2 + 6\left(2\bar{\theta}-\underline{\theta}\right)\Delta_E\left(\Delta+\Delta_E\right)^2 + 9\Delta_E\left(\theta^*+\underline{\theta}\right)\left(\Delta^2\left(-\left(\Delta_E\left(\theta^*+\underline{\theta}\right)+4\right)\right) - 6\Delta\right.}{36\Delta_E\left(\Delta+\Delta_E\right)^2}$$

A.c.3 vs A.c.2 for H $\Pi_H^{A.c.3}\left(P_L^*, P_H^*\right) \geq \Pi_H^{A.c.2}\left(P_L^*, P_H^d\right)$

$\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)^2}{9} \geq P_H''\left(\bar{\theta} - \theta'' + \theta^* - \theta'\right)$

if $P_L^* = \frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_H^d = \left(\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3} + \theta^*\Delta_E + \frac{\Delta_{\mathrm{E}}\left(\bar{\theta}-2\underline{\theta}\right)}{3} + \bar{\theta}\Delta_E\right)\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}$$

or after some manipulation $P_H^d = \frac{\Delta}{6}\frac{3\bar{\theta}\Delta_E+\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)+3\theta^*\Delta_{\mathrm{E}}+\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{\left(\Delta_E+\Delta\right)}$

on this case restiction on θ^*we have:

$$\theta^* \leq \left[\left(\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3} + \frac{\Delta_{\mathrm{E}}\left(\bar{\theta}-2\underline{\theta}\right)}{3} + \bar{\theta}\Delta_E\right)\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E} - \frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3}\right]\left(\frac{2\Delta+2\Delta_E}{2\Delta^2+\Delta\Delta_E}\right)$$

$$\left[\left(\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3} + \frac{\Delta_{\mathrm{E}}\left(\bar{\theta}-2\underline{\theta}\right)}{3} + \bar{\theta}\Delta_E\right)\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E} - \frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3}\right]\left(\frac{2\Delta+2\Delta_E}{2\Delta_E^2+\Delta\Delta_E}\right) \leq \theta^*$$

profits:

$$\Pi_H^d = \frac{\Delta\left[\left(\bar{\theta}-2\underline{\theta}\right)\left(\Delta+\Delta_{\mathrm{E}}\right) + 3\Delta_E\left(\theta^*+\bar{\theta}\right)\right]^2}{36\left(\Delta+\Delta_E\right)} - \alpha q_H^2$$

A.d.1

A.d.1 vs A.d.2

A.d.1 vs A.d.2 for L $\Pi_L^{A.d.1}\left(P_L^*, P_H^*\right) \geq \Pi_L^{A.d.2}\left(P_L', P_H^*\right)$

$\frac{\Delta(1-\theta^*)^2}{9} \geq P_L'\left(\theta' - \underline{\theta} + \theta'' - \theta^*\right)$

if $P_H^* = \frac{\Delta(2+\theta^*)}{3}$ from optitimization proces we optain:

$$P_L^d = \left[\frac{\Delta\left(2+\theta^*\right)}{3} - \underline{\theta}\Delta_E + \frac{\Delta_E\left(2+\theta^*\right)}{3} - \theta^*\Delta_E\right]\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}$$

comment is like an average weight price

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{2\Delta\left(\alpha-1\right)+3\underline{\theta}\Delta_E\left(1-\alpha\right)+2\Delta_E\alpha}{\Delta\left(1-\alpha\right)+2\Delta_E\alpha}, \frac{2\Delta\left(1-\alpha\right)+3\underline{\theta}\Delta_E\alpha-2\Delta_E\alpha}{\Delta\alpha-2\Delta_E\alpha+2\Delta}\right\}$$

$$\alpha = \frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}$$

and profit:

$$\Pi_L^d = \frac{\Delta\left(\left(2+\theta^*\right)\left(\Delta+\Delta_E\right)-3\Delta_E\left(\underline{\theta}+\theta^*\right)\right)^2}{36\Delta_E\left(\Delta+\Delta_E\right)} - \alpha q_L^2$$

A.d.1 vs A.d.2 for H $\Pi_H^{A.d.1}\left(P_L^*, P_H^*\right) \geq \Pi_H^{A.d.2}\left(P_L^*, P_H''\right)$

$\frac{\Delta(2+\theta^*)^2}{9} \geq P_H''\left(\bar{\theta} - \theta'' + \theta^* - \theta'\right)$

if $P_L^* = \frac{\Delta(1-\theta^*)}{3}$from optitimization proces we optain:

$$P_H^d = \left[\frac{\Delta\left(1-\theta^*\right)}{3} + \bar{\theta}\Delta_E + \frac{\Delta_E\left(1-\theta^*\right)}{3} + \theta^*\Delta_E\right]\frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}$$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{\Delta\left(1-\alpha\right)-3\bar{\theta}\Delta_E\alpha-\Delta_E\alpha+3\underline{\theta}\Delta_E}{\Delta\left(1-\alpha\right)+2\Delta_E\alpha}, \frac{\Delta\left(\alpha-1\right)+3\bar{\theta}\Delta_E\alpha+\Delta_E\alpha}{2\Delta+\alpha\left(\Delta-2\Delta_E\right)}\right\}$$

$$\alpha = \frac{1}{2}\frac{\Delta}{\Delta+\Delta_E}$$

profit:

$$\Pi_H^d = \frac{\Delta\left(\left(1-\theta^*\right)\left(\Delta+\Delta_E\right)-3\Delta_E\left(\underline{\theta}+\theta^*\right)\right)^2}{36\Delta_E\left(\Delta+\Delta_E\right)} - \alpha q_H^2$$

A.d.1 vs A.d.3

A.d.1 vs A.d.3 for L $\Pi_L^{A.d.1}(P_L^*, P_H^*) \geq \Pi_L^{A.d.3}(P_L', P_H^*)$

$\frac{\Delta(1-\theta^*)^2}{9} \geq P_L''(1-\theta^*+\theta')$

if $P_H^* = \frac{\Delta(2+\theta^*)}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta(2+\theta^*)}{6} + \frac{\Delta_E}{2} - \frac{\Delta_E\theta^*}{2}$$

or in an other form $P_L^d = \frac{\Delta_E\underline{\theta}}{6} + \frac{\Delta_E}{3} - \frac{\underline{\theta}\Delta}{2}$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{3\Delta_E - 2\Delta}{\Delta - 3\Delta_E}, \frac{2\Delta - 6\bar{\theta}\Delta - 3\Delta_E}{2 - 3\Delta_E - 2\Delta}\right\}$$

and profit:

$$\Pi_L^d = \frac{[(2+\theta^*)\Delta - 3\theta^*\Delta_E + 3\Delta_E][-(2+\theta^*)\Delta - 9\theta^*\Delta_E + 9\Delta_E]}{36\Delta_E} - \alpha q_L^2$$

A.d.1 vs A.d.3 for H $\Pi_H^{A.d.1}(P_L^*, P_H^*) \geq \Pi_H^{A.d.3}(P_L^*, P_H'')$

$\frac{\Delta(2+\theta^*)^2}{9} \geq P_H''(\theta^*-\theta')$

if $P_L^* = \frac{\Delta(1-\theta^*)}{3}$from optitimization proces we optain:

$$P_H^d = \frac{\Delta(1-\theta^*)}{6} + \frac{\Delta_E\theta^*}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{-6\bar{\theta}\Delta - \Delta}{-3\Delta_E - \Delta}, \frac{\Delta}{\Delta - 3\Delta_E}\right\}$$

profits is:

$$\Pi_H^d = \frac{[(1-\theta^*)\Delta + 3\theta^*\Delta_E]^2}{36\Delta_E} - \alpha q_H^2$$

A.d.2

A.d.2 vs A.d.1

A.d.2 vs A.d.1 for L $\Pi_L^{A.d.2}(P_L^*, P_H^*) \geq \Pi_L^{A.d.1}\left(P_L^d, P_H^*\right)$

$\frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)^2}{9(\Delta_E+\Delta)} \geq P_L^d(\theta''-\theta^*)$

if $P_H^* = \frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta\Delta_{\mathrm{E}}(2+\underline{\theta}+\theta^*)}{6(\Delta+\Delta_{\mathrm{E}})} - \frac{\theta^*\Delta}{2}$$

or after some manipulation $P_L^d = \frac{\Delta_E}{6}\frac{2\Delta - 2\underline{\theta}\Delta + \Delta\theta^* - 3\underline{\theta}\Delta_E}{\Delta+\Delta_E}$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{-\Delta\Delta_{\mathrm{E}}\underline{\theta}-\Delta\Delta_{\mathrm{E}}+6\underline{\theta}\Delta_E\left(\Delta+\Delta_{\mathrm{E}}\right)}{\Delta\Delta_{\mathrm{E}}+3\Delta\left(\Delta+\Delta_{\mathrm{E}}\right)},\frac{2\Delta\Delta_{\mathrm{E}}+\underline{\theta}\Delta\Delta_{\mathrm{E}}}{3\Delta^2+2\Delta\Delta_{\mathrm{E}}}\right\}$$

and profits is:

$$\Pi_L^d=\frac{\Delta\left(\underline{\theta}\Delta_E-3\theta^*\Delta-2\theta\Delta_E+2\Delta_E\right)^2}{36\left(\Delta+\Delta_E\right)}-\alpha q_L^2$$

A.d.2 vs A.d.1 for H $\Pi_H^{A.d.2}\left(P_L^*,P_H^*\right)\geq\Pi_H^{A.d.1}\left(P_L^*,P_H^d\right)$

$\frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)^2}{9(\Delta_E+\Delta)}\geq P_H^d\left(1-\theta''+\theta^*\right)$

if $P_L^*=\frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_H^d=\frac{\Delta_E\Delta\left(1-\left(\underline{\theta}+\theta^*\right)\right)}{6\left(\Delta_E+\Delta\right)}+\frac{\theta^*\Delta}{2}+\frac{\Delta}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{-3\Delta^2-2\Delta\Delta_E+5\Delta\Delta_E\underline{\theta}+6\Delta_E^2\underline{\theta}}{3\Delta^2+4\Delta\Delta_E},\frac{2\Delta_E+\Delta_E\underline{\theta}+3\Delta}{2\Delta_E+3\Delta}\right\}$$

and profit:

$$\Pi_H^d=\left[\frac{\Delta_E\Delta\left(1-\left(\underline{\theta}+\theta^*\right)\right)}{6\left(\Delta_E+\Delta\right)}+\frac{\theta^*\Delta}{2}+\frac{\Delta}{2}\right]^2-\alpha q_H^2$$

A.d.2 vs A.d.3

A.d.2 vs A.d.3 for L $\Pi_L^{A.d.2}\left(P_L^*,P_H^*\right)\geq\Pi_L^{A.d.3}\left(P_L^d,P_H^*\right)$

$\frac{\Delta_E\Delta\left(1-(\underline{\theta}+\theta^*)\right)^2}{9(\Delta_E+\Delta)}\geq P_L^d\left(1-\theta^*+\theta'\right)$

if $P_H^*=\frac{\Delta\Delta_E\left(\underline{\theta}+\theta^*+2\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_L^d=\frac{\Delta\Delta_{\mathrm{E}}\left(2+\underline{\theta}+\theta^*\right)}{6\left(\Delta+\Delta_{\mathrm{E}}\right)}-\frac{\theta^*\Delta_E}{2}+\frac{\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\frac{\Delta\underline{\theta}-\Delta-3\Delta_{\mathrm{E}}}{3\Delta_E-2\Delta}\leq\theta^*\leq\frac{-7\Delta\Delta_E-3\Delta_E^2-5\Delta\Delta_E\underline{\theta}-6\underline{\theta}\Delta^2-6\Delta^2}{-4\Delta\Delta_E-3\Delta_E^2}$$

profits:

$$\Pi_L^d=\left[\frac{\Delta\Delta_{\mathrm{E}}\left(2+\underline{\theta}+\theta^*\right)}{6\left(\Delta+\Delta_{\mathrm{E}}\right)}-\frac{\theta^*\Delta_E}{2}+\frac{\Delta_E}{2}\right]^2-\alpha q_L^2$$

A.d.2 vs A.d.3 for H $\Pi_H^{A.d.2}(P_L^*, P_H^*) \geq \Pi_H^{A.d.3}\left(P_L^*, P_H^d\right)$

$\frac{\Delta_E \Delta\left(\underline{\theta}+\theta^*+2\right)^2}{9(\Delta_E+\Delta)} \geq P_H^d(\theta^* - \theta')$

if $P_L^* = \frac{\Delta_E \Delta\left(1-(\underline{\theta}+\theta^*)\right)}{3(\Delta_E+\Delta)}$ from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E \Delta \left(1 - (\underline{\theta} + \theta^*)\right)}{6\left(\Delta_E + \Delta\right)} + \frac{\theta^* \Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{-7\Delta\Delta_E - 5\Delta\Delta_E\underline{\theta} - 6\Delta^2 - 6\underline{\theta}\Delta^2}{4\Delta\Delta_E - 3\Delta_E^2}; \frac{\Delta\Delta_E - 3\Delta\Delta_E\underline{\theta}}{3\Delta_E^2 - 2\Delta\Delta_E}\right\}$$

profits:

$$\Pi_H^d = \left[\frac{\Delta_E \Delta \left(1 - (\underline{\theta} + \theta^*)\right)}{6\left(\Delta_E + \Delta\right)} + \frac{\theta^* \Delta_E}{2}\right]^2 - \alpha q_H^2$$

A.d.3

A.d.3 vs A.d.1

A.d.3 vs A.d.1 for L $\Pi_L^{A.d.3}(P_L^*, P_H^*) \geq \Pi_L^{A.d.1}\left(P_L^d, P_H^*\right)$

$\frac{\Delta_E(2-\theta^*)^2}{9} \geq P_L'(\theta'' - \theta^*)$

if $P_H^* = \frac{\Delta_E(1+\theta^*)}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta_E\left(1 + \theta^*\right)}{6} - \frac{\theta^* \Delta}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \leq min\left\{\frac{\Delta_E\left(6\underline{\theta} - 1\right)}{\Delta_E\theta^* + 3\Delta}; \frac{\Delta_E}{3\Delta - \Delta_E}\right\}$$

and profits is:

$$\Pi_L^d = \frac{\left(-9\Delta\theta^* + \theta^*\Delta_E + \Delta_E\right)\left(-3\Delta\theta^* + \theta^*\Delta_E + \Delta_E\right)}{36\Delta} - \alpha q_L^2$$

A.d.3 vs A.d.1 for H $\Pi_H^{A.d.3}(P_L^*, P_H^*) \geq \Pi_H^{A.d.1}\left(P_L^*, P_H^d\right)$

$\frac{\Delta_E(1+\theta^*)^2}{9} \geq P_H^d(1 - \theta'' + \theta^*)$

if $P_L^* = \frac{\Delta_E(2-\theta^*)}{3}$ from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E\left(2 - \theta^*\right)}{6} + \frac{\theta^* \Delta}{2} + \frac{\Delta}{2}$$

on this case restiction on θ^*we have:

$$\theta^{*} min \left\{ \frac{3\Delta - 2\Delta_E}{3\Delta - \Delta_E} ; \frac{(6\underline{\theta} + 2)\,\Delta_E - 3\Delta}{3\Delta + \Delta_E} \right\}$$

and profit:

$$\Pi_H^d = \frac{(-9\Delta\theta^{*} + 3\Delta - \Delta_E\theta^{*} + 2\Delta_E)\,(3\Delta\theta^{*} + 3\Delta - \Delta_E\theta^{*} + 2\Delta_E)}{36\Delta} - \alpha q_H^2$$

A.d.3 vs A.d.2

A.d.3 vs A.d.2 for L $\Pi_L^{A.d.3}\,(P_L^{*}, P_H^{*}) \geq \Pi_L^{A.d.2}\left(P_L^d, P_H^{*}\right)$

$\frac{\Delta_E (2-\theta^{*})^2}{9} \geq P_L' \left(\theta' - \underline{\theta} + \theta'' - \theta^{*}\right)$

if $P_H^{*} = \frac{\Delta(1+\theta^{*})}{3}$ from optitimization proces we optain:

$$P_L^d = \left(\frac{\Delta\,(1+\theta^{*})}{3} - \theta^{*}\Delta + \frac{\Delta_{\mathrm{E}}\,(1+\theta^{*})}{3} - \underline{\theta}\Delta \right) \frac{1}{2} \frac{\Delta_E}{\Delta + \Delta_E}$$

on this case restiction on θ^{*}we have:

$\gamma = \frac{1}{2}\frac{\Delta_E}{\Delta+\Delta_E}$

$$\theta^{*} \leq min \left\{ \frac{2\Delta + \gamma\,(\Delta + \Delta_E) + 3\Delta\underline{\theta}\,(1-\gamma)}{\gamma\,(2\Delta - \Delta_E) + \Delta} ; \frac{+3\underline{\theta}\Delta\,(\gamma - 1) - \Delta\gamma - \delta\Delta_E}{-2\Delta\gamma + \Delta_E\gamma - \Delta} \right\}$$

profits:

$$\Pi_L^d = \frac{\Delta\,(-3\underline{\theta}\Delta - 2\Delta\theta^{*} + \Delta + \Delta\Delta_E + \Delta_E)\,(-6\underline{\theta}\Delta\Delta_E + 3\underline{\theta}\Delta - 6\Delta\Delta_E\theta^{*} + \Delta\Delta_E + 4\Delta\Delta_E + \Delta + \Delta_E)}{36\Delta_E\,(\Delta + \Delta_E)} - \alpha q$$

A.d.3 vs A.d.2 for H $\Pi_H^{A.d.3}\,(P_L^{*}, P_H^{*}) \geq \Pi_H^{A.d.2}\left(P_L^{*}, P_H^d\right)$

$\frac{\Delta_E (1+\theta^{*})^2}{9} \geq P_H'' \left(\bar{\theta} - \theta'' + \theta^{*} - \theta'\right)$

if $P_L^{*} = \frac{\Delta_E(2-\theta^{*})}{3}$ from optitimization proces we optain:

$$P_H^d = \left(\frac{\Delta\,(2-\theta^{*})}{3} + \theta^{*}\Delta + \frac{\Delta_{\mathrm{E}}\,(2-\theta^{*})}{3} + \bar{\theta}\Delta \right) \frac{1}{2} \frac{\Delta_E}{\Delta + \Delta_E}$$

on this case restiction on θ^{*}we have:

$$\theta^{*} \leq min \left\{ \frac{3\underline{\theta}\,(\gamma\Delta - \Delta_E) + 5\Delta\gamma + 2\Delta_E\gamma}{-\Delta - 2\Delta\gamma + \Delta_E\gamma} ; \frac{5\Delta\,(1-\gamma) - 2\Delta_E\gamma - 3\underline{\theta}\Delta\,(1-\gamma)}{2\Delta\gamma + \Delta - \Delta_E\gamma} \right\}$$

$\gamma = \frac{1}{2}\frac{\Delta_E}{\Delta+\Delta_E}$

profits:

$$\Pi_H^d = -\frac{\Delta\,(3\underline{\theta}\Delta + 4\Delta\theta^{*} - 6\underline{\theta}\Delta_E + \Delta - 5\Delta_E\Delta - 8\Delta_E)\,(3\underline{\theta}\Delta + 2\Delta\theta^{*} + 5\Delta - \Delta_E\theta^{*} + 2\Delta_E)}{36\,(\Delta + \Delta_E)\,\Delta_E} - \alpha q_H^2$$

Case B (pessimist)

B.a.1

B.b.1 vs B.a.2

B.a.1 vs B.a.2 for L $\Pi_L^{B.a.1}\left(P_L^*,P_H^*\right)\geq\Pi_L^{B.a.2}\left(P_L',P_H^*\right)$

$\frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)^2}{9}\geq P_L'\left(\theta^*-\underline{\theta}\right)$

if $P_H^*=\frac{\Delta_E\left(2\bar{\theta}-\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_L^d=\frac{\Delta_\mathrm{E}\left(2\bar{\theta}-\underline{\theta}\right)}{3}-\theta^*\Delta_E$$

comment is like an average weight price
on this case restiction on θ^*we have:

$$\frac{\Delta}{\Delta_E}\geq 1$$

and profit:

$$\Pi_L^d=\left(\frac{\Delta_\mathrm{E}\left(2\bar{\theta}-\underline{\theta}\right)}{3}-\theta^*\Delta_E\right)\left(\theta^*-\underline{\theta}\right)-\alpha q_L^2$$

B.a.1 vs B.a.2 for H $\Pi_H^{B.a.1}\left(P_L^*,P_H^*\right)\geq\Pi_H^{B.a.2}\left(P_L^*,P_H''\right)$

$\frac{\Delta_E\left(2\bar{\theta}-\underline{\theta}\right)^2}{9}\geq P_H''\left(\bar{\theta}-\theta^*\right)$

if $P_L^*=\frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)}{3}$from optitimization proces we optain:

$$P_H^d=\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3}+\theta^*\Delta$$

on this case restiction on θ^*we have:

$$\frac{\Delta}{\Delta_E}\geq 1$$

profit:

$$\Pi_H^d=\left(\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3}+\theta^*\Delta\right)\left(\bar{\theta}-\theta^*\right)-\alpha q_H^2$$

B.a.1 vs B.a.3

B.a.1 vs B.a.3 for L $\Pi_L^{B.a.1}(P_L^*, P_H^*) \geq \Pi_L^{B.a.3}(P_L', P_H^*)$
$\frac{\Delta_E(\bar{\theta}-2\underline{\theta})^2}{9} \geq P_L''(\theta'' - \underline{\theta})$
if $P_H^* = \frac{\Delta_E(2\bar{\theta}-\underline{\theta})}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta_E\left(2\bar{\theta} - \underline{\theta}\right)}{6} - \frac{\underline{\theta}\Delta}{2}$$

or in an other form $P_L^d = \frac{\Delta_E \underline{\theta}}{6} + \frac{\Delta_E}{3} - \frac{\underline{\theta}\Delta}{2}$
on this case restiction on θ^*we have:

$$\theta^* \leq \frac{\Delta_E(\underline{\theta} + 2) - 3\underline{\theta}\Delta}{6\Delta}$$

and profit:

$$\Pi_L^d = \frac{\left[\Delta_E\left(2\bar{\theta} - \underline{\theta}\right) - 3\underline{\theta}\Delta\right]^2}{36\Delta} - \alpha q_L^2$$

B.a.1 vs B.a.3 for H $\Pi_H^{B.a.1}(P_L^*, P_H^*) \geq \Pi_H^{B.a.3}(P_L^*, P_H'')$
$\frac{\Delta_E(2\bar{\theta}-\underline{\theta})^2}{9} \geq P_H''\left(\bar{\theta} - \theta''\right)$
if $P_L^* = \frac{\Delta_E(\bar{\theta}-2\underline{\theta})}{3}$from optitimization proces we optain:

$$P_H^d = \frac{\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right)}{6} + \frac{\bar{\theta}\Delta}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \leq \frac{3\Delta(\underline{\theta} + 1) + \underline{\theta}\Delta_E - \Delta_E}{6\Delta}$$

profits is:

$$\Pi_H^d = \frac{\left[\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right) + 3\underline{\theta}\Delta\right]^2}{36\Delta} - \alpha q_H^2$$

B.a.2

B.a.2 vs B.a.1

B.a.2 vs B.a.1 for L $\Pi_L^{B.a.2}(P_L^*, P_H^*) \geq \Pi_L^{B.a.1}\left(P_L^d, P_H^*\right)$
$\underline{\theta}q_0(\theta^* - \underline{\theta}) \geq P_L^d(\theta' - \underline{\theta})$
if $P_H^* = \underline{\theta}q_0 + \theta^*\Delta$ from optitimization proces we optain:

$$P_L^d = \frac{\underline{\theta}q_0 + \theta^*\Delta}{2} - \frac{\underline{\theta}\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\frac{2\underline{\theta}\Delta-\underline{\theta}\Delta_E-\underline{\theta}q_0}{\Delta}\leq\theta^*\leq\frac{-\underline{\theta}\Delta_E-\underline{\theta}q_0}{\Delta-2\Delta_E}$$

this constraint is hard to fit
and profits is:

$$\Pi_L^d=\frac{\left(\underline{\theta}q_0+\theta^*\Delta-\underline{\theta}\Delta_E\right)^2}{4\Delta_E}-\alpha q_L^2$$

B.a.2 vs B.a.1 for H $\Pi_H^{B.a.2}\left(P_L^*,P_H^*\right)\geq\Pi_H^{B.a.1}\left(P_L^*,P_H^d\right)$
$\left(\underline{\theta}q_0+\theta^*\Delta\right)\left(\bar{\theta}-\theta^*\right)\geq P_H^d\left(\bar{\theta}-\theta'\right)$
if $P_L^*=\underline{\theta}q_0$ from optitimization proces we optain:

$$P_H^d=\frac{\underline{\theta}q_0}{2}+\frac{\bar{\theta}\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\theta^*\geq\frac{\bar{\theta}\Delta_E-\underline{\theta}q_0}{2\Delta_E}$$

and profit:

$$\Pi_H^d=\frac{\left(\underline{\theta}q_0+\bar{\theta}\Delta_E\right)^2}{4\Delta_E}-\alpha q_H^2$$

B.a.2 vs B.a.3

B.a.2 vs B.a.3 for L $\Pi_L^{B.a.2}\left(P_L^*,P_H^*\right)\geq\Pi_L^{B.a.3}\left(P_L^d,P_H^*\right)$
$\underline{\theta}q_0\left(\theta^*-\underline{\theta}\right)\geq P_L^d\left(\theta''-\underline{\theta}\right)$
if $P_H^*=\underline{\theta}q_0+\theta^*\Delta$ from optitimization proces we optain:

$$P_L^d=\frac{\underline{\theta}q_0+\theta^*\Delta}{2}-\frac{\underline{\theta}\Delta}{2}$$

on this case restiction on θ^*we have:

$$\theta^*\leq min\left\{\frac{2\underline{\theta}\Delta_E+2\Delta_E-\underline{\theta}\Delta-\underline{\theta}q_0}{\Delta},\frac{\underline{\theta}\Delta+\underline{\theta}q_0}{\Delta}\right\}$$

profits:

$$\Pi_L^d=\frac{\left(\underline{\theta}q_0+\theta^*\Delta-\underline{\theta}\Delta\right)^2}{4\Delta}-\alpha q_L^2$$

B.a.2 vs B.a.3 for H $\Pi_H^{B.a.2}(P_L^*, P_H^*) \geq \Pi_H^{B.a.3}\left(P_L^*, P_H^d\right)$
$(\underline{\theta} q_0 + \theta^* \Delta)\left(\bar{\theta} - \theta^*\right) \geq P_H^d\left(\bar{\theta} - \theta''\right)$
if $P_L^* = \underline{\theta} q_0$ from optitimization proces we optain:

$$P_H^d = \frac{\underline{\theta} q_0}{2} + \frac{\bar{\theta}\Delta}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \leq \frac{\underline{\theta}\Delta + \Delta - \underline{\theta} q_0}{2\Delta}$$

profits:

$$\Pi_H^d = \frac{\left(\underline{\theta} q_0 + \bar{\theta}\Delta\right)^2}{4\Delta} - \alpha q_H^2$$

B.a.3

B.a.3 vs B.a.1

B.a.3 vs B.a.1 for L $\Pi_L^{B.a.3}(P_L^*, P_H^*) \geq \Pi_L^{B.a.1}\left(P_L^d, P_H^*\right)$
$\frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)^2}{9} \geq P_L'(\theta' - \underline{\theta})$
if $P_H^* = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{6} - \frac{\underline{\theta}\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \geq \frac{\underline{\theta}\Delta + 3\underline{\theta}\Delta_E + 2\Delta}{6\Delta_E}$$

and profits is:

$$\Pi_L^d = \frac{\left[\Delta\left(2\bar{\theta} - \underline{\theta}\right) - 3\underline{\theta}\Delta_E\right]^2}{36\Delta_E} - \alpha q_L^2$$

B.a.3 vs B.a.1 for H $\Pi_H^{Ba.3}(P_L^*, P_H^*) \geq \Pi_H^{B.a.1}\left(P_L^*, P_H^d\right)$
$\frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)^2}{9} \geq P_H^d\left(\bar{\theta} - \theta'\right)$
if $P_L^* = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_H^d = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{6} + \frac{\bar{\theta}\Delta_E}{2}$$

on this case restiction on θ^*we have:

$$\theta^* \geq \frac{\underline{\theta}\Delta + 3\underline{\theta}\Delta_E + 3\Delta_E - \Delta}{6\Delta_E}$$

and profit:

$$\Pi_H^d = \frac{\left[\Delta\left(\bar{\theta} - 2\underline{\theta}\right) + 3\bar{\theta}\Delta_E\right]^2}{36\Delta_E} - \alpha q_H^2$$

B.a.3 vs B.a.2

B.a.3 vs B.a.2 for L $\Pi_L^{B.a.3}\left(P_L^*, P_H^*\right) \geq \Pi_L^{B.a.2}\left(P_L^d, P_H^*\right)$
$\frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)^2}{9} \geq P_L'\left(\theta^* - \underline{\theta}\right)$
if $P_H^* = \frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_L^d = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3} - \theta^*\Delta_E$$

on this case restiction on θ^*we have:

$$\frac{\Delta}{\Delta_E} \geq 1$$

profits:

$$\Pi_L^d = \left(\frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3} - \theta^*\Delta_E\right)\left(\theta^* - \underline{\theta}\right) - \alpha q_L^2$$

B.a.3 vs B.a.2 for H $\Pi_H^{B.a.3}\left(P_L^*, P_H^*\right) \geq \Pi_H^{B.a.2}\left(P_L^*, P_H^d\right)$
$\frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)^2}{9} \geq P_H''\left(\bar{\theta} - \theta^*\right)$
if $P_L^* = \frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)}{3}$ from optitimization proces we optain:

$$P_H^d = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \theta^*\Delta$$

on this case restiction on θ^*we have:

$$\frac{\Delta}{\Delta_E} \geq 1$$

profits:

$$\Pi_H^d = \left(\frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} + \theta^*\Delta\right)\left(\bar{\theta} - \theta^*\right) - \alpha q_H^2$$

5 Appendix B

5.1 B1 First Best Solution

The solution to the following optimization program,

$$\max_{p_i,c_i} W(p_i,c_i) = (1+\alpha_i)\int_{p_i}^{\infty} q(x)d(x) + (p_i - c_i)q(p_i) + S(c_i)$$

must satisfy the following FOCs:

$$\frac{\partial W_i}{\partial p_i} = -q\left(p_i\right) - \alpha_i q\left(p_i\right) + q\left(p_i\right) + \left(p_i - c_i\right) q'\left(p_i\right) = 0$$

$$\frac{\partial W_i}{\partial p_i} = -\alpha_i q\left(p_i\right) + \left(p_i - c_i\right) q'\left(p_i\right) = 0 \tag{5.1}$$

$$\Rightarrow p_i = c_i + \alpha_i \frac{q\left(p_i\right)}{q'\left(p_i\right)}$$

$$\frac{\partial W_i}{\partial c_i} = -q\left(p_i\right) + S'\left(c_i\right) = 0 \tag{5.2}$$

So that p^* and c^* denote the optimal level of price and cost that solve the system of equations defined by (5.1) and (5.2).

With some manipulation and dividing both sides of (5.1) by $p_i$we obtain

$$\frac{p_i - c_i}{p_i} = -\frac{\alpha_i}{|\epsilon_i|} \tag{5.3}$$

With $\varepsilon_i$the price elasticity. For $0 < \alpha_i \leq 1$ we have a negative mark up $p_i < c_i$.

Note that and for different levels of altruism regulator fixes different prices for different firms, so if $\alpha_1 \leq \alpha_2$ we have$p_2^* \leq p_1^*$ and by equation (5.2) $c_2^* \leq c_1^*$.

Indeed, let,

$F_1 = p_i - c_i - \alpha_i \frac{q(p_i)}{q'(p_i)} = 0$

$F_2 = -q\left(p_i\right) + S'\left(c_i\right) = 0$

Total differentiation leads to:

$$\begin{cases} \frac{\partial F_1}{\partial c_i} dc_i + \frac{\partial F_1}{\partial p_i} dp_i + \frac{\partial F_1}{\partial \alpha_i} d\alpha_i & = 0 \\ \frac{\partial F_2}{\partial c_i} dc_i + \frac{\partial F_2}{\partial p_i} dp_i + \frac{\partial F_2}{\partial \alpha_i} d\alpha_i & = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{\partial F_1}{\partial c_i}\frac{dc_i}{d\alpha_i} + \frac{\partial F_1}{\partial p_i}\frac{dp_i}{d\alpha_i} & = -\frac{\partial F_1}{\partial \alpha_i} \\ \frac{\partial F_2}{\partial c_i}\frac{dc_i}{d\alpha_i} + \frac{\partial F_2}{\partial p_i}\frac{dp_i}{d\alpha_i} & = -\frac{\partial F_2}{\partial \alpha_i} \end{cases}$$

In matrix format

$$\begin{bmatrix} \partial F_1/\partial c_i & \partial F_1/\partial p_i \\ \partial F_2/\partial c_i & \partial F_2/\partial p_i \end{bmatrix} \begin{bmatrix} \partial c_i/\partial \alpha_i \\ \partial p_i/\partial \alpha_i \end{bmatrix} = \begin{bmatrix} -\partial F_1/\partial \alpha_i \\ -\partial F_2/\partial \alpha_i \end{bmatrix}$$

Using Cramer's rule we obtain: $\frac{\partial c_i}{\partial \alpha_i} = \frac{A_1}{j}; \frac{\partial p_i}{\partial \alpha_i} = \frac{A_2}{j}$

Where:

$$J = \begin{bmatrix} \partial F_1/\partial c_i & \partial F_1/\partial p_i \\ \partial F_2/\partial c_i & \partial F_2/\partial p_i \end{bmatrix};\ A_1 = \begin{bmatrix} -\partial F_1/\partial \alpha_i & \partial F_1/\partial p_i \\ -\partial F_2/\partial \alpha_i & \partial F_2/\partial p_i \end{bmatrix};\ A_2 = \begin{bmatrix} \partial F_1/\partial c_i & -\partial F_1/\partial \alpha_i \\ \partial F_2/\partial c_i & -\partial F_2/\partial \alpha_i \end{bmatrix}$$

Therefore computing the partial derivatives in A_1, A_2 and J:
$^{\partial F_1}/_{\partial c_i} = -1 < 0$
$^{\partial F_1}/_{\partial p_i} = 1 - \alpha_i \left[\frac{q'(p_i)^2 - q''(p_i)q(p_i)}{q'(p_i)^2}\right] \leq 1$
$^{\partial F_1}/_{\partial \alpha_i} = -\frac{q(p_i)}{q'(p_i)} > 0$
$^{\partial F_2}/_{\partial c_i} = S''(c_i) < 0$
$^{\partial F_2}/_{\partial p_i} = -q'(p_i) > 0$
$^{\partial F_2}/_{\partial \alpha_i} = 0$

in case of a linear demand function we have $\frac{\partial F_1}{\partial p_i} = 1 - \alpha_i$
and

$$J = q'(p_i) - (1 - \alpha_i) S''(c_i)$$

From (5.2) and assumption 1 we know that $|q'(p_i)| \leq |S''(c_i)|$ therefore if $\alpha_i < 1 - \frac{q'(p_i)}{S''(c_i)}$ then J is positive. The condition $\alpha_i < 1 - \frac{q'(p_i)}{S''(c_i)}$ ensures a negative definite Hessian matrix (for the existence of a maximum) and therefore an interior solution.

With $J > 0$ the sign of will be determined by the sign of A_1 and A_2.

$A_1 = \left(-^{\partial F_1}/_{\partial \alpha_i}\right)\left(^{\partial F_2}/_{\partial p_i}\right) - \left(^{\partial F_1}/_{\partial p_i}\right)\left(-^{\partial F_1}/_{\partial \alpha_i}\right) = -q(p_i) < 0$
$A_2 = \left(^{\partial F_1}/_{\partial c_i}\right)\left(-^{\partial F_2}/_{\partial \alpha_i}\right) - \left(-^{\partial F_1}/_{\partial \alpha_i}\right)\left(^{\partial F_2}/_{\partial c_i}\right) = -\frac{q(p_i)}{q'(p_i)} S''(c_i) < 0$

Therefore it follows that:

$$^{\partial c_i}/_{\partial \alpha_i} < 0\,,\ ^{\partial p_i}/_{\partial \alpha_i}$$

Therefore for $\alpha_1 \leq \alpha_2$ we have that $p_2^* \leq p_1^*$ and $c_2^* \leq c_1^*$

5.2 B2 Free price scenario

Proof of Proposition 1: Maximizing

$$U_i\left(\hat{p_i}, c_i; \alpha_i\right) = \left(p_i - c_i\right) q\left(p_i\right) + S\left(c_i\right) + \alpha_i \int_{p_i}^{\infty} q\left(x\right) d\left(x\right)_i \tag{5.4}$$

With respect to pi and c_i the optimal price p_i^f and cost c_i^f are the solution for the following FOCs: are:

$$\frac{\partial U_i}{\partial p_i} = \left(p_i - c_i\right) q'\left(p_i\right) - \alpha_i q\left(p_i\right) + q\left(p_i\right) = 0 \tag{5.5}$$

$$\frac{\partial U_i}{\partial c_i} = -q\left(p_i\right) + S'\left(c_i\right) = 0 \tag{5.6}$$

Rearranging (5.5)

$$\frac{p_i - c_i}{p_i} = \frac{1 - \alpha_i}{|\epsilon_i|} \tag{5.7}$$

We have a positive mark up, if $\alpha_i < 1$ (i.e. $p_i^f > c_i^f$); zero mark-up for $\alpha_i = 1$ (i.e. $p_i^f = c_i^f$). In the latter when $\alpha_i = 1$ the free price solution is the same as in the first best solution with zero altruism.

Comparing (5.7) with (5.3) $\forall \alpha_i$, $-\alpha_i / |\varepsilon_i| \leq 0$ and $(1 - \alpha_i\ )/|\varepsilon_i| \geq 0$, it follows that $p_i^f > p_i^*$. Moreover given that the mark-up decreases with the altruism level it follows that $p_1^f \geq p_2^f$ for $\alpha_1 \leq \alpha_2$. Furthermore given (A2.35.6) it follows that $c_1^f \geq c_2^f$.

Indeed, let,

$F_1 = p_i - c_i - \alpha_i \frac{q(p_i)}{q'(p_i)} + \frac{q(p_i)}{q'(p_i)} = 0$

$F_2 = -q\left(p_i\right) + S'\left(c_i\right) = 0$

Total differentiation leads to:

$$\begin{cases} \frac{\partial F_1}{\partial c_i} dc_i + \frac{\partial F_1}{\partial p_i} dp_i + \frac{\partial F_1}{\partial \alpha_i} d\alpha_i & = 0 \\ \frac{\partial F_2}{\partial c_i} dc_i + \frac{\partial F_2}{\partial p_i} dp_i + \frac{\partial F_2}{\partial \alpha_i} d\alpha_i & = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{\partial F_1}{\partial c_i} \frac{dc_i}{d\alpha_i} + \frac{\partial F_1}{\partial p_i} \frac{dp_i}{d\alpha_i} & = -\frac{\partial F_1}{\partial \alpha_i} \\ \frac{\partial F_2}{\partial c_i} \frac{dc_i}{d\alpha_i} + \frac{\partial F_2}{\partial p_i} \frac{dp_i}{d\alpha_i} & = -\frac{\partial F_2}{\partial \alpha_i} \end{cases}$$

In matrix format

$$\begin{bmatrix} \partial F_1/\partial c_i & \partial F_1/\partial p_i \\ \partial F_2/\partial c_i & \partial F_2/\partial p_i \end{bmatrix} \begin{bmatrix} \partial c_i/\partial \alpha_i \\ \partial p_i/\partial \alpha_i \end{bmatrix} = \begin{bmatrix} -\partial F_1/\partial \alpha_i \\ -\partial F_2/\partial \alpha_i \end{bmatrix}$$

Using Cramer's rule we obtain: $\frac{\partial c_i}{\partial \alpha_i} = \frac{A_1}{j}; \frac{\partial p_i}{\partial \alpha_i} = \frac{A_2}{j}$

Where:

$J = \begin{bmatrix} \partial F_1/\partial c_i & \partial F_1/\partial p_i \\ \partial F_2/\partial c_i & \partial F_2/\partial p_i \end{bmatrix}$; $A_1 = \begin{bmatrix} -\partial F_1/\partial \alpha_i & \partial F_1/\partial p_i \\ -\partial F_2/\partial \alpha_i & \partial F_2/\partial p_i \end{bmatrix}$; $A_2 = \begin{bmatrix} \partial F_1/\partial c_i & -\partial F_1/\partial \alpha_i \\ \partial F_2/\partial c_i & -\partial F_2/\partial \alpha_i \end{bmatrix}$

Therefore computing the partial derivatives in A_1, A_2 and J:

$\partial F_1/\partial c_i = -1 < 0$
$\partial F_1/\partial p_i = 1 - \alpha_i \left[\frac{q'(p_i)^2 - q''(p_i)q(p_i)}{q'(p_i)^2}\right] + \left[\frac{q'(p_i)^2 - q''(p_i)q(p_i)}{q'(p_i)^2}\right]$
$\partial F_1/\partial \alpha_i = -\frac{q(p_i)}{q'(p_i)} > 0$
$\partial F_2/\partial c_i = S''(c_i) < 0$
$\partial F_2/\partial p_i = -q'(p_i) > 0$
$\partial F_2/\partial \alpha_i = 0$

in case of a linear demand function we have $\frac{\partial F_1}{\partial p_i} = 2 - \alpha_i$
and

$$J = q'(p_i) - (2 - \alpha_i) S''(c_i)$$

From (5.2) and assumption 1 we know that $|q'(p_i)| \leq |S''(c_i)|$ therefore if $0 \leq \alpha_i \leq 1$ then J is positive. The condition $J > 0$ ensures a negative definite Hessian matrix (for the existence of a maximum) and therefore an interior solution.

With $J > 0$ the sign of will be determined by the sign of A_1 and A_2.

$A_1 = (-\partial F_1/\partial \alpha_i)(\partial F_1/\partial p_i) - (\partial F_1/\partial p_i)(-\partial F_2/\partial \alpha_i) = -q(p_i) < 0$
$A_2 = (\partial F_1/\partial c_i)(-\partial F_2/\partial \alpha_i) - (-\partial F_1/\partial \alpha_i)(\partial F_2/\partial c_i) = -\frac{q(p_i)}{q'(p_i)} S''(c_i) < 0$

Therefore it follows that:

$$\partial c_i/\partial \alpha_i < 0\,,\ \partial p_i/\partial \alpha_i < 0$$

Therefore for $\alpha_1 \leq \alpha_2$ we have that $p_2^f \leq p_1^f$ and $c_2^f \leq c_1^f$

Furthermore since (5.6) is the same as (5.2). Knowing that $p_i^f > p_i^*$ Since $q'(p_i) < 0$and $S''(c_i) < 0$ it follows from (5.6) that $c_i^f > c_i^*$.

5.3 B3 Non-Cooperative Scenario: The Provider's problem

Proof of Proposition 2 and Corollary 1

Given the utility function being maximized (5.4), the FOCs with respect to cost are given by,

$$\frac{\partial U_i}{\partial c_i} = \left(\frac{\partial \hat{p}_i}{\partial c_i} - 1\right) q\left(\hat{p}_i\right) + \left(\hat{p}_i - c_i\right) \frac{\partial q\left(\hat{p}_i\right)}{\partial \hat{p}_i} \frac{\partial \hat{p}_i}{\partial c_i} + S'\left(c_i\right) - \alpha_i q\left(\hat{p}_i\right) \frac{\partial \hat{p}_i}{\partial c_i} = 0 \tag{5.8}$$

$\forall \alpha_i \in [0,1], i = 1,2$

Given the regulatory rule $\hat{p}_i = c_{-i}$ (where $\hat{p}_i$ is the regulated price), (A3.15.8) becomes,

$$\frac{\partial U_i}{\partial c_i} = S'\left(c_i\right) - q\left(c_{-i}\right) = 0 \qquad \forall \alpha_i \in [0,1], i = 1,2 \tag{5.9}$$

Given *assumption 1*, for $i, -i = \{1,2\}, i \neq -i$ we have: (i) $\partial U_i^2/\partial c_i^2 < 0$; (ii)$|\partial U_i^2/\partial c_i^2| > |\partial U_i^2/\partial c_i \partial c_{-i}|$, i.e. $|S''(c_i)| < |q'(c_{-i})|$.

From (5.9) we see that price do not depend on altruism and the price is given under regulated price. Therefore the condition for profit maximization (5.9) becomes:

$$S'\left(c_i\right) = q\left(c_{-i}\right)$$

We argue that the optimal solution (c_i^{nc}, c_{-i}^{nc}) is the symmetric solution to (5.9), i.e. $c_i^{nc} = c_{-i}^{nc} = c^{nc}$.

To prove that there is no asymmetric solution we will look for profitable deviations from the asymmetric equilibrium. Consider the asymmetric solution $c_1^{nc} > c_2^{nc} = c^{nc}$.

Suppose that firm 1 decreases its cost by $\Delta c_1 1$). From (5.9) we can see that firm 1 by varying c_1 in Δc_1 gains $q(c_2^{nc})\Delta c_1$ at the cost of $S'(c_1)\Delta c_1$. By the SOCs we know that $S'(c_1) < q(c_2^{nc})$ therefore it is always profitable for firm 1 to decrease its cost.

Now suppose that firm 1 increases its cost by Δc_1 . From (5.9) we can see that firm 1 by varying c_1 in Δc_1 gains $S'(c_1)\Delta c_1$ at the cost of $q(c_2^{nc})\Delta c_1$. Given that by SOCs $S'(c_1) < q(c_2^{nc})$ then it is not profitable for firm 1 to increase its cost.

Consider the asymmetric solution $c_1^{nc} < c_2^{nc} = c^{nc}$.

Suppose that firm 1 deviates by decreasing its cost by Δc_1. From (5.9) we can see that firm 1 by varying c_1 in Δc_1 gains $q(c_2^{nc})\Delta c_1$ at the cost of $S'(c_1)\Delta c_1$. Now for $c_1^{nc} < c_2^{nc} = c^{nc}$ and given the SOCs we know that $S'(c_1) > q(c_2^{nc})$ therefore it is never profitable to deviate by decreasing the cost.

Now suppose that firm 1 increases its cost by Δc_1 . From (5.9) we can see that by varying c_1 in Δc_1 it gains $S'(c_1)\Delta c_1$ at the cost of $q(c_2^{nc})\Delta c_1$. Now for $c_1^{nc} < c_2^{nc} = c^{nc}$ and given the SOCs we know that $S'(c_i) > q(c_2^{nc})$ therefore it is always profitable to deviate by increasing the cost.

Therefore the only possible solution is the symmetric equilibrium $c_1^n c = c_2^{nc} = c^{nc}$.

Comparing costs and prices across the different scenarios.

First note that in the non-comparative scenario c^{nc} does not vary with the altruism level (neither the price given the price rule $\hat{p} = c_{-i}$) while prices and costs in the first best and free price scenarios (i.e. c_i^f, c_i^*, p_i^f and p_i^*) are all decreasing in the level of altruism (see proof above).

Symmetric case

For the case of selfish firms, i.e. $\alpha_i = \alpha_{-i} = 0$, then$p^f > c^f > c^* = p^* = p^{nc} = c^{nc}$ so that prices and costs are the same in the first best and non-cooperative scenario and higher in the free price scenario. Indeed, comparing the non-cooperative with the first best solution. We know that for $\alpha_i = 0, \forall i = 1, 2$, from (5.1) it follows that $p_i^* = c_i^*$. Therefore given that the cost FOC in the non-cooperative scenario (5.9) is equal to the cost FOC in the first best (5.2) it follows that $c_i^{nc} = c_i^* = p_i^*$. The remainder inequalities have been demonstrated above.

When $\alpha_i = \alpha_{-i} = 1$, then $p^f = c^f = p^{nc} = c^{nc} > c^* > p^*$, in this case the free price scenario and the non-cooperative scenario under yardstick competition give the same result.

Indeed, suppose now that α_i increases from zero such that $\alpha_i > 0, \forall i = 1, 2$. We know that, for $\alpha_i > 0 \forall i = 1, 2$ and from (5.1) $p_i^* < c_i^*$. For FOC (5.2) to hold it must follow that $c_i^*|_{(\alpha_i=0)} > c_i^*|_{(\alpha_i>0)}$. Since we know that $c_i^*|_{(\alpha_i=0)} = c_i^{nc}$ it follows that $c_i^{nc} > c_i^*|_{(\alpha_i>0)}$.

Comparing the non-cooperative with the free price scenario. We know that for $\alpha_i = 1, \forall i = 1, 2$, from (5.5) it follows that $p_i^f = c_i^f$. Therefore given that the cost FOC in the non-cooperative scenario (5.9) is equal to the cost FOC in free price scenario (5.6) it follows that $c_i^f = p_i^f = c_i^{nc} = p_i^{nc}$. The remainder inequalities have been demonstrated above.

Asymmetric case

Finally for $0 \leq \alpha_1 \leq \alpha_2 \leq 1$ we know that:

a. From the non-cooperative scenario (appendix A3) $p_1^{nc} = p_2^{nc} = c_1^{nc} = c_2^{nc}$

b. From appendix A1 we know that $c_1^* \geq c_2^*$

c. From appendix A2 that $c_1^f \geq c_2^f \geq c_1^* \geq c_2^*$

d. Above in this section we have also shown that $c_1^{nc} = c_2^{nc} \geq c_1^* \geq c_2^*$

e. For $\alpha_i < 1 \ \forall i = 1, 2$, Comparing the free price with the non-cooperative scenario, from (5.5) it follows that $p_i^f > c_i^f$. Therefore given that the cost FOC in the non-cooperative scenario (5.9) is equal to the cost FOC in free price scenario (5.6) it follows that $c_1^f > c_1^{nc} = c_2^{nc} = p_1^{nc} = p_2^{nc}$.

f. For $\alpha_i = 1 \ \forall i = 1, 2$, as shown above we know that $c_i^f = p_i^f = c_i^{nc} = p_i^{nc}$.

g. Given c), e) and f) it follows $c_1^f \geq c_2^f \geq c_1^{nc} = c_2^{nc} = p_1^{nc} = p_2^{nc}$

h. Finally given d) and g) it follows that $c_1^f \geq c_2^f \geq c_1^{nc} = c_2^{nc} = p_1^{nc} = p_2^{nc} \geq c_1^* \geq c_2^*$

Therefore given a)-h) it follows that the costs of the different scenarios are ranked in the following way: $c_1^f \geq c_2^f \geq c_1^{nc} = c_2^{nc} \geq c_1^* \geq c_2^*$, while the prices ranking is as follows: $p_1^f \geq p_2^f \geq p_1^{nc} = p_2^{nc} \geq p_1^* \geq p_2^*$.

Comparative statics and equilibrium payoffs

Recall from (5.9) that the optimal costs must satisfy the FOCs:

$$\frac{\partial U_i}{\partial c_i} = S'(c_i) - q(c_{-i}) = 0 \qquad \forall \alpha_i \in [0,1], i = 1, 2$$

let,
$F_1 \equiv \frac{\partial U_1}{\partial c_1} = -q(c_2) + S'(c_1) = 0$
$F_2 \equiv \frac{\partial U_2}{\partial c_2} = -q(c_1) + S'(c_2) = 0$

Total differentiation leads to:

$$\begin{cases} \frac{\partial F_1}{\partial c_1} dc_1 + \frac{\partial F_1}{\partial c_2} dc_2 + \frac{\partial F_1}{\partial \alpha_i} d\alpha_i & = 0 \\ \frac{\partial F_2}{\partial c_1} dc_1 + \frac{\partial F_2}{\partial c_2} dc_2 + \frac{\partial F_2}{\partial \alpha_i} d\alpha_i & = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{\partial F_1}{\partial c_1}\frac{dc_1}{d\alpha_i} + \frac{\partial F_1}{\partial c_2}\frac{dc_2}{d\alpha_i} = -\frac{\partial F_1}{\partial \alpha_i} \\ \frac{\partial F_2}{\partial c_1}\frac{dc_1}{d\alpha_i} + \frac{\partial F_2}{\partial c_2}\frac{dc_2}{d\alpha_i} = -\frac{\partial F_2}{\partial \alpha_i} \end{cases}$$

In matrix format

$$\begin{bmatrix} \partial F_1/\partial c_1 & \partial F_1/\partial c_2 \\ \partial F_2/\partial c_1 & \partial F_2/\partial c_2 \end{bmatrix} \begin{bmatrix} \partial c_1/\partial \alpha_i \\ \partial c_2/\partial \alpha_i \end{bmatrix} = \begin{bmatrix} -\partial F_1/\partial \alpha_i \\ -\partial F_2/\partial \alpha_i \end{bmatrix}$$

Using Cramer's rule we obtain: $\frac{\partial c_i}{\partial \alpha_i} = \frac{A_1}{j}; \frac{\partial p_i}{\partial \alpha_i} = \frac{A_2}{j}$

Where:

$$J = \begin{bmatrix} \partial F_1/\partial c_1 & \partial F_1/\partial c_2 \\ \partial F_2/\partial c_{1i} & \partial F_2/\partial c_2 \end{bmatrix};\ A_1 = \begin{bmatrix} -\partial F_1/\partial \alpha_i & \partial F_1/\partial c_2 \\ -\partial F_2/\partial \alpha_i & \partial F_2/\partial c_{2i} \end{bmatrix};\ A_2 = \begin{bmatrix} \partial F_1/\partial c_1 & -\partial F_1/\partial \alpha_i \\ \partial F_2/\partial c_1 & -\partial F_2/\partial \alpha_i \end{bmatrix}$$

Therefore computing the partial derivatives in A_1, A_2 and J:
$\partial F_1/\partial c_1 = S''(c_1^{nc}) < 0$
$\partial F_1/\partial c_2 = -q'(c_2^{nc}) > 0$
$\partial F_1/\partial \alpha_i = 0$
$\partial F_2/\partial c_1 = -q'(c_1^{nc}) > 0$
$\partial F_2/\partial c_2 = S''(c_2^{nc}) < 0$
$\partial F_2/\partial \alpha_i = 0$

With J>0 the sign of $\partial c_1/\partial \alpha_i, \partial c_2/\partial \alpha_i$ will be determined by the sign of A_1 and A_2.

$A_1 = (-\partial F_1/\partial \alpha_i)(\partial F_1/\partial c_2) - (\partial F_1/\partial c_2)(-\partial F_2/\partial \alpha_i) = 0$
$A_2 = (\partial F_1/\partial c_1)(-\partial F_2/\partial \alpha_i) - (-\partial F_1/\partial \alpha_i)(\partial F_2/\partial c_1) = 0$

Therefore it follows that:

$$\partial c_1/\partial \alpha_i = \partial c_2/\partial \alpha_i = 0 \tag{5.10}$$

Utility payoffs

The utility payoff of provider i as a result of its maximization behaviour is given by,

$$U_i^{nc} = \left(c_{-i}^{nc} - c_i\right) q\left(c_{-i}^{nc}\right) + S\left(c_i^{nc}\right) + \alpha_i \int_{c_{-i}^{nc}}^{\infty} q(x)\, d(x) = \tag{5.11}$$

$$= S\left(c_i^{nc}\right) + \alpha_i \int_{c_{-i}^{nc}}^{\infty} q(x)\, d(x)$$

When $\alpha_i = \alpha_{-i} = \alpha$, in equilibrium by symmetry $c_i^{nc} = c_{-i}^{nc} = c^{nc}$ and FOC (A3.25.9) becomes,

$$\frac{\partial U}{\partial c} = -q(c) + S'(c) = 0 \tag{5.12}$$

Therefore, for both ownership types in equilibrium providers gain the same utility payoff

$$U^{nc} = S\left(c^{nc}\right) + \alpha \int_{c^{nc}}^{\infty} q(x)\, d(x)$$

5.4 B4 Cooperative Scenario

Proof of Proposition 3

In a cooperative scenario providers optimally choose c_i by maximizing their joint profits JU:

$$\max_{c_i} JU = \sum_{i=1}^{2} \left[(\hat{p}_i - c_i) q(\hat{p}_i) + S(c_i) + \alpha_i \int_{\hat{p}_i}^{\infty} q(x) dx \right]$$

Thus for $\hat{p}_i = c_{-i}$

$$\max_{c_i, c_{-i}} JU = \sum_{i=1}^{2} \left[(c_{-i} - c_i) q(c_{-i}) + S(c_i) + \alpha_i \int_{c_{-i}}^{\infty} q(x) dx \right] \tag{5.13}$$

The FOCs are given by,

$$\frac{\partial JU}{\partial c_i} = -q(c_{-i}) + S'(c_i) + (c_i - c_{-i}) q'(c_i) + (1 - \alpha_{-i}) q(c_i) = 0 \; i = 1, 2 \tag{5.14}$$

These conditions can be rewritten as:

$$\frac{\partial JU}{\partial c_2} = (1 - \alpha_1) q(c_2^c) + S'(c_2^c) = q(c_1^c) + (c_1^c - c_2^c) q'(c_2^c) \tag{5.15}$$

and

$$\frac{\partial JU}{\partial c_1} = (1 - \alpha_2) q(c_1^c) + S'(c_1^c) = q(c_2^c) + (c_2^c - c_1^c) q'(c_1^c) \tag{5.16}$$

Suppose that $c_2^c < c_1^c$. Given $S''(c_i) < 0$ and $\alpha_1 \leq \alpha_2 \Rightarrow (1 - \alpha_1) q(c_2^c) + S'(c_2^c) > (1 - \alpha_2) q(c_1^c) + S'(c_1^c)$.

Therefore, $q(c_1^c) + (c_1^c - c_2^c) q'(c_2^c) > q(c_2^c) + (c_2^c - c_1^c) q'(c_1^c) \Rightarrow q(c_1^c) - q(c_2^c) + (c_1^c - c_2^c)[q'(c_2^c) - q'(c_1^c)] > 0$

However for $q'(.) < 0$ and $q''(.) > 0$ this inequality never holds for $c_2^c < c_1^c$. Now suppose that $c_2^c < c_1^c$. Then rewriting and subtracting (5.15) and (5.16) we obtain:

$$q(c^c)(\alpha_2 - \alpha_1) = 0$$

For $q(.) > 0$ and $\alpha_1 \leq \alpha_2$ this can only hold for $\alpha_1 = \alpha_2$. Therefore for $\alpha_1 \leq \alpha_2$ it must be that $c_2^c > c_1^c$. For $\alpha_1 = \alpha_2$ by symmetry $c_2^c = c_1^c$.

Comparative Statics

Proceeding with comparative statics let:

$$\begin{cases} \frac{\partial F_1}{\partial c_1}dc_1 + \frac{\partial F_1}{\partial c_2}dc_2 + \frac{\partial F_1}{\partial \alpha_1}d\alpha_1 + \frac{\partial F_1}{\partial \alpha_2}d\alpha_2 & = 0 \\ \frac{\partial F_2}{\partial c_1}dc_1 + \frac{\partial F_2}{\partial c_2}dc_2 + \frac{\partial F_2}{\partial \alpha_1}d\alpha_1 + \frac{\partial F_2}{\partial \alpha_2}d\alpha_2 & = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{\partial F_1}{\partial c_1}\frac{dc_1}{d\alpha_1} + \frac{\partial F_1}{\partial c_2}\frac{dc_2}{d\alpha_1} = -\frac{\partial F_1}{\partial \alpha_1} \\ \frac{\partial F_2}{\partial c_1}\frac{dc_1}{d\alpha_1} + \frac{\partial F_2}{\partial c_2}\frac{dc_2}{d\alpha_1} = -\frac{\partial F_2}{\partial \alpha_1} \end{cases}$$

In matrix format

$$\begin{bmatrix} \partial F_1/\partial c_1 & \partial F_1/\partial c_2 \\ \partial F_2/\partial c_1 & \partial F_2/\partial c_2 \end{bmatrix} \begin{bmatrix} \partial c_1/\partial \alpha_1 \\ \partial c_2/\partial \alpha_1 \end{bmatrix} = \begin{bmatrix} -\partial F_1/\partial \alpha_1 \\ -\partial F_2/\partial \alpha_1 \end{bmatrix}$$

Using Cramer's rule we obtain: $\frac{\partial c_i}{\partial \alpha_1} = \frac{A_1}{j}; \frac{\partial p_i}{\partial \alpha_1} = \frac{A_2}{j}$

Where:

$$J = \begin{bmatrix} \partial F_1/\partial c_1 & \partial F_1/\partial c_2 \\ \partial F_2/\partial c_{1i} & \partial F_2/\partial c_2 \end{bmatrix};\ A_1 = \begin{bmatrix} -\partial F_1/\partial \alpha_i & \partial F_1/\partial c_2 \\ -\partial F_2/\partial \alpha_i & \partial F_2/\partial c_{2i} \end{bmatrix};\ A_2 = \begin{bmatrix} \partial F_1/\partial c_1 & -\partial F_1/\partial \alpha_1 \\ \partial F_2/\partial c_1 & -\partial F_2/\partial \alpha_1 \end{bmatrix}$$

Therefore computing the partial derivatives in A_1, A_2 and J:

$\partial F_1/\partial c_1 = S''(c_1) + q'(c_1) + (1-\alpha_2)\, q'(c_1) + (c_1 - c_2)\, q''(c_1) < 0$

$\partial F_1/\partial c_2 = -q'(c_1) - q'(c_2)$

$\partial F_1/\partial \alpha_i = 0$

$\partial F_2/\partial c_1 = -q'(c_1) - q'(c_2)$

$\partial F_2/\partial c_2 = S''(c_2) + q'(c_2) + (1-\alpha_1)\, q'(c_2) + (c_2 - c_1)\, q''(c_2) < 0$

$\partial F_2/\partial \alpha_i = -q(c_2) < 0$

$$J = [S''(c_2) + q'(c_2) + (1-\alpha_1)\, q'(c_2)] * [S''(c_1) + q'(c_1) + (1-\alpha_2)\, q'(c_1)] - [-q'(c_1) - q'(c_2)]^2$$

Under the assumption 1 $q'(p_i) \leq S''(c_i)$ and linear demand function $J > 0$ so the sign of $\partial c_1/\partial \alpha_1$,$\partial c_2/\partial \alpha_1$ will be determined by the sign of A_1 and A_2.

$A_1 = (-\partial F_1/\partial \alpha_1)(\partial F_1/\partial c_2) - (\partial F_1/\partial c_2)(-\partial F_2/\partial \alpha_1) < 0$

$A_2 = (\partial F_1/\partial c_1)(-\partial F_2/\partial \alpha_1) - (-\partial F_1/\partial \alpha_1)(\partial F_2/\partial c_1) < 0$

For a linear demand function $A_2 < 0$ and $A_1 < 0$. It follows that:

$$\partial c_1^c/\partial \alpha_1 < 0,\ \partial c_2^c/\partial \alpha_1 < 0 \tag{5.17}$$

Analogously for α_2:

$$\partial c_1^c/\partial \alpha_2 < 0,\ \partial c_2^c/\partial \alpha_2 < 0 \tag{5.18}$$

Proof or Corollary 2

In order to show , consider (5.16) for $i = 1$ and rewrite it as:

$$\frac{\partial JU}{\partial c_1} = -q\left(c_2^c\right) + S'\left(c_1^c\right) + \left(c_1^c - c_2^c\right) q'\left(c_1^c\right) + \left(1-\alpha_2\right) q\left(c_1^c\right) = 0$$

Comparing with the non-cooperative FOC the latter has an extra term:$\left(c_1^c - c_2^c\right) q'\left(c_1^c\right) + \left(1-\alpha_2\right) q\left(c_1^c\right)$. As in equilibrium $c_2^c > c_1^c$ it follows that $\left(c_1^c - c_2^c\right) q'\left(c_1^c\right) + \left(1-\alpha_2\right) q\left(c_1^c\right) > 0$. Consequently $c_1^c > c^{nc} \geq c^*$. As $c_2^c > c_1^c \Rightarrow c_2^c > c^{nc} \geq c^* \; \forall \alpha_i \in [0,1]$.

In the symmetric case, when the providers have the same level of altruism, i.e. when $\alpha_1 = \alpha_2 = \alpha$, in equilibrium by symmetry, $c_1 = c_2 = c^c$, (5.14) becomes,

$$\frac{\partial JU}{\partial c} = S'\left(c^c\right) - \alpha q\left(c^c\right) = 0 \tag{5.19}$$

Also, in the symmetric case, when $\alpha \to 1$ (5.19) becomes the same as (5.9), therefore $c^c \to c^{nc}$.

Now consider $\alpha_1 = \alpha_2 = 1$. The FOC in the cooperative scenario becomes $-q\left(c_{-i}^c\right) + S'\left(c_i^c\right) + \left(c_i^c - c_{-i}^c\right) q'\left(c_i^c\right) = 0$. By symmetry $c_2^c = c_1^c = c^c$ that is the solution to the FOC that simplifies to $S'\left(c_i^c\right) - q\left(c_{-i}^c\right) = 0$. Comparing the latter with the (5.12) it follows that $c^c = c^{nc} > c^*$

Consider now $\alpha_1 = \alpha_2 = 0$. Since $c_2^c = c_1^c = c^c$ the FOCs in the cooperative scenario (5.15) and (5.16) become $S'\left(c^c\right) = 0$. Comparing the latter with (5.12) since $-q(c) < 0$ it follows that $c^c > c^{nc} > c^*$.

Comparing the cooperative solution with the with the first best solution for the symmetric case, analysing the first best FOC $\frac{\partial W_i}{\partial c_i} = -q\left(p_i\right) + S'\left(c_i\right) = 0$ we have that:

$-q\left(p^*\right) + S'\left(c^*\right) = -q\left(c^c\right) + S'\left(c^c\right) = 0 \Leftrightarrow q\left(p^*\right) - q\left(c^c\right) = S'\left(c^*\right) - S'\left(c^c\right)$ as in the first best the price is below the marginal cost c^* then $q\left(p^*\right) - q\left(c^c\right) > 0 \Rightarrow S'\left(c^*\right) - S'\left(c^c\right) > 0$. As $S''(.) < 0 \Rightarrow c^c > c^*$.

For the symmetric case $\alpha_1 = \alpha_2 = \alpha$, proceeding with some comparative static analysis, consider the FOC (5.14) by the implicit function theorem $\frac{dc^c}{d\alpha} = \frac{\partial^2 JU/\partial c^c \partial \alpha}{\partial^2 JU/\partial^2 c^c}$.

Differentiating (5.14) for c^c and α we find that $\partial^2 JU/\partial c^c \partial \alpha = -q\left(c\right)$ and $\partial^2 JU/\partial^2 c^c = S''\left(c\right) - \alpha q'\left(c\right)$. Given that by the second order conditions the latter is negative then it follows that

$$\frac{dc^c}{d\alpha} = -\frac{-q\left(c\right)}{S''\left(c\right) - \alpha q'\left(c\right)} < 0 \tag{5.20}$$

i.e., the higher is the providers' altruism level the lower is the collusive cost.

Utility Payoffs

Substituting the optimal cost strategies on the utility function the utility payoff earned by provider i under collusion is given by,

$$U^c = \left(c_{-i}^c - c_i^c\right) q\left(c_{-i}^c\right) + S\left(c_i^c\right) + \alpha \int_{c_{-i}^c}^{\infty} q(x)\, dx \tag{5.21}$$

In the symmetric case for homogeneous providers i.e. $\alpha_1 = \alpha_2 = \alpha$, the utility payoff is given by,

$$U^c = S\left(c^c\right) + \alpha \int_{c^c}^{\infty} q(x)\, dx \tag{5.22}$$

5.5 B5 Defection Scenario

Proof of Proposition 4

The defecting provider i will revert to behaving accordingly to the best response function as in (5.9)

$$S'\left(c_i\right) - q\left(c_{-i}^c\right) = 0 \tag{5.23}$$

In the non-cooperative scenario we know that $-q\left(c^{nc}\right) + S'\left(c^{nc}\right) = 0 \Rightarrow S'\left(c^{nc}\right) - S'\left(c_i^d\right) = q\left(c^{nc}\right) - q\left(c_i^c\right)$. With $c_i^c > c^{nc}$ and $q' < 0$ then $q\left(c^{nc}\right) - q\left(c_i^c\right) > 0 \Rightarrow S'\left(c^{nc}\right) - S'\left(c_i^d\right) > 0$. As $S''(.) < 0 \Rightarrow c^{nc} < c_i^d$.

consider now

$$\begin{cases} S'\left(c_1^d\right) - q\left(c_2^c\right) = 0 \\ S'\left(c_2^d\right) - q\left(c_1^c\right) = 0 \end{cases} \Rightarrow -q\left(c_2^c\right) + q\left(c_1^c\right) + S'\left(c_1^d\right) - S'\left(c_2^d\right) = 0$$

Given $q'\left(c_i\right) < 0$, it follows that $-q\left(c_2^c\right) + q\left(c_1^c\right) > 0$, then it must be $S'\left(c_1^d\right) - S'\left(c_2^d\right) < 0$. As $S''(.) < 0$ it follows that $c_2^d < c_1^d$ must hold true.

Consider equation (5.23) evaluated at c_{-i}^c and c_i^c and compare it with the FOC in collusion (5.14). For $i = 1$, the last two terms in (5.14) are positive, thus it must be $S'\left(c_i^c\right) - q\left(c_{-i}^c\right) < 0$ for the FOC to be satisfied. It follows that the cost chosen in defection is less than the cooperation cost. Thus the defection cost falls within, $c_1^{nc} < c_1^d < c_1^c$.

With regards to provider 2, rewrite FOC (5.14) as:

$$\frac{\partial JU}{\partial c_1} \equiv -\alpha_2 q\left(c_1\right) + S'\left(c_1\right) + \left(c_1 - c_2\right) q'\left(c_1\right) + q\left(c_1\right) - q\left(c_2\right) = 0$$

$$\frac{\partial JU}{\partial c_2} \equiv -\alpha_1 q\left(c_2\right) + S'\left(c_2\right) + \left(c_2 - c_1\right) q'\left(c_2\right) + q\left(c_2\right) - q\left(c_1\right) = 0$$

As $c_2 > c_1$ and $q'(.) < 0$ then $\left(c_1 - c_2\right) q'\left(c_1\right) + q\left(c_1\right) + q\left(c_2\right) > 0$. For $\partial JU / \partial c_1 = 0$ to be verified it must be that $-\alpha_2 q\left(c_1\right) + S'\left(c_1\right) < 0$. Analogously, as $\left(c_2 - c_1\right) q'\left(c_2\right) + q\left(c_2\right) - q\left(c_1\right) < 0$. For $\partial JU / \partial c_2 = 0$ then $-\alpha_1 q\left(c_2\right) + S'\left(c_2\right) > 0$. As $\alpha_2 \leq 1$ then $-\alpha_2 q\left(c_1\right) + S'\left(c_1\right) < 0 \Leftrightarrow S'\left(c_1\right) < q\left(c_1\right)$. Also as $S'(.)$ is decreasing for $c_2 > c_1 \Rightarrow S'\left(c_2\right) < S'\left(c_1\right) < q\left(c_1\right)$. Therefore if $S'\left(c_2^c\right) - q\left(c_1^c\right) < 0 \Rightarrow c_2^d < c_2^d$.

It is now possible to rank all the costs under the yardstick regulation, $c_2^* \leq c_1^* \leq c_1^{nc} = c_2^{nc} \leq c_2^d \leq c_1^d \leq c_1^c \leq c_2^c$

Comparative Statics

Recall from (5.23) that the optimal costs must satisfy the FOCs:

$$\frac{\partial U_i}{\partial c_i} = S'\left(c_i^d\right) - q\left(c_{-i}^c\right) = 0 \quad \forall \alpha_i \in [0,1], i = (1,2)$$

let,

$F_1 \equiv \frac{\partial U_1}{\partial c_1} = -q\left(c_2^c\right) + S'\left(c_1^d\right) = 0$
$F_2 \equiv \frac{\partial U_2}{\partial c_2} = -q\left(c_1^c\right) + S'\left(c_2^d\right) = 0$

We know from equation A.3.9 that $\partial c_i^c / \partial \alpha_i \leq 0$ so total differentiation leads to:

$$\begin{cases} \frac{\partial F_1}{\partial c_1} dc_1 + \frac{\partial F_1}{\partial c_2} dc_2 + \frac{\partial F_1}{\partial \alpha_1} d\alpha_1 + \frac{\partial F_1}{\partial \alpha_2} d\alpha_2 & = 0 \\ \frac{\partial F_2}{\partial c_1} dc_1 + \frac{\partial F_2}{\partial c_2} dc_2 + \frac{\partial F_2}{\partial \alpha_2} d\alpha_1 + \frac{\partial F_2}{\partial \alpha_2} d\alpha_2 & = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{\partial F_1}{\partial c_1} \frac{dc_1}{d\alpha_1} + \frac{\partial F_1}{\partial c_2} \frac{dc_2}{d\alpha_1} = -\frac{\partial F_1}{\partial \alpha_1} \\ \frac{\partial F_2}{\partial c_1} \frac{dc_1}{d\alpha_1} + \frac{\partial F_2}{\partial c_2} \frac{dc_2}{d\alpha_1} = -\frac{\partial F_2}{\partial \alpha_1} \end{cases}$$

In matrix format

$$\begin{bmatrix} \partial F_1/\partial c_1 & \partial F_1/\partial c_2 \\ \partial F_2/\partial c_1 & \partial F_2/\partial c_2 \end{bmatrix} \begin{bmatrix} \partial c_1/\partial \alpha_1 \\ \partial c_2/\partial \alpha_1 \end{bmatrix} = \begin{bmatrix} -\partial F_1/\partial \alpha_1 \\ -\partial F_2/\partial \alpha_1 \end{bmatrix}$$

Using Cramer's rule we obtain: $\frac{\partial c_i}{\partial \alpha_1} = \frac{A_1}{j}; \frac{\partial p_i}{\partial \alpha_1} = \frac{A_2}{j}$

Where:

$$J = \begin{bmatrix} \partial F_1/\partial c_1 & \partial F_1/\partial c_2 \\ \partial F_2/\partial c_{1i} & \partial F_2/\partial c_2 \end{bmatrix};\ A_1 = \begin{bmatrix} -\partial F_1/\partial \alpha_1 & \partial F_1/\partial c_2 \\ -\partial F_2/\partial \alpha_1 & \partial F_2/\partial c_{2i} \end{bmatrix};\ A_2 = \begin{bmatrix} \partial F_1/\partial c_1 & -\partial F_1/\partial \alpha_1 \\ \partial F_2/\partial c_1 & -\partial F_2/\partial \alpha_1 \end{bmatrix}$$

Therefore computing the partial derivatives in A_1, A_2 and J:

$\partial F_1/\partial c_1 = S''\left(c_1^d\right) < 0$
$\partial F_1/\partial c_2 = -q'\left(c_2^c\right) > 0$
$\partial F_1/\partial \alpha_i = -q'\left(c_2^c\right) \frac{\partial c_2^c}{\partial \alpha_1}$
$\partial F_2/\partial c_1 = -q'\left(c_1^c\right) > 0$
$\partial F_2/\partial c_2 = S''\left(c_2^d\right) < 0$
$\partial F_2/\partial \alpha_i = -q'\left(c_1^c\right) \frac{\partial c_1^c}{\partial \alpha_1}$

With $J > 0$ the sign of $\partial c_1/\partial \alpha_i$,$\partial c_2/\partial \alpha_i$ will be determined by the sign of A_1 and A_2.

$A_1 = \left(-\partial F_1/\partial \alpha_i\right)\left(\partial F_1/\partial c_2\right) - \left(\partial F_1/\partial c_2\right)\left(-\partial F_2/\partial \alpha_i\right) < 0$
$A_2 = \left(\partial F_1/\partial c_1\right)\left(-\partial F_2/\partial \alpha_i\right) - \left(-\partial F_1/\partial \alpha_i\right)\left(\partial F_2/\partial c_1\right) < 0$

Therefore it follows that:

$$\partial c_1^d/\partial \alpha_i < 0 \ \partial c_2^d/\partial \alpha_i < 0 \tag{5.24}$$

Utility payoffs

The utility payoff provider i earns in defection is given by:

$$U^d = \left(c_{-i}^c - c_i^d\right) q\left(c_{-i}^c\right) + S\left(c_i^d\right) + \alpha_i \int_{c_{-i}^c}^{\infty} q(x)\, dx \tag{5.25}$$

When the providers have the same level of altruism, i.e. $\alpha_i = \alpha_{-i} = \alpha$, by symmetry the cooperative solution is such that $c_i^c = c_{-i}^c = c^c$ and the defection cost $c_i^d = c_{-i}^d = c^d$ is the solution to $S'\left(c^d\right) - q\left(c^c\right) = 0$. If we evaluate this at $c^c = c$ and compare it with

FOC (5.19a), we obtain $S'(c^c) - q(c^c) \leq 0$. As $S'(.)$ is a decreasing function, therefore $c^d < c^c$. Also $S'(c^*) - q(c^c) > 0$, then $c^d > c^*$.

The utility payoff both providers earn is given by:

$$U^d = \left(c^c - c^d\right) q(c^c) + S\left(c^d\right) + \alpha \int_{c^c}^{\infty} q(x)\, dx \tag{5.26}$$

5.6 B6 Payoff Utility Ranking

RANK OF UTILITY PAYOFFS UNDER THE DIFFERENT SCENARIOS

Symmetric altruism

Suppose $\alpha_1 = \alpha_2 = 0$ the payoff functions at the optimum are:

$$U_i^{nc}|_{\alpha_i=0} = S(c^{nc}) \tag{5.27}$$

$$U_i^{c}|_{\alpha_i=0} = S(c^{c}) \tag{5.28}$$

$$U_i^{d}|_{\alpha_i=0} = (c_{-i}^{c} - c_i^{d})q(c_i^{c}) + S(c^{d}) \tag{5.29}$$

Therefore it follows that: $U_i^{nc} \leq U_i^{c} \leq U_i^{d}$.

Now we consider the case in which $\alpha_i = \alpha_{-i} = 1$. We have shown before if $\alpha_i \to 1$ then $c_i^c \to c_i^{nc}$ (see proof of Proposition 3) and at c_i^{nc} there are no profitable deviations. Therefore it follows that:

$$U_i^{c} = U_i^{nc} = U_i^{d} = S(c_i^{nc}) + \int_{c_{-i}^{nc}}^{\infty} q(x)dx \tag{5.30}$$

Assymmetric altruism

Case 1: $\alpha_1 = 0$ and $\alpha_2 = 1$

The utility payoffs for these levels of altruism are:

$$U_1^{nc}|_{\alpha_1=0} = S(c^{nc}) \tag{5.31}$$

$$U_1^{c}|_{\alpha_1=0} = (c_2^{c} - c_1^{c})q(c_2^{c}) + S(c_1^{c}) \tag{5.32}$$

$$U_1^{d}|_{\alpha_1=0} = (c_2^{c} - c_1^{d})q(c_2^{c}) + S(c_1^{d}) \tag{5.33}$$

$$U_2^{nc}|_{\alpha_2=1} = S(c^{nc}) + \int_{c^{nc}}^{\infty} q(x)dx \tag{5.34}$$

$$U_2^{c}|_{\alpha_2=1} = (c_1^{c} - c_2^{c})q(c_1^{c}) + S(c_2^{c}) + \int_{c_1^{c}}^{\infty} q(x)dx \tag{5.35}$$

$$U_2^{d}|_{\alpha_2=1} = (c_1^{c} - c_2^{d})q(c_1^{c}) + S(c_2^{d}) + \int_{c_1^{c}}^{\infty} q(x)dx \tag{5.36}$$

Since $c_1^{nc} < c_1^{c} < c_2^{c}$ then $S(c_1^{c}) > S(c_1^{nc})$ and $(c_2^{c} - c_1^{c})q(c_2^{c}) > 0$, implying that $U_1^{nc} < U_1^{c}$.

Since $c_1^{nc} < c_1^d < c_2^c$ analysing (5.32) and (5.33) it follows that $U_1^c < U_1^d$. Therefore: $U_1^{nc} < U_1^c < U_1^d$

Subtracting equation (5.34) to (5.35) we obtain:

$U_2^{nc}|_{\alpha_2=1} - U_2^c|_{\alpha_2=1} = -(c_1^c - c_2^c)q(c_1^c) + (S(c^{nc}) - S(c_2^c)) + \int_{c_1^{nc}}^{\infty} q(x)dx - \int_{c_1^c}^{\infty} q(x)dx$

Recall that $c_1^{cn} = c_2^{cn} < c_2^d < c_1^c < c_2^c$. The term$-(c_1^c - c_2^c)q(c_1^c) > 0$ is positive for $\alpha_2 > \alpha_1$. Furthermore since $c^{cn} < c^c$ (see proof of Proposition 3) the term $\int_{c_1^{nc}}^{\infty} q(x)dx - \int_{c_1^c}^{\infty} q(x)dx > 0$ is positive since the consumer surplus is bigger in the non-cooperative game. Finally, since $c_2^{nc} < c_2^c$ the term $(S(c^{nc}) - S(c_2^c)) < 0$ is negative since the slackness is bigger in the cooperative than in the non-cooperative scenario.

Therefore it follows that $U_2^{nc} < U_2^c$ if and only if:

$$(S(c_2^{nc}) - S(c_2^c)) \leq (c_1^c - c_2^c)q(c_1^c) - \int_{c_1^{nc}}^{\infty} q(x)dx + \int_{c_1^c}^{\infty} q(x)dx \tag{5.37}$$

A more general formulation of equation (5.37) for $0 \leq \alpha_2 \leq 1$ is

$$(S(c_2^{nc}) - S(c_2^c)) \leq (c_1^c - c_2^c)q(c_1^c) - \alpha_2 \int_{c_1^{nc}}^{\infty} q(x)dx - +\alpha_2 \int_{c_1^c}^{\infty} q(x)dx \tag{5.38}$$

Therefore for firm 2 cooperation is profitable only if we assume that the benefit from slack is big enough to offset the financial loose and the decrease in consumer surplus that more altruistic firms have to bear in cooperation. So $U_2^c > U_2^{nc}$, only if equation (A6.115.37) holds, otherwise $U_2^c < U_2^{nc}$.

For the less altruistic firm, i.e. firm 1 condition (5.37) is easier to hold since $(c_2^c - c_1^c)q(c_2^c) > 0$.

5.7 B7 Proof of proposition 5 and corollary 3

Let δ^* denote the discount rate above which collusion is sustainable such that:

$$\delta_i \geq \frac{(U_i^d - U_i^c)}{(U_i^d - U_i^{nc})} \qquad \delta_{-i} \geq \frac{(U_{-i}^d - U_{-i}^c)}{(U_{-i}^d - U_{-i}^{nc})}$$

Consider firm i. In order to understand how the each firm collusive behaviour varies with the level of altruism we need to assess the $\Delta\delta_i$ given $\Delta\alpha_i > 0$. Consider the payoff utilities:

$$U_i^{nc} = S(c_i^{nc}) + \alpha_i \int_{c_{-i}^{nc}}^{\infty} q(x)dx \tag{5.39}$$

$$U_i^c = (c_{-i}^c - c_i^c)q(c_{-i}^c) + S(c_i^c) + \alpha_i \int_{c_{-i}^c}^{\infty} q(x)dx \tag{5.40}$$

$$U_i^d = (c_{-i}^c - c_i^d)q(c_{-i}^c) + S(c_i^d) + \alpha_i \int_{c_{-i}^c}^{\infty} q(x)dx \tag{5.41}$$

Derivation with respect to α_i:

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$$\frac{\partial U_i^{nc}}{\partial \alpha_i} = \int_{c_{-i}^{nc}}^{\infty} q(x)dx \tag{5.42}$$

$$\frac{\partial U_i^c}{\partial \alpha_i} = \left(\frac{\partial c_{-i}^c}{\partial \alpha_i} - \frac{\partial c_i^c}{\partial \alpha_i}\right) q(c_{-i}^c) + (c_{-i}^c - c_i^c)q'(c_{-i}^c)\frac{\partial c_{-i}^c}{\partial \alpha_i} + S'(c_i^c)\frac{\partial c_i^c}{\partial \alpha_i} + \int_{c_{-i}^c}^{\infty} q(x)dx - \alpha_i q(c_{-i}^c)\frac{\partial c_{-i}^c}{\partial \alpha_i} \tag{5.43}$$

$$\frac{\partial U_i^d}{\partial \alpha_i} = \left(\frac{\partial c_{-i}^c}{\partial \alpha_i} - \frac{\partial c_i^d}{\partial \alpha_i}\right) q(c_{-i}^c) + (c_{-i}^c - c_i^d)q'(c_{-i}^c)\frac{\partial c_{-i}^c}{\partial \alpha_i} + S'(c_i^d)\frac{\partial c_i^d}{\partial \alpha_i} + \int_{c_{-i}^c}^{\infty} q(x)dx - \alpha_i q(c_{-i}^c)\frac{\partial c_{-i}^c}{\partial \alpha_i} \tag{5.44}$$

Consider for now the symmetric case $\alpha_i = \alpha_{-i} = 0$. As seen above the payoff utilities can be ranked as: $U_i^{nc} \leq U_i^c \leq U_i^d$ (see Graph 1).

Now consider the case for which $\alpha_i = \alpha_{-i} = 1$, as we have shown before if $\alpha_i \to 1 \Rightarrow c_i^c \to c_i^{nc}$ implying$U_i^c = U_i^{nc} = U_i^d$.

Therefore (5.42), (5.43) and (5.44) cam be written as:

$$\frac{\partial U_i^{nc}}{\partial \alpha_i} = \int_{c_{-i}^{nc}}^{\infty} q(x)dx \tag{5.45}$$

$$\frac{\partial U_i^c}{\partial \alpha_i} = \left(\frac{\partial c_{-i}^c}{\partial \alpha_i} - \alpha_i \frac{\partial c_{-i}^c}{\partial \alpha_i}\right) q(c_{-i}^c) + (c_{-i}^c - c_i^c)q'(c_{-i}^c)\frac{\partial c_{-i}^c}{\partial \alpha_i} + \frac{\partial c_i^c}{\partial \alpha_i}\left(S'(c_i^c) + q(c_{-i}^c)\right) + \int_{c_{-i}^c}^{\infty} q(x)dx \tag{5.46}$$

$$\frac{\partial U_i^d}{\partial \alpha_i} = \left(\frac{\partial c_{-i}^c}{\partial \alpha_i} - \alpha_i \frac{\partial c_{-i}^c}{\partial \alpha_i}\right) q(c_{-i}^c) + (c_{-i}^c - c_i^c) q'(c_{-i}^c) \frac{\partial c_{-i}^c}{\partial \alpha_i} + \frac{\partial c_i^d}{\partial \alpha_i}\left(S'(c_i^d) + q(c_{-i}^d)\right) + \int_{c_{-i}^c}^{\infty} q(x)dx \tag{5.47}$$

Consider (5.45) and (5.46). We know that for $\alpha_i = 0$, $U_i^{nc} < U_i^c$ also that $\alpha_i \to 1 \Rightarrow U_i^c \to U_i^{nc}$ therefore it must hold that $\partial U_i^c/\partial \alpha_i \leq \partial U_i^{nc}/\partial \alpha_i$ meaning that U_i^{nc} grows faster than U_i^c. Therefore since for $\alpha_i \to 1 \Rightarrow c_i^c \to c_i^{nc}$ it follows that $\partial U_i^c/\partial \alpha_i \to \partial U_i^{nc})/\partial \alpha_i$ (see Graph 1). If U_i^{nc} increases faster than U_i^c the it follows that δ_i increases with α_i, i.e. for firm i cooperation becomes more difficult for higher altruism levels.

Consider now the impact of $\Delta\alpha_i > 0$ on δ_{-i}. Consider the payoff utilities for firm $-i$

$$U_{-i}^{nc} = S(c_{-i}^{nc}) + \alpha_{-i} \int_{c_i^{nc}}^{\infty} q(x)dx \tag{5.48}$$

$$U_{-i}^c = (c_i^c - c_{-i}^c) q(c_i^c) + S(c_{-i}^c) + \alpha_{-i} \int_{c_i^c}^{\infty} q(x)dx \tag{5.49}$$

$$U_{-i}^d = (c_i^c - c_{-i}^d) q(c_i^c) + S(c_{-i}^d) + \alpha_{-i} \int_{c_i^c}^{\infty} q(x)dx \tag{5.50}$$

Derivation with respect to α_i:

$$\frac{\partial U_{-i}^{nc}}{\partial \alpha_i} = 0 \tag{5.51}$$

$$\frac{\partial U_{-i}^c}{\partial \alpha_i} = \left(\frac{\partial c_i^c}{\partial \alpha_i} - \frac{\partial c_{-i}^c}{\partial \alpha_i}\right) q(c_i^c) + (c_i^c - c_{-i}^c) q'(c_i^c) \frac{\partial c_i^c}{\partial \alpha_i} + S'(c_{-i}^c) \frac{\partial c_{-i}^c}{\partial \alpha_i} - \alpha_i q(c_{-i}^c) \frac{\partial c_i^c}{\partial \alpha_i} \tag{5.52}$$

$$\frac{\partial U_i^d}{\partial \alpha_i} = \left(\frac{\partial c_i^c}{\partial \alpha_i} - \frac{\partial c_{-i}^d}{\partial \alpha_i}\right) q(c_i^c) + (c_i^c - c_{-i}^d) q'(c_i^c) \frac{\partial c_i^c}{\partial \alpha_i} + S'(c_{-i}^d) \frac{\partial c_{-i}^d}{\partial \alpha_i} - \alpha_i q(c_{-i}^c) \frac{\partial c_i^c}{\partial \alpha_i} \tag{5.53}$$

It is straightforward to see that $\partial U_{-i}^c/\partial \alpha_i \geq \partial U_{-i}^{nc}/\partial \alpha_i$, i.e. when the opponent's altruism increases the firm cooperation utility payoff also increases. Since $\partial U_{-i}^{nc}/\partial \alpha_i = 0$ it follows that δ_{-i} decreases with the opponent's altruism level.

Summarizing, we have shown that $\Delta\alpha_i > 0$leads to a decrease in δ_{-i} and an increase in δ_i. Since:

$$\delta^* = max\{\delta_i, \delta_{-i}\}$$

We need to assess $max\{\delta_i, \delta_{-i}\}$. If $\alpha_i = \alpha_{-i}$ we know that $\delta_i = \delta_{-i}$. Departing from the symmetric case, consider a positive increase in firm's 2 altruism level, i.e. $\Delta\alpha_2$. As shown above this will increase δ_2 and decrease δ_1, implying that $\delta_2 \geq \delta_1$.

Still, departing from the symmetric case, consider a decrease in firm's 1 altruism level, i.e. $\Delta\alpha_1 < 0$. As shown above this will increase δ_2 and decrease δ_1, implying that $\delta_2 \geq \delta_1$.

Therefore we conclude that $\delta^* = \delta_2$ implying that $\delta^*|_{\alpha_1=\alpha_2} \leq \delta^*|_{\alpha_1'<\alpha_2}$ for $\alpha_1 \geq \alpha_1'$ and $\delta^*|_{\alpha_1=\alpha_2} \leq \delta^*|_{\alpha_1<\alpha_2'}$ for $\alpha_2 \leq \alpha_2'$.

Note that departing from a symmetric case an increase in α_1 and a decrease in α_2 are not feasible since in our model $\alpha_1 \leq \alpha_2$.