PHYSICAL REVIEW D **85,** 126003 (2012)

# NUT-charged black holes in matter-coupled $\mathcal{N} = 2$, $D = 4$ gauged supergravity

Marta Colleoni[1] and Dietmar Klemm[1,2]

[1]*Dipartimento di Fisica, Università di Milano, Via Celoria 16, I-20133 Milano, Italy*

[2]*INFN, Sezione di Milano, Via Celoria 16, I-20133 Milano, Italy*

(Received 3 April 2012; published 11 June 2012)

Using the results of Cacciatori, Klemm, Mansi, and Zorzan [J. High Energy Phys. 05 (2008) 097], where all timelike supersymmetric backgrounds of $\mathcal{N} = 2$, $D = 4$ matter-coupled supergravity with Fayet-Iliopoulos gauging were classified, we construct genuine NUT-charged BPS black holes in anti-de Sitter$_4$ with nonconstant moduli. The calculations are exemplified for the SU(1, 1)/U(1) model with prepotential $F = -iX^0X^1$. The resulting supersymmetric black holes have a hyperbolic horizon and carry two electric, two magnetic, and one NUT charge, which are however not all independent, but are given in terms of three free parameters. We find that turning on a NUT charge lifts the flat directions in the effective black hole potential, such that the horizon values of the scalars are completely fixed by the charges. We also oxidize the solutions to 11 dimensions, and find that they generalize the geometry found in the work of Gauntlett, Kim, Pakis, and Waldram [Phys. Rev. D **65**, 026003 (2001)] corresponding to membranes wrapping holomorphic curves in a Calabi-Yau fivefold. Finally, a class of NUT-charged Nernst branes is constructed as well, but these have curvature singularities at the horizon.

DOI: 10.1103/PhysRevD.85.126003 PACS numbers: 04.70.Dy, 04.65.+e, 11.30.Pb

## I. INTRODUCTION

Black holes in anti-de Sitter (AdS) spaces provide an important test ground to address fundamental questions of quantum gravity like holography. These ideas originally emerged from string theory, but became then interesting in their own right, for instance, in recent applications to condensed matter physics (cf. [1] for a review), where black holes play again an essential role, since they provide the dual description of certain condensed matter systems at finite temperature. In particular, models of the type that we shall consider here, that contain Einstein gravity coupled to U(1) gauge fields and neutral scalars,[1] have been instrumental to study transitions from Fermi-liquid to non-Fermi-liquid behavior, cf. [2,3] and references therein.

On the other hand, among the extremal black holes (which have zero Hawking temperature), those preserving a sufficient amount of supersymmetry are of particular interest, as this allows (owing to nonrenormalization theorems) to extrapolate an entropy computation at weak string coupling (when the system is generically described by a bound state of strings and branes) to the strong-coupling regime, where a description in terms of a black hole is valid [4]. However, this picture, which has been essential for our current understanding of black hole microstates, might be modified in gauged supergravity (arising from flux compactifications) due to the presence of a potential for the moduli, generated by the fluxes. This could even lead to a stabilization of the dilaton, so that one cannot extrapolate between weak and strong coupling anymore. Obviously, the explicit knowledge of supersymmetric black hole solutions in AdS is a necessary ingredient if one wants to study this new scenario.

A first step in this direction was made in [5,6], where the first examples of extremal static or rotating BPS black holes in AdS$_4$ with nontrivial scalar field profiles were constructed. Thereby, essential use was made of the results of [7], where all supersymmetric backgrounds (with a timelike Killing spinor) of $\mathcal{N} = 2$, $D = 4$ matter-coupled supergravity with Fayet-Iliopoulos gauging were classified. This provides a systematic method to obtain BPS solutions, without the necessity to guess some suitable *Ansätze*. Perhaps one of the most important results of [5] was the construction of genuine static supersymmetric black holes with spherical symmetry in the stu model. A crucial ingredient for the existence of these solutions is the presence of nonconstant scalar fields. These black holes were then further studied and generalized in [8,9].

In this paper, we shall go one step further with respect to [5], and include also NUT charge. Apart from the supersymmetric Reissner-Nordström-Taub-NUT-AdS family in minimal gauged supergravity [10], there are, to the best of our knowledge, no other known BPS solutions of this type. In addition to providing an interesting scenario to study holography [11–13], these are intriguing for the following reason: In gauged supergravity, electric-magnetic duality invariance is obviously broken due to the minimal coupling of the gravitinos to the vector potential (unless one introduces also a magnetic gauging, but we shall not do this in what follows). Nevertheless, it was discovered in [10] that supersymmetric solutions of minimal gauged supergravity

[1]The necessity of a bulk U(1) gauge field arises, because a basic ingredient of realistic condensed matter systems is the presence of a finite density of charge carriers. A further step in modeling strongly coupled holographic systems is to include the leading relevant (scalar) operator in the dynamics. This is generically uncharged, and is dual to a neutral scalar field in the bulk.

 

still enjoy a sort of electric-magnetic duality invariance in which electric and magnetic charges and mass and NUT charge are rotated simultaneously. A deeper understanding of this mysterious duality might reveal unexpected geometric structures underlying gauged supergravity theories.

In addition to the motivation given above, a further reason for considering supersymmetric NUT-charged AdS black holes is the attractor mechanism [14–18], that has been the subject of extensive research in the asymptotically flat case, but for which only little is known for spacetimes that asymptote to AdS. First steps toward a systematic analysis of the attractor flow in gauged supergravity were made in [19,20] for the non-BPS and in [5,9,21,22] for the BPS case, but it would be interesting to generalize these results to include also NUT charge. In fact, what we shall find here is that (at least for the simple prepotential considered below) the flat directions in the effective black hole potential (which generically occur in the BPS flow in gauged supergravity [5]) are lifted by turning on a NUT charge.

The remainder of this paper is organized as follows: In the next section, we briefly review $\mathcal{N}=2, D=4$ gauged supergravity coupled to Abelian vector multiplets [presence of U(1) Fayet-Iliopoulos terms] and give the general recipe to construct supersymmetric solutions found in [7]. In Sec. III, the equations of [7] are solved for the SU(1, 1)/U(1) model with prepotential $F=-iX^0X^1$, and a class of one-quarter BPS black holes carrying two electric, two magnetic, and one NUT charge is constructed. We also discuss the attractor mechanism for this solution and its near-horizon limit. Moreover, it is shown how the results of [10] are recovered in the case of constant moduli. In Sec. IV, we oxidize the solution to 11 dimensions and comment on its M-theory interpretation. Section V contains our conclusions and some final remarks.

## II. THE SUPERSYMMETRIC BACKGROUNDS OF $\mathcal{N}$ = 2, $D$ = 4 GAUGED SUPERGRAVITY

We consider $\mathcal{N}=2$, $D=4$ gauged supergravity coupled to $n_V$ Abelian vector multiplets [23].[2] Apart from the vierbein $e^a_\mu$, the bosonic field content includes the vectors $A^I_\mu$ enumerated by $I=0,\ldots,n_V$, and the complex scalars $z^\alpha$ where $\alpha=1,\ldots,n_V$. These scalars parametrize a special Kähler manifold, i.e., an $n_V$-dimensional Hodge-Kähler manifold that is the base of a symplectic bundle, with the covariantly holomorphic sections

$$\mathcal{V}=\begin{pmatrix} X^I \\ F_I \end{pmatrix}, \quad \mathcal{D}_{\bar\alpha}\mathcal{V}=\partial_{\bar\alpha}\mathcal{V}-\tfrac{1}{2}(\partial_{\bar\alpha}\mathcal{K})\mathcal{V}=0, \tag{2.1}$$

where $\mathcal{K}$ is the Kähler potential and $\mathcal{D}$ denotes the Kähler-covariant derivative. $\mathcal{V}$ obeys the symplectic constraint

$$\langle \mathcal{V}, \bar{\mathcal{V}}\rangle = X^I\bar F_I - F_I\bar X^I = i. \tag{2.2}$$

To solve this condition, one defines

$$\mathcal{V} = e^{\mathcal{K}(z,\bar z)/2}v(z), \tag{2.3}$$

where $v(z)$ is a holomorphic symplectic vector,

$$v(z)=\begin{pmatrix} Z^I(z) \\ \frac{\partial}{\partial Z^I}F(Z)\end{pmatrix}. \tag{2.4}$$

$F$ is a homogeneous function of degree two, called the prepotential, whose existence is assumed to obtain the last expression. The Kähler potential is then

$$e^{-\mathcal{K}(z,\bar z)} = -i\langle v, \bar v\rangle. \tag{2.5}$$

The matrix $\mathcal{N}_{IJ}$ determining the coupling between the scalars $z^\alpha$ and the vectors $A^I_\mu$ is defined by the relations

$$F_I=\mathcal{N}_{IJ}X^J, \qquad \mathcal{D}_{\bar\alpha}\bar F_I=\mathcal{N}_{IJ}\mathcal{D}_{\bar\alpha}\bar X^J. \tag{2.6}$$

The bosonic action reads

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm bos} &= \tfrac{1}{2}R+\tfrac{1}{4}({\rm Im}\mathcal{N})_{IJ}F^I_{\mu\nu}F^{J\mu\nu} \\ &\quad -\tfrac{1}{8}({\rm Re}\mathcal{N})_{IJ}e^{-1}\epsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}F^J_{\rho\sigma} \\ &\quad - g_{\alpha\bar\beta}\partial_\mu z^\alpha\partial^\mu\bar z^{\bar\beta}-V, \end{aligned} \tag{2.7}$$

with the scalar potential

$$V=-2g^2\xi_I\xi_J[({\rm Im}\mathcal{N})^{-1|IJ}+8\bar X^IX^J], \tag{2.8}$$

that results from U(1) Fayet-Iliopoulos gauging. Here, $g$ denotes the gauge coupling and the $\xi_I$ are constants. In what follows, we define $g_I=g\xi_I$.

The most general timelike supersymmetric background of the theory described above was constructed in [7], and is given by

$$ds^2=-4|b|^2(dt+\sigma)^2+|b|^{-2}(dz^2+e^{2\Phi}dwd\bar w), \tag{2.9}$$

where the complex function $b(z,w,\bar w)$, the real function $\Phi(z,w,\bar w)$, and the one-form $\sigma=\sigma_w dw+\sigma_{\bar w}d\bar w$, together with the symplectic section (2.1),[3] are determined by the equations

$$\partial_z\Phi = 2ig_I\left(\frac{\bar X^I}{b}-\frac{X^I}{\bar b}\right), \tag{2.10}$$

$$\begin{aligned} &4\partial\bar\partial\left(\frac{X^I}{\bar b}-\frac{\bar X^I}{b}\right)+\partial_z\left[e^{2\Phi}\partial_z\left(\frac{X^I}{\bar b}-\frac{\bar X^I}{b}\right)\right] \\ &\quad -2ig_J\partial_z\left\{e^{2\Phi}\left[|b|^{-2}({\rm Im}\mathcal{N})^{-1|IJ}\right.\right. \\ &\quad \left.\left.+2\left(\frac{X^I}{\bar b}+\frac{\bar X^I}{b}\right)\left(\frac{X^J}{\bar b}+\frac{\bar X^J}{b}\right)\right]\right\}=0, \end{aligned} \tag{2.11}$$

[2] Throughout this paper, we use the notations and conventions of [24].

[3] Note that also $\sigma$ and $\mathcal{V}$ are independent of $t$.

$$4\partial\bar\partial\left(\frac{F_I}{\bar b}-\frac{\bar F_I}{b}\right)+\partial_z\left[e^{2\Phi}\partial_z\left(\frac{F_I}{\bar b}-\frac{\bar F_I}{b}\right)\right] - 2ig_J\partial_z\left\{e^{2\Phi}\left[|b|^{-2}\,\mathrm{Re}\mathcal{N}_{IL}\,(\mathrm{Im}\mathcal{N})^{-1|JL} + 2\left(\frac{F_I}{\bar b}+\frac{\bar F_I}{b}\right)\left(\frac{X^J}{\bar b}+\frac{\bar X^J}{b}\right)\right]\right\} - 8ig_Ie^{2\Phi}\left[\langle\mathcal{I},\partial_z\mathcal{I}\rangle-\frac{g_J}{|b|^2}\left(\frac{X^J}{\bar b}+\frac{\bar X^J}{b}\right)\right]=0, \tag{2.12}$$

$$2\partial\bar\partial\Phi = e^{2\Phi}\left[ig_I\partial_z\left(\frac{X^I}{\bar b}-\frac{\bar X^I}{b}\right)+\frac{2}{|b|^2}g_Ig_J\,(\mathrm{Im}\mathcal{N})^{-1|IJ} + 4\left(\frac{g_IX^I}{\bar b}+\frac{g_I\bar X^I}{b}\right)^2\right], \tag{2.13}$$

$$d\sigma+2\star^{(3)}\langle\mathcal{I},d\mathcal{I}\rangle-\frac{i}{|b|^2}g_I\left(\frac{\bar X^I}{b}+\frac{X^I}{\bar b}\right)e^{2\Phi}dw\wedge d\bar w=0. \tag{2.14}$$

Here, $\star^{(3)}$ is the Hodge star on the three-dimensional base with metric[4]

$$ds_3^2=dz^2+e^{2\Phi}dwd\bar w, \tag{2.15}$$

and we defined $\partial=\partial_w$, $\bar\partial=\partial_{\bar w}$, as well as

$$\mathcal{I}=\mathrm{Im}(\mathcal{V}/\bar b). \tag{2.16}$$

Given $b$, $\Phi$, $\sigma$, and $\mathcal{V}$, the fluxes read

$$\begin{aligned}F^I={}&2(dt+\sigma)\wedge d[bX^I+\bar b\bar X^I]+|b|^{-2}dz\wedge d\bar w[\bar X^I(\bar\partial\bar b+iA_{\bar w}\bar b)+(\mathcal{D}_\alpha X^I)b\bar\partial z^\alpha-X^I(\bar\partial b-iA_{\bar w}b)-(\mathcal{D}_{\bar\alpha}\bar X^I)\bar b\bar\partial\bar z^{\bar\alpha}]\\&-|b|^{-2}dz\wedge dw[\bar X^I(\partial\bar b+iA_w\bar b)+(\mathcal{D}_\alpha X^I)b\partial z^\alpha-X^I(\partial b-iA_wb)-(\mathcal{D}_{\bar\alpha}\bar X^I)\bar b\partial\bar z^{\bar\alpha}]\\&-\tfrac12|b|^{-2}e^{2\Phi}dw\wedge d\bar w[\bar X^I(\partial_z\bar b+iA_z\bar b)+(\mathcal{D}_\alpha X^I)b\partial_zz^\alpha-X^I(\partial_zb-iA_zb)-(\mathcal{D}_{\bar\alpha}\bar X^I)\bar b\partial_z\bar z^{\bar\alpha}-2ig_J(\mathrm{Im}\mathcal{N})^{-1|IJ}].\end{aligned} \tag{2.17}$$

In (2.17), $A_\mu$ is the gauge field of the Kähler U(1),

$$A_\mu=-\frac{i}{2}(\partial_\alpha\mathcal{K}\partial_\mu z^\alpha-\partial_{\bar\alpha}\mathcal{K}\partial_\mu\bar z^{\bar\alpha}). \tag{2.18}$$

## III. CONSTRUCTING NUT-CHARGED BLACK HOLES

In this section, we shall obtain supersymmetric NUT-charged black holes, which have nontrivial moduli turned on. In order to solve the system (2.10), (2.11), (2.12), (2.13), and (2.14), we shall assume that both $z^\alpha$ and $b$ depend on the coordinate $z$ only, and use the separation *Ansatz* $\Phi=\psi(z)+\gamma(w,\bar w)$. Then, (2.10) becomes

$$\psi'=2i\left(\frac{\bar X}{b}-\frac{X}{\bar b}\right), \tag{3.1}$$

where a prime denotes differentiation with respect to $z$ and $X\equiv g_IX^I$. Furthermore, we can integrate (2.11) once, with the result

$$e^{2\psi}\partial_z\left(\frac{X^I}{\bar b}-\frac{\bar X^I}{b}\right)-2ie^{2\psi}\left[|b|^{-2}(\mathrm{Im}\mathcal{N})^{-1|IJ}g_J + 2\left(\frac{X^I}{\bar b}+\frac{\bar X^I}{b}\right)\left(\frac{X}{\bar b}+\frac{\bar X}{b}\right)\right]=-4\pi ip^I, \tag{3.2}$$

where $p^I$ are related to the magnetic charges, as we shall see later. Using the contraction of (3.2) with $g_I$, (2.13) boils down to

$$-4\partial\bar\partial\gamma=\kappa e^{2\gamma},\qquad \kappa=-8\pi g_Ip^I. \tag{3.3}$$

This is the Liouville equation, which implies that the metric $e^{2\gamma}dwd\bar w$ has constant curvature $\kappa$, determined by the $p^I$.

### A. $SU(1,1)/U(1)$ model

In what follows, we shall specialize to the $SU(1,1)/U(1)$ model with prepotential $F=-iX^0X^1$, that has $n_V=1$ (one vector multiplet), and thus just one complex scalar $\tau$. Choosing $Z^0=1$, $Z^1=\tau$, the symplectic vector $v$ becomes

$$v=\begin{pmatrix}1\\ \tau\\ -i\tau\\ -i\end{pmatrix}. \tag{3.4}$$

The Kähler potential, metric, and kinetic matrix for the vectors are given, respectively, by

$$e^{-\mathcal{K}}=2(\tau+\bar\tau),\qquad g_{\tau\bar\tau}=\partial_\tau\partial_{\bar\tau}\mathcal{K}=(\tau+\bar\tau)^{-2}, \tag{3.5}$$

$$\mathcal{N}=\begin{pmatrix}-i\tau & 0\\ 0 & -\frac{i}{\tau}\end{pmatrix}. \tag{3.6}$$

Note that positivity of the kinetic terms in the action requires $\mathrm{Re}\tau>0$. For the scalar potential, one obtains

$$V=-\frac{4}{\tau+\bar\tau}(g_0^2+2g_0g_1\tau+2g_0g_1\bar\tau+g_1^2\tau\bar\tau), \tag{3.7}$$

which has an extremum at $\tau=\bar\tau=|g_0/g_1|$. In what follows, we assume $g_I>0$. Notice also that $F_I=-i\eta_{IJ}X^J$, where

[4]Whereas in the ungauged case, this base space is flat and thus has trivial holonomy, here we have U(1) holonomy with torsion [7].

$$\eta_{IJ} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{3.8}$$

Moreover, $(\mathrm{Im}\mathcal{N})^{-1} = -4\mathrm{diag}(|X^0|^2, |X^1|^2)$.

For this model, (2.12) becomes

$$\partial_z\Big[e^{2\psi}(-2i\eta_{IJ})\partial_z \mathrm{Re}\Big(\frac{X^J}{\bar b}\Big)\Big] - 2i\partial_z\Big\{e^{2\psi}\Big[|b|^{-2}\,\mathrm{Re}\mathcal{N}_{IL}(\mathrm{Im}\mathcal{N})^{-1|JL}g_J + 8\,\mathrm{Re}\Big(\frac{F_I}{\bar b}\Big)\mathrm{Re}\Big(\frac{X}{\bar b}\Big)\Big]\Big\}$$
$$- 8ig_I e^{2\psi}\Big[-\frac{i}{2}\alpha_{KJ}\Big(\frac{\bar X^K}{b}\partial_z\frac{X^J}{\bar b} - \frac{X^K}{\bar b}\partial_z\frac{\bar X^J}{b}\Big) - |b|^{-2}\Big(\frac{X}{\bar b} + \frac{\bar X}{b}\Big)\Big] = 0. \tag{3.9}$$

We now make the *Ansatz*

$$\frac{X^I}{\bar b} = \frac{\alpha^I z + \beta^I}{Az^2 + Bz + C}, \tag{3.10}$$

where $A$, $B$, $C$, $\alpha^I$, and $\beta^I$ are complex constants.[5] Without loss of generality, we can take $A = 1$ and $B = iD$, with $D \in \mathbb{R}$, since we are free to shift $z \mapsto z - \mathrm{Re}B/2$. As a consequence, (3.1) reduces to

$$\psi' = 4\frac{\mathrm{Im}\alpha z^3 + z^2(\mathrm{Im}\beta - D\,\mathrm{Re}\alpha) - z(\mathrm{Im}(\bar\alpha C) + D\,\mathrm{Re}\beta) - \mathrm{Im}(\bar\beta C)}{z^4 + z^2(2\,\mathrm{Re}C + D^2) + 2Dz\,\mathrm{Im}C + |C|^2}, \tag{3.11}$$

with $\alpha \equiv g_I\alpha^I$, $\beta \equiv g_I\beta^I$. Inspired by minimal gauged supergravity [25], we choose

$$\mathrm{Im}\,\beta - D\,\mathrm{Re}\alpha = 0, \tag{3.12}$$

$$-4(\mathrm{Im}(\bar\alpha C) + D\,\mathrm{Re}\beta) = 2\,\mathrm{Im}\alpha(2\,\mathrm{Re}C + D^2), \tag{3.13}$$

$$-4\,\mathrm{Im}(\bar\beta C) = 2D\,\mathrm{Im}\alpha\,\mathrm{Im}C, \tag{3.14}$$

so that (3.11) simplifies to

$$\psi' = \frac{4z^3 + 2(2\,\mathrm{Re}C + D^2)z + 2D\,\mathrm{Im}C}{z^4 + z^2(2\,\mathrm{Re}C + D^2) + 2Dz\,\mathrm{Im}C + |C|^2}\,\mathrm{Im}\alpha, \tag{3.15}$$

which can be integrated once to give

$$\psi = \mathrm{Im}\alpha(\ln[z^4 + z^2(2\,\mathrm{Re}C + D^2) + 2Dz\,\mathrm{Im}C + |C|^2] + \ln\check C), \tag{3.16}$$

where $\check C$ denotes an integration constant that can be set to 1 without loss of generality by using the scaling symmetry $\psi \mapsto \psi - \ln\lambda$, $\gamma \mapsto \gamma + \ln\lambda$, $\kappa \mapsto \kappa/\lambda^2$, $p^I \mapsto p^I/\lambda^2$, with $\ln\lambda = \mathrm{Im}\alpha\ln\check C$, that leaves (3.1), (3.2), and (3.3) invariant.

In order to solve (3.2), we take into account that

$$|b|^{-2}(\mathrm{Im}\mathcal{N})^{-1|IJ}g_J = -4\frac{|X^I|^2}{|b|^2}g_I,$$

where there is of course no summation over $I$ on the right-hand side. Then, (3.2) becomes

$$-4\pi i p^I = [(z^2 + iDz + C)(z^2 - iDz + \bar C)]^{2\,\mathrm{Im}\alpha} \times\Big\{\Big[\frac{-\alpha^I z^2 - 2\beta^I z + \alpha^I C - \beta^I iD}{(z^2 + iDz + C)^2} + \frac{\bar\alpha^I z^2 + 2\bar\beta^I z - \bar\alpha^I\bar C - \bar\beta^I iD}{(z^2 - iDz + \bar C)^2}\Big]$$
$$- 2i\Big[-4g_I\Big|\frac{\alpha^I z + \beta^I}{z^2 + iDz + C}\Big|^2 + 8\,\mathrm{Re}\Big(\frac{\alpha^I z + \beta^I}{z^2 + iDz + C}\Big) \times \mathrm{Re}\Big(\frac{\alpha z + \beta}{z^2 + iDz + C}\Big)\Big]\Big\}. \tag{3.17}$$

In order to simplify the calculations further, we shall also take $\mathrm{Im}\alpha = 1/2$, so that (3.17) boils down to a sixth-order polynomial equation,

$$A_0 + A_1 z + A_2 z^2 + A_3 z^3 + A_4 z^4 + A_5 z^5 + A_6 z^6 = 0, \tag{3.18}$$

where $A_6 = 0$ iff

$$-\mathrm{Im}\alpha^I + 4g_I|\alpha^I|^2 - 8\,\mathrm{Re}\alpha\,\mathrm{Re}\alpha^I = 0, \tag{3.19}$$

and thus

$$\mathrm{Im}\,\alpha^I = \frac{1 \pm \sqrt{1 - 16g_I(4g_I\,\mathrm{Re}^2\alpha^I - 8\,\mathrm{Re}\alpha\,\mathrm{Re}\alpha^I)}}{8g_I}. \tag{3.20}$$

Using $\mathrm{Im}\alpha = 1/2$, and defining $8g_0\,\mathrm{Re}\alpha^0 \equiv x$, $8g_1\,\mathrm{Re}\alpha^1 \equiv y$, this yields

$$x^4 + y^4 - 8(x^2 + y^2) - 2x^2y^2 - 32xy = 0. \tag{3.21}$$

To proceed further, recall that

[5] Note that (3.10) generalizes the *Ansatz* used in [5] to obtain black holes without NUT charge.

$$\tau = \frac{Z^1}{Z^0} = \frac{X^1}{X^0} = \frac{\alpha^1 z + \beta^1}{\alpha^0 z + \beta^0}. \tag{3.22}$$

If we require that the scalar asymptotically approaches the AdS vacuum, that is, $\tau \to g_0/g_1$ for $z \to \infty$, we must have $\alpha^1/\alpha^0 = g_0/g_1$, and thus $x = y$. (3.21) implies then $x = 0$, hence $\mathrm{Re}\alpha^I = 0$. Plugging this into (3.20) gives[6]

$$\mathrm{Im}\,\alpha^I = \frac{1}{4g_I}. \tag{3.23}$$

(3.12) and (3.13) reduce, respectively, to

$$\mathrm{Im}\,\beta = 0, \qquad \mathrm{Re}\beta = -\frac{D}{4}, \tag{3.24}$$

implying

$$\beta = -\frac{D}{4}. \tag{3.25}$$

Using the above results, one finds that (3.14) is identically satisfied.

Let us go back to (3.18). Requiring $A_0 = 0$ leads to

$$-4\pi p^I = \frac{\mathrm{Re}C}{2g_I} + 8g_I|\beta^I|^2 + 2D\,\mathrm{Re}\beta^I, \tag{3.26}$$

which gives the magnetic charges in terms of some numerical constants. Note that the above equation, together with (3.25), implies

$$g_0 p^0 = g_1 p^1, \quad \kappa = -16\pi g_0 p^0 = -16\pi g_1 p^1. \tag{3.27}$$

Eventually, one finds that $A_6 = 0$ and $A_0 = 0$ are sufficient conditions for (3.18) to be satisfied.

We now turn to (2.12). After some lengthy calculations, one gets

$$e^{2\psi}\left[\langle \mathcal{I}, \partial_z \mathcal{I}\rangle - |b|^{-2}\left(\frac{X}{\bar{b}} + \frac{\bar{X}}{b}\right)\right] = -\frac{D}{16 g_0 g_1}, \tag{3.28}$$

and thus (2.12) can be integrated once to give

$$\begin{aligned} e^{2\psi}\partial_z\left[2i\,\mathrm{Im}\left(\frac{F_I}{\bar{b}}\right)\right] &- 2ig_J e^{2\psi}\left[|b|^{-2}\,\mathrm{Re}\mathcal{N}_{IL}(\mathrm{Im}\mathcal{N})^{-1|JL}\right. \\ &\left. + 8\,\mathrm{Re}\left(\frac{F_I}{\bar{b}}\right)\mathrm{Re}\left(\frac{X^J}{\bar{b}}\right)\right] + i\frac{g_I D}{2g_0 g_1} z = -4\pi i q_I, \end{aligned} \tag{3.29}$$

where $q_I$ are related to the electric charges. In order to solve (3.29), notice that

$$\mathrm{Re}\mathcal{N} = \frac{\bar{X}^0 X^1 - \bar{X}^1 X^0}{2i}\begin{pmatrix} |X^0|^{-2} & 0 \\ 0 & -|X^1|^{-2}\end{pmatrix}, \tag{3.30}$$

from which

$$\mathrm{Re}\,\mathcal{N}_{IL}(\mathrm{Im}\mathcal{N})^{-1|JL} g_J = 2i(\bar{X}^0 X^1 - \bar{X}^1 X^0)(-1)^I g_I$$

(no summation over $I$).

[6]Taking the lower sign yields $\mathrm{Im}\alpha^I = 0$, and thus a constant scalar.

Using this, (3.29) boils down to a fifth-order polynomial equation,

$$B_0 + B_1 z + B_2 z^2 + B_3 z^3 + B_4 z^4 + B_5 z^5 = 0. \tag{3.31}$$

One finds that $B_5$ vanishes identically provided that (3.24) holds. Requiring $B_0 = 0$ yields

$$\eta_{IJ}\left(\frac{\mathrm{Im}C}{4g_J} - D\,\mathrm{Im}\beta^J\right) + 4(-1)^I g_I\,\mathrm{Im}(\beta^1\bar{\beta}^0) = 2\pi q_I, \tag{3.32}$$

which determines the electric charges. Given that (3.25) holds, the above equation leads to

$$g_I^{-1}\,\mathrm{Im}C = 8(\pi\eta^{IJ}q_J + \mathrm{Im}\beta^I D), \tag{3.33}$$

where $\eta^{IJ}$ denotes the inverse of $\eta_{IJ}$. Note that (3.32) implies also

$$\mathrm{Im}\,C = 4\pi(g_1 q_0 + g_0 q_1), \tag{3.34}$$

which, combined with (3.33), yields (no summation over $I$)

$$(-1)^I g_I\,\mathrm{Im}\beta^I = \frac{\pi(g_1 q_0 - g_0 q_1)}{2D}. \tag{3.35}$$

It turns out that then all coefficients in (3.31) vanish, and thus (3.29) is satisfied.

Finally, taking

$$e^{2\gamma} = \left(1 + \frac{\kappa}{4}w\bar{w}\right)^{-2} \tag{3.36}$$

as a solution of the Liouville equation (3.3), one can compute the shift vector from (2.14), with the result

$$\sigma = \frac{iD}{32 g_0 g_1}\frac{w d\bar{w} - \bar{w} dw}{1 + \frac{\kappa}{4}w\bar{w}}. \tag{3.37}$$

Note that $d\sigma$ is proportional to the Kähler form on the two-space with metric $e^{2\gamma}dw d\bar{w}$. The four-dimensional line element reads

$$\begin{aligned} ds^2 = -4|b|^2(dt + \sigma)^2 + \frac{dz^2}{|b|^2} + \frac{z^2 + 16g_0 g_1\mathrm{Re}(\beta^1\bar{\beta}^0)}{4g_0 g_1} \\ \times \frac{dw d\bar{w}}{(1 + \frac{\kappa}{4}w\bar{w})^2}, \end{aligned} \tag{3.38}$$

where

$$|b|^2 = 4g_0 g_1\frac{|z^2 + iDz + C|^2}{z^2 + 16g_0 g_1\,\mathrm{Re}(\beta^1\bar{\beta}^0)}. \tag{3.39}$$

As we said, positivity of the kinetic terms in the action requires $\mathrm{Re}\tau > 0$. From (3.22), one sees that this is equivalent to

$$z^2 > -16 g_0 g_1\,\mathrm{Re}(\beta^1\bar{\beta}^0). \tag{3.40}$$

As can be seen from (3.39), $|b|$ diverges when $\mathrm{Re}\tau = 0$, signaling the presence of a curvature singularity at the point where ghost modes appear. The solution we have found will have an event horizon for some $z = z_h$, with $z_h^2 + iDz_h + C = 0$, and thus $z_h^2 = -\mathrm{Re}C$ and $Dz_h = -\mathrm{Im}C$, which in turn imply

$$\mathrm{Im}^2 C = -D^2\,\mathrm{Re}C, \tag{3.41}$$

and therefore $\mathrm{Re}C < 0$. Putting these results together, we can be more specific about the geometry of the horizon. First of all, contracting (3.26) with $g_I$ and taking into account (3.25) and the second equation of (3.3) yields

$$\kappa = 2\,\mathrm{Re}C + 16\sum_I g_I^2|\beta^I|^2 - D^2. \tag{3.42}$$

If we want the dangerous point where ghost modes appear to be hidden behind the horizon, we must have $-\mathrm{Re}C > -16g_0g_1\,\mathrm{Re}(\beta^1\bar{\beta}^0)$. Using this in (3.42) gives

$$\kappa < 16|\beta|^2 - D^2, \tag{3.43}$$

which, together with (3.25), yields $\kappa < 0$, so that the horizon must be hyperbolic. Note that one can also have solutions with spherical instead of hyperbolic symmetry, but these are naked singularities. A special case occurs for $\kappa = 0$, i.e., for a flat horizon. Then, the point where ghost modes appear coincides with the horizon. The resulting geometry describes a Nernst brane [26], whose entropy vanishes at zero temperature. Solutions of this type have potential applications in AdS/cond-mat, but unfortunately for $\kappa = 0$ the spacetime (3.38) has a curvature singularity at $z = z_h$, where

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \sim (z - z_h)^{-2}. \tag{3.44}$$

Coming back to the case of arbitrary $\kappa$, the fluxes can be computed from (2.17), with the result (no summation over $I$)

$$\begin{aligned} F^I = {}& (dt+\sigma)\wedge dz\frac{16g_0g_1}{(z^2+16g_0g_1\,\mathrm{Re}(\beta^0\bar{\beta}^1))^2}\bigg[\bigg(\frac{\mathrm{Im}C}{4g_I} + D\,\mathrm{Im}\beta^I\bigg)(16g_0g_1\,\mathrm{Re}(\beta^0\bar{\beta}^1) - z^2) \\ & + 2z\bigg(16g_0g_1\,\mathrm{Re}(\beta^0\bar{\beta}^1)\bigg(\mathrm{Re}\beta^I + \frac{D}{4g_I}\bigg) - \mathrm{Re}(\beta^I\bar{C})\bigg)\bigg] - \frac{ie^{2\gamma}dw\wedge d\bar{w}}{z^2+16g_0g_1\,\mathrm{Re}(\beta^0\bar{\beta}^1)}\cdot\bigg[\bigg(\frac{D^2}{4g_I} + D\,\mathrm{Re}\beta^I + \frac{\kappa}{8g_I}\bigg)z^2 \\ & + D\bigg(\frac{\mathrm{Im}C}{4g_I} + D\,\mathrm{Im}\beta^I\bigg)z + D\,\mathrm{Re}(\beta^I\bar{C}) + \frac{\kappa}{8g_I}\bigg(\mathrm{Re}C - \frac{\kappa}{2}\bigg)\bigg]. \end{aligned} \tag{3.45}$$

To sum up, the metric is given by (3.38), the U(1) field strengths by (3.45), and the complex scalar $\tau$ reads

$$\tau = \frac{g_0}{g_1}\frac{z - 4ig_1\beta^1}{z - 4ig_0\beta^0}, \tag{3.46}$$

where the constants $\beta^I \in \mathbb{C}$ are constrained by (3.25). *A priori*, the solution is labeled by the 7 real parameters $\beta^I$, $C$, $D$, but (3.25), together with (3.42), leave 4 independent constants. Note that $\kappa$ can be set to 0, $\pm 1$ without loss of generality by using the scaling symmetry $(t, z, w, C, D, \beta^I, \kappa) \mapsto (t/\lambda, \lambda z, w/\lambda, \lambda^2 C, \lambda D, \lambda\beta^I, \lambda^2\kappa)$ leaving the metric, fluxes, and scalar invariant. A convenient way of parametrizing the constraint (3.25) is

$$\begin{aligned} g_0\,\mathrm{Im}\beta^0 = -g_1\,\mathrm{Im}\beta^1 = \frac{\nu}{4}, \qquad g_0\,\mathrm{Re}\beta^0 = \frac{\mu - n}{4}, \\ g_1\,\mathrm{Re}\beta^1 = -\frac{\mu + n}{4}, \end{aligned} \tag{3.47}$$

where $n = D/2$. Then, (3.39) becomes

$$|b|^2 = 4g_0g_1\frac{|z^2 + 2inz + C|^2}{z^2 - \mu^2 - \nu^2 + n^2}, \tag{3.48}$$

with

$$\mathrm{Re}\,C = \frac{\kappa}{2} - \mu^2 - \nu^2 + n^2. \tag{3.49}$$

If, in addition, we want the metric to have a horizon, the additional constraint (3.41) must be satisfied. We have thus obtained a three-parameter family $(\mu, \nu, n)$ of black holes, whose NUT charge is given by $n$.

The magnetic and electric charges read, respectively,

$$\begin{aligned} P^I &= \frac{1}{4\pi}\int_{\Sigma_\infty} F^I = \bigg[p^I - \frac{D^2}{8\pi g_I} - \frac{D}{2\pi}\,\mathrm{Re}\beta^I\bigg]V, \\ Q_I &= \frac{1}{4\pi}\int_{\Sigma_\infty} G_I = \bigg[q_I + \frac{D}{2\pi}\eta_{IJ}\,\mathrm{Im}\beta^J\bigg]V, \end{aligned} \tag{3.50}$$

where $G_{+I} = \mathcal{N}_{IJ}F^{+J}$ [24], $\Sigma_\infty$ denotes a surface of constant $t$, $z$ for $z \to \infty$, and $V$ is defined by

$$V = \frac{i}{2}\int e^{2\gamma}dw\wedge d\bar{w}. \tag{3.51}$$

For $\kappa = -1$, this yields in terms of the parameters $\mu$, $\nu$, $n$,

$$\begin{aligned} P^0 &= -\frac{V}{4\pi g_0}\bigg[n^2 + n\mu - \frac{1}{4}\bigg], \\ P^1 &= -\frac{V}{4\pi g_1}\bigg[n^2 - n\mu - \frac{1}{4}\bigg], \\ Q_0 &= \frac{nV}{4\pi g_1}\bigg[\sqrt{\frac{1}{2} + \mu^2 + \nu^2 - n^2} + \nu\bigg], \\ Q_1 &= \frac{nV}{4\pi g_0}\bigg[\sqrt{\frac{1}{2} + \mu^2 + \nu^2 - n^2} - \nu\bigg]. \end{aligned} \tag{3.52}$$

The values of the scalar field on the horizon and the entropy are

$$\tau_{\rm h} = \frac{g_0}{g_1}\frac{\sqrt{\frac{1}{2}+\mu^2+\nu^2-n^2}-\nu+i(\mu+n)}{\sqrt{\frac{1}{2}+\mu^2+\nu^2-n^2}+\nu-i(\mu-n)},$$
$$S=\frac{A_{\rm h}}{4G}=\frac{\pi V}{4g_0g_1}, \tag{3.53}$$

where we have taken into account that $8\pi G = 1$ in our conventions. If the horizon is compactified to a Riemann surface of genus $h > 1$, we can use Gauss-Bonnet to get $V = 4\pi(h-1)$, and thus

$$S=\frac{\pi^2(h-1)}{g_0g_1}. \tag{3.54}$$

For a noncompact horizon, $V$ is infinite, but the entropy and charge densities are finite. If we define the complex charge

$$z^I = P^I + i\eta^{IJ}Q_J, \tag{3.55}$$

as well as the symplectic vector

$$\mathcal{Z} = \begin{pmatrix} z^I \\ -i\eta_{IJ}z^J \end{pmatrix}, \tag{3.56}$$

the Bekenstein-Hawking entropy can be rewritten in the form

$$S = -\frac{16i\pi^3}{V}\langle \mathcal{Z}, \bar{\mathcal{Z}}\rangle, \tag{3.57}$$

where $\langle\cdot,\cdot\rangle$ denotes the symplectic product. For nonvanishing NUT parameter $n$, one can express $\tau_{\rm h}$ in terms of the charges,

$$\tau_{\rm h}=\frac{g_0}{g_1}\frac{1-16\pi g_0 z^0/V}{1-16\pi g_1 z^1/V}. \tag{3.58}$$

If the NUT charge is zero, both the nominator and the denominator of (3.58) vanish, and $\tau_{\rm h}$ ceases to be a function of the charges: In this case, we have $Q_I = 0$, $P^I = V/(16\pi g_I)$, while $\tau_{\rm h}$ depends on the two parameters $\mu$, $\nu$ which are independent of the charges. The scalar field is thus not stabilized for $n = 0$; $\tau_{\rm h}$ takes values in the moduli space SU(1, 1)/U(1).[7] These flat directions are lifted by turning on a NUT parameter, since then $\tau_{\rm h}$ is completely fixed by the charges, cf. (3.58).

### B. Near-horizon limit

The near-horizon limit is obtained by setting $z = z_{\rm h} + \epsilon\hat{z}$, $t = \hat{t}/(2\epsilon)$, and taking the limit $\epsilon \to 0$. Then, the metric (3.38) boils down to

$$ds^2 = -\frac{\hat{z}^2}{L^2}d\hat{t}^2 + L^2\frac{d\hat{z}^2}{\hat{z}^2} + \frac{e^{2\gamma}dwd\bar{w}}{8g_0g_1}, \tag{3.59}$$

which is $\mathrm{AdS}_2\times\mathrm{H}^2$, with the AdS length scale $L$ set by

$$L^{-2} = 16g_0g_1(1+2\mu^2+2\nu^2).$$

Note that the shift vector $\sigma$ is scaled away in this limit. The near-horizon limit of the fluxes (3.45) can be cast into the form

$$F^I = -8\,\mathrm{Im}(X^I\bar{X}^Jg_J)d\hat{t}\wedge d\hat{z} + 2\pi i p^I e^{2\gamma}dw\wedge d\bar{w}. \tag{3.60}$$

### C. Constant scalars

In order to shed further light on the physical meaning of the parameters appearing in (3.38), and to compare with the results of [10], we will now consider the case of constant scalars. As we are interested in solutions with genuine horizons, we take $\kappa = -1$ in what follows.

First of all, from (3.46) it is clear that $\tau$ is constant iff $g_0\beta^0 = g_1\beta^1$. Taking into account (3.25), this implies

$$\beta^I = -\frac{D}{8g_I}. \tag{3.61}$$

Since $\tau = g_0/g_1$, the scalar potential $V$ in (2.7) reduces to a cosmological constant $\Lambda = -3/l^2$, with $l^{-2} = 4g_0g_1$. Setting

$$z=\frac{r}{l}, \qquad w = 2e^{i\phi}\tanh\frac{\theta}{2}, \tag{3.62}$$

as well as[8]

$$\mathrm{Re}C = \frac{N^2}{l^2}-\frac{1}{2}, \quad \mathrm{Im}C = -\frac{M}{2N}, \quad D=\frac{2N}{l}, \tag{3.63}$$

and transforming the time coordinate according to $t \mapsto l(N\phi - t/2)$, the metric (3.38) becomes

$$ds^2 = -\frac{\lambda}{r^2+N^2}(dt - 2N\cosh\theta d\phi)^2 + \frac{r^2+N^2}{\lambda}dr^2 + (r^2+N^2)(d\theta^2+\sinh^2\theta d\phi^2), \tag{3.64}$$

with $\lambda$ given by

$$\lambda = \frac{1}{l^2}(r^2+N^2)^2 + \left(-1+\frac{4N^2}{l^2}\right)(r^2-N^2) - 2Mr + \left(\frac{2N^2}{l}-\frac{l}{2}\right)^2 + \frac{l^2M^2}{4N^2}. \tag{3.65}$$

This represents a subclass of the (hyperbolic) Reissner-Nordström-Taub-NUT-AdS spacetime. The fluxes (3.45) boil down to

[7] Nevertheless, the entropy is independent of the values of the moduli on the horizon not fixed by the charges, in agreement with the attractor mechanism [14–18].

[8] Notice that, with (3.61) and (3.63), the constraint (3.42) is automatically satisfied.

$$F^I = -(dt - 2N\cosh\theta d\phi)\wedge\frac{dr}{2lg_I(r^2+N^2)^2}\left[\frac{Ml}{2N}(r^2-N^2) + 2rN\left(\frac{2N^2}{l}-\frac{l}{2}\right)\right] - \frac{\sinh\theta d\theta\wedge d\phi}{2lg_I(r^2+N^2)}\left[\left(\frac{2N^2}{l}-\frac{l}{2}\right)(r^2-N^2) - Mlr\right]. \tag{3.66}$$

It is not difficult to see that the action (2.7) reduces to the one of minimal gauged supergravity considered in [10] for $g_0F^0 = g_1F^1 \equiv F/(2l)$. The field strength $F$ computed this way from (3.66) coincides exactly with the expression following from the RN-TN-AdS gauge potential (2.4) of [10] if we identify

$$Q = -\frac{Ml}{2N}, \qquad P = \frac{2N^2}{l} - \frac{l}{2}. \tag{3.67}$$

These are precisely the conditions on the electric and magnetic charge found in [10], for which the Reissner-Nordström-Taub-NUT-AdS solution is supersymmetric.[9] Moreover, if (3.67) holds, the function (3.65) reduces to Eq. (2.1) of [10]. As a nontrivial consistency check, we have thus reproduced the known BPS conditions of minimal gauged supergravity. As we said, in order to have a horizon, the additional constraint (3.41) must be satisfied. In this case, (3.41) leads to

$$M = \frac{4N^2}{l}\left(\frac{1}{2} - \frac{N^2}{l^2}\right). \tag{3.68}$$

This leaves a one-parameter family of supersymmetric black holes, labeled by the NUT charge $N$. From (3.63), it is also clear that the imaginary part of $C$ is related to the black hole mass.

## IV. LIFTING TO M THEORY

We now want to uplift the black hole solutions obtained in Sec. III A to M theory, and comment on their higher-dimensional interpretation. The Kaluza-Klein *Ansatz* given in [27] allows to reduce 11-dimensional supergravity to $\mathcal{N} = 4$ SO(4) gauged supergravity in four dimensions, which can be further truncated to the $F = -iX^0X^1$ model of Sec. III A. The reduction *Ansatz* for the metric reads [27]

$$ds_{11}^2 = \Delta^{2/3}ds_4^2 + \frac{2\Delta^{2/3}}{g^2}d\xi^2 + \frac{\Delta^{2/3}}{2g^2}\left[\frac{c^2}{c^2X^2+s^2}\sum_{i=1}^3(h^i)^2 + \frac{s^2}{s^2\tilde{X}^2+c^2}\sum_{i=1}^3(\tilde{h}^i)^2\right], \tag{4.1}$$

where

$$\tilde{X} = X^{-1}q, \quad q^2 = 1+\chi^2X^4, \quad \Delta = [(c^2X^2+s^2)(s^2\tilde{X}^2+c^2)]^{1/2}, \quad c = \cos\xi, \quad s = \sin\xi, \quad h^i = \sigma_i - gA^i_{(1)}, \quad \tilde{h}^i = \tilde{\sigma}_i - g\tilde{A}^i_{(1)}. \tag{4.2}$$

Here, the $\sigma_i$ are left-invariant 1-forms on $S^3 = SU(2)$, and $\tilde{\sigma}_i$ are left-invariant 1-forms on a second $S^3$. They satisfy

$$d\sigma_i = -\tfrac{1}{2}\epsilon_{ijk}\sigma_j\wedge\sigma_k, \qquad d\tilde{\sigma}_i = -\tfrac{1}{2}\epsilon_{ijk}\tilde{\sigma}_j\wedge\tilde{\sigma}_k. \tag{4.3}$$

$A^i_{(1)}$, $\tilde{A}^i_{(1)}$ denote the $SU(2)\times SU(2) \cong SO(4)$ Yang-Mills potentials, $g$ is the gauge-coupling constant, and $X = \exp(\phi/2)$. $\phi$ and $\chi$ are the dilaton and axion of the $\mathcal{N} = 4$, $D = 4$ theory, respectively. The *Ansatz* for the 4-form is given by [27]

$$F_{(4)} = -g\sqrt{2}U\epsilon_{(4)} - \frac{4sc}{g\sqrt{2}}X^{-1} * dX\wedge d\xi + \frac{\sqrt{2}sc}{g}\chi X^4 * d\chi\wedge d\xi + F'_{(4)} + F''_{(4)}, \tag{4.4}$$

with $*$ the Hodge dual operator of $ds_4^2$, and $\epsilon_{(4)}$ the corresponding volume form. The expressions for $F'_{(4)}$ and $F''_{(4)}$ are rather lengthy, and can be found in Eqs. (9) and (10) of [27]. $U$ is defined by

$$U = X^2c^2 + \tilde{X}^2s^2 + 2. \tag{4.5}$$

Plugging the above reduction *Ansätze* into the 11-dimensional equations of motion gives rise to the equations of motion of $\mathcal{N} = 4$, $D = 4$ gauged supergravity. If we truncate further by setting $A^1_{(1)} = A^2_{(1)} = \tilde{A}^1_{(1)} = \tilde{A}^2_{(1)} = 0$ [which corresponds to considering only the Cartan subgroup $U(1)\times U(1)$ of SO(4)], the bosonic Lagrangian in four dimensions becomes [27]

$$\mathcal{L}_4 = R*1 - \frac{1}{2}*d\phi\wedge d\phi - \frac{1}{2}e^{2\phi}*d\chi\wedge d\chi - V*1 - \frac{1}{2}e^{-\phi}*F^3_{(2)}\wedge F^3_{(2)} - \frac{1}{2}\frac{e^{\phi}}{1+\chi^2e^{2\phi}}*\tilde{F}^3_{(2)}\wedge\tilde{F}^3_{(2)} - \frac{1}{2}\chi F^3_{(2)}\wedge F^3_{(2)} + \frac{1}{2}\frac{\chi e^{2\phi}}{1+\chi^2e^{2\phi}}\tilde{F}^3_{(2)}\wedge\tilde{F}^3_{(2)}, \tag{4.6}$$

where $F^3_{(2)} = dA^3_{(1)}$, $\tilde{F}^3_{(2)} = d\tilde{A}^3_{(1)}$, and the scalar potential reads

$$V = -2g^2(4 + 2\cosh\phi + \chi^2e^{\phi}). \tag{4.7}$$

This is (up to a constant prefactor) equal to the Lagrangian (2.7) for the prepotential $F = -iX^0X^1$, if we identify

$$F^0 = \frac{1}{\sqrt{2}}F^3_{(2)}, \qquad F^1 = \frac{1}{\sqrt{2}}\tilde{F}^3_{(2)}, \qquad \tau = e^{-\phi} - i\chi, \tag{4.8}$$

and take $g_0 = g_1 = g/\sqrt{2}$ for the gauge-coupling constants. This allows to oxidize the solution (3.38), (3.45), and (3.46) to 11 dimensions. The functions $X$, $\tilde{X}$ are then given by

[9] Actually, the conditions given in [10] are $Q = \mp Ml/(2N)$ and $P = \pm(2N^2/l - l/2)$, corresponding to vanishing $\mathcal{B}_{\mp}$ in (3.10), (3.12) of [10]. We have here the upper sign, but the lower one can easily be generated by the *CPT* transformation $\phi \mapsto -\phi$, $t \mapsto -t$ (that leaves the metric invariant).

$$X^2 = \frac{(z+\nu)^2 + (n-\mu)^2}{z^2 - \mu^2 - \nu^2 + n^2}, \qquad \tilde{X}^2 = \frac{(z-\nu)^2 + (n+\mu)^2}{z^2 - \mu^2 - \nu^2 + n^2}. \tag{4.9}$$

Choosing Euler angles $\psi$, $\vartheta$, $\varphi$ on the first $S^3$ and $\Psi$, $\Theta$, $\Phi$ on the second $S^3$, we have for the left-invariant 1-forms

$$\sigma_1 = \sin\psi d\vartheta - \cos\psi \sin\vartheta d\varphi,$$
$$\sigma_2 = \cos\psi d\vartheta + \sin\psi \sin\vartheta d\varphi,$$
$$\sigma_3 = d\psi + \cos\vartheta d\varphi,$$

and similar for $\tilde{\sigma}_i$. After that, the expressions $\sum_i (h^i)^2$ and $\sum_i (\tilde{h}^i)^2$ in (4.1) simplify in our case to

$$\sum_{i=1}^{3} (h^i)^2 = d\vartheta^2 + \sin^2\vartheta d\varphi^2 + (d\psi + \cos\vartheta d\varphi - gA^3_{(1)})^2,$$
$$\sum_{i=1}^{3} (\tilde{h}^i)^2 = d\Theta^2 + \sin^2\Theta d\Phi^2 + (d\Psi + \cos\Theta d\Phi - g\tilde{A}^3_{(1)})^2, \tag{4.10}$$

where

$$A^3_{(1)t} = 2g\frac{\kappa(n-\mu) + 4n(-\mu^2 - \nu^2 + n^2) - 2z(\mathrm{Im}C + 2n\nu)}{z^2 - \mu^2 - \nu^2 + n^2},$$
$$A^3_{(1)w} = -\frac{i\bar{w}}{2g(1+\frac{\kappa}{4}w\bar{w})(z^2-\mu^2-\nu^2+n^2)}\bigg[\Big(n^2 + n\mu + \frac{\kappa}{4}\Big)z^2 + nz(2n\nu + \mathrm{Im}C) + \Big(\frac{\kappa}{2} - \mu^2 - \nu^2 + n^2\Big)\Big(\frac{\kappa}{4} - n^2 + n\mu\Big) + n\nu\,\mathrm{Im}C - \frac{\kappa^2}{8}\bigg],$$

$A^3_{(1)\bar{w}} = (A^3_{(1)w})^\star$, and $A^3_{(1)z} = 0$. The expressions for $\tilde{A}^3_{(1)}$ result from those for $A^3_{(1)}$ by replacing $\mu \to -\mu$ and $\nu \to -\nu$.

For $\mu = \nu = n = 0$, the solution (4.1) can be interpreted as the gravity dual corresponding to membranes wrapping holomorphic curves in a Calabi-Yau fivefold [28]. It would be interesting to see whether the general solution (4.1) (for $\mu$, $\nu$, $n \neq 0$) has a similar interpretation. This might allow for a microscopic entropy computation of the four-dimensional black hole (3.38), which can then be compared with the macroscopic Bekenstein-Hawking result (3.54).

## V. FINAL REMARKS

In this paper, we constructed a family of one-quarter BPS black holes in $\mathcal{N} = 2$, $D = 4$ Fayet-Iliopoulos-gauged supergravity carrying two electric, two magnetic, and one NUT charge. The solution is given in terms of three free parameters, and has a hyperbolic horizon. We saw that for vanishing NUT charge, there are flat directions in the effective black hole potential, in agreement with the results of [5], where a general near-horizon analysis was done. Turning on a NUT parameter lifts these flat directions, so that the horizon value of the moduli are completely fixed in terms of the charges.

A possible extension of our work would be to use the 11-dimensional interpretation of our solution, cf. the oxidized metric obtained in Sec. IV, to compute microscopically the entropy, which can then be compared with the classical Bekenstein-Hawking result (3.53). Moreover, it would be interesting to consider other prepotentials, for instance, the $t^3$ model, which allows for supersymmetric black holes with spherical symmetry [5], and try to add rotation and NUT charge to the known static black holes [5,8,9]. We hope to come back to these points in a future publication.

## ACKNOWLEDGMENTS

This work was partially supported by INFN and MIUR-PRIN Contract No. 2009-KHZKRX.

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