PETRI NETS APPLICABILITY TO MUSIC ANALYSIS AND COMPOSITION

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ABSTRACT

This paper addresses the adoption of Petri Nets (PNs) in order to describe music processes. On the one hand, such formalism can be used to represent the results coming from the analysis of already existing music works. On the other hand, Petri Nets provide a formal tool to generate new music pieces. This paper is the evolution of an earlier work presented at CMMR 2005, where analytical and descriptive possibilities of Petri Nets were explored in music field. The focus of this paper is writing music by Petri Nets, and the last section will show a case study of PN-based composition.

1. INTRODUCTION

A music piece can be described in a number of different ways, depending on the information we want to convey. For instance, nowadays Common Western Notation is considered the most appropriate formalism to represent a score to be performed by occidental musicians.

If we address a higher level of description, we are no more interested in melodic, rhythmic, and harmonic aspects related to single music events (such as notes and rests), but to characteristics shared by groups of elementary events. Thus, we are interested in describing music objects together with their mutual relationships, where the term music object refers to every music phenomenon or event perceived as a whole, i.e. as a coherent entity.

As demonstrated in [1], Petri Nets can be employed to capture some relevant high-level processes in music composition. In particular, such formalism is able to highlight the repetition of music objects, the causality relationships among them and the concurrent development of voices, parts and sections inside a music piece.

Sometimes it is cumbersome to provide a PN-based description of music, and – in any case – this operation requires a deep understanding of music processes involved in composition. On the other hand, a successful formalization results in a very clear and effective way to make music processes revealed.

2. AN OVERVIEW ABOUT PETRI NETS

A Petri Net is an abstract and formal model to represent the dynamic behaviour of a system with asynchronous and concurrent activities ([7], [8]).

Petri Nets consist in a set of basic objects: *places*, *transitions* and *arcs*, whose graphical representations are circles, rectangles, and oriented lines respectively. Both places and transitions are called *nodes*.

In Figure 1, an elementary Petri Net is shown. P1, P2, P3, P4, P5 are places, T1 is a transitions, and the oriented lines represent arcs. An arc can connect only nodes of different kind, i.e. places to transitions or vice versa. The number sometimes associated to arcs is called *arc weight*, and its meaning will be soon explained.
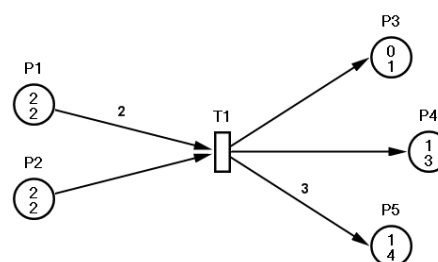

Figure 1. Example of a Petri Net. The transition T1 is enabled to fire.

A key concept that lets Petri Nets evolve and self-modify is the idea of *marking*, realized by using *tokens*: at a given time, any place holds a non-negative number of tokens, indicated by the upper numerical value inside the place. The lower value inside places indicates their *capacity*, i.e. the maximum number of housed tokens.
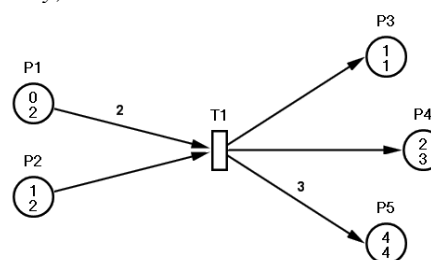

Figure 2. Evolution of the Petri Net represented in Figure 1.

Tokens can be transferred from place to place according to policies known as *firing rules*:

- A transition is enabled when all the incoming places of that transition present a number of tokens greater or equal to the weights of the corresponding incoming arcs, and – after the fire of the transition – the marking of all the output places will be less than or equal to their capacities.

- When a transition is enabled, the fire drops from the incoming places a number of tokens equal to the weights of the incoming arcs and adds to each outgoing place a number of tokens equal to the weights of the corresponding outgoing arc.

Finally, let us remark that Petri Nets can be described using the concept of *refinement*, a simple morphism to describe complex Petri Nets in terms of simpler ones, as shown in Figure 3.
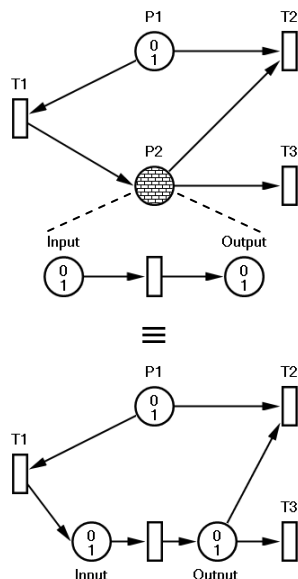

Figure 3. Refinements and subnets

3. PETRI NETS AND MUSIC

Petri Nets can be applied to various fields of information management and description. A regards music, a specific extension known as Music Petri Nets is available, where music objects are associated to places and music operators are associated to transitions. As mentioned before, a *music object* may be anything with a musical meaning which can be thought as an entity, whereas a *music operator* provides a method to transform music objects.

In this approach, when a place receives a token, the associated music object (if present) is played, and the token will remain locked until the end of the fragment. When a transition fires, its music operator changes the music objects associated to the incoming places and transfers them to the outgoing places. Transitions without music operators are used for mere net evolution.

At LIM,[1] Petri Nets were applied to music since 1982. In particular, early papers such as [3] investigated the possibility to describe causality in music processes through the formal approach of Petri Nets.

After the first attempts, a number of researches were conducted on the aforementioned matter. The early approach was the analytical one, where the key problem

was: Are Petri Nets a valid tool to highlight relationships among music objects? On the one hand, [6] demonstrated a great efficacy in describing the structure of Ravel's *Bolero*; on the other hand, [2] pointed out some limitations of this approach, providing opportune counterexamples such as Stravinskij's *The Rite of Spring*.

From the analytical perspective, excellent or poor results in representing music analysis through Petri Nets formalism mainly depend on three factors:
1. The intrinsic characteristics of the piece to be described.
2. The ability in confining those music objects which prove to be efficient from Petri Nets point of view.
3. The degree of detail the analysis wants to reach.

This last statement can justify the contradictory results obtained when considering different music pieces. The test case presented at CMMR 2005 [1] has taken into consideration the first movement of a sonata by W.A. Mozart. Intentionally, this case study represents an intermediate situation between a strongly structured piece and a completely unstructured one. Considering this test case, Petri Nets proved to be:
- a very synthetic instrument to describe the macro-structure of sonata form, but limited to a very small amount of information;
- an efficient description instrument at a medium degree of abstraction (repetition of rhythmic and melodic patterns, transformation of themes in order to generate new music material);
- a completely inadequate tool to depict the status and the evolution of atomic music elements (chords, notes, rests).

4. ANALYSIS-ORIENTED VS. COMPOSITION-ORIENTED APPROACH

Research about Petri Nets and music can follow two directions, namely the analytical and the compositional one. The latter approach is aimed at creating new music pieces according to layouts defined by Petri Nets, whereas the former has the purpose of making music structures emerge as concurrent processes or causal relationships represented through Petri Nets.

In music literature, there is at least one significant example which allows the application of both approaches, namely the diptych formed by prelude and fugue in Baroque music. As stated in [9], Baroque preludes often represented the imaginative, instinctual, unrehearsed side, whereas fugues expressed the geometric, rational, sought core of music.

In general terms, when we want to apply Petri Nets to existing music forms, the whole process can be summarized as follows:
- analysis of the form, in order to discover a number of music objects and to catch their relationship;
- description of the results coming from the analysis step in terms of Petri Net formalism;
- creation of a set of music objects;

- population of the obtained Petri Net by the introduction of those music objects.

Such process will be described in detail in the following section and exemplified through a case study.

5. A CASE STUDY: PRELUDE AND FUGUE FOR BRONZE CYLINDERS

Even if the diptych formed by prelude and fugue is typical of Baroque music, we have chosen to revise it according to a contemporary meaning. Far from literature of western notated music, this work is inspired by P. Schaeffer's *musique concrete* (concrete music): in the creative process we use real sounds, sampled and computer-manipulated.

5.1. Preparation of the Sound Material

In order to generate the original material, we have sampled the sound of three bronze objects struck by a flexible drumstick. Having different dimensions, the cylinders provide different "pitches": the fundamental frequencies roughly correspond to E6 (668 Hz), G6 (776 Hz), and F#7 (1444 Hz). The result consists in three elementary sound objects, that we have afterwards manipulated through computer-based techniques in order to generate other sound material. In particular, we employed filters, echoes, reverberations and composite processes to obtain a number of derived objects of the following lengths: 0.5, 2, and 3.5 seconds.

5.2. Petri Nets Design for the Prelude

Even if this composition is far from traditional preludes as regards the aural result, we tried to reflect the structures employed in Baroque music.

On the one hand, the general layout of this first piece has a high degree of freedom. Even in Baroque literature such kind of composition did not present a well defined, standard and commonly accepted structure. This aspect is reflected by our composition, that employs a plausible structural organization but does not cite an existing layout.

On the other hand, the prelude plays with the original sound material in a contrapuntal fashion. All the sound events are derived from the same base objects, and the non-linear evolution of sections is clearly illustrated in Figure 4.
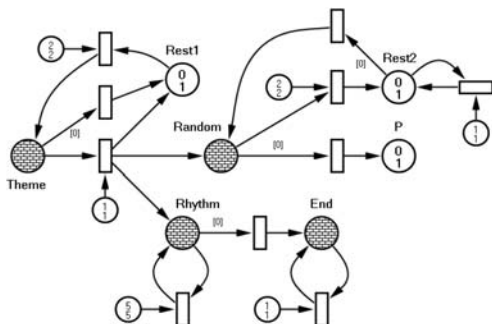
Figure 4. The general structure of the prelude.

It is worth to spend a few words about two sections of the piece that we consider particularly interesting. This discussion could help understanding the way Petri Nets work.

The first part that we will analyze in detail is the so-called *random section*. Figure 5 illustrates its structure in terms of Petri Nets. There are 6 sound objects, namely MX1..MX6, which have the same probability to be selected. However, the capacity of each place ensures that at any time a fragment is not playing more than twice. The selection process is repeated 5 times and generates a random section whose length is not fixed. In fact, MX1, MX2, MX4 and MX5 are 2 seconds long, whereas MX3 and MX6 are 3.5 seconds long. The function of Rest place is inserting a short time interval (1 second) between the execution of a sound object and the proposition of the next one.
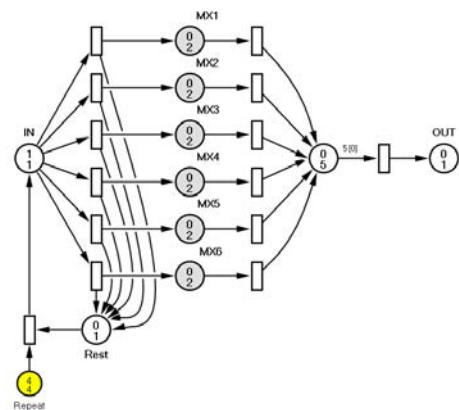
Figure 5. The random section of the prelude.

The section shown in Figure 6 manipulates MX7, MX8 and MX9 sound objects. Each object is 0.5 seconds long. Before exiting, the whole process is repeated 11 times, for a total timing of 5.5 seconds. The choice of the sound objects to play is established by the deterministic weight of the arcs: the probabilities are 10/15 for MX7, 3/15 for MX8, and 2/15 for MX9 respectively. The transitions following Load place have the same probability to be executed, and their function is altering the rhythmic value of the selected sound object. The first transition maintains the original rhythm, the second one halves and duplicates the object, the third one generates a triplet and so on.
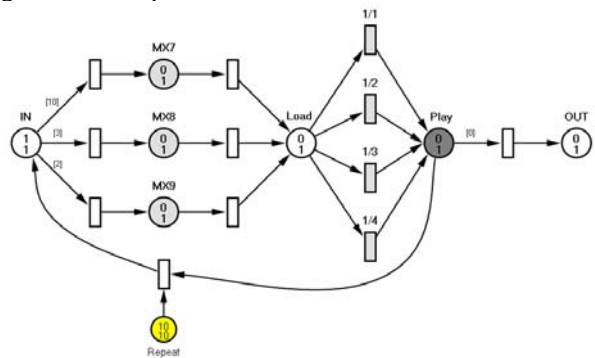
Figure 6. A subnet to choose sound objects and to transform their rhythm.

5.3. Petri Nets Design for the Fugue

The fugue requires at first an analysis of the historical form. We based our study on a particular standard structure, namely the so-called scholastic fugue coded by A. Gédalge in [4]. Our fugue reflects Gédalge's proposal and presents the following sequence of sections: E1 (1st exposition), D1 (1st developmental episode), E2 (2nd exposition or re-exposition), D2 (2nd episode), E3 (3rd exposition or re-exposition), D3 (3rd developmental episode), P (episode of pedal point), and finally S (strettos section).
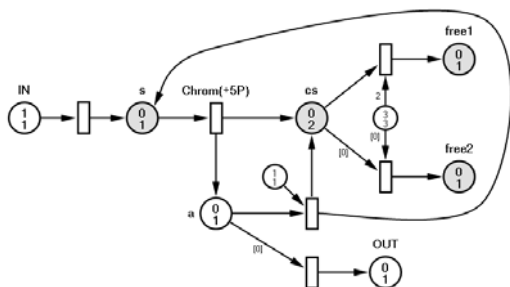

Figure 7. First exposition of the fugue.

The main elements of a traditional fugue – namely the subject s, the answer a, and the optional countersubject cs – are here revised according to the new sound objects. Figure 7 provides the detailed representation of the exposition E1 in terms of Petri Nets.

6. CONCLUSIONS

The case study *Prelude and Fugue for Bronze Cylinders* remarks the applicability of Petri Nets to the description and generation of music processes. The two pieces unveil two different applicability fields: Petri Nets allow both the representation of existing music structures, at different degrees of abstraction, and their (self)evolution in order to generate new pieces. In particular, it is interesting to apply such formalism to the analysis of well-known music forms (such as Baroque fugue) or to single music pieces (such as Ravel's *Bolero*), in order to maintain structures but changing music objects: as a result, we obtain a formal model that can be employed to originate a sort of variations on a theme.

Finally, please note that in this paper we have considered only composition-oriented aspects of Petri Nets. However, the application of such formalis to music could range from artistic experiments, like stochastic composition or contemporary counterpoint, to commercial purposes, such as advertising jingles or movie trailers. This is the reason why we consider such formalism as a promising matter for the generation of multimedia products.

7. ACKNOWLEDGEMENTS

8. REFERENCES

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