

# Equivalence checking for NuSMV specifications

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**Abstract.** We present a technique for checking the equivalence of NuSMV specifications. The approach is founded on the notion of equivalence between Kripke structures. The necessity to tackle this problem arisen working on using mutation to asses the static analysis fault detection capability. Indeed, *mutation*, consisting into introducing simple syntactic changes – representing typical mistakes designers often make – into specifications, may produce equivalent *mutants*, namely models behaving as the original one. Equivalent mutants should be detected since they do not represent actual faults. In program mutation, detecting equivalent mutants is an undecidable problem and, when possible, is a time-consuming activity, difficult to automatize. In this work we focus on how detecting equivalence of NuSMV specifications. The novel technique we propose, consists in building a merging unique specification and proving by model checking a series of CTL properties.

## 1 Introduction

The problem of detecting equivalent NuSMV specifications is connected to the problem of identifying equivalent mutants. *Mutation* consists in introducing small modifications, called *mutations*, into models; these simple syntactic changes should represent typical mistakes a designer may make during the modeling activity.

Mutation and the problem of checking equivalent mutants is well-known in the context of program, and common fault classes have been defined by Kuhn in [10]. From fault classes it is easy to derive *mutation operators* [1], that produce copies of the original program each containing a single fault: these faulty programs are called *mutants*. More recently, mutation has been applied to specifications like FSMs [6], Petri nets [7], Statecharts [8], Estelle specifications [5], Object-Z specifications [11], etc. We have focused our attention on the mutation of Kripke structures and, in particular, of their representation as NuSMV specifications [13].

Most mutation operators can produce equivalent mutants, namely models behaving as the original one. Equivalent mutants pose a challenge, since they do not represent actual faults and cannot be detected by observing the model behavior. Therefore, it is important to identify and remove from the set of mutants the equivalent ones.

In program mutation, detecting equivalent mutants is an undecidable problem [1] and, when possible, is a time-consuming activity [9], difficult to automatize. In this work we focus on how detecting equivalence of NuSMV specifications obtained by mutation. We propose a novel procedure for checking the equivalence, which is based on the notion of equivalence between Kripke structures.

Section 2.1 presents the NuSMV syntax and Section 2.2 introduces some definitions about Kripke structures. Section 3 presents the notion of equivalence between Kripke structures and how the problem of proving the equivalence between two Kripke structures  $M_1$  and  $M_2$  can be reduced to the problem of proving some properties over a single Kripke structure  $M_{12}$ , obtained by *merging*  $M_1$  and  $M_2$ ; Section 4 shows how to build the merging specification when the Kripke structures are represented as NuSMV specifications and how to prove the equivalence using some CTL properties.

## 2 Background

### 2.1 NuSMV and its notation

NuSMV [4,13] is known as a model checker derived from the CMU SMV [12]. It allows for the representation of synchronous and asynchronous finite state systems, and for the analysis of specifications expressed in *Computation Tree Logic* (CTL) and *Linear Temporal Logic* (LTL).

A NuSMV specification contains a **VAR** section for variable declarations. A variable type can be Boolean, integer defined over intervals or sets, or an enumeration of symbolic constants. A *state* of the model is an assignment of values to variables.

A NuSMV specification describes the behavior of a Finite State Machine (FSM) in terms of a “possible next state” relation between states that are determined by the values of variables. Transitions between states are determined by the updates of the variables declared in the **ASSIGN** section, that contains the initialization (by the instruction **init**) and the update mechanism (by the instruction **next**) of variables. A **DEFINE** statement can also be used as a macro to syntactically replace an *identifier* with the *expression* it is associated with. There exist the following four ways to explicitly assign values to a variable:

```
ASSIGN identifier := simple_expression -- simple assignment
ASSIGN init(identifier) := simple_expression -- init value
ASSIGN next(identifier) := next_expression -- next value
DEFINE identifier := simple_expression -- macro definition
```

where *identifier* is a variable identifier; *simple\_expressions* are built only from the values of variables in the current state and they cannot have a **next** operation inside; *next\_expression* relates current and next state variables to express transitions in the FSM (see the NuSMV User Manual [3] for more details on the assignment syntax and restriction rules for assignments). In both *simple-* and *next-* expressions, a variable’s value can be determined either unconditionally or conditionally, depending on the form of the expression. Conditional expressions can be:

1. An **if-then-else** expression `cond1? exp1: exp2` which evaluates to *exp1* if the condition *cond1* evaluates to true, and to *exp2* otherwise.
2. A condition **case** expression:

```
case
  left_expression_1 : right_expression_1 ;
  ...
  left_expression_N : rightexpression_N;
esac
```

which returns the value of the first *right\_expression\_i* such that the corresponding *left\_expression\_i* condition evaluates to TRUE, and the previous *i-1* left expressions evaluate to FALSE. The type of expressions on the left hand side must be boolean. An error occurs if all expressions on the left hand side evaluate to FALSE. To avoid these kinds of errors, NuSMV performs a static analysis and, if it believes that in some states no left expression may be true, it forces the user to add a *default case* with *left\_expression* equal to TRUE.

NuSMV offers another more declarative way of defining initial states and transition relations. Initial states can be defined by the keyword **INIT** followed by characteristic properties that must be satisfied by the variables values in the initial states. Transition relations can be expressed by constraints, through the keyword **TRANS**, on a set of *current state/next state* pairs. *Invariant conditions* can be expressed by the command **INVAR**.

Temporal properties are specified in the **CTLSPEC** (resp. **LTLSPec**) section that contains the CTL (resp. LTL) properties to be verified.

## 2.2 Kripke structures

### Definition 1 (Kripke structure).

A *Kripke structure* is a quadruple  $M = \langle S, S^0, T, \mathcal{L} \rangle$  where

- $S$  is a set of states;
- $(S^0 \subseteq S) \neq \emptyset$  is the set of initial states;
- $T \subseteq S \times S$  is the transition relation that must be left-total, i.e.,  $\forall s \in S, \exists s' \in S: (s, s') \in T$ ;
- $\mathcal{L}: S \rightarrow \mathcal{P}(AP)$  is the proposition labeling function, where  $AP$  is a set of atomic propositions; we require  $\mathcal{L}$  to be injective, i.e.,  $\forall s_1, s_2 \in S, s_1 \neq s_2 \rightarrow \mathcal{L}(s_1) \neq \mathcal{L}(s_2)$ : this means that a state is uniquely identified by its labels.

**Definition 2 (Computation tree).** Given a Kripke structure  $M = (S, S^0, T, \mathcal{L})$ , a computation tree of  $M$  is a tree structure where the root is an initial state  $s_0 \in S_0$ , and the children of a node  $s \in S$  in the computation tree are all the states  $s' \in S$  such that there exists a transition  $(s, s') \in T$ .

### Definition 3 (Structure equivalence).

Two Kripke structures  $M_1$  and  $M_2$  with the same set of atomic propositions are equivalent iff they have the same computation trees.

### Definition 4 (Path).

A path  $\psi$  is a sequence of states in  $S$

$$\psi = s_1, s_2, \dots, s_n$$

such that

$$\forall i \in [1, n-1] \quad (s_i, s_{i+1}) \in T$$

Let's identify with  $\Psi$  the (infinite) set of all the paths in  $M$ .

Let's identify with  $\Psi^0 \subseteq \Psi$  the (infinite) set of all the paths such that the starting state  $s_1 \in S^0$ .

### Definition 5 (Reachability).

A state  $s \in S$  is reachable in  $M$  if there exists a path  $\psi^0 = s_1, \dots, s_n \in \Psi^0$  such that  $s_n = s$ , i.e.

$$isReach(s) \triangleq \exists \psi^0 = s_1, \dots, s_n \in \Psi^0: s_n = s$$

We denote by  $reach(M) \subseteq S$  the set of reachable states of the machine  $M$ .

### Definition 6 (Successor state).

A state  $s'$  is a successor of another state  $s$  if  $(s, s') \in T$ . We denote by  $next(s)$  the set of the successor states of  $s$ , i.e.

$$next(s) = \{s' \in S: (s, s') \in T\}$$

## 3 Equivalence of Kripke structures

In this section we give the notion of equivalence between two Kripke structures [2].

Let  $M_1 = \langle S_1, S_1^0, T_1, \mathcal{L}_1 \rangle$  and  $M_2 = \langle S_2, S_2^0, T_2, \mathcal{L}_2 \rangle$  be two Kripke structures with the same set of atomic propositions  $AP$ . A relation  $E$  can be defined on  $S_1 \times S_2$  to express the equivalence between states of the two structures  $M_1$  and  $M_2$ ; two states are equivalent if they have the same labels and bring to next states having the same labels.

**Definition 7 (State equivalence).**

$\forall s_1 \in S_1 \forall s_2 \in S_2$  we say  $s_1 E s_2$  iff the following condition holds:

$$\begin{aligned} & \mathcal{L}_1(s_1) = \mathcal{L}_2(s_2) \\ & \forall s'_1 \in \text{next}(s_1) \exists s'_2 \in \text{next}(s_2) : \mathcal{L}_1(s'_1) = \mathcal{L}_2(s'_2) \wedge \\ & \forall s'_2 \in \text{next}(s_2) \exists s'_1 \in \text{next}(s_1) : \mathcal{L}_2(s'_2) = \mathcal{L}_1(s'_1) \end{aligned} \quad (1)$$

**Theorem 1 (Structure equivalence).**

Let  $M_1$  and  $M_2$  be two Kripke structures with the same set of atomic propositions. If the following properties hold (initial states have same labeling and reachable states are equivalent):

$$\forall s_1^0 \in S_1^0, \exists s_2^0 \in S_2^0 : [\mathcal{L}_1(s_1^0) = \mathcal{L}_2(s_2^0)] \quad (2)$$

$$\forall s_2^0 \in S_2^0, \exists s_1^0 \in S_1^0 : [\mathcal{L}_2(s_2^0) = \mathcal{L}_1(s_1^0)] \quad (3)$$

$$\forall s_1 \in \text{reach}(M_1) \exists s_2 \in \text{reach}(M_2) : s_1 E s_2 \quad (4)$$

$$\forall s_2 \in \text{reach}(M_2) \exists s_1 \in \text{reach}(M_1) : s_2 E s_1 \quad (5)$$

then  $M_1$  and  $M_2$  are equivalent.

The problem of checking the equivalence of two Kripke structures  $M_1, M_2$  can be reduced to the problem of proving some properties over a new *merging* Kripke structure  $M_{12}$  derived from  $M_1$  and  $M_2$ . In Section 3.1 we show how to build  $M_{12}$ .

### 3.1 Construction of merging Kripke structure $M_{12}$

Let  $M_1 = \langle S_1, S_1^0, T_1, \mathcal{L}_1 \rangle$  and  $M_2 = \langle S_2, S_2^0, T_2, \mathcal{L}_2 \rangle$  be two Kripke structures with the same set of atomic propositions  $AP$ .

Let  $M_{12} = \langle S_{12}, S_{12}^0, T_{12}, \mathcal{L}_{12} \rangle$  be a Kripke structure built upon  $M_1$  and  $M_2$ , satisfying the following conditions.

**C1:** condition over the states  $S_{12}$ . There exist two projection functions:

$$\pi_1 : S_{12} \rightarrow S_1 \quad (6)$$

$$\pi_2 : S_{12} \rightarrow S_2 \quad (7)$$

such that

$$\forall s_1 \in S_1, \forall s_2 \in S_2, \exists s_{12} \in S_{12} [\pi_1(s_{12}) = s_1 \wedge \pi_2(s_{12}) = s_2] \quad (8)$$

**C2:** condition over the initial states  $S_{12}^0$ .

$$\forall s \in S_{12} [s \in S_{12}^0 \iff \pi_1(s) \in S_1^0 \wedge \pi_2(s) \in S_2^0] \quad (9)$$

**C3:** condition over the transition relation  $T_{12}$ .

$$\forall s_{12} \in S_{12}, \forall s'_{12} \in S_{12} \left[ s'_{12} \in \text{next}_{M_{12}}(s_{12}) \iff \begin{aligned} & \pi_1(s'_{12}) \in \text{next}_{M_1}(\pi_1(s_{12})) \wedge \\ & \pi_2(s'_{12}) \in \text{next}_{M_2}(\pi_2(s_{12})) \end{aligned} \right] \quad (10)$$

**3.1.1 Corollary** A state  $s_{12} \in S_{12}$  is reachable in  $M_{12}$  iff its projections  $\pi_1(s_{12})$  and  $\pi_2(s_{12})$  are, respectively, reachable in  $M_1$  and  $M_2$ .

$$\text{isReach}_{M_{12}}(s_{12}) \iff \text{isReach}_{M_1}(\pi_1(s_{12})) \wedge \text{isReach}_{M_2}(\pi_2(s_{12})) \quad (11)$$

**Proof** Let's suppose that

$$\exists s_{12} \in S_{12} [isReach_{M_{12}}(s_{12}) \wedge \neg isReach_{M_2}(\pi_2(s_{12}))] \quad (12)$$

If  $s_{12}$  is reachable in  $M_{12}$ , it means that there exists a path  $\psi_{12}^0 = s_1, \dots, s_n \in \Psi_{12}^0$  such that  $s_n = s_{12}$ .  
Let's now consider the following sequence of states in  $M_2$ :

$$\pi_2(s_1), \dots, \pi_2(s_n)$$

By Formula 12 we know that the projection of state  $s_n$  in  $M_2$  is not reachable, i.e.,  $\neg isReach_{M_2}(\pi_2(s_n))$ <sup>3</sup>; this means that  $\exists i \in [1, n-1]: \pi_2(s_{i+1}) \notin next(\pi_2(s_i))$ . But this contradicts the condition on the construction of  $T_{12}$  (formula 10). A similar contradiction is achieved if, at the beginning of the proof, we suppose that

$$\exists s_{12} \in S_{12} [isReach_{M_{12}}(s_{12}) \wedge \neg isReach_{M_1}(\pi_1(s_{12}))]$$

### 3.2 Equivalence checking of Kripke structures

#### Definition 8 (Equivalence of the projections).

We say that a state  $s_{12} \in S_{12}$  is labelly equivalent iff the labelings of the projections are equivalent, i.e

$$le(s_{12}) \triangleq \mathcal{L}_1(\pi_1(s_{12})) = \mathcal{L}_2(\pi_2(s_{12})) \quad (13)$$

We say that two states  $s_{12}, s'_{12} \in S_{12}$  are labelly equivalent with respect to the projection  $\pi_1/\pi_2$  iff the labelings of the projections are equivalent, i.e

$$le_1(s_{12}, s'_{12}) \triangleq \mathcal{L}_1(\pi_1(s_{12})) = \mathcal{L}_1(\pi_1(s'_{12})) \quad (14)$$

$$le_2(s_{12}, s'_{12}) \triangleq \mathcal{L}_2(\pi_2(s_{12})) = \mathcal{L}_2(\pi_2(s'_{12})) \quad (15)$$

#### Definition 9 (Mirror state).

For all states  $s_{12} \in S_{12}$  we define the predicate mirror as:

$$mirror(s_{12}) \triangleq le(s_{12}) \rightarrow \forall s'_{12} \in next(s_{12}) \left( \begin{array}{l} \exists s''_{12} \in next(s_{12}) [le(s''_{12}) \wedge le_1(s'_{12}, s''_{12})] \wedge \\ \exists s'''_{12} \in next(s_{12}) [le(s'''_{12}) \wedge le_2(s'_{12}, s'''_{12})] \end{array} \right) \quad (16)$$

#### Theorem 2 (Equivalence between $M_1$ and $M_2$ ).

$M_1$  and  $M_2$  are equivalent iff the following properties

$$\forall s_{12}^0 \in S_{12}^0, \exists s_{12}^{0'} \in S_{12}^0 [le(s_{12}^{0'}) \wedge le_1(s_{12}^0, s_{12}^{0'})] \quad (17)$$

$$\forall s_{12}^0 \in S_{12}^0, \exists s_{12}^{0''} \in S_{12}^0 [le(s_{12}^{0''}) \wedge le_2(s_{12}^0, s_{12}^{0''})] \quad (18)$$

$$\forall s_{12} \in reach(M_{12}) [mirror(s_{12})] \quad (19)$$

hold in  $M_{12}$ .

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<sup>3</sup>  $s_{12} = s_n$

## 4 Equivalence checking of NuSMV specifications

### Definition 10 (NuSMV model as Kripke structure).

A NuSMV model is a Kripke structure  $M = \langle S, S^0, T \rangle$  where each state of  $S$  is labeled by a predicate  $\bigwedge_{i=1}^r (v_i = d_i)$ , being  $\text{var}(M) = \{v_1, \dots, v_r\}$  a finite fixed set of variables and  $\{d_1, \dots, d_r\}$  their interpretation values over domains  $D_1, \dots, D_r$ ; the transition relation  $T$  expresses the updating of the state variables interpretation by the syntax given in Section 2.1.

### 4.1 Mutated specification

Given a NuSMV model  $M_o = \langle S_o, S_o^0, T_o \rangle$ , we apply a *mutation* to it: we mutate the initial assignments and/or the next state assignments of a set of variables, that is the way in which their initial/next value is calculated. We obtain a machine  $M_{mu} = \langle S_{mu}, S_{mu}^0, T_{mu} \rangle$  with the same state space and the same variables, i.e.,  $S_o = S_{mu}$  and  $\text{var}(M_o) = \text{var}(M_{mu})$ , but, maybe, with a different transition relation  $T_{mu}$  and/or a different set of initial states  $S_{mu}^0$ . If  $S_o^0 = S_{mu}^0 \wedge T_o^0 = T_{mu}^0$ , the two models  $M_o$  and  $M_{mu}$  are said to be *equivalent*, otherwise they are *not equivalent*.

**Partitioning of the variables** The variables  $\text{var}(M_o)$  can be decomposed in subsets depending on the fact that they are *affected* by the mutation or not.

Let  $MV = \{\tilde{v}_1, \dots, \tilde{v}_k\}$  be the set of variables whose initial/next assignment has been mutated. Let  $\tilde{D}_1, \dots, \tilde{D}_k$  be their domains.

Let  $NV$  be the set of variables whose initial/next assignment has not been mutated.  $NV$  can be decomposed in two parts:

- the set of variables  $DV = \{v_{k+1}, \dots, v_n\}$ : a variable  $v_j \in DV$  iff there is a variable  $v_i \in MV$  whose value, in some state, is determined according to the value of  $v_j$  in the current/previous state. Let  $D_{k+1}, \dots, D_n$  be their domains;
- the set of variables  $IN$  that are not considered in the evaluation of the value of any variable in  $MV$ .

The variables of  $M_o$  and of  $M_{mu}$  are the same, i.e.:

$$\text{var}(M_o) = \text{var}(M_{mu}) = MV \cup NV = MV \cup DV \cup IN$$

*Example* Model 1 shows a NuSMV specification and Model 2 another specification obtained from the previous one applying a mutation to it: the relation operator  $\geq$  in the second branch of the case expression in the assignment of variable  $amPm$  has been replaced with the relational operator  $>$ . The obtained partition of the variables is:

- $MV = \{amPm\}$
- $DV = \{h\}$
- $IN = \{h12\}$

Model 3 shows the specification obtained from the merging of the original specification (Model 1) and the mutated one (Model 2). Assignments of variables  $MV$  ( $amPm$ ) and  $DV$  ( $h$ ) are those defined in the original specification. Variables  $MV'$  ( $amPmMut$ ) have been obtained introducing a copy of variables in  $MV$ , appending the suffix *Mut* to their names; their assignments are those defined in the mutated specification. Variables in  $IN$  ( $h12$ ) have not been exported.

<pre> <b>MODULE</b> main <b>VAR</b>   h: 0..23;   h12: 1..12;   amPm: {AM, PM}; <b>ASSIGN</b>   <b>init</b>(h) := 0;   <b>next</b>(h) := (h + 1) <b>mod</b> 24;   h12 :=     <b>case</b>       h <b>in</b> {1, 12}: h;       h &gt; 12: h <b>mod</b> 12;       <b>TRUE</b>: 12;     <b>esac</b>;   amPm :=     <b>case</b>       h &lt; 12: AM;       h &gt;= 11: PM;     <b>esac</b>; </pre>	<pre> <b>MODULE</b> main <b>VAR</b>   h: 0..23;   h12: 1..12;   amPm: {AM, PM}; <b>ASSIGN</b>   <b>init</b>(h) := 0;   <b>next</b>(h) := (h + 1) <b>mod</b> 24;   h12 :=     <b>case</b>       h <b>in</b> {1, 12}: h;       h &gt; 12: h <b>mod</b> 12;       <b>TRUE</b>: 12;     <b>esac</b>;   amPm :=     <b>case</b>       h &lt; 12: AM;       h &gt; 11: PM;     <b>esac</b>; </pre>	<pre> <b>MODULE</b> main <b>VAR</b>   h: 0..23;   amPm: {AM, PM};   amPmMut: {AM, PM}; <b>ASSIGN</b>   <b>init</b>(h) := 0;   <b>next</b>(h) := (h + 1) <b>mod</b> 24;   amPm :=     <b>case</b>       h &lt; 12: AM;       h &gt;= 11: PM;     <b>esac</b>;   amPmMut :=     <b>case</b>       h &lt; 12: AM;       h &gt; 11: PM;     <b>esac</b>; </pre>
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**Model 1.** Original specification

**Model 2.** Equivalent mutant

**Model 3.** Merging specification

## 4.2 Merging specification

Given the NuSMV specifications  $M_o$  and  $M_{mu}$ , we define **merging** specification the NuSMV model  $M_e = \langle S_e, S_e^0, T_e \rangle$  built as follows:

- $var(M_e) = MV \cup MV' \cup DV$ , being
  - $MV = \{\tilde{v}_1, \dots, \tilde{v}_k\}$  the set of all variables of  $M_o$  whose initial assignment and/or next state assignment has been mutated in  $M_{mu}$ .  $\tilde{D}_1, \dots, \tilde{D}_k$  are their domains.
  - $MV' = \{\tilde{v}'_1, \dots, \tilde{v}'_k\}$  a renamed copy of  $MV$ . Their domains are the same of the variables in  $MV$ , that is  $\tilde{D}_1, \dots, \tilde{D}_k$ . There exists a bijective function

$$mut : MV \rightarrow MV'$$

such that  $\forall \tilde{v}_i \in MV (mut(\tilde{v}_i) = \tilde{v}'_i)$ .

- $DV = \{v_{k+1}, \dots, v_n\}$  the set of all non mutated variables of  $M_o$  upon which the value of some mutated variable depends on.
- The initial state assignments of variables in  $MV \cup DV$  are those defined in  $M_o$ , while variables in  $MV'$  have initial assignments as in  $M_{mu}$ .
- The next state assignments of variables in  $MV \cup DV$  are those defined in  $M_o$ , while variables in  $MV'$  have next state assignments as in  $M_{mu}$ .

## 4.3 Equivalence of the projections and mirror state

Since a state in a NuSMV model is identified by the values of its variables, for NuSMV specifications, the predicates  $le$ ,  $le_1$  and  $le_2$  (see Definition 8) can be defined in the following way:

$$\begin{aligned}
 le(s_e) &\triangleq \forall v \in MV \llbracket v \rrbracket_{s_e} = \llbracket mut(v) \rrbracket_{s_e} \\
 &\triangleq \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s_e}
 \end{aligned} \tag{20}$$

$$\begin{aligned}
le_1(s_e, s'_e) &\triangleq \forall v \in (MV \cup DV) \left[ \llbracket v \rrbracket_{s_e} = \llbracket v \rrbracket_{s'_e} \right] \\
&\triangleq \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}_i \rrbracket_{s'_e} \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s_e} = \llbracket v_j \rrbracket_{s'_e}
\end{aligned} \tag{21}$$

$$\begin{aligned}
le_2(s_e, s'_e) &\triangleq \forall v \in (MV' \cup DV) \left[ \llbracket v \rrbracket_{s_e} = \llbracket v \rrbracket_{s'_e} \right] \\
&\triangleq \bigwedge_{i=1}^k \llbracket \tilde{v}'_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s'_e} \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s_e} = \llbracket v_j \rrbracket_{s'_e}
\end{aligned} \tag{22}$$

Applying formulas 20 and 21, the formula  $le(s'_e) \wedge le_1(s_e, s'_e)$  can be written in the following way:

$$\begin{aligned}
le(s'_e) \wedge le_1(s_e, s'_e) &\triangleq \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s'_e} = \llbracket \tilde{v}'_i \rrbracket_{s'_e} \wedge \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}_i \rrbracket_{s'_e} \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s_e} = \llbracket v_j \rrbracket_{s'_e} \\
&\triangleq \bigwedge_{i=1}^k \left( \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}_i \rrbracket_{s'_e} \wedge \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s'_e} \right) \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s_e} = \llbracket v_j \rrbracket_{s'_e}
\end{aligned} \tag{23}$$

Applying formulas 20 and 22, the formula  $le(s'_e) \wedge le_2(s_e, s'_e)$  can be written in the following way:

$$\begin{aligned}
le(s'_e) \wedge le_2(s_e, s'_e) &\triangleq \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s'_e} = \llbracket \tilde{v}'_i \rrbracket_{s'_e} \wedge \bigwedge_{i=1}^k \llbracket \tilde{v}'_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s'_e} \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s_e} = \llbracket v_j \rrbracket_{s'_e} \\
&\triangleq \bigwedge_{i=1}^k \left( \llbracket \tilde{v}'_i \rrbracket_{s_e} = \llbracket \tilde{v}_i \rrbracket_{s'_e} \wedge \llbracket \tilde{v}'_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s'_e} \right) \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s_e} = \llbracket v_j \rrbracket_{s'_e}
\end{aligned} \tag{24}$$

Finally, the predicate *mirror* (see Definition 9) for NuSMV specifications can be defined using formulas 20, 23 and 24.

$$\begin{aligned}
\text{mirror}(s_e) &\triangleq \\
&\left( \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s_e} \right) \rightarrow \\
&\forall s'_e \in \text{next}(s_e) \\
&\left( \begin{aligned} &\exists s''_e \in \text{next}(s_e) \left[ \bigwedge_{i=1}^k \left( \llbracket \tilde{v}_i \rrbracket_{s'_e} = \llbracket \tilde{v}_i \rrbracket_{s''_e} \wedge \llbracket \tilde{v}_i \rrbracket_{s'_e} = \llbracket \tilde{v}'_i \rrbracket_{s''_e} \right) \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s'_e} = \llbracket v_j \rrbracket_{s''_e} \right] \wedge \\ &\exists s'''_e \in \text{next}(s_e) \left[ \bigwedge_{i=1}^k \left( \llbracket \tilde{v}'_i \rrbracket_{s'_e} = \llbracket \tilde{v}_i \rrbracket_{s'''_e} \wedge \llbracket \tilde{v}'_i \rrbracket_{s'_e} = \llbracket \tilde{v}'_i \rrbracket_{s'''_e} \right) \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s'_e} = \llbracket v_j \rrbracket_{s'''_e} \right] \end{aligned} \right)
\end{aligned} \tag{25}$$

#### 4.4 Equivalence checking through CTL properties

**Definition 11 (Both and Either predicates).**

Let

$$\begin{aligned}
\text{Both}(\tilde{d}_{i=1}^k, d_{j=k+1}^n) &\triangleq \bigwedge_{i=1}^k \left( \tilde{d}_i = \tilde{v}_i \wedge \tilde{d}_i = \tilde{v}'_i \right) \wedge \bigwedge_{j=k+1}^n d_j = v_j \\
\text{Either}(\tilde{d}_{i=1}^k, d_{j=k+1}^n) &\triangleq \left( \bigwedge_{i=1}^k \tilde{d}_i = \tilde{v}_i \vee \bigwedge_{i=1}^k \tilde{d}_i = \tilde{v}'_i \right) \wedge \bigwedge_{j=k+1}^n d_j = v_j
\end{aligned}$$

be two predicates such that, given a  $n$ -upla of values  $d = (\tilde{d}_{i=1}^k, d_{j=k+1}^n)$ , *Both*( $d$ ) means that both machines  $M_o$  and  $M_{mu}$  are in the same state  $d$ , while *Either*( $d$ ) means that at least one machine is in state  $d$ .

Let's see now how the formulas described in Theorem 2 can be checked through some CTL properties. Section 4.4.1 describes how to prove properties 17 and 18, Section 4.4.2 how to prove property 19.



#### 4.4.1 Condition on the initial states

##### Definition 12 (Initial state as tuple of values).

Let  $IS$  be the tuples of values of the variables in the initial states, i.e.,

$$IS = \left\{ \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, \tilde{d}'_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) : \right. \\ \left. \exists s_e^o \in S_e^o \left[ \bigwedge_{i=1}^k \left( \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s_e^o} \wedge \tilde{d}'_i = \llbracket \tilde{v}'_i \rrbracket_{s_e^o} \right) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e^o} \right] \right\}$$

Let's also define

$$IS_{MV} = \left\{ \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) : \exists s_e^o \in S_e^o \left[ \bigwedge_{i=1}^k \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s_e^o} \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e^o} \right] \right\}$$

$$IS_{MV'} = \left\{ \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) : \exists s_e^o \in S_e^o \left[ \bigwedge_{i=1}^k \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s_e^o} \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e^o} \right] \right\}$$

By definition of  $IS$ ,  $IS_{MV}$  and  $IS_{MV'}$ , it holds that

$$\forall \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, \tilde{d}'_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) \\ \left( \tilde{d}_{i=1}^k, \tilde{d}'_{i=1}^k, d_{j=k+1}^n \right) \in IS \iff \left( \tilde{d}_{i=1}^k, d_{j=k+1}^n \right) \in IS_{MV} \wedge \left( \tilde{d}'_{i=1}^k, d_{j=k+1}^n \right) \in IS_{MV'}$$

**First condition on the initial states** Using formula 23, formula 17 becomes

$$\forall s_e^o \in S_e^o, \exists s_e^{o'} \in S_e^{o'} \left[ \bigwedge_{i=1}^k \left( \llbracket \tilde{v}_i \rrbracket_{s_e^o} = \llbracket \tilde{v}_i \rrbracket_{s_e^{o'}} \wedge \llbracket \tilde{v}_i \rrbracket_{s_e^o} = \llbracket \tilde{v}'_i \rrbracket_{s_e^{o'}} \right) \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s_e^o} = \llbracket v_j \rrbracket_{s_e^{o'}} \right] \quad (26)$$

Formula 26 can be rewritten, substituting the quantification over the initial states with the quantification over the values of the variables in the initial states (i.e.,  $IS$ ), in the following way

$$\forall \left( \tilde{d}_{i=1}^k, \tilde{d}'_{i=1}^k, d_{j=k+1}^n \right) \in IS, \exists s_e^{o'} \in S_e^{o'} \left[ \bigwedge_{i=1}^k \left( \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s_e^{o'}} \wedge \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s_e^{o'}} \right) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e^{o'}} \right] \quad (27)$$

Note that the interpretations of the variables in state  $s_e$  have been replaced with the actual values of the variables in the state.

Formula 27 can be further simplified, observing that the values of the variables in  $MV'$  (i.e.,  $\tilde{d}'_{i=1}^k$ ) are not used in the propositional formula (matrix) of the existentially quantified subformula. Quantifying over  $IS_{MV}$ , formula 27 can be rewritten in the following way

$$\forall \left( \tilde{d}_{i=1}^k, d_{j=k+1}^n \right) \in IS_{MV}, \exists s_e^{o'} \in S_e^{o'} \left[ \bigwedge_{i=1}^k \left( \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s_e^{o'}} \wedge \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s_e^{o'}} \right) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e^{o'}} \right] \quad (28)$$

**Second condition on the initial states** Using formula 24, formula 18 becomes

$$\forall s_e^0 \in S_e^0, \exists s_e^{0''} \in S_e^0 \left[ \bigwedge_{i=1}^k (\llbracket \tilde{v}'_i \rrbracket_{s_e^0} = \llbracket \tilde{v}_i \rrbracket_{s_e^{0''}} \wedge \llbracket \tilde{v}'_i \rrbracket_{s_e^0} = \llbracket \tilde{v}'_i \rrbracket_{s_e^{0''}}) \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s_e^0} = \llbracket v_j \rrbracket_{s_e^{0''}} \right] \quad (29)$$

Formula 29 can be rewritten, substituting the quantification over the initial states with the quantification over the values of the variables in the initial states (i.e.,  $IS$ ), in the following way

$$\forall (\tilde{d}_{i=1}^k, \tilde{d}'_{i=1}^k, d_{j=k+1}^n) \in IS, \exists s_e^{0''} \in S_e^0 \left[ \bigwedge_{i=1}^k (\tilde{d}'_i = \llbracket \tilde{v}_i \rrbracket_{s_e^{0''}} \wedge \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s_e^{0''}}) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e^{0''}} \right] \quad (30)$$

Formula 30 can be further simplified, observing that the values of the variables in  $MV$  (i.e.,  $\tilde{d}_{i=1}^k$ ) are not used in the matrix of the existentially quantified subformula. Quantifying over  $IS_{MV'}$ , formula 30 can be rewritten in the following way

$$\forall (\tilde{d}_{i=1}^k, d_{j=k+1}^n) \in IS_{MV'}, \exists s_e^{0''} \in S_e^0 \left[ \bigwedge_{i=1}^k (\tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s_e^{0''}} \wedge \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s_e^{0''}}) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e^{0''}} \right] \quad (31)$$

**Unique formula for checking formulas 28 and 31** The matrices of the universally quantified formulas 28 and 31 are the same. So, it is possible to prove both properties, using the following formula.

$$\forall (\tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j) \left( \begin{array}{l} \exists s_e^0 \in S_e^0 \left[ \left( \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e^0} = \tilde{d}_i \vee \bigwedge_{i=1}^k \llbracket \tilde{v}'_i \rrbracket_{s_e^0} = \tilde{d}_i \right) \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s_e^0} = d_j \right] \rightarrow \\ \exists s_e^{0'} \in S_e^0 \left[ \bigwedge_{i=1}^k (\tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s_e^{0'}} \wedge \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s_e^{0'}}) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e^{0'}} \right] \end{array} \right) \quad (32)$$

The proof of the correctness is based on the following theorem.

**Theorem 3.** *Being  $A$ ,  $B$  and  $C$  three domains such that  $A \cup B \subseteq C$ , it holds that*

$$\forall x \in A [f(x)] \wedge \forall y \in B [f(y)] \equiv \forall z \in C [(z \in A \vee z \in B) \rightarrow f(z)]$$

In our case  $IS_{MV} \subseteq \left( \times_{i=1}^k \tilde{D}_i \times \times_{j=k+1}^n D_j \right)$  and  $IS_{MV'} \subseteq \left( \times_{i=1}^k \tilde{D}_i \times \times_{j=k+1}^n D_j \right)$ . So, we can take as  $C$  the domain  $\times_{i=1}^k \tilde{D}_i \times \times_{j=k+1}^n D_j$ .

Formula 32 can be rewritten using the *Both* and *Either* predicates (see Definition 11), in the following way

$$\forall (\tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j) \left( \begin{array}{l} \exists s_e^0 \in S_e^0 \llbracket \text{Either}(\tilde{d}_{i=1}^k, d_{j=k+1}^n) \rrbracket_{s_e^0} \rightarrow \\ \exists s_e^{0'} \in S_e^0 \llbracket \text{Both}(\tilde{d}_{i=1}^k, d_{j=k+1}^n) \rrbracket_{s_e^{0'}} \end{array} \right) \quad (33)$$

**4.4.1.1 Checking equivalence of initial states in NuSMV** In NuSMV, a CTL property  $\varphi$  is true iff it is true starting from each initial state, i.e.,

$$M \models \varphi \quad \text{iff} \quad \forall s_0 \in S^0 (M, s_0) \models \varphi$$

So, if we want to know if a property is true in *at least* an initial state, we must check  $\neg\varphi$ ; if  $M \not\models \neg\varphi$ , it means that there exists an initial state in which  $\varphi$  is true, i.e.,

$$M \not\models \neg\varphi \quad \text{iff} \quad \exists s_0 \in S^0 (M, s_0) \models \varphi$$

So, in order to check the validity of Property 33,  $\forall (\tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j)$ , we first check the CTL property  $\neg\text{Either}(\tilde{d}_{i=1}^k, d_{j=k+1}^n)$ ; if it is false, then we must also check that the CTL property  $\neg\text{Both}(\tilde{d}_{i=1}^k, d_{j=k+1}^n)$  is false.

**Example** In the following, we report some of the CTL properties that must be checked over the specification shown in Model 3 in order to prove the equivalence in the initial states of the specifications shown in Models 1 and 2.

```

CTLSPEC NAME isNotInitState_1 := !((AM = amPm | AM = amPmMut) & h = 0)
CTLSPEC NAME notEqInitState_1 := !((AM = amPm & AM = amPmMut) & h = 0)
CTLSPEC NAME isNotInitState_2 := !((AM = amPm | AM = amPmMut) & h = 1)
CTLSPEC NAME notEqInitState_2 := !((AM = amPm & AM = amPmMut) & h = 1)
--...
CTLSPEC NAME isNotInitState_24 := !((PM = amPm | PM = amPmMut) & h = 0)
CTLSPEC NAME notEqInitState_24 := !((PM = amPm & PM = amPmMut) & h = 0)
CTLSPEC NAME isNotInitState_25 := !((PM = amPm | PM = amPmMut) & h = 1)
CTLSPEC NAME notEqInitState_25 := !((PM = amPm & PM = amPmMut) & h = 1)
...

```

We must check that, if a CTL property *isNotInitState.i* is false, then also the CTL property *notEqInitState.i* is false. In the example, we checked that *isNotInitState\_1* and *notEqInitState\_1* are false, and all the properties *isNotInitState.i*, with  $i = 2, \dots, 48$ , are true: so the two specifications are equivalent in the initial states. Totally we had to check 49 over 96 properties.

## 4.4.2 Condition on the transitions

**Definition 13 (Next state as tuple of values).**

Let  $NS(s)$  be the set of tuples of values of the variables in the next states of  $s \in S$ , i.e.

$$NS(s) = \left\{ \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) : \right. \\ \left. \exists s' \in \text{next}(s) \left[ \bigwedge_{i=1}^k (\tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s'}) \wedge \tilde{d}'_i = \llbracket \tilde{v}'_i \rrbracket_{s'} \right] \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s'} \right\}$$

Let's also define

$$NS_{SMV}(s) = \left\{ \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) : \exists s' \in \text{next}(s) \left[ \bigwedge_{i=1}^k \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s'} \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s'} \right] \right\}$$

$$NS_{MV'}(s) = \left\{ \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) : \exists s' \in next(s) \left[ \bigwedge_{i=1}^k \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s'} \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s'} \right] \right\}$$

By definition of  $NS$ ,  $NS_{MV}$  and  $NS_{MV'}$ , it holds that

$$\forall s \in S, \forall \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, \tilde{d}'_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) \\ \left( \tilde{d}_{i=1}^k, \tilde{d}'_{i=1}^k, d_{j=k+1}^n \right) \in NS(s) \iff \left( \tilde{d}_{i=1}^k, d_{j=k+1}^n \right) \in NS_{MV}(s) \wedge \left( \tilde{d}'_{i=1}^k, d_{j=k+1}^n \right) \in NS_{MV'}(s)$$

**Condition in the transition relation** Applying the definition of the predicate *mirror* (see formula 25), formula 19 becomes

$$\forall s_e \in reach(M_e) \\ \left( \left( \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s_e} \right) \rightarrow \right. \\ \left. \forall s'_e \in next(s_e) \left( \begin{array}{l} \exists s''_e \in next(s_e) \left[ \bigwedge_{i=1}^k (\llbracket \tilde{v}_i \rrbracket_{s'_e} = \llbracket \tilde{v}_i \rrbracket_{s''_e} \wedge \llbracket \tilde{v}_i \rrbracket_{s'_e} = \llbracket \tilde{v}'_i \rrbracket_{s''_e}) \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s'_e} = \llbracket v_j \rrbracket_{s''_e} \right] \wedge \\ \exists s'''_e \in next(s_e) \left[ \bigwedge_{i=1}^k (\llbracket \tilde{v}'_i \rrbracket_{s'_e} = \llbracket \tilde{v}_i \rrbracket_{s'''_e} \wedge \llbracket \tilde{v}'_i \rrbracket_{s'_e} = \llbracket \tilde{v}'_i \rrbracket_{s'''_e}) \wedge \bigwedge_{j=k+1}^n \llbracket v_j \rrbracket_{s'_e} = \llbracket v_j \rrbracket_{s'''_e} \right] \end{array} \right) \right) \right) \quad (34)$$

Formula 34 can be simplified, replacing the universal quantification over the next states of  $s_e$  with the universal quantification over the values of the variables in the next states of  $s_e$  (i.e.,  $NS(s_e)$ ), in the following way

$$\forall s_e \in reach(M_e) \left( \left( \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s_e} \right) \rightarrow \right. \\ \left. \forall \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, \tilde{d}'_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) \in NS(s_e) \right. \\ \left. \left( \begin{array}{l} \exists s''_e \in next(s_e) \left[ \bigwedge_{i=1}^k \left( \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s''_e} \wedge \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s''_e} \right) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s''_e} \right] \wedge \\ \exists s'''_e \in next(s_e) \left[ \bigwedge_{i=1}^k \left( \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s'''_e} \wedge \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s'''_e} \right) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s'''_e} \right] \end{array} \right) \right) \right) \quad (35)$$

In formula 35, in the first existentially quantified subformula, the values of the variables  $MV'$  (i.e.,  $\tilde{d}'_{i=1}^k$ ) are never used, and, in the second existentially quantified subformula, the values of the variables  $MV$  (i.e.,  $\tilde{d}_{i=1}^k$ ) are never used. So, formula 35 can be rewritten, splitting the universal quantification over  $NS(s_e)$  in two universal quantification over  $NS_{MV}(s_e)$  and  $NS_{MV'}(s_e)$ , in the following way

$$\forall s_e \in \text{reach}(M_e) \left( \left( \left( \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s_e} \right) \rightarrow \right. \right. \\ \left. \left( \forall \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) \in NS_{MV}(s_e), \right. \right. \\ \left. \left. \left( \exists s_e'' \in \text{next}(s_e) \left[ \bigwedge_{i=1}^k \left( \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s_e''} \wedge \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s_e''} \right) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e''} \right] \wedge \right. \right. \\ \left. \left( \forall \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) \in NS_{MV'}(s_e), \right. \right. \\ \left. \left. \left( \exists s_e''' \in \text{next}(s_e) \left[ \bigwedge_{i=1}^k \left( \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s_e'''} \wedge \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s_e'''} \right) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e'''} \right] \right) \right) \right) \right) \quad (36)$$

In formula 36, the matrices of the two universally quantified subformulas over  $NS_{MV}(s_e)$  and  $NS_{MV'}(s_e)$  are the same. According to Theorem 3, the conjunction of the two universally quantified subformulas can be replaced by a single formula universally quantified over the bigger domain  $\times_{i=1}^k \tilde{D}_i \times \times_{j=k+1}^n D_j$ . Note that, for any  $s_e \in S_e$ ,  $\times_{i=1}^k \tilde{D}_i \times \times_{j=k+1}^n D_j \supseteq NS_{MV}(s_e) \cup NS_{MV'}(s_e)$ . This is the obtained formula:

$$\forall s_e \in \text{reach}(M_e) \left( \left( \left( \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s_e} \right) \rightarrow \right. \right. \\ \left. \forall d = \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) \right. \\ \left. \left( (d \in NS_{MV}(s_e) \vee d \in NS_{MV'}(s_e)) \rightarrow \right. \right. \\ \left. \left. \left( \exists s_e'' \in \text{next}(s_e) \left[ \bigwedge_{i=1}^k \left( \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s_e''} \wedge \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s_e''} \right) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e''} \right] \right) \right) \right) \right) \quad (37)$$

Formula 37 can be rewritten, transforming the antecedent of the rightmost implication, in the following way

$$\forall s_e \in \text{reach}(M_e) \left( \left( \left( \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s_e} \right) \rightarrow \right. \right. \\ \left. \forall d = \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) \right. \\ \left. \left( \left( \exists s_e' \in \text{next}(s_e) \left[ \left( \bigwedge_{i=1}^k \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s_e'} \vee \bigwedge_{i=1}^k \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s_e'} \right) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e'} \right] \rightarrow \right) \right. \right. \\ \left. \left. \left( \exists s_e'' \in \text{next}(s_e) \left[ \bigwedge_{i=1}^k \left( \tilde{d}_i = \llbracket \tilde{v}_i \rrbracket_{s_e''} \wedge \tilde{d}_i = \llbracket \tilde{v}'_i \rrbracket_{s_e''} \right) \wedge \bigwedge_{j=k+1}^n d_j = \llbracket v_j \rrbracket_{s_e''} \right] \right) \right) \right) \right) \quad (38)$$

Using the *Both* and *Either* predicates (see Definition 11), formula 38 can be rewritten in the following way

$$\forall s_e \in \text{reach}(M_e) \left( \left( \left( \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s_e} \right) \rightarrow \right. \right. \\ \left. \forall \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right) \left[ \begin{array}{l} \exists s_e' \in \text{next}(s_e) \llbracket \text{Either}(\tilde{d}_{i=1}^k, d_{j=k+1}^n) \rrbracket_{s_e'} \rightarrow \\ \exists s_e'' \in \text{next}(s_e) \llbracket \text{Both}(\tilde{d}_{i=1}^k, d_{j=k+1}^n) \rrbracket_{s_e''} \end{array} \right] \right) \right) \quad (39)$$

The inner universal quantifier can be extracted in the following way

$$\forall \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right), \forall s_e \in \text{reach}(M_e) \left( \left( \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s_e} \right) \rightarrow \left[ \begin{array}{l} \exists s'_e \in \text{next}(s_e) \llbracket \text{Either}(\tilde{d}_{i=1}^k, d_{j=k+1}^n) \rrbracket_{s'_e} \rightarrow \\ \exists s''_e \in \text{next}(s_e) \llbracket \text{Both}(\tilde{d}_{i=1}^k, d_{j=k+1}^n) \rrbracket_{s''_e} \end{array} \right] \right) \quad (40)$$

Finally the two implications can be simplified in the following way<sup>4</sup>

$$\forall \left( \tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j \right), \forall s_e \in \text{reach}(M_e) \left( \left( \bigwedge_{i=1}^k \llbracket \tilde{v}_i \rrbracket_{s_e} = \llbracket \tilde{v}'_i \rrbracket_{s_e} \wedge \exists s'_e \in \text{next}(s_e) \llbracket \text{Either}(\tilde{d}_{i=1}^k, d_{j=k+1}^n) \rrbracket_{s'_e} \right) \rightarrow \left( \exists s''_e \in \text{next}(s_e) \llbracket \text{Both}(\tilde{d}_{i=1}^k, d_{j=k+1}^n) \rrbracket_{s''_e} \right) \right) \quad (41)$$

**4.4.2.1 Checking equivalence of the transition relation in NuSMV** In NuSMV, checking property 41 means checking that the following formula

$$\text{AG} \left( \left( \bigwedge_{i=1}^k \tilde{v}_i = \tilde{v}'_i \wedge \text{EX} \left( \text{Either} \left( \tilde{d}_{i=1}^k, d_{j=k+1}^n \right) \right) \right) \rightarrow \text{EX} \left( \text{Both} \left( \tilde{d}_{i=1}^k, d_{j=k+1}^n \right) \right) \right) \quad (42)$$

holds in  $M_e$ ,  $\forall (\tilde{d}_{i=1}^k \in \tilde{D}_i, d_{j=k+1}^n \in D_j)$ .

Formula 42 has been obtained from formula 41 simply applying the semantics of the CTL operators **AG** and **EX**:

- $M \models \text{AG}(\varphi)$  iff  $\forall s \in \text{reach}(M) ((M, s) \models \varphi)$
- $M, s \models \text{EX}(\varphi)$  iff  $\exists s' \in \text{next}(s) ((M, s') \models \varphi)$

**Example** In the following, we report some of the CTL properties that must be checked over the specification shown in Model 3 in order to prove the equivalence of the transition relations of the specifications shown in Models 1 and 2.

**CTLSPEC** NAME `transRelOk.1` :=

**AG** ((`amPm = amPmMut` & **EX** ((`AM = amPm` | `AM = amPmMut`) & `h = 0`)) -> **EX** ((`AM = amPm` & `amPmMut = AM`) & `h = 0`))

**CTLSPEC** NAME `transRelOk.2` :=

**AG** ((`amPm = amPmMut` & **EX** ((`AM = amPm` | `AM = amPmMut`) & `h = 1`)) -> **EX** ((`AM = amPm` & `amPmMut = AM`) & `h = 1`))

...

**CTLSPEC** NAME `transRelOk.24` :=

**AG** ((`amPm = amPmMut` & **EX** ((`PM = amPm` | `PM = amPmMut`) & `h = 0`)) -> **EX** ((`PM = amPm` & `amPmMut = PM`) & `h = 0`))

**CTLSPEC** NAME `transRelOk.25` :=

**AG** ((`amPm = amPmMut` & **EX** ((`PM = amPm` | `PM = amPmMut`) & `h = 1`)) -> **EX** ((`PM = amPm` & `amPmMut = PM`) & `h = 1`))

...

We must check that all the CTL properties *transRelOk.i* are true. As soon as we find a property false, we can stop checking since we will have found that the two specifications are not equivalent. In the example, we checked that all the properties *transRelOk.i*, with  $i = 1, \dots, 48$  are true. So, the two specifications are equivalent (in Section 4.4.1.1 we have also checked that they are equivalent in the initial states): the specification shown in Model 2 is an *equivalent mutant* of the specification shown in Model 1.

<sup>4</sup>  $P \rightarrow (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$

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