Vector Boson Production at Hadron Colliders: A Fully Exclusive QCD Calculation at Next-to-Next-to-Leading Order

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We consider QCD radiative corrections to the production of W and Z bosons in hadron collisions. We present a fully exclusive calculation up to next-to-next-to-leading order (NNLO) in QCD perturbation theory. To perform this NNLO computation, we use a recently proposed version of the subtraction formalism. The calculation includes the γ-Z interference, finite-width effects, the leptonic decay of the vector bosons, and the corresponding spin correlations. Our calculation is implemented in a parton level Monte Carlo program. The program allows the user to apply arbitrary kinematical cuts on the final-state leptons and the associated jet activity and to compute the corresponding distributions in the form of bin histograms. We show selected numerical results at the Fermilab Tevatron and the LHC.

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The production of W and Z bosons in hadron collisions through the Drell-Yan (DY) mechanism [1] is extremely important for physics studies at hadron colliders. These processes have large production rates and offer clean experimental signatures, given the presence of at least one high-p_T lepton in the final state. Studies of the production of W bosons at the Fermilab Tevatron lead to precise determinations of the W mass and width [2]. The DY process is also expected to provide standard candles for detector calibration during the first stage of the LHC running.

Because of the above reasons, it is essential to have accurate theoretical predictions for the vector-boson production cross sections and the associated distributions. Theoretical predictions with high precision demand detailed computations of radiative corrections. The QCD corrections to the total cross section [3] and to the rapidity distribution [4] of the vector boson are known up to the next-to-next-to-leading order (NNLO) in the strong coupling α_S. The fully exclusive NNLO calculation, including the leptonic decay of the vector boson, has been completed more recently [5]. Full electroweak corrections at $\mathcal{O}(\alpha)$ have been computed for both W [6] and Z production [7].

In this Letter we present a new computation of the NNLO QCD corrections to vector-boson production in hadron collisions. The calculation includes the γ-Z interference, finite-width effects, the leptonic decay of the vector bosons, and the corresponding spin correlations. Our calculation parallels the one recently completed for Higgs boson production [8,9], and it is performed by using the same method.

The evaluation of higher-order QCD corrections to hard-scattering processes is complicated by the presence of infrared (IR) singularities at intermediate stages of the calculation that prevents a straightforward implementation of numerical techniques. Despite this difficulty, general methods have been developed in the past two decades, which allow us to handle and cancel IR singularities [10–12] appearing in next-to-leading order (NLO) QCD calculations. In the past few years, several research groups have been working on extensions of these methods to NNLO [13–17], and, recently, the NNLO calculation for $e^+e^- \to$ 3 jets was completed by two groups [18,19]. Parallely, a new general method [20], based on sector decomposition [21], has been proposed and applied to the NNLO calculations of $e^+e^- \to 2$ jets [22], Higgs [23] and vector [5] boson production in hadron collisions, and to some decay processes [24]. Our method [8] applies to the production of colorless high-mass systems in hadron collisions, and is based on an extension of the subtraction formalism [11,12] to NNLO that we briefly recall below.

We consider the inclusive hard-scattering reaction

$$h_1 + h_2 \to V(q) + X, \tag{1}$$

where the collision of the two hadrons h_1 and h_2 produces the vector boson V ($V = Z/\gamma^*$, W^+, or W^-), with four-momentum q and high invariant mass $\sqrt{q^2}$. At NLO, two kinds of corrections contribute: (i) *real* corrections, where one parton recoils against V, and (ii) *one-loop virtual* corrections to the leading order (LO) subprocess. Both contributions are separately IR divergent, but the divergences cancel in the sum. At NNLO, three kinds of corrections must be considered: (i) *double real* contributions, where two partons recoil against V, (ii) *real-virtual* corrections, where one parton recoils against V at one-loop order, and (iii) *two-loop virtual* corrections to the LO subprocess. The three contributions are still separately divergent, and the calculation has to be organized so as to explicitly achieve the cancellation of the IR divergences.

We first note that, at LO, the transverse momentum q_T of V is exactly zero. As a consequence, as long as $q_T \neq 0$, the (N)NLO contributions are actually given by the (N)LO contributions to V + jet(s). Thus, we can write the cross section as

$$d\sigma^{V}_{\rm (N)NLO}|_{q_T\neq 0} = d\sigma^{V+\rm jets}_{\rm (N)LO}. \tag{2}$$

This means that, when $q_T \neq 0$, the IR divergences in our NNLO calculation are those in $d\sigma^{V+\rm jets}_{\rm NLO}$: they can be treated by using available NLO methods to handle and cancel IR singularities (e.g., the general NLO methods in Refs. [10–12]). The only remaining singularities of NNLO type are associated to the limit $q_T \to 0$. We treat them by an additional subtraction, whose general structure [8] can be worked out by exploiting the known singular behavior of $d\sigma^{V+\rm jets}_{\rm (N)LO}$ when $q_T \to 0$. Our extension of Eq. (2) to include the contribution at $q_T = 0$ is [8]

$$d\sigma^{V}_{\rm (N)NLO} = \mathcal{H}^{V}_{\rm (N)NLO} \otimes d\sigma^{V}_{\rm LO} + [d\sigma^{V+\rm jets}_{\rm (N)LO} - d\sigma^{\rm CT}_{\rm (N)LO}]. \tag{3}$$

Comparing with the right-hand side of Eq. (2), we have subtracted the (N)LO counterterm $d\sigma^{\rm CT}_{\rm (N)LO}$ and added a contribution at $q_T = 0$, which is needed to obtain the correct total cross section. The coefficient $\mathcal{H}^{V}_{\rm (N)NLO}$ does not depend on q_T and is obtained by the (N)NLO truncation of the hard-scattering perturbative function

$$\mathcal{H}^{V} = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{V(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{V(2)} + \cdots. \tag{4}$$

According to Eq. (3), the NLO calculation of $d\sigma^V$ requires the knowledge of $\mathcal{H}^{V(1)}$ and the LO calculation of $d\sigma^{V+\rm jets}$. The general (process-independent) form of the coefficient $\mathcal{H}^{(1)}$ is known: the precise relation between $\mathcal{H}^{(1)}$ and the IR finite part of the one-loop correction to a generic LO subprocess is explicitly derived in Ref. [25]. At NNLO, the coefficient $\mathcal{H}^{V(2)}$ is also needed, together with the NLO calculation of $d\sigma^{V+\rm jets}$. The calculation of the general structure of the coefficients $\mathcal{H}^{(2)}$ is in progress. Meanwhile, by using the available analytical results at $\mathcal{O}(\alpha_S^2)$ for the total cross section [3] and the transverse-momentum spectrum [26] of the vector boson, we have explicitly computed the coefficient $\mathcal{H}^{V(2)}$ of the DY process. Since the NLO corrections $d\sigma^{V+\rm jets}_{\rm NLO}$ to $q\bar{q} \to V +$ jet(s) are also available [27], using Eq. (3) we are able to complete our fully exclusive NNLO calculation of vector-boson production.

We have encoded our NNLO computation in a parton level Monte Carlo program, in which we can implement arbitrary IR safe cuts on the final-state leptons and the associated jet activity.

In the following we present an illustrative selection of numerical results for Z and W production at the Tevatron and the LHC. We consider u, d, s, c, b quarks in the initial state. In the case of $W^\pm$ production, we use the (unitarity constrained) Cabibbo-Kobayashi-Maskawa matrix elements $V_{ud} = 0.974\,19$, $V_{us} = 0.2257$, $V_{ub} = 0.003\,59$, $V_{cd} = 0.2256$, $V_{cs} = 0.973\,34$, $V_{cb} = 0.0415$ from the Particle Data Group 2008 [28]. In the case of Z production, additional Feynman diagrams with fermionic triangles should be taken into account. Their contribution cancels out for each isospin multiplet when massless quarks are considered. The effect of a finite top-quark mass in the third generation has been considered and found extremely small [29], so it is neglected in our calculation. As for the electroweak couplings, we use the so-called G_μ scheme, where the input parameters are G_F, m_Z, m_W. In particular we use the values $G_F = 1.166\,37 \times 10^{-5}$ GeV^{-2}, $m_Z = 91.1876$ GeV, $\Gamma_Z = 2.4952$ GeV, $m_W = 80.398$ GeV, and $\Gamma_W = 2.141$ GeV. We use the Martin-Stirling-Thorne-Watt (MSTW)2008 [30] sets of parton distributions, with densities and α_S evaluated at each corresponding order [i.e., we use $(n+1)$-loop α_S at N^{n}LO, with $n = 0, 1, 2$]. For the sake of brevity, we do not show detailed results about the dependence on the renormalization (μ_R) and factorization (μ_F) scales. The distributions presented below are obtained by fixing the scales at the value $\mu_R = \mu_F = m_V$, where m_V is the mass of the vector boson. We limit ourselves to quoting the scale dependence of total and accepted cross sections as obtained by setting $\mu_R = \mu_F = \mu$ and varying μ between $m_V/2$ and $2m_V$.

We start the presentation of our results by considering the inclusive production of e^+e^- pairs from the decay of an on shell Z boson at the LHC. In Fig. 1 (left-hand panel) we show the rapidity distribution of the e^+e^- pair at LO, NLO, and NNLO, computed by using the MSTW2008 partons. The corresponding cross sections are $\sigma_{\rm LO} = 1.761 \pm 0.001$ nb, $\sigma_{\rm NLO} = 2.030 \pm 0.001$ nb, and $\sigma_{\rm NNLO} = 2.089 \pm 0.003$ nb. (Throughout the Letter, the errors on the values of the cross sections and the error bars in the plots refer to an estimate of the numerical errors in the Monte Carlo integration.) The NNLO cross section decreases by about 1.7% setting $\mu = m_Z/2$, and it increases by about 1.5% setting $\mu = 2m_Z$. In Fig. 1 (right-

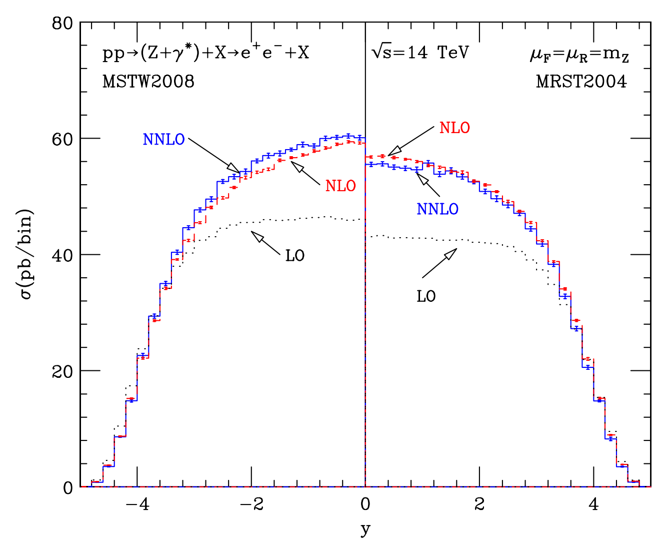

FIG. 1 (color online). Rapidity distribution of an on shell Z boson at the LHC. Results obtained with the MSTW2008 set (left-hand panel) are compared with those obtained with the MRST2004 set (right-hand panel).

hand panel) we also show the results obtained by using the Martin-Roberts-Stirling-Thorne (MRST)2002 LO [31] and MRST2004 [32] sets of parton distribution functions. The corresponding cross sections are $\sigma_{\rm LO} = 1.629 \pm 0.001$ nb, $\sigma_{\rm NLO} = 1.992 \pm 0.001$ nb, and $\sigma_{\rm NNLO} = 1.954 \pm 0.003$ nb. We note that, in going from NLO to NNLO, the total cross section increases by about 3% in the case of the MSTW2008 partons, whereas it decreases by about 2% in the case of the MRST2004 partons. Although the MRST2004 partons are now superseded [30] by the MSTW2008 partons, these results on the rapidity distribution of the Z boson clearly illustrate the nontrivial interplay between QCD radiative corrections and parton distributions. The qualitative behavior of the QCD radiative corrections (and not only their size) can definitely depend on the parton distributions.

We next consider the production of e^+e^- pairs from Z/γ^* bosons at the Tevatron. For each event, we classify the lepton transverse momenta according to their minimum and maximum values, $p_{T\,\rm min}$ and $p_{T\,\rm max}$. The leptons are required to have a minimum p_T of 20 GeV and pseudorapidity $|\eta| < 2$. Their invariant mass is required to be in the range $70 < m_{e^+e^-} < 110$ GeV. The accepted cross sections are $\sigma_{\rm LO} = 103.37 \pm 0.04$ pb, $\sigma_{\rm NLO} = 140.43 \pm 0.07$ pb, and $\sigma_{\rm NNLO} = 143.86 \pm 0.12$ pb. Setting $\mu = m_Z/2$ ($\mu = 2m_Z$), $\sigma_{\rm NNLO}$ varies by about -0.6% ($+0.3\%$). In Fig. 2 we plot the distributions in $p_{T\,\rm min}$ and $p_{T\,\rm max}$ at LO, NLO, and NNLO. We note that at LO the $p_{T\,\rm min}$ and $p_{T\,\rm max}$ distributions are kinematically bounded by $p_T \le Q_{\rm max}/2$, where $Q_{\rm max} = 110$ GeV is the maximum allowed invariant mass of the e^+e^- pairs. The NNLO corrections have a visible impact on the shape of the $p_{T\,\rm min}$ and $p_{T\,\rm max}$ distribution and make the $p_{T\,\rm min}$ distribution softer and the $p_{T\,\rm max}$ distribution harder.

We finally consider the production of a charged lepton plus missing p_T through the decay of a W boson ($W = W^+, W^-$) at the Tevatron. The charged lepton is selected to have $p_T > 20$ GeV and $|\eta| < 2$ and the missing p_T of the event should be larger than 25 GeV. We define the transverse mass of the event as $m_T = \sqrt{2p_T^l p_T^{\rm miss}(1-\cos\phi)}$, where ϕ is the angle between the p_T of the lepton and the missing p_T. The accepted cross sections are $\sigma_{\rm LO} = 1.161 \pm 0.001$ nb, $\sigma_{\rm NLO} = 1.550 \pm 0.001$ nb, and $\sigma_{\rm NNLO} = 1.586 \pm 0.002$ nb. Setting $\mu = m_W/2$ ($\mu = 2m_W$), the accepted cross section at NNLO varies by about -0.8% ($+0.6\%$).

In Fig. 3 we show the m_T distribution at LO, NLO, and NNLO. We note that at LO the distribution has a kinematical boundary at $m_T = 50$ GeV. This is due to the fact that at LO the W is produced with zero transverse momentum: therefore, the requirement $p_T^{\rm miss} > 25$ GeV sets $m_T \ge 50$ GeV. Around the region where $m_T = 50$ GeV there are perturbative instabilities in going from LO to NLO and to NNLO. The origin of these perturbative instabilities is well known [33]: since the LO spectrum is kinematically bounded by $m_T \ge 50$ GeV, each higher-order perturbative contribution produces (integrable) logarithmic singularities in the vicinity of the boundary. We also note that, below the boundary, the NNLO corrections to the NLO result are large; for example, they are about $+40\%$ at $m_T \sim 30$ GeV. This is not unexpected, since in this region of transverse masses, the $\mathcal{O}(\alpha_S)$ result corresponds to the calculation at the first perturbative order and, therefore, our $\mathcal{O}(\alpha_S^2)$ result is actually only a calculation at the NLO level of perturbative accuracy.

As previously mentioned, vector-boson production at NNLO was already considered in Ref. [5]. Our calculation uses a different and completely independent method. Performing some computations of cross sections and acceptances, we have checked that our results and those of the program in Ref. [5] agree within the corresponding numerical accuracy. More detailed numerical comparisons between the results of the two calculations are beyond the scope of this Letter. Owing to the different underlying methods and the ensuing different structures of the two programs, accurate numerical comparisons require a siz-

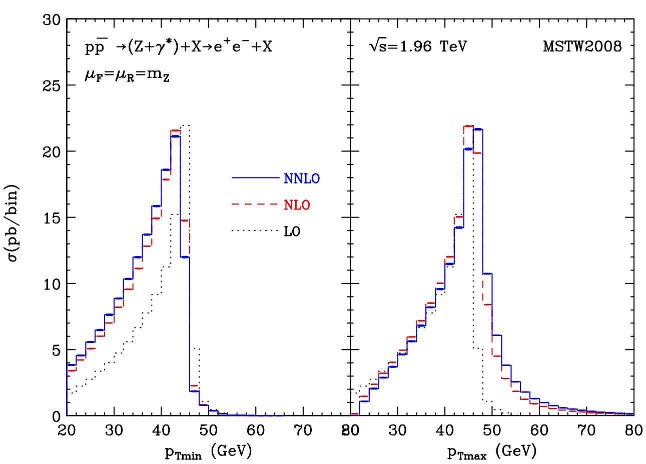

FIG. 2 (color online). Distributions in $p_{T\,\rm min}$ and $p_{T\,\rm max}$ for the Z signal at the Tevatron.

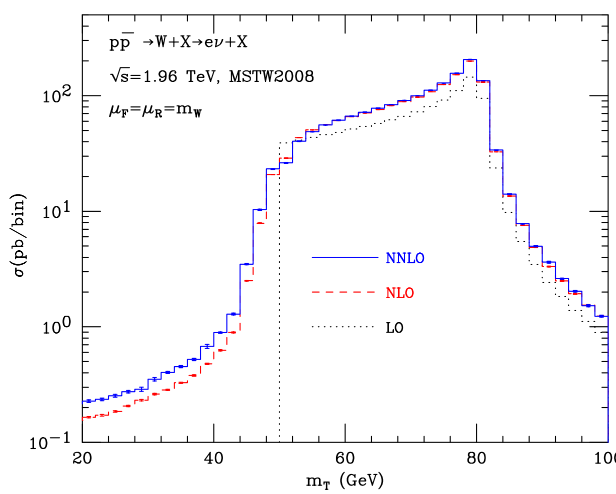

FIG. 3 (color online). Transverse mass distribution for W production at the Tevatron.

able amount of computational work, especially in the case of calculations of kinematical distributions. Notwithstanding, in view of their application to high-precision physics at hadron colliders, these studies are certainly of interest and deserve future investigations.

In this work, we have illustrated a calculation of the cross section for W and Z boson production up to NNLO in QCD perturbation theory. The calculation is directly implemented in a parton level event generator. This feature makes it particularly suitable for practical applications to the computation of distributions in the form of bin histograms. Our program produces numerically stable NNLO results for cross sections and associated distributions. For example, the typical size of the error bars of the NNLO results in the plots of Figs. 1–3 is at the level of about 1%. Higher numerical accuracy is achieved in the case of integrated distributions and cross sections. A public version of the program will be available in the near future.

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