
Discrete Matter, Far Fields, and Dark Matter

A. CARATI¹, S.L. CACCIATORI² and L. GALGANI¹

¹ *Department of Mathematics, University of Milano - Via Saldini 50, I-20133 Milano, Italy*

² *Department of Physical and Mathematical Sciences, Insubria University - Via Valleggio 11, I-22100 Como, Italy*

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Abstract. - We show that in cosmology the gravitational action of the far away matter has quite relevant effects, if retardation of the forces and discreteness of matter (with its spatial correlation) are taken into account. The expansion rate is found to be determined by the density of the far away matter, i.e., by the density of matter at remote times. This leads to the introduction of an effective density, which has to be five times larger than the present one, if the present expansion rate is to be accounted for. The force per unit mass on a test particle is found to be of the order of $0.2cH_0$. The corresponding contribution to the virial of the forces for a cluster of galaxies is also discussed, and it is shown that it fits the observations if a decorrelation property of the forces at two separated points is assumed. So it appears that the gravitational effects of the far away matter may have the same order of magnitude as the corresponding local effects of dark matter.

Introduction. – In most cosmological models usually considered, the matter is dealt with as a continuous medium. The aim of the present paper is to point out how relevant are the gravitational effects which are due to the discreteness of matter, thought of as constituted of galaxies described as point particles, if one takes into account both the role of retardation of the forces (as required by general relativity) and the correlated nature of the positions of the galaxies. Concerning the role of discreteness, the key point is that the gravitational force on a test particle due to a continuous matter with a spherically symmetric density vanishes. Instead, for a matter constituted of point particles whose positions are dealt with as random variables with a spherically symmetric probability distribution, it is only the mean gravitational force that vanishes, while the fluctuations can be very large. This actually is the point of view that was taken by Chandrasekhar and von Neumann in connection with the motions of stars (see the review [1]). They showed that, if the positions of the stars about a test particle are considered as (independent) random variables, then the force on the test particle may be very large; actually, this happens with so huge a probability that the variance of the force is even divergent. It will be shown here that very large fluctuations of the force on a test particle occur also in the case of galaxies. However, while in the case of stars this is due to the occurrence of close encounters, in the case of galaxies the largeness of the fluctuations is instead

due to the gravitational contribution of the far galaxies, when one takes into account both the retarded character of their action and the correlated nature of the positions of the galaxies.

A probabilistic approach in a cosmological context, with galaxies described as point particles, whose positions are dealt with as random variables presenting correlations, is a familiar one since a rather long time; see for example the book of Mandelbrot [2], the book [3] by Peebles and the work [4] by Davis and Peebles. Particular emphasis on the possible fractal nature of matter distribution was given, in addition to Mandelbrot, by several authors. See for example the reviews [5] by Sylos Labini et al., [6] by Coleman and Pietronero and [7] by Combes, and the works [8] by Ruffini et al., [9] and [10] by Gabrielli et al. and finally the work [11] by Joyce et al. Now, in all such papers the nonrelativistic approximation for gravitation was used, and retardation was altogether neglected, so that one is actually dealing with purely static Newtonian gravitational forces.

The main original contribution of the present paper consists in showing that, if retardation is taken into account (together with Hubble's law and the correlated nature of the positions of the galaxies), then the gravitational action of far away matter enters the game and may, in some cases, be the dominant one.

This will be shown by considering an extremely simplified model of the Universe, with the Hubble constant hold

fixed at its present value H_0 . Two results will be obtained. First we show that the influence of the far away galaxies can be described as corresponding to the existence of an effective density which is about five times larger than the present barionic one, i.e., about equal to the usually estimated density of dark matter. Then we look at the force on a test particle. We show that, if the correlated nature of the positions of the galaxies is taken into account, the force (per unit mass) can be estimated as given by $0.2 cH_0$ (c being the speed of light), which is about the value of the acceleration at which the influence of dark matter starts to be felt. Such results thus appear to indicate that far away matter may produce gravitational effects comparable to those usually attributed to local dark matter.

We finally give a preliminary discussion of the problem whether such an estimate of the gravitational action of far away matter may account, through the virial theorem, for the observed velocity dispersion in clusters of galaxies. We show that this is possible, provided the gravitational force of far away matter has a suitable property concerning its dependence on position. Namely, the force should not be smooth, and its values at two separated points should rather be uncorrelated. We point out how the extremely simplified model here considered may not suffice to settle the question whether such a decorrelation property should hold, because the answer may require the introduction of a more realistic model, in which the time variation of Hubble's constant is taken into account. We leave the discussion of this interesting point for future work, and in the present paper we limit ourselves to exhibit, through the simplest conceivable model, how relevant the role of far away matter may be for cosmology, if retardation of the forces (in addition to the correlated nature of the positions of the galaxies) is taken into account.

Definition of the model. – In order to fully take the discrete character of matter into account, one should in principle deal with an N -body problem, in which each particle is coupled to the gravitational field through the Einstein equation having all the other particles as sources. This is however a formidable task. So we introduce first of all the approximation in which one looks at the motion of a test particle, when the motion of the sources is assigned, as given by observational cosmology. This will naturally lead to a compatibility problem, because the test particle too will have to move according to the same law. It will be shown how this compatibility condition is solved through the introduction of a suitable effective density.

As the simplest model for the motion of the sources, we take a velocity field which corresponds to Hubble's law, i.e., we neglect altogether the peculiar velocities (a further comment on this point will be given later). Taking a locally Minkowskian coordinate system centered about an arbitrary point, a particle with position vector \mathbf{q} will then have a velocity

$$\dot{\mathbf{q}} = H_0 \mathbf{q} . \quad (1)$$

For the sake of simplicity of the model, the Hubble con-

stant H_0 will be assumed to be independent of time. On this point we will come back later on. It is easily established that the chart has a local Hubble horizon $R_0 = c/H_0$, where the galaxies have the speed of light. Notice furthermore that the form (1) of Hubble's law is the one appropriate to our choice of Minkowskian coordinates. For example, one could choose, as Davis and Peebles (but not Joyce *et al.*) do, coordinates “*expanding with the background cosmological model*”, with respect to which the galaxies have zero velocity (the peculiar velocities having been neglected); see formula (1), page 426, of [4]. Our choice of coordinates is perhaps more convenient in the present case, but obviously, just by definition, the results do not depend on the choice of the coordinates at all.

So we investigate the gravitational action due to a system of N galaxies whose motions $\mathbf{q}_j(t)$, $j = 1, \dots, N$, are assigned. The energy-momentum tensor $T^{\mu\nu}$ then is

$$T^{\mu\nu} = \sum_{j=1}^N \frac{1}{\sqrt{g}} \frac{M_j}{\gamma_j} \delta(\mathbf{x} - \mathbf{q}_j) \dot{\mathbf{q}}_j^\mu \dot{\mathbf{q}}_j^\nu \quad (2)$$

where M_j , and γ_j are the mass and the Lorentz factor of the j -th particle, g is the determinant of the metric tensor (which is considered as an unknown of the problem), δ the Dirac delta function, and the dot denotes derivative with respect to proper time along the worldline of the source. The velocities of the galaxies are assumed to satisfy Hubble's law (1), while their position vectors \mathbf{q}_j are considered as random variables, whose statistical properties will be discussed later.

The perturbation approach. – The study of the solutions of Einstein's equations with the energy-momentum tensor (2) as a source still is a formidable task, and so we limit ourselves to a perturbation approach, in which the energy-momentum tensor $T^{\mu\nu}$ (2) is considered as a perturbation of the vacuum. Following the standard procedure (see [12] or [13]), we have to determine a zero-th order solution (the vacuum solution), and solve the Einstein equations, linearized about it. The simplest consistent zero-th order solution is the flat metric, because it will be shown that, coherently, the perturbation turns out to be small (at least if the free parameters are chosen in accordance with the observations). We did not investigate whether there do exist other *ansatzs* for the vacuum which give better results. Some further comments on the perturbation procedure will be given later.

Thus the metric tensor $g_{\mu\nu}$ is written as a perturbation of the Minkowskian background $\eta_{\mu\nu}$, namely, as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, and it is well known that in the linear approximation the perturbation $h_{\mu\nu}$ has to satisfy essentially the wave equation with $T^{\mu\nu}$ as a source. More precisely, one gets

$$\square [h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h] = -\frac{16\pi G}{c^4} T_{\mu\nu} , \quad (3)$$

where G is the gravitational constant, h the trace of $h_{\mu\nu}$, and $\square = (1/c^2)\partial_t^2 - \Delta_2$. The solutions are the well known retarded potentials

$$h_{\mu\nu} = \frac{-2G}{c^4} \sum_{j=1}^N \frac{M_j}{\gamma_j} \frac{2\dot{q}_\mu^{(j)}\dot{q}_\nu^{(j)} - c^2\eta_{\mu\nu}}{|\mathbf{x} - \mathbf{q}_j|} \Big|_{t=t_{\text{ret}}} \quad (4)$$

(with $\mathbf{q}^{(j)} \equiv \mathbf{q}_j$).

The mean metric, the compatibility condition and the effective density. – In order to implement in a suitable sense the compatibility condition previously mentioned, we now make reference to the mean metric, which is obtained by averaging over the position vectors of the galaxies, considered as random variables. For a spherically symmetric probability distribution it is immediately seen that the mean of each of the off-diagonal terms vanishes, and that the means of the spatial diagonal components are all equal. Denoting the mean by $\langle \cdot \rangle$, the mean metric at the origin is then

$$ds^2 = \langle g_{\mu\nu} \rangle dx^\mu dx^\nu = (1 - \alpha - 3\beta) c^2 dt^2 - (1 + \alpha + \beta) dl^2$$

where $dl^2 = dx^2 + dy^2 + dz^2$ and

$$\alpha = \frac{2G}{c^2} \left\langle \sum_j \frac{M_j}{|\mathbf{q}_j|} \right\rangle, \quad \beta \lesssim \frac{4GH_0^2}{3c^4} \left\langle \sum_j M_j |\mathbf{q}_j| \right\rangle. \quad (5)$$

This actually is a spatially flat Friedmann–Robertson–Walker metric. We can now formulate the compatibility condition as the requirement that the expansion rate corresponding to such a metric coincide with the one (H_0) that was assumed for the sources. The condition then takes the form

$$\frac{1}{2} \frac{d}{dt} \log \frac{1 + \alpha + \beta}{1 - \alpha - 3\beta} = H_0. \quad (6)$$

The sums (5) defining the coefficients α and β might be estimated through integrals involving a suitable effective matter density. There arises however the problem that, due to the retarded character of the time entering the expressions for α and β , the galaxies lying near the border of the chart are to be taken at times near that of the big bang, at which the density diverges. This by the way shows that only the galaxies at the border are the relevant ones. This very fact, however, also allows one to solve the problem just mentioned, because one can then introduce an effective density ρ_{eff} having the property that both relations

$$\left\langle \sum_j \frac{M_j}{|\mathbf{q}_j|} \right\rangle \simeq 4\pi\rho_{\text{eff}} \frac{R_0^2}{2}, \quad \left\langle \sum_j M_j |\mathbf{q}_j| \right\rangle \simeq 4\pi\rho_{\text{eff}} \frac{R_0^4}{4}$$

hold, with the same effective density. This gives

$$\alpha \simeq 4\pi G \rho_{\text{eff}} R_0^2 / c^2, \quad \beta < (2/3)\alpha. \quad (7)$$

Using $\dot{R}_0 = c$, one then gets

$$\dot{\alpha} \simeq \frac{8\pi G}{c^2} \rho_{\text{eff}} R_0 c, \quad \dot{\beta} \simeq \frac{2}{3} \dot{\alpha}.$$

With these expressions for $\dot{\alpha}$ and $\dot{\beta}$, the consistency condition (6) then becomes an algebraic one, which gives for ρ_{eff} the value

$$\rho_{\text{eff}} \simeq \frac{1}{4} \frac{3H_0^2}{8\pi G} \simeq 5\rho_0, \quad (8)$$

where $\rho_0 = \Omega_0 (3H_0^2)/(8\pi G)$, with $\Omega_0 \simeq 0.05$, is the observed barionic density at the present time. Notice by the way that the perturbation procedure appears to be qualitatively consistent, because the first-order perturbation turns out to be small, of the order of one tenth the unperturbed one.

This is the first result of the present paper. Due to the retarded nature of the potentials, it turns out that the far away galaxies are the ones that give the dominant contribution to the mean metric of the Universe. Moreover, the consistency condition that the expansion rate obtained with such a mean metric be equal to H_0 , determines the value of a corresponding effective density, which is about five times the observed barionic one, i.e., about equal to the estimated density of the dark matter.

Form of the force due to the far away galaxies. –

So far for what concerns the mean metric. We now come to the problem of estimating the effects of the fluctuations on the dynamics of a test particle. The force per unit mass on a test particle is obtained in the familiar way through the equation for the geodesics. Notice that the Hubble relation (1) has here an essential impact. Indeed, the force contains both a term decreasing as $1/r^2$, which is proportional to the velocity of the source, and a term decreasing as $1/r$, which is proportional to the acceleration of the source. Thus, estimating the acceleration too through Hubble's law, the latter term actually turns out not to depend on distance at all, and thus it is again the far away matter that is found to give the dominant contribution. Compare this with the way in which Mach's principle was dealt with in [12] (see page 102). There, lacking Hubble's law, the velocities of the sources were neglected. Thus, only the Newtonian, fast decaying, potential was considered, and so only the near matter, and not the far one, appeared to play a role.

So we address our attention to the dominant term of the gravitational force per unit mass, namely, the one proportional to the acceleration of the source. Such a term, which we denote by \mathbf{f} , has, at the origin, the form

$$\mathbf{f} = \frac{4GH_0^2 M}{c^2} \mathbf{u}, \quad \mathbf{u}(N) = \sum_{j=1}^N \frac{\mathbf{q}_j}{|\mathbf{q}_j|} \quad (9)$$

with the positions \mathbf{q}_j of the N galaxies are evaluated at corresponding retarded times. Here, the masses of the galaxies were all put equal to a common value M , and the Lorentz factors γ_j were put equal to 1, for the reasons to be illustrated later. So, apart from a multiplicative factor, such a force just reduces to the sum of the unit vectors pointing to each of the galaxies at the corresponding retarded time. Actually, our attention was addressed

to the component of such a force \mathbf{f} along a given direction. Such a component will be simply denoted by f , and the corresponding component of \mathbf{u} by u .

Estimate of the force. Role of the probabilistic assumptions for the distribution of galaxies. – Having determined the quantity f of interest (or equivalently u), we now come to the problem of how to describe the distribution of the galaxies. It is immediately seen that f exactly vanishes (at any point) if the matter is described as a continuous medium with a spherically symmetric density. From the probabilistic point of view considered here, such a result (the vanishing of f for a spherically symmetric matter density) now reads as the vanishing of the mean value of f for a spherically symmetric probability density of the position of a galaxy.

We thus come to an estimate of the variance of the force f (or of u). It will be seen that the result depends on the further assumptions one introduces concerning the spatial distribution of the galaxies. Assume first that the positions \mathbf{q}_j of the N galaxies are independent random variables, uniformly distributed with respect to the Lebesgue measure. Then the sum defining u is found to grow as \sqrt{N} , just in virtue of the central limit theorem. For what concerns the estimate of the force on a test particle, one easily sees that with the present assumption it is completely negligible, just because the considered sum behaves as \sqrt{N} rather than as N (see later).

So we modify such an assumption and, following all the previously mentioned authors, we consider the case in which the position vectors of the galaxies present a correlation, i.e., are no more independently distributed. Thus, the sum defining u is no more constrained to grow as \sqrt{N} , and can have a faster growth. Just for the sake of concreteness, we fix our model by requiring that the probability density corresponds to a fractal of dimension 2. In such a way, however, the analytical computation of the probability distribution of the force becomes a quite nontrivial task with respect to the Poissonian case considered by Chandrasekhar and von Neumann, and also with respect to the fractal, but purely Newtonian, case considered by Gabrielli *et al.* in the papers [9] and [10]. So we are forced, at least provisionally, to investigate the problem by numerical methods.

We proceeded as follows. In order to estimate the sum defining u , the positions of the N galaxies were extracted (with the method described in [2]) in such a way that the mass distribution has fractal dimension 2. We then studied the dependence of u on the number N of galaxies, which was made to vary in the range $1000 \leq N \leq 512,000$, the density being kept constant. This means that the positions of the N points were taken to lie inside a cutoff sphere whose volume was made to increase as N . For the values of N investigated, the corresponding radius turns out to be so small with respect to the present horizon, that the Lorentz factors γ could altogether be put equal to 1 (as was previously assumed), and more in general the special

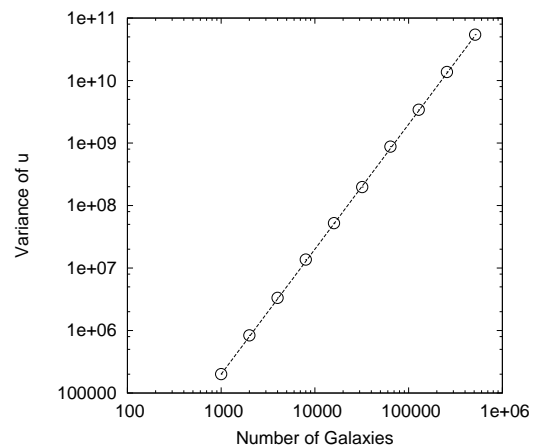


Fig. 1: The variance σ_u^2 of u versus the number N of galaxies in log–log scale. The dashed line is the curve $\sigma_u^2 = 0.2 N^2$.

relativistic character of our model was actually justified.

The mean of u turns out to practically vanish for all N , while its variance σ_u^2 is found to grow as N^2 (actually, as $0.2 N^2$), rather than as N , as occurs in the uniform case. This is shown in Fig. 1. The standard deviation σ_f is thus proportional to N , being given by

$$\sigma_f \simeq \sqrt{0.2} \frac{4GH_0^2}{c^2} MN = \sqrt{0.2} \frac{4G}{R_0^2} MN. \quad (10)$$

We now take such a result, which was obtained for extremely small values of N , and extrapolate it up to the present horizon $R_0 = c/H_0$, i.e., we insert in formula (10) the actual value of N , so that the quantity MN can be identified with the total visible mass of the Universe. The latter can be written as $MN = (4/3) \pi \rho_{\text{eff}} R_0^3$, in terms of the effective density ρ_{eff} previously discussed. This gives $\sigma_f \simeq 0.2 cH_0$.

On the other hand, if a random variable f has zero mean and a finite variance σ_f^2 , with great probability its modulus will take on values very near to its standard deviation σ_f . In such a sense we may say to have found

$$|f| \simeq 0.2 cH_0, \quad (11)$$

which constitutes the second result of the present work. Namely, in our oversimplified model the force per unit mass, i.e., the acceleration, exerted by the far matter on a test particle, is found to have a value of the order of cH_0 , which is the one that is met in most cases in which the presence of a dark matter is advocated. Notice that the assumption of a uniform, rather than correlated, distribution of matter would lead instead to $|f| \simeq cH_0/\sqrt{N}$, i.e., essentially to $f \simeq 0$. Namely, as previously mentioned, without the correlation hypothesis for the positions of the galaxies, the usual procedure of neglecting at all the gravitational contribution of the far away matter, would be justified. We expect that the coefficient $\sqrt{0.2}$ in (10) may depend on the degree of correlation chosen for the positions of the galaxies. This point will be investigated elsewhere.

Notice that this force, acting on each test particle, also acts on each galaxy itself, thus producing an acceleration which should be added to the one given by Hubble's law. On the other hand, such an acceleration was neglected in our model, because the peculiar velocities were assumed to vanish, so that we have here a consistency problem. In this connection we notice that this acceleration is small with respect the Hubble acceleration $H_0 c$ of the far away galaxies (which are the relevant ones), so that our procedure seems to be consistent. A more accurate discussion of this point is left for future work.

Possible application to the virial of the forces for a cluster of galaxies. – We now address the problem whether the previous result may be applied to estimating the virial of the external forces for a cluster of galaxies. We have in mind the work of Zwicky [14] for the Coma cluster, in which the contribution of the internal matter was found to be negligible, that of the external galaxies was not even mentioned (perhaps, in the spirit of the continuum approximation), and the presence of a dark matter was proposed. Let us recall that, according to the virial theorem, for a confined system constituted by n particles (think of a cluster of galaxies) one has $\overline{\sigma_v^2} = -\overline{\mathcal{V}}/n$. Here, $\sigma_v^2 = (1/n) \sum_i v_i^2$ is the variance of the velocity distribution of the particles (the galaxies of the cluster), whereas $\mathcal{V} = \sum_i \mathbf{f}_i \cdot \mathbf{x}_i$ is called the virial of the forces (per unit mass), \mathbf{x}_i denoting the position vector of the i -th internal particle with respect to the center of mass of the cluster, while overline denotes time-average. It was shown by Zwicky that the contribution of the internal forces is negligible, so that in estimating the virial we can just consider the force due to the external galaxies.

It is well known that the virial of the external forces (per unit mass) equals the virial of the tidal force (per unit mass) $f - f^*$ where f^* is the force (per unit mass) at the center of mass, because the contribution of f^* vanishes. The key point now is that the contribution of the tidal forces depends on the smoothness properties of the field of force f . Indeed it turns out that, if the field is smooth, so that the tidal force can be estimated through a Taylor expansion, then one finds $\overline{\mathcal{V}}/n \simeq H_0^2 L^2$ where L is the linear dimension of the cluster. For the Coma cluster this contribution turns out to be negligible.

If one instead assumes that the forces at different points be uncorrelated, then it turns out that the contribution may be of the correct order of magnitude.

Indeed this assumption has two deep consequences. The first one is that it makes conceivable that locally, in some regions, the random field of force may form patterns of a central-like type, which are attractive towards a center, with a nonvanishing force at the center. By the way, this is equivalent to the fact that locally, in such special regions, the external far away matter produces a pressure. The second consequence is that in such a case the variance of the tidal force $f - f^*$ just equals $\sqrt{2}$ the variance of the force f , the estimate of which was given in formula (11).

So, having assumed that the tidal force be of central-like type, the terms of the sum $\sum_{i=1}^n (\mathbf{f}_i - \mathbf{f}^*) \cdot \mathbf{x}_i$ can be estimated as $(\mathbf{f}_i - \mathbf{f}^*) \cdot \mathbf{x}_i \simeq -\sqrt{2}|f| |\mathbf{x}_i|$, with f given by (11), and with $|\mathbf{x}_i| \simeq L/4$, where L is the diameter of the cluster. So, for the velocity variance one gets

$$\overline{\sigma_v^2} \simeq \sqrt{2} 0.05 c H_0 L \simeq 0.07 c H_0 L . \quad (12)$$

In the case of Coma one thus finds a value $\simeq 8 \cdot 10^5 \text{km}^2/\text{sec}^2$, which is very near to the value $5 \cdot 10^5 \text{km}^2/\text{sec}^2$ reported by Zwicky.

The prediction that the velocity variance depends linearly on L , according to (12), may be of interest, and apparently is in agreement with the observations (see [15], Fig. 2, page 539, and [7]). Notice that, with the parameters entering the problem, the square of a velocity can be formed only as c^2 , or as $c H_0 L$ or as $(H_0 L)^2$. But the first term is by far too large, the last term (as previously pointed out) by far too small, while the term linear in L is indeed about of the correct order of magnitude. Thus, the previous considerations appear to indicate that the decorrelation assumption for the forces at different points is necessary in order that the observed velocity dispersion in a cluster may be ascribed to the gravitational action of the far away galaxies.

We now briefly address the question of understanding which mechanisms might be responsible for such a decorrelation. We have in mind two mechanisms. The first one is suggested by the analysis made in the paper [11] of Joyce et al., in which the Newtonian contribution to the tidal force is estimated, albeit in a different context. Indeed in such a paper it is shown (see page 418) that the Newtonian contribution to the tidal force is finite, whereas the purely Newtonian non tidal contribution would be divergent, at least for certain values of the fractal dimension. On the other hand, the latter quantity is just of the same order of magnitude of the tidal force corresponding to our far fields, and so such a result suggests that the tidal force due to the far fields may be divergent. This in turns may be considered as an indication of decorrelation. The second mechanism has instead a cosmological character, and is related on the one hand to the fact that the cosmological horizons relative to different galaxies do not coincide, and on the other hand to the fact that the main contribution to the force comes from the matter near the horizon. Remark in addition that the distributions of matter about two different horizons should be considered as independent ones (the horizons being non causally connected), one is led to conceive that also the corresponding forces might be independent.

A consistent discussion of this point would require the consideration of a more realistic model, in which the time dependence of Hubble's constant be taken into account. So we leave a discussion of this point for future work.

A comment on the perturbation approach. – In the present paper we have chosen a perturbation approach in which the zero-th order solution is the flat metric,

and consistently the first-order correction was found to be small with respect to the unperturbed one.

One might imagine that a better approximation be obtained if the mean metric, i.e., the Friedmann–Robertson–Walker one, is taken directly as zero-th order solution. One easily sees, however, that such an approximation scheme meets with two difficulties. The first one is that in such a case the source entering the equation for the first order solution is of the same order as the zero-th order source. Indeed, the source is proportional to the quantity

$$T^{\mu\nu} - \langle T^{\mu\nu} \rangle$$

which is not small, as its modulus is almost everywhere equal to that of $\langle T^{\mu\nu} \rangle$. A more serious difficulty is the fact that the first-order perturbation has to satisfy essentially d'Alembert's equation with the source just mentioned, while such an equation cannot be solved by elementary methods, and it is not even known whether it admits bounded solutions at all. So, in paper [4], Davis and Peebles, who use such a perturbation procedure for the analogous nonrelativistic case, have to introduce a suitable resummation procedure. Now it is not clear whether a similar resummation procedure can be introduced also in our relativistic case, and furthermore in our case a discussion of the boundedness of the solution would be required because, at variance with Davis and Peebles, we are not restricting ourselves to the case of short distances.

Now, neither of the mentioned difficulties comes in with our perturbation procedure. In addition, it seems to us that the procedure of David and Peebles eventually is equivalent to the nonrelativistic version of our procedure. Indeed, their formula (12) is equivalent to our formula (3), taken in the nonrelativistic approximation, while their formula (14) just gives the contribution to the force due to the near galaxies. This contribution occurs also in our case, and does not appear explicitly in our formula (9), only because in the latter we just retained the dominant contribution due to the far away galaxies.

It is worth mentioning that a perturbation about the FRW metric is performed also by Joyce *et al.* in the paper [11]. But in their case they take into account the fact that the zero-th order solution is due to the radiation energy density, so that the energy–momentum tensor due to matter is not a perturbation of the vacuum. Thus they do not meet with the previously mentioned problems.

Conclusions. – In conclusion, we have studied the retarded gravitational action of the far away galaxies. Such an action vanishes if the matter in the Universe is described in terms of a continuous spherically symmetric continuum. We have pointed out that such an action is instead quite relevant if the discrete character of matter, as constituted of galaxies with correlated positions, is taken into account. Some gravitational effects were estimated, and were found to have the same order of magnitude as the corresponding local ones of dark matter.

It is sometimes stated [16] that the fractal picture of the Universe may be incompatible with the framework of the standard cosmological theories, and in the paper [11] by Joyce *et al.* a solution was proposed, based on the idea that the contribution of matter to Einstein's equations should be considered as a perturbation to the contribution of radiation. Perhaps the present approach, in which a perturbation to the vacuum is performed, and the FRW metric is obtained in the mean (even if radiation is altogether neglected), may be considered as providing an alternative complementary solution to the problem.

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