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EQUILIBRIUM AND *TÂTONNEMENT* IN WALRAS'S *ELÉMENTS*

Franco Donzelli*
University of Milano
Department of Economics, Business and Statistics

Two alternative interpretations of the equilibrium notion employed by Walras in the *Eléments* and related writings have been recurrently suggested in the literature: a 'stationary' interpretation and an 'instantaneous' (specifically, temporary) one. Such persistent oscillations in the reading of Walras's equilibrium concept lead one to suspect that some ambiguity or inconsistency may possibly exist within Walras's original formulation of general equilibrium theory. In this paper we explore this conjecture focussing, in particular, on the relationship between the theory of the *tâtonnement*, regarded by Walras as an essential part of his overall equilibrium approach, and Walras's own interpretation of the equilibrium concept as employed in the *Eléments*.

To this end we analyze the evolution of the *tâtonnement* construct over the four editions of the *Eléments* published during Walras's lifetime (1874-1877, 1889, 1896, 1900), separately referring to each of the four nested equilibrium models (exchange, production, capital formation, circulation and money) developed by Walras therein. From our analysis it turns out that either interpretation can indeed be traced back to Walras's original treatment of the *tâtonnement* process and associated equilibrium notion in one or the other of the equilibrium models he puts forward in the various editions of the *Eléments*: specifically, while the 'instantaneous' interpretation goes back to the exchange model, as dealt with in the second (1889) and subsequent editions, the 'stationary' interpretation is instead grounded in Walras's theory of the *tâtonnement* process in production, as developed in the second (1889) and third (1896) editions of the *Eléments*. Yet, it is only the 'instantaneous' interpretation to pass the internal consistency test, as Walras himself eventually realizes in preparing the fourth (1900) edition of his work, where in fact only the 'instantaneous' (specifically, temporary) equilibrium notion survives.

1. Introduction

In July 1874 Walras eventually managed to send to press the first installment of the first edition of his *magnum opus*, the *Eléments d'économie politique pure*, a book that was bound to become, in the course of time, the source of one of the most impressive theoretical constructions in economics, namely, general equilibrium theory (shortly, GET). Yet, more than one hundred and thirty years after that fateful event, and in spite of its enduring influence on the subsequent developments of economic theory, Walras's own conception of the way in which the

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* Address for correspondence: e-mail: franco.donzelli@unimi.it

working of a competitive economy should be modelled is still surrounded by a number of obscurities: in particular, the exact nature of the equilibrium notion actually employed by Walras in his own version of GET is still controversial.

Indeed, about one century after the first appearance of the *Eléments,* a consensus apparently emerged among the practitioners of GET concerning the interpretation of the equilibrium concept originally used by Walras in his theoretical writings: as a matter of fact, in the late 1970s and early 1980s, authors conversant with both economic theory and history of economic analysis, such as Morishima (1977 and 1980) and Diewert (1977), forcefully stressed, in the context of formalized reconstructions of Walras's conception, that the equilibrium notion permeating the *Eléments* ought to be interpreted as an early, but none the less sophisticated, instance of that equilibrium notion that, after Hicks 1939, had come to be referred to as «temporary equilibrium» (Grandmont 1982). A few years later, during the 1980s and the 1990s, the temporary equilibrium nature of Walras's theory was further clarified, with particular reference to its place in the historical evolution of GET, by van Witteloostuijn and Maks (1988 and 1990), and Donzelli (1990, 1993 and 1997).

Yet, quite recently, when the whole interpretative issue might appear to have been satisfactorily settled, an alternative interpretation of Walras's theory has made once again its appearance among the students of GET. One interesting instance of such an alternative interpretation can be found in Magill and Quinzii (1998), an influential advanced textbook which, while reviewing the modern analytical developments of the theory of incomplete markets, also provides a historical reconstruction, unusually extensive for contemporary standards, of the origins of GET. There we can find the following statement:

> Walras (1874) conceived his general equilibrium model as a genuine intertemporal model, that is, time and capital were to play an essential role. In the end, analytical difficulties forced him to confine his attention to a special equilibrium, namely a steady state in which all prices remain unchanged and the sole price linking adjoining periods is the rate of interest (Magill and Quinzii 1998, 132).

The idea that Walras's theory can be (or, better, ought to be) interpreted as either a steady-state or a stationary equilibrium theory is deeply rooted in the history of GET: as a matter of fact, such an idea can be traced back at least to Wicksell 1954 [1893], 164-168, and 1934 [1901], 226, fn. 1; after that, it has been further adumbrated, albeit with much caution and a number of provisoes, by such authorities on Walras's thought as Schumpeter 1961 [1912] and Hicks (1934, 346). William Jaffé, the translator of Walras's *Eléments* into English (Walras 1954), and certainly one of the best connoisseurs of Walras's works, has occasionally ended up by wrongly translating some crucial words or passages (in particular, the word «statique», which is sometimes mistakenly rendered as «stationary» (Walras 1954, 269) in order to support his own stationary interpretation of Walras's equilibrium theory, an interpretation that is frequently betrayed by Jaffé's own notes and comments (see, *e.g.*, Walras 1954, 537, 542, 550). Ironically, Jaffé's occasionally wrong translations have in turn originated bewilderment and further mistakes in later commentators, such as Collard (1973, 472-473).

As is well-known, however, the most influential source of the stationary equilibrium interpretation of Walras's theory, at least as far as the inter-war period is concerned, is represented by Cassel's textbook, *The Theory of Social Economy*, published in English version in 1932. Even if the *Eléments* are the manifest source of Cassel's inspiration, in *The Theory of Social Economy* Walras's works are nowhere cited. Owing to this curious omission, Cassel is charged with plagiarism by Wicksell (1919, 225). Indeed, Wicksell's charge is partly justified: in fact, from a purely formal point of view, the stationary equilibrium model put forward by Cassel in chapter IV of his book is very similar to the production model developed by Walras in *Section* IV of the *Eléments* (*Section* III in the second and third editions). Owing to such similarity, Cassel's stationary equilibrium model, soon renamed as the Walras-Cassel model, was often read as a simplified version of Walras's own theory, thereby concurring to the diffusion and strengthening of the stationary interpretation of the latter.

From what has just been said, it appears that, since the publication of the *Eléments*, the stationary equilibrium interpretation of Walras's theory has exhibited an extraordinary resilience, alternating over time with the temporary equilibrium interpretation of the same theory and effectively competing with the latter for the primacy within the economics profession. In the light of this, one is led to suspect that some ambiguity or inconsistency may possibly exist within Walras's original formulation of GET, or even within the fundamental structure of GET as such, explaining the persistent oscillations in the interpretation of Walras's equilibrium concept over time.

In the following we intend to explore this conjecture. A natural starting point for such inquiry is represented by the examination of a special theory playing a fundamental role in the characterization of Walras's equilibrium notion, namely, the theory of the *tâtonnement*. Such a theory, dating back to the very origin of Walras's approach, *i.e.*, to the preparatory writings preceding the publication of the first edition of the *Eléments*, is regarded by Walras as an essential part of his equilibrium theory, as he repeats over and over again in a number of letters to his followers and scientific correspondents (Jaffé 1965, vol. II, 364, 370, 385, 387, 404). Yet, Walras will never be fully satisfied with his own handling of this issue: in fact, the theory of the *tâtonnement* is probably the part of the *Eléments* which goes through the most frequent and radical revisions over the years. However, Walras's unremitting efforts notwithstanding, the treatment of the *tâtonnement* remains obscure, self-contradictory and, on some occasions, definitely wrong in all the editions of the *Eléments*.

Controversy over this theory has been long-lasting and extensive. With his review of the second edition of the *Eléments* in 1889, Edgeworth starts a series of three notes, critically commenting upon various aspects of Walras's theory, among which the theory of the *tâtonnement* has the right of place (Edgeworth 1889a, 1889b, 1891); more than three decades later, Edgeworth (1925, vol. II, 310-312) will come back once again to the same topic. Spurred by Walras himself, who is seriously annoyed by Edgeworth's attacks to the theory of the *tâtonnement* and is looking for a possible advocate of his own theoretical approach (Jaffé 1965, vol. II, 364), Bortkiewicz (1890, 358-359) counteracts Edgeworth's initial criticisms, developing what will become the standard reply to the objections concerning the *tâtonnement* in the small community of Walras's followers; in fact, a few years later Pareto 1964 [1896-1897], 24-25, in taking Walras's part in the *tâtonnement* controversy, will embrace Bortkiewicz's line of defence without any change. Yet, if Edgeworth's 1889 review of the second edition of the *Eléments* is the first published paper where Walras's theory of the *tâtonnement* is explicitly criticized, it is actually Bertrand's 1883 review-article of Walras's *Théorie mathématique de la richesse sociale* which opens the fire against the substance (if not the name, since the word *tâtonnement* is nowhere mentioned in Bertrand's paper) of Walras's treatment of the equilibration process.

Half a century after Bertrand's early attack, very similar questions concerning Walras's *tâtonnement* theory are raised by Hicks (1934, 342-343), who will thereafter repeatedly come back to the same issue in his later writings. After Hicks's rediscovery of the *tâtonnement* story in the mid-1930s, the debate about the reconstruction and possible formalization of such highly controversial part of Walras's theory tends to spread everywhere and to become endemic. Just to mention a few authors who have been directly concerned with the interpretation of Walras's *tâtonnement* theory, we may recall the following: Jaffé 1935, 1954, 1965, 1967, 1980, 1981; Lange 1944, esp. 94-97; Goodwin 1951, 1953; Patinkin 1956, esp. 377-385, and 1965, esp. 531-540; Uzawa 1960; Newman 1965, 101-103; Walker 1970, 1972, 1987a, 1987b, 1988, 1996; Arrow and Hahn 1971, esp. 263-323; Morishima 1977, esp. 27-45, and 1980; Varian 1984, 244-249; Huck 2001; Bridel and Huck 2002a, 2002b; Rebeyrol 2002; Costa 2002.

The whole debate is full of conflicting views and rich in *coups de théâtre*, including a few unexpected recantations of the positions previously held by some authoritative interpreters of Walras's approach: of the latter, in fact, we count at least three, starting from Hicks's unexplained change of mind about the true meaning of Walras's original assumptions concerning the *tâtonnement* (Hicks 1939, 128, and 1976, 151), to arrive at Jaffé's (1981, 315-316) and Walker's (1988, 306) explicit recognitions that some of the claims they had previously made about Walras's analysis of the adjustment process were mistaken. Yet, in spite of the duration of the controversy and the variety of the stances advocated therein, to the best of our knowledge none of the participants in this apparently never-ending hermeneutical and theoretical exercise has ever suggested that there might exist some connection between the uncertain epistemological status of Walras's theory of *tâtonnement* and the similarly dubious status of the equilibrium notion employed by the same author and his followers. It is our intention to fill this gap in the literature.

The paper is organized as follows. In section 2, after surveying the main steps in the evolution of Walras's theoretical ideas and reviewing the intricate vicissitudes of the various editions of the *Eléments* and related works, we shall focus on the peculiar structure of Walras's theoretical system, which is made up of four nested equilibrium models, arranged in order of increasing scope and usually referred to as the models of exchange, production, capital formation, and circulation and money (the last model being introduced only in the fourth edition of the *Eléments*). Further, after explaining what Walras exactly means by 'solving' each one of these models, we shall investigate the relationships between what Walras regards as the two constituent 'halves' of equilibrium theory: the 'mathematical' or 'theoretical' solution, provided by a model in equational form satisfying certain determinacy requirements, on the one hand, and the 'practical' or 'empirical' solution, directly provided by the market by means of the *tâtonnement* process, on the other.

Sections 3, 4, and 5 will analyze Walras's treatment of equilibrium and *tâtonnement* in the models of exchange, production, and capital formation, respectively, with exclusive reference to the first two editions of the *Eléments* and the other writings of the same period (1874-1889). Section 3, in particular, will focus on the exchange model. In this regard, we shall first specify the 'Walrasian rule of price adjustment', underlying the *tâtonnement* process in exchange. Then we shall show that, after a relatively imprecise and realistically naïve handling of the disequilibrium process in the first edition, paving the way to alternative and contradictory interpretations of the *tâtonnement* in exchange, Walras makes the analysis much more precise in the second edition, by explicitly introducing a 'no trade out of equilibrium' assumption. The final outcome is that the *tâtonnement* becomes a virtual process, where no observable disequilibrium behavior is allowed; hence, while the *tâtonnement* in exchange must be supposed to take place in a purely 'logical' time, distinct from the 'real' time over which the economy evolves, the associated equilibrium, supposedly reached in one single instant of 'real' time, becomes 'instantaneous' in nature.

Section 4 will examine Walras's treatment of equilibrium and *tâtonnement* in the production model in the first two editions of the *Eléments*. In this regard, after specifying the 'Walrasian rule of quantity adjustment', underlying the *tâtonnement* process in production, we shall show that, over the whole period under discussion, and even beyond, Walras keeps to the assumption that disequilibrium production entails observable behavior, taking place in 'real' time and producing actual consequences. This assumption, which cannot be easily reconciled with the coexisting treatment of the *tâtonnement* process in exchange, gives also rise to further specific difficulties, which Walras desperately endeavors to face: to this end, in the first edition he makes the unpalatable assumption of a 'foreign market', accommodating all 'domestic' quantitative disequilibrium; in the second edition, instead, while giving up all recourse to an external *deus ex machina*, he resorts to the alternative assumption of a stationary economy, identically repeating itself period after period under unchanging conditions. When applied to the production model, this assumption gives rise to the prototype of the stationary equilibrium model, later resumed under the species of the Walras-Cassel model, on which the stationary interpretation of Walras's equilibrium theory is actually founded. Yet, when applied to the capital formation model, as Walras tries to do, the stationarity assumption turns out to be self-contradictory, as we shall show in section 5.

Section 6 will examine Walras's final stance on equilibrium and *tâtonnement*, as can be found in the fourth edition of the *Eléments* (1900). We shall show that in this edition Walras, in trying to remedy the difficulties and contradictions engendered by his previous treatment of the *tâtonnement* process in production, introduces a new assumption, known as the «hypothèse des *bons*», with the purpose of extending to the *tâtonnement* in production those same features (virtuality, unobservability, evolution in a merely 'logical' time set) that had been characterizing the *tâtonnement* in exchange for a long time, namely, since the second edition of the *Eléments*. In this new context, all stationarity assumptions can and must be dropped, so that the equilibrium notion emerging from a generalized virtual *tâtonnement* process, in both exchange and production, cannot but be of the 'instantaneous', specifically 'temporary', type. Section 7 concludes.

2. THE STRUCTURE OF THE *ELÉMENTS*: WALRAS'S NESTED EQUILIBRIUM MODELS AND THE TWOFOLD CONCEPT OF 'SOLUTION'

During Walras's lifetime, four successive editions of the *Eléments* were sent to press: the first one, subdivided into two installments, appeared in 1874 and 1877; the second, third, and fourth editions, instead, were each published as a unitary volume in 1889, 1896, and 1900, respectively. There was also a posthumous edition, arranged by Walras himself before his death, which was published in 1926; this edition, known in the past as the «quatrième édition définitive», is now more simply indicated as the fifth edition. The second and third editions present only minor differences, which (barring one exception, to be mentioned in due time) are irrelevant for our present purposes. The fourth and fifth editions are almost indistinguishable. Hence, for our purposes it will be enough to focus attention on the first, second, and fourth editions only.[1]

Apart from the *Eléments*, occasionally it will be necessary to refer to other writings, whose drafting is strictly intertwined with that of the *Eléments*. In particular, the content of the first edition of the *Eléments* partially overlaps with the content of four *mémoires*, separately written and published in the bulletins of learned societies, after presentation by Walras of the corresponding texts at public meetings of the same societies. Of the two *mémoires* partly covering the topics dealt with in the first installment of the first edition, only the first one («Principe d'une théorie mathématique de l'échange», January 1874) was actually published before the appearance, in July 1874, of the first installment of the first edition, thereby becoming the first published instance of Walras's equilibrium theory; the second («Equations de l'échange») appeared only in October 1876. The other two *mémoires*, «Equations de la production» and «Equations de la capitalisation», partly covering the topics dealt with in the second installment of the first edition, appeared in October 1876 and March 1877, respectively. In 1877 the four *mémoires* were collected by Walras in a brochure, which was given the title of *Théorie mathématique de la richesse sociale*. This brochure was reprinted in 1883, under the same title, in a revised and augmented version, containing three more *mémoires*, written in the meanwhile.[1] Finally, we shall have occasion to mention two further papers, separately written and published in 1891 and 1892, respectively, which were later on merged, with some revisions, in a single text, appearing under the title «Théorie géométrique de la détermination des prix» as «Appendice I» of the third and following editions of the *Eléments*.[2]

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[1] Since a *variorum* edition of the *Eléments* is now available, allowing easy comparisons among the texts of the various editions, all the quotations will refer to this edition, contained in vol. VIII of the *Œuvres économiques complètes d'Auguste et de Léon Walras*, published in 1988. Page and edition references will be given in the following way: WALRAS 1988, page number(s), number(s) of the edition(s) involved; if no edition number is specified, this means that the text quoted has remained unaltered over all the editions. Jaffé's English edition (WALRAS 1954), though a valuable tool for scholars and historians, is frequently unreliable, for the reasons mentioned in the introduction.

In elaborating his theoretical system, Walras adopts a particular constructive and expository method, partly surviving in the subsequent evolution of GET. Walras's method consists in formulating a number of equilibrium models, arranged in order of increasing descriptive reach: the models are nested in the sense that the scope of each model is wider than the scope exhibited by the previous one (if any) in the sequence. Three models are fully developed in all the editions of the *Eléments*, as well as in the four *mémoires* mentioned above: a model of exchange, dealing first with the problem of the exchange of two commodities (consumers' goods) for one another, and then of any finite number of commodities for each other; a model of production, containing the previous model (in a sense to be qualified) and further dealing with the problem of the transformation of productive services into consumers' goods; and finally a model of capital formation, containing the previous model (still in a sense to be qualified) and further dealing with the problem of the transformation of productive services into newly produced durable capital goods. The structure of these three models, as well as the mutual relations among them, remain basically unaltered in all the editions of the *Eléments*, as Walras himself underlines in the preface to the fourth edition (1988, 5, 4-5). In the fourth edition, however, one can also find a fourth model which, in addition to the problems already tackled in the third, also deals with circulating capital and money. Money had already been discussed at length in all the previous editions, but only in the fourth one, as Walras once again underlines in the corresponding *Preface* (1988, 8-9, 4-5), an attempt is made to integrate the issues concerning money into the main formal apparatus of the theory.

Walras's purpose is to 'solve' each one of the above equilibrium models. But for Walras 'solving' a formal model implies a special two-step procedure, which is well illustrated by the following passage:

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[1] This latter work, which encompasses, with minor textual revisions, all the *mémoires*, as well as the 1877 brochure, is now available in vol. XI of the *Œuvres économiques complètes d'Auguste et de Léon Walras*, published in 1993. When quoting from the various *mémoires*, the 1877 brochure or the 1883 collection, we shall identify the original source of the quotation by specifying the year of first publication of the corresponding text; page numbers, anyhow, will always refer to the pagination of vol. XI of the *Œuvres économiques complètes d'Auguste et de Léon Walras*, where all the above materials can be found in a critical *variorum* edition.

[2] When quoting from this text, we shall refer to the *variorum* edition of the *Eléments*, *i.e.*, to WALRAS 1988, adopting the conventions already specified at p. 91, fn. 1.

> Or, pour démontrer que des prix de marchandises, qui sont des quantités, résultent effectivement de telles ou telles données ou conditions, il est absolument indispensable à mon sens: 1° de formuler, d'après ces données ou conditions, un système d'équations, en nombre rigoureusement égal à celui des inconnues, dont les quantités en question soient les racines, et 2° d'établir que l'enchaînement des phénomènes de la réalité constitue bien la résolution empirique de ce système d'équations. C'est ce que j'ai fait en ce qui concernait successivement l'échange, la production et la capitalisation.[1]

As can be seen, 'solving' a model in Walras's sense entails satisfying two distinct requirements, each implying a specific step. Let us consider the first step first.

Given an economic problem, identified by certain 'data' or 'conditions' and certain 'unknowns', a formal model of the problem can be constructed by specifying a system of equations, suitably embodying the given 'data' and 'conditions' (parameters, functions, relations); further, the number of the equations must be rigorously equal to the number of the unknowns. Apparently, according to what Walras seems to suggest on a number of occasions, such equality is enough to provide the «solution théorique et mathématique» or the «solution scientifique» of the problem or, more precisely, of the model representing it (1988, 93, 307, 375; 1988, 189, 2-5; 1988, 461, 4-5). But how Walras's statements should be interpreted is not perfectly clear.

Sometimes, by insisting on the equation counting procedure, Walras seems to imply that the ascertained equality between the number of the equations and the number of the unknowns of a particular model is enough to ensure the determinateness of the model, hence the existence, and even the uniqueness, of a solution to the system of equations representing it. Yet, this reading of Walras's statements is disproved by the following simple observation: Walras explicitly allows for the possibility that a model, which satisfies the equation counting requirement, either admits of no solution (1988, 96-97) or, on the contrary, has multiple solutions (1988, 97). But even this remark needs further qualification. On the one hand, what Walras interprets as a situation where no solution exists would be read nowadays as a situation where the model exhibits a perfectly admissible solution (namely, a trivial no-trade equilibrium). On the other, he tends to restrict the possibility of multiple equilibria to the elementary case of the exchange of two commodities for one another, belittling the probability of such occurrence in more complex situations, such as the case of the exchange of many commodities for each other (1988, 242); further, even when the possibility of multiple solutions is allowed for, their number is always assumed to be finite, so that the solutions are locally unique.

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[1] WALRAS 1988, 651. The omission of money in the above list of topics and corresponding models is simply due to the fact that the quoted passage dates back to the first edition of the *Eléments*, remaining unaltered thereafter; hence, it was written at a time when money had not yet become the object of a formal model, calling for a 'solution' in Walras's sense.

To conclude on this point, Walras's prevailing stance on the first step of his own 'solution' procedure, the step which is supposed to provide the «solution théorique et mathématique» of the equilibrium model under question, may be summarized as follows: given a model represented by a system of equations, if the number of the equations is equal to the number of the unknowns, then the model is determinate, which implies that 1) a solution exists (allowing also for the trivial solutions of the no-trade type), and 2) either there exists a unique solution, or there exist finitely many solutions, which are therefore locally unique. This proposition is never proved by Walras, and of course, as will become clear since the mid-1930s, it cannot be proved without further specific assumptions. Yet Walras consistently behaves in the *Eléments* as if it were a proven theorem. But this is not enough from Walras's point of view, since the 'solution' procedure can be regarded as complete only if the first step is supplemented by the second one.

Before turning to the second step, however, we can take advantage of the preceding discussion to clarify in which sense, and within which limits, the sequence of Walras's four models should be regarded as nested. Given the equational approach taken by Walras, each of the four models is described by a system of equations, equal in number to the variables considered therein. Hence, when one passes from a smaller model in the sequence to a larger one (if any such model exists), the number of the equations describing the model should increase by as much as the number of the new variables considered therein. In this sense the larger model can be said to contain the smaller. But it would be a serious mistake to think that the larger system of equations, describing the larger model, can be obtained by simply adding new equations, equal in number to the newly introduced variables, to the smaller system of equations, which can be left unchanged otherwise: for, owing to the general interrelation among all the variables, which is after all the distinguishing feature of GET, the introduction of new variables is generally bound to affect some of the functions and relations entering the smaller system of equations, which cannot therefore remain unchanged.

Walras, being the founder of GET, is of course aware of the analytical problems posed by the interrelation among all the variables of the economy. Yet, he is frequently tempted by the short cut neatly described in the following passage:

> Théoriquement, toutes les inconnues du problème économique dépendent de toutes les équations de l'équilibre économique. Toutefois, même au point de vue statique

> et théorique, il est bien permis de considérer certaines de ces inconnues comme dépendant plus spécialement des équations qui s'introduisent avec elles dans le problème pour les déterminer. Et à bien plus fort raison a-t-on ce droit quand on passe du point de vue statique au point de vue dynamique.
>
> (Walras 1988, 430, 4-5)

In effect, as we shall see in section 4 below, Walras often appears to yield to the above temptation in discussing the 'solution' of the production model: in fact, given that the production model 'contains' in some sense the exchange model,[1] Walras is often led to consider the equations of the exchange model as if they were not only separable from the larger system of equations describing the production model, but also independently solvable; but this is a mistake, giving rise to momentous consequences.[2]

Let us now turn to the second step in Walras's 'solution' procedure. To clarify the meaning of the second step, as well as its relationship with the first, it may be useful to quote a passage to be found towards the end of *Leçon* 6 of the *Eléments*, where Walras presents the 'solution' of the problem of the exchange of two commodities for one another, the simplest problem tackled in Walras's theoretical system and, consequently, the first problem to which Walras's 'solution' procedure is applied in the *Eléments*. The 'solution' of this problem, referred to by Walras (1988, 93) as the «loi de l'offre et de la demande effectives» or the «loi d'établissement des prix d'équilibre», is expressed as follows:

> Deux marchandises étant données, pour qu'il y ait équilibre du marché à leur égard, ou prix stationnaire de l'une en l'autre, il faut et il suffit que la demande effective de chacune de ces deux marchandises soit égale à son offre effective. Lorsque cette égalité n'existe pas, il faut, pour arriver au prix d'équilibre, une hausse du prix de la marchandise dont la demande effective est supérieure à l'offre effective, et une baisse du prix de celle dont l'offre effective est supérieure à la demande effective.

In the simple problem under discussion, one can define two reciprocal relative prices, one for either commodity in terms of the other; hence there is only one unknown independent price variable, namely, either relative price (arbitrarily chosen). In this case, therefore, the first requirement for a 'solution' in Walras's sense is satisfied by positing one single equation, namely, the market-clearing equation obtained by equalling the aggregate demand for either commodity with the aggregate supply of the same commodity (the market-clearing equation relative to the other commodity being then necessarily satisfied, as Walras elsewhere proves). Hence, the first sentence in the above quoted passage summarizes the first step in Walras's 'solution' procedure, providing the «solution théorique et mathématique» of the problem; the second sentence, instead, summarizes the second step in the procedure, providing the «solution pratique» or the «résolution empirique» of the same problem (1988, 93, 307, 375; 1988, 189, 2-5; 1988, 461, 4-5).

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[1] As WALRAS 1988, 283 himself writes: «L'état d'équilibre de la production, contenant implicitement l'état d'équilibre de l'échange, est à présent facile à définir».

[2] As we shall see in section 4, a similar, but even more serious, problem arises in connection with Walras's discussion of the *tâtonnement* in the production model, where he *de facto* regards the dynamic equations governing the adjustment process in exchange as separable from the dynamic equations governing the adjustment process in production. Analogous problems arise also in connection with the capital formation model (see, in particular, WALRAS 1988, 430 ff., 4-5) and the model of circulation and money (see, in particular, *ibidem*, 465 ff., 4-5). On the separability issue in the latter model, see also Patinkin's motivated criticism (1965, 558-572).

The second sentence states what might rightfully be called the «Walrasian rule of price adjustment», according to which the price of a commodity changes over time if and only if the market for that commodity does not clear, the rate of change in the price being a sign-preserving function of the aggregate excess demand for the commodity in question. Since, in the case under discussion, the excess demand for either commodity is a function of the only independent price variable, namely, the arbitrarily selected relative price, in this case the Walrasian price adjustment rule defines a simple dynamical system, consisting in one functional equation in which the state (or adjustment) variable consists precisely in the selected relative price. The precise nature of the time set over which the dynamical system evolves cannot be discussed at this stage; this crucial issue will be taken up in detail in following sections.

The process engendered by this dynamical system represents the simplest form of *tâtonnement* process, namely, the form appropriate to the elementary problem under discussion; but, in spite of its simplicity, this introductory example embodies the essential features of Walras's *tâtonnement* in the more complex models as well (Walras 1988, 189, 2-5; 1988, 698, 3-5). In particular, here as elsewhere in the *Eléments*, it is precisely the theory of the *tâtonnement* that represents the second step in Walras's 'solution' procedure, for it is the *tâtonnement* process that provides that «solution pratique» or «empirique», identical with the «solution théorique et mathématique», which fulfils the second requirement of Walras's methodology.

The relationship between the two solutions, the «théorique» and the «pratique», is well illustrated by the following passage (Walras 1988, 93), whose import certainly transcends the simple case to which it concretely refers:

> On voit clairement à présent ce qu'est le mécanisme de la concurrence sur le marché; c'est la solution pratique, et par hausse et baisse des prix, du problème de l'échange dont nous avons fourni la solution théorique et mathématique. On doit comprendre d'ailleurs que notre intention n'est aucunement de substituer une solution à l'autre. La solution pratique est d'une rapidité et d'une sûreté qui ne laissent rien à désirer. On peut voir, sur de grands marchés fonctionnant même sans courtiers ni crieurs, le prix courant d'équilibre se déterminer en quelques minutes, et des quan-

> tités considérables de marchandise s'échanger à ce prix en deux ou trois quarts d'heure. Au contraire, la solution théorique serait, dans presque tous les cas, absolument impraticable.

The main reason why, according to Walras, the mathematical solution cannot be computed is that the theorist does not actually know the data of the problem under question. But this limitation is not really relevant, for the analytical solution of the system of equations describing a particular model is not the real purpose of the theory, whose aim is rather to clarify the general principle governing the determination of equilibrium prices and quantities (*ibidem*). On the other hand, even if the analytical solution could actually be computed, there would be no point in calculating it, since the practical solution provided by the market is reached no less quickly and precisely than any analytical solution.[1]

However, the practical solution provided by the market can be regarded as a sufficient 'solution' of the overall problem only if it coincides with the theoretical solution: for only in the case of identity between the two solutions can the practical one be taken as an efficient substitute for the other. What needs then to be done «pour établir que la solution théorique et la solution du marché sont identiques» (Walras 1988, 189, 2-5)? In principle, what ought to be proved is that the dynamical system describing the adjustment process converges to an equilibrium (in the sense of dynamical systems theory) which coincides with either the unique solution of the system of equations describing the model, or one of the finitely many solutions of the same system of equations (which of the solutions the process converges to depending in this case on the initial conditions). Such a proof, however, can be conveniently split into two steps: first, one ought to prove that the adjustment process does not change the data (parameters, functions, relations) of the economy; second, one ought to prove that either the unique solution is a globally asymptotically stable equilibrium, with respect to the specified dynamical process, or each of the finitely many solutions is an asymptotically stable equilibrium, with respect to the specified dynamical process, for appropriately given initial conditions.

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[1] In his first *mémoire*, WALRAS 1874, 37 takes a more optimistic view about both the mathematical solvability of the exchange problem and the precision, though not the speed, of the analytical solution: «*A priori,* ce problème est évidemment soluble, du moins en principe, par le procédé mathématique, comme il est soluble, en fait, sur le marché, par le procédé empirique de la hausse et de la baisse». In particular, supposing that all the necessary information about the data of the problem may be conveyed to a «calculateur», then «ce calculateur déterminera le prix d'équilibre non pas certes aussi rapidement, mais à coup sûr plus rigoureusement que cela ne pourrait se faire par le mécanisme de la hausse et de la baisse».

As to the change in the data, Walras is aware of the need to avoid its occurrence during the *tâtonnement* process. The assumptions he makes to this end will be discussed in the following sections.

As to the stability of the solution(s) with respect to the specified dynamical process, Walras once again is aware that a formal proof is needed, but his treatment remains highly unsatisfactory in all the editions of the *Eléments*. Postponing a more detailed discussion of this issue to the following sections, we confine ourselves here to considering the simplest possible case for illustrative purposes. In the case of the exchange of two commodities for one another, Walras's 'proof' simply consists in observing that, starting from any initial condition (a price «crié au hasard»), hence from a corresponding value of the excess demand function, the 'Walrasian price adjustment rule' is bound to produce the 'right' change in price, namely, a change reducing the value of the excess demand in the next round of *tâtonnement*, not only in the case of a unique equilibrium, but also in the case of any stable equilibrium (in Walras's sense), when the equilibria are more than one (Walras 1988, 98-99).

This observation is certainly not a proof of the convergence of the *tâtonnement* process to equilibrium in the simple case under consideration. But even less satisfactory are the alleged 'proofs' put forward by Walras in relation to the other more complex cases and models. In 1884 a critical remark in a letter received from Wicksteed (Jaffé 1965, vol. II, 16-18) reveals to Walras the defective character of his 'proof' of convergence to equilibrium of the *tâtonnement* process in exchange (of finitely many commodities for each other) and in production. After taking due notice of Wicksteed's observation (Jaffé 1965, vol. II, 24-26), Walras revises all the passages of the first edition of the *Eléments* where he had boldly claimed the «certain» convergence of the *tâtonnement* process to the «solution théorique»: in the light of his more mature reflections, since the second edition he replaces the demanding term «certain» with the less compromising term «probable» (Walras 1988, 195, 326, 328, 2-5; 1988, 698, 3-5).

But this weakening of his previous confidence does not lead Walras to change his mind as to the need of supplementing the «solution théorique» of each particular model with the «solution pratique» provided by the appropriate *tâtonnement*: in fact, in all the editions of the *Eléments*, an almost standardized formula, repeated over and over again, confirms such a need with reference not only, as we have already seen, to the model of exchange of two (Walras 1988, 93) and then finitely many (189, 2-5) commodities for each other, but also to the models of production (307), capital formation (375), and finally circulation and money (461, 4-5). The relation between the two kinds of 'solutions', the «théorique» and the «pratique», or between equilibrium theory and the theory of the *tâtonnement*, still remains at the center of Walras's theoretical thought. And to the analysis of this relation in the various equilibrium models we now turn, focussing first of all on the two related aspects whose discussion has been deliberately postponed: the nature of the time set over which the *tâtonnement* process is supposed to take place and the nature of the assumptions made by Walras to ensure the invariance of the economic data during the *tâtonnement*.

3. Equilibrium and *tâtonnement* in the first two editions of the *Eléments*: pure exchange

This section and the following three will analyse the relationship between the equilibrium concept and the *tâtonnement* construct, focussing on the evolution of Walras's ideas over the various editions of the *Eléments* and related writings. Sections 3, 4 and 5 will examine the changes in Walras's ideas on equilibrium and *tâtonnement* occurring between the first and the second edition of the *Eléments*, that is, between 1874 and 1889 (with a marginal extension to the following two years): in particular, section 3 will concentrate on the exchange model, section 4 on the production model, while section 5 will be devoted to the model of capital formation. Section 6, instead, will examine the final changes in Walras's conception concerning equilibrium and *tâtonnement* introduced in 1900, with the appearance of the fourth edition of the *Eléments*.

Let us then start from the exchange model. Let us suppose that in the exchange economy under question (or in the 'market' under question, to use Walras's prevailing expression) there are I traders, who are also consumers, indexed by $i = 1, \dots, I$, where $2 \leq I < \infty$. The traders exchange quantities of L perfectly divisible commodities, to be regarded as consumers' goods in this model, indexed by $l = 1, \dots, L$, where $2 \leq L < \infty$. $\mathbb{R}^L$ is the commodity space.

Each trader i is characterized by: a consumption set $X_i = \{x_i\} = \mathbb{R}^L_+$, where x_i is a consumption plan, or simply a consumption, possible for i; a utility function $u_i : X_i \rightarrow \mathbb{R}$; an endowment $\omega_i^c \in \mathbb{R}^L_+ \setminus \{0\}$, where the superscript c stands for 'consumers' goods' (a different sort of endowment will be defined in the next section, in examining the production model). Given $x_i \in X_i$, $(x_i - \omega_i^c) \in \mathbb{R}^L$ is a trade plan, or simply a trade, possible for trader i. Walras makes a number of specific assumptions on the characteristics of the traders, which do not occasionally satisfy contemporary standards of rigor or generality. However, since Walras's most questionable original assumptions on traders' characteristics are actually irrelevant for achieving his own results, they will be dispensed with in the following; on the contrary, we shall feel free to make all those assumptions that subsequent theoretical developments have proved to be necessary in order to justify some results taken for granted by Walras.[1] Summing up, therefore, the data of the exchange economy consist in an I – tuple of characteristics $(X_i, u_i(\cdot), \omega^c_i)_{i=1,\ldots,I}$.

For most of the following discussion it will be enough to confine our attention to an economy with only two commodities. Hence, in order to simplify the exposition, and following for that matter Walras's own lead, let us provisionally assume $L=2$.

Then let $p = (p_1, p_2) \in \mathbb{R}^L_{++}$ be the price system, where the commodity prices, expressed in units of account, are assumed always to be strictly positive (prices can be bounded away from zero by making some desirability assumption on the utility functions). From this we get $p_{12} \equiv \frac{p1}{p2} \equiv \frac{1}{p21}$, the relative price of commodity 1 in terms of commodity 2, p_{21} being the reciprocal relative price of commodity 2 in terms of commodity 1. Since p_{12} and p_{21} are not independent of one another, we can and must focus attention on one of them only. Then, let us select p_{12} for consideration; that price will be called for short 'the relative price' in the following.

Traders are assumed 'to behave competitively': this means that, in choosing a consumption or trade plan, they are supposed to take prices (or, in this model, the relative price) as given. Since, in making their choices, traders take prices as given parameters, they are said to be price-takers and to be characterized by a sort of rationality which is often labelled as 'parametric'. Traders are also assumed to maximize their utility functions, subject to their budget constraints. Hence, given the relative price, $p_{12} \in \mathbb{R}_{++}$, any trader i is supposed to solve the following optimization problem:

$$\begin{gathered} \max_{x_I \in X_I} u_i(x_i) \\ s.t. \\ p_{12}x_{1i} + x_{2i} = p_{12}\omega^c_{1i} + \omega^c_{2i} \end{gathered} \tag{3.1}$$

Let us assume, for simplicity, that trader i's characteristics are so specified that, $\forall p_{12} \in \mathbb{R}_{++}$, the optimization problem has a unique solution, $x_1(p_{12}) \in \mathbb{R}^2_+$. The function $x_i : \mathbb{R}_{++} \longrightarrow \mathbb{R}^2_+$ is trader i's demand function, while the function $z_i \equiv x_i - \omega^c_1 : \mathbb{R}_{++} \longrightarrow \mathbb{R}^2$ is i's excess demand function. Let $\omega^c \equiv \Sigma^I_{i=1}\, \omega^c_1$ be the aggregate endowments (of consumers' goods) of the economy; further, let $x(\cdot) \equiv \Sigma^I_{i=1}\, x_i(\cdot)$ and $z(\cdot) \equiv \Sigma^I_{i=1}\, z_i(\cdot) \equiv$

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[1] In particular, we shall dispose of Walras's inessential assumption that the traders' utility functions be additively separable in their arguments; on the contrary, we shall assume all those properties of preferences (such as strict convexity) which are required to generate well-defined individual excess demand functions.

$x(\cdot) - \omega^c$ be the aggregate demand and the aggregate excess demand function, respectively. Of course, the aggregate demand and excess demand functions, as well as the aggregate endowment vector, can be written coordinatewise as follows: $x(\cdot) \equiv (x_1(\cdot), x_2(\cdot))$, $z(\cdot) \equiv (z_1(\cdot), z_2(\cdot))$, and $\omega^c \equiv (\omega^c_1, \omega^c_2)$.

An equilibrium relative price, $p^\star_{12}$, is then a root of the following market-clearing equation for commodity 1:

$$z_1(p^\star_{12}) = x_1(p^\star_{12}) - \omega^c_1 = 0,$$

the market-clearing equation for commodity 2 being necessarily satisfied, in view of the budget constraints assumed to hold in the traders' optimization problems, when the equation for commodity 1 is satisfied. Positing the equation allows the theorist to define the equilibrium concept appropriate to the exchange model at hand, thereby providing what Walras calls the «solution théorique et mathématique» of the corresponding problem. But this is not enough for Walras, since for him the theorist is also required to explain how an equilibrium is actually reached or 'established' in the market; to this end, one has to resort to the theory of the *tâtonnement*, which provides the «solution pratique» or «empirique» of the problem. As we have already seen, in this elementary case the process is governed by the simplest possible application of the 'Walrasian rule of price adjustment', according to which the adjustment variable, namely, the relative price p_{12}, starting from a value which is randomly quoted («crié au hazard») at the beginning of the process, rises or falls over a suitably specified time set according to whether the excess demand for commodity 1 as a function of the relative price, $z_1(p_{12})$, is greater or less than zero.

From the very beginning of his scientific activity, Walras is firmly convinced that the 'Walrasian rule of price adjustment', governing the *tâtonnement* process in the exchange model, is a natural, almost self-evident, and basically irreplaceable formalization of the way in which any real-world competitive market actually operates in the establishment of a competitive equilibrium. Moreover, he does not lay any particular claim concerning the novelty or the originality of his own theory of the *tâtonnement* in exchange, convinced as he is that such a theory is nothing but an embellished and slightly more formalized version of the old classical idea, already discussed at length by, *e.g.*, J. S. Mill, of the price adjustment process in a competitive market. As a matter of fact, when Edgeworth (1889a, 268), in his review of the second edition of the *Eléments*, attacks the Walrasian theory of the *tâtonnement* by asserting that «Prof. Walras's laboured lessons indicate *a* way, not *the* way, of descent to equilibrium», Walras's immediate reaction is one of true astonishment, since he cannot believe that any unprejudiced person may ever call into question either the general validity, or the realistic foundation of his approach.[1]

Walras's naïve realistic epistemology, at least as far as the *tâtonnement* process is concerned, while promptly embraced by Bortkiewicz[2] and Pareto 1964 [1896-1897], 24-25, fails of course to convince Edgeworth, who reiterates his accusations, with almost identical wording, in both Edgeworth 1891, 370 and 1925, vol. II, 311. In effect, Walras's claims about his theory of the *tâtonnement* in exchange are wholly unfounded from a methodological point of view. First of all, Walras's theory of the *tâtonnement* in exchange is all but an objective description of what really happens in a competitive market: contrary to what Walras himself probably believes, and anyhow would have us to believe, his *tâtonnement* construct is at least as theory-laden as any other alternative construct aiming at explaining the working of competitive markets.[3] In the second place, as to the alleged generality of Walras's *tâtonnement*, it is really hard to decide whether Walras's approach to disequilibrium analysis is more or less general than, say, Edgeworth's model of recontracting (1881) or Marshall's model of barter (1961 [1890], Appendix F. Barter, 791-793).

If Walras's theory of the *tâtonnement* in exchange cannot plausibly be said to provide the most general or realistic explanation of competitive disequilibrium processes, it can certainly be said to provide the most natural explanation of such processes that is consistent with the theoretical premises underlying Walras's exchange model. It can even be argued that the 'Walrasian rule of price adjustment', as well as the ensuing *tâtonnement* process in exchange, represent the only possible approach to disequilibrium analysis within Walras's exchange model. To avoid unnecessary complications, in the following discussion we shall keep to the two-commodity model, as summarized above. As a matter of fact, the analysis of the exchange problem can be generalized to models with any finite number ($L > 2$) of commodities, and, indeed, it has been so generalized by Walras himself since the first edition of the *Eléments*. Yet, such extensions of the exchange model necessarily raise some additional difficulties, which Walras is unable to solve in a way that would fully meet contemporary standards of rigor and generality. In fact, when the number of traded commodities is $L > 2$, the further difficulties to confront are twofold: in the first place, concerning the so-called «solution théorique et mathématique», in Walras's sense, one has to specify what exactly are the prices to be taken as given parameters by the traders; secondly, concerning the so-called «solution pratique» or «empirique», still in Walras's sense, one has to specify how exactly the 'Walrasian rule of price adjustment' works in a multi-commodity exchange economy.

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[1] Walras's stance can be inferred, in particular, from his letters to Bortkiewicz and Pareto, published in JAFFÉ 1965, vol. II, 434, 630.

[2] In striving to counteract Edgeworth assertion that Walras's *tâtonnement* is just «*a* way, not *the* way, of descent to equilibrium», BORTKIEWICZ 1890, 358-359 writes:

> Certes, il peut y avoir plus d'une méthode pour résoudre un systéme d'equations donné; mais nous n'avons pas, dans le cas présent, un probléme d'algebre devant nous; il s'agit plutôt de montrer quel est le procedée réel, effectivement employé sur le marché, qui constitue le mode de résolution des équations données. Y aurait-il peut-être, d'après M. Edgeworth, un autre phénomène économique se produisant sur le marché et pouvant être regardé comme mode de résolution des équations en question? Non.

[3] Sometimes even Walras seems to realize that a few peculiar traits of the *tâtonnement* construct are in fact dictated by the need to satify some compelling (for him) analytical requirements, rather than suggested by an open-minded observation of the way in which the markets of the real world actually work. A few instances of Walras's oscillating attitude towards the true epistemological status of the *tâtonnement* construct will be given later in this Section, concerning the exchange model with many (*i.e.*, more than two) commodities, and in the next one, concerning the production model.

As to the first problem, Walras finds it convenient to suggest a two-step solution procedure, which is clearly enunciated since the second edition of the *Eléments*. First he supposes that only direct trades between pairs of commodities are allowed to take place on the $\frac{L(L-1)}{2}$ separate 'trading posts', or «marchés spéciaux» (Walras 1988, 163, 2-5), corresponding to all possible commodity pairs; such trades are assumed to lead to the establishment of $\frac{L(L-1)}{2}$ independent rates of exchange between pairs of commodities. According to Walras, however, such rates can only correspond to a constrained or 'imperfect' equilibrium, which would be disrupted if the pairwise trade constraint were to be relaxed. Assuming then all kinds of indirect exchanges to be allowed to take place, 'arbitrage' activities would lead the economy to a 'perfect or general' equilibrium, whose distinctive feature would be that the $\frac{L(L-1)}{2}$ independent rates of exchange between pairs of commodities characteristic of the *interim* 'imperfect' equilibrium would shrink to only $L - 1$ independent rates of exchange between $L - 1$ commodities, arbitrarily chosen among the L commodities existing in the economy, and the remaining one, taken as a *numéraire* (Walras 1988, 161-173, 2-5). If, *e.g.*, the first commodity were selected as the *numéraire*, the price system resulting from 'arbitrage' activities might be written as $p = (1, p_2, \ldots, p_l, \ldots, p_L) \in \{1\} \times \mathbb{R}_+^{L-1}$ where p_l is the price of commodity l in terms of commodity 1, for $l = 2, \ldots, L$.

Now, the solution procedure suggested by Walras to tackle the first problem is not entirely satisfactory, for the following two reasons: in the first place, the *interim* 'imperfect' equilibrium, assumed to be reached in the first step of the procedure, can only make sense under Walras's special assumptions about the traders' utility functions, which, as already mentioned, are supposed to be additively separable in their arguments; secondly, contrary to what Walras occasionally, but not invariably, seems to suggest, one should refrain from confusing the issue of the establishment of a 'general' equilibrium of the L-commodity exchange economy, in the sense of the occurrence of a simultaneous clearing of all the L commodity markets, with the issue of the formation of a consistent price system, as represented, *e.g.*, by a vector of $L - 1$ prices for as many commodities, each price being expressed in terms of the remaining commodity chosen as a *numéraire*: in fact, a consistent price system, in the sense specified, is perfectly compatible with market disequilibrium. Yet, in spite of these shortcomings, Walras's solution of the first additional problem arising in modeling a multi-commodity exchange economy is not far from that adopted in the subsequent developments of general equilibrium theory.

Much more idiosyncratic and controversial, instead, is the approach taken by Walras in confronting the second additional problem arising in a multi-commodity exchange model. In such a context, the most natural way to generalize the 'Walrasian rule of price adjustment' would appear to be the following: at any given moment of time, given a price system expressed in terms of a *numéraire*, barring the price of the *numéraire* commodity, which is identically set equal to one, one should suppose the prices of all the other commodities whose aggregate excess demands are different from zero to simultaneously move in the directions prescribed by the 'Walrasian rule'. In view of the postulated simultaneity of all price changes, this sort of price adjustment process has been labelled as the «simultaneous *tâtonnement* process» by Uzawa (1960, 184-185), who has also proved its global stability under relatively restrictive conditions, which are however not unusual in the modern analysis of general equilibrium stability. Yet, though explicitly recognizing that the «simultaneous *tâtonnement* process» is the natural candidate for 'realistically' representing the real-word functioning of a set of interrelated markets, hence for extending the 'Walrasian rule of price adjustment' to a multi-commodity exchange model, in the more formalized part of his discussion of the equilibration process Walras (1988, 188-197, 1 and 2-5) eventually opts for an alternative market-by-market procedure, which «consists of a price adjustment which successively clears the markets of commodities». In view of the postulated sequentiality of the price changes, which are assumed to occur one at a time in a definite order, this sort of price adjustment process has been labelled as the «successive *tâtonnement* process» by Uzawa (1960, 186-188),[1] who has also proved its global stability under conditions similar to the ones invoked in proving the stability of the «simultaneous process». In referring to the

[1] As we shall see in a moment, the labels «simultaneous» and «successive», suggested by Uzawa to describe the two alternative *tâtonnement* processes which are discussed, either informally or formally, in Walras's *Eléments*, can actually be traced back to Walras himself, though not specifically to the *Eléments*.

system of difference equations formalizing such «successive *tâtonnement* process», Uzawa (1960, 186, fn. 1) further reminds the reader that such «system defines an iterative method for solving systems of equations which is known as the Gauss-Seidel method in numerical analysis» (on this issue, see also Arrow and Hahn 1971, 305-306).

The reason why Walras, though regarding the «simultaneous *tâtonnement* process» as a more faithful representation of the true working of real competitive markets, ends up by selecting the alternative «successive *tâtonnement* process» for the purposes of his formal analysis, is synthetically hinted at by Walras himself in a letter to the Italian economist Maffeo Pantaleoni, dated «2 septembre 1889» and published in Jaffé 1965, vol. II, 343-347. There, in referring to his market-by-market iterative *tâtonnement* process, Walras writes:

> Reprenez encore les tâtonnements que je vous présente ainsi *successivement* pour les besoins de l'analyse comme s'operant *simultanément* sur le marché, n'avez-vous pas exactement dans son ensemble le fait de la determination des prix de plusieurs marchandises sous l'émpire de la libre concurrence?
>
> (Jaffé 1965, vol. II, 345; italics in the original)

From this passage one can infer that it is only for allegedly analytical reasons («pour les besoins de l'analyse») that Walras reluctantly opts for the admittedly unrealistic «successive *tâtonnement* process». The fact is that Walras, being wholly unaware of the very existence of such branches of mathematics as vector analysis and multivariate calculus, is unwillingly forced to drop the «simultaneous *tâtonnement* process», whose very description would require the use of such tools, contenting himself with the «successive *tâtonnement* process» which, owing to its peculiar market-by-market nature, can at least be described (though certainly not solved) by means of scalar methods. Thus, in order to explain Walras's expository choice no more profound reason needs to be invoked than his poor mastership of mathematics.[1] Yet, chiefly because of the unfortunate coincidence of Walras's «successive *tâtonnement* process» with the so-called Gauss-Seidel iterative method of solving a system of simultaneous equations, a whole line of literature has developed where the simple product of Walras's poor mathematical skills has been occasionally mistaken for the result of deep reflections concerning the relationship between Walras's two 'solution' concepts, the nature of statics and dynamics, as well as the role of time in the theory of the *tâtonnement*.[1] Precisely for these reasons, however, in examining the general epistemological properties of the process engendered by the application to the exchange model of the 'Walrasian rule of price adjustment', it is wiser to stick to the two-commodity exchange model, which, for its very nature, is free of all the idiosyncratic aspects encumbering the multi-commodity one; on the other hand, the two-commodity exchange model is general enough for discussing all the fundamental issues we are interested in at this stage.

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[1] It is certainly not accidental that, more than half a century later, when the analysis of *tâtonnement* stability is taken up again from a much higher mathematical standpoint, it will be the «simultaneous process», rather than the «successive one», to attract the attention of general equilibrium theorists and to become the basis for the modern analysis of general equilibrium stability: see, in particular, SAMUELSON 1941 and 1942, for local stability; ARROW and HURWICZ 1958, ARROW, BLOCK, and HURWICZ 1959, and UZAWA 1961 for global stability; NEGISHI 1962, and ARROW and HAHN 1971, 263-323 for surveys of results on *tâtonnement* stability.

As we have seen above, in the model under question the traders are assumed to be price-takers and utility-maximizers. This means that the relative price is the only information signal commonly known to all the traders and taken into account by all of them in making their choices; hence, the relative price is the only possible candidate to play the role of a state variable in the adjustment process towards equilibrium. Further, for any given relative price, each trader's optimal choice of a plan of action, that is, a consumption or trade plan, turns out to be well defined, both at equilibrium and out of equilibrium, in accordance with one and the same choice procedure, which is independent of the state of the economy. It is precisely this property of the Walrasian choice procedure which allows the theorist to construct the individual demand and excess demand functions, from which, by aggregation over the traders, the aggregate demand and excess demand functions can be derived. Finally, from the latter the theorist can obtain, for any quoted price, a measure of market disequilibrium (as represented, *e.g.*, by the value of the aggregate excess demand for either commodity at that price), which in turn primes the appropriate feedback reaction, implicit in the 'Walrasian rule of price adjustment'.

In the above argument a crucial role is played by the fact that the Walrasian choice procedure is the same whether the economy finds itself at an equilibrium or a disequilibrium state. This means that, in Walras's exchange model, equilibrium and disequilibrium are symmetrical states, as far as the optimal choices of plans of action are concerned. But they are not symmetrical states, as far as actual behavior is concerned. As a matter of fact, in Walras's exchange model, an equilibrium is so defined that the plans of action optimally chosen by all the traders, being mutually compatible, can be actually carried out (the mutual compatibility of the chosen plans is supposedly granted by the market-clearing condition, which evidently encompasses, albeit implicitly, some sort of orderly market assumption). At an equilibrium state, therefore, the optimally chosen plans of action, as such unobservable entities, give rise to the corresponding actions, which are instead observable. On the contrary, at a disequilibrium state, the optimally chosen plans of action, which still are unobservable entities, cannot give rise to the corresponding actions: in fact, since the optimally chosen plans are not mutually compatible, not all of them can be carried out, so that the traders' actual behavior cannot fully correspond to the planned one. From this a fundamental conclusion can be drawn: even if the Walrasian choice procedure allows one to explain or predict the traders' optimally chosen plans of action, as such unobservable, both at equilibrium and disequilibrium states, Walras's theory allows one to explain or predict the traders' optimal actions, that is, their observable behavior, at equilibrium only. This means, however, that observable disequilibrium behavior, granting that it can actually occur in the economy, cannot be explained or predicted by the Walrasian theory of the *tâtonnement* in exchange: from the point of view of such theory, that sort of behavior is unpredictable.

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[1] See, in particular: GOODWIN 1951 and 1953, JAFFÉ 1981, and WALKER 1988.

But, even if Walras's theory of the *tâtonnement* in exchange cannot predict any observable disequilibrium behavior, can it at least allow for the occurrence of any such behavior? This question naturally raises another: What would be the effect of that sort of behavior on the data of the economy, specifically on the traders' endowments?

Now, the economic data can change for both exogenous and endogenous reasons. Disequilibrium behavior, which in this case basically means disequilibrium trading, or trading «at 'false' prices», to use Hicks's expression (1939, 128), would indeed affect the traders' endowments, even if, as we have just seen, Walras's theory of the *tâtonnement* in exchange would be unable to predict the ensuing changes. Yet, as we have seen in section 2 above, the data of the economy must not be permitted to change during the *tâtonnement* process, since otherwise the «solution pratique» could no longer coincide with the «solution théorique». From this it follows that the theory of the *tâtonnement* in exchange should not allow for the occurrence of any observable disequilibrium behavior.

As to Walras, he is perfectly aware that the data of the problem must not change during the *tâtonnement*, for he writes:

> C'est le droit du théoricien du supposer les éléments des prix invariables durant le temps qu'il emploie à formuler la loi d'établissement des prix d'équilibre.
>
> (Walras 1988, 146)

However, though reserving this right to himself, he seems to be in doubt about how to exercise it. In fact, just at the beginning of his treatment of the theory of exchange, Walras (1988, 71-72) exemplifies the working of competition by considering a well-organized competitive market, the Paris Stock Exchange, where a specific state bond, the «rente française 3%», is traded.[1] Walras discusses three alternative hypotheses, exhausting all the possibilities, as to the state of the market at a given money price of the bond under question: either equality between demand and supply, or excess demand, or excess supply. In accordance with the 'Walrasian rule of price adjustment', the price remains unchanged in the first case, while it rises in the second and falls in the third. But Walras does not seem to suppose that trades can take place only in the first case, that is, when the market is in equilibrium; on the contrary, his treatment of the three cases is entirely symmetrical.[2]

In 1883 Bertrand reviews Walras's recently published *Théorie mathématique de la richesse sociale*, which, as mentioned above, contains an example almost identical to the one just commented upon. Bertrand reads Walras's statements as implying that trades can actually take place not only at equilibrium, but also out of equilibrium. From this he infers that, by repeatedly changing the data of the problem, such trading at disequilibrium prices will prevent the process from converging to the «solution théorique et mathématique» of the system of equations representing the model, since those equations are based on the original data. Hence, according to Bertrand, the problem of exchange is indeterminate.

Walras reacts to this criticism in a paper published in 1885, where he asserts that, from a theoretical point of view, in an ideal market («sur le marché théorique»), all exchanges are suspended as long as the market is out of equilibrium.[3] Further, in the second edition of the *Eléments*, he modifies the passage discussed above in three different places, to the effect that trade is now suspended when there is either excess demand or excess supply in the market, while it is assumed to take place when the market is in equilibrium. Precisely, while leaving the remainder of the paragraph wholly unaltered, in the case of excess demand Walras adds just six words: «Théoriquement, l'échange doit être suspendu»; in the case of excess supply he inserts four words: «Suspension de l'échange»; finally, in the case of market clearing or market equilibrium, he adds the lapidary sentence: «L'échange a lieu» (Walras 1988, 71-72, 2-5).

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[1] It should be noted that a quite similar example, though concerning a corn market, rather than an asset market, can be found at the beginning of Walras's 1874 *mémoire* (WALRAS 1874, 32), later reprinted without changes in both WALRAS 1877b and 1883.

[2] In an applied paper on the working of the Stock Exchange written a few years later, Walras points out that the practice prevailing at the Paris Stock Exchange at that time is that no trade should take place out of equilibrium (WALRAS 1880, 408, 432). This fact, however, is nowhere commented upon in Walras's previous or contemporary theoretical writings.

[3] See WALRAS 1985, 312, fn. 1. A similar statement can also be found in a *mémoire* attached to a letter sent by Walras to Pareto in 1895 (JAFFÉ 1965, vol. II, 630); a draft of the *mémoire*, however, had probably been written a few years before.

In the second edition of the *Eléments*, therefore, the explicit assumption of 'no trade out of equilibrium' allows Walras to solve some of the problems, though not all the problems, that had marred the theory of the *tâtonnement* in exchange in the first edition: the assumption of invariance of the economic data during the adjustment process is now made rigorous; further, it is now made clear that the only kind of disequilibrium that is consistent with the 'Walrasian rule of price adjustment' and the associated theory of the *tâtonnement* in exchange is unobservable: we can still have disequilibrium plans of action, but we can no longer have disequilibrium actions or observable behavior. The explicit exclusion of all observable disequilibrium behavior, due to the 'no trade out of equilibrium' assumption, might appear to entail a significant loss in terms of predictive power. But this fear is deceptive: for, as explained above, no observable disequilibrium behavior could be predicted anyway.

With the 'no trade out of equilibrium' assumption, the time structure of the analysis becomes clearer than before. From a temporal point of view, the 'market' (that is, the exchange economy) under question becomes something self-contained, lacking any necessary connection with either the preceding or the following 'markets' (if any); as Walras (1988, 77) says, when the market ends, the traders «s'en vont chacun de leur coté». The data are unchanging from the beginning to the end of the 'market', that is, until an equilibrium is established; but subsequently they typically change, since «les éléments des prix sont essentiellement variables» (Walras 1988, 146). During the *tâtonnement* process nothing observable can happen, since trades are ruled out and all other possible sources of changes in the data are equally excluded: this means, however, that the time over which the *tâtonnement* evolves is essentially a 'logical' time, not a 'real' one, and that the *tâtonnement*, being a purely virtual process, requires only one instant of 'real' time to carry its effects through. Moreover, when an equilibrium is reached, all the planned trades take place at the same instant, at the equilibrium price. As Walras himself writes:

> Le prix courant théorique est essentiellement un prix unique résultant, *à un moment donné*, d'un échange général.
>
> (Walras 1885, 312, fn. 1; italics added)

To sum up, the sort of exchange equilibrium which emerges from the 'no trade out of equilibrium' assumption appears to be so strictly related to one specific instant of 'real' time, namely, the instant to which the data defining the equilibrium can be referred, as to deserve the name of 'instantaneous' equilibrium. The expression 'instantaneous equilibrium' is never used by Walras, even if, as we have just seen, Walras explicitly connects the equilibrium prices and trades to 'a given instant' («un moment donné») in the history of the economy. Similarly, the distinction between a 'logical' and a 'real' time is never explicitly drawn by Walras in discussing the exchange model in either the second edition of the *Eléments* or his other writings dating back to the same period.[1] Yet, in spite of this omission, that expression and that distinction are useful, and even necessary, to correctly reconstruct Walras's system of thought, as it emerges from the introduction of the 'no trade out of equilibrium' assumption in the exchange model of the second edition of the *Eléments*.

In this respect, it should also be noted that in the same second edition Walras keeps unaltered the expressions «état stationnaire» and «prix stationnaire», already employed in the first edition to qualify respectively the 'market equilibrium state' and the sort of price which should be expected to rule in such an equilibrium state (Walras 1988, 71-72, 93; 196, 1; 197, 2-5). But such state and price are «stationnaire» only in the 'logical' time over which the *tâtonnement* takes place. The use of these expressions in no way implies that the data of the economy should be taken to be stationary in the 'real' time over which the economy evolves. In fact, immediately after the passage quoted above, claiming the theorist's right to assume the data unchanging during the *tâtonnement* process, Walras (1988, 146) goes on by saying:

> Mais c'est son devoir, une fois cette opération terminée, de se souvenir que les éléments des prix sont essentiellement variables et de formuler en conséquence la loi de variation des prix d'équilibre.

4. EQUILIBRIUM AND *TÂTONNEMENT* IN THE FIRST TWO EDITIONS OF THE *ELÉMENTS*: PRODUCTION

Let us consider now the production model. In this model the quantities of the consumers' goods available in the economy are no longer viewed as given amounts, as they were in the exchange model of the previous section, where they were taken to coincide with the given endowments ω^c_{li}, for $l = 1, \ldots, L$ and $i = 1, \ldots, I$; rather, they are regarded as the variable outcome of an endogenous process, the production process, transforming productive services (inputs) into consumers' goods (outputs).

The services issue from various types of factors or 'capitals' («capitaux», in Walras's terminology), grouped into the three categories of lands, persons, and capital goods proper. A service simply consists in the use of a certain quantity of capital for a specified period of time. Walras's choice of units is such that the quantity of a service turns out to be numerically equal to the quantity of capital from which the service flows. In the production model the quantities of the various types of capitals are part of the data of the problem. Hence, the quantities of the services available in the economy are regarded as given amounts, taking the place of the endowments of consumers' goods of the exchange model.

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[1] As we shall see in section 6 below, however, in the fourth edition of the *Eléments* Walras, though using a different terminology, will explicitly draw a similar distinction, concerning the use of the time concept in all his equilibrium models, including the pure-exchange one.

The owners of the various types of capitals coincide with the consumers. The latter can still be identified with the traders of the exchange model; however, since in the production model there also exists another category of agents, the entrepreneurs or producers, who are also involved in the trading activity, in this context we prefer to avoid the term 'traders' altogether, using instead the expression 'consumers' or 'owners', to designate one category of agents, and 'entrepreneurs' or 'producers', to designate the other. In the production model the owners of the various types of capitals are not allowed to sell them; yet they can lease them, that is, they can sell their services. The services can be sold either to the entrepreneurs as productive services or to other consumers as consumers' services; otherwise the owners can make a direct use of the services of the capitals they own as consumers' services.

As far as consumers are concerned, Walras keeps to the assumption of 'competitive behavior', duly extended to the new context. Consumers are still indexed by $i = 1, \ldots, I$, and consumers' goods by $l = 1, \ldots, L$. Further, let us suppose that there are M distinct services in the economy, indexed by $m = 1, \ldots, M$, where $0 < M < \infty$.

Consumer i's consumption set is given by $X_i \times Y_i = \{(x_i, y_i)\} = \mathbb{R}_+^{L+M}$, where (x_i, y_i) is a consumption plan of consumer i, $x_i = (x_{1i}, \ldots, x_{Li}) \in \mathbb{R}_+^L$ consumer i's demand for the L consumers' goods, and $y_i = (y_{1i}, \ldots, y_{Mi}) \in \mathbb{R}_+^M$ consumer i's demand for the M services. Let $u_i : \mathbb{R}_+^{L+M} \longrightarrow \mathbb{R}$ be consumer i's utility function. Finally, let $\omega_i^s = (\omega_{1i}^s, \ldots, \omega_{Mi}^s) \in \mathbb{R}_+^M$ be owner i's endowment of the M services. Consumer i's characteristics are then represented by the triple $(X_i \times Y_i, u_i(\cdot), \omega_i^s)$. Let $p = (1, \ldots, p_L) \in \{1\} \times \mathbb{R}_+^{L-1}$ be the prices of consumers' goods, expressed in terms of commodity 1, an always desired consumers' good, taken as the *numéraire*; let $w = (w_1, \ldots, w_M) \in \mathbb{R}_+^M$ be the prices of the M services, expressed in terms of the same *numéraire*. The price system is then $(p, w) \in \{1\} \times \mathbb{R}_+^{L-1+M}$.

Given a price system $(p, w) \in \{1\} \times \mathbb{R}_+^{L-1+M}$, consumer i solves the following optimization problem:

$$\max_{(x_i,\, y_i) \in X_i \times Y_i} u_i(x_i, y_i)$$

$$s.t. \tag{4.1}$$

$$px_i + wy_i = w\omega_i^s \text{ or } px_i = w(\omega_i^s - y_i)$$

Let us assume, for simplicity, that consumer i's characteristics are so specified that, $\forall (p, w) \in \{1\} \times \mathbb{R}_+^{L-1+M}$, the optimization problem has a unique solution, $(x_i, y_i)(p, w) \in \mathbb{R}_+^L \times \mathbb{R}_+^M$. The function $(x_i, y_i) : \{1\} \times \mathbb{R}_+^{L-1+M} \rightarrow \mathbb{R}_+^L \times \mathbb{R}_+^M$ is consumer i's demand function, where the first component function, $x_i\ (\cdot)$, refers to the demand for consumers' goods, while the second component function, $y_i\ (\cdot)$, refers to the demand for consumers' services. Let $\omega^s \equiv \sum_{i=1}^{I} \omega_i^s$ be the aggregate endowments (of services) of the economy; further, let $x\ (\cdot) \equiv \sum_{i=1}^{I} x_i\ (\cdot)$ and $y\ (\cdot) \equiv \sum_{i=1}^{I} y_i\ (\cdot)$ be the aggregate demand function for consumers' goods and the aggregate demand function for consumers' services, respectively.

As we have already remarked in discussing the exchange model, consumers' optimal choices of consumption plans are well defined when both equilibrium and disequilibrium prices are quoted; further, consumers' optimal choices are arrived at by following one and the same procedure in either case. It should also be noted that profits (or losses) do not appear in consumers' budget constraints. Those profits (or losses) that should accrue during the production process are not credited (or debited) to consumers, but are retained (or made good) by producers.

Each entrepreneur produces one output only, employing a constant quantity of each input per unit of output, whatever the scale of production. While the single-output assumption is never called into question by Walras, the fixed-coefficients assumption, instead, is occasionally questioned by him. An alternative hypothesis of variable coefficients, resulting from the choice of the cost-minimizing production technique, is adumbrated in the first edition of the *Eléments* (Walras 1988, 305), and exposed in a quite rudimentary and confused way since the third (1988, 720-2, 3; 586-591, 4, 4-5, or 5). Anyhow, since the alternative hypothesis is nowhere employed in the analytical part of the *Eléments*, we shall keep to the fixed-coefficients assumption in the following.

Let $A = [a_{ml}] = [a_1, \ldots, a_l, \ldots, a_L]$ be the $M \times L$ matrix of the technical coefficients of the economy, where a_{ml} is the quantity of input m needed to produce one unit of output l, while a_l is the M-dimensional vector whose elements represent the quantities of the M inputs needed to produce one unit of output l.

Given (p, w), the unit profit vector is $\pi = (\pi_1, \ldots, \pi_l, \ldots, \pi_L) = p - wA$. Under the stipulated conditions, the assumption that each producer of consumers' good l will choose the quantity of output to produce, say $q_l^{\star}$, and correspondingly the quantities of inputs to employ, $a_l q_l^{\star}$, in such a way as to maximize profits, does not generally give rise to well defined choices. For, if $\pi_l > 0$, then there exists no profit-maximizing choice; if $\pi_l = 0$, then the profit-maximizing choice is indeterminate, that is,

$q_l^\star \in [0, \infty)$; finally, if $\pi_l < 0$, then $q_l^\star = 0$ (hence, the last is the only case where the choice is well defined).

In all probability, this is the main reason why Walras does not assume the producers to be profit maximizers.[1] This theoretical choice is full of consequences, for it entails an asymmetrical treatment, within the same model, of the behavior of consumers and producers, both at and out of equilibrium. Since producers are not supposed 'to behave competitively', namely, to act as price-taking profit maximizers, the 'Walrasian rule of price adjustment' cannot be generalized to the whole economy, that is, to the totality of the agents; on the other hand, Walras cannot either give up completely his 'price adjustment rule', as in the economy there still exists a category of agents, namely, the consumers, who 'behave competitively', and consequently depend on that 'rule' for adjusting their choices. The solution devised by Walras consists in duplicating the rules, that is, in placing a new rule, that might be called the 'Walrasian rule of quantity adjustment', to the side of the already existing one, the 'Walrasian rule of price adjustment': in the production model, therefore, the revisions of the choices of plans of action (consumption plans), made by consumers during their own specific *tâtonnement* process, continue to depend on the 'price adjustment rule', exactly as they did before in the exchange model; on the contrary, the revisions of the aggregate behavior of producers during their own specific *tâtonnement* process are made to depend on the newly devised 'quantity adjustment rule'.

The 'quantity adjustment rule' works in a way that is quite different from that of the 'price adjustment rule'. In the case of production, in fact, given the single-output assumption, Walras takes into consideration the whole set of producers supplying a certain output, that is, in Marshallian language, the 'industry' producing it. For each consumers' good l to be produced, he supposes the quantity to be supplied by the industry l to be randomly assigned («criée au hazard») at the start of the specific *tâtonnement* process concerning production, and subsequently increased or decreased, over a time set to be suitably specified, according to whether the unit profit π_l is greater or less than zero; over the time set of the *tâtonnement* in production, the output of good l will not change if and only if π_l is nil, this being the sign that an equilibrium output has been reached. For each consumers' good *l*, therefore, the output of good *l* as a function of time (in the appropriate time set) is the state variable of the dynamic system governing the production adjustment process.

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[1] Many clues suggest that Walras would have adopted profit maximization as a possible explanation of producers' choices, were it not for the nonexistence of solutions to the maximization problem when unit profits are positive. In effect, when unit profits are negative, and consequently the profit-maximizing solution exists and is perfectly determinate ($q_l^\star = 0$), Walras is frequently led to adopt the profit-maximizing solution, implying the suspension of production (WALRAS 1988, 329, 399, 709, 4-5), even if, as we shall see in a moment, this breaks the symmetry of his own 'quantity adjustment rule', causing a discontinuity in producers' behavior that it is quite difficult to accept.

As can be seen, this *tâtonnement* is really something different from the *tâtonnement* in exchange discussed in the previous section: first, attention is focussed on a collective entity, the industry, rather than on the individual producers composing it; second, as an almost unavoidable consequence of the previous modeling decision, little room is left to the discussion of individual choices (the only individual decision explicitly mentioned by Walras being the decision to suspend production in the case of losses, on which we have dwelled in the previous footnote); third, once again as an almost necessary consequence of what has just been said, the equilibrium concept is not immediately related to a state where the plans of action optimally chosen by the individual agents of the economy are mutually compatible, hence executable. In a sense, and contrary to what Walras himself occasionally seems to suggest, the 'quantity adjustment rule' and the ensuing *tâtonnement* process in production appear to be more directly inspired by the pre-existent classical approach than the 'price adjustment rule' and the associated *tâtonnement* process in exchange.

In any case, as far as the production model is concerned, the outcome of Walras's modeling decisions, perfectly outlined since the first edition of the *Eléments*, is the following: within the same model there coexist two distinct categories of agents, consumers and producers, obeying different sets of choice or behavior assumptions, as well as two distinct rules of adjustment, the 'price' and the 'quantity adjustment rule', governing two different and separate *tâtonnement* processes, the *tâtonnement* in exchange and the *tâtonnement* in production, respectively. Not surprisingly, such coexistence of alternative rules and assumptions gives rise to a number of serious problems, marring both equilibrium and disequilibrium analysis. At the equilibrium, however, most problems turn out to be concealed, owing to the special features of the equilibrium concept adopted by Walras for this model.

In the production model, an equilibrium is defined as an array of prices, $(p^\star, w^\star) \in \{1\} \times \mathbb{R}_+^{L-1+M}$, and quantities, $(x^\star, y^\star) \in \mathbb{R}_+^L \times \mathbb{R}_+^M$, satisfying the following two systems of equations:[1]

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[1] Following Walras, we give here the equilibrium conditions in equational form. As the subsequent debate has demonstrated, it would be easy, and theoretically more satisfactory, to transform the equation systems (4.2) and (4.3) into systems of inequalities, accompanied by the appropriate complementary slackness conditions. However, since this change is irrelevant for our present purposes, we prefer to keep to Walras's original formulation.

$$p^\star = w^\star A \tag{4.2}$$

$$Ax^\star + y^\star = \omega^s \tag{4.3}$$

where $x^\star = x(p^\star, w^\star)$ and $y^\star = y(p^\star, w^\star)$.

When equations (4.2) and (4.3) above are satisfied, the state variables of the dynamical systems expressing the two 'Walrasian adjustment rules', of 'prices' and 'quantities', respectively, take on stationary values in their respective time sets. As $\pi^\star = p^\star - w^\star A = 0$, the problem of the allotment of profits (losses) cannot arise, so that the consumers' budget constraints duly take into account all the incomes generated in the economy. On the other hand, the fact that the equilibrium demand for consumers' goods, $x^\star$, equals the equilibrium supply of such goods, what in turn implies that the equilibrium demand for capital services, $Ax^\star$, equals the equilibrium supply of such services coming from the owners of the various types of capitals, $\omega^s - y^\star$, may be interpreted, if one so wishes, as the outcome of optimizing choices made not only by consumers, as one would expect *a priori*, but also by producers: for, if $\pi^\star = 0$, then the equilibrium aggregate input-output vector $(Ax^\star, x^\star)$ may indeed be viewed as the aggregate outcome of profit-maximizing choices made by individual producers, each producing an otherwise indeterminate proper fraction of such equilibrium aggregate input-output vector. Hence, after all, the differences in the behavioral rules governing the choices of the two groups of agents in the economy are somewhat disguised at the equilibrium.

Yet, what may be concealed at the equilibrium, cannot be hidden out of equilibrium. Due to this, in developing his theory of the *tâtonnement* with specific reference to the production model, Walras gets entangled in a number of inaccuracies, mistakes, and self-contradictory statements, which he tries to correct by repeatedly changing his exposition. In fact, this is the only part of Walras's overall theory of the *tâtonnement* for which no less than four significantly different versions are put forward in the *Eléments* and related writings: beyond the three alternative versions to be found, as is usual concerning the theory of the *tâtonnement*, in the first, second, and fourth edition of the *Eléments*, respectively, in this case one can also find a fourth variant in the «Théorie géométrique de la détermination des prix».

We shall now summarize that part of Walras's analysis of the *tâtonnement* process in the production model which is common to the versions developed in the first two editions of the *Eléments* and, with one qualification, to the variant contained in the «Théorie géométrique» as well. The differences between these versions will be examined later in this section, when we shall also critically discuss Walras's approach. Until that moment, we shall deliberately abstain from any critical remark.

The first point to be stressed is the following: since in the model under discussion Walras makes use of two distinct rules of adjustment, the 'price' and the 'quantity adjustment rule', to which two distinct types of *tâtonnement* are associated, the *tâtonnement* in exchange and the *tâtonnement* in production, here the analysis of the overall adjustment process necessarily takes the form of a sequence of alternating partial adjustment processes, each governed by its specific adjustment rule and developing in its appropriate time set.[1] How all such different time sets should be interpreted, and what sort of relation should be postulated among them, is a problem that we shall discuss in due time.

Walras assumes that, at the beginning of the overall *tâtonnement* process, let us say at time 0, a vector $w_0 \in \mathbb{R}^M_+$ of prices of services and a vector $q_0 \in \mathbb{R}^L_+$ of quantities of consumers' goods to be produced be randomly announced; in order to supply the output vector q_0 the producers need to employ an input vector Aq_0. The produced quantities of consumers' goods are sold at prices that clear the corresponding markets, that is, assuming of course a solution to exist, the quantities q_0 are sold at selling prices («prix de vente») $\hat{p}_0 \in \{1\} \times \mathbb{R}^{L-1}_+$ such that $x(\hat{p}_0, w_0) = q_0$.[2]

At input prices w_0, the vector of the unit costs of consumers' goods, called by Walras «prix de revient», is $c_0 = w_0A$. In general it will turn out that $c_0 \neq p_0$. For each good l such that, $c_{l0} \neq p_{l0}$, the 'Walrasian rule of quantity adjustment' calls for an increase or a decrease of the quantity produced with respect to q_{l0}, over the appropriate time set, according to whether p_{l0} is greater or less than c_{l0}, or, what is the same, according to whether $\pi_{l0} = p_{l0} - c_{l0}$ is greater or less than 0. According to Walras, the change in the output of good l will induce a change in the opposite direction in the corresponding price, via the market-clearing mechanism discussed in the previous paragraph, while the unit cost of the same good will remain unaltered so long as the prices of the inputs are fixed at their initial values w_0; therefore the divergence between price and unit cost will tend to be reduced. According to Walras the quantity adjustment process will go on until the quantities produced of the various goods converge to a vector $\hat{q}_1$ with the following properties: when the quantities produced are $\hat{q}_1$, the corresponding selling prices, that is, the market-clearing prices $\hat{p}_1$ such that $x(\hat{p}_1, w_0) = \hat{q}_1$, are also equal to the unit costs, so that one has $\hat{p}_1 = w_0 A$ as well.

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[1] Once again, referring to this sequence of alternating partial adjustment processes, Walras seems to recognize that this peculiar feature of his theory of the *tâtonnement* is due to «les bésoins de la démonstration», rather than to descriptive realism. In fact, according to WALRAS 1988, 704, 3-5, in order to get «une image exacte et complète du phénomène général de l'établissement de l'équilibre économique sous l'empire de la libre concurrence», it is necessary that «on se représente comme s'effectuant simultanément toutes les opérations que, pour les bésoins de la démonstration, nous avons dû supposer s'effectuant successivement».

[2] This is the assumption made in the first two editions of the *Eléments*. In the «Théorie géométrique», instead, WALRAS 1988, 700-702, 3-5 assumes that, at time 0, beyond w_0 and q_0, also a vector of prices of consumers' goods, p_0, be randomly announced. Such a vector is then supposed to converge to $\hat{p}_0$ through a *tâtonnement* process, based on the 'price adjustment rule' and quite similar in principle to the *tâtonnement* of the exchange model. What Walras is trying to do here is clear: he is striving to incorporate the *tâtonnement* in exchange into the overall *tâtonnement* of the production model. To this end, he constructs an artificial exchange model, by supposing that q_0, the quantities of consumers' goods to be produced, randomly announced at time 0 in the production model, can play the role of ω_c, the given aggregate endowments of consumers' goods in the exchange model discussed in section 3 above. His analysis is flawed, however, and he gets nowhere. In particular, he forgets that in whatever exchange model, even an artificial one, it is not enough to specify the aggregate endowments of the economy to make the model solvable: for one has also to know, among other things, the distribution of such endowments among the traders.

This point being reached, however, one can only be sure that $w_0 A\hat{q}_1$, the value of the services bought by the producers, is equal to $w_0 [\omega^s - y(\hat{p}_1, w_0)]$, the value of the services that their respective owners wish to sell to the producers.[1] But, in general, one will have $A\hat{q}_1 \neq \omega^s - y(\hat{p}_1, w_0)$, that is, the quantities of the services bought by the producers will generally be different from the quantities of the services that their respective owners wish to sell to the producers. Therefore, in order to bring to equality demand for and supply of services, one further *tâtonnement* turns out to be necessary in the market for services: this time the rule to apply is the 'Walrasian rule of price adjustment', according to which the price of a service increases or decreases according to whether the excess demand for that service is greater or less than zero. The working of the mechanism is rather obscure in this case, since changes in the prices of the services can only exert their effects on the demands for services coming from the consumers; but all the same Walras proves confident that the process will converge to a market-clearing vector of prices of the services, say to a vector $\hat{w}_1$ such that $A\hat{q}_1 = \omega^s - y(\hat{p}_1, \hat{w}_1)$.

Now, when the quantities produced are $\hat{q}_1$ and the service prices $\hat{w}_1$, the prices $\hat{p}_1$, which cleared the markets for consumers' goods at the initial quantities q_0 and service prices w_0, no longer clear their respective markets. Hence the overall *tâtonnement* will have to start again from the same point from which it had initially started. According to Walras, however, the extent of the disequilibrium existing in the economy will diminish at every round of the overall *tâtonnement*, so that in the end the process will 'certainly' (or at least 'probably') converge to a full price and quantity equilibrium of the production model.

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[1] This equality can be proved by first summing over the consumers the budget constraint equations appearing in the maximization problems (4.1), getting $\Sigma_{i=1}^{I} \hat{p}_1 x_i(\hat{p}_1, w_0) = \Sigma_{i=1}^{I} w_0 [\omega_i^s - y_i(\hat{p}_1, w_0)]$, hence $\hat{p}_1 x(\hat{p}_1, w_0) = w_0 [\omega^s - y(\hat{p}_1, w_0)]$. Then, by recalling that $x(\hat{p}_1, w_0) = \hat{q}_1$ and $\hat{p}_1 = w_0 A$, a simple substitution into the previous equation gives the desired result. It should be noted that, before reaching this stage of the overall *tâtonnement*, not even this result can be granted.

Walras's 'proof' of the convergence to an equilibrium of the *tâtonnement* process in the production model is no more convincing than the similar 'proofs' that he provides with regard to all his other models. But it is not to this deficiency of Walras's theory, already mentioned in section 2, that we want to draw the readers' attention now. Rather we want to stress that the *tâtonnement* process in the production model, as imagined by Walras and summarized above, is logically faulty and cannot work, even in principle, since it stumbles from the very beginning against an insuperable obstacle, namely, the unenforceability of undesired actions in an economy based on voluntary exchange.

In fact suppose, as no doubt Walras does in the first two editions of the *Eléments*, that the announced output q_0 be actually produced and sold at market-clearing prices $\hat{p}_0$. As already mentioned, the input quantities required to produce q_0 are Aq_0. But who can be supposed to sell such quantities? Certainly not the owners of the services, for, at prices $(\hat{p}_0, w_0)$, they wish to sell to the producers quantities of services $\omega^s - y\ (\hat{p}_0, w_0)$, which typically differ from Aq_0; so that the process envisaged by Walras cannot apparently even start. And of course, even if the process could be started somehow, the same obstacle would present itself over and over again, during the whole duration of the *tâtonnement*, since the equality between demand for and supply of services would be reached only at the end of the process, when the last *tâtonnement* in the market for services were eventually to converge.

Walras proves to be partly aware of the existence of this problem, as witnessed by the repeated attempts to contrive some solution he makes in the various editions of the *Eléments* and related writings. Anyhow, even if Walras's proposed way-outs are far from satisfactory (at least before the final solution put forward in the 1900 edition of the *Eléments*), his understanding of the special difficulties surrounding the theory of *tâtonnement* in production is much clearer than that exhibited by most of the later critics and commentators of that same theory.[1]

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[1] In particular, Walker 1972, 1987, and 1996 probably too busy at praising the realistic flavor of Walras's theory of *tâtonnement* in production in the first three editions of the *Eléments*, as contrasted with the despicable approach embraced by him in the fourth one, does not even realize the existence of the very serious difficulty pointed out in the text, a difficulty which in the last analysis explains both Walras's twistings in the first three editions of the *Eléments* and his final change of course in the fourth one. On the other hand, Huck 2001, and Bridel and Huck 2002, engrossed by the idea that Walras's problem in the first three editions of the *Eléments* is to neutralize the so-called 'distributional effects of disequilibrium production', do not apparently realize that Walras's real problem in those editions is how to dodge the issue of the unenforceability of unintended actions in a disequilibrium economy resting on the voluntary exchange assumption.

The first device concocted by Walras, in both the *mémoire* «Equations de la production» (1876b) and the first edition of the *Eléments*, consists in assuming the existence of 'a foreign market' («un marché étranger»), where there are people willing to make available whatever amounts of services were to be required by the domestic entrepreneurs (Walras 1988, 312, 1). Not only are these foreign people assumed to be ready to deliver the required amounts of services on demand, but they are also supposed to be satisfied with being paid, in exchange for the quantities of services supplied, an aggregate value which is different from that realized by selling the goods produced with the employment of those same services; further, according to Walras, this story must go on for as long as necessary, that is, until the equalities between prices and unit costs have been brought about by the appropriate *tâtonnement*. In conclusion, therefore, the problem arising here is not only that of specifying how profits and losses should be allotted among consumers, a problem already stressed by Morishima (1977, 49) with reference to Walras's *tâtonnement* in production; the real question here is that the transactions envisaged by Walras are nothing but 'swindles',[1] as they do not even respect the basic condition of voluntary exchange, namely, the *quid pro quo* clause.

The confused hypothesis that the producers shall buy the required amounts of services on 'a foreign market' is a device that badly conceals the real problem. Probably Walras soon realizes that the suggested contrivance is not only inadequate, but also counterproductive, for it contradicts the true spirit of general competitive analysis. Yet, it is only since the second edition of the *Eléments* that he drops it, adopting instead a significantly new set of assumptions.

First of all, an attempt is made to ground the *tâtonnement* in the production model on the same basic foundations as the *tâtonnement* in the exchange model:

> Il s'agit d'arriver à l'équilibre de la production de la même façon que nous sommes arrivés à l'équilibre de l'échange, c'est-à-dire en supposant les données du problème invariables pendant tout le temps que dureront nos tâtonnements, sauf à supposer ensuite ces données variables en vue d'étudier les effets de leurs variations.
>
> (Walras 1988, 308, 2-3)

Yet, the approach cannot be exactly the same in the two cases, since there also exists an important difference between the *tâtonnement* in production and that in exchange, which cannot be neglected:

> Mais le tâtonnement en matière de production rencontre une complication qui n'existait pas en matière d'échange. Dans l'échange, il n'y a pas de modification des marchandises. Un prix étant crié, et la demande et l'offre effective correspondant à

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[1] Solow 1956, 554 is the first to use the word «swindle» with reference to Walras's *tâtonnement*, though in a sense different from the present one.

> ce prix n'étant pas égales, on crie un autre prix auquel correspondent une autre demande et une autre offre effectives. Dans la production il y a transformation des services producteurs en produits. Certain prix des services étant criés, et certain quantités de produits étant fabriquées, si ces prix et ces quantités ne sont pas prix et quantités d'équilibre, il faudra non seulement crier d'autres prix, mais fabriquer d'autres quantités de produits.
>
> (Walras 1988, 308, 2-5)

This passage, which will remain unaltered until the last edition of the *Eléments*, has been frequently misinterpreted, in particular by Patinkin (1955, 378-380), as we shall see later in section 6. To fully understand its meaning, one should recall that, already in the second edition of the *Eléments*, Walras definitely adopts the idea that, in the *tâtonnement* in exchange, no actual transaction can take place, nor any natural or economic event can occur out of equilibrium, which is capable of altering the data of the problem: in fact, as Walras makes it clear in the above passage, all that occurs in disequilibrium is that new prices are announced and new plans of action (not actions) are correspondingly formulated. But when it comes to production the situation is wholly different: in this case, according to Walras, during the *tâtonnement* real actions are carried out and physical transformations take place which give rise to observable outcomes. Of such difference one has to take account, as the following passage witnesses:

> Acceptant cette nécessité, nous devons supposer que, pour chaque reprise du tâtonnement, nos entrepreneurs trouveront, dans le pays, des propriétaires fonciers, travailleurs et capitalistes possédant les mêmes quantités de services et ayant le mêmes besoins des services et des produits.
>
> (Walras 1988, 308, 2-3)

The new assumptions introduced by Walras aim at solving two problems at a time. In the first place, he wants to get rid of the indefensible *deus ex machina* of the first edition (the 'foreign market'): hence, he accepts to close his model, focussing attention on 'the domestic economy' («le pays») only. In the second place, however, he has also to take care of another requirement: even if something real and observable is occurring during the *tâtonnement* in production, the data of the economy must remain unchanged. To this purpose Walras assumes the economy under consideration to be stationary, that is, to repeat itself identically, period after period, as far as its data are concerned. The requirement that the data remain unchanged during the *tâtonnement* in production is here satisfied by means of the novel assumption that the data of the economy remain unchanged in the 'real' time set, that is, in that same time set over which the economy evolves; this implies that a 'period' in the history of the economy coincides with a 'round' of the *tâtonnement* process under question.

The assumption of invariance of the data of the economy in the 'real' time set or, in short, the assumption of stationarity of the economy, is here legitimate, since the data are all exogenous variables, functions, or relations, which cannot undergo any endogenous change as a result of what occurs in the economy, either at equilibrium or out of equilibrium. On the one hand, utility functions and technology are exogenous by assumption; therefore, if one so wishes, they can legitimately be taken as invariant in the 'real' time set. On the other hand, in the economy envisaged in the production model, only two types of commodities are allowed to be the object of economic activity: consumers' goods and services. Both types of commodities have the nature of flow variables (or «revenus», as Walras calls them), which cannot be stored and do not outlast their first use (Walras 1988, 280). An economy with these characteristics may be conveniently called a 'pure flow economy'. Hence, no economic activity is capable of endogenously affecting the data of a pure flow economy, by permanently changing the quantities of unstorable flow variables.

While the assumption of stationarity of the economy, in the sense specified, solves the two problems mentioned above, such assumption cannot of course solve all the remaining problems of the *tâtonnement* in production. In particular, at each resumption («reprise») of the *tâtonnement* in production, the same difficulty immediately emerges in the 'closed' stationary pure flow economy of the second edition, as it emerged in the 'open' economy of the first: who is going to take care of the mismatch between the required quantities of inputs and the quantities that the 'domestic' owners are willing to supply? In a sense, the situation is even worse here, since, in the absence of the accommodating foreign market of the first edition, there is no longer any external agency on which to unload the burden of the unavoidable 'swindles'. As a matter of fact, one ought honestly to admit (but Walras doesn't) that, in this case, nobody can really predict what will actually happen in the economy out of equilibrium: so, even if here some sort of observable disequilibrium behavior is allowed to take place in the 'real' time over which the economy evolves, no *theory* is actually available to predict such behavior. Yet this outcome is less disturbing than it might appear at first sight: for, in a pure flow stationary economy, any sort of disequilibrium activity, whatever it might be, cannot leave any mark on the data of the economy; it can only affect, hopefully in the right direction, the state variables of the adjustment process. Such sort of observable disequilibrium behavior, which cannot possibly affect the evolution of the data of the economy in the 'real' time set, may be conveniently called 'inessential'; but, precisely because it is inessential, its unpredictability is after all irrelevant.

As far as the second edition of the *Eléments* is concerned, our findings up to this point can be summed up as follows: while in this edition the *tâtonnement* in exchange appears to be a virtual process, in 'logical' time, with unobservable disequilibrium, in the same edition the *tâtonnement* in production appears as an actual process, in 'real' time, with observable (though, rigorously, both inessential and unpredictable) disequilibrium. These two interpretations of the *tâtonnement*, though coexisting, as we have just seen, in the same edition of the *Eléments*, are referred by Walras to two different activities, exchange and production, or even, with one qualification, to two different models: the exchange and the production model, respectively.[1]

Such two interpretations will be repeatedly taken up again in the later literature on the *tâtonnement*. Yet, they will be typically seen as alternative interpretations of one and the same model. Thus Hicks (1934, 343), referring to Walras's exchange model (freely interpreted), writes:

> Walras' system of prices will be reached, either if contracts are made provisionally or (a more important case) if people come on to the market on successive 'days' with the same dispositions to trade, and there is no carry-over of stocks (or a constant carry-over) from one day to the next.

Many years later Varian (1984, 247), referring to a general equilibrium model which should probably be interpreted as an exchange model, but might also be read as a suitably specified production model, will write:

> The *tâtonnement* story makes sense in two sorts of situations, one being the situation where no trade takes place until equilibrium is reached, so that the adjustment process is really just an 'auctioneer's rule'. The other situation is where all goods are unstorable so that each day the market reopens with new goods and all agents start their attempts to trade all over again.

What really matters, here, is to stress something that is not fully understood either by Walras, or, for that matter, by most of his successors: the two alternative interpretations of the *tâtonnement* support two alternative interpretations of the equilibrium concept, which will be called respectively the 'instantaneous' and the 'stationary' interpretation in the following. As we have seen at the end of the previous section, since the virtual *tâtonnement* in exchange does not entail the carrying out of any action out of equilibrium, it may be supposed to take place in a 'logical' time which can be distinguished from the 'real' time over which the economy evolves; hence, from a formal point of view, such virtual process can be imagined to take only one instant of 'real' time to carry its effects through. As a consequence, the sort of equilibrium which is supported by this adjustment process can be interpreted as an 'instantaneous' equilibrium, that is, an equilibrium referred to that instant of 'real time' in which one can observe (or infer) the data on which the equilibrium itself depends. On the contrary, since the actual *tâtonnement* in production develops in the same 'real' time set as that over which the economy evolves, under the assumption, however, that the data remain unaltered over such 'real' time set, the sort of equilibrium which is supported by this adjustment process can be interpreted as a 'stationary' equilibrium, that is, an equilibrium referring to the whole (potentially infinite) sequence of 'real' time instants in each of which one can observe (or infer) the unchanging data on which the equilibrium itself depends.

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[1] The qualification is due to this: while in the model of exchange only one interpretation of the *tâtonnement* (namely, the *tâtonnement* in exchange) is actually present, in the model of production both interpretations of the *tâtonnement* (namely, both the *tâtonnement* in exchange and the *tâtonnement* in production) are actually employed by Walras, even if, as we have seen above, he does not succeed in satisfactorily coordinating them.

We can conclude, therefore, that the production model of the second edition of the *Eléments* does not rule out a stationary interpretation of the equilibrium concept. In effect, as already anticipated in the introductory section, Cassel, in putting forward his own simplified version of Walras's production model, wholeheartedly embraces such an interpretation (1932, ch. 4). The ensuing Walras-Cassel model will then powerfully contribute to the diffusion of the stationary equilibrium interpretation of Walras's own theory.

As for Walras himself, he seems to be quite uncertain as to which interpretation to espouse. Take, in particular, the following passage where Walras (1988, 308, 2-3) explains what will happen when the *tâtonnement* in production is eventually finished:

> [Les entrepreneurs] pourront ou s'acquitter et en rester là, ou plutôt continuer indéfiniment la production dont la marche sera dès lors réglée pour autant qu'aucune variation ne surviendra dans les données [...].

Here Walras appears to oscillate between a purely instrumental and a more substantive reading of the assumption of stationarity of the data in the 'real' time set: in the first case, the only use of the assumption would be to justify the establishment of an equilibrium, which is not supposed to really last over time; in the second case, instead, the eventually reached stationary equilibrium would describe an authentically stationary economy, repeating itself indefinitely over time.

Walras's ambiguity as to the interpretation of the equilibrium concept employed in the production model of the second edition of the *Eléments* surely depends on the coexistence of two alternative interpretations of the equilibrium concept in that edition, and even in that same model. But there is also another reason for Walras's oscillations, which will be explained in the next section.

5. EQUILIBRIUM AND *TÂTONNEMENT* IN THE FIRST TWO EDITIONS OF THE *ELÉMENTS*: CAPITAL FORMATION

As already remarked in section 2, since the first edition of the *Eléments* Walras puts forward a third model, more comprehensive in scope than both the exchange and the production model: such third model is the model of capital formation, whose aim is to extend the equilibrium analysis to a new set of variables, that is, prices and produced quantities of newly produced capital goods as well as the so called 'rate of net income' («taux du revenu net»).[1] Moreover, since the first edition the analysis of the *tâtonnement* is extended to this model with a view to explaining how the equilibrium values of all the variables, including the newly introduced ones, are reached on the market by means of the *tâtonnement* process.

The *tâtonnement* associated to the model with capital formation engenders a number of specific problems, adding to those already pointed out. Yet we intend here to focus attention on one new problem only, directly concerning the interpretation of the *tâtonnement* construct and the equilibrium concept.

Newly produced capital goods present both similarities and dissimilarities with consumers' goods: the similarity lies in that newly produced capital goods, exactly like consumers' goods, are the outcome of a production process; the dissimilarity, on the contrary, is due to the fact that, unlike consumers' goods, newly produced capital goods outlast their first use, that is, they render a whole stream of services over time – it is precisely this property that makes them «capitaux», instead of «revenus», in Walras's sense (1988, 280).

As we have seen in the previous section, in the second edition of the *Eléments*, in contrasting the *tâtonnement* in production with the *tâtonnement* in exchange, Walras points out that the mere fact that in the production model consumers' goods are regarded as the result of a production process, rather than being taken as given endowments as it happens in the exchange model, forces the theorist to develop an analysis of the *tâtonnement* in production which is necessarily different from the analysis of the *tâtonnement* in exchange. But what applies to produced consumers' goods, also applies, according to Walras (1988, 377, 2-5), to newly produced capital goods as well:

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[1] The «rate of net income» in Walras's sense coincides with the common ratio of the net price of the service of any newly produced capital good to the price of the same capital good. The net price of a service, in turn, is obtained by subtracting from the price of that service the unit depreciation and insurance premiums on the corresponding capital good, which are taken by Walras as proportional to the price of the same capital good. The assumptions underlying this definition, as well as the whole model of capital formation, are questionable and have indeed been repeatedly questioned over time. But these problems need not retain us here, for they are irrelevant for our present purposes.

> Dans la capitalisation, du reste, il y a transformation des services en capitaux neufs, comme dans la production il y a transformation des services en produits. Un certain taux du revenu net et certains prix des services étant criés, et certaines quantités de produits et de capitaux neufs étant fabriquées, si ce taux, ces prix et ces quantités ne sont pas taux, prix et quantités d'équilibre, il faut non seulement crier un autre taux et d'autres prix, mais fabriquer d'autres quantités de produits et de capitaux neufs.

The problems posed by the *tâtonnement* relative to the production of new capital goods, being similar to the problems posed by the *tâtonnement* relative to the production of consumers' goods, must be confronted in a similar way. Hence, in the second edition of the *Eléments*, also as far as the production of new capital goods is concerned, Walras postulates the occurrence of a *tâtonnement* process in 'real' time, taking place in a stationary economy that identically repeats itself period after period, where, as in the production model, a 'period' in the history of the economy is supposed to coincide with a 'round' of the *tâtonnement* process:

> Acceptant cette nécessité, nous devons supposer que, pour chaque reprise du tâtonnement, nos entrepreneurs de produits et de capitaux neufs trouveront, dans le pays, des propriétaires fonciers, travailleurs et capitalistes possédant les mêmes quantités de services, ayant les mêmes besoins des services et des produits et les mêmes dispositions à l'épargne.
>
> (Walras 1988, 376, 2-3)

Yet, by focussing attention on the similarities between produced capital goods and produced consumers' goods, Walras ends up by overlooking that there exists a fundamental difference between the two types of commodities, a difference that he himself emphasizes elsewhere in the *Eléments*: for, as already recalled, capital goods are durables, surviving their first use, while consumers' goods are nondurables, perishing with their first use; shortly, capital goods are storable stocks, while consumers' goods are unstorable flows. Hence, the economy investigated in the model with capital formation, unlike the economy analyzed in the production model, is not a pure flow economy: rather, it is an economy where both stocks and flows can be produced.

However, if actual production is allowed to take place out of equilibrium in the *tâtonnement* relative to the production of new capital goods, as Walras would have us to believe, the newly produced capital goods add up to the already existing capital goods, necessarily changing the endowments of capital goods, hence also the quantities of the services they render. In short, in an economy where storable goods can actually be produced, an endogenous source of change in the data of the economy is at work which makes it illegitimate to assume the invariance of the data in the 'real' time over which the economy evolves.[1]

To sum up on this point, in the second edition of the *Eléments* Walras extends the assumption of an actual *tâtonnement* process, taking place in 'real' time under the assumption of stationary data, from the production model to the model with capital formation; in the latter model, however, such assumption, as well as the ensuing stationary interpretation of the equilibrium notion, becomes self-contradictory. Walras might easily have realized the absurdity of extending the assumption of stationary data to the model with capital formation, had he recalled what he himself had written on this point a few years before:

> Pour avoir une offre, une demande et des prix de capitaux, il faut substituer à la conception d'un état économique stationnaire celle d'un état économique progressif.
>
> (Walras 1988, 350, 1-3)

In effect, Walras seems sometimes to perceive that there is something wrong with a stationary model where capital goods can actually be produced, even out of equilibrium. In effect, in explaining what will happen in the economy when the *tâtonnement* is eventually finished, he writes:

> La production et la capitalisation pourront alors continuer, mais, bien entendu, avec les changements provenant de l'existence des capitaux neufs.
>
> (Walras 1988, 376, 2-3)

When literally taken, this statement would imply that capital goods, newly produced during the various rounds of the *tâtonnement* process in 'real' time, can only be used when the *tâtonnement* is finished. But what is the point in producing new capital goods, which can be stored somewhere in the economy, but cannot be used till the end of the *tâtonnement*, lest the data of the economy should change?

6. EQUILIBRIUM AND *TÂTONNEMENT* IN THE FOURTH EDITION OF THE *ELÉMENTS*

After the publication of the second edition of the *Eléments* (or, for that matter, the third, which is almost identical to the second in this respect), Walras's theory of equilibrium and *tâtonnement* is still left in a quite unsatisfactory status: in fact, it is marred by a number of epistemological and theoretical inconsistencies, giving rise to true and proper analytical mistakes. Among the epistemological inconsistencies, the most evident is revealed by the dualistic character of Walras's theoretical system: for in this system, as expounded in the just mentioned editions of the *Eléments*, one can find, side by side, two alternative interpretations of the *tâtonnement*, the virtual and the actual one, taking place in two different time sets, the 'logical' and the 'real' one, supporting two alternative notions of equilibrium, the 'instantaneous' and the 'stationary' one, and referring to different models, the exchange model, on the one hand, and the models of production and capital formation, on the other (with the aggravating circumstance that each of the last two models encompasses a sort of exchange sub-model, or better a number of such sub-models, obeying an adjustment rule and exhibiting a *tâtonnement* process that are different from the rule and process characterizing the larger encompassing model). Among the analytical inconsistencies, the most serious ones are perhaps the following two: first, the impossibility of even starting the *tâtonnement* processes concerning production (of both consumers' and capital goods), due to the unenforceability of unintended actions in an economy based on voluntary exchange; second, the logical impossibility of assuming the invariance of the data in 'real' time in the model with capital formation.

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[1] This is the reason why VARIAN 1984, 247, in the passage quoted above in the text, after distinguishing between the two types of *tâtonnement* (the virtual one, in 'logical' time, and the actual one, in 'real' time), continues as follows:

> The first story makes sense if goods are stocks or flows, but the second makes sense only if the goods are flow goods. For if stock goods are available, unsold goods will accumulate and endowments will change from day to day. This change in endowments will presumably affect agents' demands and supplies.

All the problems appear to arise from the attempt, still made by Walras in the second and third edition of the *Eléments*, to preserve a degree of realism in the analysis of the *tâtonnement* processes concerning production, when he had already abandoned any claim to realism (granting he had laid any such claim before) in the analysis of the *tâtonnement* process in exchange. By 'realistic' we mean here an analysis of the *tâtonnement* process trying to model it as an actual, observable, essential disequilibrium process in 'real' time: exactly what Walras is still striving to do in the second and third edition as far as production is concerned.

But Walras's efforts are not crowned with success: even putting aside all the inconsistencies mentioned above, and confining attention to the less controversial side of Walras's theory, one has to conclude that the results he gets on the mere ground of realism are very modest indeed. Let us take the production model, where at least the stationarity assumption is not logically inconsistent. Not even with respect to this model is Walras capable of producing a real *theory* of observable disequilibrium behavior. We are indeed told that such observable disequilibrium behavior can actually take place during the *tâtonnement* process, but we are not given any theoretical hint on how to predict it: such behavior may well be observable, but it remains unpredictable in Walras's world. Moreover, even if it were predictable, any effort to predict it would be misplaced: for, as we have seen above, such disequilibrium behavior is necessarily inessential, as it cannot affect the data; hence there is no point in predicting it.

As can be seen, the doubtful gains in terms of realism are far from sufficient to compensate the serious consistency losses caused by the survival of a 'real' time interpretation of the *tâtonnement* processes concerning production. So, when Walras, in preparing his last comprehensive revision of the *Eléments*, makes a final effort to polish his theory of the *tâtonnement*, his course of action is practically traced out: what he has to do is just to remorselessly suppress any sort of realistic pretence in the analysis of the *tâtonnement* processes concerning production. This is what Walras does by means of the so-called «hypothèse des *bons*», which is discussed in three different places in the fourth edition of the *Eléments*: in *Leçon* 20, «Equations de la production»; in *Leçon* 24, «Equations de la capitalisation et du crédit»; and, finally, in *Leçon* 29, «Equations de la circulation et de la monnaie». In the first two *Leçons* Walras corrects his previous treatment of the *tâtonnement* in the production of consumers' and capital goods, respectively; in the last one, which is one of the few completely new parts of the fourth edition, Walras systematically expounds his final view concerning statics, dynamics, *tâtonnement*, and equilibrium.[1]

The «hypothèse des *bons*» is first introduced towards the end of *Leçon* 20, immediately after the passage, preserved without change since the second edition and already quoted in section 4 above, where Walras contrasts the *tâtonnement* in exchange, where all that is required is a mere change in prices, with the *tâtonnement* in production, where instead an actual transformation of inputs into outputs is involved in the process. As will be recalled, this difference gives rise to a special difficulty that, according to Walras, is peculiar to the *tâtonnement* in production. As we have seen in section 4, in the second edition of the *Elé-*

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[1] While the most exhaustive and compelling statement of the consequences of assuming the «hypothèse des *bons*» on the time structure of equilibrium analysis can be found in *Leçon* 29 of the fourth edition of the *Eléments*, *i.e.*, in the first of the two lessons of that edition where Walras eventually puts forward a formal model of circulating capital and money, viewed as an integral part of his general equilibrium theory, the introduction of that «hypothèse» has hardly anything to do with either the model with circulating capital and money *per se* or, more specifically, with the *tâtonnement* process especially devised by Walras with reference to that model. As a matter of fact, a highly questionable assumption about the allegedly dicothomic (or almost dicothomic) charater of the economy, an assumption explicitly made by Walras in *Leçon* 30 of the fourth edition of the *Eléments* (1988, 465, 4-5), allows him to treat the *tâtonnement* process underlying the model with circulating capital and money as if it simply were a slightly modified version of the *tâtonnement* process underlying the model with capital formation. From Walras's own viewpoint, therefore, no further special difficulties can arise from the *tâtonnement* process associated with the model with circulating capital and money, on top of the difficulties already implicit in the models with production and capital formation of the second and third editions of the *Eléments*.

ments such a difficulty is taken care of by introducing an assumption of stationarity of the economic data, hence of the *tâtonnement* process in 'real' time. In the fourth edition, the passage containing the stationarity assumption is bodily replaced by the following one, introducing an alternative assumption, precisely, the «hypothèse des *bons*»:

> Pour réaliser un *tâtonnement* rigoureux en matière de production comme en matière d'échange, tout en tenant compte de cette circonstance, il n'y a qu'à supposer les entrepreneurs représentant par des *bons* des quantités successives de *produits* déterminées d'abord au hasard puis en augmentation ou diminution suivant qu'il y aura excédent du prix de vente sur le prix de revient ou réciproquement, jusqu'à égalité de ces deux prix; et les propriétaires fonciers, travailleurs et capitalistes représentant de même par des *bons* des quantités successives de *services* à des prix criés d'abord au hasard puis en hausse ou baisse suivant qu'il y aura excédent de la demande sur l'offre ou réciproquement, jusqu'à égalité de l'une et de l'autre.
>
> (Walras 1988, 309, 4-5)[1]

Though the above passage does not satisfy ideal standards of rigor and completeness,[2] its general import is all the same perfectly clear: the purpose of the «hypothèse des *bons*» is simply to turn an actual process involving observable disequilibrium in 'real' time, namely, the *tâtonnement* in production of the second edition, into a virtual process involving unobservable disequilibrium in 'logical' time, namely, the new *tâtonnement* in production of the fourth edition. To this end the «hypothèse des *bons*» effectively provides the conclusive answer: in the previous editions, as we have seen, the virtual character of the *tâtonnement* in exchange coexisted with the actual character of the *tâtonnement* in production (of both consumers' and capital goods); in the fourth edition, instead, by means of the «hypothèse des *bons*», which applies to the *tâtonnement* in production (of both consumers' and capital goods), all sorts of *tâtonnement* processes take on the same virtual nature already characterizing the *tâtonnement* process in exchange.

In the light of our previous observations, the severe criticisms levelled at the above quoted passage, particularly by Hicks and Patinkin, appear basically unfounded. Patinkin (1956, 378-380) objects that, by applying the «hypothèse des *bons*» to the *tâtonnement* in production only, and by forgetting to extend a similar assumption to the *tâtonnement* in exchange, Walras gives rise to an asymmetry between the two types of *tâtonnement*, which is unjustifiable from a logical point of view. But Patinkin's argument is really based on a misunderstanding: for, as we have seen, at the time of the introduction of the «hypothèse des *bons*», Walras had already been adopting for a long time a virtual interpretation of the *tâtonnement* in exchange; so that the application of the «hypothèse des *bons*» to the *tâtonnement* in production in the fourth edition has the effect of restoring a broken symmetry between the two types of *tâtonnement*, exactly the opposite of Patinkin's assertion. Hicks's criticism is even more ferocious, given that he qualifies the above passage as «rather pathetic» (Hicks 1934, 346, fn. 19). The reason for such a contemptuous epithet is that, when writing his 1934 paper, Hicks is still personally leaning towards a 'stationary' interpretation of the equilibrium concept (1934, 343, 346); and he realizes that Walras's «hypothèse des *bons*», by generalizing the assumption of a virtual *tâtonnement* in 'logical' time to the whole economy, strikes a fatal blow to the 'stationary' interpretation of the equilibrium concept in favor of the 'instantaneous' one.[1]

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[1] A very similar passage can be found in the fourth edition of the *Eléments* towards the end of *Leçon* 24, where the «hypothèse des bons» is introduced with reference to the production of new capital goods (Walras 1988, 377, 4-5). Also in this case the passage concerned replaces a passage dating back to the second edition, quoted in section 5 above, where Walras had introduced an assumption of stationarity of the economic data, hence of the *tâtonnement* process in 'real' time, in order to take care of the allegedly peculiar difficulty concerning the *tâtonnement* in the production of new capital goods.

[2] In particular Walras forgets to explicitly mention, among the quantities that should be represented by means of *bons* during the newly devised *tâtonnement* process, the quantities of consumers' goods demanded by the consumers as well as the quantities of services demanded by the entrepreneurs. The reason for such oversight, which can anyhow be easily corrected by any unprejudiced reader, is probably related to the twisted history of this passage, which is inserted into a pre-existing context which is left unaltered otherwise.

The last remarks lead us eventually to consider the general implications of the adoption of the «hypothèse des *bons*», implications which are systematically stated by Walras himself in the compact methodological passage contained in *Leçon* 29 of the fourth edition of the *Eléments*:

> Au moyen de l'hypothèse des *bons*, on peut distinguer nettement, surtout si l'on les suppose successives, les trois phases suivantes:
>
> 1° La phase des *tâtonnements préliminaires* en vue de l'établissement de l'équilibre en principe;
>
> 2° La phase *statique* de l'établissement effectif *ab ovo* de l'équilibre relatif à la livraison des services producteurs et des produits pendant la période de temps considérée, aux conditions convenues, sans changements dans les données du problème;
>
> 3° Une phase *dynamique* de trouble continuel de l'équilibre par des changements dans ces données et de rétablissement continuel de l'équilibre ainsi troublé.
>
> En conséquence de ces définitions, il doit être bien entendu que les *capitaux neufs*, *fixes ou circulants*, qui seront livrés pendant la seconde phase, ne fonctionneront que dans la troisième phase, constituant ainsi un premier changement dans les données du problème.
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> (Walras 1988, 447, 449, 4-5)

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[1] A few years later, without acknowledging it explicitly, Hicks will change his mind about what should be regarded as the most appropriate equilibrium concept or the most acceptable *tâtonnement* process. In fact, in *Value and Capital* (1939, 122 ff.), Hicks will envisage an adjustment process, based on the famous distinction «Monday»-«week», which is nothing but a rewriting, in a pseudo-realistic, Marshallian language, of Walras's virtual *tâtonnement* process, a process based *inter alia* on that «hypothèse des *bons*» that Hicks had qualified as «pathetic» only five years before. Moreover, Hicks's newly discovered notion of «temporary equilibrium» is nothing but a specialization of Walras's notion of 'instantaneous' equilibrium.

The time structure of Walras's analysis comes out very clearly of the above passage, which dispels all possible doubts concerning the nature of the economy that Walras intends to take as the object of his theoretical investigations. In modern language, the economy that Walras wants to study is a competitive sequence economy, that is, an economy where markets open sequentially over time and the agents of the economy make their choices at any time at which the markets open.

More precisely, the economy envisaged by Walras evolves over a chronologically ordered sequence of non-degenerate connected time intervals of equal length, which may be called dates, covering the whole real line. The time length of each date is arbitrary, but to fix ideas it may be convenient, following Walras (1988, 577, 4-5), to suppose that a date lasts just one year. Let t be the initial instant of the corresponding date. Then we can let $t \in \mathbb{Z}$, where $\mathbb{Z}$ is the set of the integers. To each instant $t \in \mathbb{Z}$ there corresponds the half-open interval $[t, t + 1)$, which can be identified with date t. Each instant $t \in \mathbb{Z}$ is characterized by a certain array of data, which remain unchanged over the whole date t.

In the initial instant of each date t a virtual *tâtonnement* process takes place, allowing the economy to instantaneously reach the equilibrium corresponding to the data prevailing at instant t. By using the expression «*tâtonnements préliminaires*» Walras makes it clear that the *tâtonnement* process he is envisaging here develops in a 'logical' time set and takes just one instant of 'real' time to carry its effects through. The equilibrium supported by such virtual *tâtonnement* process in 'logical' time is 'instantaneous' in nature; more precisely, since the economy under investigation is a sequence economy, each 'instantaneous' equilibrium associated with the initial instant of a particular date takes the nature of a «temporary» equilibrium, in Hicks's sense. The evolution of the economy over time can then be described by means of a chronologically ordered sequence of «temporary» equilibria: Walras (1988, 447) denotes such a descriptive procedure by the expression «équilibre variable ou mobile». No assumption of stationarity of the data, hence of the sequence of «temporary» equilibria, is either necessary or logically possible, and no such assumption is in effect made by Walras.

No specific limitation as to the nature of the commodities that can be traded or produced in the economy needs to be imposed here: in particular both flows and stocks can be the object of economic activity. At the initial instant of any given date t, given the equilibrium prices instantaneously reached at t by means of the «*tâtonnements préliminaires*» in 'logical' time, the agents optimally choose their equilibrium plans of action (trade, consumption, and production plans) that, being mutually compatible, can all be actually carried out. According to Walras, the «phase *statique*», spanning the whole date, precisely consists in the execution of the equilibrium plans of action. The quantities of flow goods and services whose delivery is implied by the equilibrium plans are actually delivered and used during the whole time span of the date. The quantities of newly produced capital goods, instead, may well be delivered during the time span of the date, but cannot be used until the first instant of the subsequent date:[1] such quantities represent the first (endogenous) source of change in the data of the economy. According to Walras, the 'dynamic phase' consists precisely in the change in the data, which is supposed to take place at the initial instant of each date. Hence, the «phase *dynamique*» is no less instantaneous than the «phase des *tâtonnements préliminaires*»: as a matter of fact, from a temporal (though not a logical) point of view, the two phases necessarily coincide, since they unavoidably overlap at the initial instant of each date.

This construction is no doubt artificial and contrived, but it can take care of all the analytical problems left unsolved by the previous editions of the *Eléments*: in particular, no unintended disequilibrium action needs to be enforced, since no observable disequilibrium behavior is allowed to take place; no self-contradictory assumption mars the capital formation model, since no stationarity assumption is any longer required. Further, some of the epistemological problems besieging the previous editions of the *Eléments* disappear as well: in particular, the disturbing coexistence of two different equilibrium concepts, the 'instantaneous' and the 'stationary' one, is no longer required in Walras's final comprehensive model, since in that model the 'instantaneous' notion is the only one to survive. The price to be paid, in terms of loss of descriptive realism and predictive power, for finally getting rid of all the surviving remnants of the 'stationary' interpretations of both the *tâtonnement* construct and the equilibrium concept is not high, after all; but indeed the price to be paid is low essentially because the gains granted by the previous pseudo-realistic interpretations of the *tâtonnement* and the equilibrium concept were almost insignificant. Not irrelevant, instead, is the methodological cost associated with the generalization of the idea of an 'instantaneous' *tâtonnement* process: as a matter of fact, the very idea of an 'instantaneous process', which simply reflects the coexistence in the same analytical framework of two distinct time concepts (the 'logical' time of the virtual *tâtonnement* process and the 'real' time of the actual economic process), is very hard to swallow, for it may legitimately appear as a true and proper contradiction in terms. But this is a cost that Walras is willing to pay in the end, especially in view of his unswerving faith in the speed and effectiveness of the market in providing its «solution pratique»: for, if the competitive markets are indeed so quick and reliable in delivering their «solution» that the equilibrium prices can be reached «en quelques minutes», as Walras (1988, 93) apparently believes, then to suppose that the equilibration process be 'instantaneous' cannot appear, after all, as an unbearable distortion of what happens in the real world.

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[1] In modern language, therefore, one would say that the quantities of the capital goods that are newly produced during a certain date are in effect contracted for forward delivery, at the initial instant of the subsequent date.

7. Conclusions

Over the long time period elapsed since the appearance, in 1874, of the first installment of the first edition of Walras's *Eléments*, two alternative interpretations of the equilibrium notion employed by Walras in his *magnum opus* have been recurrently suggested by scholars and theorists conversant with both Walras's work and the history of general equilibrium theory: a stationary equilibrium interpretation, first advocated by Wicksell and later popularized by Cassel's influential reformulation of Walras's production model, and a temporary equilibrium interpretation, first embraced by Pareto and the small group of Walras's immediate followers and much later defended, in the context of formalized reconstructions of Walras's theory, by Morishima and others in the last quarter of the twentieth century. Such persistent oscillations in the interpretation of Walras's equilibrium concept lead one to suspect that some ambiguity or inconsistency may possibly exist within Walras's original formulation of general equilibrium theory in the *Eléments* and related writings. In this paper we have explored this conjecture focussing, in particular, on the relationship between the theory of the *tâtonnement*, regarded by Walras as an essential part of his equilibrium approach, and Walras's own interpretation of the equilibrium concept as employed in the *Eléments*.

The theory of the *tâtonnement* is probably the part of the *Eléments* which undergoes the most frequent and radical revisions over the years. Hence, our first task in this paper has been to explain how such theory evolves over the four editions of the *Eléments* published during Walras's lifetime (1874-1877, 1889, 1896, 1900). Moreover, since Walras develops a special version of the *tâtonnement* for each of the four nested equilibrium models put forward in the *Eléments* (exchange, production, capital formation, circulation and money), it has been necessary to separately analyze the evolution of the *tâtonnement* construct with reference to each model in turn.

As regards the model of exchange, we have shown that, after a somewhat naïve handling of the disequilibrium process in the first edition of the *Eléments* (1874), Walras makes his analysis much more precise in the second edition (1889), by explicitly introducing a 'no trade out of equilibrium' assumption. By virtue of this assumption, since the second edition of the *Eléments* the *tâtonnement* process in exchange becomes a virtual process, where no observable disequilibrium behavior is allowed to occur. Hence, while the *tâtonnement* process in exchange must be supposed to take place in a purely 'logical' time, distinct from the 'real' time over which the economy evolves, the associated equilibrium, supposedly reached in one single instant of 'real' time, becomes 'instantaneous' in nature.

Concerning the production model, however, we have shown that, for a much longer period, Walras keeps faithful to his original assumption that disequilibrium production entails observable behavior, taking place in 'real' time and producing actual consequences. This assumption – which of course, from the second edition onwards, cannot be easily reconciled with the coexisting treatment of *tâtonnement* in exchange – gives also rise to further specific difficulties, which Walras desperately endeavors to overcome over the years: to this end, in the first edition he makes the unpalatable assumption of the existence of a 'foreign market', accommodating all 'domestic' quantitative disequilibrium; in the second edition, instead, while giving up all recourse to an external *deus ex machina*, he resorts to the alternative assumption that the economy under question is a stationary economy, identically repeating itself period after period under unchanging conditions. When applied to the production model, this assumption gives rise to the prototype of the stationary equilibrium model, later resumed under the species of the Walras-Cassel model, on which the stationary interpretation of Walras's equilibrium theory is actually founded. Yet, when applied to the capital formation model, as Walras tries to do, the stationarity assumption turns out to be self-contradictory. In view of this, in trying to remedy the difficulties and contradictions engendered by his previous treatment of the *tâtonnement* in production and capital formation, in the fourth edition (1900) Walras eventually makes a new assumption, known as the «hypothèse des *bons*», with the purpose of extending to the *tâtonnement* in production and capital formation those same features (virtuality, unobservability, evolution in a merely 'logical' time set) that had already been characterizing the *tâtonnement* in exchange for a long time, namely, since the appearance of the second edition of the *Eléments*. In this new context, all stationarity assumptions can and must be dropped, so that the equilibrium notion emerging from a generalized virtual *tâtonnement* process, in both exchange and production and capital formation, cannot but be of the 'instantaneous', specifically «temporary», type.

The twisted history of the *tâtonnement* construct, hence of the equilibrium concept, over the various editions of the *Eléments* can thus help explain the recurrent oscillations in the interpretation of Walras's equilibrium notion that can be found in the literature. From our reconstruction it appears that the stationary equilibrium interpretation, which has been playing such an important role in the subsequent history of general equilibrium theory, can indeed be traced back to the stationarity assumption made by Walras in the second edition of the *Eléments*, in the attempt to overcome the serious difficulties arising from his own original assumption of a 'real' time, observable disequilibrium *tâtonnement* process in production. Yet, as we have also shown, Walras himself eventually realizes that only a virtual *tâtonnement* process in 'logical' time is consistent with the basic assumptions underlying his comprehensive model, so that, in the last analysis, the 'instantaneous' equilibrium notion is the only one to pass the internal consistency test and consequently to survive in the last edition of the *Eléments* during Walras's lifetime.

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