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MATHEMATICAL MODELS OF 2D PLOTTER COMPONENTS

MATEMATICKÉ MODELY SOUČÁSTÍ 2D PLOTRU

Abstract

This paper deals with most common 2D plotter components and provide identification procedure and exact parameters needed for mathematical modelling. Since a lot of Open-Source CNC machines use the same components, this information can be used to build-upon and improve performance of millions of machines.

Abstrakt

Tato práce se zabývá postupem identifikace a získáním přesných paramterů nejběžnějších součástí 2D polohovacího plotru, které jsou potřebné pro matematické modelování. Jelikož značná část Open-Source CNC strojů využívá totožné komponenty, může být tato informace využita k vylepšení vlastností miliónů strojů.

Keywords

Mathematical Model, 2D Plotter, Identification, Motor, Pulley, Belt, NEMA17, GT2

1 INTRODUCTION

Mathematical models are essential components of design and development of every machine. Deep knowledge of every single component can significantly reduce resources needed for testing and optimization. It is also easier to confront limitations and weaknesses of a whole machine. There are components available on today's market whose mathematical models are well known. However large portion of suppliers are still not willing to share their exact information with end user.

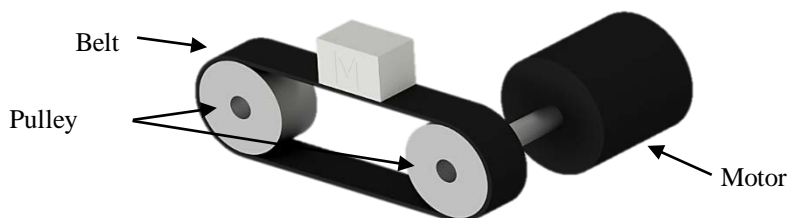


Fig. 1 Typical drive solution for one axis of 2D plotters

Most common solution of 2D plotter drive consist of belt, motor and set of pulleys. Simplified visualization is shown in Fig. 1. Next chapter will deal with most crucial components in order to provide all necessary identification methods which lead to mathematical models of motors and belts. In this paper, GT2 6mm belts and NEMA17 hybrid stepper motors were used to obtain essential parameters.

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2 MATHEMATICAL MODEL OF A STEPPER MOTOR

Hybrid stepper motor is combination of PM (Permanent Magnet) motor and VR (Variable Reluctance) motor. It consists of core with permanent magnet and fine teeth on stator's coils. The stepper motor operates on the principle of electromagnetic and magnetic force, formed between the stator and rotor. The basis for modelling such a system is found in the intensity of the magnetic field. The intensity of the magnetic field is a vector quantity describing the degree of force effects of the magnetic field. If we are interested in the force effects inside the stepper motor, we will get the intensity of the field in the gap between the rotor and stator: [1],[2]

$$H_g = \frac{nl}{g} \quad (1)$$

where:

- n – number of coil windings,
- l – current [A],
- g – gap between stator and rotor [m].

Considering the permeability environment in the gap between the rotor and stator, we obtain the magnetic induction in the given gap:

$$B_g = \frac{\mu_0 nl}{g} \quad (2)$$

where:

- μ_0 – permeability of air $\left[\frac{H}{m}\right]$.

The stator and rotor are attracted to each other by specific gears. Considering the mutual area where x is the longitudinal length w thickness, on the self-acting teeth, we can express their reciprocal magnetic flux:

$$\Phi = \frac{xw\mu_0 nl}{g} \quad (3)$$

Furthermore, it is necessary to consider that the magnetic flux is created by each thread of the coil along its length. The resulting magnetic flow linkages are:

$$\Psi = n\Phi = \frac{xw\mu_0 n^2 l}{g} \quad (4)$$

where:

- n – number of teeth.

Current pulses flowing through pairs of coils A, \bar{A} , a, B, \bar{B} creates a torque. Individual rotor and stator teeth and their arrangement is causing fine stepping motion. Control pulses are generated in phases A and B separately by the driver circuitry and can be described by equations:

$$\tau_A = -pn\Phi I_A \sin p\theta \quad (5)$$

$$\tau_B = -pn\Phi I_B \sin p(\theta - \lambda) \quad (6)$$

where:

- p – is number of magnetic pole pairs (equal to number of teeth in case of hybrid motors)
- θ – is rotation angle [rad]
- λ – tooth pitch [rad]

The equation of motion of the rotor is as follows: [2], [3]

$$J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + pn\Phi I_A \sin p\theta + pn\Phi I_B \sin p(\theta - \lambda) = 0 \quad (7)$$

where:

- D – Viscous damping coefficient
- J – moment of inertia [kg m²]

After linearization of equation of motion and while considering disturbances δi_B , δi_A and $\delta\theta$ as extremely small, linear differential equation is obtained:

$$J \frac{d^2(\delta\theta)}{dt^2} + D \frac{d(\delta\theta)}{dt} + 2p^2 \Phi n I_0 \left(\cos \frac{p\lambda}{2} \right) (\delta\theta) + pn\Phi \left(\sin \frac{p\lambda}{2} \right) (\delta i_A - \delta i_B) = 0 \quad (8)$$

The ratio of the Laplace transforms of the output and the input will provide the transfer function of the stepper motor:

$$G(s) = \frac{\theta_o}{\theta_i} = \frac{2p^2 \Phi n I_0}{(Js^2 + Ds + 2p^2 \Phi n I_0)} = \frac{\omega_{np}^2}{s^2 + \frac{D}{J}s + \omega_{np}^2} \quad (9)$$

where:

- ω_{np}^2 – natural frequency of motor which is equal to: [2]

$$\omega_{np} = \sqrt{\left(\frac{2p^2 \Phi n I_0}{J} \right)} \quad (10)$$

2.1 Identification of Motor parameters

In order to acquire precise mathematical model of a real motor, corresponding physical parameters should be incorporated into the transfer function. Most common parameters are shipped with motor in form of datasheets. Other parameters can be obtained from sales or technical support; however, some parameters will not be possible to obtain and needs to be physical or mathematically identified.

The first parameter is number of coil windings. The approximate number of coils can be determined based on the geometric knowledge of the coil. First, we calculate the length of the wire from the relationship:[1]

$$l = \frac{RS}{\rho} \quad (11)$$

where:

- R – is electrical resistance of a coil
- ρ – is resistivity [$\Omega \cdot m$]
- S – is cross-sectional area [$\Omega \cdot m$]
- l – is length of a wire [m]

With mentioned parameters the length of wire can be calculated:

$$l = \frac{RS}{\rho} = \frac{5,9 \cdot 3,14 \cdot 10^{-8}}{0,0169 \cdot 10^{-7}} = 10,962 \text{ m} \quad (12)$$

Following step is to determine the approximate number of coils windings. The mean length of the thread of one coil is 72mm. The resulting number of coils will be obtained by ration of the length of the wire and length required for one winding of the given coil.

$$n = \frac{l}{l_n} = \frac{10,962}{72 \cdot 10^{-3}} = 152,25 \cong 152 \text{ coil windings} \quad (13)$$

Next parameter is the gap between the rotor and stator. The smaller the gap, the better the performance of the engine, however it is more demanding for production (precision) and construction (vibration, speed). The gaps between rotor and stator in today's engines range between 0.02 -0.05mm depending on their application. For NEMA17 hybrid stepper motor we choose a gap width of 0.05mm.

The parameters of individual teeth are measurable by a sliding meter after disassembly of the engine and removal of the rotor from the stator. The parameters are as follows:

x – length horizontal with the axis of rotation = 5x2 mm

w – width = 1mm

The tooth pitch λ is calculated from the number of stator teeth following the equation:

$$\lambda = \frac{2\pi}{N_r} = 0,126 \text{ rad} \quad (14)$$

Another important parameter is the magnetic flux. It is obtained by: [2]

$$\Phi = \frac{xw\mu_0 nI}{g} = \frac{10 \cdot 10^{-3} \cdot 1 \cdot 10^{-3} \cdot 1,257 \cdot 10^{-6} \cdot 152 \cdot 1}{0,05 \cdot 10^{-3}} = 0,382 \cdot 10^{-4} \text{ Tm}^2 \quad (15)$$

We also calculate the constant torque and viscous friction:

$$K_T = 2nN_r\Phi = 2 \cdot 152 \cdot 50 \cdot 0,382 \cdot 10^{-4} = 0,581 \text{ NmA}^{-1} \quad (16)$$

$$D = \frac{K_T}{R} = \frac{0,581}{5,9} \cong 0.1 \text{ Nms}^{-1}\text{rad}^{-1} \quad (17)$$

2.2 Simulation model

Simulation model is implemented in the environment MATLAB Simulink. The aim of this model is to simulate an XY plotter driven by stepper motors. Entering the desired position at any given time is divided into corresponding axis. The signal goes through the transfer function of given axis and its output is combined with the second axis signal and it is plotted in an XY chart. The models of individual axes are similar. They differ only in mechanical transmission, mass and moment of inertia. As a basis of transfer function for both axes, we will use the formula derived above:

$$G(s) = \frac{\Theta_o}{\Theta_i} = \frac{2p^2\Phi nI_0}{(Js^2 + Ds + 2p^2\Phi nI_0)} = \frac{\omega_{np}^2}{s^2 + \frac{D}{J}s + \omega_{np}^2} \quad (18)$$

Number of stator teeth of hybrid motor SX17-1005VLQCEF is 50. In the previous chapter we calculated the number of coils, which is 152 in this case. Driving current is given by manufacturer's datasheet and is equal to 1 Amp. The magnetic flux was calculated for this specific stepper motor in previous chapter and equals to $0.382 \cdot 10^{-4} \text{ Tm}^2$. Natural frequency of each motors is obtained:

$$\omega_{npx} = \sqrt{\left(\frac{2p^2\Phi nI_0}{J_x}\right)} = \sqrt{\left(\frac{2 \cdot 50^2 \cdot 0,382 \cdot 10^{-4} \cdot 152 \cdot 1}{29,621 \cdot 10^{-6}}\right)} = 990.008 \text{ Hz} \quad (19)$$

$$\omega_{npy} = \sqrt{\left(\frac{2p^2\Phi nI_0}{J_y}\right)} = \sqrt{\left(\frac{2 \cdot 50^2 \cdot 0,382 \cdot 10^{-4} \cdot 152 \cdot 1}{300,177 \cdot 10^{-6}}\right)} = 310.992 \text{ Hz} \quad (20)$$

The resulting transfer function for the X and Y axes are:

$$G(s)_{mx} = \frac{990,008^2}{s^2 + \frac{1}{29,621 \cdot 10^{-6}}s + 990,008^2} = \frac{1}{1,02 \cdot 10^{-6}s^2 + 0,0345s + 1} \quad (21)$$

$$G(s)_{my} = \frac{310,992^2}{s^2 + \frac{1}{300,177 \cdot 10^{-6}}s + 310,992^2} = \frac{1}{1,034 \cdot 10^{-5}s^2 + 0,0345s + 1} \quad (22)$$

Simulink model is shown in Fig. 2. Each axis is mathematically divided and is merged together in XY graph. Both axes have its own transfer function which is defined by motor type, weight of axis and moment of inertia. Gain block in model schematics represents conversion from rotational to linear motion and it is achieved by simple pulley.

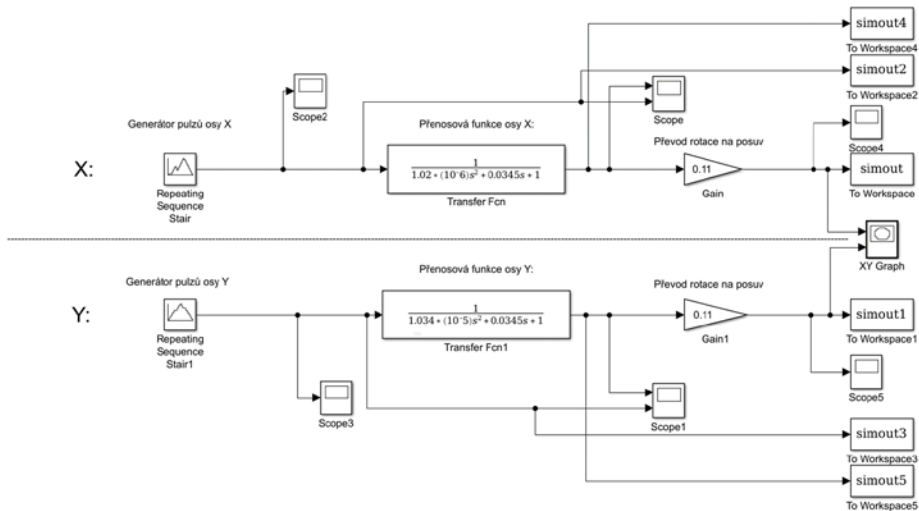


Fig. 2 - Simulink model schematics

Resulting graphs shows input signal and simulated move of an axis. Each axis has its own input signal and graph. Fig. 3 represents X axis and Fig 4. shows simulation of Y axis. Red colour represents input pulses generated by stepper motor driver and blue colour shows expected rotational motion of a motor. The third graph (Fig. 5) is XY graph which is obtained by merging output signals from both axes. Thanks to simple conversion from rotational to linear motion last graph displays simulated move of XY plotter.

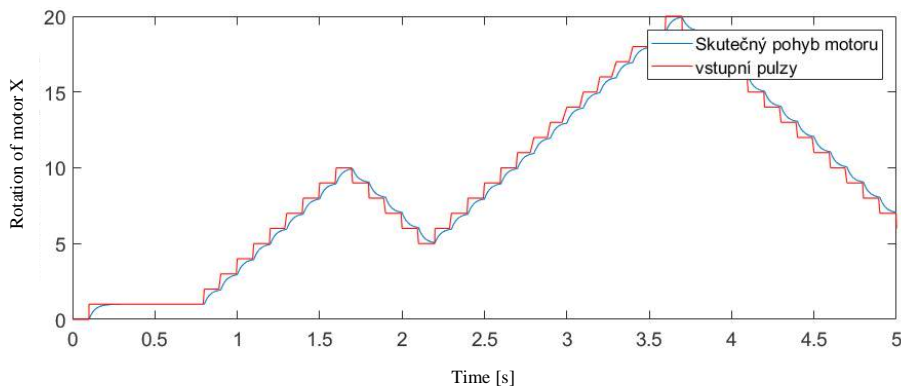


Fig. 3 - Simulation of X axis

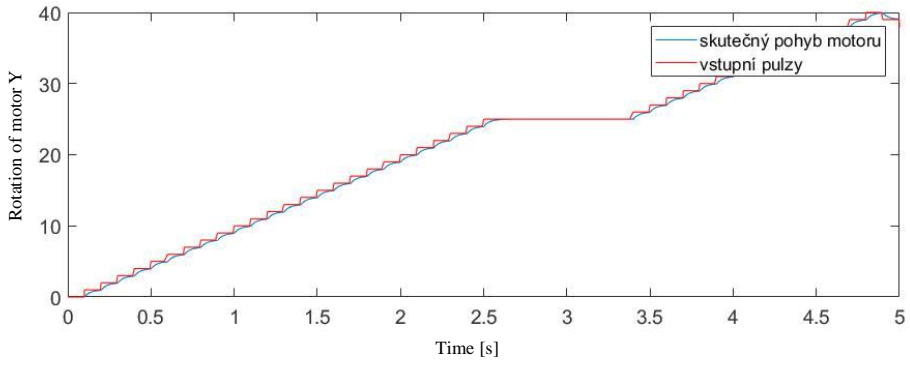


Fig. 4 - Simulation of Y axis

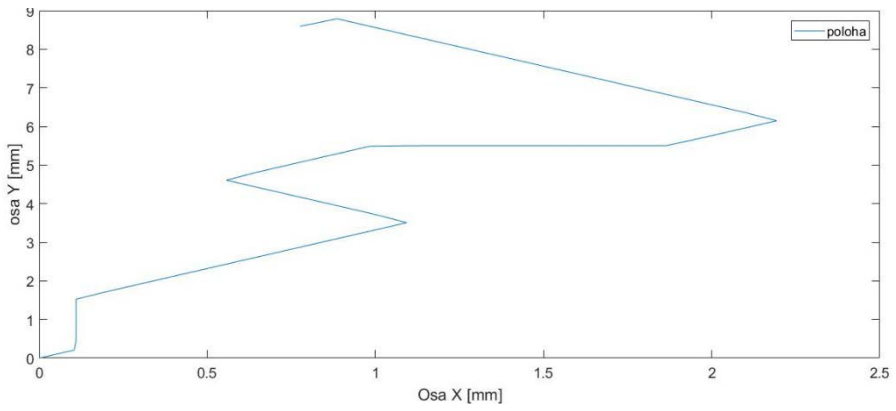


Fig. 5 - Simulation of XY plotter trajectory

3 CONCLUSIONS

In this paper the mathematical model of hybrid stepper motor was obtained with step by step description. From the basic relationships I got to the transfer function of individual axes of the machine and specified their parameters. I then applied this model to my machine and its real components. After disassembling and measuring all necessary parameters of the stepper motor the final forms of transfer functions were obtained. I created a Simulink model in which I put the parameters of my machine. As input, I used the input signal generator and carried out a verification of the dynamics that protrude from the stepper motor model. Future work will be aimed to belt transfer functions and more complex simulation model which would provide more realistic trajectories. The comparison of real and simulated machine will be included as well.

4 ACKNOWLEDGEMENT

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