PHYSICAL REVIEW D **96,** 086018 (2017)

Holographic bulk reconstruction with α' corrections

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(Received 8 August 2017; published 23 October 2017)

We outline a *holographic* recipe to reconstruct α' corrections to anti–de Sitter (AdS) (quantum) gravity from an underlying CFT in the strictly planar limit ($N \to \infty$). Assuming that the boundary CFT can be solved in principle to all orders of the 't Hooft coupling λ, for scalar primary operators, the λ^{-1} expansion of the conformal dimensions can be mapped to higher curvature corrections of the dual bulk scalar field action. Furthermore, for the metric perturbations in the bulk, the AdS/CFT operator-field isomorphism forces these corrections to be of the Lovelock type. We demonstrate this by reconstructing the coefficient of the leading Lovelock correction, also known as the Gauss-Bonnet term in a bulk AdS gravity action using the expression of stress-tensor two-point function up to subleading order in λ^{-1}.

DOI: 10.1103/PhysRevD.96.086018

I. INTRODUCTION

In the absence of a fully nonperturbative formulation of (super)string theory, it is pragmatic to think of the Maldacena duality [1,2] as furnishing a manifestly holographic *definition* of quantum gravity in asymptotically anti–de Sitter (AdS) spaces in terms of a conformal field theory. This definition is in terms of a large-N conformal field theory supported on the conformal boundary of the AdS space. Now we have examples of AdS holography which are neither supersymmetric, nor require ten or eleven spacetime dimensions. This is the case for e.g. the duality of higher spin gravity in AdS_3 with W_N minimal models in two dimensions [3] and that of (Vasiliev) higher spin gravity theory [4] in AdS_4 with the $O(N)$ vector model in $2+1$ dimensions [5]. Thus, we have come to realize that AdS/CFT is more general than the original string theory examples where it was first discovered, and that AdS/CFT can be elevated to a constructive principle or a starting point/definition for quantum gravity in asymptotically AdS spaces in terms of CFT degrees of freedom. Although this definition is manifestly background dependent, it is completely nonperturbative.[1] For quantum gravity practitioners, the task then becomes to extract the quantum gravitational degrees of freedom from the boundary CFT Hilbert space.[2]

In a series of papers [9–11], a reformulation of the Lorentzian version of the AdS_{d+1}/CFT_d correspondence [12,13] was worked out in the leading semiclassical (super) gravity approximation, $N \to \infty, \lambda \to \infty$. This reformulation was based on mapping *normalizable* bulk fields $\Phi(z,x)$ with asymptotic falloffs, $\Phi(z,x) \overset{z\to 0}{\sim} z^{\Delta}\phi_0(x)$, to local CFT operators $\mathcal{O}_\Delta(x)$ with scaling dimensions Δ [14]. Namely,

$$\phi_0(x) \leftrightarrow \mathcal{O}_\Delta(x).$$

Here the boundary is located at $z \to 0$ and the boundary coordinates have been collectively denoted by x. The central aim of this reformulation was to recover approximate locality in the bulk in the most transparent manner—by mapping on-shell bulk insertions to a delocalized (*smeared*) boundary (CFT) operator with compact support on the boundary,

$$\phi(z,x) \leftrightarrow \int dx' K(x'|x,z)\mathcal{O}_\Delta(x').$$

This was an improvement over earlier attempts [13,15,16], which generally involved representation of a local bulk insertion in terms of a nonlocal CFT operator

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[1]Perhaps this is the paradigm in constructing quantum gravity in general, i.e. background dependence (through its asymptotic symmetries) is an essential ingredient akin to the choice of a global symmetry group in an ordinary quantum field theory. Perhaps, just as it does not make much sense to talk about quantum field theories with different (global) symmetry groups in the same Hilbert space, it does not make sense to talk of an arbitrarily background-*independent* formulation of quantum gravity. See [6] for similar arguments.

[2]However, the more prevalent use of this duality has been to do with the exact obverse, i.e. extract CFT observables (correlation functions) from semiclassical gravity using the GKPW prescription [7,8], for example, applications of AdS/CFT in condensed matter or QCD.

with support over the *entire* boundary and hence required delicate cancellations to recover bulk locality at leading order in $1/N$ expansion. The smearing function immediately reproduces the bulk correlators in terms of the boundary correlators, for example, via

$$\langle\Phi(x_1 z_1)\Phi(x_2, z_2)\rangle = \int dx_1' dx_2' K(x_1'|x_1, z_1)K(x_2'|x_2, z_2)\langle\mathcal{O}_\Delta(x_1')\mathcal{O}_\Delta(x_2')\rangle.$$

This boundary-to-bulk map or the *smearing function* $K(x'|x, z)$ has been constructed not just for (spinless) scalar fields, but also for massive and massless (vector) gauge fields, as well as the spin-two graviton [17] and free higher spin fields [18]. Generically, the smearing functions are nonvanishing only for points on the boundary which are spacelike separated[3] from the local bulk insertion. Further, perturbative quantum gravity corrections, i.e. $1/N$ nonplanar effects to the smearing picture, were worked out subsequently [19–21]. All this was done in the supergravity (SUGRA) approximation $\lambda \to \infty$, where the gravity action is just the cosmological Einstein-Hilbert action. In the original string theory examples, the Regge slope α' is related to the inverse powers of λ, and such α' corrections are expected to give rise to "stringy" higher derivative corrections to the (cosmological) Einstein-Hilbert action. However, in the generic case, these $1/\lambda$ corrections, i.e. "stringy" corrections to the AdS gravity, are yet to be worked out from the CFT. The aim of this paper is to exactly supply that.

The outline of the paper is as follows. In Sec. II and Subsection II A, we introduce massive scalar fields in the bulk interacting with higher curvature terms in a fixed background. We point out how the concept of a "string length" emerges from the λ-corrected anomalous dimensions of the corresponding boundary primary operators. This makes clear the connection between the higher curvature terms in the bulk and the $1/\lambda$ corrections at the boundary. Section III and subsequently Subsection III A deal with the higher derivative corrections towards the usual Einstein-Hilbert action. We point out that the equality of the components of the bulk fields and the number of degrees of freedom of the dual operator in the CFT forces these corrections to be of the Lovelock type. In Sec. IV we solve for the metric perturbation equation obtained in the previous section in order to construct the first subleading order (in $1/\lambda$ expansion) smearing function for the gravitons. Finally, we conclude in Sec. V. Appendix A collects some important formulas.

II. MATTER CORRECTIONS BEYOND (SUPER)GRAVITY

The standard lore in AdS/CFT is that field operators $\Phi(z, x)$ in the quantum gravity side are described using a bulk (AdS) Langrangian (action) with parameters determined by the anomalous (conformal) dimension Δ of the dual operator in the CFT, $\mathcal{O}_\Delta(x)$. In particular, in the planar limit, i.e. $N \to \infty$, only connected correlators survive, which generates purely *quadratic* terms in the bulk (AdS) lagrangian,[4]

$$\mathcal{L} = -\frac{1}{2}\partial_M\Phi\partial^M\Phi - \frac{1}{2}m^2(\lambda)\Phi^2.$$

When nonplanar corrections are included, then even in the leading planar limit, anomalous dimensions of CFT operators receive corrections from the marginal coupling, N, which could be both perturbative and nonperturbative in nature. Of course, for *supersymmetric* CFTs there are special cases of BPS-operators which are protected against such corrections, but since we are considering generic CFTs or even non-BPS operators in a supersymmetric CFT, we will not restrict ourselves to such special or protected operators. Computing such anomalous dimensions of gauge-invariant operators, at finite values of the coupling, is *the* fundamental problem in gauge field theories. In general, accomplishing this solution requires fundamentally new insights or new methods into solving gauge field theories nonperturbatively. However, for the case of *large-N* gauge theories, it might be that the field theory is integrable in the planar limit, for example, as is the case for the $\mathcal{N} = 4$ Super Yang-Mills theory or the ABJ(M) theory [22,23].

In a generic large-N CFT we have two parameters, a (planar) factorization parameter N and an exactly marginal coupling λ. AdS/CFT isomorphism demands an equality of dimensionless parameters on either side. In the (super) gravity limit, the emergent AdS spacetime has two *dimensionful* parameters, the AdS_{d+1} radius R and Newton's gravitational constant G_N. Their ratio constitutes a single dimensionless parameter which is defined to be

$$\frac{R^{d-1}}{b(d)G_N} \equiv N^2. \tag{1}$$

Here $b(d)$ is a numerical constant dependent on the spacetime dimensionality.

For example, in the most extensively explored case of the duality between type IIB strings on $\mathrm{AdS}_5 \times S^5$ and $\mathcal{N} = 4$ SYM [1], the bulk-boundary dictionary between the string coupling g_s, string length l_s and Yang-Mills coupling g_{YM} is (omitting *all* numerical factors which are dependent on the spacetime dimensions),

$$g_s = g_{\mathrm{YM}}^2, \quad \left(\frac{R}{l_s}\right)^4 \sim N g_{\mathrm{YM}}^2 = \lambda. \tag{2}$$

[3]For bulk gauge field insertions, the support is over lightlike separated points on the boundary.

[4]Also the strictly planar limit in the CFT implies the vanishing bulk Newton's constant limit $G_N \to 0$, so the matter fields do not back-react on the AdS geometry. Thus one can safely operate in the *probe* approximation for matter fields.

Also, we have the following relation between Planck length (related to ten-dimensional Newton's constant), string coupling and string length:

$$l_p^8 = g_s^2 l_s^8 \tag{3}$$

Combining (2) and (3), we have

$$\frac{R^8}{l_p^8} \sim N^2. \tag{4}$$

Ten-dimensional Newton's constant is l_p^8, but after dimensionally reducing the S^5 directions, the five-dimensional (AdS) Newton's constant becomes $\tilde{l}_p^3 = l_p^8/R^5$. Substituting $l_p^8 = \tilde{l}_p^3 R^5$ into (4), we thus obtain

$$\frac{R^3}{\tilde{l}_p^3} \sim N^2,$$

modulo factors depending on dimensionality of the AdS spacetime.

The dependence on the coupling of the dual CFT λ is much more nontrivial to deduce for an arbitrary CFT, i.e. understanding the second dimensionless parameter or the emergence of a third length scale. However, as in (2), it turns out that in the well-known examples of string theory/CFT dualities, λ corresponds to string length (squared) α'. The existence of a dual (super)gravity theory is obtained when the limit $\lambda \to \infty$ is taken. However, from the perspective that AdS/CFT is much more general and examples are known which do not require the existence of supersymmetry or ten spacetime dimensions, the equality of "fundamental" parameters on the CFT side and gravity side tells us that for each marginal coupling λ there should be a new length scale l_s in the bulk, which captures effects of *extended classical probes* of the bulk geometry:

$$l_s^2 \sim \frac{R^2}{\lambda^\alpha}.$$

Here α is determined by the dimensionality of the CFT. In the case of $d = 4$, $\alpha = 1/2$ as has been derived from methods exploiting planar integrability—the anomalous dimensions of operators, such as the Konishi operator, receive their first corrections to be of the order $1/\sqrt{\lambda}$ [24]. Since we are reconstructing the bulk/gravity side from the CFT, our work intends to take as an input the expressions of anomalous dimensions of some operator in arbitrary order of $1/\lambda$ (obtained by some pure CFT method) and as an output determines the form of the correction terms to be added to the dual bulk field action. However, in practice what we do in the following is to make an educated guess of the bulk correction terms (curvature corrections) and then use the HKLL dictionary to relate the coefficients of these correction terms to the dimensionless coefficients appearing in the expression for scaling dimensions or some other dimensionless coefficients of the correlation functions of the dual boundary operators. Once this relation is established, they uniquely determine the coefficients of the bulk terms in terms of the boundary correction. Thus, we are reverse-engineering the CFT data to constrain the correction terms added to the bulk action.[5]

A. Defining a string length

Here we revisit the case of a bulk scalar, $\Phi \leftrightarrow \mathcal{O}_\Delta$. Duality relates mass m of scalar field in bulk to conformal dimension Δ of boundary primaries. If the bulk Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2} g^{MN} \partial_M \Phi \partial_N \Phi - \frac{1}{2} m^2(\Delta) \Phi^2, \tag{5}$$

then it is well-known that at $N, \lambda \to \infty$

$$\Delta_\infty = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2 R^2}.$$

Stringy corrections can sense the curvature, so on general grounds including $1/\lambda$ effects, we can write down a modified bulk Lagrangian for the scalar which includes higher curvature corrections,

$$\mathcal{L} = -\frac{1}{2} g^{MN} \partial_M \Phi \partial_N \Phi - \frac{1}{2} m^2 \Phi^2 + l_s^2 (a \mathcal{R}^{MN} \partial_M \Phi \partial_N \Phi + b \mathcal{R} g^{MN} \partial_M \Phi \partial_N \Phi + c m^2 \mathcal{R} \Phi^2). \tag{6}$$

Here $\mathcal{R}^{MN}$ and $\mathcal{R}$ are, respectively, the Ricci tensor and Ricci scalars for the background and a, b, c are constant, dimensionless coefficients which do not depend on any bulk parameters.[6] Also in what follows, the capitalized latin indices will denote bulk coordinates and we will reserve the greek indices μ, ν etc. to denote the boundary coordinates. Around pure AdS, which is maximally symmetric,

$$\mathcal{R}_{AB} \propto g_{AB}$$

and the above modified scalar action simplifies to (on restoring the canonical normalization of the kinetic term)

[5]Note that the smearing functions can also be extracted purely from the boundary data in some simple cases just from symmetry considerations. See e.g. [25–29]. This is also true in HKLL construction in some sense, where the smearing function is the unique kernel one can write down which will satisfy AdS covariance [17,20,21].

[6]Note that in what follows, we are neglecting terms such as $\Phi \nabla \mathcal{R} \nabla \Phi$ for two reasons. First, we contain ourselves with only higher curvature interactions and secondly, due to expansion around pure AdS, for which $\mathcal{R}$ is a constant, such terms drop out. It can also be partially integrated out to give a term going as $\mathcal{R}(\partial\Phi)^2$, which we already considered.

$$\mathcal{L} = -\frac{1}{2}g^{MN}\partial_M\Phi\partial_N\Phi - \frac{1}{2}m^2\left(1 + \bar{c}\frac{l_s^2}{R^2} + O\left(\frac{l_s}{R}\right)^4\right)\Phi^2, \tag{7}$$

where $\bar{c}$ is an dimensionless order-one constant dependent on the spacetime dimensions and so are the coefficients a, b, c appearing in the higher curvature correction terms in (6). Thus, the overall effect is just a correction of the mass parameter in leading order (infinite λ) Lagrangian (5). This then immediately provides the change in the conformal dimension through the asymptotic falloff

$$\Delta(\lambda) = \Delta_\infty\left(1 + \bar{c}f(\Delta_\infty)\frac{l_s^2}{R^2} + \cdots\right), \tag{8}$$

where $f(\Delta_\infty)$ is a function of only Δ_∞ and is given by

$$f(\Delta_\infty) = \frac{(\Delta_\infty - \frac{d}{2})^2 - \frac{d^2}{4}}{\Delta_\infty(\Delta_\infty - \frac{d}{2})}.$$

Note that it goes to an order-one constant as well, when we consider conformal primaries with large operator dimensions. In principle, this is what one expects by directly working with the CFT itself, namely, one can compute the conformal dimension as

$$\Delta(\lambda) = \Delta_\infty\left(1 + O\left(\frac{1}{\lambda^\alpha}\right)\right). \tag{9}$$

This is, for example, what was done for the Konishi operator [24]. This is of course the holy grail of field theorists, to solve the spectral dimension for arbitrary coupling, λ. However, as quantum gravity practitioners, we will assume that the CFT has been solved exactly and the spectral dimensions are known to all orders in $1/\lambda$. Comparing the two expressions for the conformal dimensions (8) and (9), we can identify "string length"

$$\frac{l_s^2}{R^2} \equiv \frac{1}{\lambda^\alpha}. \tag{10}$$

Thus, finally the smearing function in this case is simply modified to (keeping the SUGRA and boundary normalizations the same as in [11])

$$\Phi(t,x,z) = \frac{\Gamma[\Delta(\lambda) - \frac{d}{2} + 1]}{\pi^{d/2}\Gamma[\Delta(\lambda) - d + 1]}\int_{t'^2+y'^2<z^2} dt' d^{d-1}y' (2\sigma z')^{\Delta(\lambda)-d}\mathcal{O}_{\Delta(\lambda)}(t+t', x+iy'), \tag{11}$$

for AdS covariant bulk-boundary distance

$$\sigma(z,x|z',x') = \frac{z^2 + z'^2 + (x-x')^2}{2zz'}$$

in e.g. Poincaré AdS.

As we mentioned before, at this point one should treat the above smearing function as the prescription to reverse-engineer the bulk fields and correlators from their boundary counterparts.

III. GRAVITY ACTION FROM λ^{-1} CORRECTIONS IN CFT

After getting an intuition behind the equivalence between the boundary λ^{-1} corrections and bulk higher curvature corrections, we now directly consider modified gravity actions in the bulk. Here we will neglect contributions from any other matter fields and set the stage for computing metric perturbation and hence the modified graviton smearing function in the next sections.

The λ corrections in the bulk are better not thought of as quantum corrections, but *classical* nonlocality induced contributions due to *extended probes of geometry*. For example, in the $\mathcal{N} = 4$ SYM/ type IIB case, these are *classical stringy nonlocal* effects. Such effects are manifested in local Lagrangian field theory by an infinite number of higher derivative terms. Thus, one needs to turn them on in the gravity action to precisely capture the nonlocalities arising out of extended probes in the bulk

$$I_{\text{bulk}} = \frac{1}{16\pi G_N}\int d^{d+1}x\sqrt{-g}(\mathcal{R} - 2\Lambda + \alpha_1\mathcal{R}^2 + \alpha_2\mathcal{R}_{\mu\nu}^2 + \alpha_3\mathcal{R}_{\mu\nu\rho\sigma}^2 + \alpha_4\Box\mathcal{R} + \cdots). \tag{12}$$

In general, the parameters in the bulk action Λ and α_i's are functions of the gauge theory parameters N and λ,[7]

$$\frac{\alpha_i}{R^2} \sim \frac{1}{\lambda^\alpha} + O\left(\frac{1}{\lambda^{2\alpha}}\right),$$
$$\Lambda R^2 \sim \Lambda_0 R^2 + \frac{a}{\lambda^\alpha} + O\left(\frac{1}{\lambda^{2\alpha}}\right), \quad \text{with} \quad \Lambda_0 R^2 = -\frac{d(d-1)}{2}. \tag{13}$$

These dependences are not arbitrary but constrained by the following principles:

(a) The new higher derivative gravity action admits an *exact* pure AdS solution. This is because the

[7] We will later see how to interpret α_i's as a purely boundary quantity without making any reference to AdS radius R.

symmetries should remain intact on both the AdS side and CFT side. On the CFT side, since the conformal symmetry of the vacuum does not get overhauled by the λ^{-1} corrections, hence a pure AdS space must be a solution to the λ^{-1} corrected bulk field equations as well.

(b) This pure AdS solution to the field equations of higher derivative gravity with the cosmological constant Λ is identical to AdS space with radius R, which is a solution to the (two-derivative) Einstein's field equations with cosmological constant $\Lambda_0 = -\frac{d(d-1)}{2R^2}$. This is necessary so that the definition of Newton's constant (1) does not get revised/renormalized.[8] In the $\mathcal{N}=4$ SYM/ type IIB duality, it is known that the AdS radius does not get renormalized by stringy α' corrections owing to supersymmetry [30]. This demand then implies [31]:

$$\Lambda = \Lambda_0 + \frac{d(d-3)}{2R^4}[d(d+1)\alpha_1 + d\alpha_2 + 2\alpha_3], \quad \text{with} \quad \Lambda_0 = -\frac{d(d-1)}{2R^2}. \tag{14}$$

(c) The λ^{-1} corrections in the bulk, being classical corrections due to nonlocality/extended probes, must admit consistent semiclassical quantizations of gravity (about AdS space). In particular these corrections must not change the number of (on-shell) degrees of freedom associated with the graviton as AdS/CFT demands that the number of degrees of freedom of the graviton must be same as that of the CFT stress tensor. This is directly manifest in the *holographic* gauge of [17], where there is a *direct* isomorphism between components of the CFT stress-tensor and local graviton insertions in AdS,

$$h_{\mu\nu} \leftrightarrow T_{\mu\nu} \quad \text{with} \quad h_{\mu z} = h_{zz} = 0. \tag{15}$$

Generically, higher derivative actions modify the degrees of freedom due to the presence of higher derivatives. Isomorphism with gauge theory/CFT forces the requirement of keeping intact the number of graviton on-shell degrees of freedom, i.e. forces us to choose a very particular form of the λ-corrected action, one which was obtained in [32,33]

$$\alpha_1 = \alpha_3 = -\frac{1}{4}\alpha_2.$$

α_4 can be consistently set to zero as it is the coefficient of a total derivative term. This term is called a Gauss-Bonnet term [34], and the specialty of such a term is that the resulting equations of motion contain only second derivatives. For higher order λ^{-1} corrections one generates/adds higher order Lovelock terms [34] in the bulk AdS action. Note that if the stress tensor of the CFT is itself Lorentz anomalous, then of course one can consider general higher derivative terms which are not Lovelock. As an example, consider in AdS_3, a topologically massive gravity term. It corresponds to an extra stress tensor (generally called the anomalous stress tensor) in the dual chiral log CFT.

(d) The stress-tensor two-point function (and its higher order functions in general) determine the coefficient α_1, since in the boundary limit

$$\lim_{z,z'\to 0}\langle h_{\mu\nu}(x,z)h_{\rho\sigma}(x',z')\rangle_{(SU)GRA} = z^{d-2}z'^{d-2}\langle T_{\mu\nu}(x)T_{\rho\sigma}(x')\rangle_{(S)\mathrm{CFT}}, \tag{16}$$

as was done in [17]. The left-hand-side graviton two-point function in the Gauss-Bonnet gravity is a function of α_1. But there is one subtlety to note here: usually the normalization convention on the left-hand side, i.e. the (super)gravity side, is different from the right-hand or the S(CFT) side. For (super)gravity the loop expansion parameter is $\kappa^2 = 16\pi G_N$ and the graviton is defined as a perturbation $h_{\mu\nu}$ around some background $g^{(0)}_{\mu\nu}$ like

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \kappa h_{\mu\nu},$$

and as a result the Newton's constant *does not* show up in the SUGRA two-point function for gravitons. In the large-N CFT side, however, two-point functions, particularly of boundary stress tensors (and in general all connected correlators), are usually taken to have a norm which scales as N^2,

$$\langle \mathcal{O}_\Delta(x)\mathcal{O}_\Delta(x')\rangle = \frac{C_\mathcal{O}}{(x-x')^{2\Delta}}, \quad \text{with} \quad C_\mathcal{O} \sim N^2.$$

The usual practice in a large-N CFT is to set the norm of the two-point function to unity, by defining

$$\mathcal{O} \to \mathcal{O}/\sqrt{C_\mathcal{O}}.$$

But the stress-tensor two-point function is not normalized to unity. In fact, the norm gives the *central charge*, which is a characteristic of the field theory [heuristically speaking, it is an indicator of the field content of the (S)CFT]. For an (S)CFT one has [35,36]

$$\langle T_{\mu\nu}(x)T_{\rho\sigma}(x')\rangle = C(N^2,\lambda)\frac{I_{\mu\nu;\rho\sigma}}{(x-x')^{2d}}. \tag{17}$$

Here $I_{\mu\nu,\rho\sigma}$ is some (universal) conformally covariant structure depending on $(x-x')$ independent of the

[8] If we allow the AdS radius to change with λ, then (1) implies that Planck length needs to change identically, so as to keep their ratio fixed and equal to N^2. But change in Planck length implies that they are *quantum/loop* corrections in the bulk, not classical string corrections.

CFT field content. The central charge has an asymptotic expansion about $\lambda \to \infty$,[9]

$$C(N^2, \lambda) = C(N^2) f(\lambda), \qquad f(\lambda) = 1 + O(\lambda^{-\alpha}).$$

(of course recall that we are talking in the strictly planar limit; otherwise, the central charge would, in addition, have terms subleading in $1/N$). Thus, we need to make the identification (15) in a way so that the boundary limit of the AdS graviton two-point function reproduces the CFT stress-tensor two-point function with the normalization $\sim N^2$ (central charge). This is easily achieved by the GKPW recipe [7,8] of extracting boundary stress-tensor correlators or perhaps more directly for us, through the BDHM (extrapolate) or HKLL dictionary of AdS/CFT in the *holographic gauge*, where we arrive at the boundary CFT stress-tensor two-point functions by taking the boundary limit of the bulk graviton two-point function,

$$\langle T_{\mu\nu}(x) T_{\lambda\rho}(z, y)\rangle_{\rm CFT} = \lim_{z\to 0} z^{2d} \langle h_{\mu\nu}(z, x) h_{\lambda\rho}(z, y)\rangle_{\rm SUGRA}.$$

One *always* identifies the central charge in the supergravity limit ($N, \lambda \to \infty$) in terms of SUGRA parameters

$$C(N^2) = n(d) \frac{R^{d-1}}{16\pi G_N}, \qquad (18)$$

where $n(d)$ is a numerical factor depending on the dimensionality of the field theory.

A. CFT-induced (four)-derivative corrections to bulk action (Gauss-Bonnet)

We now perform the required calculations in order to justify various points that we discussed above. The variation of the action (12) restricted to first order in $\lambda^{-\alpha}$ gives the field equations for four-derivative corrected gravity [37],[10]

$$\begin{aligned}0 = {}& \frac{g_{AB}}{2}(\mathcal{R} - 2\Lambda + \alpha_1 \mathcal{R}^2 + \alpha_2 Ric^2 + \alpha_3 Rie^2) \\ & - \mathcal{R}_{AB} + 2\alpha_1(\nabla_A \nabla_B \mathcal{R} - g_{AB} \Box \mathcal{R} - \mathcal{R}_{AB}\mathcal{R}) \\ & + \alpha_2(\nabla^C \nabla_A \mathcal{R}_{BC} + \nabla^C \nabla_B \mathcal{R}_{AC} - \Box \mathcal{R}_{AB} \\ & - g_{AB} \nabla_C \nabla_D \mathcal{R}^{CD} - 2\mathcal{R}_{AC}\mathcal{R}_B C) \\ & - 2\alpha_3(\mathcal{R}_A{}^{LMN}\mathcal{R}_{BLMN} + \nabla^L \nabla^M \mathcal{R}_{ALBM} + \nabla^M \nabla^L \mathcal{R}_{ALBM}). \end{aligned} \qquad (19)$$

Obviously, *Ric* and *Rie* signify the Ricci and Riemann tensors for the background. Next, using the (Bianchi) identities,

$$\begin{aligned} \nabla^A \mathcal{R}_{AB} &= \frac{1}{2} \nabla_B \mathcal{R}, \\ \nabla^A \nabla^B \mathcal{R}_{AB} &= \frac{1}{2} \Box \mathcal{R} \quad \text{and} \\ \nabla^L \nabla^M \mathcal{R}_{ALBM} &= \Box \mathcal{R}_{AB} - \frac{1}{2}\nabla_A \nabla_B \mathcal{R} - \mathcal{R}_{AC}\mathcal{R}_B D \\ &\quad + \mathcal{R}_{ACBD}\mathcal{R}^{CD}, \end{aligned}$$

we can recast the field equations in a nicer form

$$\begin{aligned} 0 = {}& \mathcal{R}_{AB} - \frac{1}{2} g_{AB}(\mathcal{R} - 2\Lambda + \alpha_1 \mathcal{R}^2 + \alpha_2 Ric^2 + \alpha_3 Rie^2) \\ & + 2\alpha_1 \mathcal{R}\mathcal{R}_{AB} - 4\alpha_3 \mathcal{R}_{AC}\mathcal{R}_B{}^C + (2\alpha_2 + 4\alpha_3)\mathcal{R}_{ACBD}\mathcal{R}^{CD} \\ & + 2\alpha_3 \mathcal{R}_{ALMN}\mathcal{R}_B{}^{LMN} + \left(2\alpha_1 + \frac{\alpha_2}{2}\right) g_{AB} \Box \mathcal{R} \\ & - (2\alpha_1 + \alpha_2 + 2\alpha_3)\nabla_A \nabla_B \mathcal{R} + (\alpha_2 + 4\alpha_3)\Box \mathcal{R}_{AB}. \end{aligned} \qquad (20)$$

Since we must have AdS spacetime as a solution to these four-derivative gravity field equations, we plug in an AdS space ansatz with the AdS radius R,[11]

$$\mathcal{R}_{ABCD} = -\frac{1}{R^2}(g_{AC} g_{BD} - g_{AD} g_{BC}).$$

The derivative terms become derivatives of the metric and vanish, and the nonvanishing contributions to the field equations are

$$\begin{aligned} & \mathcal{R}_{AB} - \frac{1}{2} g_{AB}(\mathcal{R} - 2\Lambda + \alpha_1 \mathcal{R}^2 + \alpha_2 Ric^2 + \alpha_3 Rie^2) \\ & \quad + 2\alpha_1 \mathcal{R}\mathcal{R}_{AB} - 4\alpha_3 \mathcal{R}_{AC}\mathcal{R}_B{}^C \\ & \quad + (2\alpha_2 + 4\alpha_3)\mathcal{R}_{ACBD}\mathcal{R}^{CD} + 2\alpha_3 \mathcal{R}_{ALMN}\mathcal{R}_B{}^{LMN} = 0. \end{aligned}$$

After contracting with g^{AB} on both sides, we get the *revised* cosmological constant parameter in the Lagrangian,

[9]Note that the $1/\lambda$ order correction to the graviton two-point function can be absorbed in a redefinition of Newton's constant G_N, instead of the running of the central charge C. (We should not confuse this renormalization of G_N with $1/N$, i.e. quantum gravity loop corrections.) However, when we go to the graviton three-point function level, there are three "central charges" corresponding to the coefficients of three distinct tensor structures allowed for the three-point function. Thus, at the three-point level one cannot just absorb the $1/\lambda$ running of three distinct central charges into a single Newton's constant. We thank Dan Kabat for pointing this out to us.

[10]Recall that for noncompact spaces such as AdS, all total derivative terms vanish and there is no need to add boundary terms such as the Gibbons-Hawking term.

[11]This radius is unchanged while the "bare" parameters in the Lagrangian are changed as higher and higher derivatives terms are added.

$$\Lambda = -\frac{d(d-1)}{2R^2} + \frac{d(d-3)}{2R^4}(d(d+1)\alpha_1 + d\alpha_2 + 2\alpha_3). \tag{21}$$

In general, for the higher derivative contributions to vanish one needs to arrange the α_i's to set the coefficients of the three *independent* higher derivative terms $\Box\mathcal{R}$, $\nabla_A\nabla_B\mathcal{R}$ and $\Box\mathcal{R}_{AB}$ in the field equations (20) to vanish, i.e.

$$\begin{aligned} 2\alpha_1 + \frac{1}{2}\alpha_2 &= 0, \\ 2\alpha_1 + \alpha_2 + 2\alpha_3 &= 0, \\ \alpha_2 + 4\alpha_3 &= 0. \end{aligned}$$

Evidently, this is achieved for *arbitrary* spacetime dimensions for the Gauss-Bonnet combination, i.e.

$$\alpha_2 = -4\alpha_1; \alpha_3 = \alpha_1.$$

For this case the field equations take the form

$$\begin{aligned} &\mathcal{R}_{AB} - \frac{1}{2}g_{AB}[\mathcal{R} - 2\Lambda + \alpha_1(\mathcal{R}^2 - 4Ric^2 + Rie^2)] \\ &\quad + 2\alpha_1[\mathcal{R}\mathcal{R}_{AB} - 2\mathcal{R}_{AC}\mathcal{R}_B{}^C - 2\mathcal{R}_{ACBD}\mathcal{R}^{CD} \\ &\quad + \mathcal{R}_{ALMN}\mathcal{R}_B{}^{LMN}] = 0. \end{aligned} \tag{22}$$

Note that the cosmological constant for (asymptotically) AdS backgrounds in Gauss-Bonnet gravity becomes

$$\Lambda = -\frac{d(d-1)}{2R^2} + \frac{d(d-1)(d-2)(d-3)}{2R^4}\alpha_1. \tag{23}$$

Below, following [17,38], we shall use a slightly more convenient form of the field equations,

$$\begin{aligned} &\mathcal{R}_{AB} + \frac{d}{R^2}\left(1 - \frac{\alpha_1}{R^2}(d-2)(d-3)\right)g_{AB} \\ &\quad - \frac{\alpha_1}{d-1}g_{AB}(\mathcal{R}^2 - 4Ric^2 + Rie^2) \\ &\quad + 2\alpha_1(\mathcal{R}\mathcal{R}_{AB} - 2\mathcal{R}_{AC}\mathcal{R}_B{}^C - 2\mathcal{R}_{ACBD}\mathcal{R}^{CD} \\ &\quad + \mathcal{R}_{ALMN}\mathcal{R}_B{}^{LMN}) = 0. \end{aligned} \tag{24}$$

IV. CORRECTIONS TO BULK (SU)GRA FROM $1/\lambda$ RUNNING OF CFT STRESS-TENSOR CORRELATIONS

In this section we finally turn to linearizing the above equation of motion in order to obtain the smearing function to the first subleading order in λ^{-1} expansion. The graviton equation of motion is obtained by linearizing the field equations (22) and (24) around the pure AdS solution,

$$g_{AB} = g^{(0)}_{AB} + h_{AB},$$

using the following linearized forms around AdS space,

$$\begin{aligned} \mathcal{R}^{(1)A}{}_{LMN} &= \frac{1}{2}(\nabla_M\nabla_N h^A_L + \nabla_M\nabla_L h^A_N \\ &\quad - \nabla_M\nabla^A h_{LN} - M \leftrightarrow N), \\ \mathcal{R}^{(1)}_{AB} &= \nabla_{(A}\nabla^C h_{B)C} - \frac{1}{2}\Box h_{AB} - \frac{1}{2}\nabla_B\nabla_A h \\ &\quad - \frac{d+1}{R^2}h_{AB} + \frac{1}{R^2}g^{(0)}_{AB}h, \\ \mathcal{R}^{(1)} &= \nabla^C\nabla^D h_{CD} - \Box h + \frac{d}{R^2}h. \end{aligned}$$

At the end of the day, quite expectedly, the linearized Ricci equation remains the same,

$$\begin{aligned} &\left(1 - \frac{2(d-2)(d-3)}{R^2}\alpha_1\right)\left(\nabla_{(A}\nabla^C h_{B)C} - \frac{1}{2}\Box h_{AB}\right. \\ &\quad \left. - \frac{1}{2}\nabla_B\nabla_A h - \frac{1}{R^2}h_{AB} + \frac{1}{R^2}g^{(0)}_{AB}h\right) = 0, \end{aligned} \tag{25}$$

i.e. identical to that of (super)gravity [38], except for the appearance of an *overall* coefficient which doesn't alter anything. However, the bulk Green's function (e.g. Feynman or retarded/advanced ones) for the metric perturbations $G_{AB}(x-y)$ will be affected,

$$\begin{aligned} &\left(1 - 2\alpha_1\frac{(d-2)(d-3)}{R^2}\right)\left(\nabla_{x^{(A}}\nabla^{x^C}G_{x^{B)}x^C}(x-y)\right. \\ &\quad - \frac{1}{2}\Box_x G_{AB}(x-y) - \frac{1}{2}\nabla_{x^B}\nabla_{X^A}G(x-y) - \frac{1}{R^2}G_{AB}(x-y) \\ &\quad \left. + \frac{1}{R^2}g^{(0)}_{AB}G(x-y)\right) = \frac{1}{\sqrt{g}}g_{AB}\delta^{d+1}(x-y) \end{aligned}$$

because the right-hand side is a delta function. In particular, the Green's function will get rescaled by this prefactor,

$$G_{GB} \sim \frac{1}{(1 - 2\alpha_1\frac{(d-2)(d-3)}{R^2})}G_{\mathrm{EH}},$$

where the subscript GB denotes Gauss-Bonnet theory in the bulk, while the subscript denotes Einstein-Hilbert in the bulk. This then implies that the smearing functions obtained in [17] from the spacelike supported Green's function will also get rescaled by the α_1-dependent normalization factor.[12] Hence, following [17], the final graviton smearing expression becomes ($i = 1, 2$)

[12]This can also be expected in other intuitive ways. For example, we can derive (25) by redefining the initial metric perturbation h_{AB}, which has the Einstein-Hilbert form, upon absorbing the extra α_1-dependent factor in it.

$$z^2\langle h_{\mu\nu}(t_1,\mathbf{x_1},z_1)h_{\mu\nu}(t_2,\mathbf{x_2},z_2)\rangle(\alpha_1)$$
$$=\frac{1}{\mathrm{vol}(B^d)(1-2\alpha_1\frac{(d-2)(d-3)}{R^2})}\int_{t_i'^2+|\mathbf{y}_i'|^2<z_i^2}\int dt_i'd^{d-1}y_i'\langle T_{\mu\nu}(t_1+t_1',\mathbf{x}_1+i\mathbf{y}_1')T_{\mu\nu}(t_2+t_2',\mathbf{x}_2+i\mathbf{y}_2')\rangle,$$
$$\text{where the volume of a unit d ball} = \mathrm{vol}(B^d)=\frac{2\pi^{d/2}}{d\Gamma(d/2)} \tag{26}$$

Now if we use the HKLL dictionary of AdS/CFT in the *holographic gauge*, then we arrive at the boundary CFT stress-tensor two-point functions,

$$\langle T_{\mu\nu}(x)T_{\lambda\rho}(y)\rangle=\lim_{z\to 0}z^{2d}\langle h_{\mu\nu}(z,x)h_{\lambda\rho}(z,y)\rangle.$$

Thus, we see that the stress-tensor two-point functions of the $1/\lambda$-corrected CFT is related to the leading $\lambda\to\infty$ result by

$$\langle T_{\mu\nu}(x)T_{\lambda\rho}(y)\rangle_\lambda=\frac{1}{(1-2\alpha_1\frac{(d-2)(d-3)}{R^2})}\langle T_{\mu\nu}(x)T_{\lambda\rho}(y)\rangle_{\lambda=\infty}. \tag{27}$$

Here α_1/R^2 is to be expressed in powers of λ^{-1} as in (13) or in terms of CFT central charges as in (28) below. Further, since the central charge is defined to be the coefficient of the leading singularity of the two-point function of the CFT stress tensor (17), this overall coefficient of the Gauss-Bonnet correction to the bulk determines the "λ running" of the central charge as we turn on the marginal coupling λ,[13]

$$\frac{\alpha_1}{R^2}=\frac{1}{2(d-2)(d-3)}\left(1-\frac{C(\lambda\to\infty)}{C(\lambda)}\right). \tag{28}$$

Above, $C(\lambda)$ denotes the central charge appearing in front of the stress-tensor correlator, but only expanded up to first subleading order in λ^{-1} expansion. Alternatively, upon defining

$$C(\lambda)\equiv C_\infty+\frac{1}{\lambda^\alpha}C^{(1)},$$

we can obtain

$$\frac{\alpha_1}{l_s^2}=\frac{C^{(1)}}{2C_\infty(d-2)(d-3)}. \tag{29}$$

[13]Similar expressions have also been found in e.g. [39] where they consider AdS/CFT for Gauss-Bonnet theory in the bulk from a "bottom-up" phenomenological approach. Their Gauss-Bonnet term could have contributions from $1/N$ order since the AdS radius after adding the GB term changes compared to the AdS solution in the pure EH gravity. Here we reconstruct the bulk action from the CFT order by order, in an $1/\lambda^\alpha$ expansion.

This equation predicts that for $d=2$, 3, the correction vanishes, $C^{(1)}=0$, which makes sense because in those dimensions the bulk Gauss-Bonnet term either vanishes or is a topological term (not local). Thus, knowing the boundary CFT data, i.e. the stress-tensor two-point function order by order in $1/\lambda$ expansion, we can determine the coefficient of the respective Lovelock terms that we need to add in the gravity Lagrangian/action to reconstruct the bulk. Using (28) or (29), the right-hand side of (26) becomes a purely boundary quantity.

V. CONCLUSION AND OUTLOOK

In this paper we have taken the first step towards incorporating $\frac{1}{\lambda}$ corrections (i.e. finiteness of the marginal coupling in a CFT) in the construction of smeared boundary operators which play the roles of local fields in the AdS bulk. The construction is performed while holding N infinite, i.e. when only connected two-point correlators in the CFT are turned on, or equivalently in the limit of Newton's constant $G_N\to 0$ in the AdS bulk, i.e. the bulk theory is at tree-level in quantum gravity. We have studied the effect of the $1/\lambda$ corrections in two cases. In the first case, we looked at changes in the bulk theory resulting from $1/\lambda$ corrections to (two-point) correlators of a CFT scalar primary (unprotected). The anomalous dimensions of the CFT primary operators develop a dependence on λ, and we showed that this leads to new tree-level/classical interactions to the dual bulk AdS scalar theory via nonminimal couplings to the background, i.e. couplings to higher orders of the curvature tensors and scalars. These new interactions can be thought to be arising out of massive string modes since in the context of the gauge-string duality, the $1/\lambda$ corrections are expected to be equivalent to perturbative worldsheet effects (α' corrections). In the second case, we looked at the pure gravity sector in the AdS bulk (dual to the CFT stress-tensor multiplet), and again we found out that CFT $1/\lambda$ corrections to the TT correlators transpire into higher curvature correction terms, but only those which are of the special Lovelock form. We have thus shown that the (HKLL) map from local bulk operators to nonlocal boundary operators via the smearing functions can be easily and very naturally extended from the $\lambda\to\infty$ case to include $1/\lambda$ corrections [Eqs. (11) and (26)].

There are various avenues for future directions. Our work is only the first step (the next to leading order in $1/\lambda$) towards understanding the emergence of higher curvature and higher derivative terms on the bulk, i.e. stringy physics from the underlying CFT. The logical next step would be to extend our results to incorporate higher order $1/\lambda$ corrections. It remains to be seen how the standard HKLL AdS/CFT bulk-boundary map morphs. Another natural generalization would be to consider both the scalars and the gravitational degrees of freedom interacting at a certain order of α'. Our results seem to suggest that as long as we have full AdS isometry, the scalar and metric smearing functions will again have the same structures; however, the overall normalization factors and powers of the AdS covariant distance function $\sigma(x, x')$ will change. The AdS isometry considerations dictate that it is also straightforward to generalize our situation for asymptotically AdS spacetimes which are global quotients of pure AdS, viz. AdS_3-Rindler, BTZ and higher dimensional hyperbolic black holes. We expect our results to be easily adapted for such backgrounds.

However, one of the main goals of our studies of bulk locality (microcausality) from boundary is to better understand AdS quantum gravity itself and perhaps precisely derive the restrictions placed on the dual CFT in order to have a local causal bulk AdS. Along this line, one of the recent interesting developments was in [40], which pointed out the need for an infinite tower of higher spin particles in the AdS bulk in order to construct a consistent (causality respecting) theory of quantum gravity. In AdS/CFT, one should see such a structure purely from the dual CFT. In [40] the authors found their results by looking at the bulk three-point graviton vertex, or, equivalently, in the CFT stress-tensor three-point function. This is an effect which is manifest at an order subleading in $1/N$. As we mentioned in the introduction, within the HKLL program itself, a lot of literature already exists that deals with $1/N$ corrections to fields of various spins such as spin 2 [21] and higher [18], and thus the next step will be to understand such effects simultaneously with our $1/\lambda$ corrections. However, we believe that deriving the results of [40] would require us to consider another criterion in addition to microcausality in the bulk. Microcausality is a feature of local quantum field theories (in this curved space QFT), and it guarantees causal propagation of information. However, this is not expected to be enough when one considers stringy physics, which is not described by a local QFT Lagrangian, i.e. containing only a finite number of terms. For causal propagation in such stringy physics, one requires additional consistency/causality criteria in the bulk. The criterion of a gravitational Shapiro delay used by [40] could exactly be such a constraint on a nonlocal yet causal bulk theory. Another standard expectation that comes from the studies of these massive stringy modes is that for a sub-AdS duality one needs to have a gap in conformal dimensions between fields of spin 2 and higher [41]. In fact, in [42] it was shown that a key result of [40] [Eq. (5.20) of [40]] can be rederived from a CFT by *assuming* this gap and assuming the chaos bound on out of time ordered four-point functions of [43]. So far, our prescriptions, depending solely on bulk microcausality, are insensitive to these extra constraints. However, from our point of view, i.e. from the point of *ab initio* reconstructing the bulk from the CFT, these conditions or restrictions should emerge naturally from the existence of a perturbative expansion of OPEs in two parametrically large dimensionless quantities, namely N and λ. It would be nice to see the emergence of a gap in the spectrum which is related to the marginal coupling of the CFT (without using a bulk stringy spectrum).

ACKNOWLEDGMENTS

We thank Dan Kabat for helpful discussions and for giving his valuable feedback on the draft. S. R. thanks Justin David and Aninda Sinha for enlightening discussions and Chethan Krishnan for pointing out reference [30]. Special thanks are due to Arpan Bhattacharyya for pointing out many references in higher derivative gravity literature, and to Kallol Sen for telling us about their interesting work in [44]. S. R. also wishes to thank Prof. Shibaji Roy for their hospitality at the Saha Institute of Nuclear Physics (SINP), Kolkata during summer 2014, where part of this work was being conducted. S. R. is supported by IIT Hyderabad seed Grant No. SG/IITH/F171/2016-17/SG-47. The work of S. R. in Israel was partially supported by the American-Israeli Bi-National Science Foundation, the Israel Science Foundation Center of Excellence and the I-Core Program of the Planning and Budgeting Committee, and the Israel Science Foundation's "The Quantum Universe". The work of D. S. is funded by the ERC grant "Selfcompletion."

APPENDIX: GRAVITATIONAL PERTURBATION RESULTS

The form of the gravitational perturbation theory used in this paper is[14]

[14]In metric perturbation theory it is customary to multiply the perturbation by $\kappa = \sqrt{8\pi G}$,

$$g_{AB} = g^{(0)}_{AB} + \kappa\delta g_{AB}.$$

However, we will not follow this convention in this paper.

$$
\begin{aligned}
g_{AB} &= g^{(0)}_{AB} + \delta g_{AB},\\
g^{AB} &= g^{(0)AB} - \delta g^{AB} + \delta g^{AL}\delta g^{B}_{L} + \cdots,\\
\mathcal{R}_{AB} &= \mathcal{R}^{(0)}_{AB} + \Delta^{(1)}\mathcal{R}_{AB} + \Delta^{(2)}\mathcal{R}_{AB} + \cdots,\\
\Delta^{(1)}\mathcal{R}_{AB} &= \nabla_{(A}\nabla^{C}\delta g_{B)C} - \frac{1}{2}\Box\delta g_{AB} - \frac{1}{2}\nabla_B\nabla_A\delta g^{C}_{C} + \mathcal{R}^{(0)}_{AC}\delta g^{C}_{B} - \mathcal{R}^{(0)}_{ACBD}\delta g^{CD}\\
\Delta^{(2)}\mathcal{R}_{AB}g^{(0)AB} &= \delta g_{AB}\left(\frac{g^{(0)AB}g^{(0)CD} - g^{(0)AC}g^{(0)BD}}{4}\Box + \frac{\nabla^A\nabla^C g^{(0)BD} - g^{(0)CD}\nabla^A\nabla^B}{2} + \frac{\mathcal{R}^{(0)AC}g^{(0)BD} - \mathcal{R}^{(0)ACBD}}{2}\right)\delta g_{CD}\\
\Delta^{(1)}\mathcal{R}_{AB}\delta g^{AB} &= \delta g_{AB}\left(\nabla^A\nabla^C g^{(0)BD} - \frac{1}{2}g^{(0)AC}g^{(0)BD}\Box - \frac{1}{2}\nabla^A\nabla^B g^{(0)CD} + \mathcal{R}^{(0)AC}g^{(0)BD} - \mathcal{R}^{(0)ACBD}\right)\delta g_{CD}\\
\Delta^{(2)}\mathcal{R} &= \frac{1}{4}(g^{(0)AB}g^{(0)CD} + g^{(0)AC}g^{(0)BD})\Box + \frac{1}{2}\nabla^A\nabla^C g^{(0)BD} + \frac{1}{2}(\mathcal{R}^{(0)AC}g^{(0)BD} + \mathcal{R}^{(0)ACBD}).
\end{aligned}
$$

1. First-order perturbation expressions

We have used the following first-order perturbations of gravitational quantites[15]:

$$
\begin{aligned}
\delta\Gamma^{\mu}_{\nu\sigma} &= \frac{1}{2}g^{\mu\alpha}(\nabla_\nu\delta g_{\alpha\sigma} + \nabla_\sigma\delta g_{\nu\alpha} - \nabla_\alpha\delta g_{\nu\sigma}),\\
\delta\mathcal{R}^{\sigma}{}_{\rho\mu\nu} &= \nabla_\mu\delta\Gamma^{\sigma}_{\nu\rho} - \nabla_\nu\delta\Gamma^{\sigma}_{\mu\rho},\\
&= \frac{1}{2}(\nabla_\mu\nabla_\nu\delta g^{\sigma}_{\rho} + \nabla_\mu\nabla_\rho\delta g^{\sigma}_{\nu} - \nabla_\mu\nabla^{\sigma}\delta g_{\rho\nu} - \mu\to\nu).\\
\delta\mathcal{R}_{\mu\nu} &= \nabla_\rho\delta\Gamma^{\rho}_{\mu\nu} - \nabla_\nu\delta\Gamma^{\rho}_{\rho\mu}\\
&= \frac{1}{2}(\nabla^\sigma\nabla_\mu\delta g_{\sigma\nu} + \nabla^\sigma\nabla_\nu\delta g_{\mu\sigma} - \Box\delta g_{\mu\nu} - g^{\rho\sigma}\nabla_\nu\nabla_\mu\delta g_{\rho\sigma}),\\
\delta\mathcal{R} &= \nabla^\mu\nabla^\nu\delta g_{\mu\nu} - g^{\mu\nu}\Box\delta g_{\mu\nu} - R^{\mu\nu}\delta g_{\mu\nu},\\
\delta\sqrt{-g} &= \frac{1}{2}\sqrt{-g}g^{\mu\nu}\delta g_{\mu\nu}.
\end{aligned}
$$

One also needs the following results using the Bianchi Identity,

$$
\begin{aligned}
\nabla^A\mathcal{R}_{AB} &= \frac{1}{2}\nabla_B\mathcal{R},\\
\nabla^A\nabla^B\mathcal{R}_{AB} &= \frac{1}{2}\Box\mathcal{R}
\end{aligned}
$$

and

$$
\begin{aligned}
\nabla^M\nabla^L\mathcal{R}_{ALBM} &= \Box\mathcal{R}_{AB} - \nabla^M\nabla_B\mathcal{R}_{AM}\\
&= \Box\mathcal{R}_{AB} - \nabla_B\nabla_M\mathcal{R}_A{}^M - [\nabla_M,\nabla_B]\mathcal{R}_A{}^M\\
&= \Box\mathcal{R}_{AB} - \frac{1}{2}\nabla_A\nabla_B\mathcal{R} - \mathcal{R}_{AC}\mathcal{R}_B{}^D\\
&\quad + \mathcal{R}_{ACBD}\mathcal{R}^{CD}.
\end{aligned}
$$

[15] A collection of relevant formulas can also be found in e.g. [45].

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