Chapter 2
A Model Predictive Control Approach to AUVs Motion Coordination

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2.1 Motivation

This chapter concerns the decentralized coordinated control of a formation of autonomous underwater vehicles (AUVs) subject to a given set of constraints. The need of AUV motion coordination is due to observation and actuation requirements, such as, spatial and temporal distribution, persistence, event detection and monitoring, etc., which are critical to address a wide range of applications, and can only be achieved by distributing sensors and actuators by a number of distributed fixed and mobile platforms. Examples of application areas are climate change, environment sustainability, natural resources management, surveillance, and security. A selected sample of a vast literature is [1, 4, 9, 14, 20, 24, 26, 27, 31].

Thus, the vast research effort undertaken to design systems for the coordinated control of multiple autonomous vehicles is not surprising. The cooperative control of a team of distributed agents with decoupled nonlinear dynamics and exchanging delayed information has been addressed in a number of works, notably, [2, 6, 7, 10, 11, 16, 19, 22, 23, 29, 32]. The last reference is a chapter of the recently published book edited by Lunze referred to in Sect. 2.5 in which multiple issues pertinent to networked control are considered. The schemes proposed in the above references

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are decentralized in that each agent computes its control law locally by exchanging, possibly delayed, state information with neighboring agents.

Model predictive control (MPC)-like schemes have been widely adopted to formulate decentralized cooperative control problems. The seminal work of Mayne and co-workers reported in the two Automatica articles cited in Sect. 2.5 address fundamental MPC stability, optimality, and robustness issues that lay down important foundations for further research effort on decentralized coordinated control. Typically, in the approaches to decentralized control, control laws depend on the local state variables and on, possibly delayed, information from neighboring agents. Information exchange strategies that improve the formation stability and performance and, at the same time, are robust to changes in the communication topology are considered in [3]. The sensed and communicated information flow is modeled by a graph, and stability conditions are obtained in terms of the eigenvalues of the graph Laplacian. The problem of unreliable communication channels between the MPC controller output and the actuator input has been addressed in, among others, [8]. Here, the mechanism for compensation of packet dropouts has been incorporated in the MPC scheme for discrete time problems. This article also includes some stability and sub-optimality analysis under an asymptotic controllability assumption. In order to show stability, the authors prove that, under the considered assumptions, the value function associated with the underlying optimal control problem exhibits Lyapunov properties.

Although very significant to motion coordinated control challenges, these approaches are not tailored for the specific requirements arising in the marine environment. Highly nonlinear and complex dynamics due to hydrodynamic effects, [5], huge variability of underwater phenomena, severe communication constraints, and scarcity of onboard resources compound to make the networked AUV formation control problem a formidable one, [27]. Due to the fact that radio waves are strongly attenuated in the underwater milieu, acoustics are the most common form of communication but, unfortunately, not only exhibits low bandwidth, high-noise level, and low reliability, but also requires relatively high-power consumption, [25].

2.2 The AUV Formation Control Problem

The AUV formation control problem considered here is based on a MPC scheme and targets field demonstrations with NAUV vehicles from LSTS—The Laboratory for Underwater Systems and Technologies of Porto University—(http://lsts.fe.up.pt) and consists in tracking a given trajectory while maintaining a given formation pattern and satisfying state, control, and communications constraints. The key reason to choose an MPC scheme relies on the fact that it enables to combine the highly desired optimization of scarce onboard resources with the feedback control nature of the scheme that allows to cope with the significant perturbations and with the wide variability of the underwater milieu.

Fig. 2.1 The AUV pose and velocity coordinates are, respectively, in external and in body-fixed reference frameworks. The *line in black* is the reference trajectory
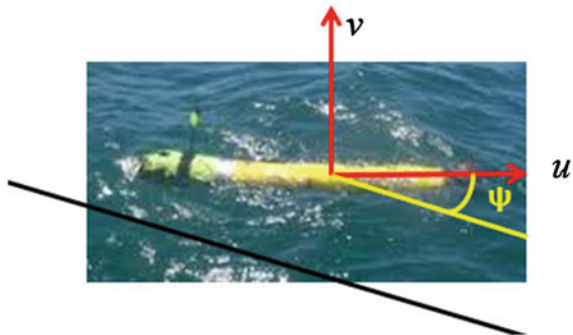
The NAUV is a small torpedo-shaped vehicle with one propeller and four control fins. It is equipped with an advanced miniaturized onboard computer system with a real-time Linux kernel, a Benthos acoustic modem, and an accurate positioning system comprising an ADCP and an IMU,[1] [28]. The model of the AUV NAUV for the motion in the horizontal plane, depicted in Fig. 2.1, is given by (2.1), [5].

The value of the model coefficients was extracted from elaborated identification procedures combining data from [21] coupled with data from LSTS field experiments. The AUV state $x^T = [\eta^T, v^T]$[2] satisfies

$$\dot{\eta} = \begin{bmatrix} u\cos(\psi) - v\sin(\psi) \\ u\sin(\psi) + v\cos(\psi) \\ r \end{bmatrix}, \quad \dot{v} = \begin{bmatrix} \frac{\tau_u - (m-Y_{\dot{v}})vr - X_{u|u|}u|u|}{m-X_{\dot{u}}} \\ \frac{(m-X_{\dot{u}})ur - Y_{v|v|}v|v|}{m-Y_{\dot{v}}} \\ \frac{\tau_r - (Y_{\dot{v}}-X_{\dot{u}})uv - N_{r|r|}r|r|}{I_{zz}-N_{\dot{r}}} \end{bmatrix}, \qquad (2.1)$$

where $\eta = [x, y, \psi]^T \in \mathbf{R}^3$, $v = [u, v, r]^T \in \mathbf{R}^3$, $\tau = [\tau_u, \tau_r] \in \mathbf{R}^2$, and m are, respectively, the vehicle's pose (position and yaw), velocity (surge, sway, and yaw rate), input forces (surge and yaw), and mass. In these equations, the parameter I_{zz} is the rotational mass, and while the triple $(X_{\dot{u}}, Y_{\dot{v}}, N_{\dot{r}})$, represents the surge, sway, and yaw hydrodynamic added mass, the triple $(X_{u|u|}, Y_{v|v|}, N_{r|r|})$, are the surge, sway, and yaw hydrodynamic quadratic drag coefficients.

The control strategy for AUV i, $i = 1, \dots, n_v$, should minimize, over a given time interval, a cost functional penalizing the tracking error relative to the reference trajectory, η_{ref}^i, and the control effort, i.e.,

$$\int_t^{t+T} \left[(\eta^i(s) - \eta_{\text{ref}}^i(s))^T \mathbf{Q}(\eta^i(s) - \eta_{\text{ref}}^i(s)) + \tau^{iT}(s)\mathbf{R}\tau^i(s) \right] ds, \qquad (2.2)$$

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[1] ADCP and IMU stand by acoustic Doppler current profiler, and inertial measurement unit, respectively. While the former provides water current velocity measurements, the former measures position, velocity, and orientation.

[2] From now on, "T" in upper script will denote transposed.

subject to the vehicle dynamics (2.1), (i) position endpoints constraints, $\eta^i(t + T) \in C_{t+T}$, (ii) pointwise control constraints, $\tau^i(s) \in \mathscr{U}^i$, (iii) state constraints, $(\eta^i(s), v^i(s)) \in \mathscr{S}^i$, and (iv) two graph constraints specifying, respectively, the communication links $g^c_{i,j}(\eta^i(s), \eta^j(s)) \in C^c_{i,j}, \forall j \in \mathscr{G}^c$, and the formation pattern, $g^f_{i,j}(\eta^i(s), \eta^j(s)) \in C^f_{i,j}, \forall j \in \mathscr{G}^f$. While control constraints reflect saturations, state constraints incorporate safety, and communication constraints ensure the AUVs connectivity. The severe power constraints impose the need of each AUV to communicate only with its neighbors and thus imposes the need of decentralization. The communications structure is described by a triple $(g^c, C^c, \mathscr{G}^c)$, where $g^c \colon \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}^M$, $C^c \in \mathbf{R}^M$, and $\mathscr{G}^c$ is a graph specifying the communication links among AUVs. The formation constraints specify the AUVs relative positions and are described by triple $(g^f, C^f, \mathscr{G}^f)$ where $g^f : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}^M$, $C^f \in \mathbf{R}^M$, and $\mathscr{G}^f$ is a graph representing the vehicles' formation relations.

2.3 The Approach

Outline. Our approach is based on a MPC scheme, being the information exchanged over acoustic communication channels. To deal with the bandwidth limitation that precludes closing low-level (fast) feedback loops over acoustic communications, the following two-layer control framework distributed over the AUVs in formation is considered: The lower layer deals with the fast low-level control in each vehicle. The upper layer deals with acoustic communications and provides control corrections to the lower layer. Each vehicle has a fast low-level formation controller. This is a feedback controller for the whole formation. We use a model-based approach to close the control loop around state estimates from the vehicle and from models of the other vehicles. This is done without communications with the other AUVs. We use MPC for the high-level controller, which runs in each vehicle. The model state value is reset when a message with the true state data of other AUVs is received. The MPC is run with the model updates to generate a sequence of control inputs for the AUVs in the formation. These control inputs are sent to the other AUVs for coordination. The MPC cost function is targeted at balancing the control effort and the quadratic error to the given formation reference trajectory and to the given formation pattern. While control constraints reflect control saturations and other model features, state constraints preclude collisions with obstacles.

Implementation. The main features of the implemented discrete time overall MPC controller of the AUVs formation are as follows:

- Decentralization. Each vehicle runs its own MPC scheme (which are identical for all vehicles) and communicates only with its neighbors;
- Computational efficiency. The MPC optimal control problem is approximated by a LQ optimization problem involving: (i) quadratic cost functionals, (ii) AUV

dynamics approximated by a discrete time linear model, and (iii) state and control
constraints given by linear inequalities;

- Easy incorporation of communication delays and packet dropouts; and
- Accommodation of noise and disturbances in the vehicles-simulated motion.

Let N_p, n_v, and T be, respectively, the prediction horizon, the number of vehicles,
and the sampling period. Then, the optimal formation control problem to be solved
in AUV i involves data from all its neighbors as defined by the formation graph and,
for some reference time t, can be stated as follows:

$$\text{Minimize} \quad \sum_{k=1}^{N_p} \left[\|y_k^{\text{ref},i} - y_k^i\|_{\mathbf{Q}^i}^2 + \|\tau_{k-1}^i\|_{\mathbf{R}^i}^2 + \sum_{j \in \mathscr{G}(i)} \|\mathbf{D}^{ij}(y_k^i - y_k^j) - d^{ij}\|_{\mathbf{L}^{ij}}^2 \right] \quad (2.3)$$

$$\text{subject to} \quad x_{k+1}^j = \Phi^j(T)x_k^j + \Psi^j(T)\tau_k^j, \quad x_0^j = x_t^j \quad (2.4)$$

$$y_k^j = \mathbf{C}^j x_k^j \quad (2.5)$$

$$x_k^j \in [x_{LB,t}^j, x_{UB,t}^j], \quad \tau_k^j \in [\tau_{LB}^j, \tau_{UB}^j]. \quad (2.6)$$

We observe that the cost functional consists in a weighted sum of three terms:
trajectory error, control effort, and a penalization of the deviation form the formation
configuration. We observe that (2.4) represents not only the discrete time linearized
dynamics of vehicle i,[3] but also those of all the vehicles with which the vehicle i is
linked through the graph $\mathscr{G}(i)$. Notice that the constraints hold for $j \in \{i\} \cup \mathscr{G}(i)$,
being, for each time k, $\mathscr{G}(i)$ the set of nodes of the graph specifying the vehicles
linked to AUV i.

Here, y_k^i and $y_k^{\text{ref},i}$ are, respectively, the vector of outputs of vehicle i and its
reference, x_t^j is the initial state of vehicle j at the initial time t, $\mathbf{D}^{ij}$ is the adjacency
matrix reflecting the formation relation between vehicles i and j, d^{ij} is a vector
parameter specifying distances between vehicles i and j, and $x_{LB,t}^j$, $x_{UB,t}^j$, τ_{LB}^j, and
τ_{UB}^j are bounds for state and control at time t, respectively. The matrices $\mathbf{Q}^i$, $\mathbf{R}^i$, and
$\mathbf{L}^{ij}$ are the chosen performance weights for AUV i.

Now, we describe the MPC scheme running onboard each AUV. If communication
packets dropout or arrive late, then the vehicles will not share the same data and there
will be differences in the control strategies computed by the various vehicles. This
issue is mitigated by replacing the missing sampled data by simulated data. The MPC
scheme for AUV i is as follows:

1. Initialization: prediction and control horizons, and other optimal control prob-
   lem parameters that depend on specific mission requirements, such as, level of
   perturbations, existence of obstacles, relative weight of trajectory tracking, and
   formation pattern errors.

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[3] The matrices $\Phi^j(T)$ and $\Psi^j(T)$ are obtained by integrating the piecewise constant linear system
in (x, u) approximating the original system over the sampling period T.

2. Sample the state variable, compute its estimate, and communicate it to its neighbors via acoustic modem.
3. Obtain the state variable of its neighbors via acoustic modem.
   – If data are available go to step 4.
   – Otherwise, generate estimates of the neighbors' state by running their models.
4. Solve the linear quadratic optimization problem (LQP^i) at the current time t, for the current prediction horizon and for the given reference output trajectory. This yields the optimal control for vehicle i.
5. Apply the control τ^i for the current control horizon.
6. Slide time for the optimization problem and adjust parameters as needed.
7. Let time elapse until the end of the current control horizon and go to step 2.

Results. We evaluated the MPC controller by taking into account conditions which are representative of field operations. We introduce the following four metrics for performance evaluation: trajectory tracking (*TM*), formation tracking (*FM*), control effort (*CM*), and total cost (*C*). While the first two are the L_2 norm of trajectory and formation tracking errors, the third one is the total control fuel consumption. In this assessment, three different scenarios were considered for a side-by-side formation of two vehicles along a sine wave trajectory with a nominal velocity of 1 m/sec: no communication, communication without delays, and communication with a 0.1 sec that corresponds to a 150 m distance between vehicles. In this last scenario, a prediction model was used to mitigate the impact of the delay. For each scenario, Gaussian noise with mean and variance values (0, 0.1), (0, 0.25), and (0.1, 0.02) is considered. In the case of no noise and no delay, the values *TM* = 0.7, *FM* = 0.2, *CM* = 0.2, and C = 34.4 were obtained. In Table 2.1, it is shown how our MPC controller performed in the various situations. Its entries were obtained by averaging the performances of 10 runs with independent realizations of the random variables.
    From the data in the table, some conclusions emerge as follows:

- The value of the cost function and performance measures of the formation controller degrades as the noise level increases whatever simulation scenario, being the impact of the noise mean far greater than that of its variance.
- The performance of the controller improves significantly with enabled communications relatively to open-loop case.

Table 2.1 MPC performance table

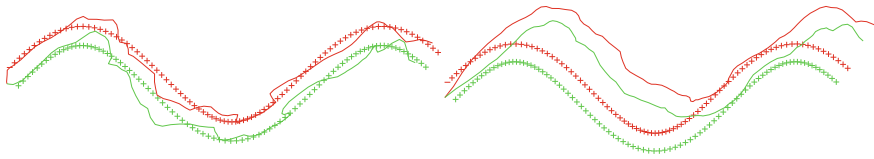| Noise (Mean, Var.)	(0, 0.1)	(0, 0.25)	(0.1, 0.02)
Criteria	TM FM CM C	TM FM CM C	TM FM CM C
Comms disabled	11.8 2.8 40.6 524.9	33.5 4.8 48.2 1158.0	211.7 39.6 57.7 8197.0
Comms enabled No delay	0.8 0.3 14.7 48.4	1.0 0.4 25.9 70.3	1.1 0.4 17.6 81.3
Comms enabled Delay = 1 s	0.9 0.3 24.5 52.5	1.2 0.4 34.7 74.9	1.6 0.8 18.3 105.5

Fig. 2.2 Side-by-side AUV formation trajectories with communications and noise: (Mean = 0, Variance = 0.25), (Mean = 0.1, Variance = 0.02)
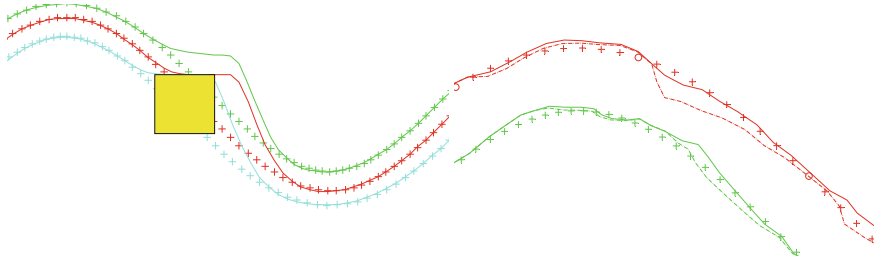

Fig. 2.3 *In the left* Obstacle avoidance of a three AUV triangle formation control and obstacle avoidance. *In the right* Effect of communications dropouts in two AUV formation control

- The use of prediction mitigates the impact of delay, as it significantly prevents performance degradation.

These conclusions are backed by a cursory inspection of the trajectories obtained with simulation runs shown in Fig. 2.2 where solid lines represent actual trajectories and the "+" the reference trajectory waypoints. Figure 2.3 shows the flexibility and robustness of our MPC approach. In the left, three AUVs moving in a triangle formation are able to avoid collision with an obstacle whose emergence can be regarded as perturbation forcing the vehicles to deviate from their originally nominal trajectories. In the right, the impact of random communication dropouts, are marked with "o," of the red AUV in receiving messages from the green AUV in the controller performance is shown. It is clear from the trajectory with communication dropouts, represented by the dash-dot line that the MPC controller is able to recover after a certain transient.

2.4 The Reach Set MPC Research Challenges

Although the conclusions in the previous section are extremely relevant for control design, there is still plenty of room to improve the control performance. One consists in improving state estimates relatively to the ones provided by the linear approximation of the AUV dynamics by taking into account the nonlinear nature of the system. Unfortunately, this will imply a much higher computational complexity. In order

to address both these issues, we propose a new formulation of the MPC scheme, [15], that relies in the observation[4] that the optimal control problem in Sect. 2.2 is equivalent to

$$\text{Minimize } \{V(t + \Delta, \bar{x}(t + \Delta)): \bar{x}(t + \Delta) \in \mathcal{R}(t + \Delta, (t, \bar{x}(t)))\}, \qquad (2.7)$$

where $\mathcal{R}(t_2, (t_1, x_1))$ is the Reach set of the extended system $\dot{\bar{x}} = [l(x, \tau), f(x, \tau)]^T$, where l is the integrand in (2.2) and f is specified by (2.1), i.e., set of points that the extended system can reach at t_2 when x departs from x_1 at $t_1 \le t_2$, [12], and $V(t, z)$ is the Value function, i.e., the minimum cost from the point (t, z) onward, [13, 30]. Under appropriate assumptions, $V(t, z)$ can be computed as a solution to the following Hamilton–Jacobi–Bellman equation with an appropriate boundary condition, [13, 30],

$$\frac{\partial}{\partial t} V(t, \bar{x}) + \min_{\tau \in \mathcal{U}} \left\{ \left\langle \frac{\partial}{\partial \bar{x}} V(t, \bar{x}), (f(x, \tau), l(x, \tau)) \right\rangle \right\} = 0. \qquad (2.8)$$

For time invariant systems, both Reach set and Value functions can be computed off-line, being, with respect to the former, the online computational burden reduced to (i) rotations and translations of the Reach set to take into account the pose of the vehicle at t, and (ii) the computation of the optimal control in $[t, t + \Delta]$. Moreover, both computational complexity and amount of information to be shared among the vehicles can be further reduced by considering polyhedral approximations to the Reach sets. In spite of powerful tools available, [17, 18], solving (2.8) with state constraints, even off-line and for systems with a moderate dimension, remains a huge challenge. The book by Stanley Osher and Ronald Fedkiw mentioned in Sect. 2.5 provides a good overview on level set methods to generate pertinent computational schemes. Another challenge concerns the "online" update of $V(\cdot, \cdot)$ which depends strongly on the types of perturbations. We note that, for the case of obstacle emergence, $V(\cdot, \cdot)$ has to be updated only in the region encompassing all the possible paths joining the current pose and the best one for which the obstacle is overcome.

2.5 Most Relevant Literature

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[4] State constraints are omitted to facilitate the exposition.

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