Asymmetric complements in a vertically differentiated market: competition or integration?*

Ornella Tarola† and Cecilia Vergari‡

September 17, 2013

Abstract

We study the effects of integration of asymmetric complements when they are vertically differentiated. While confirming the standard effects of integration, namely the internalization of the double marginalization externality and the reduction of competition, we point out a new *positive quality effect*, due to an increase in the average quality of the goods on sale. We also characterize the conditions under which integration turns out to be optimal for both firms' and consumers. We thus provide valuable directions for competition agencies when considering the joint ownership in vertically differentiated markets.

JEL classification: L1, L4.
Keywords: Integration, monopoly, duopoly, vertical differentiation

1 Introduction

In high tech industries there is an increasing evidence of asymmetric systems consisting of a *core good* and *complementary products*.[1] Typical examples of asymmetric systems are operating systems and internet browsers, computers and printers, televisions and video players, mobile phones and apps, *inter alia*. Although in some circumstances base good and complementary products are

*We thank two anonimuous referees for constructive criticisms. We also benefitted from useful comments from Jean J. Gabszewicz, Michele Grillo and Andrea Mantovani. Any remaining errors are ours.

†DISS, University of Rome "La Sapienza", Piazzale Aldo Moro 5, 00100 Rome, Italy, corresponding author, e-mail: ornella.tarola@uniroma1.it, phone num: +39(0)6-49910253.

‡Department of Economics, University of Bologna, Strada Maggiore, 45 I-40125 Bologna Italy. email: cecilia.vergari@unibo.it.

[1] Core goods are goods that combined with another (complementary good) to create a system can also be purchased (and used) separately. For the purpose of the paper, the terms "core good" and "base good" are used as synonimous.

produced within the same firm, as keeping high standard of production requires specific capabilities, many small and highly specialized companies are only concerned with the production of top-quality complementary components. For example, while Microsoft and Apple produce both operating system and internet browser, Linux restricts itself to the production of operating system, and Mozilla and Google specialize in Internet browsers. Interestingly, there are circumstances in which producers of the complementary components are fostered by the producers of core goods to enter the market. This is the case of Intel Corporation: leader in the production of microprocessor, it has a long history of inducing entry into complementary markets through the development and royalty-free dissemination of intellectual property. Quite often, instead, the business philosophy of well established firms turns out to be the acquisition of complements' producers. Recent examples is Apple acquiring Chomp in 2012, a producer of a search engine compatible with Apple's iPhone, or Microsoft acquiring Lionhead Studios in 2006, a video-games producer, and Skype Communication in 2011.

In spite of the huge amount of empirical evidence on these practices and the relevance of their competition policy implications, a theoretical analysis of the incentives for asymmetric producers to prefer an arrangement compared with another and the effects on competition resulting from such a choice is still missing.[2]

The aim of this theoretical paper is to identify under which conditions a core good producer finds it optimal to acquire a complementary good seller and to evaluate the competition policy implications of such a choice.

Among researchers, the view that an integrated monopoly may be beneficial to consumers is generally shared. The joint ownership of complements benefits consumers as it removes a problem of double marginalization while possibly passing through to consumers further gains.[3] This clear-cut merger's implication is mitigated in oligopoly markets, as clearly argued by Economides and Salop (1992). In oligopolies, "production and distribution networks are often composed of both competing and complementary brands of components. The complementary components then can be combined to produce products or system, which are substitutes for one another."(Economides and Salop 1992, 105). Although the joint ownership determines "vertical" integration between producers of different components, it also decreases competition as it induces "horizontal" integration between producers of a given component. While the

[2] In general, "there is not a consensus regarding appropriate enforcement policy with respect to vertical mergers. This is reflected in controversy over the merits of cases pursued by the Federal Trade Commission (FTC) and the Department of Justice (DOJ) in the United States and by the EuropeanCommission (EC) in the EuropeanUnion over the last decade." (Church 2008, p. 1455).

[3] For instance, when the quality of a two-product system is determined by the minimum of the qualities of its components, an *integrated monopoly* in which both complementary goods are sold by a single integrated firm dominates complementary monopolies (namely, independent ownership) in terms of welfare implications: in fact, the first entails higher quality goods' and higher market coverage (Economides 1999, Maruyama et al. 2011) while neutralizing the vertical externality of double marginalization.

former effect pushes prices downwards, the latter pushes prices upward so that the net effect of joint ownership turns out to be ambiguous.

Close in spirit to Economides and Salop (1992), our purpose is to identify how the basic ingredients of the integration problem change when a system consists of *asymmetric complements* and competition develops along a *quality dimension.* Although to the best of our knowledge we are the first to combine the notion of asymmetric system with vertical differentiation, the empirical evidence gathered so far shows that producers of complementary components may have different skills and experience in production with immediate consequences on the quality of components.[4] Several examples can be found. In the new category of smart phones, leader firms in designing the device and the operating platform (e.g Apple with the iPhone) do not meet success in producing apps (Apple with iMessage), these latter being better developed by others (e.g. Whats-App Messenger). In the computer industry, Android, a Java-based operating system that runs on the Linux 2.6 kernel, launched by Google in 2007, provides another example. Google does not produce a mobile handset while investing in a software platform for devices against Nokia and Microsoft, both leader in device.[5] In order to capture this skill-gap among firms, we put the argument in the simplest possible vertically differentiated setting. There is an incumbent monopolist that produces a base good. This good can be equipped with a complementary component, which is produced by the monopolist itself and by a potential rival. The value of the base good increases with the quality of the complementary component with which it is bundled. In line with the above considerations, *we assume that, the monopolist produces a complementary component whose quality is lower than that produced by the potential rival.*[6] Of course, one can easily find some counter examples where the producer of the base good is also producing the high quality component. We discuss later this alternative possibility and provide some theoretical arguments for justifying this scenario.

When facing the rival, the monopolist can decide to sell the base good and its low quality component as a system, thereby preventing entry. Otherwise, the monopolist can put on sale the base good alone and the low quality system separately. In this scenario entry is allowed so that both the low quality component and the high quality one are on sale, while the *producers are disintegrated.* Finally, the monopolist can pursue an *integration strategy*, thus acquiring the rival at some acquisition price. We derive the equilibrium values in the whole set of possibilities and identify their main properties from both a private and a social viewpoint. Further, we study under which conditions integration can be

[4] Alvisi, Carbonara and Parisi (2011), assume that *symmetric* components are vertically differentiated and in Brito and Catalao-Lopes (2010) the asymmetric complementarity issue is faced in case of autonomous markets.

[5] Incidentally, this discrepancy among producers' capabilities is quite often evoked to justify the increasing pattern of innovation by acquisition phenomenon taking place in the high-tech sector: well established companies acquire innovating start-up firms as to exploit their innovations and capabilities, thereby avoiding to pursue internal research activities.

[6] Gawer and Anderson (2007) discuss this matter at lenght. They state that one of the reasons for Intel to foster small firms to enter complementary markets is indeed represented by the skill-gap in producing complements between these firms and Intel.

welfare enhancing, thereby drawing some competition policy implications.

We find that from the firms' viewpoint, the incentive to integration with the high quality system on sale always dominates the alternatives. Moreover, we define circumstances such that integration is optimal also from the consumers' point of view. In this regard, we confirm the *horizontal and vertical effects* described by Economides and Salop (1992) and describe some further forces determined by the vertical product differentiation. In particular, we identify a *positive quality effect* taking place whenever the market is provided with the top quality system and a *negative composition effect* arising whenever those consumers willing to buy when the producers do not integrate, stop buying under integration. So, whenever the positive effects dominate the negative ones, integration turns out to be optimal for both firms and consumers.

Our model also displays several properties that are interesting in the literature on entry in complementary markets, thereby contributing to the open dilemma for the base good owner to enter a complementary market or leave another firm to produce components.[7] Indeed, we both provide a rationale for the incumbent to enter the complementary market and identify circumstances such that credibly committing to accommodate entry turns out to be the most profitable strategy.

It is worth remarking that we develop the analysis under two specific assumptions, namely zero production costs and uniform consumers' distribution. These assumptions have strong theoretical implications, as it will be discussed at lenght in the paper. Still, we feel that they can capture some specific features observed in high tech market, thereby making the model useful to analyse the interaction among many relevant players in this sector (e.g. Google, Apple). For example, the assumption of zero production costs can be easily adjusted to consider sunk or fixed costs, which represent a significant component in the high tech industries. Also, the uniform distribution of consumers allows to model situations where the willingness to pay does not get more concentrated somewhere. No doubt, there exist circumstances where tastes are related to particular features of consumers with immediate consequences on their willingness to pay in some range of the distribution. Neverthless, the massive concern of people toward high tech products makes the traditional uniform distribution rather satisfactory.

The structure of the paper is as follows. In section 2 we present the model. In Section 3 the game is defined and the possible scenarios are presented. We study the equilibrium and welfare analysis in Sections 4 and 5, respectively. Then, we discuss in Section 6 the scenario where it is the producer of the base good to develop a high-quality component. This further case can be observed for example if the monopolist has some experience economies so to overcome possible compatibility pitfalls between the base good and the complementary components.[8] Finally, we briefly conclude.

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[7] A huge strand of literature is concerned with this issue. We refer the interested reader to Gawer and Henderson (2007).

[8] Typically, such a type of analysis is developed in the mix-and match literature originated from Matutes and Regibeau (1988) and Economides (1989).

2 The model

We consider a market with an incumbent monopolist I and a potential rival E. The monopolist sells a base good of quality $\bar{u}$ to a population of consumers identified by the parameter $\theta \in [a, b]$, with $0 = a < b$ and uniformly distributed with density equal to $\frac{1}{b}$.

For future reference, notice that, given a, the parameter b is proportional to the willingness to pay for quality defined as a function of b. Accordingly, given a, the higher b, the higher the heterogeneity among consumers captured by the dispersion of the willingness to pay. Also, once fixed a, namely the lower bound of the market, moving b changes the average valuation of quality in the market and thus the corresponding willingness to pay for quality. In particular, the higher b, the higher this willingness to pay. So, increasing b affects both the dispersion of the willingness to pay and the average willingness to pay along the same direction.[9]

Both the monopolist I and the potential rival E can sell a complementary asset of quality v_I and v_E, respectively, with $v_E > v_I$. The complementary good does not bring any value to the consumers who do not buy the base good. Still, it allows the base good to perform better. Denoting by u_i the overall quality of the base good when equipped with the complementary variant v_i, $i = I, E$, $u_E > u_I > \bar{u}$ holds, where $\bar{u}$ is the quality of the base good alone.[10] Accordingly, the utility for consumer θ when buying either of the goods is:[11]

$$U(\theta) = \begin{cases} \theta u_E - P_E \text{ when buys the high quality system} \\ \theta u_I - P_I \text{ when buys the low quality system} \\ \theta \bar{u} - p_u \text{ when buys the base good alone} \\ 0 \text{ when refrains from buying} \end{cases}$$

where P_i (resp. p_u) is the price paid by the consumer θ for getting the base good equipped with variant i (resp. the base good alone). Note that under separate ownership the price paid by a consumer to use the high quality system, P_E, will be the sum of the price of the base good alone plus the price of the high quality complementary variant.

The average cost of production of both the base good and the complementary assets are assumed to be constant and equal to zero.[12] Depending on the strategy of the monopolist when facing the rival, different arrangements can

[9] Consider instead the case $a > 0$. In this case, suppose we move both bounds of the market: a decreases and b increases. Then the above considerations about the dispersion of the reservation prices still hold (that is the dispersion increases). In constrast, the effect of these changes of a and b on the average valuation of quality is ambiguous: namely when the decrease in a is relatively larger than the increases in b, this valuation decreases.

[10] We discuss the opposite case in which the incumbent can offer a high quality system (i.e. $u_I > u_E > \bar{u}$) in Section 6.

[11] Note that the assumption $a = 0$ implies that at equilibrium the market is always uncovered.

[12] While the zero cost assumption allows us to keep the analysis neat and to focus on the private and social incentives to allow entry and/or integrate, it can affect the equilibrium analysis. We discuss about this in Section 6.

be observed in the market. There can be a case, where the monopolist sells a system consisting of the base good and the low quality component. In this scenario, entry is prevented and the high quality variant is never marketed at equilibrium. Otherwise, the monopolist can offer both the base good alone and the low quality system separately. Then, there is room for the rival to sell the high quality complementary variant and separate ownership emerges at equilibrium. Finally, the optimal strategy for players can consist of joint ownership or integration. In this case, at some acquisition price the monopolist acquires the rival and thus sells on its behalf the complementary high quality variant v_E. In the following section, we define the game and analyse the above described scenarios in turn.

3 The game

The equilibrium path is defined in a two-stage game which develops as follows. First the incumbent decides whether to allow entry. Then, in case of entry, the incumbent and the rival compete in prices or possibly integrate (if this is the most profitable strategy). We solve the game by backward induction and find the subgame perfect Nash equilibria.

Monopoly Let us consider the first scenario, where the incumbent sells only the system of quality $u_I > \bar{u}$, consisting of the base good and the low quality component. It follows that entry is not allowed and competition is kept out from the market. Then, each consumer θ can either buy the low quality system and get utility $\theta u_I - P_I$ or not buying at all and get a nil utility. As now the monopolist extends the monopoly power to the complementary market, from standard computations one immediately obtains the equilibrium price P_I^m, profit Π_I^m and consumer surplus CS^m:[13]

$$P_I^m = \frac{1}{2} b u_I, \Pi_I^m = \frac{1}{4} u_I b^2, CS^m = \frac{1}{8} u_I b^2.$$

Separate ownership Otherwise, the monopolist can sell the base good and the low quality system, separately. In this case, the rival can enter the market and sell the high quality component. Thus, it follows that, each consumer can either buy only the base good at price p_u and get utility $\theta\bar{u} - p_u$; or buy the base good equipped with the low quality (resp. high quality) complementary variant at some price P_I (resp. $P_E = p_u + r_E$, where r_E is the price of v_E sold by the high quality complementary producer) in which case the utility is $\theta u_I - P_I$ (resp. $\theta u_E - p_u - r_E$); or refrain from buying. As we prove in the Appendix, at equilibrium, depending on the consumers' dispersion of quality valuation (b) and the quality gap between the systems ($u_G = u_E/u_I$), there can be a configuration with positive demands for the low quality system as well as for the high quality system; a further configuration can be such that all active

[13] Given the demand $b - \frac{P_I}{u_I}$, it is straightforward to find these equilibrium values.

consumers buy either the low quality system or the high quality system; finally, one can observe a configuration with a positive demand for the base good alone and the high quality system as well.[14] To specify the equilibrium configurations, we proceed as follows.

We start defining the consumer indifferent between buying something and not buying at all, say $\bar{\theta}$. We can prove that $\bar{\theta} = \frac{P_I}{u_I} = \min\left\{\frac{p_u}{\bar{u}}, \frac{P_I}{u_I}, \frac{p_u+r_E}{u_E}\right\}$ for $b \leq \bar{b}(u_G)$, where $\bar{b}(u_G) = \frac{2u_G+1}{(u_G+2)}$. Under this market configuration, we show that $\frac{P_I}{u_I} = \frac{p_u+r_E}{u_E}$ and $\frac{P_I}{u_I} < \frac{p_u+r_E}{u_E}$ whenever $b = \bar{b}(u_G)$, and $b < \bar{b}(u_G)$, respectively. Notice that $\frac{\partial \bar{b}}{\partial u_G} > 0$. That is to say that the higher u_E (resp. the lower u_I), the wider the range of parameters such that the condition $b < \bar{b}$ is satisfied. Then, let us denote by $\breve{\theta}$ the consumer indifferent between buying the low quality system and the high quality system, namely $\breve{\theta} = \frac{p_u+r_E-P_I}{u_E-u_I}$. We find that $\breve{\theta} = b$ and $\breve{\theta} < b$ whenever $b = \underline{b}(u_G)$ and $b > \underline{b}(u_G)$, respectively, where $\underline{b}(u_G) = \frac{2u_G+1}{4u_G-1}$. From the above, it emerges that the demand for the high quality bundle turns out to be nil in the case when the quality differential between bundles does not suffice to compensate their price gap so that not even consumers with high reservation prices are willing to buy the high quality system. Traditionally, in vertically differentiated markets, depending on the dispersion of the willingness to pay for quality, an upper bound to the number of surviving variants at equilibrium can be identified (*finiteness property*). Furthermore, the surviving variants are those at the top of the quality ladder, those lying at the bottom being pushed out from the market (Shaked and Sutton, 1983; Gabszewicz and Thisse, 1979). In our model, at first sight this property no longer holds. Even worst, it seems that the ratio underlying the finiteness property is reversed, as we find that, for certain ranges of parameters, at equilibrium only the lowest quality variant is sold. Notice however that, the above finding has nothing to do with the notion of natural oligopoly where the finiteness property arises.[15] Rather, it follows from the fact that the price of the high quality bundle is affected by both the price r_E defined by the high quality producer, and the price p_u defined by the competing firm I. So, if p_u is set very high by the incumbent and the reservation price b for the high quality system is not sufficiently high, given the price of the low quality bundle, it may well happen that the high quality bundle does not face demand. In other words, our result derives from the asymmetric nature of the base good versus the complementary variant: as this latter cannot be consumed without the former, the producer of the base good can prevent consumers from buying the high quality bundle by quoting an extremely high price for the base good alone. It is worth noting that, $\frac{\partial \underline{b}}{\partial u_G} < 0$. This implies that, *ceteris paribus* a lower u_E (resp. a higher u_I) increases the set of parameters such that consumers are not willing to buy the

[14] We exclude the possibility that all active consumers buy the base good alone as it is not incentive compatible with the entrant possibility to produce the high quality complementary component.

[15] As the market is always uncovered at equilibrium, we do not have a natural monopoly/duopoly.

high quality system compared to the low quality alternative as the quality gap between bundles decreases.

Accordingly, we can conclude that.

Lemma 1 *Under separate ownership, at equilibrium, for any $b < \underline{b}$, the demand for the high quality system is nil, and all active consumers buy the low quality system; for any $\underline{b} \leq b \leq \overline{b}$ both the high quality system and the low quality system have a positive demand.*

Proof. See the Appendix, Section 9.1. ■

As a consequence of the above statement, whenever $b < \underline{b}$ the market is kept monopolized by the incumbent and provided with the low quality system, as in the monopoly scenario. Accordingly, the demand function for the low quality system writes as $\left(b - \frac{P_I}{u_I}\right)$ and equilibrium price $P_I^{so,I}$, profit $\Pi_I^{so,I}$ and consumer surplus $CS^{so,I}$ turn out to be:

$$P_I^{so,I} = \frac{1}{2}bu_I,\ \Pi_I^{so,I} = \frac{1}{4}u_I b^2,\ CS^{so,I} = \frac{1}{8}u_I b^2,$$

where superscript so, I denotes this separate ownership scenario. In this case, at equilibrium the incumbent's profits under separate ownership $\Pi_I^{so,I}$ coincide with those observed under monopoly, Π_I^m. In the case when $\underline{b} \leq b \leq \overline{b}$, both demands turn out to be positive at equilibrium. In particular, consumer types $\theta \in (\breve{\theta} - \frac{P_I}{u_I})$ buy the low quality system, whereas consumer types $\theta \in (b - \breve{\theta})$ buy the base good as well as the high quality component. In this market configuration, that we label so, equilibrium prices p_u^{so}, P_I^{so} and r_E^{so}, profits Π_I^{so} and Π_E^{so} and consumer surplus CS^{so} turn out to be

$$p_u^{so} = \frac{1}{6}u_I + \frac{1}{3}u_E,\ P_I^{so} = \frac{1}{2}bu_I,\ r_E^{so} = \frac{1}{3}b\left(u_E - u_I\right),$$

$$\Pi_I^{so} = \frac{1}{36}b^2\left(5u_I + 4u_E\right),\ \Pi_E^{so} = \frac{1}{9}\left(u_E - u_I\right)b^2,$$

$$CS^{so} = \frac{4u_E^2(2b-1)^2 + u_I u_E (b-2)^2 + u_I^2\left(2b - 8b^2 + 1\right)}{72(u_E - u_I)}.$$

It is worth noting that even for $\underline{b} \leq b \leq \overline{b}$, it is reasonable that no consumer buys the base good alone. In order to serve the low consumer types, the monopolist would be required to quote a very low price for the base good. As no price discrimination is allowed, the incumbent should quote a unique low price for both the low and the high consumer types. Thus, when selling the base good to those consumers willing to buy the high quality complementary variant, it would be prevented from taking advantage of the high quality component. We show in the Appendix that this latter price strategy is never profitable, thereby proving that the condition that $\frac{P_I}{u_I} = \min\left\{\frac{p_u}{\bar{u}}, \frac{P_I}{u_I}, \frac{p_u + r_E}{u_E}\right\}$ is an equilibrium strategy.

On the contrary, in the case when $b > \overline{b}$, the condition $\frac{P_I}{u_I} = \min\left\{\frac{p_u}{\bar{u}}, \frac{P_I}{u_I}, \frac{p_u + r_E}{u_E}\right\}$ is not always satisfied. In particular, for $b > \overline{b}$, it emerges that the active consumers choose to buy the high quality system rather than the low quality system.

Nevertheless, depending on the quality gap between u_E and $\bar{u}$, the nature of price competition changes. In particular:

- whenever the quality gap between the high quality system and the base good is not very significant, namely $u_E < 2\overline{u}$, the monopolist cannot exclude that some potential consumers are willing to buy the base good alone. In this case, the demand for the monopolist and that for the high quality producer write as $(b - \frac{p_u}{\overline{u}})$ and $\left(b - \frac{r_E}{u_E - \overline{u}}\right)$, respectively. It is immediate to show that, these demands, at the equilibrium prices $r'_E = \frac{1}{2}b\left(u_E - \overline{u}\right)$ and $p'_u = \frac{1}{2}b\overline{u}$, are both positive and the equilibrium profits are:
$$\Pi_I = \frac{1}{4}b^2\overline{u},\ \Pi_E = \frac{1}{4}b^2\left(u_E - \overline{u}\right).$$
It follows that, the assumption that no consumer is willing to buy the base good alone, namely $\frac{P_I}{u_I} = \min\left\{\frac{p_u}{u}, \frac{P_I}{u_I}, \frac{p_u + r_E}{u_E}\right\}$ no longer holds.

- Whenever the complementary variant brings a substantial improvement to the performance of the base good, namely $u_E > 2\overline{u}$, the monopolist targets *only* those consumers willing to use the high quality system. As a consequence of this monopolist's choice, the demand for the base good and that for the high quality system coincide and write as $(b - \frac{p_u + r_E}{u_E})$. Price competition leads to the following equilibrium price, profits and consumer surplus:
$$\begin{aligned} p_u^{so,E} &= \frac{1}{3}bu_E,\ r_E^{so,E} = \frac{1}{3}bu_E, \\ \Pi_i^{so,E} &= \frac{1}{9}b^2u_E, i = I, E, \\ CS^{so,E} &= \frac{1}{18}u_Eb^2. \end{aligned}$$

We can conclude the following.

Lemma 2 *Under separate ownership, at equilibrium, for $b > \overline{b}$: (i) whenever $u_E < 2\overline{u}$, there are consumers willing to buy both the base good alone and the high quality system; rather (ii) whenever $u_E > 2\overline{u}$, all active consumers buy the high quality system.*

Proof. See the Appendix, Section 9.1. ■

Integration strategy Let us now move to consider the case when the incumbent acquires at some acquisition price P_A the high quality producer. In this scenario, that we denote *int*, the monopolist can avoid to compete against the rival and sell the high quality system on its behalf while paying an acquisition price for the integration. *A priori,* under integration the monopolist can sell all three variants, combinations of two of them, or only one of them, namely the top combination u_E at price P_E.[16] We find the following.

[16] Selling only the low combination u_I coincides with the monopoly scenario.

Lemma 3 *Under integration, at equilibrium only the high quality system is on sale.*

Proof. Suppose that the incumbent sells all the three variants. Then, the profit maximization problem writes as

$$\max_{p_u, P_I, P_E} \left((\widetilde{\theta} - \bar{\theta}) p_u + (\breve{\theta} - \widetilde{\theta}) P_I + (b - \breve{\theta}) P_E \right)$$

where P_E is the price of the high quality system; the indifferent consumer types are $\bar{\theta} = \frac{p_u}{\bar{u}}$, $\widetilde{\theta} = \frac{P_I - p_u}{u_I - \bar{u}}$ and $\breve{\theta} = \frac{P_E - P_I}{u_E - u_I}$. Equilibrium prices, p_u^{int}, P_E^{int} and P_I^{int}, profit Π_I^{int} and consumer surplus CS^{int} are then:

$$P_E^{int} = \frac{1}{2} b u_E, P_I^{int} = \frac{1}{2} b u_I, p_u^{int} = \frac{1}{2} b \bar{u},$$
$$\Pi_I^{int} = \frac{1}{4} u_E b^2,$$
$$CS^{int} = \frac{1}{8} u_E b^2.$$

The demands for the base good alone and the low quality system corresponding to the equilibrium prices P_I^{int} and p_u^{int} are equal to zero. Thus, only the high quality system turns out to be sold at equilibrium. ■

Selling more than one quality would entail a positive market expansion effect, as some consumers refraining from buying the high quality system would be willing to buy an alternative good, either the base good or the low quality system. Nevertheless, this positive effect is overcompensated by a negative cannibalization effect. Indeed, when expanding the range of variants on sale, the monopolist would lose those consumers moving from the high quality system to the competing good, whatever it is. As the equilibrium price of the alternative good would be lower than that of the high quality system, the monopolist would suffer a profit loss from this.[17]

4 Equilibrium analysis

It remains now to set the equilibrium analysis, namely to see whether the incentive to acquisition dominates the alternative choices. To this aim, notice that, in order to be preferred over the alternatives, the acquisition proposal should yield the high quality producer E a gain P_A at least equal to the profits it would get under turning off the proposal. Furthermore, it is convenient for the incumbent I to make such a proposal if, and only if, profits obtained when acquiring the high quality producer after paying the acquisition price P_A are larger than the profits it would get in the alternative scenario, whatever it is. We know from the previous section that:

[17] This is a well established result in the literature studying monopoly and product quality, see *inter alia* Acharyya (1998).

- For $b < \underline{b}$, the demand for the high quality system is nil. Thus, no room is left to the high quality producer and the acquisition price turns out to be zero.[18] Of course, in this scenario, integration dominates the alternative monopoly choice as $\Pi_I^{int} = \frac{1}{4}u_E b^2 > \Pi_I^m = \frac{1}{4}u_I b^2$.

- For $\underline{b} \leq b \leq \overline{b}$, both the players get positive profits at equilibrium. Indeed, the incentive for the incumbent to allow entry, namely Π_I^{so}, always dominates that to prevent the rival from selling the high quality component, namely Π_I^m, as $\Pi_I^{so} - \Pi_I^m = \frac{1}{9}b^2(u_E - u_I) > 0$. Accordingly, the acquisition price P_A turns out to be equal to the equilibrium profits that the high quality producer would get under separate ownership, namely $\Pi_E^{so} = \frac{1}{9}(u_E - u_I)\, b^2$. As $\Pi_I^{int} - P_A(= \Pi_E^{so}) > \Pi_I^{so}$, then integration is preferred over the alternative.

- Finally, for $b > \overline{b}$, we find that whenever $u_E < 2\overline{u}$, the incumbent prefers the monopoly scenario as $\Pi_I(= \frac{1}{4}b^2\overline{u}) < \Pi_I^m(= \frac{1}{4}u_I b^2)$; whereas for $u_E > 2\overline{u}$, the incumbent allows entry if and only if $u_E > \frac{9}{4}u_I$. Indeed, when the high quality system is by far higher than the low quality system, equilibrium profits under separate ownership $\Pi_I^{so,E}$ are higher than those under monopoly Π_I^m. Accordingly, for $u_E < \frac{9}{4}u_I$, the acquisition price is zero, as in this range of parameters entry would be blockaded; whereas for $u_E > \frac{9}{4}u_I$ the acquisition price results to be $P_A = \Pi_E^{so,E}(= \frac{1}{9}b^2 u_E)$, as now the high quality producer would be allowed to enter the market. Therefore, whatever the quality gap between variants, for $b > \overline{b}$, integration is the optimal strategy.

Thus, the above comparisons prove the following result.

Proposition 4 *At equilibrium, integration always prevails over the other scenarios.*

Let us spend now a few words on the role of the uniform distribution of consumers in the analysis. Although the vertical differentiation model has been developed often under the assumption of uniform continuous distribution, recent works have shown how different consumers' distributions can alter the equilibrium analysis.[19]

In particular, two basic mechanisms can be activated when the willingness to pay gets more concentrated somewhere compared with a uniform distribution: first, some of the consumers with the lowest willingness to pay, who were not willing to buy at all (resp. willing to buy) under uniform distribution, purchase (resp. stop purchasing); further, some consumers whose willingness to pay is

[18] This can be seen as an *irrelevance result*: the market equilibrium is independent of the incumbent decision to allow or not entry. Formally, $\Pi_I^{dis,I} = \Pi_I^m$.

[19] For example, Acharyya (1998) considers the effect of a discrete consumers' distributions on the quality menu choice by a monopolist. More recently, Bonisseau and Lahmandi-Ayed (2007) show that there exist particular non uniform consumers' distribution such that the equilibrium does not exist.

now higher (resp. lower) are willing to move from a low-quality to a high-quality variant (resp. from a high-quality to a low-quality variant).[20] Accordingly, the incentives for consumers to buy, and if so, to choose a particular variant, change with the distribution and so well the incentives for firms to select the product lines. In our model, assuming a uniform distribution allows to clearly disentangle the two effects taking place when competition develops along a quality dimension (the quality effect and the composition effect) besides the basic ingredients of the dilemma between joint ownership versus separate ownership. Nevertheless, it prevents us from considering a consumers' driven incentive to production, namely to analyse how firms could react to some distributional shock in terms of production menu. Let us assume for example a distributional shock such that consumers get more concentrated toward a. This could increase the profitability of producing the low quality variant compared with the high quality complementary product, with immediate consequences on the profitability of the integration choice. Symmetrically, the same rationale can be applied to the scenario with consumers more concentrated toward b. In this case, some consumers willing to buy the low quality variant under uniform distribution could be willing to buy the high quality system now. Also, some others not buying before could now enter the market. Thus, the equilibrium configuration would change depending on the relative weight of these incentives.

5 Welfare analysis

We have shown above that the privately optimal strategy is integration between firms. We next wonder whether a competition authority would allow such an acquisition. As a measure of welfare we consider consumer surplus (CS). We discuss in Section 7 what changes when moving to the social welfare as alternative measure of welfare.[21]

Let us remind here that, if integration would be banned, then at equilibrium one could observe either monopoly or separate ownership, depending on the dispersion of consumers' willingness to pay for quality, and on the qualities of the systems. In particular, the outside option is separate ownership whenever $\underline{b} \leq b \leq \overline{b}$, or $\overline{b} < b$ and $u_E > \frac{9}{4}u_I$; and monopoly otherwise, namely for $\underline{b} > b$, or $\overline{b} < b$ and $u_E < \frac{9}{4}u_I$.

In this latter case, consumer surplus under integration ($CS^{int} = \frac{1}{8}u_E b^2$) is higher than that under monopoly ($CS^m = \frac{1}{8}u_I b^2$). This result is rather intuitive as in both cases the market is monopolized, however under integration the product quality is higher than under monopoly ($u_E > u_I$).

In the case when the outside option is separate ownership, comparing the equilibrium consumer surplus under separate ownership versus integration, we find that for b and u_E sufficiently high, consumers are better off under integra-

[20] Benassi et al. (2006) clearly disentagle these mechanisms in the case of 'purely distributive' shocks to the distribution of the consumers' characteristics, namely shocks that do not modify the mean and the support of the distribution itself.

[21] Formal details are in the Appendix, Section 9.2.

tion than under separate ownership. Indeed, in both cases, at equilibrium all active consumers would buy the high quality system; however under integration the double marginalization problem is removed, as the incumbent sells both the base good and the high quality complementary component.[22] As for $\underline{b} \leq b \leq \overline{b}$, the sign of the difference between consumer surplus under separate ownership CS^{so} and under integration CS^{int} depends on the average valuation of quality in the market b and the quality differential $u_G = u_E/u_I$. In particular, $CS^{so} - CS^{int} > 0$ whenever $b < \widetilde{b}$, where $\widetilde{b}(u_G) = \frac{1+2u_G}{7u_G-4} < \overline{b}$. Furthermore, $\widetilde{b} > \underline{b} \iff u_G < \frac{5}{2}$, while the reverse is true, namely $\widetilde{b} < \underline{b}$, otherwise. The following Proposition summarizes our previous considerations.

Proposition 5 *From the consumers' viewpoint: integration is always improving for high values of the quality gap; in contrast, whenever the quality gap is not so significant, integration is improving if and only if the average valuation of quality in the market is sufficiently high.*

The idea underlying the above findings goes as follows. When the quality gap is extremely high, then the integration scenario is socially optimal for any value of b. Indeed, for low values of b, under both integration and separate ownership the market would be monopolized. However, under integration the product quality is higher than under monopoly ($u_E > u_I$). This is a positive *quality effect* of integration. For high values of b, instead, under separate ownership, namely in the so, E scenario, only the high quality system is put on sale. This is the same result as under integration. So, moving from separate ownership to integration does not affect the average quality of the goods on sale. However under integration, the firm is able to internalize the double marginalization externality. This is a positive *pricing effect* of integration.

On the contrary, in the case of a low quality gap ($u_G < \frac{5}{2}$), moving from separate ownership to integration entails on one hand the two positive effects detected above (the positive quality and pricing effects); on the other hand, two negative effects linked to the reduction of competition become relevant. Indeed, under *int*, only the high quality system is marketed, while under separate ownership both qualities are on sale. As a result of this reduction in the number of qualities, some of the consumers that under *dis* were buying the low quality system move to the high quality system under *int*; however some others stop buying. This latter negative *composition effect* of demand is weaker, the higher is $\boldsymbol{b}$. Finally, when switching from *dis* to *int*, competition becomes milder as we now have only one firm rather than two; this negative *competition effect* is not very strong when consumers are very heterogeneous (it is as if each firm had its own segment of the market). Accordingly, the two negative effects of moving from *dis* to *int* dominate as long as b and the quality gap are sufficiently small, thus driving the consumers' preference for *so* over *int*.

[22] This is the Cournot (1938) result according to which when an intergrated monopoly sells two complementary goods prices are lower than when two separate monopolists sell each one of these complementary goods.

6 Reversed qualities in the complementary market

In the equilibrium analysis we have shown that integration always takes place and, as a result, only the high quality system is on sale. These conclusions are drawn under the crucial assumption that the quality of the incumbent's complement is lower than that of the (potential) rival. One may wonder whether our findings still hold in the case when the reverse takes place, namely it is the monopolist to produce the top quality. At least two arguments could justify this assumption. First, the monopolist could have some experience economies, thereby being able to produce a better complement than the rival. Further, he could be allowed to combine in a better way the base good and the complementary variant so as to escape from possible compatibility pitfalls. The purpose of this section is to discuss this further scenario, thereby considering that the quality of the rival's complement is lower than the quality of the incumbent's complement.

Zero quality costs Let us start first keeping the assumption of *zero quality costs*. Borrowing from the existing literature that with a uniform distribution of consumers, "if no cost is attached to quality improvement, the monopolist will always select the top quality" (Gabszewicz and Wauthy, 2002, 2), we can guess that in our model the incumbent would find it profitable to offer only the high quality system and entry would be blockaded. Indeed, having at its disposal also the high quality complementary good, the monopolist would not have incentive to let the low quality complement's producer enter the market, as this latter would cannibalize the incumbent' profits in the case of entry. Rather interestingly, this argument is not only detailed by the existing theoretical literature (see also Bonisseau and Lahamandi-Ayed, 2006) but also validated by the empirical evidence. To give an example, Gawer and Henderson (2007) explore Intel's strategy with respect to complements. They find that "Intel's behavior with respect to complementary markets is greatly shaped by whether the firm can match the competencies of potential entrants" (p. 3). Namely, the incentive for the base good producer to accommodate or rather blockade entry in the complementary market is mainly related to the skills of the complementary variant' s producer: as long as this latter is very skilled (competent), the former can benefit from allowing entry!

Positive quality costs Notice however that, *costs for quality* improvements are not always negligible. So, it can be interesting to extend the above discussion to the scenario with quality-specific production costs. Once more, we can refer to the existing literature on vertical product differentiation to discuss this further possibility. The pioneering contribution by Mussa and Rosen (1978) shows that, under monopoly, quality discrimination takes place and quality is underprovided with respect to the competitive outcome, when quality-specific production costs are taken into account. So, *a priori* one could be induced to state that the pooling menu (namely the top quality good to all consumers) we found at equilibrium in our model is no longer observed under the assumption of positive quality costs. Acharyya (1998), however, relaxes the Mussa-Rosen statement and shows that it depends, *inter alia*, on the specification of the cost function. If the cost function is not sufficiently convex in quality, the monopolist offers a pooling menu.[23] This case can be observed for example when marginal costs of quality are constant. In light of this analysis, we can extend our findings to the case where quality costs do not increase too fast, thereby concluding that, under this condition, offering the top quality bundle is still the most profitable choice. Indeed, if a separating menu (namely different qualities to different consumers) is not observed at equilibrium, the effects inducing the incumbent to allow or prevent the rival from offering the complementary variant are still at work, so that our qualitative findings do not change.[24]

Admittedly, it is not clear whether the equilibrium we find is robust to the alternative case when the cost function is highly convex in quality. We know from Acharyya (1998) that in this alternative scenario, there are circumstances such that the incentive to a separating menu may dominate that to a pooling menu. Thus, in our model one should contemplate the possibility that it is profitable for the incumbent to offer both the base good and the high quality bundle at equilibrium. Still, such a type of strategy would allow the rival to enter the market and to offer the low quality complementary variant. In particular, with the two producers being active in the market, two main effects would be observed. On the one hand, the incumbent would be penalized by a competition effect as the market would move from a monopoly to a duopoly. On the other hand, there could be a positive effect.[25] More precisely, when the average consumers' willingness to pay for quality, b, is sufficiently high with respect to the quality costs, with the low quality system on sale, the incumbent could at least partially appropriate of the benefit deriving from those low-type consumers that now buy this low quality system rather than the base good alone. Of course, this positive effect can only take place when the price of the base good is set sufficiently high that no consumer buys the base good alone.[26] More precisely, as no price discrimination is allowed, in order to serve both consumers buying the base good alone and those buying the low quality bundle, the incumbent should quote a very low base good price. Still, at this price, it could not take advantage of the increased willingness to pay measured by $(u_E - \bar{u}) > 0$. So, when the consumers' average willingness to pay is relatively high, serving at a

[23] More precisely, he shows that the extent of market coverage under a pooling menu depends on the distribution of consumers across different types.

[24] Rather interestingly, a pooling menu can also be observed when there are economies of scope in production. Kim and Kim (1996) consider a vertically differentiated market where a monopolist facing a discrete distribution of consumers can reduce total costs of production by jointly producing two products. In this circumstance, they prove that there exist conditions on cost relatedness such that the monopolist finds it profitable a separating menu. Still, a pooling menu can arise at equilibrium under other conditions.

[25] See Appendix 9.3.2 for details.

[26] As shown in Appendix 9.3.2 at the separate ownership equilibrium the demand for the base good alone is zero for b sufficiently high.

high price only consumers willing to buy the low quality system is by far more profitable than serving even those willing to buy the base good alone at lower price. On the contrary, when b is relatively low with respect to the quality costs so that the profits from the high quality system are *ceteris paribus* less significant (or nil for very low values of b),[27] having the low quality complementary variant on sale allows the incumbent to sell the base good to a larger share of the market with a positive effect on profits.

It follows that the market structure under separate ownership depends on the balance between the above effects. When the negative competition effect overcompensates the positive effect, entry is blockaded. Otherwise, namely when the positive effect prevails over the negative, there is room in the market for the low quality producer.[28]

Consider now the integration scenario. The acquisition price depends on the low quality firm's outside option. If the monopolist has incentive to blockade entry, the acquisition price is nil and we can reasonably conclude that acquisition takes place. In this case, as far as the quality menu choice is concerned, in line with the existing literature, we can show that the monopolist has incentive to quality discriminate. In particular, when b is sufficiently high, the monopolist offers both the base good alone and the two systems. On the contrary, for low values of b, the high quality system is not put on sale.[29] If instead the monopolist has incentive to allow entry, the acquisition price turns out to be positive. In this case, if acquisition is profitable compared to the outside option, we are again in the integration scenario when the monopolist can quality discriminate.

Notice that, such a type of setting would entail different welfare implications compared to the original setting with zero costs and the monopolist producing the low quality variant. Indeed, with positive and sufficiently convex production costs, under integration one can observe the whole set of variants on sale at equilibrium. This selling choice departs from that observed when production costs are nil: under integration only the high quality system is on sale when costs do not play any role. So, moving from the assumption of zero costs to that of highly convex quality costs with the incumbent being the producer of the high quality system, determines, under integration, a wider set of variants on sale, with a lower average quality level. Rather interestingly, with such a type of quality-specific production costs, integration expands the set of variants in the market also with respect to both the alternative scenarios of separate ownership and blockaded entry. Accordingly, while the rationale behind the *vertical* and *horizontal effects* observed in Economides and Salop (1992) still holds, that underlying both the *quality effect* and *the composition effect*, evoked in the introduction, can change in this new framework, their extent and direction being not clear-cut *a priori*. Finally, not even an unambiguous conclusion on the welfare properties of integration can be reached when evaluating the *price effect*. Indeed, the optimal equilibrium prices are now depending not only on

[27] As shown in Appendix 9.3.2 at the separate ownership equilibrium the demand for the high quality system is zero for b sufficiently low.

[28] In Appendix 9.3.1 we show that there are cases in which blocking entry is unprofitable.

[29] See Appendix 9.3.3 for further details.

the competition between firms or cannibalization phenomenon in the case of integration, but also on the quality-related costs incurred by each firm when producing its own quality.

7 Some policy implications

The result stated in Proposition 5 represents something to chew on by competition agencies when evaluating proposed mergers. Indeed, it shows that in vertically differentiated markets, the traditional welfare enhancing effect (taking place when the double marginalization is removed) can be magnified by a positive quality effect. Further, as the negative competition effect, if any, can play a minor role when consumers are very heterogeneous, it may well happen that, even when enhancing a monopoly structure, integration can benefit consumers. We have considered consumer surplus as a measure of welfare, showing that firms' and consumers' interests can be diverging in some circumstances. It follows that, if a competition authority would refer to consumer surplus in order to decide whether preventing firms from integration, this latter would be banned in several cases, namely whenever integration would harm consumers. This is no longer true when identifying the effects of integration based on the social welfare (consumer surplus plus producer surplus). Indeed, integration is found to be always social welfare improving.[30] This is due to the fact that the firms' profits under integration are so high to neutralize the possible damage to consumers. Accordingly, there are circumstances such that integration could be banned or rather allowed depending on which welfare measure would be taken into account!

One could question our arguments while saying that the above considerations hold under the assumption that the monopolist can only offer a low quality system when preventing the rival to enter the market. Still, we have discussed at length the case when the reverse holds, namely the monopolist in the base market can also produce a high quality variant in the complementary market. It is interesting to notice that, even under the alternative scenario, there exist circumstances such that integration is welfare improving at least from consumers' viewpoint. No doubt, in order to draw further insights into the consumers' versus social welfare properties of this case, an in-depth formal analysis would be necessary. Still, our preliminary results can be intended as a natural entry point for further research as they show that, from a competition policy perspective, integration can be allowed on a welfare ground whatever the specific skills of firms to the extent they are involved in complementary productions.

8 Conclusion

This paper sheds light on the effects of joint ownership of asymmetric complements when they are vertically differentiated. Although our model is highly

[30] Formal details are in Appendix, Section 9.2.

stylized and introduces some specific assumptions on the cost functions and the distribution of consumers, it provides strong arguments for the positive nature of network integration among firms in vertically differentiated markets, while showing at the same time that, under some circumstances, anti-competitive consequences can be observed. Although our arguments depend on the assumptions of the model, they can represent a starting valuable point for competition agencies when considering the joint ownership in vertically differentiated markets.

Two possible extensions are worth discussing. First of all, our findings were proved to hold under the assumption that *price discrimination* is not allowed. There are at least three main justifications for this assumption. First, competition authorities could show hostile attitude to price discrimination practices because a dominant firm may "exploit" final consumers by means of price discrimination, with negative effects on consumers' welfare. Second, and especially in Europe, it is sometimes a policy objective to attain a "single market" across the region. Arguably, one manifestation of a single market is that a firm does not set different prices in different regions. Third, price discrimination can be used by a dominant firm to "exclude" (or weaken) actual or potential rivals from the market, thereby preventing them from entering or forcing to exit. Of course, there exist circumstances such that price discrimination allows to extend market coverage with possibly a positive effect on welfare, ceteris paribus. In this case, the authorities should put in balance this latter welfare-enhancing effect with the above evoked forces leading to a less competitive market structure. When allowing for price discrimination in our model, the incumbent could quote different prices for different consumers, depending on their specific willingness to pay. Accordingly, under the separate ownership scenario he could find it profitable to serve some low-type consumers willing to buy only the base good which would be otherwise excluded from the market. This price strategy could increase the profitability of the separate ownership scenario compared with the acquisition choice and possibly, depending on the value of parameters, change the market structure observed at equilibrium.

The second remark concerns our choice of the vertical differentiation model compared with a *horizontally differentiated model*. Our interest has been in studying the economic incentives for an incumbent producing both a base good and a complementary variant to compete or integrate with a rival in the complementary market under the assumption of different competencies in producing the same (complementary) good. Thus, vertical differentiation in the complementary market allows us to perfectly suit this scenario. Nevertheless, casual observation points out the presence of more than one complementary (imperfectly substitutable) good. Thus an interesting extension would be to study the potential introduction of many complementary goods. Horizontal differentiation with consumers' love for variety would then allow us to describe such a framework where reasonably the value of the base good would increase not only with the quality of a complementary component but also with the available number of these components. Also, under an alternative approach to horizontal differentiation, one could consider that a complementary variant could be preferred over the other, depending on consumers' sex, educational level and so on. Take the example of video-game market: there exist video-games mainly suited for male consumers, while others better meet the requirements coming from a young female audience. The wider the range of goods provided by a firm, the larger the market share this firm can a priori satisfy. Such a type of setting would allow us to study the incentives for the monopolist to integrate, depending on the rival's production line and consumers' features.

While these new elements go beyond the aim of this paper, they open the door to further research.

9 Appendix

9.1 Separate ownership equilibrium candidates

In the separate ownership scenario, each consumer can either buy only the base good at price p_u and get utility $\theta\bar{u}-p_u$; or buy the base good equipped with the low quality (resp. high quality) complementary variant at some price P_I (resp. p_u+r_E, where r_E is the price of v_E sold by the low-quality producer) in which case the utility is $\theta u_I - P_I$ (resp. $\theta u_E - p_u - r_E$); or refrain from buying. Different market configurations can arise at the price equilibrium. In order to find the Nash equilibrium in the price subgame, we proceed in two steps. First, we compute equilibrium candidates for each possible market configuration; second, we identify the parameters constellation for which candidates effectively yield the corresponding market outcome.

Let $\bar{\theta}$ be the lowest consumer willing to buy something, then $\bar{\theta}=\min\left\{\frac{p_u}{\bar{u}},\frac{P_I}{u_I},\frac{p_u+r_E}{u_E}\right\}$. In the following, we first prove that at equilibrium, for any value of the parameters, no consumer buys the base good alone (H1), and we find the equilibrium market configurations for $b\leq\bar{b}$ (H2). This is the proof of Lemma 1. We then find the equilibrium market configurations for $b>\bar{b}$ (H3): this is the proof of Lemma 2.

9.1.1 Proof of Lemma 1

H1 First, we assume that $\bar{\theta}=\frac{p_u}{\bar{u}}=\min\left\{\frac{p_u}{\bar{u}},\frac{P_I}{u_I},\frac{p_u+r_E}{u_E}\right\}$, namely there are at least some consumers buying only the base good. Then, one can observe either

$$\frac{P_I}{u_I}<\frac{p_u+r_E}{u_E}\text{ or }\frac{p_u+r_E}{u_E}<\frac{P_I}{u_I}.$$

a We start assuming that $\frac{p_u}{\bar{u}}<\frac{P_I}{u_I}<\frac{p_u+r_E}{u_E}$ namely $\widetilde{\theta}=\frac{P_I-p_u}{u_I-\bar{u}}>\frac{p_u}{\bar{u}};\breve{\theta}=\frac{p_u+r_E-P_I}{u_E-u_I}>\frac{P_I-p_u}{u_I-\bar{u}}$. So, profits accruing to the monopolist are:

$$\Pi_I=(\widetilde{\theta}-\bar{\theta})p_u+(\breve{\theta}-\widetilde{\theta})P_I+(b-\breve{\theta})p_u$$

while the high-quality producer's profits write as:

$$\Pi_E=(b-\breve{\theta})r_E.$$

Price competition leads to the following equilibrium prices:

$$\begin{aligned} P_I &= \frac{b\left(2u_I\left(u_E-u_I\right)+\bar{u}\left(2u_E+u_I-3\bar{u}\right)\right)}{8u_E-6\bar{u}-2u_I} \\ r_E &= \frac{2b\left(u_E-\bar{u}\right)\left(u_E-u_I\right)}{4u_E-3\bar{u}-u_I} \\ p_u &= \frac{1}{2}b\bar{u}. \end{aligned}$$

At this candidate equilibrium, $\bar{\theta}-\widetilde{\theta}=\frac{1}{2}\frac{(u_I-3\overline{u}+2u_E)b}{(4u_E-u_I-3u)}>0$ which is in contrast with the initial assumption. So, we conclude that this candidate is not the actual equilibrium outcome.

b Then, let us assume that $\frac{p_u}{\overline{u}}<\frac{p_u+r_E}{u_E}<\frac{P_I}{u_I}$. In this case, the consumer indifferent between buying the base good and buying the high quality system is defined as $\hat{\theta}=\frac{r_E}{u_E-\overline{u}}$. So profit functions for the incumbent and the high-quality producer write, respectively, as

$$\begin{aligned} \Pi_I &= (b-\bar{\theta})p_u, \\ \Pi_E &= (b-\hat{\theta})r_E. \end{aligned}$$

From the FOCs, we can show that the candidate equilibrium prices of the high quality component and the base good, are respectively, $r_E'=\frac{1}{2}b\left(u_E-\overline{u}\right)$ and $p_u'=\frac{1}{2}b\overline{u}$; and candidate equilibrium profits of the incumbent and the high-quality producer are: $\Pi_I=\frac{1}{4}b^2\overline{u}$ and $\Pi_E=\frac{1}{4}b^2\left(u_E-\overline{u}\right)$. Notice however that, at these prices $\hat{\theta}=\bar{\theta}=\frac{1}{2}b$. Accordingly, it derives that those consumers buying the base good are willing to buy the high quality component as well. That is to say that at these candidate equilibrium prices the market configuration with some consumers buying only the base good never arises. So, we can conclude that this candidate is not an equilibrium outcome.

H2 Then, we move to the following assumption: $\bar{\theta}=\frac{P_I}{u_I}=\min\left\{\frac{p_u}{\overline{u}},\frac{P_I}{u_I},\frac{p_u+r_E}{u_E}\right\}$. Also, let $\breve{\theta}=\frac{p_u+r_E-P_I}{u_E-u_I}$, where *a priori* $\breve{\theta}\lesseqqgtr b$.[31]

a Let us assume first that $\breve{\theta}<b$. It follows that:

$$\Pi_I=(\breve{\theta}-\bar{\theta})P_I+(b-\breve{\theta})p_u$$
$$\Pi_E=(b-\breve{\theta})r_E.$$

[31] In this case, it does not matter whether $\frac{p_u}{u}>\frac{p_u+r_E}{u_E}$ or $\frac{p_u}{u}<\frac{p_u+r_E}{u_E}$ as in both cases the utility from buying the base good alone is always dominated by the utility of buying either the low or the high quality system. That is no consumer is willing to buy the base good alone.

Price competition leads to the following candidate equilibrium:

$$\begin{aligned} p_u &= \frac{1}{6}u_I + \frac{1}{3}u_E, P_I = \frac{1}{2}bu_I, r_E = \frac{1}{3}b\left(u_E - u_I\right); \\ \Pi_I^{so} &= \frac{1}{36}b^2\left(5u_I + 4u_E\right), \Pi_E^{so} = \frac{1}{9}\left(u_E - u_I\right)b^2; \\ CS^{so} &= \frac{4u_E^2(2b-1)^2 + u_I u_E (b-2)^2 + u_I^2\left(2b - 8b^2 + 1\right)}{72(u_E - u_I)}. \end{aligned}$$

At this candidate equilibrium we have *(i)* $\frac{P_I}{u_I} < \frac{p_u}{\overline{u}} \iff b < \widehat{b} = \frac{1+2u_G}{3\overline{u}}$ and *(ii)* $\frac{P_I}{u_I} < \frac{p_u + r_E}{u_E} \iff b < \overline{b} = \frac{2u_G+1}{(2+u_G)}$. As $\overline{b} < \widehat{b}$, then for $b < \overline{b}$, the assumption that $\frac{P_I}{u_I} = \min\left\{\frac{p_u}{\overline{u}}, \frac{P_I}{u_I}, \frac{p_u+r_E}{u_E}\right\}$ is satisfied. Furthermore, one gets that $b > \breve{\theta} \iff b > \underline{b} = \frac{2u_G+1}{4u_G-1}$. So, one can conclude that for any $b \in \left[\underline{b}, \overline{b}\right]$, the above candidate equilibrium is an equilibrium, with positive market shares for both the high and the low quality systems.

b Let us assume now that $\breve{\theta} \geq b$. We know from the above that this assumption is met whenever $b \leq \underline{b} = \frac{2u_G+1}{(4u_G-1)}$. In this case, no consumer is willing to buy the high quality system, and profit functions for the incumbent and the high-quality producer write as

$$\begin{aligned} \Pi_I^{so,I}(P_I, p_u, r_E) &= (b - \bar{\theta})P_I, \\ \Pi_E^{so,I}(P_I, p_u, r_E) &= 0. \end{aligned}$$

From price competition, one immediately gets the equilibrium profit for the incumbent

$$\Pi_I^{so,I} = \frac{1}{4}u_I b^2.$$

So, one can conclude that for any $b \leq \underline{b}$, the high-quality producer does not face any demand as $b \leq \breve{\theta}$, and $\Pi_I^{so,I} = \Pi_I^m$.

9.1.2 Proof of Lemma 2

H3 Let us finally assume that $\bar{\theta} = \frac{p_u + r_E}{u_E} = \min\left\{\frac{p_u}{u}, \frac{P_I}{u_I}, \frac{p_u+r_E}{u_E}\right\}$.[32] In this market configuration all consumers buy the base good bundled with the high quality complementary variant. From H2, where we prove that the equilibrium is *so* for $b \in \left[\underline{b}, \overline{b}\right]$ and it is so, I for $b \leq \underline{b}$, it follows that this is possible iff $b > \overline{b}$. Then, profit functions write as

$$\begin{aligned} \Pi_I &= (b - \bar{\theta})p_u, \\ \Pi_E &= (b - \bar{\theta})r_E. \end{aligned}$$

It is easy to show that the candidate equilibrium prices are $p_u^{so,E} = \frac{1}{3}bu_E$ and $r_E^{so,E} = \frac{1}{3}bu_E$, so that at equilibrium $\bar{\theta} < b$ and $\Pi_i^{so,E} = \frac{1}{9}b^2 u_E$,

[32] As in case H2, it does not matter whether $\frac{p_u}{u} > \frac{P_I}{u_I}$ or $\frac{p_u}{u} < \frac{P_I}{u_I}$.

with $i = I, E$. At this candidate equilibrium, it emerges that $\frac{p_u + r_E}{u_E} < \frac{p_u}{\overline{u}} \iff (u_E - 2\overline{u}) > 0$. So, one can conclude that whenever $b > \overline{b}$ this candidate equilibrium is an equilibrium iff $u_E > 2\overline{u}$. It remains to study the equilibrium configuration in the case when $u_E < 2\overline{u}$. As in this range of parameters, $\frac{p_u + r_E}{u_E} > \frac{p_u}{\overline{u}}$, then the analysis in H1b applies. At the candidate equilibrium H1b, the assumption in H3 that all active consumers buy the high quality complementary system, namely $\frac{p_u + r_E}{u_E} = \min\left\{\frac{p_u}{\overline{u}}, \frac{P_I}{u_I}, \frac{p_u + r_E}{u_E}\right\}$ is satisfied in the limit as $\frac{p_u + r_E}{u_E} = \frac{p_u}{\overline{u}}$. Thus, we conclude that the candidate equilibrium H1b is an equilibrium in the range of parameters $b > \overline{b}$ and $u_E < 2\overline{u}$.

9.2 Social welfare

Social welfare in each equilibrium scenario is:

$$SW^{so} = \frac{\left(4u_I u_E - 4bu_I u_E + u_I^2 + 4u_E^2 + 2bu_I^2 - 16bu_E^2 - 13b^2 u_I u_E - 10b^2 u_I^2 + 32b^2 u_E^2\right)}{72\left(u_E - u_I\right)}$$

$$SW^{so,I} = SW^m = \frac{3}{8} u_I b^2$$

$$SW^{so,E} = \frac{5}{18} u_E b^2$$

$$SW^{int} = \frac{3}{8} u_E b^2$$

As to verify whether integration dominates the alternative from a social welfare viewpoint, we proceed as in Section 5. Thus, we first recall that for $b < \underline{b}$, or $b > \overline{b}$ and $u_G < \frac{9}{4}$, the outside option is monopoly; while it is separate ownership for $b \in \left[\underline{b}, \overline{b}\right]$ or $b > \overline{b}$ and $u_G > \frac{9}{4}$. It is immediate to see that SW under integration $\left(SW^{int} = \frac{3}{8} u_E b^2\right)$ is higher than that under monopoly $\left(SW^m = \frac{3}{8} u_I b^2\right)$ in the range of $b-$parameters such that monopoly is the outside option.

Rather, in the case when the outside option is separate ownership, comparing the equilibrium SW under separate ownership *versus* integration, we find that, for $b \in \left[\underline{b}, \overline{b}\right]$:

$$SW^{so} - SW^{int} = \frac{(u_I + 2u_E)^2 - 2b(4u_E - u_I)(u_I + 2u_E) + b^2\left(14u_I u_E - 10u_I^2 + 5u_E^2\right)}{72(u_E - u_I)}$$

The above difference is negative, $SW^{so} - SW^{int} < 0$ as the two b-roots are outside the admissible interval ($b \in \left[\underline{b}, \overline{b}\right]$). In the case when $b > \overline{b}$ and $u_G > \frac{9}{4}$, then

$$SW^{so,E} - SW^{int} = -\frac{7}{72} b^2 u_E < 0 \; .$$

Integration is thus always socially optimal.

9.3 Reversed quality

Assume that the incumbent monopolist produces the high quality complementary variant whereas the rival produces the low quality variant, i.e. $v_I > v_E$, so that $u_I > u_E > \overline{u}$. Also, producing these variants is costly, in particular the unit cost is convex in quality.

9.3.1 Entry is blockaded

Whenever the incumbent sells only the high quality system u_I, entry is blockaded. In this case, the profit function for firm I writes as:

$$\Pi_I = \left(P_I - \frac{u_I^2}{2}\right)\left(b - \frac{P_I}{u_I}\right)$$

Then, price P_H^*, demand D_H^* and profit Π_H^* at equilibrium are

$$\begin{aligned} P_I^* &= \frac{u_I\left(2b+u_I\right)}{4} \\ D_I^* &= \frac{\left(2b-u_I\right)}{4} > 0 \iff b > \frac{u_I}{2} \\ \Pi_I^* &= \frac{1}{16}\left(u_I-2b\right)^2 u_I \end{aligned}$$

Note that $D_I^* > 0 \iff b > \frac{u_I}{2} \equiv b_M$: this proves that this strategy may not be profitable (namely, if $b < b_M$ it is not profitable to produce only the high quality system). This opens the door to the possibility to observe at equilibrium either separate ownership or integration at some positive acquisition price.

9.3.2 Separate ownership

In this scenario, the high quality system is entirely produced by the incumbent, the low quality system is produced by both firms, the incumbent providing the base good, while the rival the low quality variant. In order to capture this asymmetry, we assume that firm I incurs a cost equal to $\frac{u_I^2}{2}$ when providing the high quality system, while the cost for firm E when producing the low quality variant is $c\frac{u_E^2}{2}$, with $0 < c \leq 1$.[33]

Let us write the incumbent's and the rival's maximization problems in the separate ownership scenario:

$$\max_{p_u, P_H} \left((\widetilde{\theta} - \bar{\theta})(p_u - \frac{\overline{u}^2}{2}) + (\breve{\theta} - \widetilde{\theta})(p_u - \frac{\overline{u}^2}{2}) + (b - \breve{\theta})(P_I - \frac{u_I^2}{2}) \right)$$

$$\max_{r_L} \left((\breve{\theta} - \widetilde{\theta})(r_E - c\frac{u_E^2}{2}) \right)$$

[33] One could question this assumption on cost asymmetry. However, the results we provide in this section do not qualitatively change when removing this assumption, thereby setting $c = 1$.

where $\overline{\theta} = \frac{p_u}{u}$, $\widetilde{\theta} = \frac{r_E + p_u - p_u}{u_L - u}$ and $\breve{\theta} = \frac{P_I - (r_E + p_u)}{u_I - u_E}$. P_I and r_E are the prices of the high quality system and the low quality complementary variant, respectively. From the F.O.C.s, we derive the following equilibrium prices:

$$P_I = \frac{4\overline{u}u_I^2 - 4u_I^3 - \overline{u}^3 + \overline{u}^2 u_E + b(-2)\left(3\overline{u}u_E - 4u_I u_E - 3\overline{u}^2 + 4u_I^2\right) - 2cu_E^2(u_I - \overline{u})}{4(3\overline{u} - 4u_I + u_E)}$$

$$r_E = \frac{(u_I - \overline{u})\left(\overline{u}u_E - \overline{u}u_I + u_I u_E - \overline{u}^2 + 2cu_E^2\right) + b2(u_E - \overline{u})(u_I - u_E)}{8u_I - 6\overline{u} - 2u_E}$$

$$p_u = \frac{1}{4}\overline{u}\left(2b + \overline{u}\right).$$

Notice that, the demand for the base good alone D_u at the equilibrium prices writes as

$$D_u = \frac{(u_E - \overline{u})\overline{u}(\overline{u} + u_E - 2u_I) + 2(u_I - \overline{u})\left(u_I(u_E - \overline{u}) + 2cu_E^2\right) + b(-2)(u_E - \overline{u})(2u_I + u_E - 3\overline{u})}{4(4u_I - 3\overline{u} - u_E)(u_E - \overline{u})}.$$

Let us denote by $\breve{b}$

$$\breve{b} = \frac{\left(4u^2 u_I - \overline{u}^3 - 2\overline{u}u_I^2 - 4\overline{u}u_I u_E + \overline{u}u_E^2 - 4c\overline{u}u_E^2 + 2u_I^2 u_E + 4cu_I u_E^2\right)}{2(2u_I - 3\overline{u} + u_E)(u_E - \overline{u})}$$

the value of b such that $D_u(\breve{b}) = 0$ and $\frac{dD_u}{db} < 0$. Accordingly, for any $b \geqslant \breve{b}$, the base good alone is not bought or $D_u = 0$, while the reverse holds for $b < \breve{b}$. The demand for the high quality system D_H at equilibrium prices writes as

$$D_I = \frac{\left(4bu_E - 4bu_I + \overline{u}u_I - \overline{u}u_E - u_I u_E - \overline{u}^2 + 2u_I^2 - cu_E^2\right)}{2(3\overline{u} - 4u_I + u_E)}.$$

Denoting by $\ddot{b}$ the value of b such that $D_H(\ddot{b}) = 0$,

$$\ddot{b} = \frac{\left(\overline{u}u_E - \overline{u}u_I + \overline{u}_I u_E + \overline{u}^2 - 2u_I^2 + cu_E^2\right)}{4(u_E - u_I)}$$

with $\frac{dD_I}{db} > 0$, we can conclude that for any $b \geqslant \ddot{b}$, the high quality system is bought or $D_I > 0$, while the reverse holds, namely $D_I = 0$ for $b < \ddot{b}$. Finally, the demand for the low quality system is

$$D_E = \frac{\left(2b\overline{u}u_I - 2b\overline{u}u_E - 2bu_I u_E - \overline{u}^3 + 2bu_E^2 - cu_E^3 + \overline{u}u_I^2 + \overline{u}^2 u_E - c\overline{u}u_E^2 - u_I^2 u_E + 2cu_I u_E^2\right)(u_I - \overline{u})}{2(4u_I - 3\overline{u} - u_E)(u_E - u_I)(u_E - \overline{u})}$$

with $D_E > 0 \iff c < \frac{(u_E - \overline{u})((u_I - \overline{u})(u + u_I) + b2(u_I - u_E))}{u_L^2(2u_I - \overline{u} - u_E)} \equiv \widehat{\widehat{c}}$.

When comparing these b thresholds, we find:

$$\breve{b} - \ddot{b} = \frac{\left(\overline{u}^2 u_I - cu_E^3 - \overline{u}^3 + \overline{u}u_E^2 - c\overline{u}u_E^2 - u_I u_E^2 + 2cu_I u_E^2\right)(4u_I - 3\overline{u} - u_E)}{4(2u_I - 3\overline{u} + u_E)(u_I - u_E)(u_E - \overline{u})}$$

$> 0 \iff c > \widehat{c} \equiv \frac{(\overline{u} - u_E)(u_I - \overline{u})(\overline{u} + u_E)}{(\overline{u} - 2u_I + u_E)u_E^2} \in (0,1) < \widehat{\widehat{c}}$.

Therefore, in the range where $b < \frac{u_H}{2} \equiv b^M$, the incumbent allows entry and, under separate ownership $D_u \geq 0 \iff b \leq \breve{b}$ and $D_H \geq 0 \iff b \geq \ddot{b}$.

As for the other b thresholds:

$$\check{b}-b^{M} = \frac{\left(\overline{u}^{2}-u_{E}^{2}+4cu_{E}^{2}\right)\left(u_{I}-\overline{u}\right)}{2\left(2u_{I}-3\overline{u}+u_{E}\right)\left(u_{E}-\overline{u}\right)}>0\text{, always if }c>\frac{1}{4};$$

$$\ddot{b}-b^{M} = \frac{\left(\overline{u}u_{E}-\overline{u}u_{I}-u_{I}u_{E}+\overline{u}^{2}+cu_{E}^{2}\right)}{4\left(u_{E}-u_{I}\right)}>0$$

Therefore, if $c\in\left(\frac{1}{4},1\right)$, in the range of $b<b^{M}$, we observe that $D_u>0$, $D_H=0$ and $D_L>0$.[34]

On the contrary, in the range of b, where $b>b^{M}$, blocking entry gives the incumbent positive profits. However, one cannot exclude that entry would be more profitable than blocking entry even in this range of $b-$parameters. In order to conclude that blocking entry is the optimal strategy, one needs to verify that monopoly profits $\Pi_{I}^{*}=\frac{1}{16}\left(u_{I}-2b\right)^{2}u_{I}$ are higher than the profits it could earn when allowing entry under separate ownership. The incumbent profit under separate ownership depends on b as well as on c. We next provide an example where the incumbent prefers to blockade entry. Assume $c<\widehat{c}<\widehat{\widehat{c}}$, then

$$\check{b}<\ddot{b}$$

For $b\in\left(\max\{b_M,\check{b}\},\ddot{b}\right)$, we observe $D_u=0$, $D_E>0$ and $D_I=0$. Then the incumbent's profit writes as:[35]

$$\frac{\left(2b\overline{u}u_{I}-2b\overline{u}u_{E}-2bu_{I}u_{E}-\overline{u}^{3}+2bu_{E}^{2}-cu_{E}^{3}+\overline{u}u_{I}^{2}+\overline{u}^{2}u_{E}-c\overline{u}u_{E}^{2}-u_{I}^{2}u_{E}+2cu_{I}u_{E}^{2}\right)\left(u_{I}-\overline{u}\right)}{2\left(3\overline{u}-4u_{I}+u_{E}\right)\left(u_{I}-u_{E}\right)\left(u_{E}-\overline{u}\right)}\left(\frac{1}{4}\overline{u}\left(2b-\overline{u}\right)\right).$$

This profit is decreasing in c. Comparing this profit with $\Pi_{I}^{*}=\frac{1}{16}\left(u_{I}-2b\right)^{2}u_{I}$, we find that blocking entry is profitable iff $c>\widetilde{c}$, where

$$\widetilde{c}\equiv-\left(u_{E}-\overline{u}\right)\frac{A-4bB+4\left(u_{E}-u_{I}\right)b^{2}C}{2\left(2b-\overline{u}\right)\left(2u_{I}-\overline{u}-u_{E}\right)\left(u_{I}-\overline{u}\right)u_{E}^{2}\overline{u}}$$

with $A=2\overline{u}^{5}+4u_{I}^{5}-3\overline{u}u_{I}^{4}-2\overline{u}^{4}u_{I}-5u_{I}^{4}u_{E}+3\overline{u}u_{I}^{3}u_{E}+2\overline{u}^{2}u_{I}^{3}-2\overline{u}^{3}u_{I}^{2}+u_{I}^{3}u_{E}^{2}$, $B=\left(\overline{u}^{4}+4u_{I}^{4}-2\overline{u}u_{I}^{3}-\overline{u}^{3}u_{E}-5u_{I}^{3}u_{E}+3\overline{u}u_{I}^{2}u_{E}+\overline{u}^{2}u_{I}u_{E}-2\overline{u}^{2}u_{I}^{2}+u_{I}^{2}u_{E}^{2}\right)$ and $C=\left(5\overline{u}u_{I}+u_{I}u_{E}-2\overline{u}^{2}-4u_{I}^{2}\right)$. Therefore, we conclude that for $c>\widetilde{c}$, entry is blockaded. Note that the sign of $\widetilde{c}-\widehat{c}$ is the sign of the following polynomial: $\left(u_{I}^{3}\left(3\overline{u}-4u_{I}+u_{E}\right)-2\overline{u}^{2}\left(u_{I}-\overline{u}\right)^{2}\right)+b^{2}4\left(\left(\overline{u}-u_{I}\right)\left(4u_{I}-\overline{u}\right)+u_{I}u_{E}-\overline{u}^{2}\right)+b\left(-4\right)\left(2\overline{u}\left(u_{I}-\overline{u}\right)\left(\overline{u}+u_{I}\right)+u_{I}\left(u_{I}u_{E}+3\overline{u}^{2}-4u_{I}^{2}\right)\right)$. So, there are parameter values such that $\widetilde{c}<\widehat{c}$. In the range of $c-$parameters where $\widehat{c}>c>\widetilde{c}$, blocking entry is profitable.[36]

[34] $\ddot{b}-b^{M}\iff c<\frac{\left(u_{I}-\overline{u}\right)\left(\overline{u}+u_{E}\right)}{u_{E}^{2}}$, where this threshold is larger than 1.

[35] Note that $p_u-\frac{\overline{u}^{2}}{2}>0$, always as we are in $b>b_M>\frac{\overline{u}}{2}$.

[36] One can find numerical examples to prove that this condition holds for some parameter values.

9.3.3 Integration scenario

We here assume that $c = 1$ as the integrated firm produces the entire low quality system. The maximization problem writes as:

$$\max_{p_u, P_I, P_E} \left((\widetilde{\theta} - \bar{\theta})(p_u - \frac{\overline{u}^2}{2}) + (\breve{\theta} - \widetilde{\theta})(P_I - \frac{u_E^2}{2}) + (b - \breve{\theta})(P_I - \frac{u_I^2}{2}) \right)$$

with $\bar{\theta} = \frac{p_u}{u}$, $\widetilde{\theta} = \frac{P_E - p_u}{u_E - \overline{u}}$ and $\breve{\theta} = \frac{P_I - P_E}{u_I - u_E}$. Price competition implies:

$$\begin{aligned} P_I &= \frac{1}{4} u_I \left(2b + u_I\right) \\ P_E &= \frac{1}{4} u_E \left(2b + u_E\right) \\ p_u &= \frac{1}{4} \overline{u} \left(2b + \overline{u}\right). \end{aligned}$$

At these prices, demand for the high quality system at equilibrium is:

$$D_I^I = \frac{(2b - u_I - u_E)}{4}.$$

Notice that, while demand for the base good ($D_u^I = \frac{1}{4} u_E$) and that for the low quality system ($D_E^I = \frac{1}{4}\left(u_I - \overline{u}\right)$) turn out to be always positive, $D_I^I > 0$ iff $b > \frac{(u_I - u_E)}{2}$.

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