ECONOMETRIC TESTS OF THE EXPECTATIONS THEORY OF THE TERM STRUCTURE IN NEW ZEALAND

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This paper attempts to account for the empirical failure of the expectations theory of the term structure when it is tested using a variety of methods based on single-equation and vector autoregressive (VAR) models. It is argued that the failure of the spread to forecast future short-term rate changes is due to the omission from the regression of a time-varying term premium that is correlated with the spread. The inclusion of a white-noise error term in spread regressions is thought to take account of any random components in the term premium, and thus enable better judgement to be made about the expectations hypothesis of the term structure.

This investigation finds there is strong empirical support for the *long-run* implications of the expectations theory. However, the empirical evidence does not support the *short-run* predictions of the expectations theory when these predictions are tested by imposing restrictions on the parameters of single-equation and VAR models. These results are inconsistent with the view that the inclusion of a white-noise error term in spread regressions is enough to reconcile the expectations theory with the data.

Key words: expectations theory, term structure of interest rates, econometric tests of rationality, term premium.

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Chapter One

Introduction

1.1 Background

A traditional expression of the expectations hypothesis of the term structure of interest rates (EHTS), relates to the approximate equilibrium relationship between long-term and short-term yields[1]. Often this relationship is augmented by considerations about risk, alleging that the yields on all bonds are equal up to a constant term premium. Under this representation the yield on a longer-term n-period bond is a constant, plus an arithmetic average of yields on current and expected future shorter-term m-period bonds up to $n - m$ periods in the future (Campbell and Shiller, 1991). While some recent cointegration studies appear to support the *long-run* validity of the expectations hypothesis (see Campbell and Shiller, 1987; McDonald and Speight, 1991; Hall, Anderson and Granger, 1992; Shea, 1992), it is a widely documented fact that almost all empirical studies reject the *short-run* predictions of the expectations theory (Shiller, 1990; Campbell and Shiller, 1991).

There have been a number of explanations offered for the empirical failure of the expectations hypothesis. According to Mankiw and Miron (1986), Hardouvelis (1988) and Roberds, Runkle and Whiteman (1996), the failure may be related to the possible role of monetary policy. In the USA, adherents of this view argue that the persistence of changes in the Federal funds rate (i.e. the instrument used by the Federal Reserve to carry out its policy decisions), can help explain why the yield spread has had negligible forecasting power. Another alternative suggested by Mankiw and Summers (1984), Mankiw and Miron (1986), Hardouvelis (1988), Simon (1989), Tzavalis and Wickens (1997) and Driffill, Psaradakis and Sola (1997) is that the failure of the spread to forecast future short-

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[1] Since this analysis is undertaken using pure discount bonds the yields in question are spot yields. Pure discount bonds (i.e. Treasury bills) have no coupon payments and their redemption price is fixed and known at the time of issue. The return earned over the life of the bill is therefore the difference between the issue price and redemption price (expressed as a percent) and is known as the spot yield or spot rate (Cuthbertson, 1996).

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term rates is due to the omission from the regression of a time-varying term premium that is correlated with the spread.

In light of these possibilities, this paper examines whether the inclusion of a time-varying term premium alone can rehabilitate the EHTS. As with Driffill, Psaradakis and Sola (1997), the inclusion of a white noise-error term on the long-term rate of interest is thought to take account of any random components in the term premium. By including this proxy in spread regressions, any bias due to omitting the term premium should be reduced. This would enable better judgement to be made about the validity of the EHTS.

1.2 Problem statement and research objectives

The purpose of this paper is to determine if the New Zealand term structure of interest rates is consistent with a weaker form of the expectations theory for the 1990 to 1997 period. In this weaker representation the term premium is allowed to vary through time, and this is not thought to destroy the overall ability of the expectations hypothesis to describe the relation among long-term and short-term rates. If the expectations theory is valid, this relation suggests that the yield on an n-period bond should be a term premium, plus an arithmetic average of the yields on current and expected m-period bonds up to $n - m$ periods in the future (Campbell and Shiller, 1991).

The overall objective of this paper is to investigate the validity of this less restrictive expectations theory. To achieve this objective, this paper focuses on an equivalent expression of the expectations hypothesis: the yield spread between an n-period and m-period bond, $S_t^{(n,m)} = R_t^n - r_t^m$. According to the expectations theory, the spread is a term premium, plus an optimal predictor of a weighted average of future changes in m-period rates over the life of the n-period bond (Campbell and Shiller, 1991). Therefore, a more specific objective is to:

- determine if the spread is a term premium, as well as an optimal predictor of a weighted average of subsequent short-term yield changes over the horizon of the n-period bond.

It should be noted that the spread relation provides a number of alternative metrics for evaluating the expectations model. One such metric involves an application of the theory of cointegration to the term structure or yield curve. If the expectations theory is valid short-term and long-term yields must be cointegrated with a cointegrating vector of $(-1, 1)$

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(Campbell and Shiller, 1987). Testing for cointegration is not a test of the weak form of the expectations theory, although it is crucial as the lack of cointegration means the expectations theory can immediately be rejected. Thus, a secondary objective is to:

- determine if short-term and long-term rates are cointegrated with a cointegrating vector of $(-1, 1)$.

1.3 Research justification

The expectations theory is, as the name suggests, a theory of the term structure that is founded on the notion of 'expectations'. It contains the view that the spread, which serves as a measure of the shape of the term structure, depends exclusively upon the market's expectations about future short-term rates. The theory leads to a straightforward explanation of the term structure, and it produces a series of important observations on what, for given states of expectations, will be the actual spread at a point in time, and how it might change from one point in time to another (Lutz, 1942).

Lloyd (1986) argues that if the expectations theory is valid, it will have important implications for government policy as it implies the monetary authorities cannot alter the term structure unless they affect expectations. Lloyd suggests that the immediate return to equilibrium, equalising returns across all maturities, means that only the actual level of rates can be influenced and not the relevant positioning of rates within the term structure. He points out that the selling of long-term bond stock will not increase the long-term rate relative to the short-term rate. Lloyd contends that the resulting excess supply of long-term bonds will reduce the price and push up the long-term rate, while a fall in price will induce investors to switch from the short-end to the long-end of the market, thus creating an excess supply of short-term bonds. The excess supply of short-term bonds will decrease the price and increase the rate on short-term bonds. Lloyd suggests that equilibrium will be restored when the term structure returns to its original form (i.e. based on what the market expects). The term structure will not have changed, although its position in interest space will have altered; in this case shifted upwards. Therefore, according to the expectations theory, any attempts aimed at affecting the term structure will be unsuccessful unless policy is targeted at changing expectations about future short-term rates.

This finding is particularly important as the structural relationship between short-term and long-term rates will determine both the nature of monetary transmission and the ability of governments to influence the real economy. According to the expectations theory, even

though policy authorities can accurately control short-term rates, the authorities can affect long-term rates, which play a critical role in a number of economic decisions, only insofar as they influence a long average of present and expected future short-term rates (Lutz, 1942). One conclusion that Lutz draws from this relationship is that a change in the short-term rate will bring about a change in the long-term rate only if a general conviction is created that the short-term rate will remain low for a considerable period of time. If the expectations theory is a valid representation of the term structure, the monetary authorities have to create such a conviction if they want to bring down the long-term rate and stimulate the economy.

1.4 Thesis overview

In order to ascertain if a time-varying term premium is enough to reconcile the expectations theory with the data, this paper adopts the following format. Chapter Two provides a brief discussion of the alternative forms of expectations theory, and identifies several ways in which the modified expectations theory (i.e. the pure expectations theory augmented by a constant term premium) can be tested. The empirical literature on each of these methods is summarised and two explanations for the documented failure of the spread to predict short-term rate changes are identified. The empirical literature on each of these possibilities is also considered and a conclusion drawn from this evidence provides the justification for this paper. Chapter Three outlines the approach that will be used to evaluate if the spread is a term premium, plus an optimal predictor of a weighted average of expected future changes in short-term rates over n-periods. This chapter begins with a consideration of the time series properties of the data, and then introduces a number of methods that will be used to assess the validity of the less restrictive form of the expectations theory. Chapter Four presents the results from applying these methods to the New Zealand interest rate data, and Chapter Five discusses these results and presents conclusions as to whether the weaker form of the expectations theory is valid.

Chapter Two

Literature Review

2.1 Introduction

This chapter provides the justification for investigating the weaker form of the expectations theory and sets the scene for the ensuing analysis. The chapter begins with a discussion on the term structure theories and their relationship among the long-term and short-term rates of interest. The focus then shifts to the ways in which the expectations theory of the term structure can be evaluated. Tests are presented using the single-equation, vector autoregressive (VAR) and vector error-correction (VEC) models, and the empirical evidence under each approach is reviewed. The studies considered strongly reject the market efficiency hypothesis when it is tested using the former two models. Several explanations for this lack of empirical support are proposed, and the empirical evidence on each of these possibilities is also examined. In light of this evidence, an area for further investigation is identified.

2.2 Term structure theories

The term structure of interest rates deals with the relationship between the yields on bonds of different maturities. Two major theories have evolved to account for the observed shape of the yield curve at different points in time, namely, the expectations theory and the market segmentation theory of Culbertson (1957)[2]. There are three forms of the expectations theory, including the pure expectations theory of Fisher (1896), Hicks (1946) and Lutz (1942), the liquidity theory of Keynes (1930) and Hicks (1946) and the preferred habitat theory of Mondigliani and Sutch (1966). The term structure theories are the focus of sections 2.2.1 to 2.2.4.

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[2] The graphical depiction of the relationship between the yields on bonds of different maturities is known as the yield curve (Mondigliani, 1996).

2.2.1 The pure expectations theory

The pure expectations theory dates back to early statements made by Fisher (1896) where he implicitly suggests the market has perfect foresight. Fisher alleges that the rate for a loan contracted today and payable in two years, is the actuarial average of the rate for a loan contracted today and payable in one year, and the rate for a loan to be contracted in one year and payable within two. Despite these early statements the expectations theory cannot be attributed to one individual. Instead, most of the underlying theory was not developed until the late 1930s and early 1940s by, notably, Hicks (1946) and Lutz (1942). The theory as devised by these and other authors rests on three basic assumptions. These assumptions, which are detailed at the outset of Lutz's paper, are: (1) everybody concerned knows what future short-term interest rates will be; (2) there are no costs of investment either for lenders or borrowers; (3) there is complete shiftability for lenders and borrowers. The lender who wants to lend for ten years is equally well prepared to buy a ten-year bond or to lend on a one-year contract and to re-lend ten times. Similarly, a lender who wants to invest for only one-year is in principle prepared to buy a ten-year bond or a bond of any other maturity and sell it after one year. The same shiftability is assumed for the borrower.

In view of assumptions (2) and (3) investors and borrowers will be willing to shift from the long end to the short end of the market (and vice versa), as the opportunity presents itself. Clearly, an opportunity to switch from one end of the market to the other will present itself if the expected returns from investing in these two types of assets should be different for a given investment period. For example, if the expected return on a short-term bonds exceeds that on a given long-term bond then investors will purchase short-term bonds in preference to the long-term bond. Likewise, for an investor already in long-term bonds there will be an incentive to move out of them and into short-term bonds. In regards to assumption (1) the direction of these asset switches will be known, for all investors will operate in the same way. This kind of asset switching behaviour must result in an adjustment in prices, and thus in returns on the two types of bonds such that their expected returns become equivalent.

Hicks (1946) and Lutz (1942) formulated a simple theorem about the equalisation of returns for any conceivable investment strategy over any particular holding period. This conclusion can be epitomised in the kind of formula developed by Hicks. In advancing his formula, Hicks argued that a loan for six months is equivalent to a loan for one-month,

combined with a series of forward loan transactions, each renewing the loan (re-lending the principal, or principal and interest) for a successive month. He contends that if one decides upon some minimum period of time, loans for less than which time one is prepared to discount, every loan of every duration can be reduced to a loan for the minimum period, combined with a given number of renewals for subsequent periods of the same length, contracted forward. Looking at it this way, the rate of interest for an n-period loan is compounded out of the spot rate of interest for loans of one-period and the forward rates of interest, also for one-period loans, but to be executed in the 2, 3, ..., n periods. Hicks argues that if no interest is to be paid until the conclusion of a loan transaction, then the same capital sum must be arrived at by accumulating for n-periods at the n-period rate of interest, or alternatively by accumulating for one-period at the one-period rate, and then by accumulating for subsequent periods at the respective forward rates. Hicks obtains the following formula for expressing the equalisation theorem:

$$(1 + R_t^n)^n = (1 + r_t^1)(1 + f_t^2)...(1 + f_t^n) \tag{2.1}$$

where R_t^n is the current n-period rate (ie the long-term rate), r_t^1 is the current one-period rate and $f_t^2,...,f_t^n$ are the forward rates[3]. Hicks notes that if one assumes simple interest at the outset that this relation becomes:

$$nR_t^n = r_t^1 + f_t^2 + ... + f_t^n \tag{2.2}$$

and one finds *that the long-term rate is a simple arithmetic average of the current one-period rate and all relevant forward, expected, one period rates.* Hence the obvious name, the expectations theory, and commonplace observation that the long-term rate of interest depends upon future expectations about the short-term rate[4]. As a result of this relationship, Lutz (1942) finds that if short-term rates are expected to remain the same for some time in

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[3] The intuition behind the term forward rate is that a market participant who can borrow and lend at currently quoted short-term and long-term rates can fix the rate at which s/he borrows or lends in future periods by an appropriate set of current transactions (Cook and Hahn, 1990).

[4] Should one have data on the holding period returns (ie the $R_t^2,...,R_t^n$) for the outset of a given period, one can calculate a set of forward rates from the Hicksian formula, as:

$$f_t^\tau = \frac{(1 + R_t^\tau)}{(1 + R_t^{\tau-1})} - 1, \tau = 2, 3, ..., n.$$

Meiselman (1962) argues that it is these forward rates which can be regarded as the true, unbiased, estimates of the future one-period rates.

the future, then both current short-term and long-term rates will be equal and the yield curve will be a straight line. Alternatively, expectations of higher (lower) short-term rates in the future will be built into long-term bonds making the yield curve upward (downward) sloping. As such, the theory can account for any shape of the yield curve by assuming any expected movements in future short-term rates. In particular, a humped yield curve will reflect expectations of a rise in short-term rates over the next i-years and a fall thereafter.

Lutz presented an alternative representation of the equalisation theorem that differs from the Hicksian in its treatment of interest payments. Lutz computed a formula that is based on the assumption that long-term interest payments are made regularly at the same intervals as those at which the short-term rate are paid. The Lutzian formula is:

$$R_t^n = \frac{(1+r_t^1)(1+f_t^2)...(1+f_t^n)-1}{(1+f_t^2)(1+f_t^3)...(1+f_t^n)+(1+f_t^3)...(1+f_t^n)+...+(1+f_t^n)+1} \quad (2.3)$$

where the notation is the same as used by Hicks (1942). Lutz adds that the arithmetic average can, however, be used as a sufficiently close approximation of the equalisation theorem for most purposes.

2.2.2 Liquidity premium theory

A variant of the expectations theory, of Keynesian inspiration (see Keynes (1930)) but articulated largely by Hicks (1946), is the liquidity premium or risk premium theory. Recall that the expectations theory assumed the markets for bonds exhibit complete shiftability between a given bond and any other bond. Hicks argued that in reality such perfect substitutability does not exist. He suggests that borrowing is typically undertaken to finance long-term projects and that such borrowers prefer to issue long-term bonds so as to hedge against the risk of fluctuations in interest costs. Lenders, on the other hand, prefer to hold short-term bonds so as to avoid the fluctuations in portfolio value associated with holding long-term bonds. Hicks argued that in this situation, the forward market for loans may be expected to have a constitutional weakness on one side, a weakness which offers an opportunity for speculation. He suggests that if no extra returns were offered for long lending then most lenders would prefer to lend short, and that such a situation would leave a large excess of demands to borrow long that would not be met. He claims that borrowers would tend to offer better terms in order to persuade lenders to switch over to the long end of the market. Hicks argues that a lender who did this would be in a position exactly

analogous to that of a speculator, as s/he would only enter the long market because s/he expected to gain by doing so, and to gain sufficiently to offset the risks incurred. Hicks alleges that the forward rate of interest for any particular period is thus determined as being the rate that just tempts a sufficient number of speculators to undertake the forward contract. He contends that the forward rate must be higher than the short-term rate expected to rule in the future period, since otherwise these speculators would get no compensation for the risks incurred. Hicks states that the forward rate will thus exceed the expected short rate by a liquidity premium which corresponds exactly to the normal backwardation of the commodity market[5, 6]. This would mean that the rate on long-term bonds would be above the expected rates on short-term bonds by a liquidity premium, and thus the actual return from investing in an n-year bond would be higher than the expected returns from investing consecutively in n one-year bonds.

2.2.3 Preferred habitat theory

Another variant of the expectations theory that has been proposed by Modigliani and Sutch (1966) and which, in essence, blends the pure expectations theory, the liquidity theory and the market segmentation theory, is the so-called preferred habitat theory[7]. The Modigliani and Sutch model shares with the Hicksian approach the notion that the term structure is basically controlled by the principle of the equality of expected returns, but modified by the term premium. Recall that Hicks (1946) assumes that all lenders are concerned with the short period return and that all lenders who go long are bearing the risk associated with the uncertainty of the short period return from longer-term investments. Modigliani and Sutch point out that it is not rational for lenders to prefer to lend short or be concerned with short-term capital losses. They contend that this view would only be correct if one assumes that lenders intend to liquidate their investment at the shortest possible date (i.e. s/he has a short habitat). Modigliani and Sutch note that in reality different transactors are likely to have different habitats, as the market segmentation theory asserts. Their reasoning is that if an investor has an n-period habitat (in that s/he has funds which s/he will not need for n-

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[5] Hicks's own view was that the liquidity premiums are an increasing function of the length of the loan undertaken. The reason for this is the alleged increasing riskiness of the investor committing her/himself to longer and longer investment periods.

[6] The difference between the spot price and the currently fixed future price is called normal backwardation (Hicks 1946).

[7] The market segmentation theory is discussed in the following section.

periods and thus s/he intends to keep invested for n-periods), and if that investor purchases such a security, s/he will know the exact outcome (as measured by the terminal value) with certainty. If, however, the investor stays short, the outcome will be uncertain as it depends on the future course of short rates in periods 2, 3, ..., n. Modigliani and Sutch argue that if an investor has risk aversion, s/he will prefer to stay long unless the average of expected short rates exceeds the long rate by an amount

sufficient to compensate s/he for the extra risk of going short. They note that risk aversion should not lead investors to stay short but, instead, should lead them to hedge by staying in their preferred habitat, unless other maturities (shorter or longer) offer a premium to compensate for the risks and costs of moving out of their chosen habitat. In this particular model the n-period rate could differ from the rate implied by the pure expectations hypothesis by a positive or negative risk premium, reflecting the extent to which the supply of funds with habitat n differs from the demand for n-period loans at that rate. Hence, in situations where the n-period demand exceeds the supply of funds within that habitat, there would tend to arise a premium in the n-period maturity, and conversely. Such a premium would tend to bring about a shift of funds from different maturities, by tempting traders out of their natural habitat by the lure of higher expected returns.

2.2.4 Market segmentation theory

Culbertson (1957) proposed an alternative to the expectations theory which has been labelled the market segmentation theory. Culbertson suggested that both lenders and borrowers have definite preferences for instruments of a specific maturity, and for various reasons will tend to stick to bonds of the corresponding maturity, without paying attention to returns on other bonds[8]. He alleges that the rates for bonds of different terms to maturity tend to be determined, each in their separate market, by the independent demand and supply schedules. Culbertson argues that these rates so set might well imply wide differences in expected returns, but such differences will not induce traders to move out of their preferred habitat unless the discrepancies become extreme. As a result, Culbertson's

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[8] Culbertson identifies the following factors as underlying the decisions of lenders and borrowers as to the maturity of the debt they create or hold. These are (1) the liquidity differences between long-term and short-term debt; (2) the attractiveness of debts of different securities on basis of expected future changes in debt prices; (3) changes in the maturity structure of the supply of debt coupled with rigidities in the maturity structure of demand; (4) differences in lending costs related to debt maturity.

theory does not support the view that long-term interest rates should equal the average of short-term interest rates expected over the maturity of the long-term bond, or the view that market expectations can logically be inferred from the term structure of interest rates.

2.3 Identification of testing methods and review of the related literature

In this section the evidence for the validity of the expectations theory of the term structure of interest rates is examined. Tests of the linear expectations model can be divided into two major types: those that use single-equation and VAR analysis to test the rationality (unbiasedness) hypothesis of the expectations theory, and those that use cointegration analysis to check the long-run implications of the theory. The following sections identify the various methods of testing the expectations theory in the single-equation, VAR and VEC frameworks. The literature for each approach is reviewed.

2.3.1 Empirical tests and evidence using single equation models

According to Dziwura and Green (1996), a great deal of empirical research on the term structure of interest rates has been concerned with determining the information content the yield curve provides regarding future interest rates. Specifically, *do forward rates derived from the term structure represent an unbiased forecast of expected future short-term rates?* In the pure form of the expectations theory there is no allowance made for term premiums, and changes in the short-term rate are equivalent to what the market expects rates to be at a particular point in time. Dziwura and Green suggest that in this case one would not expect any variation between the forward rate and the future short-term rate, and the one-period return over any investment horizon is certain and independent of the maturity of the bond. All bonds should therefore have a one-period expected return equal to the short-term rate for that period, and the excess return (the holding period return minus the current short-term rate) should be zero. That is:

$$E_t h_{t+1}^\tau - r_t = 0 \qquad\qquad (2.4)$$

where E_t is the expectations operator conditional on information at time t, h_{t+1}^τ is the one-period return from t to $t + 1$ on a bond with τ-periods to maturity and r_t is the current short-term rate. Dziwura and Green note that this strict interpretation of the expectations theory presumes that no compensation for longer-term investments, which are generally considered more risky than short-term investments, are demanded by investors. They

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maintain that a more realistic version of the theory (i.e. modified expectations) assumes that forward rates are equivalent to expected short-term rate changes, plus a constant and non-varying term premium required by investors to compensate them for the risks of holding longer-term bonds. This implies that all bonds should have a one-period return equal to the short term rate plus a constant term premium, defined as the difference between the forward rate and the corresponding expected future short-term rate, that reflects excess returns[9].

Dziwura and Green show that one can test the validity of the expectations theory by regressing changes in the short-term rate against the *forward rate premium*, which is the difference between the forward rate, f_t^τ, at time t, τ-periods ahead, and the current short-term rate, i.e. $(f_t^\tau - r_t)$, to determine how well forward rates predict future short-term rates. If one defines the term premium, θ, as the difference between the forward rate and the corresponding expected future short-term rate, $E_t(r_{t+\tau})$:

$$\theta_t^\tau = f_t^\tau - E_t(r_{t+\tau}),$$ (2.5)

then this can be seen by rearranging terms and subtracting the short-term rate from both sides:

$$f_t^\tau - r_t = E_t(r_{t+\tau}) - r_t + \theta_t^\tau.$$ (2.6)

This equation now decomposes the forward rate premium into the expected change in the short-term rate, $(E_t(r_{t+\tau}) - r_t)$, plus a term premium.

Dziwura and Green add that the expectations hypothesis is based on two assumptions about the behaviour of participants in the money market. These are that the term premium participants' demand for investing in one maturity rather than another is constant over time, and that interest rate expectations are formed rationally, so that:

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[9] Shiller (1990) notes that applied workers have rarely taken seriously the risk neutrality expectations hypothesis as it has been defined in the theoretical literature, and that the theoretical discussion of the expectations hypothesis may be something of a red herring. He contends that the applied literature has defined the expectations hypothesis to represent constancy through time of differences in expected holding period returns, or constancy through time of the differences in forward rates and expected future spot rates, and not that these constants are zero. This paper follows this convention by defining the expectations theory to be consistent with this more generalised form.

$$r_{t+\tau} = E_t(r_{t+\tau}) + \varepsilon_{t+\tau},$$ (2.7)

where $r_{t+\tau}$ is the actual future short-term rate and $\varepsilon_{t+\tau}$ is a forecast error which is uncorrelated with other information at time t. Dziwura and Green then substitute equation (2.7) into equation (2.8) to yield a theoretical equation which is estimated by:

$$r_{t+\tau} - r_t = \alpha + \beta(f_t^\tau - r_t) + \varepsilon_{t+\tau},$$ (2.8)

where under the rational expectations assumption the error term in equation (2.8) is uncorrelated with the right-hand side variable so that the β coefficient can be estimated consistently. The expectations theory predicts that β should not differ significantly from unity, as all variation in the future short-term rate should be reflected in current forward rates. Dziwura and Green allege that a significantly different value would contradict the assumption of a constant term premium, while a slope coefficient β equal to zero would suggest that the forward rate premium has no power to forecast the change in the short-term rate τ-periods ahead.

Dziwura and Green estimate *cumulative power regressions* with one-month and one-year Treasury bill securities for maturities of up to twelve-months and five-years in the future, respectively. These regressions measure the cumulative predictive power of the slope of the yield curve between a one-period rate and a longer-term rate at various maturities. For example, one can estimate the predictive power of the yield curve from one to six-months with the following regression:

$$r_{t+5} - r_t = \alpha + \beta(f_t^5 - r_t) + \varepsilon_{t+5}$$ (2.9)

where the dependent variable is the change in the one-month rate over the following five-months and the independent variable is the difference between the forward rate for a one-month bill five-months in the future and the current one-month rate. Dziwura and Green estimate cumulative power regressions for the 1982 to 1995 period and report that β coefficients are well under unity and significantly different from zero. In the one-month and one-year regression sets they find that forward rates explain a great deal of the variance in the subsequent short-term rates as indicated by coefficients that lie mostly between 0.34 and 1.22. Dziwura and Green conclude that forward rates have significant forecasting power for subsequent short-term rates, but even the strongest results cannot support the assumptions of the expectations theory.

Cook and Hahn (1990) conduct a similar analysis with one-month Treasury bill rates with maturities up to six-months in the future using McCulloch data for the 1 / 1952 to 8 / 1986 period. Cook and Hahn estimate cumulative power regressions and report positive and steadily declining coefficients over the yield curve out to six-months. Cook and Hahn find coefficients which range from 0.5 to 0.02 and report that only the coefficient in the regression covering the cumulative change in one-month rate one month in the future is significant. Cook and Hahn also estimate *standard regressions* in which the long-term rate has a maturity that is equal to twice that of the short-term rate. They estimate standard regressions with Treasury bill rates and private sector rates for the maturities of three, six and twelve-months using McCulloch and Salomon Brothers data for the 12 / 1966 to 10 / 1986 period, respectively. Cook and Hahn find little support for the expectations theory using both rates. Specifically, they find that regressions using Treasury bill rates yield mainly negative coefficients which are all insignificantly different from zero, whereas regressions that use private sector rates produce coefficients that are all positive, but only one is significant and the explanatory power is negligible. Cook and Hahn conclude that contrary to the expectations theory, the forward rate premium has almost no explanatory power to forecast the future change in the short-term rate τ-periods ahead.

Fama (1984) examines a new approach to measuring the information in forward rates about term premiums and future short-term rates. Fama replaces the change in short-term rates in equation (2.8) with the holding period premium, which is the difference between the one-period return on a bond with τ-periods to maturity and the current short-term rate, $h_{t+1}^{\tau} - r_t$, to test if there is information in forward rates about the variation in expected term premiums. This can be written as:

$$h_{t+1}^{\tau} - r_t = \mu + \delta(f_t^{\tau} - r_t) + \varepsilon_{t+\tau}, \tag{2.10}$$

where rational expectations posit that δ should not differ significantly from zero, since all variation in the forward rate premium should be reflected in expected short-term rate changes and not in variations in the term premium. Fama notes that a value of δ greater than zero is evidence that the forward rate premium has forecasting power for excess returns (i.e. term premiums) that vary through time. Likewise, a value of δ equal to one would indicate that all variation in the forward rate premium is due to excess returns and none is due to expected short-term rate changes. Fama uses prices of one to six-month Treasury bills from the Center for Research in Security Prices at the University of

Chicargo, to estimate equation (2.10) for the 1959 to 1982 period. He estimates equation (2.10) over the total sample period and for shorter (generally five years) sub-periods for values of $\tau > 1$. Fama finds strong evidence that expected term premiums vary through time in a way that is captured in the forward rate premium. This is particularly apparent as slope coefficients more than four standard errors from zero are common both in the overall period and the shorter five-year sub-periods. Fama's evidence therefore implies that forward rates contain variation in expected returns on multi-period bills. Hence it offers little support to the expectations theory.

Fama (1986) replicates his earlier work using market quotes for one, three, six and twelve-month bills, prime quality commercial paper, bankers' acceptances and certificates of deposits from the Salomon Brothers *Analytical Record of Yields and Yield spreads* for the 1967 to 1985 period. He estimates equation (2.10) using Treasury bills and private issuer securities for τ values of three, six and twelve with corresponding short rates of one, three and six-months, respectively. Fama finds strong evidence that forward rate premiums contain time-varying term premiums, since all of the regression slopes are more than two standard errors above zero, and estimates more than four standard errors are common. He also finds that *only* the slope in the B3/S1 (i.e. the one-month return from buying a three-month security now and selling it at two-months to maturity) regression for bills is more than two standard errors below unity, and all but one of the regression slopes are 0.79 or greater, and half are greater than 0.9. Fama suggests that these regressions support the conclusion that most of the variation in current forward rate premiums is variation in expected return premiums rather than in forecasts of future changes in rates. He argues that if forward rates are just expected returns, the humps and inversions in term structures of forward rates during recessions imply that the ordering of risks and rewards across maturities change with business conditions and are not always constant or monotonic. Fama concludes that this behaviour is inconsistent with simple term structure models.

Campbell and Shiller (1991) note that if one assumes the expected total return over m-periods on buying an n-period bond and selling it m-periods later equals the return on holding a m-period bond plus a constant, then one finds that the expectation of a non-linear expression in R_t^n and $R_{t+m}^{(n-m)}$ equals r_t^m plus a constant. Linearising this expression around $R_t^n = R_{t+m}^{(n-m)} = 0$, they argue that one gets a rational expectations model that if solved forward yields:

$$R_t^n = \frac{1}{k}\sum_{i=0}^{k-1} E_t r_{t+mi}^m + \theta, \qquad\qquad k = n/m \qquad\qquad (2.11)$$

where R_t^n is the longer-term n-period interest rate and r_t^m is the shorter-term m-period interest rate. The expectations hypothesis then states that the n-period rate is a constant term premium, θ, plus a simple average of the current m-period rate and expected m-period rates up to $n - m = (k - 1)m$ periods in the future.

Campbell and Shiller present two rearrangements of equation (2.11) that can be shown to imply that the current spread between an n-period and m-period bond, $S_t^{(n,m)} = R_t^n - r_t^m$, is a constant, plus an optimal predictor of future changes in interest rates. First, the spread predicts the m-period change in long-term rates:

$$s_t^{(n,m)} \equiv \left(\frac{m}{(n-m)}\right) S_t^{(n,m)} = E_t R_{t+m}^{(n-m)} - R_t^n, \qquad\qquad (2.12)$$

where $R_{t+m}^{(n-m)} - R_t^n$ denotes the m-period change in long-term rates, $s_t^{(n,m)}$ is a maturity-specific multiple of the yield spread and the constant is suppressed for simplicity. The intuition behind equation (2.12) is that if the yield on an n-period bond is expected to rise over the next m-periods, then holders of the bond will suffer a capital loss. Thus, for the equality to hold over the next m-periods, the n-period bond has to have a higher current yield than the m-period bond. As a result of this relationship, the spread which serves as a measure of the shape of the term structure can be used to reflect the market's current expectation of future long-term rates. If the spread between a long-term and short-term bond is relatively high (low), then agents would expect the yield on the longer-term bond to rise (fall) over the life of the shorter-term bond.

Second, by subtracting r_t^m from both sides of equation (2.11) and rearranging terms, Campbell and Shiller show that the spread is a constant term premium, plus an optimal predictor of a weighted average $S_t^{(n,m)*}$ of changes in shorter-term interest rates over n-periods:

$$S_t^{(n,m)} = E_t S_t^{(n,m)*} + \theta, \qquad\qquad (2.13)$$

where $S_t^{(n,m)*} \equiv \frac{1}{k}\sum_{i=1}^{k-1}\left(\sum_{j=1}^{i} \Delta r_{t+jm}^m\right) = \sum_{i=1}^{k-1}\left(1 - \frac{i}{k}\right)\Delta r_{t+im}^m$.

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Campbell and Shiller denote $S_t^{(n,m)*}$ as the *perfect foresight spread*, since it is the spread one would obtain, given the model, if there was perfect foresight about future interest rates. With perfect foresight, if m-period rates are going to rise over the life of the n-period bond, then the n-period bond rate needs to be higher than the current m-period rate to equate the returns on the n-period bond and a sequence of m-period bonds. As before, the spread can be used to reflect the market's current expectation of future interest rates. If the spread is relatively high (low) then agents would expect, on average, future short-term rates to rise (fall) over the life of the long-term bond.

Campbell and Shiller suggest that a straightforward test of these implications is to regress either the realised value of $R_{t+m}^{n-m} - R_t^n$ or $S_t^{(n,m)*}$ onto a constant and $s_t^{(n,m)}$ or $S_t^{(n,m)}$, respectively. Under rational expectations, the expectations theory predicts that the estimated slope coefficient on $s_t^{(n,m)}$ or $S_t^{(n,m)}$, for the former and later regressions, should be unity. Campbell and Shiller tested both equations (2.12) and (2.13) using McCulloch's (1990) monthly data on US Treasury bills for all possible pairs of maturities in the range one, two, three, four, five, six and nine-months and one, two, three, four, five and ten-years, for the 1 / 1952 to 2 / 1987 period. Campbell and Shiller find that estimates of equation (2.12) offer almost no support to the expectations theory as the coefficients on $s_t^{(n,m)}$ are almost always negative and always significantly different from unity. This confirms that the spread provides the wrong direction of forecast for the change in the yield of the longer-term bond over the horizon of the short-term bond. Campbell and Shiller find that the estimates of equation (2.13) are somewhat more promising for the theory as the coefficients on $S_t^{(n,m)}$ are almost always positive and insignificantly different from unity when the maturity of the long-term bond is below three or four years. This means that the current spread between n-period and m-period bonds predicts how the average m-period rate will change over the next n-periods. Campbell and Shiller conclude that certain statements can be made quite generally. They suggest that for almost any pair of maturities between one-month and ten-years that the following is true: when the spread is relatively high the yield on the longer-term bond tends to fall over the life of the shorter-term bond (i.e. counter to the expectations theory), and at the same time shorter-term rates tend to rise over the life of the longer-term bond (i.e. in accordance with the theory).

Hardouvelis (1994) tested equations (2.12) and (2.13) using post-war, end-of-quarter data on three-month and ten-year bonds for seven countries. The data set extends as far back as

possible for each country and ends in the second quarter of 1992. In the US the sample begins in 1953, in Canada it begins in 1950, in Japan, the UK, Germany and France it begins in the 1960s and in Italy the early 1970s. Hardouvelis' paper examines the relationship between the spread and the future evolution of long-term and short-term rates to establish if there is a puzzle. The puzzle Hardouvelis refers to is the frequently reported finding that the spread fails to correctly predict future movements in the long-term rate, yet it does forecast short-term rate movements in the way implied by the expectations theory. Hardouvelis finds that in the US, the UK, Canada, Germany and Japan the long-term rate moves in the opposite direction from the one predicted by the expectations theory, and that only in the US is the coefficient on $s_t^{(n,m)}$ statistically significant. In France and Italy the long-term rate moves in the correct direction, but the estimated slope coefficients are not statistically significant. The expectations theory fares slightly better with estimates obtained from equation (2.13), as the coefficients on $S_t^{(n,m)}$ are all positive and significant in five of the seven countries, although, as before, the coefficients on $S_t^{(n,m)}$ differ significantly from the value predicted by the expectations theory.

Mankiw and Miron (1986) examine the expectations theory using three and six-month Treasury bills from 1890 to 1979. Mankiw and Miron divide their sample into five different monetary regimes to examine if the failure of the expectations theory is robust. The first sample runs from 1890 to 1958 and within this sample four different monetary regimes are examined. The first regime is from the fourth week of the quarter from 4 / 1890 to 4 / 1914 (ending at the founding of the Federal Reserve), the second regime is from 1 / 1915 to 4 / 1933 (ending at the introduction of the New Deal banking reforms), the third regime is from 1 / 1934 to 1 / 1951 (ending at the Accord) and the fourth regime is from 2 / 1951 to 4 / 1958 (ending at the time when an active market for three and six months Treasury bills begins). The second sample runs from 1 / 1959 to 2 / 1979 and is used as a contrast for the results using data from 1890 to 1958. Mankiw and Miron conduct tests on the following equation:

$$r_{t+1} - r_t = \alpha + \beta(R_t - r_t) + \varepsilon_{t+1} \tag{2.14}$$

where the dependent variable is the one-period change in the three-month rate and the independent variable is the spread between the six-month and three-month rates. According

to the expectations theory $\alpha = -2\theta$ and $\beta = 2$[10]. Mankiw and Miron estimate equation (2.14) and find that prior to the founding of the Federal Reserve, the slope of the yield curve exhibits strong predictive power for the path of the three-month rate. In fact, the estimated coefficient on the spread between the six-month and three-month rates is substantially high and only slightly below the value predicted by the theory. These results contrast sharply with those obtained for the remaining four monetary regimes. Mankiw and Miron find estimated slope coefficients for the 1915 to 1958 period that are always significantly different from two and not usually significant from zero, indicating that post 1915 the spread contains little information for the path of short-term rates. This interesting result will be considered in more detail in a following section.

2.3.2 Empirical tests and evidence using VAR models

Recently, tests of the expectations theory of the term structure have included the use of the VAR methodology. Campbell and Shiller (1987) were the first to derive and test the implications of the expectations theory using a VAR model comprising Δr_t^m and $S_t^{(n,m)}$. Campbell and Shiller found that a weak test of the expectations model is to establish if $S_t^{(n,m)}$ linearly Granger-causes Δr_t^m. The intuition for this result is as follows. Since, from equation (2.13), $S_t^{(n,m)}$ is an optimal predictor of a weighted average of future Δr_t^m conditional on the full information of agents, if agents have information useful for forecasting short-term changes other than that contained in the history of that variable, it will be reflected in $S_t^{(n,m)}$. If agents do not have such information, then $S_t^{(n,m)}$ is an exact linear function of current and lagged Δr_t^m [11].

A further test of the expectations theory is to compare the forecasts of future changes in short-term rates embodied in the spread to an unrestricted VAR forecast that is easily computed from the VAR system. The theoretical spread $S_t^{(n,m)'}$, defined as the optimal forecast of the right-hand-side of equation (2.13) given the information subset, can then be calculated and compared with the actual spread. As the information subset includes the spread, the two forecasts should be equal if the expectations theory is true. The equality of

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[10] This equation is equivalent to a regression of the perfect foresight spread onto a constant and the actual spread except for the scaling factor of two.

[11] Which, in Campbell and Shiller's words, is a stochastic singularity which we do not observe in the data.

these two variables can be shown to impose a set of cross-equation restrictions on the estimated coefficients of the VAR model. Only if these restrictions hold is it the case that $S_t^{(n,m)}$ is an optimal predictor of future Δr_t^m.

Campbell and Shiller also compare the behaviour of the spread and the theoretical spread with the following two volatility measures:

$$VR = \frac{\text{var}(S_t^{(n,m)})}{\text{var}(S_t^{(n,m)'})} \qquad (2.15)$$

$$S_t^{(n,m)'} = \alpha + \beta S_t^{(n,m)} + \varepsilon_t \qquad (2.16)$$

which are designed to test against the alternative that the spread moves 'too much'. The first test gauges the validity of the expectations theory with the ratio of the variance of the observed spread to the variance of the theoretical spread. If the expectations theory is valid $S_t^{(n,m)}$ should not be excessively volatile relative to $S_t^{(n,m)'}$ and the levels variance ratio in equation (2.15) should be unity. The second test involves a comparison of the movements in the spread with that predicted by the model. If the expectations theory holds, $S_t^{(n,m)'}$ should mimic movements in $S_t^{(n,m)}$ and the correlation coefficient in equation (2.16) should be one.

Campbell and Shiller applied the VAR tests of the expectations theory to monthly US Treasury bonds with maturities of one-month and twenty-years for both the full sample 1 / 1959 to 10 / 1983, and for a short sample ending in 8 / 1978. They find the results are somewhat mixed for the weak test of the expectations theory. Empirical tests of this proposition find that spreads Granger-cause changes in short-term rates, although changes in short-term rates also Granger-cause spreads (i.e. contrary to the expectations theory's predictions). Campbell and Shiller also find a lack of support when formally testing the expectations theory's implied restrictions. These tests suggest that one can firmly reject the null hypothesis that the theoretical spread equals the actual spread for both periods. Despite these negative results, Campbell and Shiller do find support for the expectations theory with tests that are based on the variance ratio and the correlation between the theoretical spread and the actual spread. These tests produce variance ratios and correlation coefficients that do not differ significantly from unity.

MacDonald and Speight (1991) test similar implications of the expectations theory on a multi-country database, and also consider a weaker form of the expectations theory that incorporates the influence of a possible time-varying term premium. MacDonald and Speight use quarterly interest rate data on a three-month Treasury bill and a representative single government long bond for Belgium, Canada, Germany, the UK and the US. The sample period begins in 1 / 1964 and ends in 4 / 1984 for each of the five countries. MacDonald and Speight find that the spread does not often include information that is useful for forecasting future changes in short-term rates, specifically spreads Granger-cause changes in short-term rates only in Belgium and Germany. MacDonald and Speight also find limited support when testing if the spread is equal to the unrestricted forecast of future changes in short-term rates. MacDonald and Speight find that tests of the restrictions implied by equating $S_t^{(n,m)}$ with $S_t^{(n,m)'}$ cannot be rejected for the UK, but can be for the remaining countries. However, interestingly, non-rejection can also be extended to the US when allowing for a time-varying term premium (which requires the restriction tests to be implemented using information prior to the period in which anticipations were formed). This lack of support is also evident in tests of fitted and actual spread variance measures in that the levels variance ratios are indicative of excess volatility for all countries. MacDonald and Speight conclude that the results are somewhat mixed for some countries (particularly the UK and the US), whilst for other countries the expectations model is strongly rejected.

Campbell and Shiller (1991) test for co-movements in the spread and the theoretical spread using monthly data on US Treasury bills for maturities in the range one, two, three, four, five, six and nine months and one, two, three, four, five and ten years, for the 1 / 1952 to 2 / 1987 period. Campbell and Shiller first test for co-movements by computing the ratio of the standard deviation of $S_t^{(n,m)}$ to the standard deviation of $S_t^{(n,m)'}$. A general finding is that the coefficients are typically around one-half, regardless of the maturity of the long-term and short-term bonds. This contrasts with results obtained from regressions involving the theoretical spread and the actual spread. These regressions produce correlation coefficients that are almost always positive and around unity when the maturity of the long-term bond exceeds three-years. Campbell and Shiller conclude that the VAR procedures give strong evidence that $S_t^{(n,m)}$ is excessively variable relative to $S_t^{(n,m)'}$, and generally weaker evidence that $S_t^{(n,m)}$ and $S_t^{(n,m)'}$ are imperfectly correlated.

Shea (1992) tests the VAR cross-equation restrictions over portions of McCulloch's (1990) zero-coupon yield curves. These restrictions were derived (although not directly tested) in Campbell and Shiller (1991), who instead concentrated on the study of the correlations between $S_t^{(n,m)}$ and $S_t^{(n,m)'}$ [12]. Since these tests are reflections of the expectations hypothesis he employs Campbell and Shiller's (1991) data set to investigate these restrictions. Shea finds similarities between his results and those of Campbell and Shiller when they regressed $S_t^{(n,m)'}$ onto $S_t^{(n,m)}$ and found the predictive power of the spread increased the longer the maturity of the long-term bond whose yield was included in the yield spread. In particular, Shea finds the cross-equation restrictions can be rejected for almost all maturities when the long-term bond included in the yield spread is twelve-months or less. But for the remaining maturities of two, three, four, five and ten years the cross-equation restrictions are generally accepted, providing the maturity of the short-term bond is less than six-months. Shea concludes that the slope of the yield curve is better suited to predicting changes in one-month to five-month yields over a two-year to ten-year time span.

Cuthbertson (1996) conducts VAR tests of the expectations theory using weekly London interbank (offer) rates with maturities of one, four, thirteen, twenty-six and fifty-two weeks for the 1987 to 1992 period. Cuthbertson finds that tests of Granger-causality afford limited support to the expectations theory. He finds that spreads Granger-cause changes in short-term rates, but changes in short term rates often Granger-cause spreads. Cuthbertson also tests the VAR cross-equation restrictions and finds support for the theory at only long-term maturities. In Particular, he finds the restrictions can be rejected when the maturity of the long-term bond included in the spread is four, thirteen and twenty-six weeks. For the remaining long-term maturity of fifty-two weeks, the restrictions are easily accommodated. Unlike earlier findings, he does find clear support for the expectations theory using the volatility measures in equations (2.15) and (2.16). These measures produce level variance ratios that are within two standard deviations of unity in five of the eight cases, and correlation coefficients that do not differ significantly from unity.

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[12] Campbell and Shiller (1991) present a modified version of the VAR to test the expectations theory. This approach differs from earlier work, as the VAR is modified to handle a finite rather than infinite value of n.

2.3.3 Empirical tests and evidence using VEC models

More recently, tests for evaluating the expectations theory have evolved to include the recently developed theory of cointegration. Campbell and Shiller (1987) were the first to apply this approach to the expectations theory of the term structure. Campbell and Shiller found that if short-term and long-term rates are integrated of order one then equation (2.13) implies that the spread is stationary, or alternatively that short-term and long-term rates are cointegrated with a cointegrating vector of (−1, 1). At present the literature is testing cointegration restrictions that are consistent with the expectations hypothesis using maximum likelihood methods developed by Johansen (1988). The advantage of this procedure is that it allows one to view the entire yield curve as a cointegrated system and perform a sequence of tests for its conformity to the expectations hypothesis. Hall, Anderson and Granger (1992) were the first to point out the cointegrating implications in this context. They found that if there is a non-stationary yield curve of length n, then the expectations hypothesis implies that $(n - 1)$ linearly independent spreads are cointegrated.

MacDonald and Speight (1991) apply cointegration tests to a multi-country data base using quarterly interest rate data on a three-month Treasury bill and a representative single government long-term bond for five countries over the 1 / 1964 to 4 / 1984 period. MacDonald and Speight test for cointegration with both the Engel and Granger (1987) and Johansen (1988) methodologies, and find their results are clearly mixed. On the one hand the Engel and Granger tests are only clearly supportive of cointegration in Belgium and Germany, while the attendant statistics are generally mixed for the UK, the US and Canada. This contrasts with Johansen's maximum likelihood approach in which test statistics are supportive of one cointegrating vector in each country. MacDonald and Speight also use the Johansen technique to test that the corresponding cointegrating vector equals the spread vector. MacDonald and Speight find that this hypothesis cannot be rejected for Germany, the US and the UK, but can be rejected for Canada and is rejected at the 1% level for Belgium. MacDonald and Speight conclude that on balance the results support the contention that short-term and term-rates are cointegrated, with the cointegrating vector equal to (−1, 1), as implied by the expectations theory.

Hall, Anderson and Granger (1992) conduct a cointegration analysis on eleven US yield series which are taken from the Fama twelve-month Term Structure File that runs from January 1970 to December 1988. The file contains one series for bills with one month to

maturity, and so on to a series with twelve months to maturity. This analysis is also repeated for three monetary regimes in view of empirical evidence that these have caused structural changes in the term structure. These regimes include a period where the Federal Reserve targeted interest rates (the period up to September 1979), implemented its new operating procedures and ceased targeting interest rates (October 1979 to September 1982) and abandoned these new operating procedures to resume interest rate targeting (October 1982 onwards). Hall, Anderson and Granger examine implications of the expectations hypothesis for the entire yield curve. This includes testing whether a set of n yields are cointegrating with $(n-1)$ cointegrating vectors, and that these cointegrating vectors are the spread vectors. The results support the earlier proposition of the expectations theory, but reject that the ten linearly independent spreads comprise a basis for cointegrating space. Hall, Anderson and Granger suggest that the rejection may have been caused by problems associated with changes in monetary regimes. They investigate this possibility with an analysis of the four shortest yields over the three sub-samples. They find that the results from an analysis of the first and third sub-sample are consistent with the theory. These relationships appear to have broken down during the period of the new operating procedures, as there are only two cointegrating vectors and none of the possible spread vectors are cointegrating. Hall, Anderson and Granger conclude that only during periods in which the Federal Reserve targeted interest rates are the tests supportive of the expectations theory.

Shea (1992) tests the cointegrating restrictions with US monthly term structure data with maturities of one and six-months and three, five, ten, fifteen, twenty and twenty five years, for the sample 1 / 1952 to 2 / 1987, and for a shorter sample ending in 2 / 1987. He examines implications of the expectations hypothesis for the entire yield curve to see in greater detail how the expectations hypothesis succeeds and fails as a description of a cointegrated system of interest rates. Shea begins with a yield curve containing the one-month and six-months yields and proceeds to the longest yield curve he can observe. He finds that although yields appear to be cointegrated, they can often have too many common trends (too few cointegrating vectors) to support the expectations hypothesis, especially if one tries to model the very long maturities with the short and intermediate yields. Shea also investigates if the spreads are the components of the cointegrating vectors. Among twelve-monthly yield curve data sets he finds that the spread restriction can be rejected four and three times when pre and post-1979 data are used, respectively. In general he finds the

spread restriction is easily accommodated except when the three, twenty and twenty-five-year rates are added to the system. This restriction is also rejected when intermediate maturities (maturities less than five-years) are coupled with very long maturities. Shea concludes that his results provide only partial support for the expectations theory in that he is unable to include short-term yields in a system containing intermediate along with long-term yields.

Pesaran and Wright (1996) test the expectations theory with London interbank rates with maturities of one, three, six and twelve-months for the January 1980 to September 1994 period. The main aim of their paper is to build on the recent work on the term structure of interest rates and specifically to use the results obtained by Hall, Anderson and Granger (1992) to construct medium term forecasting models for the UK interbank market. These results allege that the cointegrating vectors, or spreads, can be used as error-correction terms in forecasting interest rates. However, before estimating forecasting models in the context of the UK market, it is necessary to confirm the cointegrating implications of the expectations hypothesis. Pesaran and Wright find the results are consistent with the theory of cointegration developed by Hall, Anderson and Granger, as there are three cointegrating vectors between the four interest rates, and each of these cointegrating vectors represents a spread. Pesaran and Wright conclude in favour of the expectations hypothesis as these spreads can contribute as error-correction terms in setting up forecasting equations for predicting rates.

2.4 General explanations for lack of empirical support

The studies considered strongly reject the short-run predictions of the expectations theory, especially when it is tested with standard regressions using the spread or the forward rate premium as an explanatory variable. A number of explanations for the failure of the rational expectations assumption have been provided, and these explanations generally involve assertions that the term premium is not constant, or that monetary policy has in some way affected the nature of the empirical tests. Section 2.4.1 investigates the effect a time-varying term premium can have on empirical tests, along with empirical evidence on the expectations model augmented by a time-varying term premium. Section 2.4.2 considers the empirical evidence on the role monetary policy might play.

2.4.1 Empirical tests and evidence of a time-varying term premium

Most explanations for the lack of empirical support for the expectations theory have focused on the possibility that the expected term premium is not constant (as assumed by the theory), but varies substantially over time. Previous investigators, including Mankiw and Summers (1984), Mankiw and Miron (1986) and Hardouvelis (1988), show formally that a time-varying risk premium can bias downward the coefficient on the spread or forward rate premium. As shown in Mankiw and Miron (1986), if the correlation between the term premium θ_t and the expected change in the short rate $E_t \Delta r_{t+1}$ is ρ, then the estimate of β in equation (2.14) converges to:

$$\frac{2\sigma^2(E_t\Delta r_{t+1}) + 4\rho\sigma(E_t\Delta r_{t+1})\sigma(\theta)_t}{\sigma^2(E_t\Delta r_{t+1}) + 4\sigma^2(\theta_t) + 4\rho\sigma(E_t\Delta r_{t+1})\sigma(\theta_t)} \tag{2.17}$$

where $\sigma^2(x)$ denotes the variance and $\sigma(x)$ denotes the standard deviation. It can be seen from this expression that in the presence of a time-varying term premium (ie $\sigma(\theta_t) > 0$) that plim $\hat{\beta} \neq 2$, although the extent of the departure depends on the variance of expected changes in the short rate, as when the variance of this term approaches infinity the plim $\hat{\beta}$ goes to two[13].

Simon (1989) examines rational expectations in the three-month and six-month sector of the Treasury bill market from January 1961 to March 1988 with a risk premium that is specified to be proportional to the volatility of excess returns. Simon also breaks this period into sub-periods which run from January 1961 to December 1971, January 1972 to September 1979, October 1979 to September 1982 and October 1982 to March 1988 in order to examine the predictive power of the yield spread at different periods. He carries out tests of the expectations hypothesis on the following equation:

$$r_{t+13} - r_t = b_0 + b_1(R_t - r_t) + b_2\Theta_t + u_{t+13} \tag{2.18}$$

where $\Theta_t = E_t(2R_t - r_t - r_{t+13})^2$ is the time-varying risk premium which is specified to be proportional to the expectation, formed at time t, of the square of the excess 13-week holding period return on six-month bonds over three month bills. Under the joint

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[13] More generally, $\hat{\beta}$'s deviation from two will depend on the ratio of the variance of the term premium to the variance of the expected change in the short rate.

hypothesis of rational expectations and the risk premium specification, $b_0 = -2\phi$, $b_1 = 2$ and $b_2 = -2\alpha$. Simon finds that tests of the expectations model without a time-varying term premium yield estimates of b_1 which differ significantly from two for both the entire sample period and all the sub-sample periods. However, including the expected volatility of excess returns dramatically improves the forecasting power of the yield curve from 1961 to 1972 and from 1972 to 1979. Simon finds that rational expectations cannot be rejected and the yield curve has significant forecasting power. Moreover, the term premium has the expected sign and is statistically significant at the 1-percent level. The results are less favourable from 1979 to 1982 and from 1982 to 1988. He finds that rational expectations cannot be rejected (because of large standard errors), although the yield curve does not have significant predictive power in either sub-period. He also finds the term premium is negative and statistically significant at a 1-percent level from 1979 to 1982, and is negative but not significant from 1982 to 1988. Simon concludes that the conflicting results between the expectations hypothesis with, and then without, a time-varying risk premium highlight the importance of modelling the risk premium as an optimal forecast.

Tzavalis and Wickens (1997) use US monthly term structure data with maturities of three, six, nine and twelve-months for the 12 / 1941 to 2 / 1991 period, to see if a time-varying term premium can explain the puzzling behaviour of yield spread models. They introduce a single factor representation of the term premium which provides a formal connection between term premium associated with different maturities and allows one term premium to determine another. Since the term premium is specified as being related to expected holding period returns, the excess holding period return of one maturity can determine another. Tzavalis and Wickens carry out tests of the expectations theory using Campbell and Shiller (1991) equations that are augmented by this proxy. The regression models that are used to test the expectations theory are presented below:

$$(n-1)(R_{t+1}^{n-1} - R_t^n) = \alpha_1(n) + \beta_1(n)(R_t^n - r_t) + c_1(n,m)(h_{t+1}^m - r_t) + u_{t+1}^n \quad (2.19)$$

$$\sum_{i=1}^{n-1}(1-\frac{i}{n})\Delta r_{t+1} = \alpha_2(n) + \beta_2(n)(R_t^n - r_t) + c_2(1)(\sum_{i=0}^{n-1}(h_{t+i}^{n-1-i} - r_t) + v_t^n, \quad (2.20)$$

where consistency with the expectations hypothesis requires $\beta_1 = 1$ and $c_1(n,m) = -\gamma(n,m)$ in equation (2.19), and $\beta_2 = 1$ and $c_2(1) = \gamma(1)$ in equation (2.20). They find that tests of the expectations models that ignore the term premium yield estimates of $\beta_1(n)$ that are negative

and increase in absolute value with n, and estimates of $\beta_2(n)$ that are positive and in some cases not far away from unity. On the other hand, the models with a single factor representation reveal a remarkable improvement. Tzavalis and Wickens find that the versions of the models forecasting long-term and short-term rates cannot be rejected as the estimates of $\beta_1(n)$ and $\beta_2(n)$ are not different from their theoretical values of unity. Moreover, the estimates of $c_1(n,m)$ are negative and significant and the estimates of $c_2(1)$ are not statistically different from unity, for all n. Tzavalis and Wickens conclude that their results support the finding that a time-varying term premium can explain the puzzling behaviour of the spread in failing to forecast future long-term rates even though its forecasts of future short-term rates are in the correct direction.

Driffill, Psaradakis and Sola (1997) analyse a weaker form of the expectations theory with monthly observations on the US and the UK one-month and three-month interest rates for the period of 12 / 1982 to 2 / 1991 for the US, and for the periods of 2 / 1975 to 12 / 1994 and 11 / 1982 to 12 / 1994 for the UK. Their purpose was to examine some alternative ways of testing the expectations theory, and in so doing to offer an explanation for the finding that while the spread contains some predictive power for future changes in short-term rates, the estimated spread coefficient is often of the wrong size and sometimes of the wrong sign. Driffill, Psaradakis and Sola suggest that an explanation for this failure is that a weaker form of the expectations theory may hold in which the term premium contains an element which varies randomly over time, independently of short-term rates. They argue that the inclusion of a white noise error term on the long-term rate of interest may be enough to reconcile the theory with the data. Driffill, Psaradakis and Sola investigate this proposition by testing the restrictions implied by the expectations theory in equation (2.13) with a constant, and then a time-varying, term premium. They find the restrictions appropriate to the traditional form of expectations model are quite clearly rejected for the US and the UK, while the restrictions for the weaker form of model are not. Driffill, Psaradakis and Sola also conduct similar tests on a VAR model for the yield spread and the first difference of the short-term rate. They find that the more stringent restrictions imposed by the expectations theory with a constant term premium are rejected for the US and UK, while the restrictions which allow for a random component in the term premium are not. Driffill, Psaradakis and Sola conclude that the results are consistent with a weaker form of expectations theory, and that many of the rejections of the theory appear to result from not allowing for a random element in the term premium.

2.4.2 Empirical evidence on monetary policy regime changes

A second explanation of the failure suggested by Mankiw and Miron (1986), supports the view that it is related to monetary policy regime changes. Recall that Mankiw and Miron test the expectations theory and find that the yield curve has strong predictive ability during the 1890 to 1914 period, some predictive ability during the 1915 to 1933 period, and no predictive ability after 1933. Mankiw and Miron argue that the relative success of the theory before the founding of the Federal Reserve is attributable to the greater predictable changes in the short-term rate. They suggest an explanation for the difference in performance between the two monetary regimes is that the short-term rate is approximately a random walk after the creation of the Federal Reserve but not before. Mankiw and Miron argue that in this situation the spread would always equal the term premium and that fluctuations in the spread would have no predictive power for the path of the short-term rate[14]. Mankiw and Miron speculate that the reason the short-term rate became a random walk after the creation of the Federal Reserve and remained so throughout the 1915 to 1979 period may be due to the Federal Reserve's commitment to stabilising interest rates.

Hardouvelis (1988) uses weekly Treasury bill rates with maturities of one to twenty-six weeks in the future to examine the predictive power of the term structure across recent monetary regimes that are characterised by different degrees of interest rate targeting. His purpose is to scrutinise the Mankiw-Miron hypothesis, which alleges the predictive power of forward rates should be greater in regimes with relatively low levels of interference. Hardouvelis investigates this using equation (2.8) and three separate monetary regimes which include periods in which the Federal Reserve targeted interest rates (January 1972 to October 1979), ceased targeting interest rates (October 1979 to October 1982) and only partially targeted interest rates (October 1982 to November 1985). Hardouvelis uses weekly data on Treasury bills to calculate two week forward rates at one week intervals from one to twenty-four weeks in the future. He finds that there are large differences in the predictive power of forward rates across the monetary regimes. Essentially, he finds that forward rates have predictive power that last for about seven weeks during the interest rate targeting regime, increase substantially and last for the entire twenty-four weeks during the period in which the Federal Reserve ceased targeting interest rates, with the coefficient

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[14] Equivalently, if the short-term rate is not at all predictable ($\sigma(E_t\Delta r_{t+1}) = 0$) then the estimate of β in equation (2.17) is zero.

estimates that only differ from unity in three out of twenty-four regressions, and remained strong lasting for fourteen-weeks during the period when the Federal Reserve only partially targeted interest rates. Hardouvelis concludes that overall the results appear to be consistent with the Mankiw-Miron hypothesis, as the predictive power of the term structure increased after 1979 and then decreased after 1982 when the Federal Reserve again targeted interest rates.

Roberds, Runkle and Whiteman (1996) use daily observations on Federal funds and Treasury bill rates for all possible combinations of maturities in the range one, thirty, sixty, ninety and one-hundred-and-eighty days, to better understand the predictive power of the spread and the stance of monetary policy. One of their aims is to see if differences in operating procedures can explain the Campbell-Shiller (1991) finding, which is that average future short-term rates do not change as much from the current short-term rate as the current yield spread predicts they will. Roberds, Runkle and Whiteman investigate this using equation (2.13) and three different operating regimes over the 1974 to 1991 period. These regimes include the Federal funds targeting regime (January 2, 1975, to October 3, 1979), the non-borrowed reserves targeting regime (October 11, 1979, to October 6, 1982) and the present borrowed reserves targeting with contemporaneous accounting regime (February 2, 1984, to July 24, 1991). Roberds, Runkle and Whiteman find there are important differences among the recent monetary regimes and the information content in the yield curve at the short end of the term structure. Specifically, they find the term structure was not informative during the Federal funds targeting regime, as evidenced by the low spread coefficients which are in most cases one standard error of zero. It was informative during the non-borrowed targeting regime, with slope coefficients generally being within one standard error from unity, and there is information in the more recent regime, although the spread displays the characteristic pattern found by Campbell and Shiller (1991). Roberds, Runkle and Whiteman conclude that their results are consistent with the idea advanced by Mankiw and Miron (1986), that the information content in the term structure is strongly linked to volatility in short-term rates. Roberds, Runkle and Whiteman argue that this effect shows up as the estimates of the slope coefficients are generally larger for the volatile 1979 to 1982 period than is the case in periods in which the Federal Reserve was aggressively smoothing interest rates.

2.5 Conclusion

This chapter has dealt with the relevant term structure theories and explained the implications of the expectations theory when the long-term rate is an arithmetic average of current and expected future short-term rates. One of these implications is that forward rates that are implicit in the term structure should reflect the market's expectations of future short-term rates, plus possibly a constant that is required to induce market participants to hold longer-term bonds. In this case, all bonds should have a one-period holding return equal to the short-term rate for that period plus a constant. Campbell and Shiller (1991) generalised this equality to m-periods and found that the n-period rate is a constant term premium, plus an average of current and expected m-period rates over $n - m$ periods. Campbell and Shiller also derived the implications for the expectations theory for the relation between the spread and subsequent movements in short-term and long-term rates. In the former case it was found that these implications can be tested using single-equation, VAR and VEC models. The summary of the empirical evidence under each approach reveals the general finding that almost all empirical evidence statistically rejects the short-run predictions of the expectations hypothesis, although there is general empirical support when testing the long-run validity of the theory. In fact, the former of the two findings is not just limited to the brief summary of the recent literature that is presented here. Shiller (1990), and Campbell and Shiller (1991) point out that it is a widely documented fact that empirical evidence does not support the expectations theory.

Two possible explanations for the empirical failure of the expectations theory were provided. One of these possibilities focuses on the effect that a time-varying term premium can have on yield spread regressions. Here it was shown that a non-constant term premium could bias downwards the coefficient on the spread, and thus lead to the rejection of the expectations hypothesis. The literature on yield spread regressions that include a proxy for the term premium was reviewed, to see if a time-varying term premium is enough to reconcile the theory with the data. A finding from this review is that at no time could the expectations theory be rejected. One convenient method used by Driffill, Psaradakis and Sola (1997) assumes that the inclusion of a white-noise error term in equation (2.13) can capture the effects of a time-varying term premium. This has the advantage over other approaches, as it allows one to test the expectations theory in the VAR framework. A second explanation for the failure of the expectations theory is related to the stance of

monetary policy. This explanation suggests that the monetary authorities' commitment to stabilising interest rates could influence empirical tests, and in periods where the degree of interference is large the expectations theory will perform poorly. Subsequent empirical research finds credence for this conjecture, with the expectations theory performing better in monetary regimes that are characterised by low levels of interest rate targeting.

The literature provides two possible avenues in which to explore the rehabilitation of the expectations theory. However, since this paper uses interest rate data that have been collected after the enactment of the Reserve Bank Act, and there has been no change in how the Reserve Bank implements monetary policy, only one avenue remains to be explored. This paper, therefore, intends to investigate if the data are consistent with a weaker form of expectations theory, one that includes the possibility of a time-varying term premium. As with Driffill, Psaradakis and Sola (1997) the white-noise error term will be used to proxy the term premium. This has an advantage over Simon's (1989) and Tzavalis and Wickens' (1997) approaches, as it allows one to evaluate the expectations theory using the Campbell and Shiller (1987) VAR methodology. Driffill, Psaradakis and Sola (1997) provide several tests in which one can evaluate the expectations theory using single-equation and VAR models, and these tests will be discussed in Chapter Three. In addition to Driffill, Psaradakis and Sola's tests, this thesis intends to utilise methods developed by Johansen (1988) to test the long-run validity of the expectations theory. The cointegrating literature provided two metrics for assessing the expectations theory, and these too will be explained in greater detail in Chapter Three.

Chapter Three

Methodology

3.1 Introduction

This chapter develops the approach that will be used to test the validity of the expectations theory using models that incorporate measures of a time-varying term premium. The chapter begins by introducing some of the necessary issues that arise when working with time series data. These issues concern the underlying data generating properties of the data, and it is necessary to investigate these properties as they may influence standard inference procedures and the correct approach to econometric modelling. This chapter then provides a description of the data along with the tests that will be used to evaluate models of the weaker form of the expectations theory.

3.2 Investigating the underlying properties of the data

Before econometric modelling takes place it is crucial to investigate the underlying time series properties of the variables of interest. The reason is that in order to apply standard inference procedures to a dynamic time-series model, one needs the variables to be stationary, especially since the majority of econometric theory is built upon this assumption. The following sections investigate exactly what is a stationary and non-stationary series, and what are the econometric implications from including non-stationary variables in a regression model. Section 3.2.3 then proposes a variety of methods that will be used to determine the stationarity of a variable. In the event the stationarity assumption is violated it will be necessary to use appropriate methods for transforming a non-stationary into a stationary variable. Sections 3.2.4 to 3.2.5 outline these methods and provide tests that can be used to determine the correct approach.

3.2.1 Stationary and non-stationary variables

Following Johnston and DiNardo (1997), the implications of stationarity and non-stationarity can be considered with the following model:

$$y_t = \alpha + \rho y_{t-1} + u_t \qquad (3.1)$$

which can also be expressed as:

$$(1 - L\rho)y_t = \alpha + u_t \qquad (3.2)$$

where L is the standard lag operator and u_t is the stochastic error term that is NID($0, \sigma^2$).

In order for equations (3.1) and (3.2) to be stationary around a drift, the root of $(1 - L\rho) = 0$ needs to exceed one in absolute terms. As the root is $L = |1/\rho|$, this requirement will only be met if $|\rho| < 1$. If one rewrites equation (3.1) as[15]:

$$y_t = \alpha(1 + \rho + \rho^2 + ... + \rho^{t-1}) + \rho^t y_0 + (u_t + \rho u_{t-1} + ... + \rho^{t-1}u_1), \quad (3.3)$$

it may be shown that if the stationarity condition is satisfied, the mean, variance and covariances of the y_t series are all constants, independent of time.

If $|\rho| = 1$, the y_t series is said to have a unit root or is non-stationary with a drift. In the unit root case it can easily be established that taking expectations of both sides of equation (3.3) yields:

$$E(y_t) = \alpha t + y_0, \qquad (3.4)$$

and so the mean of the y_t series changes with time. Squaring both sides of equation (3.3) and taking expectations, one can compute the variance as:

$$\text{var}(y_t) = [(y_t - E(y_t))^2] = E[(u_t + u_{t-1} + ... + u_1)^2] = t\sigma^2, \qquad (3.5)$$

which illustrates that the variance increases with t and becomes infinite as $t \to \infty$.

If a stationary series experiences a shock, the effects will dissipate and the series will return to its long-run levels. If a series is non-stationary, however, one can readily verify from equation (3.3) that shocks in the distant past get the same weight as the initial value. In this case shocks may persist so that the mean and variance cannot return to their long-run levels. The implications of non-stationarity in series to be used in regression models will be taken up in the next section.

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[15] Assuming the initial condition is y_0, this can be obtained with the iterative technique.

3.2.2 Spurious regressions

Granger and Newbold (1974) were among the first to alert many to the econometric implications of non-stationarity. Granger and Newbold found these implications by estimating:

$$y_t = \beta_0 + \beta_1 x_t + u_t,$$ (3.6)

where y_t and x_t were generated as *independent* random walks. In their Monte Carlo analysis, Granger and Newbold found they were able to reject the null of no relationship in three-quarters of all occasions. Another distinctive feature of these regressions was their tendency to have high R^2 values, and to also exhibit high degrees of serial correlation. This tendency lead Granger and Newbold to specify a convenient method for identifying a so-called spurious regression. They suggest that if the R^2 exceeds the Durbin-Watson (1951) *d*-statistic, the regression is likely to be spurious.

Granger and Newbold argue that the essence of the problem is that if $\beta_1 = 0$, and one attempts to fit equation (3.6) with non-stationary variables, then our customary tests for statistical significance no longer hold, because these statistics no longer follow their standard distributions under the null. This follows since under that hypothesis the residuals from equation (3.6):

$$u_t = y_t - \beta_0$$ (3.7)

will have the same unit root properties as the y_t series.

The preceding discussion highlights the importance of testing time series variables for the presence of a unit root before including them in a regression model. There are several methods that can be used to test a variable for its stationarity, and these methods are discussed below.

3.2.3 Testing for unit roots

According to Johnston and DiNardo (1997), one method for detecting if a series has a unit root is to evaluate a time-series graph of the series, although they add that such an approach is often highly subjective and misleading, and thus advise testing for unit roots using more formal procedures. This thesis intends to employ both of these approaches, with the latter

method including tests by Dickey and Fuller (1979; 1981) and Kwiatkowski, Phillips, Schmidt and Shin (1992). These tests are the subject of the ensuing sections.

3.2.3.1 Dickey and Fuller's unit root test

Dickey and Fuller (1979; 1981) devised a procedure which enables one to ascertain if ρ in equation (3.1) is equal to one (ie non-stationary) or not. It should be recognised that one cannot simply test for a unit root with the ratio, $\hat{\rho} - 1/s.e.(\hat{\rho})$, as under the null hypothesis the distribution of this test statistic is non-standard. Dickey (1976) and Fuller (1976) solved this problem by tabulating valid asymptotic critical values in several important cases.

If one subtracts y_{t-1} from both sides of equation (3.1) a more convenient expression is:

$$\Delta y_t = \gamma y_{t-1} + u_t , \qquad\qquad (3.8)$$

where testing the hypothesis $\rho = 1$ is now equivalent to testing $\gamma = 0$ (ie $\gamma = (\rho - 1)$). The validity of this testing procedure depends on the assumption that u_t is NID(0, σ^2). If, however, the data generating process (DGP) for y_t is higher than the autoregressive of order one process AR(1) assumed, then u_t will be serially correlated. In such cases Dickey and Fuller (1979; 1981) suggest OLS estimation of the re-parameterised model:

$$\Delta y_t = \gamma y_{t-1} + \sum_{j}^{p} \delta_j \Delta y_{t-j} + u_t \qquad\qquad (3.9)$$

where p is selected so the residuals are serially independent. A number of criteria have been proposed for allowing the data to determine the distributed lag. This paper intends to use the Schwarz (1978) Bayesian Criterion (SBC). This criterion can be calculated as:

$$SBC = T \times \ln(\text{residual sum of squares}) + n \times \ln(T), \qquad\qquad (3.10)$$

where n = the number of parameters estimated, T = the number of useable observations and p is that value which results in min SBC.

Recall the null hypothesis is that $\gamma = 0$ which, if true, implies that $\rho = 1$ in equation (3.1) or, equivalently, that y_t is non-stationary or y_t has a unit root or $y_t \sim I(1)$. The alternative hypothesis is that $\gamma < 0$ which, if true, suggests that y_t is stationary or $y_t \sim I(0)$. This null hypothesis is tested by using the results obtained from OLS estimation of equation (3.9) to compute the ADF test statistic:

$$\tau = \frac{\hat{\gamma}}{se(\hat{\gamma})} \qquad\qquad (3.11)$$

which is then compared to its Dickey-Fuller critical value under the null at the chosen significance level. Rejection of the null is indicated when the computed test statistic has a larger negative value than the one-sided critical value.

It should be mentioned that when testing the stationarity of a series it is also imperative to test for the presence of a drift, β_0, and deterministic trend, β_1, in the DGP. This is essential as differencing a deterministic series or including a trend when the series is stochastic, will not yield a stationary series. Dickey and Fuller (1979; 1981) allowed for these possibilities by generalising their model to include a drift and trend. The other class of models they considered are:

$$\Delta y_t = \beta_0 + \gamma y_{t-1} + \sum_{j}^{p} \delta_j \Delta y_{t-j} + u_t \qquad\qquad (3.12)$$

$$\Delta y_t = \beta_0 + \beta_1 t + \gamma y_{t-1} + \sum_{j}^{p} \delta_j \Delta y_{t-j} + u_t \qquad\qquad (3.13)$$

where p is not necessarily the same for each equation and the appropriate critical values now depend on the form of the autoregression (i.e. if there is a drift or trend). The appropriate statistics to use for equations (3.9), (3.12) and (3.13) are labelled τ, τ_μ and τ_τ, respectively. Dickey and Fuller (1981) also suggest three F-statistics to test joint hypotheses on the coefficients of equations (3.12) and (3.13). These statistics, which are denoted as ϕ_1, ϕ_2 and ϕ_3, test the respective joint null hypotheses, $\gamma = \beta_0 = 0$ in equation (3.12), $\gamma = \beta_0 = \beta_1 = 0$ and $\gamma = \beta_1 = 0$ in equation (3.13). The test statistic is computed with the usual F-statistic formula, but since it does not follow the standard F-distribution its critical values are tabulated in Dickey and Fuller (1981).

In light of the previous discussion, this paper will employ the following testing strategy which has been proposed by Dolado, Jenkinson and Sosvilla-Rivero (1990). This strategy begins by estimating the most unrestricted model (i.e. equation (3.13)) and using the τ_τ statistic to test the null of a unit root $H_0^1 : \gamma = 0$. If this is rejected there is no need to go further, if not then the joint hypothesis $H_0^2 : \gamma = \beta_1 = 0$ is tested using the ϕ_3 statistic. The non-rejection of H_0^2 implies the series is subject to a stochastic rather than deterministic

trend. In this case they suggest increasing the power of the test by imposing the restriction that $\beta_1 = 0$ and using equation (3.12) to test $H_0^3 : \gamma = 0$ with τ_μ. If the null is rejected the procedure terminates, if not it suggests the series is generated by a random walk with drift. To check for significant drift one tests $H_0^4 : \gamma = \beta_0 = 0$ using the ϕ_1 statistic. Again if this hypothesis is not rejected one can increase the power of the test by estimating equation (3.9) and testing $H_0^5 : \gamma = 0$ using τ. The non-rejection of this hypothesis suggests that y_t contains a unit root.

It is essential to recognise that the non-rejection of H_0^5 suggests that y_t is integrated to at least order one, although it may be integrated of higher orders. It is important to correctly ascertain the order of a series because series which are integrated of higher orders will not be stationary by first differencing. To test if the series is integrated of order two one can apply the ADF procedure to Δy_t with:

$$\Delta^2 y_t = \beta_0 + \beta_1 t + \gamma \Delta y_{t-1} + \sum_j^p \delta_j \Delta^2 y_{t-j} + u_t , \qquad (3.14)$$

where the null hypothesis is now Δy_t is non-stationary. As before one can apply the Dolado, Jenkinson and Sosvilla-Rivero (1990) testing procedure to equation (3.14) which nests both the trend stationary and difference stationary hypotheses.

3.2.3.2 KPSS unit root test

Kwiatkowski, Phillips, Schmidt and Shin (1992) propose an alternative test with a null hypothesis that a series is stationary around a deterministic trend against the alternative of a unit root. The KPSS test assumes that a series can be decomposed into the sum of a deterministic trend, a random walk and a stationary error. That is:

$$y_t = \delta t + z_t + \varepsilon_t , \qquad (3.15)$$

where $z_t = z_{t-1} + u_t ,$

the u_t are IID($0, \sigma_u^2$) and the initial value of z_0 is treated as fixed and serves the role of an intercept. The test for stationarity is a test of the hypothesis that the random walk has zero variance (i.e. $\sigma_u^2 = 0$). Since ε_t is assumed to be stationary, under the null hypothesis y_t is trend stationary. KPSS also consider the special case in which they set δ in equation (3.15)

to zero, so that under the null hypothesis y_t is now stationary around a level (i.e. z_0). KPSS test the hypothesis that $\sigma_u^2 = 0$ with the following one-sided LM test statistic:

$$\text{LM} = \sum_{t=1}^{T} S_t^2 / \hat{\sigma}_\varepsilon^2 , \tag{3.16}$$

where $S_t = \sum_{j=1}^{T} e_j$, $t = 1, 2,..., T,$

is defined as the partial sum process of the residuals and the e_t terms are the residuals obtained by regressing the series on either a constant and trend or a constant only. Similarly, $\hat{\sigma}_\varepsilon^2$ is the estimate of the error variance obtained from either the regression with or without a trend. The validity of the LM test depends on the assumption that the errors are IID(0, σ_ε^2), since this is required for $\hat{\sigma}_\varepsilon^2$ in the denominator of equation (3.16) to converge in probability to σ_ε^2. KPSS, however, question the validity of the IID assumption, alleging that series to which the stationarity tests will be applied are typically highly dependent over time. As a consequence, KPSS propose a modified version of equation (3.16) that is valid under more general conditions. The statistic is:

$$\text{LM} = \eta / s^2(l) = T^{-2} \sum S_t^2 / s^2(l) , \tag{3.17}$$

where $s^2(l) = T^{-1} \sum_{t=1}^{T} e_t^2 + 2T^{-1} \sum_{s=1}^{l} w(s,l) \sum_{t=s+1}^{T} e_t e_{t-s}$

is a consistent estimator of σ_ε^2 when the errors are not IID, and $w(s, l)$ is an optional weighting function that corresponds to the Bartlett window $w(s,l) = 1 - s/(l+1)$, as in Newey and West (1987).

To test the stationarity of a series one computes the LM statistic and compares this value to the one-sided critical value given in Kwiatkowski, Phillips, Schmidt and Shin (1992). If the LM statistic exceeds the critical value the null hypothesis of trend stationarity can be rejected in favour of the unit root alternative. As before, the rejection of the null indicates that the series is integrated to at least order one. To test for higher possibilities one can apply the KPSS test to the series in differences.

3.2.4 Treatment of non-stationary series

This section discusses several methods of removing the trend component from a non-stationary series. The trend can be expressed as having stochastic and/or deterministic components, and the form of the trend has important implications for the appropriate transformation to attain a stationary series. The following sections deal with common forms of trend along with the appropriate methods of removal.

3.2.4.1 Difference stationary and trend stationary series

Following Greene (1993), one can reasonably characterise the movements of many macroeconomic time series by a random walk with drift:

$$y_t = \alpha + y_{t-1} + u_t \tag{3.18}$$

or as a trend stationary (TS) process:

$$y_t = \alpha + \beta t + u_t, \tag{3.19}$$

where in both cases u_t is a white noise process. Clearly, both of these stochastic processes will produce strongly trending non-stationary series. If one rewrites equation (3.18) as:

$$y_t = y_0 + \alpha t + \sum_{j=1}^{t} u_j, \tag{3.20}$$

the non-stationarity components are found to consist of a deterministic trend, $y_0 + \alpha t$, and the stochastic trend, $\sum_{j=1}^{t} u_j$. In this case, subtracting the deterministic trend from each observation will not result in a stationary series, as the stochastic trend has not been eliminated. However, the non-stationarity can be removed by taking first differences of y_t. This yields:

$$\Delta y_t = \alpha + u_t \tag{3.21}$$

which is a difference stationary (DS) process. The series is then said to be integrated of order one I(1) since taking first differences yields a stationary series. In general, a series is integrated of order d, denoted I(d), if the series becomes stationary after differencing d times.

The trend stationary process in equation (3.19) differs from equation (3.18), as the source of the non-stationarity can be attributable only to a deterministic trend, $(\alpha + \beta t)$. As such, the correct strategy for attaining a stationary series would be to remove the deterministic trend, so that:

$$u_t = y_t - \alpha - \beta t \tag{3.22}$$

is then stationary. If instead one decides to take first differences of equation (3.19), the result will be to trade the trend for autocorrelation in the form of a moving average of order one MA(1) process.

It should be recognised that the appropriate method of transformation may not be immediately obvious. This, coupled with the problems of taking the incorrect approach, imply that some means of choosing between a difference stationary and trend stationary series is needed. Fortunately, this can be achieved with Dickey and Fuller's equation (3.13) which allows one to test the DS hypothesis against the TS hypothesis with the ϕ_3 statistic.

3.2.4.2 Cointegration

Granger (1981) first recognised that the treatment of variables that contain unit roots is not so straightforward. Granger points out that a vector of I(1) variables may have linear combinations that are already stationary without differencing. Engel and Granger (1987) formalised the idea of variables sharing an equilibrium relationship in terms of cointegration between time series. Following Engel and Granger, this can be seen by considering two time series y_t and x_t which are both I(d). In general, any linear combination of such variables:

$$u_t = y_t - \delta x_t \tag{3.23}$$

will also be I(d). Although it is possible that there exists a vector $(1 - \delta)'$, such that $u_t \sim$ I($d - b$), $b > 0$. If this occurs the variables are said to be cointegrated of order (d, b), as there is a special constraint operating on the long-run components of the series. The constraint can be perceived as some kind of equilibrium relationship with forces that prevent the series from diverging too significantly from one another. Thus if the variables in equation (3.23) are involved in an equilibrium relationship, then u_t can be interpreted as the equilibrium error (i.e. the deviation from long-run equilibrium at time t). If the equilibrium is to be

meaningful the equilibrium error process should be stationary. This suggests that u_t should rarely drift far from zero and often cross the zero line, implying that equilibrium should occasionally occur.

It should be noted that there is a close relationship between cointegration and error-correction models. This finding was first pointed out by Granger (1981) and later proved in Granger (1983). The significance of what became known as the Granger Representation Theorem alleges that if one finds evidence of cointegration there must be an error correction representation (ECM). Therefore, if y_t and x_t are CI(1, 1), one can express this in an ECM representation of the following form:

$$\Delta y_t = \alpha u_{t-1} + \gamma_{1i}(L)^i \Delta y_{t-i} + \beta_{1i}(L)^i \Delta x_{t-i} + \varepsilon_t, \tag{3.24}$$

where $u_{t-1} = y_{t-1} - \delta x_{t-1}$. The ECM states that changes in y_t depend not only on the lagged changes in y_t and x_t, but also on the extent of the disequilibrium between y_t and x_t. The appeal of such a formulation is that it captures the long-run dynamics of the system. This contrasts sharply with the outcome of differencing two variables which are CI(1, 1), as it is impossible to infer the long-run steady state equilibrium. To avoid the misspecification errors associated with differencing two (or more) variables that are linked in an equilibrium relationship, the ensuing section introduces tests for cointegration.

3.2.5 Testing for cointegration

This paper intends to test for cointegration using both the Engel and Granger (1987) and Johansen (1988) methodologies. These tests are explained in the subsequent sections.

3.2.5.1 The Engel-Granger test for cointegration

Engel and Granger (1987) propose a two-step estimation procedure to determine if two or more variables are cointegrated. If one considers equation (3.23) when y_t and x_t are both I(1), the variables would be said to be cointegrated if there is a cointegrating vector δ such that the residuals are I(0). In the first step one needs to estimate δ in order to use the coefficient estimates in the tests of the equilibrium relationship. Engel and Granger show that a consistent estimate of δ can be obtained by estimating:

$$y_t = \delta x_t + u_t \tag{3.25}$$

which is called the cointegrating regression. Stock (1987) proves that if the variables are cointegrated then an OLS regression yields a super-consistent estimator of δ, as the OLS estimator converges much faster in the non-stationary case.

If the variables are cointegrated, the cointegrating residual $\hat{u}_t$ should be stationary. Accordingly, the second step in the procedure is to test the residual for a unit root. Engel and Granger propose seven statistics for this purpose, although essentially they advocate testing with the ADF statistic applied to $\hat{\alpha}$ in the following equation:

$$\Delta \hat{u}_t = \alpha \hat{u}_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta \hat{u}_{t-i} + \varepsilon_t , \qquad\qquad (3.26)$$

where p is chosen to ensure ε_t is white noise[16]. As with univariate unit root tests, the null hypothesis of a unit root and thus the null of no cointegration is based on the t-ratio on $\hat{\alpha}$. However, Engel and Granger note that the critical values computed by Dickey (1976) and Fuller (1976) are no longer applicable, as the critical values reject the null of non-cointegration too often if $\hat{\delta}$ must be estimated. Fortunately, Engel and Granger (1987) and MacKinnon (1991) have tabulated the appropriate critical values for regressions involving two or more variables.

It should be pointed out that despite the relative ease in implementing this procedure there are potential problems involved in its use. In relation to this thesis, one particular problem relates to the presence of the two step estimator. In Particular, any errors introduced in generating the equilibrium errors, will also be carried into the ADF test for cointegration (Enders, 1995). Another problem is that one is unable to test restrictions on the cointegrating vector. This, however, is most important from this paper's perspective, as it allows one to test the expectations theory by drawing statistical inference concerning the magnitudes of the estimated long-run coefficients. Fortunately, Johansen (1988) has developed a maximum likelihood approach that can avoid these problems, and this approach is discussed in the following section.

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[16] The question of whether to include a constant or trend in the ADF statistic depends on whether a constant or trend appears in (3.25), as deterministic components can be added to (3.25) or (3.26) but not both (Harris, 1995).

3.2.5.2 The Johansen technique

This section deals with the Johansen maximum likelihood approach for testing for cointegration. Essentially, this involves a consideration of the procedures for determining the cointegration rank of a series, obtaining the maximum likelihood estimates of the parameters of these vectors, and carrying out valid statistical tests of restrictions on the estimated long-run equilibrium parameters and estimated speed of adjustment parameters.

3.2.5.2.1 The Johansen approach

The Johansen (1988) procedure begins by formulating the p-dimensional vector autoregressive (VAR) model of the form:

$$x_t = \Pi_1 x_{t-1} + ... + \Pi_k x_{t-k} + \mu + \varphi D_t + u_t \qquad (3.27)$$

where x_t is a vector of p variables, Π_i is a $p \times p$ matrix of parameters, μ is a vector of constants, D_t is included to take account of any short-run shocks and $u_1,..., u_t$ are $IIN_p(0, \Lambda)$[17]. Following Johansen (1995) one can re-parameterise equation (3.27) into a vector error-correction (VEC) model of the form:

$$\Delta x_t = \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Pi x_{t-k} + \mu + \varphi D_t + u_t \qquad (3.28)$$

where $\Gamma_i = -\sum_{j=i+1}^{k} \Pi_j$ and $\Pi = \sum_{i}^{k} \Pi_i - I$.

In this representation the Γ_i matrix contains the contemporaneous short-run adjustment parameters for the variables in Δx_{t-i}, whereas the parameters in the Π matrix contain information about the long-run equilibrium relationships amongst the variables in x_{t-k}. If one assumes that x_t is a vector of I(1) variables, then equation (3.28) can be used to distinguish between stationarity by differencing and by linear combinations. Essentially, the correct approach depends on the rank of Π as this gives the number of distinct cointegrating vectors.

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[17] In this case p will be selected using a mulivariate generalisation of the SBC.

Following Johansen and Juselius (1990), there are three distinct cases to consider. The first case is when Π has full rank (i.e. there are $r = p$ linear combinations of the variables in x_t which are I(0)), which implies that the vector x_t is already stationary. In this situation, there is no threat of estimating a spurious regression and the correct modelling strategy would be to estimate a VAR in levels (i.e. equation (3.27)). The second case is when the rank of Π is zero (i.e. there are no linear combinations of x_t that are I(0)) and there is no cointegration. As $x_t \sim$ I(1), this suggests that the appropriate modelling strategy would be to estimate equation (3.28) with no long-run components (i.e. with Πx_{t-k} set to zero). The final case is when Π has reduced rank, $0 < \text{rank}(\Pi) = r < p$, and can be decomposed into the product of two $p \times r$ matrices α and β such that $\Pi = \alpha\beta'$. Expressed in this way, α represents the speed of adjustment to disequilibrium while β is a matrix of long-run coefficients such that $\beta'x_{t-k}$ comprises the $r \leq (p - 1)$ cointegrating relationships (i.e. $\beta'x_{t-k} \sim$ I(0)). The subsequent section introduces the procedure by which Johansen (1988) obtains a maximum likelihood estimate (MLE) of $\Pi = \alpha\beta'$ and conducts tests for reduced rank.

3.2.5.2.2 MLE procedure and testing for reduced rank

Following Johansen (1988), a maximum likelihood estimate of Π can be obtained from the results of two sets of OLS regressions that are intended to remove any short-run dynamics on Δx_t and x_{t-k}. In both instances these effects can be removed by regressing Δx_t and x_{t-k} separately on the lagged differences, obtaining the residual matrices R_{0t} and R_{kt} for the former and latter regressions. This defines the residual product moment matrices of the residuals as:

$$S_{ij} = T^{-1}\sum_{i=1}^{T} R_{it} R'_{jt} \quad i,j = 0,\ldots, k. \tag{3.29}$$

After performing these regressions the concentrated likelihood function has the form of a reduced rank regression:

$$R_{0t} = \alpha\beta' R_{kt} + u_t. \tag{3.30}$$

For fixed β, equation (3.30) can be solved for α by regression:

$$\hat{\alpha}(\beta) = -S_{ok}\beta(\beta''S_{kk}\beta)^{-1}, \tag{3.31}$$

and the estimate of β can be obtained by solving:

$$\left| \lambda S_{kk} - S_{ko} S_{00}^{-1} S_{0k} \right| = 0 \tag{3.32}$$

for the p eigenvalues, $\hat{\lambda}_1 > \ldots \hat{\lambda}_p$ and the corresponding eigenvectors:

$$\hat{v} = (\hat{v}_1, \hat{v}_2, \ldots, \hat{v}_p), \tag{3.33}$$

in which $\hat{v}$ is normalised such that:

$$\hat{v}' S_{kk} \hat{v} = I. \tag{3.34}$$

The estimate can then be obtained by choosing $\hat{\beta}$ to be the first largest r eigenvectors of $S_{k0} S_{00}^{-1} S_{0k}$ with respect to S_{kk} (i.e. the first r columns of $\hat{v}$). These are called the canonical variates, and the eigenvalues are the squared canonical correlations between the residuals of x_{t-k} and Δx_t corrected for the effect of the lagged differences of x_t. The eigenvectors are normalised by the condition $\hat{\beta}' S_{kk} \hat{\beta} = I$ so that the estimate of the α and Π matrices is then given by:

$$\hat{\alpha} = S_{ok} \hat{\beta} \tag{3.35}$$

and $\hat{\Pi} = S_{ok} \hat{\beta} \hat{\beta}'$ (3.36)

Following Harris (1995), the preceding discussion suggests that $\hat{\lambda}_i$ serves as a measure of how strongly the cointegrating relations $\hat{v}' x_t$ ($i = 1, \ldots, r$) and the stationary $\Delta x_t \sim I(0)$ elements are correlated. Specifically, high correlations between the distinct $\hat{v}' x_t$ ($i = 1, \ldots, r$) combinations of I(1) levels in x_t and Δx_t indicate the cointegrating vectors, as in order to achieve such a high correlation they themselves must be I(0). This means that $\hat{v}' x_t$ ($i = r + 1, \ldots, p$) indicate the non-stationary combinations which are uncorrelated with Δx_t. Therefore, for the eigenvectors corresponding to the non-stationary part of the model $\hat{\lambda}_i = 0$ for $i = r + 1, \ldots, p$. Since each eigenvector, $\hat{v}_i$, has a corresponding eigenvalue, $\hat{\lambda}_i$, testing for the number of cointegrating vectors amounts to testing for the number of statistically significant $\hat{\lambda}_i$ terms. Johansen and Juselius (1990) identify two likelihood ratio test statistics for this purpose. These are the λ_{trace} and λ_{max} statistics:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{p} ln(1 - \hat{\lambda}_i) \qquad (3.37)$$

$$\lambda_{max}(r, r + 1) = -T \, ln(1 - \hat{\lambda}_{r+1}). \qquad (3.38)$$

where the λ_{trace} statistic tests the null hypothesis of r cointegrating vectors against a general alternative, and the λ_{max} statistic tests the same null against the alternative of $r + 1$. The testing procedure for both of these statistics is to start by testing the null hypothesis that $H_o^1: r = 0$, against the appropriate alternative. If the null hypothesis is rejected the procedure is then to test $H_o^2: r \leq 1$, the rejection of which implies the existence of at least one cointegrating vector. The testing procedure continues in this manner and only stops whenever the null hypothesis of r cointegrating vectors is *not rejected.*

Johansen and Juselius (1990) provide the critical values for both these statistics, and these values depend on the number of non-stationary components under the null, $(p - r)$, and the form of the μ vector. In particular, they depend on whether a constant and trend appear in equation (3.28), and if these terms enter the short or long-run components of the model. This suggests that before any such tests can be carried out it is necessary to ascertain where these deterministic terms lie. Fortunately, there exists a strategy to test for the inclusion of deterministic terms while jointly testing for the rank of Π. This is presented in the following section.

3.2.5.2.3 Testing for deterministic components and the rank of Π

Following Hansen and Juselius (1994), one can consider the necessary options by reformulating equation (3.28) as:

$$\Delta x_t = \Delta x_{t-1} + \alpha \begin{bmatrix} \beta \\ \mu_1 \\ \delta_1 \end{bmatrix} \tilde{x}_{t-k} + \alpha_{\perp}\mu_2\delta + \alpha_{\perp}\delta_2 t + u_t \qquad (3.39)$$

in which $\tilde{x}'_{t-k} = (x'_{t-2}, 1, t)$, and for notational simplicity $k = 2$ and the deterministic terms in D_t are omitted. In this representation, μ_1 and δ_1 allow for the effects of an intercept and linear trend term in the cointegration space, whereas μ_2 and δ_2 permit the effects of an intercept and quadratic trend in the short-run model. It should be noted that one can specify a model in which $\mu_1 = \mu_2 = \delta_1 = \delta_2 = 0$, although this is unrealistic as the intercept is

generally needed to account for the unit of measurement for the variables. In practice only three configurations regarding intercepts and trends are considered:

- Model two does not allow for linear trends in the data, so that $\delta_1 = \delta_2 = \mu_2 = 0$. Therefore, the only deterministic components in the model are the intercepts in the cointegrating relations.

- Model three does allow for linear trends in the data through μ_2, implying $\delta_1 = \delta_2 = 0$. In this case it is assumed that the intercept in the cointegrating relation, μ_1, is cancelled by the intercept in the short run model, μ_2, leaving only μ_2 (i.e. in estimating equation (3.39), μ_1 is combined with μ_2 providing an overall intercept in the short-run model).

- Model four does not allow for quadratic trends in the data, so that the only restriction is $\delta_2 = 0$. However, having $\delta_1 \neq 0$ means that cointegrating space now has a linear trend. This means the model will allow for trend stationary variables, and this can be in the form of a single variable or an equilibrium relation.

The strategy for testing where the deterministic terms lie while jointly testing the rank of Π is discussed in Johansen (1992), and it follows the so-called Pantula principle. Essentially, this strategy involves estimating all three models and presenting the results from the most restrictive alternative (i.e. $r = 0$ and model two) through to the least restrictive alternative (i.e. $r = p - 1$ and model four.) The procedure is then to start with the most restrictive model and move through to $r = p - 1$ and model four, and at each stage compare the λ_{trace} and/or λ_{max} test statistics with the chosen quantile of the corresponding table. The procedure only stops the first time the null hypothesis of r cointegrating vectors is *not rejected*. Osterwald-Lenum (1992) provides the critical values for models two to four, and these critical values are valid only when the deterministic terms are limited to centred seasonal dummies and intercepts.

Once the correct model and the number of cointegrating relations is selected, the chosen model is estimated and one can proceed to test restrictions on the α and β parameters. Testing these parameters and the principle behind the tests are the topic of the subsequent section.

3.2.5.2.4 Testing restrictions on the α and β parameters

Following Johansen and Juselius (1990), one can test various restrictions on the α and β parameters by comparing the number of statistically significant eigenvalues in the unrestricted estimation of the VEC model to the number in the restricted. The form of the test is:

$$LR = T\sum_{i}^{r}[\ln(1-\hat{\lambda}_i^*) - \ln(1-\hat{\lambda}_i)] \qquad (3.40)$$

in which the statistic has an asymptotic χ^2 distribution under the null with degrees of freedom equal to the number of restrictions placed on either α and β, and where $\hat{\lambda}_i^*$ are the estimated eigenvalues in the restricted version of the model.

Following Harris (1995), tests of restrictions on the α parameters can be interpreted as tests for *weak exogeneity*. Suppose that $z_t = [y_{1t},\ y_{2t},\ x_t]'$, $r = 2$ and $k = 2$ so that writing the VEC model in full gives (excluding the deterministic components):

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta x_t \end{bmatrix} = \Gamma_1 \begin{bmatrix} \Delta y_{1t-1} \\ \Delta y_{2t-1} \\ \Delta x_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix}. \qquad (3.41)$$

In this equation each of the α_{ij} parameters represents the speed at which the *ith* variable adjusts towards the *jth* error-correction term, while β_{ij} is the long-run parameter for the *ith* levels variable in the *jth* error-correction term. Thus, if $\alpha_{ij} = 0$, the first difference of the *ith* equation does not respond to the *jth* error-correction term. If the presence of zeros is extended to row i for all $\alpha_{ij}, j = 1, ..., r$, this indicates that the cointegrating vectors in β do not enter the equation determining Δz_{ij}. This means when estimating the parameters of the model there is no loss of information from not modelling the determinants of Δz_{ij}.; thus, this variable is weakly exogenous to the system and can enter the right-hand-side of the VECM. To test for weak exogeneity requires a test that H: $\alpha_{ij} = 0$ for $j = 1, ..., r$, that is, row i contains zeros. This test can be carried out with a likelihood ratio test involving the restricted and unrestricted models. The procedure is to restrict α and compare the r most significant eigenvalues for the restricted and unrestricted models using equation (3.40). If

the calculated value of equation (3.40) exceeds the χ^2 critical value, with degrees of freedom equal to the number of restrictions placed on α, the restrictions can be rejected.

A particularly important aspect of the Johansen technique is that it allows one to test restricted forms of the cointegrating vector β. As before, the procedure is to restrict β and then compare the r most significant eigenvalues for the restricted and unrestricted models using equation (3.40). The key insight into such tests is that if there are r cointegrating vectors, only these r cointegrating vectors will be stationary. This implies that all values of $\ln(1 - \hat{\lambda}_i^*)$ and $\ln(1 - \hat{\lambda}_i)$ should be equivalent if the restrictions are true. Hence, small values of the LR statistic imply that it is permissible to apply the restriction to the cointegrating vector.

3.3 Testing the expectations theory of the term structure

The purpose of this section is to present the tests that will be used to evaluate the weaker form of the expectations theory in relation to the spread and future changes in short-term rates. These tests are presented using the single-equation, VAR and VEC approaches, although only tests in the former two settings can be interpreted as tests of the less restrictive expectations model. The VEC tests are still important, as under certain conditions these tests must hold for the validity of the expectations theory. The following section provides a description of the data that will be used for this empirical investigation. Sections 3.3.2 to 3.3.4 then present the single-equation, VAR and VEC tests of the expectations model.

3.3.1 The data

This paper tests the validity of the EHTS using daily Reserve Bank of New Zealand (RBNZ) interest rate data on one-month and three-month Treasury bills, over the period 3 of January, 1990 to 6 of June, 1997. It should, however, be noted that due to necessary data transformations and the dynamic structure of the estimated models, the effective sample period is reduced to 6 March, 1997.

3.3.2 A single-equation test of the expectations theory

According to Driffill, Psaradakis and Sola (1997) a weaker form of the expectations theory occurs when the term premium contains an element which varies randomly over time, independently of short-term rates. Consequently, they suggest replacing equation (2.11) by:

$$R_t^n = \frac{1}{k}\sum_{i=0}^{k-1} E_t r_{t+mi}^m + \theta + u_t \qquad\qquad k = n / m \qquad\qquad (3.42)$$

and equation (2.13) by:

$$S_t^{(n,m)} = \sum_{i}^{k-1}(1-\frac{i}{k})\Delta r_{t+im}^m + \theta + u_t, \qquad\qquad (3.43)$$

where u_t is a white-noise error term that may be thought of as a proxy for the time-varying term premium.

Driffill, Psaradakis and Sola identify a number of methods for testing the empirical validity of equations (3.42) and (3.43). Among these methods they advise estimating equation (3.43) directly, by replacing $E_t\Delta r_{t+im}^m$ by the realisations Δr_{t+im}^m and then using an instrumental variables (IV) estimator. This corresponds to McCallum's (1976) and Wickens' (1982) 'errors in variables' method for estimating rational expectations models. The idea being to replace the expected values by their realised values so that the resulting equation can be estimated by IV using predetermined variables as instruments. Driffill, Psaradakis and Sola (1992) suggest that when u_t is serially uncorrelated valid instruments include a constant, $S_{t-i}^{(n,m)}$ and Δr_{t-i+1}^m, for $i \geq 1$.

After estimating equation (3.43) and conducting the various misspecification tests to ensure the model is well defined statistically, one can proceed to consider the validity of the expectations theory restrictions. If the expectations theory is valid, the estimated coefficients on $E_t\Delta r_{t+im}^m$, for $i = 1, ..., k - 1$, should not differ significantly from their respective weights. Therefore, when $n = 3$ and $m = 1$, equation (3.43) becomes:

$$S_t^{(3,1)} = \frac{2}{3}E_t\Delta r_{t+1}^1 + \frac{1}{3}E_t\Delta r_{t+2}^1 + \theta + u_t, \qquad\qquad (3.44)$$

and it remains to test whether the estimated coefficients on $E_t \Delta r_{t+1}$ and $E_t \Delta r_{t+2}$ differ significantly from their weights of two-thirds and one-third, respectively.

3.3.3 VAR tests of the expectations theory

Driffill, Psaradakis and Sola (1997) also provide methods for testing parameter restrictions implied by the expectations theory using the VAR approach. As discussed in section 2.3.2 these restrictions arise from projecting the expectations model onto a subset of the information set used by market participants. This of course is necessary as the true information set used to compute $E_t \Delta r_{t+im}^m$, for $i = 1, ..., k - 1$, is unobservable. If the spread is included in the information subset, then equation (3.43) takes on the same form when projected on the information subset. It says the observed spread should equal the optimal forecast of future changes in short-term rates, conditional on the information subset used in this test.

As with Driffill, Psaradakis and Sola, one can see what the implications of the weaker form of expectations model are for a VAR representation for the stationary $S_t^{(n,m)}$ and Δr_t^m series (*with their means removed*), by considering a *pth*-order VAR written in companion form as:

$$x_t = A x_{t-1} + u_t \qquad\qquad (3.45)$$

$$\text{where } A = \begin{bmatrix} \alpha_1 & ... & \alpha_p & \gamma_1 & ... & \gamma_p \\ 1 & & & & & \\ & \cdot & & & & \\ & & 1 & & & \\ \delta_1 & ... & \delta_p & \lambda_1 & ... & \lambda_p \\ & & & 1 & & \\ & & & & \cdot & \\ & & & & & 1 \end{bmatrix},$$

$x_t = (S_t^{(n,m)}, ..., S_{t-p+1}^{(n,m)}, \Delta r_t^m, ..., \Delta r_{t-p+1}^m)'$, and $u_t = (u_{1t}, 0, ..., 0, u_{2t}, 0, ..., 0)'$, with u_{1t} and u_{2t} being zero mean serially uncorrelated disturbances. In this form one can easily compute multiperiod interest rate forecasts as $E[x_{t+i} / H_t] = A^i x_t$, where H_t is the limited information set containing current and lagged values of the spread and the one-period change in the *m*-period rate. One can then use equation (3.43) to compute the VAR forecast

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of the perfect foresight spread $S_t^{(n,m)^*}$. Projecting equation (3.43) onto the information subset, H_t, can be shown to impose a set of highly non-linear, cross-equation restrictions on the estimated coefficients of the VAR. When $n = 3$ and $m = 1$, Driffill, Psaradakis and Sola show these restrictions to be:

$$b'Ax_{t-1} = \frac{1}{3}d'(2A^2 + A^3)x_{t-1},$$ (3.46)

where b and d denote $2p \times 1$ selection vectors, in which all elements are zero except for the first element of b and the $p + 1$st element of d, which are unity[18]. If the expectations theory is valid, then equation (3.46) should hold whatever information subset agents are using. If this equation is to hold for all values of x_t, it must be the case that:

$$b'A - \frac{1}{3}d'(2A^2 + A^3) = 0.$$ (3.47)

Hence, a test of the full expectation's theory restrictions simply requires one to estimate unrestricted VAR equations and apply a Wald test based on the restrictions in equation (3.47).

As observed by Campbell and Shiller (1987), a rather weak implication of the expectations model is that $S_t^{(n,m)}$ must linearly Granger-cause Δr_t^m if there is information in $S_t^{(n,m)}$ useful for forecasting future Δr_t^m other than that contained in the history of that variable. If one considers the previous VAR representation for $S_t^{(n,m)}$ and Δr_t^m, written as:

$$S_t^{(n,m)} = \phi(L) S_t^{(n,m)} + \gamma(L) \Delta r_t^m + u_{1t}$$ (3.48)

$$\Delta r_t^m = \psi(L) S_t^{(n,m)} + \lambda(L) \Delta r_t^m + u_{2t}$$ (3.49)

where $\phi(L) = \sum_{i-1}^{p} \alpha L^i$, etc.., and L is the lag operator defined by $Lx_t = x_{t-1}$. A test for Granger-causality focuses on whether the lags of one variable enter into the equation of the other. Therefore, to determine if the spread Granger-causes future changes in m-period rates one needs to determine if the lagged values of the spread enter into equation (3.49).

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[18] Driffill, Psaradakis and Sola (1992) show that the expectations model with a constant term premium imposes a set of restrictions on the VAR that are more stringent than those in equation (3.46).

This can be determined by testing $\psi_1 = \psi_2 = .. = \psi_p = 0$ with a Wald test. Thereafter, it is necessary to test for the presence of a feedback relationship, as consistency with the theory requires the lagged values of the spread to enter equation (3.49), but not for the lagged values of the change in the m-period rate to enter equation (3.48). The reason is that the EHTS postulates a uni-directional relationship for the spread to the change in the m-period rate and not from the change in the m-period rate to the spread.

3.3.4 VEC tests of the expectations theory

This section provides a method for testing the stationarity of the spread variable that is included in the single-equation and VAR models. As Campbell and Shiller (1987) point out, the stationarity of the spread is necessary for the expectations model to hold since this implies that short-term and long-term yields are cointegrated with a cointegrating vector of (−1, 1). This is, of course, a testable implication of the theory, and one that is also crucial in ascertaining whether the spread should be used in the single-equation and VAR models. The cointegration between interest rates can of course be tested with either the Engel and Granger (1987) or the Johansen (1988) methodologies. Engel and Granger's approach is the easiest to implement, although it is limited in the sense that one is unable to test for higher cointegration rank than one, or test restrictions on the cointegrating vector. Fortunately, Johansen (1988) developed a multivariate approach that can avoid these and other problems.

The Johansen approach enables one to identify the number of cointegrating vectors spanning the cointegration space for a VECM representation of the yield curve. If one assumes that x_t in equation (3.28) represents this yield curve of length p, then essentially such tests center on the rank of the Π matrix. According to Hall, Anderson and Granger (1992) and Shea (1992), one would expect the rank of Π to be $r = p - 1$ to offer validity to the expectations hypothesis. Following these authors, the reason can be seen by noting that under the expectations hypothesis and for an I(1) term structure, any long-term and short-term spread is a stationary sum of expected short-term rate changes. This implies that any n-period rate is cointegrated with the m-period rate, so if one was to consider p yields then each of the $(p - 1) \times p$ dimensional spread vectors contained in the set [(−1, 1, 0,.., 0)′, (−1, 0, 1,.., 0)′, (−1, 0,.., 0, 1)′] is cointegrating for the vector x_t. As these spread vectors are linearly independent, the cointegration space should have rank $p - 1$.

After testing the cointegration rank it is of course necessary to investigate whether $\beta'x_t$ indeed appears to be a collection of interest rate spreads. One can test if the spreads are the components of the cointegrating vectors by testing if β consists of a collection of $p-1$ linearly independent spread vectors. To investigate this one needs to restrict the β matrix and then test if this results in Πx_t being a matrix of $r = p - 1$ independent linear combinations of interest rate spreads.

3.4 Conclusions

This chapter has developed the approach that will be used to evaluate if the spread is a term premium, plus an optimal predictor of a weighted average of changes in short-term rates over n-periods. This approach begins with a consideration of the underlying time series properties of the one-month and three-month yield series, as these variables are required to be stationary. The implications of non-stationarity were discussed and it was found that inclusion of non-stationary variables in a regression model can lead to spurious or nonsensical results. To avoid the problems associated with non-stationary variables, several methods that can be used to ascertain a variables stationarity were presented. In the event the variables are non-stationary, it was found that the trend component, from a trending series, can be removed by including a trend if the series is deterministic, and by differencing if the trend is stochastic. It was also found that a set of variables may be linked in some form of long-run equilibrium relationship, such that a linear combination of the variables is already stationary without differencing. As the appropriate method for detrending a non-stationary series may not be immediately obvious, and since taking the incorrect option will not yield a stationary series, a number of tests to identify the type of trend were presented.

The chapter then introduced the approach that will be used to evaluate the expectations theory when it incorporates the possibility of a time-varying term premium. This involves testing the relation between the spread and subsequent changes in short-term rates using the single-equation and VAR approaches. In the former case, it was found that the empirical validity of the expectations theory can be tested by estimating (3.43) directly, using IV. If the expectations theory is valid the estimated coefficients on the expected future changes in m-period rates should not differ significantly from their respective weights. The VAR approach was found to provide two additional methods for testing the expectations theory. In the more restrictive case it was found that the theory can be

evaluated by comparing the forecast of future changes in short-term rates embodied in the spread with an unrestricted VAR forecast. As the information subset was specified as including the spread, the two forecasts should be the same if the expectations theory is true. The second method for evaluating the expectations theory was to establish if the spread Granger-causes the change in m-period rates. It was found that consistency with the theory requires the spread to Granger-cause the change in the m-period rate, but not the reverse.

The final method of testing the expectations theory also focused on the spread relation but it was not found to be a test of the weaker form of expectations. Here it was found that if short-term and long-term rates are both integrated of order one, then equation (3.43) implies that the spread is stationary, or equivalently that short-term and long-term rates are cointegrated with a cointegrating vector of (−1, 1). In the above circumstances a cointegration analysis should be carried out first, since lack of cointegration means the EHTS can immediately be rejected as an equilibrium model. The following chapter applies this approach to test the validity of the expectations theory, and presents the results.

Chapter Four

Results

4.1 Introduction

This chapter applies the testing strategy discussed in Chapter Three to New Zealand interest rate data to investigate the empirical validity of the expectations theory. The chapter begins with an investigation into the underlying properties of the daily yield series on one-month, r_t^1, and three month, r_t^3, Treasury bank bills. This involves an application of various methods to determine the stationarity status of the variables, and, if needed, the use of appropriate techniques to transform a non-stationary to a stationary variable. This chapter then applies a single equation test along with VAR and VEC tests of the expectations theory to determine its validity. The following sections present the results from this investigation.

4.2 The results from investigating the time series properties of the data

A number of methods that can be used to determine a variables stationarity were discussed in the previous chapter, and the results from applying these techniques to the data are presented in the following section. If the stationarity assumption is violated it is necessary to apply some form of transformation to the non-stationary variable(s). The appropriate approach depends on the form of trend, and whether the trend is deterministic or stochastic. This can be determined with the Dickey-Fuller and KPSS tests, the results from which are presented in sections 4.2.1.2 and 4.2.1.3. In cases where the variables are both integrated of the same order, differencing will induce misspecification errors if the variables are cointegrated. To avoid this possibility, two methods that can be used to test for cointegration were discussed and the results from these tests are presented in section 4.2.2.

4.2.1 The results from testing for unit roots

This section investigates the stationarity of the r_t^1 and r_t^3 variables by testing for the presence of a unit root. Two methods were used, including a visual inspection of each variable's path through time and the application of the formal testing procedures of Dickey

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and Fuller (1979; 1981) and Kwiatkowski, Phillips, Schmidt and Shin (1992). The ensuing sections present the results from the application of these techniques.

4.2.1.1 The results from the Visual inspection

The following figure displays the highly volatile nature of the daily yields on one-month and three-month Treasury bank bills for the period 3 January, 1990 to 6 March, 1997[1].

Figure 4.1: Daily yields on one-month and three-month Treasury bills
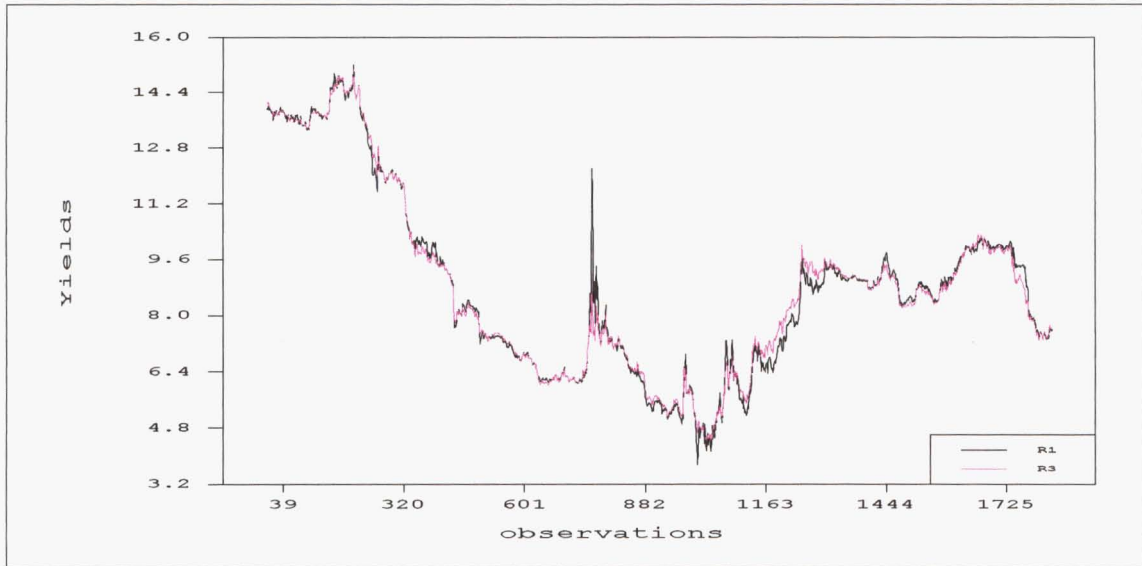
It should be expressed that visual inspection does have its perils, yet the visual pattern is one of non-stationarity for both variables. This is especially evident as the daily yields on one-month and three-month bills appear to meander through time with no tendency to revert to a long-run mean level. This type of *random walk* behaviour was found to be typical of variables which contain a unit root. It should also be pointed out that both the r_t^1 and r_t^3 variables appear to be linked together in some form of cointegrating relationship. This too is clear as there appear to be forces at work that prevent both variables from diverging to significantly from one another. The cointegration between yields is especially important as a necessary condition for the expectations theory to hold is that the spread is stationary.

Another distinctive feature of Figure 4.1 is the sharp increase in both yields beginning on 6

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[1] Note that R1 in Figure 4.1 is equal to r_t^1 in the text, and analogously for R3.

January and lasting to 9 January, 1993 (ie observations 756 – 760). According to the Reserve Bank's monetary policy statement (1993, June), interest rates significantly increased on 6 January as monetary policy reacted to the sustained downward pressure on the exchange rate in order to avoid a threat to the maintenance of price stability. By mid-afternoon on 9 January, monetary conditions had firmed sufficiently for the Reserve Bank to begin to partially unwind its tight monetary settings.

As the r_t^1 and r_t^3 series appear to contain unit roots, it is possible to investigate if taking first differences yields a stationary series by considering the time paths of the change in daily returns for both these variables. The following figure displays the daily change in yields on one-month and three-month bills for the period 4 January, 1990 to 6 March, 1997.

Figure 4.2: Daily change in yields on one-month and three-month Treasury bills
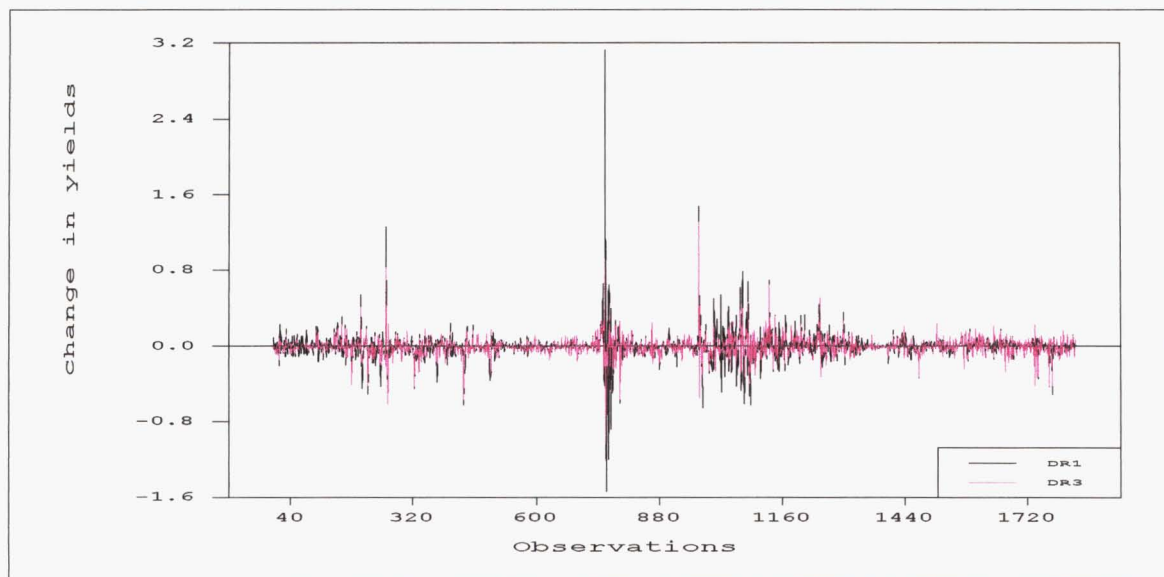
The figure shows that the change in yields for both series appear to be stationary. This is apparent as both series exhibit a constant mean (i.e. around zero) and variance with almost all observations lying within the ± 1.6 percentage bounds. As the variables appear to be stationary in first differences rather than levels, this suggests that the yield series on one-month and three-month bills are integrated of order one, therefore satisfying a necessary condition for two or more variables to be cointegrated.

Other features to note are the number of significant spikes which appear to be common to both series. The earliest spike that is shown at observation 259 coincides with a sharp

increase in interest rates relating to a publicly released statement on 11 January, 1981[20]. According to the Reserve Bank monetary policy statement (1991) this reiterated that short-term rates should generally exceed long-term rates while inflation is being brought down. Although this rise was short lived as the outbreak of the Gulf War on 17 January lowered overseas interest rates, helping to return short-term rates to their previous levels. The predominant spikes at observations 756 − 760 relate to the Reserve Bank's actions on 6 to 9 January, 1993 discussed earlier in this section. The final spike that occurs at observation 970 corresponds to a sudden rise in interest rates in response to a temporary fall in the exchange rate. According to the Reserve Bank's monetary policy statement (1993, December) this rise was brought about by an inconclusive election night result on 12 November, 1993. It should be mentioned that such shocks may cause problems for the ensuing analysis and thus may require the use of dummy variables.

4.2.1.2 The results from the Dickey-Fuller unit root test

This section presents the results of the Dickey-Fuller (1979; 1981) tests for the null of a unit root. These tests were conducted using Doldado, Jenkins and Sosvilla-Rivero's (1990) suggested procedure. This procedure begins by estimating the following general equation which nests both the trend stationary and difference stationary hypothesis:

$$\Delta y_t = \beta_0 + \beta_1 t + \gamma y_{t-1} + u_t.$$

$$(4.1)$$

The OLS results are then used to compute the τ_τ statistic in order to test the null hypothesis of a unit root (ie H_0^1: $\gamma = 0$). However, before testing the null it is important that the residuals from equation (4.1) are serially independent. In testing this prerequisite it was found that only the residuals from the Δr_t^3 equation satisfied this condition. Accordingly, it was necessary to augment the Δr_t^1 equation with lagged values of the dependent variable. The lag was chosen using the Schwarz (1978) Bayesian criterion (SBC) that is presented in equation (3.10). This criteria selected a lag of nine and at this lag the exact significance of the Ljung-Box (1978) test for autocorrelation was 0.997, thus indicating that there are no significant autocorrelations among the first five residuals.

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[20] Since the statement was released on a Friday, interest rates reacted on Monday, 14 January.

As both equations are now correctly specified one can proceed to test H_0^1. This resulted in computed τ_τ statistics of -1.603 and -1.496 for the r_t^1 and r_t^3 series, respectively. As these values are not more negative than the Dickey-Fuller's τ_τ critical value at the 0.05 level, i.e. -3.41, the null could not be rejected. Doldado, Jenkins and Sosvilla-Rivero then suggest a joint test that $H_0^2 : \beta_1 = \gamma = 0$, in order to establish if too many deterministic terms were included in equation (4.1). This was tested using a conventional F-statistic where the critical value is given by Φ_3. The F-statistic values along with the Φ_3 critical value can be found in Table 4.1. As these values do not exceed the Φ_3 critical value, H_0^2 could not be rejected. This is particularly important as it implies that neither series can be made stationary by including a deterministic trend.

As H_0^2 has not been rejected the power of the unit root test can be improved by estimating equation (4.1) without the trend, and then testing $H_0^3 : \gamma = 0$ with the τ_μ statistic. But before this hypothesis is tested, it is once again necessary to check that the residuals from the restricted equation are serially independent. The results from testing this requirement were the same as before, indicating that it is only necessary to include lags to the Δr_t^1 equation. The SBC again selected a lag of nine and at this lag the Ljung-Box (1978) test found there was no evidence of serial correlation. The results from computing the τ_μ statistic after fitting the restricted version of equation (4.1) are shown in Table 4.1. The results show that the computed ADF statistics for the r_t^1 and r_t^3 series are not more negative than the τ_μ critical value so that H_0^3 cannot be rejected. In this case one can check if there are still too many deterministic terms in the restricted equation by evaluating H_0^4. Failure to reject this hypothesis means that the variables are not stationary around a drift, and that once again the power of the test can be increased by re-estimating without the constant. The results for all steps in the testing procedure are reported in the following table.

Table 4.1: ADF Testing procedure results for the series in levels

Model	Hypothesis	Statistic	ADF value (r_t^1)	ADF value (r_t^3)	Critical value
Constant and trend					
	$H_0^1 : \gamma = 0$	τ_τ	-1.60295	-1.49654	-3.41000
	$H_0^2 : \beta_1 = \gamma = 0$	Φ_3	1.94239	2.30801	6.25000
Constant, no trend					
	$H_0^3 : \gamma = 0$	τ_μ	-1.93288	-2.00656	-2.86000
	$H_0^4 : \beta_0 = \gamma = 0$	Φ_1	2.41999	1.53536	4.59000
no constant, no trend					
	$H_0^5 : \gamma = 0$	τ	-1.55923	-1.88548	-1.95000

* Significant at the 0.05 level

Inspection of the table reveals that the null of a unit root cannot be rejected throughout the testing procedure for both series. This result is hardly surprising considering the most restricted model produces $\hat{\gamma}$ values of −0.0006 and −0.0016 for the respective yield series on one-month and three-month bills.

The rejection of H_0^5 indicates that r_t^1 and r_t^3 series are integrated to at least order one. To check for the possibility that these series are integrated of order two one can apply the ADF tests to the first difference of these series. As before these, tests can be conducted using the adopted strategy, although this time the procedure begins by estimating:

$$\Delta^2 y_t = \beta_0 + \gamma \Delta y_{t-1} + \beta_1 t + u_t.$$ (4.2)

The Ljung-Box (1978) test indicates that only the residuals from the $\Delta^2 r_t^1$ equation are serially correlated. The SBC suggests adding eight lags of $\Delta^2 r_t^1$ to equation (4.2) and at this lag there was no evidence of autocorrelation among the first five residuals. The results from applying the ADF tests to the Δr_t^1 and Δr_t^3 series are presented below.

Table 4.2: ADF Testing procedure results for the series in first differences

Model	Hypothesis	Statistic	ADF value (Δr_t^1)	ADF value (Δr_t^3)	critical value
constant and trend	$H_0^1 : \gamma = 0$	τ_τ	−14.9858*	−42.2647*	-3.41000

* Significant at the 0.05 level

Since the t-values are more negative than the one-sided critical value, the null hypothesis can be rejected in favour of a stationary alternative. Therefore, in accordance with the visual inspection, one can conclude that the yield series on one-month and three-month bills are subject to a stochastic trend that can be made stationary by differencing.

4.2.1.3 The results from the KPSS unit root test

This section presents the results of Kwiatkowski, Phillips, Schmidt and Shin's (1992) test for the null hypothesis that a series is *stationary* around a deterministic trend. Kwiatkowski, Phillips, Schmidt and Shin express the series as a sum of a deterministic trend, random walk and stationary error, and the tests corresponds to the hypothesis that the variance of the random walk equals zero. Two different tests are used to test the null of trend stationarity, with the difference relating to the way the deterministic trend is

accommodated. These tests are the $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ tests for the null hypotheses that a series is stationary around a level or around a deterministic trend.

In the *level* series case these test statistics are presented for values of the lag truncation parameter l, used in the estimation of the long-run variance, from zero to ten. A maximal value of ten has been chosen as for most series the value of the long-run variance estimate has settled down reasonably by this time. Therefore, by the time we reach $l = 10$ the value of the test statistic has also settled down[21]. Table 4.3 presents the results from applying the KPSS tests to the data.

Table 4.3: KPSS test results for the series in levels and differences

Series	Lag truncation parameter (l)
	0	1	2	3	4	5	6	7	8	9	10
η_μ: 0.05 critical value is 0.463
r_t^1	59.7*	29.9*	19.9*	15.0*	12.0*	10.0*	8.58*	7.52*	6.69*	6.02*	5.48*
r_t^3	59.5*	29.8*	19.9*	14.9*	11.9*	9.95*	8.54*	7.47*	6.65*	5.99*	5.45*
Δr_t^1	0.21	0.19	0.19	0.21	0.23						
Δr_t^3	0.48	0.47*	0.47*	0.48*	0.49*						
$\Delta^2 r_t^3$	0.00	0.00									
η_τ: 0.05 critical value is 0.146
r_t^1	38.7*	19.4*	12.9*	9.72*	7.78*	6.49*	5.57*	4.88*	4.34*	3.91*	3.56*
r_t^3	38.5*	19.3*	12.8*	9.65*	7.72*	6.44*	5.52*	4.83*	4.30*	3.87*	3.53*
Δr_t^1	0.07	0.06	0.07	0.07	0.08						
Δr_t^3	0.15*	0.15*	0.15*	0.16*	0.16*						
$\Delta^2 r_t^3$	0.00	0.00									

* Significant at the 0.05 level

Unfortunately, the values of these test statistics are fairly sensitive to the choice of l as for both series the value of the test statistic decreases as l increases. However, irrespective of l, the outcome of the test is not in much doubt, as for both r_t^1 and r_t^3 the null hypotheses of level stationarity and trend stationarity can be rejected at the 0.05 level.

The rejection of the null hypothesis indicates that the series are integrated to at least order one. In this situation one can test whether yields are integrated of order two, by applying the KPSS tests to the Δr_t^1 and Δr_t^3 variables. But this time, and for the reasons outlined earlier, the test statistics are only presented for a maximal lag truncation parameter of four. Again the results for the $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ test statistics are displayed in Table 4.3. Inspection of

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[21] Kwiatkowski, Phillips, Schmidt and Shin select l based on the same consideration (see page 174).

this table reveals that the null hypothesis of trend stationarity cannot be rejected for the Δr_t^1 variable at the designated significance level. In this case it appears that the r_t^1 variable is integrated of order one, and that the r_t^3 variable is integrated to at least order two. This latter possibility can be tested by applying the KPSS tests to $\Delta^2 r_t^3$. The reported results indicate that the null hypothesis of stationarity cannot be rejected at the 0.05 level.

To conclude, the KPSS tests finds in common with both the visual inspection and Dickey-Fuller test for the yield series on a one-month bill. Although in contrast to these methods, the KPSS tests find that the yield series on a three-month bill is integrated of order two. On the balance, however, the evidence of the visual inspection and formal tests suggest that both series are integrated of order one.

4.2.2 The results from testing for cointegration

If the r_t^1 and r_t^3 variables are both stochastically trending series of order one, then a necessary condition for the expectations theory to hold is that the spread, $S_t^{(3,1)}$, is stationary. The existence of a stationary spread series implies that r_t^1 and r_t^3 are cointegrated with cointegrating vector of (−1, 1). This thesis proposed testing for cointegration with both the Engel and Granger (1987) and Johansen (1988) methodologies, and the results from these approaches are presented in the subsequent sections.

4.2.2.1 The results from the Engel-Granger test for cointegration

This section presents the results from Engel-Granger two-step procedure, which begins by estimating the following long-run equilibrium relationship:

$$r_t^3 = \delta_0 + \delta_1 r_t^1 + u_t \tag{4.3}$$

The results from estimating this cointegrating regression with OLS are reported in Table 4.4.

Table 4.4: The results from estimating the cointegrating regression

$\hat{r}_t^3 = 0.16251 + 0.98426 r_t^1$
$R^2 = 0.992$, Durbin-Watson (1951) test = 0.14069

Inspection of the table reveals that the estimated cointegrating parameter of 0.984 is very close to the theoretically predicted value of unity. Recall that Stock (1987) found that if

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these variables are cointegrated then this estimate will be a super-consistent estimate of the true cointegrating parameter. Unfortunately, however, one cannot test if $\hat{\delta}_1$ differs significantly from unity because of the nature of the regression equation, i.e. it contains non-stationary variables[22]. This is especially clear with Granger and Newbold's (1974) rule of thumb, which suggests that an estimated regression is likely to be spurious if the R^2 exceeds the Durbin Watson (1951) d-statistic.

The essence of the Engel-Granger procedure is to test whether the cointegrating residual from equation (4.3) is stationary. This is tested with an ADF statistic from the following equation:

$$\Delta\hat{u}_t = \alpha_1\hat{u}_{t-1} + \sum_{j}^{p}\gamma_j\Delta\hat{u}_{t-j} + \varepsilon_t, \qquad (4.4)$$

where p is chosen to ensure the residuals approximate white noise. The SBC selects a lag of seven for this purpose, and at this lag the exact significance level of the Ljung-Box test is 0.994 indicating there is no evidence of serial correlation among the first five residuals and that one can proceed to test the cointegrating residual for a unit root. The results from fitting equation (4.4) are presented below.

Table 4.5: The results from estimating the auxiliary regression

| Variable	Coefficient	Std error	T-statistic	P-value
u_{t-1}	−0.047530832	0.008900774	−5.34008	0.00000010
Δu_{t-1}	0.0899440087	0.023721801	3.77037	0.00016822
Δu_{t-2}	−165825833	0.023768093	−6.97683	0.00000000
Δu_{t-3}	−0.194456838	0.024081285	−8.07502	0.00000000
Δu_{t-4}	−0.024691167	0.024458956	−1.00949	0.31287245
Δu_{t-5}	0.059397993	0.023798341	2.49589	0.01265242
Δu_{t-6}	−0.012187814	0.023304851	−0.52297	0.60105672
Δu_{t-7}	−0.125899973	0.023285421	−5.40682	0.00000007

$R^2 = 0.154229$, Ljung-Box (1978) test = 0.45727~ $\chi^2(5)$, p-value = 0.99375292

Analysis of these results shows that the regression yields an $\hat{\alpha}_1$ value of −0.0475 with an associated t-statistic of −5.3401. MacKinnon (1991) reports the t-critical when two variables appear in the equilibrium relationship at the 0.05 level as being −3.3377. Consequently, one can reject the null of a unit root and thus the null of no cointegration between the r_t^1 and r_t^3 series. The plot of the cointegrating residuals is shown below.

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[22] Estimates of the significance of δ are provided when discussing the Johansen's (1988) approach.

Figure 4.3: The cointegrating residual
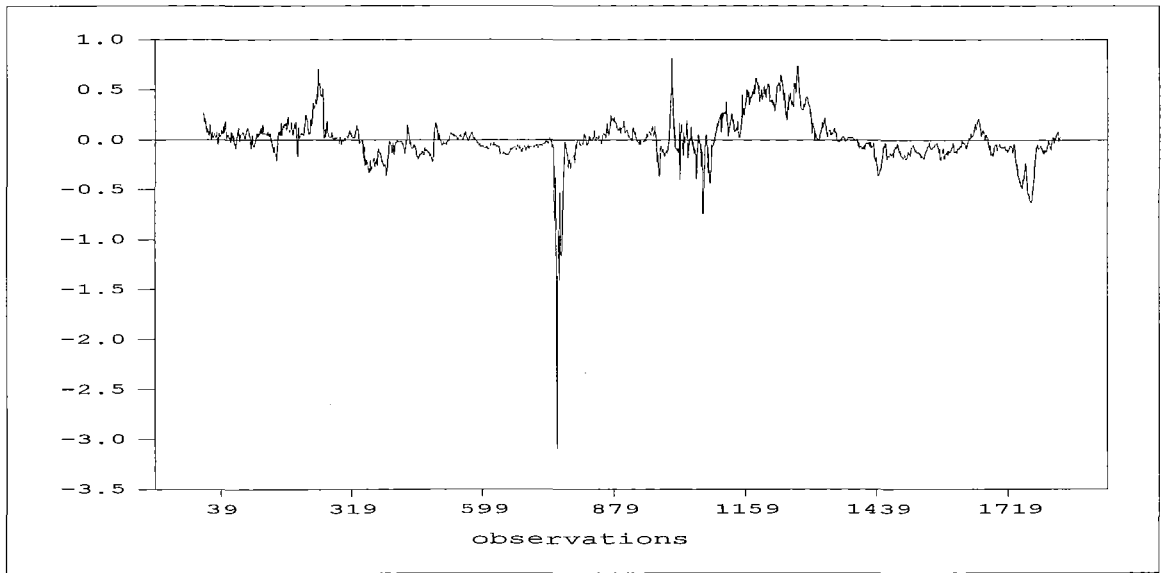
The figure shows that the equilibrium errors meet the criteria for a long-run equilibrium to be meaningful. This is clear as the cointegrating residuals rarely drift far from zero and fairly frequently cross the zero-line. The figure also shows the importance of including a dummy-variable for the 756 – 760 observations in the ensuing analysis.

The finding that the spread is stationary is particularly important as it supports the inference that the r_t^3 series is actually integrated of order one. To gain further support and insight into the relationship between the yield series on one-month and three-month bills, a more comprehensive analysis is conducted in the following section.

4.2.2.2 The results from the Johansen approach

A preliminary step in Johansen's (1988) approach is to estimate an unrestricted VAR(k) model, and determine the appropriate lag length. In regards to this study, this involved estimating the following model:

$$x_t = \Pi_1 x_{t-1} + \Pi_2 x_{t-2} + ... + \Pi_k x_{t-k} + \mu + \varphi D_t + u_t \qquad (4.5)$$

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where $x_t = [\, r_t^1,\ r_t^3\,]'$ is a vector of $I(1)$ yields, Π_i is a (2×2) matrix of parameters, μ is a (2×1) vector of constants and D_t includes three dummy variables to take account of the short-run shocks to the system[23].

The initial task is to determine the appropriate lag length. This was found to be four using the SBC and at this lag one cannot reject the null of no serial correlation with the Ljung-Box (1978) test. Consequently, the lag was set to four and equation (4.5) was re-formulated as the following model:

$$\Delta x_t = \sum_{i=1}^{3} \Gamma_i \Delta x_{t-i} + \Pi x_{t-1} + \mu + \varphi D_t + u_t \tag{4.6}$$

where $\Gamma_i = -\sum_{j=i+1}^{4} \Pi_j$ and $\Pi = \sum_{i=1}^{4} \Pi_i - I$ [24].

Interest then focuses on the rank of Π as this provides the number of cointegrating relationships. Johansen and Juselius (1990) provide the λ_{trace} and λ_{max} statistics for determining this rank. However, before any such tests are carried out it is necessary to establish the most appropriate configuration of the deterministic components in equation (4.6). Section 3.2.5.2.3 outlines a strategy for testing where these deterministic terms lie while jointly testing for the cointegration rank. The results from this strategy are presented in Table 4.6. Although it should be noted at the outset that the inclusion of the dummy variables affects the underlying distributions of the λ_{trace} and λ_{max} statistics, so that the critical values published by Osterwald-Lenum (1992) are only indicative.

Table 4.6: The results from determining the cointegrating rank and model

Statistic	H_0: r	n − r	Model two	Model three	Model Four
λ_{max}					
	0	2	86.391*→	85.849*	86.268*
	1	1	7.863	4.398	5.814
λ_{trace}					
	0	2	94.254*→	90.247*	92.082*
	1	1	7.863	4.398	5.814

* Indicates a rejection at the 0.05 level.

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[23] These shocks were found to correspond to the 259, 756-760 and 970 observations that were discussed in section 4.2.1.1.

[24] Doornick and Hendry (1994) point out that x_{t-1} in equation (4.6) is asymptotically equivalent to x_{t-k} in equation (3.28).

Recall from section 3.2.5.2.2 that the λ_{max} statistic tests the null of $r = 0$ against $r = 1$, whereas the λ_{trace} statistic tests the same null against an unrestricted alternative of $r = n$. Beginning with the null of no cointegration (ie $r = 0$) and model two, the fourth column reports the value of the λ_{max} and λ_{trace} statistics as 86.391 and 94.254, respectively. Osterwald-Lenum (1992) report the critical value for $n - r = 2$ and at the 0.05 level as 15.67 and 19.96. Hence, the null of no cointegration is rejected and one proceeds to test the same null in the next most restrictive alternative (ie model three). The strategy continues in this fashion (i.e. from left to right and row by row) until the null of r cointegrating vectors is not rejected. The table shows the first non-rejection of the λ_{max} and λ_{trace} statistics corresponds to the null of one cointegrating vector and model two[25].

In this case Π can be decomposed into the product of a 2×1 matrix α and a 3×1 matrix β such that $\Pi = \alpha\beta'$. In this representation the β matrix contains the long-run equilibrium parameters such that $\beta'x_{t-1}$ represents the single error-correction term, whereas the parameters in the matrix α measure the speed at which Δx_t adjusts towards the lagged error-correction term. The maximum likelihood estimates of α and β from fitting equation (4.6) with the constant restricted to cointegrating space are reported in Table 4.7. It should be noted that these estimates are based on one cointegrating vector which is normalised with respect to the coefficient on the r_t^3 variable.

Table 4.7: The ML estimates of α and β in full system

β'		
r_t^1	r_t^3	constant
−0.992	1.000	−0.119

α	Coefficient	t-values for α
α_{11}	0.134	8.476
α_{21}	0.056	5.049

N = 1826, Shenton-Bowman (1977) test =11610 [0.000] $\sim\chi^2(4)$, Godfrey (1988) test = 3.759 [0.44] $\sim\chi^2(4)$, Engel (1982) test = 202.236 and 221.577 $\sim \chi^2(4)$

The table also reports some residual diagnostics for evaluating the statistical adequacy of the estimated model. The test of normality is based on a multivariate version of the univariate Shenton and Bowman (1977) test. Since this test reports a significance level of 0.00, the null of normality can be rejected at any significance level. In this case one can investigate the residuals of each equation individually to try and identify the problem. The

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[25] The λ_{max} and λ_{trace} statistic's critical value for $n - r = 1$ and model two are 9.24.

univariate statistics for each of these equations reveal estimates of skewness and kurtosis which greatly exceed their respective norms of zero and three. This can be clearly seen by investigating the histogram of standardised residuals, shown in Appendix 1.a and Appendix 1.b. These and the accompanying plots indicate that the problems with kurtosis mainly relate to the substantial amount of interest rate volatility in the month of January, 1993 (as shown in figure 4.1).

The rejection of normality suggests that the results must be interpreted with caution as the maximum likelihood estimation procedure along with tests for parameter significance are not strictly valid[26]. However, standard econometrics texts such as Gujarati (1995) and Thomas (1985) note that if the sample size is sufficiently large, standard testing procedures can still be relied on[27]. This finding is based on the Central Limit Theorem which states that as the sample size approaches infinity, the sampling distributions of the OLS estimators approach the normal distribution irrespective of the form of the distribution of the residuals (Thomas, 1985). As this paper uses a sample of 1,830 observations, it will be presumed that the sample size is large enough to invoke the normality assumption required for maximum likelihood estimation and hypothesis testing.

Table 4.7 also reports Godfrey's (1988) test for autocorrelation and Engel's (1982) univariate test for an autoregressive conditional heteroscedastic (ARCH) process. Godfrey's test reports an exact significance level of 0.44, indicating that the null of no autocorrelation cannot be rejected at standard levels. Engel's univariate tests for ARCH produces values greatly in excess of the $\chi^2(4)$ critical value. In this case the null of conditional homoscedasticity can be rejected at all levels. The reason for this rejection is apparent when inspecting the plot of the standardised residuals, shown in Appendix 1.a and Appendix 1.b. These figures show that the conditional variance fluctuates greatly over time, especially for the Δr_t^3 equation. The combined influence of these two tests suggests that the MLE and conventionally computed t, χ^2, F-statistics are still valid and unbiased, although more efficient MLE can be obtained if one was to include an ARCH(k) error process in equation (4.6).

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[26]   The principle reason is that the MLE procedure along with t, χ^2 and F tests require that the disturbance term satisfies the normality assumption.

[27]   Thomas (1985) specifies a sample in excess of fifty as being large enough to rely on standard testing procedures.

Notwithstanding the effects of ARCH, the model appears adequately specified for the purpose of interpreting and testing the MLE of α and β. The lower half of Table 4.7 reports the estimated speed of adjustment parameters and associated t-values for testing the null hypothesis of weak exogeneity. As these t-values exceed the asymptotic t-critical values, the null hypothesis of weak exogeneity can be rejected at normal levels. This rejection is significant as it implies that both the Δr_t^1 and Δr_t^3 equations are influenced by the lagged error-correction term. Table 4.7 illustrates that both yield changes adjust to the following cointegrating vector (−0.992, 1, −0.119), and that the $\hat{\beta}_i$ are very close to their theoretically predicted values[28]. This of course can be tested, but before doing so it is necessary to test for exclusion from long-run space. The results from conducting individual tests for parameter significance are shown in Table 4.8.

Table 4.8: The results from tests for exclusion in the cointegrating vector

r	$\chi^2(1)$	r_t^1	r_t^3	constant
1	3.84	78.32	77.49	1.94

* Significant at the 0.05 level

The table shows that only the constant can be excluded from β as the computed likelihood ratio value of 1.78 does not exceed the χ^2 (1) asymptotic critical value of 3.84. In this situation the model contains no deterministic components in the data, and the intercept in the cointegrating vector equals zero. Hansen and Juselius (1994) warn against excluding the intercept as it is generally needed to account for the unit of measurements of the variables, and cases where the restriction is justified are exceptional. Based on their warning it was decided not to exclude the intercept in the cointegrating vector.

Using the same approach, one can also test if the spread is a component of the cointegrating vector. Essentially, this involves restricting the cointegrating vector to that hypothesised by the expectations theory (i.e. restricting the coefficient on r_t^1 to be −1), and then testing if the rank of Π is still one. The likelihood ratio test produces a value of 0.84 which is found to be insignificant when compared to the $\chi^2(1)$ critical value at conventional levels. Thus, in conformity with the expectations theory of the term structure the spread restriction is accepted.

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[28] It should also be noted that these values are very similar to the estimates recovered using the OLS methodology.

Both of the preceding methodologies have found support for the long-run validity of the expectations hypothesis. This support is particularly important as it implies the spread is stationary, and that it is now permissible to proceed in using the spread in the single-equation and VAR models. This means that one can investigate the short-run validity of the expectations theory when it includes the possibility of a time-varying term premium.

4.3 The results from testing a weaker form of expectations model

The objective of this paper is to examine if $S_t^{(3,1)}$ is a term premium, plus an optimal predictor of a weighted average of Δr_{t+1}^1 and Δr_{t+2}^1. This paper proposed testing this hypothesis with a test on a single-equation model and tests on a VAR model. The results from these tests are presented in the following section.

4.3.1 The results from the single equation model

The empirical validity of the expectations hypothesis may be evaluated by estimating equation (3.44) directly, using the errors in variables method and IV. Consistency with the hypothesis requires the coefficients on Δr_{t+1}^1 and Δr_{t+2}^1 to not differ significantly from two-thirds and one-third, respectively. IV estimation using a constant, $S_{t-1}^{(3,1)}$, $S_{t-2}^{(3,1)}$ $S_{t-3}^{(3,1)}$ and Δr_{t1}^1, Δr_{t-1}^1 .., Δr_{t-3}^1 as instruments yields the following results.

Table 4.9: The results from single equation estimation using IV

Variable	Coefficient	Standard error	T-value	P-value
Premium	1.0862	0.043047	25.2324	0.000
Δr_{t+1}^1	0.90894	0.018660	48.7094	0.000
Δr_{t+2}^1	−0.38103	0.045245	−8.4215	0.000

N = 1826, R^2 = −0.88474, GR^2 = 0.94682, Sargan (1964) test = 4.8305 [0.437] ~χ^2(5), Jarque-Bera (1980) test = 801.0194 [0.000] ~ χ^2(2), Koenker (1981) test = 147.5363 [0.000] ~ χ^2(1), Sargan (1976) test = 616.8898 [0.00] ~ χ^2(5)

An unusual feature of the summary statistics is the traditional measure of goodness-of-fit, or the R^2 of −0.88474, which is generally constrained to lie between zero and one. This peculiar result illustrates Pesaran and Smith's (1994) finding that the use of the residuals from an IV regression in the construction of goodness-of-fit measures can be highly misleading. Pesaran and Smith have proposed an alternative measure for IV regressions, known as the generalised R^2 statistic (GR^2). By this criterion the model has high explanatory power with 94 percent of the variation in the dependent variable being explained by the variation in the independent variables. Refer to Appendix 2.a for a plot of the actual and fitted values.

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The remaining statistics are misspecification tests used to check the adequacy of the model's specification. Sargan's (1964) test is a general test of the misspecification of the model and its instruments. Under the null this statistic is asymptotically distributed as χ^2 with $s - k$ degrees of freedom, where s is the number of instruments and k the number of regressors. The exact significance level for this statistic indicates that one does not reject the null that the model and its instruments are correctly specified at conventional significance levels. Jarque and Bera's (1980) test investigates if the residuals satisfy the normality assumption that is required for the validity of standard t, χ^2 and F statistics. The significance level for this test implies that the null of normality can be rejected at any level. Although as before it will be presumed that the sample size is large enough to invoke the normality assumption required for these tests. The final test is Sargan's (1967) test for serial correlation. The significance levels for this statistic indicate that the null of no autocorrelation can be rejected at any level.

In this situation the estimated coefficients are unbiased, but the variance of these coefficients is incorrectly computed making any inference based on the usual t, χ^2, F-statistics incorrect. Fortunately, the effects of autocorrelation can be purged with Newey and West's (1987) heteroscedastic and autocorrelation consistent standard errors. The results from the IV estimation of equation (3.44) based on Newey and West's adjusted standard errors are reported below.

Table 4.10: The results from single-equation estimation with Newey-West's adjusted std errors

Variable	Coefficient	Standard error	T-value	P-value
Premium	1.0862	0.090201	12.0417*	0.000
Δr_{t+1}^1	0.90894	0.043109	21.0849*	0.000
Δr_{t+2}^1	−0.38103	0.090002	−4.2336*	0.000

* significant at the 0.05 level

The results show the data are somewhat inconsistent with the expectations theory's restrictions, as the estimated coefficient on Δr_{t+2}^1 is of the wrong sign. Nevertheless one can still proceed to consider the validity of the expectations theory's restriction for the estimated coefficient on Δr_{t+1}^1. An asymptotic t-test for the null hypothesis that the coefficient on the expected next period change in the one-month rate is two-thirds yields 5.62. As this value is outside the acceptance level at conventional significance levels the null hypothesis can be rejected.

4.3.2 The results from the vector autoregressive (VAR) model

An alternative metric for evaluating the relationship between the spread and the future path of interest rates involves the use of the VAR methodology. Essentially, this approach uses an information subset, consisting of the history of $S_t^{(3,1)}$ and Δr_t^1, to generate a forecasting scheme for Δr_{t+1}^1 and Δr_{t+2}^1 which is then compared with the forecast embodied in the spread. As the information subset includes the spread, the two forecasts should be the same if the expectations theory is true.

A preliminary step before testing the restrictions implied by the expectations theory is to determine the form of the information subset (i.e. the appropriate order of the VAR). This was taken as being four, a lag that was supported by the SBC, and the results obtained from the unrestricted estimation of the VAR system are presented below. Refer to Appendix 3.a and Appendix 3.b for plots of the actual and fitted values.

Table 4.11: The results from OLS estimation of the spread equation in the VAR

Variables	Coefficients	Standard error	T-value	P-value
$S_{t-1}^{(3,1)}$	0.77225	0.034839	22.1664	0.000
$S_{t-2}^{(3,1)}$	−0.067535	0.047700	−1.4158	0.157
$S_{t-3}^{(3,1)}$	0.049266	0.047675	1.0334	0.302
$S_{t-4}^{(3,1)}$	0.20060	0.034901	5.7477	0.000
Δr_{t-1}^1	−0.18764	0.018672	−10.0494	0.000
Δr_{t-2}^1	−0.060783	0.019175	−3.1700	0.002
Δr_{t-3}^1	0.0062078	0.019193	0.32344	0.746
Δr_{t-4}^1	−0.0054175	0.012618	0.42935	0.668

$R^2 = 0.88343$, $\overline{R}^2 = 0.88298$, Jarque-Bera (1980) test = 3604851 [0.000] ~ $\chi^2(2)$, Godfrey (1978) test = 0.0038508 [0.951] ~ $\chi^2(1)$

Table 4.12 The results from OLS estimation of the Δr_t^1 equation in the VAR

Variables	Coefficients	Standard error	T-value	P-value
$S_{t-1}^{(3,1)}$	0.40618	0.065213	6.2285	0.000
$S_{t-2}^{(3,1)}$	−0.019293	0.089286	−0.021608	0.983
$S_{t-3}^{(3,1)}$	−0.011324	0.089240	−0.12690	0.899
$S_{t-4}^{(3,1)}$	−0.30070	0.065328	−4.6029	0.000
Δr_{t-1}^1	0.21627	0.034950	6.1879	0.000
Δr_{t-2}^1	0.080622	0.035892	2.2463	0.025
Δr_{t-3}^1	0.0068773	0.035927	0.19143	0.848
Δr_{t-4}^1	0.0024890	0.023618	0.10539	0.916

$R^2 = 0.10343$, $\overline{R}^2 = 0.099978$, Jarque-Bera (1980) test = 638178.9 [0.000] ~ $\chi^2(2)$, Godfrey (1978) test = 0.042279 [0.838] ~ $\chi^2(1)$

The tables report both the Jarque-Bera (1980) and Godfrey (1978) tests for the respective nulls of normality and no autocorrelation. The exact significance level for these tests

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indicates that one can reject the null of normality, but not reject the null of no autocorrelation. Again, if one invokes the normality assumption, the misspecification tests suggest the model is well defined statistically, and thus one can proceed to consider the validity of the expectations theory restrictions.

Recall that if one projects the information subset onto equation (3.44), it can be shown to impose a set of highly nonlinear cross-equation restrictions on the estimated coefficients on a VAR. These restrictions were provided in equation (3.47), and a Wald test of the validity of these restrictions results in a value of 763.023. Since under the null hypothesis this statistic is distributed as central χ^2 with eight degrees of freedom, the restrictions which allow for a random error in the model can be rejected at any significance level.

The final test of the expectations theory is to determine if $S_t^{(3,1)}$ linearly Granger-causes future Δr_t^1. In order to test this hypothesis, one needs to determine whether the lagged values of $S_t^{(3,1)}$ enter into the Δr_t^1 equation. A Wald test for the null hypothesis of no Granger-causality from $S_t^{(3,1)}$ to future Δr_t^1 resulted in a value of 117.124, thus firmly rejecting the hypothesis of no Granger-causality. Consistency with the theory also requires that Δr_t^1 not Granger-cause $S_t^{(3,1)}$. This involves testing whether the lags of Δr_t^1 enter into the $S_t^{(3,1)}$ equation. A Wald test for the null hypothesis of no Granger-causality from Δr_t^1 to $S_t^{(3,1)}$ yields a value 109.242, thus also firmly rejecting the null of no Granger-causality. Taken together, the results of the Granger-causality tests indicate that $S_t^{(3,1)}$ Granger-causes future Δr_t^1, although Δr_t^1 also Granger-causes $S_t^{(3,1)}$ (i.e. in contrast to the expectations theory's predictions).

4.4 Conclusions

This chapter has conducted an investigation into the underlying properties of the r_t^1 and r_t^3 series. This involved a visual assessment of the series time paths along with more formal testing procedures. Here it was found that r_t^1 was integrated of order one and r_t^3 was integrated to at least order one, but possibly of order two by the KPSS unit root test. On the balance of evidence it was concluded that r_t^3 was actually integrated of only order one, as if r_t^3 was integrated of order two the spread would not be stationary and the expectations theory could immediately be rejected. This was of course tested with the Engel and

Granger (1987) and Johansen (1988) methodologies. These approaches both found support for the stationarity of the spread and thus the long-run validity of the expectations theory. This too is important as it allows one to use the spread in the univariate and multivariate frameworks. Consequently, one can test the short-run predictions of the weaker form of expectations model. These results along with the conclusions drawn from this analysis will be discussed in the following chapter.

Chapter Five

Conclusions

5.1 Introduction

This chapter discusses the results from the previous chapter and provides conclusions as to whether the weaker form of the expectations theory is valid. This chapter also identifies the limitations of this investigation and provides a number of suggestions for future research.

5.2 Discussion and conclusions

The purpose of this paper was to establish if the inclusion of a time-varying term premium on the long-term rate of interest is enough to reconcile the expectations theory with the data. It was suggested that a white-noise error term can serve as a reasonable proxy for any variation in the term premium, and including such a variable in the expectations model should reduce any bias caused through its omission. In this less restrictive interpretation of the EHTS, it was found that the n-period rate is a term premium, plus an arithmetic average of current and subsequent m-period rates up to $n - m$ periods in the future.

The overall objective of this paper was to investigate the validity of this weaker form of the expectations theory. To this end the paper focused on an equivalent expression of the EHTS, the yield spread between an n-period and m-period bond. Here it was found that the spread is a term premium, as well as an optimal predictor of a weighted average of future changes in m-period rates over n-periods. The spread relation provided a number of different metrics for assessing the validity of the less restrictive expectations theory. One of these metrics involved an application of the theory of cointegration to the yield curve. It was found that if the short-term and long-term rates are both integrated of order one, the expectations theory posits that these rates should be cointegrated with a cointegrating vector of $(-1, 1)$. This was tested with both the Engel and Granger (1987) and Johansen (1988) methodologies using daily interest rate data on one-month and three-month bills for the period 3 January, 1990 to 6 March, 1997. These approaches found that r_t^1 and r_t^3 were indeed cointegrated with respective cointegrating parameters of -0.984 and -0.992. The

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Johansen approach was also used to test that the corresponding cointegrating vector equals the spread vector. Here it was found that the spread restriction could easily be accommodated at conventional significance levels. This is important as it confirms that all of the long-run components between r_t^1 and r_t^3 cancel out so that the spread is stationary. On the basis of this metric it is permissible to conclude that there is strong empirical support for the long-run implications of the expectations theory of the term structure.

The cointegration analysis was not, however, a test for the validity of the weak form of the expectations theory. Although it was found that if r_t^1 and r_t^3 are both stochastically trending processes of order one, the stationarity of the spread is a necessary condition for the expectations theory to hold. The confirmation of this requirement also meant that one was able to evaluate if there is a randomly fluctuating term premium intervening between $S_t^{(3,1)}$ and the appropriate weighted average of expected future Δr_{t+1}^1 and Δr_{t+2}^1, using the single-equation and VAR models. In the former case the EHTS was tested by estimating the term structure relationship the other way around. That is, when $S_t^{(3,1)}$ is explained by expected future Δr_{t+1}^1 and Δr_{t+2}^1, using the actual future changes in the one-month rates to proxy the expected change, and using instrumental variables. It was found that consistency with the expectations theory requires the estimated coefficients on Δr_{t+1}^1 and Δr_{t+2}^1 to not differ significantly from their respective weights of two-thirds and one-third. Subsequent tests of the expectations theory restrictions revealed that the estimated coefficient on Δr_{t+1}^1 is statistically different from its theoretically predicted value, and the estimated coefficient on Δr_{t+2}^1 is not of the hypothesised sign. The rejection of these restrictions suggests that the data examined here is inconsistent with the weaker form of expectations theory.

The expectations theory of the term structure was also tested by imposing restrictions on a VAR representation for the stationary $S_t^{(3,1)}$ and Δr_t^1 series. Here it was found that the equality of the spread and the theoretical spread imposes a set of highly non-linear cross-equation restrictions on the estimated parameters of the VAR. If the expectations theory is valid these restrictions should hold whatever the information subset agents are using. When the expectations theory was tested by imposing these restrictions on the VAR, the restrictions which allow for a random error in the model were strongly rejected. As a corollary of this implied restriction it was found that there must also be Granger-causality running from $S_t^{(3,1)}$ to Δr_t^1. The intuitive reason is that $S_t^{(3,1)}$ should be an optimal predictor

of a weighted average of future Δr_t^1. A test of this proposition revealed that $S_t^{(3,1)}$ includes information useful for forecasting future Δr_t^1, although Δr_t^1 also contains information useful for predicting $S_t^{(3,1)}$, which is in contrast to the expectations theory's predictions. As before, one can conclude that there is little or no support for the expectations theory when it includes a random element in the term premium.

On the basis of the cointegrating analysis one can conclude that there is a strong element of validity to the expectations hypothesis, in that there is an approximate equilibrium relationship between the daily yields on one-month and three-month Treasury bank bills. As such this evidence corroborates findings by Campbell and Shiller (1987), MacDonald and Speight (1991), Hall, Anderson and Granger (1992) and Shea (1992). The main objective of this paper was not, however, to investigate the long-run validity of the EHTS, but rather to ascertain the validity of the less restrictive form of expectations theory. On the basis of this evidence one can conclude that, unlike Driffill, Psaradakis and Sola (1997), the inclusion of a white-noise error term in spread regressions is not enough to reconcile the expectations theory with the data. This means that the spread is not an optimal predictor of a weighted average of expected future changes in short-term rates over the horizon of the long-term bond.

5.3 Limitations

This thesis has used the expectations theory of the term structure to test New Zealand interest rate data at the short-end of the maturity spectrum (ie one-month and three-month bills) for the January 1990 to March 1997 period. In this respect the investigation is limited in the time period and the maturities of the bonds to which it applies. One cannot claim the generality of Campbell and Shiller (1991), who tested the expectations theory over a thirty-five year period using US monthly interest rate data for all possible maturities in the range one, two, three, four, five and nine-months and one, two, three, four, five and ten years.

This paper summarised the empirical evidence on the expectations theory of the term structure when it includes a constant term premium. A finding from this review is that almost all empirical studies statistically reject the short-run predictions of the expectations theory. It was found that most explanations for this lack of support concentrate on the possibility that the term premium is not constant as assumed by the theory, but rather varies substantially through time. The purpose of this paper was thus to investigate the

validity of the expectations theory when it allows the term premium to vary through time. The investigation is limited in that it only investigates the validity of the weaker form of expectations theory. One could not explicitly claim that the inclusion of a time-varying term premium in the spread regressions is enough to reconcile the expectations theory with the data in its true sense, as this would require one to also investigate the restrictions implied by the expectations theory with a constant term premium.

This thesis investigated the validity of the expectations theory of the term structure using daily interest rate data on r_t^1 and r_t^3 that was collected from the Reserve Bank of New Zealand. As with all studies that utilise secondary data there is potential to introduce some type of bias into the analysis. One specific type of bias that is applicable to this investigation is that any errors introduced by the Reserve Bank when entering the yields on one-month and three-month bills will also be carried over to this analysis. This investigation is limited in this respect.

5.4 Suggestions for future research

The purpose of this thesis was to investigate the validity of the weaker form of the expectations theory in relation to the spread and subsequent future changes in short-term rates. This involved an evaluation of the generalised spread relation using the one-month and three-month Treasury bank bill yields. It was indicated above that this investigation is limited in that it only applies tests to the less restrictive expectations theory and that it only uses a pair of interest rate maturities. Further research can improve on this analysis by testing the restrictions implied by the expectations theory with both a constant and time-varying term premium in the spirit of Campbell and Shiller's (1991) investigation. Only such an extensive investigation can establish if the inclusion of a white-noise error term in spread regressions is enough to reconcile the expectations theory with the data for New Zealand.

This paper indicated that there are two possible explanations for the widespread failure of the expectations theory of the term structure. These explanations were found to involve assertions that the term premium is not constant, or that the stance of monetary policy has in some way affected the nature of empirical tests. In the latter case it was found that the monetary authorities commitment to stabilising interest rates could explain why the spread has had negligible forecasting power. This explanation was provided by Mankiw and

Miron (1986) and subsequent empirical work has also been conducted by Hardouvelis (1988) and Roberds, Runkle and Whiteman (1996). All of these studies were found to support the Mankiw and Miron hypothesis, with the expectations theory always performing better in monetary regimes that are characterised by low levels of interest rate targeting. This provides an avenue in which one can further explore the failings and perhaps rehabilitation of the expectations theory of the term structure. Further research will obviously contribute to the validity of the EHTS and its implications for policy makers and debt market participants.

Contemporary research has been using the maximum likelihood methods of Johansen (1988) to test the cointegrating implications of the EHTS. Here it was found that consistency with the expectations theory requires a set of p yields to be cointegrated with $(p - 1)$ cointegrating vectors, and that the corresponding cointegrating vectors equal the spread. The cointegrating implications of the EHTS have been tested by MacDonald and Speight (1991), Hall, Anderson and Granger (1992) and Shea (1992). A general finding from this empirical work is that the data are largely supportive of the cointegrating restrictions. Hall Anderson and Granger (1992) note that this type of cointegration has the important implications that the term premiums of Treasury bills are stationary processes and that a single non-stationary common factor underlies the behaviour of each yield to maturity. They add that the common factor could not be identified and that it could be a linear combination of several $I(1)$ variables. Hall, Anderson and Granger (1992) allege that further research may suggest a useful way of identifying the common non-stationary factor so that it can be estimated and studied. They declare that much could be learned if this factor can be linked to economic variables such as monetary growth and/or inflation, and that further research on the common factor interpretation will undoubtedly improve the understanding of how the term structure changes over time.

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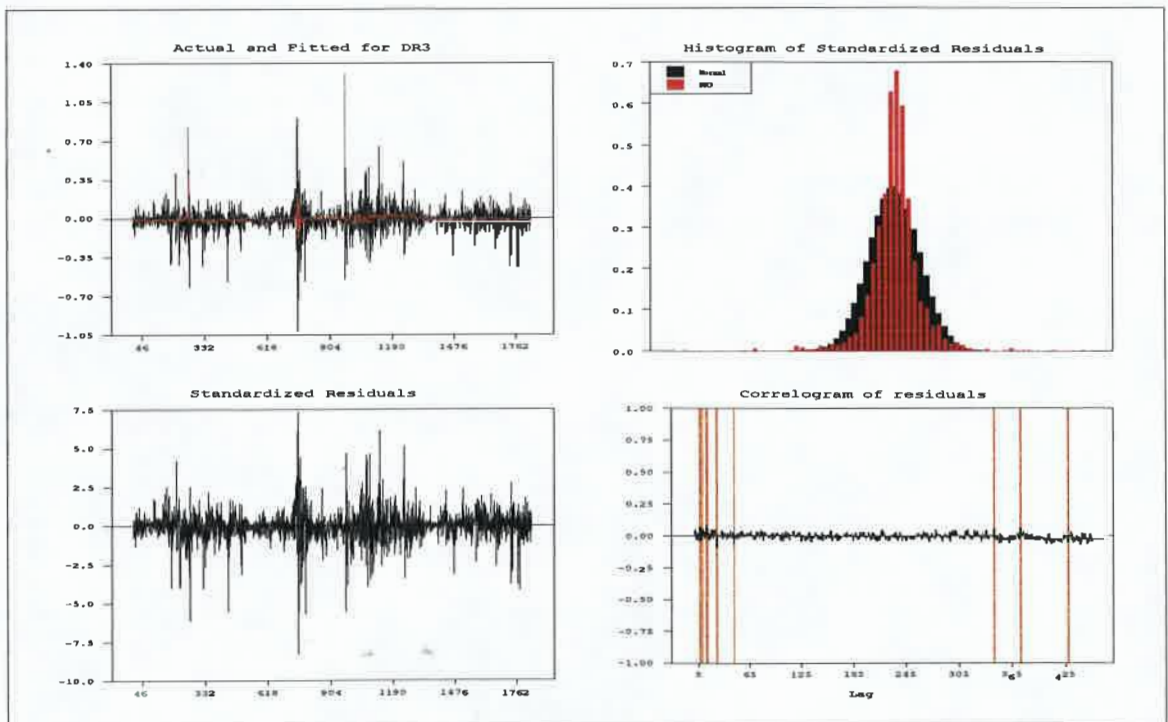
Appendices

Appendix 1: VEC estimation

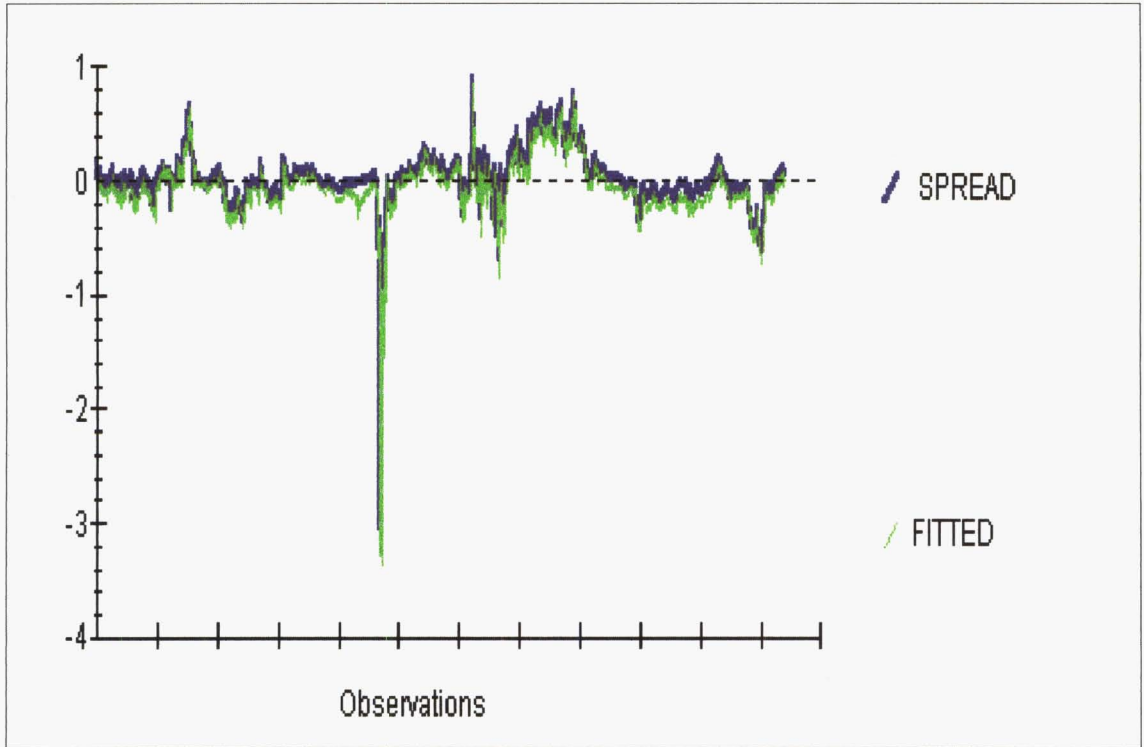### Appendix 1.a: Residual output for the Δr_t^1 series
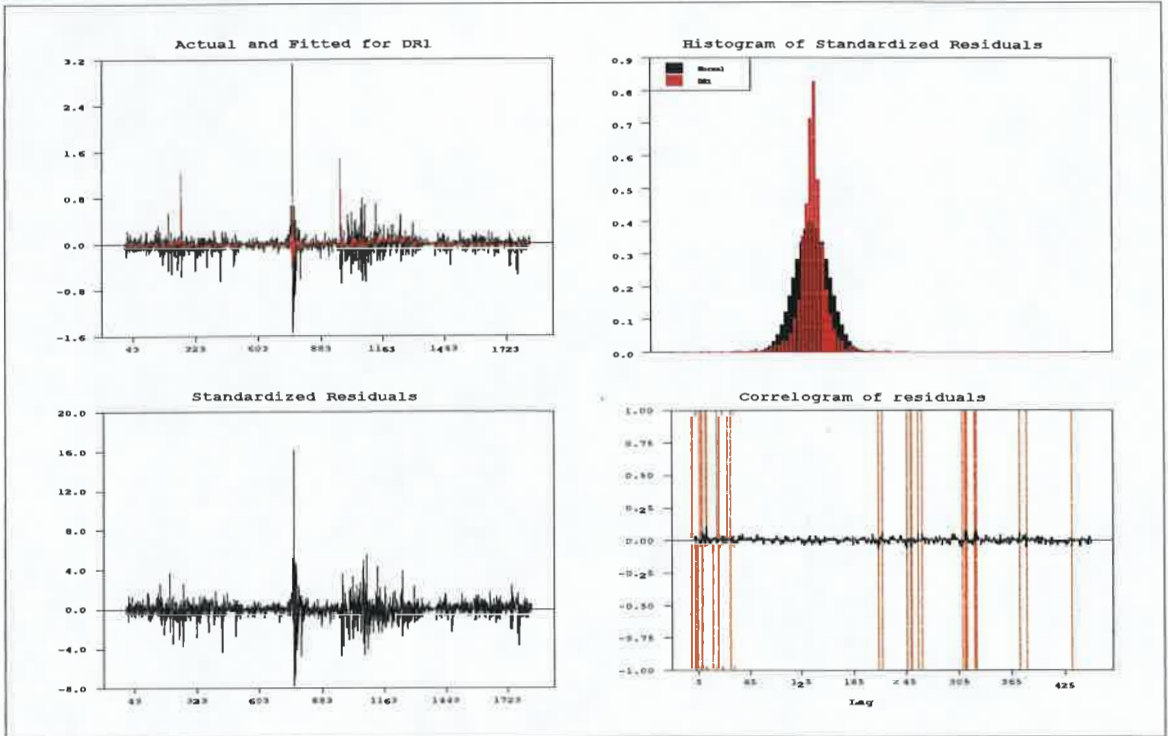
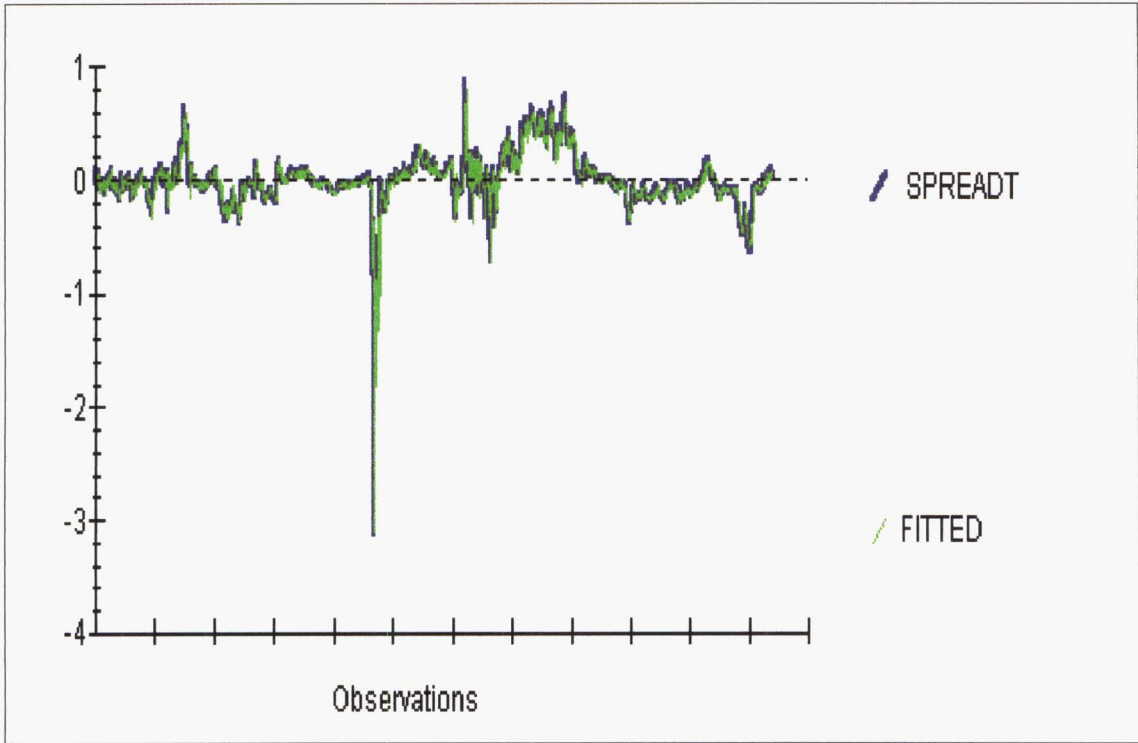### Appendix 1.b: Residual output for the Δr_t^3 series

Appendix 2: Single-equation estimation

Appendix 2.a Plot of actual and fitted values

Appendix 3: VAR estimation

Appendix 3.a: Plot of actual and fitted values for OLS estimation of the detrended S_t equation

Appendix 3.b: Plot of actual and fitted values for OLS estimation of the detrended Δr_t equation