PORTFOLIO OPTIMISATION USING THE JOHANNESBURG SECURITIES EXCHANGE TRADABLE INDICES: AN APPLICATION OF THE MARKOWITZ'S MEAN-VARIANCE FRAMEWORK

by

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ABSTRACT

The aim of this study was to assess the feasibility of constructing optimal portfolios using the Johannesburg Securities Exchange tradable sector indices. Three indices were employed, namely Financials, Industrials and Resources and were benchmarked against the JSE All Share Index for the period January 2007 to December 2017. The period was split into three, namely before the 2007-2009 global financial crises, during the global financial crises and after the global financial crises. The Markowitz's mean-variance optimisation framework was employed for the construction of global mean variance portfolios. The results of this study showed that it was feasible to construct mean-variance efficient portfolios using tradable sector indices from the Johannesburg Securities Exchange. It was also established that, on the other hand, global mean variance portfolios constructed in this study, outperformed the benchmark index in a bullish market in terms of the risk-return combinations. On the other hand, in bear markets, the global mean variance portfolios were observed to perform better than the benchmark index in terms of risk. Further, the results of the study showed that portfolios constructed from the three tradable indices yielded diversification benefits despite their positive correlation with each other. The results of the study corroborate the findings by other scholars that the mean-variance optimisation framework is effective in the construction of optimal portfolios using the Johannesburg Securities Exchange. The study also demonstrated that Markowitz's mean-variance framework could be applied by investors faced with a plethora of investment choices to construct efficient portfolios utilising the Johannesburg Securities Exchange tradable sector indices to achieve returns commensurate with their risk preferences.

Key words:

Global minimum variance portfolio; Johannesburg Securities Exchange; global financial crisis; Markowitz; modern portfolio theory

KAFUSHANE NGOCWANINGO

Inhloso yalolu cwaningo wukuhlola ukuthi kuyinto enokwenzeka ngempumelelo yini ukwakha amaphothifoliyo asebenza kahle kakhulu ngokusebenzisa izinkombamanani ze-Johannesburg Securities Exchange zemikhakha yemikhiqizo ehwebekayo. Ocwaningweni kusetshenziswe izinkombamanani ezintathu, okungama-Financials, ama-Industrials kanye nama-Resources futhi lokhu kwaqhathaniswa ngokwesilinganiso se-JSE All Share Index (iNkomabamanani Yamasheya Onke) sesikhathi esisukela kuMasingana 2007 kuya kuZibandlela 2017. Lesi sikhathi sahlukaniswa izingxenye ezintathu, eyokuqala okuyisikhathi esingaphambi kokuqala kwezinkinga zokufadalala komnotho emhlabeni wonke jikelele ngowezi-2007 kuya zowezi-2009, eyesibili yingenkathi kuqhubeka izinkinga zokufadalala komnotho womhlaba kanti ingxeye yesithathu yisikhathi esalandela emva kwezinkinga zokufadalala komnotho womhlaba. Ukwakha amaphothifoliyo evariyensi yenanimaphakathi (mean variance) yomhlaba wonke kwasetshenziswa uhlaka luka-Markowitz lokuthuthukisa ivariyensi yenanimaphakathi. Imiphumela yalolu cwaningo yabonisa ukuthi kuyinto enokwenzeka ngempumelelo ukwakha amaphothifoliyo ayisebenzisa kahle kakhulu ivarinyensi yenanimaphakathi, ngokusebenzisa izinkombamanani ze-Johannesburg Securities Exchange zemikhakha yemikhiqizo ehwebekayo. Kwatholakala futhi nokuthi, ngakolunye uhlangothi, amaphothifoliyo evariyensi yenanimaphakathi yomhlaba wonke akhiwe kulolu cwaningo, asebenza kangcono kakhulu kunenkombamanani okwakuqhathaniswa nayo kuleyo makethe okwabe kulindeleke ukuthi ikhuphuke intengo yezabelomcebo uma kubhekwa izimo zobungozi kanye nenzuzo. Ngakolunye uhlangothi, ezimakethe lapho kwabe kulindeleke ukuthi yehle intengo yezabelomcebo, amaphothifoliyo evariyensi yenanimaphakathi yomhlaba wonke abonisa ukusebenza kangconywana ngasohlangothini lobungozi kunenkombamanani okwakuqhathaniswa nayo. Ngaphezu kwalokho, imiphumela yocwaningo yabonisa ukuthi lawo maphothifoliyo akhiwa kusetshenziswa izinkombamanani ezintathu okuhwebekayo ngazo akhiqiza imihlomulo yokutshalwa kwezimali emikhakheni ehlukahlukene nakuba lawo maphothifoliyo enobudlelwano bokukhula nokwehla ngendlela efanayo noma ehambelanayo (positive correlation). Imiphumela yocwaningo isekela lokho ukwatholwa ngezinye izazi zocwaningo, okungukuthi uhlaka lokuthuthukisa ivariyensi yenanimaphakathi lusebenza kahle ekwakheni amaphothifoliyo asebenza kahle kakhulu ngokusebenzisa i-Johannesburg Securities Exchange. Ucwaningo lwabonisa futhi ukuthi uhlaka luka-Markowitz lokuthuthukisa ivariyensi yenanimaphakathi lungasetshenziswa ngabatshalizimali ababhekene nezinto eziningi futhi ezihlukahlukene eziphathelene nokutshala izimali okumele bakhethe kuzona ukuze bakhe amaphothifoliyo asebenza kahle kakhulu besebenzisa izinkombamanani ze-Johannesburg Securities Exchange zemikhakha yemikhiqizo ehwebekayo ukuze bathole inzuzo ehambelana nesimo sobungozi abasiqokile.

KAKARETSO

Sepheo sa boithuto bona ke ho lekola kgonahalo ya ho aha dipotefoliyo tse tswang pele re sebedisa diindekse tsa karolo ya kgwebisano tsa Johannesburg Securities Exchange. Ho sebedisitswe diindekse tse tharo, e leng Ditjhelete, Di-indasteri le Disebediswa mme di ile tsa bapiswa le JSE All Share Index bakeng sa nako ya ho tloha ka Pherekgong 2007 ho isa Tshitwe 2017. Nako ena e ile ya arolwa dikoto tse tharo, moo se qalang e bileng sa pele ho koduwa ya lefatshe ya ditjhelete ya 2007-2009, sa bobedi e bile sa nakong ya koduwa ya lefatshe ya ditjhelete ha sa boraro e bile sa ka mora koduwa ya lefatshe ya ditjhelete. Moralo wa Markowitz wa phapano e bohareng ya ntlafatso (*Markowitz's mean-variance optimisation framework)* o ile wa sebediswa ho aha dipotefoliyo tsa lefatshe tsa phapano e bohareng ya ntlafatso. Diphetho tsa boithuto bona di bontshitse hore ho ne ho kgoneha ho aha dipotefoliyo tsa phapano e bohareng e sebetsang ho sebediswa diindekse tsa karolo ya kgwebisano ho tswa ho Johannesburg Securities Exchange. Ho boetse ho fumanwe hore, ka lehlakoreng le leng, dipotefoliyo tsa lefatshe tsa phapano ya bohareng tse ahilweng boithutong bona, di sebeditse hantle ho feta indekse ya papiso mebarakeng e ditheko tse nyolohang ho latela ditlhopho tsa kotsi-puseletso. Ka lehlakoreng le leng, mebarakeng ya ditheko tse theohang, dipotefoliyo tsa lefatshe tsa diphapano tsa bohareng ho bonwe di sebetsa hantle ho feta indekse ya papiso ntlheng ya kotsi. Ho feta moo, diphetho tsa boithuto di bontshitse hore dipotefoliyo tse ahilweng ka diindekse tsa kgwebisano tse tharo di hlahisitse melemo ya tsetelo matseteng a fapaneng leha di ne di sebetsa hantle mmoho. Diphetho tsa phuputso di tsamaellana le diphumano tsa baithuti ba bang tsa hore moralo wa ntlafatso ya phapano ya bohareng e sebetsa hantle kahong ya dipotefoliyo tse ntlafetseng ha ho sebediswa Johannesburg Securities Exchange. Phuputso e boetse e bontshitse hore moralong wa Markowitz phapano e bohareng e ka sebediswa ke batsetedi ba lebaneng le dikgetho tse ngata haholo tsa tsetelo ho aha dipotefoliyo tse sebetsang ba sebedisa diindekse tsa karolo ya kgwebisano tsa Johannesburg Securities Exchange ho fumana dipuseletso tse nyallanang le mamello tsa bona tsa kotsi.

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DECLARATION

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I declare that the above thesis is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

S. HUNI

SIGNATURE

30 AUGUST 2018

DATE

LIST OF ABBREVIATIONS AND ACRONYMS

ALSI	All Share Index
APT	Arbitrage Pricing Theory
BVB	Romanian Bucharest Stock Exchange
CAPM	Capital Asset Pricing Model
CML	Capital Market Line
COMI	Condmag
COTR	Transilvania Constructii
DCC	Dynamic Conditional Correlation
$E_{(r)}$	Expected Return
ETF	Exchange Traded Funds
FDI	Foreign Direct Investment
FINI	SA Financial Index
FTSE	Financial Times Stock Exchange
GARCH	Generalised Autoregressive Conditional Heteroscedasticity
GMVPU	Global Minimum Variance Portfolio Uncapped
ICB	Industry Classification Benchmark
IMP	Impact Developer and Contractor
INDI	SA Industrial Index
JSE	Johannesburg Securities Exchange
MAD	Mean Absolute Deviation
MPT	Modern Portfolio Theory
MV	Minimum Variance
MVA	Mean Variance Analysis
MVO	Mean Variance Optimisation
MVP	Minimum Variance Portfolio

MVPCAP	Mean Variance Portfolio as a Weight Capped Index
NSE	Nepalese Stock Exchange
OMXBBCAPPI	OMX Baltic Benchmark as a Weight Capped Index
PMPT	Post Modern Portfolio Theory
RESI	SA Resources Index
SA	South Africa
S &P	Standard and Poor
STDEV	Standard Deviation
USA	United States of America

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CHAPTER 1

INTRODUCTION AND BACKGROUND

1.1 Introduction

The volatility of financial markets in both developed and developing economies is a major concern to stock market investors, scholars and finance field practitioners. The South African (SA) market is not immune to such volatilities. The SA market is highly integrated with other global markets and is influenced by micro and macro-economic factors such as inflation, inflation expectations, oil prices, exchange rates and real activities (Szczygielski & Chipeta, 2015:49). Such financial market integration, globalisation and technology advances have increased the importance of portfolio risk management.

Stock market movements cannot be predicted with certainty; hence, the need to hold efficient portfolios with a haven for constancy. The 2008 - 2009 global financial crisis instigated an economic tremor in the South African economy, leading to the gross domestic product (GDP) contracting by minus 1.8%. Consequently, stock market investors have become concerned about minimising risk. Markowitz (1952) proposed a minimum-variance model for portfolio construction aimed at minimising portfolio risk. The minimum-variance model has been extensively studied by scholars to determine its efficiency in minimising portfolio risk (Baker & Haugen 2012; Blitz & van Vliet 2007; Haugen & Baker, 1991; Jagannathan & Ma, 2003). The question for stock market investors remains whether such method can be instrumental in risk minimising on the Johannesburg Securities Exchange (JSE) in times of market downswings.

The JSE is the biggest of 22 African security exchanges and is among the top 20 in the world (JSE, 2017). More than 800 securities with different risk-return characteristics trade daily on the JSE equity market (JSE, 2018). For the year ending September 2017, the number of trades on the JSE equity market amounted to 26 081 with a yearly reported volume of 5 359 000 000 trades according to the JSE 2017 market statistics (JSE, 2017).

With so much trading activities and so many investments to choose from, an average investor may be overwhelmed and might be challenged on making an investment choice. As the fundamental principle of financial economics, resources are always scarce, hence choices must be made. The issue of scarcity as a key economic principle in modern society brings forth a concern of optimal allocation of the scarce resource, in this case investments so as to maximise on returns (Sims, *et al*., 2014:2).

The process of choosing an investment consist of certain considerations. Investors might need to know more about the previous performance of the security, the costs associated with the security, the regulatory issues, and the risk associated with the investment before making a choice. A big challenge that investors face is determining how well their investments will perform (Lombard 2015:9). This can be remedied by making use of a benchmark normally represented by a market index to evaluate the performance of the chosen investment (Elton & Gruber, 1999:266; Lombard 2015:9).

Using a market index benchmark will enable an investor to check on how volatile an investment is, the investment’s performance against a benchmark, and whether the benchmark used is relevant (Lombard, 2015:10). The Research Association for Savings and Investment South Africa (ASISA) carried out a study from which it was documented that 82% of actively managed general equity portfolios failed to beat their respective benchmarks (ASISA, 2015). For a 20-year period ending June 2014, only 18% of the actively managed portfolios on average were found to outperform the benchmark which was mostly the FTSE/JSE All Share Index (ALSI) (Lombard, 2015:10 - 11). However, several studies have documented that the passive trading strategy based on portfolios created on the underpinnings of the Modern Portfolio Theory (MPT) can outperform the market (Brouwer, 2015; Contreras, Lizama & Stein, 2016; Lombard, 2015; Mwamba & Suteni, 2010; Oladele & Bradfield, 2016; Du Plesis & Ward, 2009; Roopanand, 2001).

The volatility of the SA market as measured by the South African Volatility Index (SAVI) for the period 2007 to 2016 ranged from as low as 12 to 60 (Cairns, 2016:1). Considering the high volatility of the SA market, due to political instability, the weakening rand, and, lately a series of rating downgrades, portfolio construction is highly important for investors to get good returns. Holding efficient, optimally constructed and well-diversified portfolios might be of utmost importance to SA investors in such times to avoid unnecessary loss of wealth.

An efficient portfolio is a structured collection of financial vehicles held by an individual or group of investors with the aim of maximising returns corresponding to certain levels of risk (Popina & Martyniuk, 2016:160). The portfolio may include a pool of investment tools such as shares, obligations, savings certificates, bonds and gold, practically any asset traded for determination of future returns. Constructing an efficient portfolio is practically balancing a basket of risky securities with an admissible level of volatility and a desirable level of risk (Popina & Martyniuk, 2016:160). Portfolio optimisation is one of the most important considerations and has been a mainstay in finance (Clarke, De Silva & Thorley, 2006:10). The concept was originally articulated by Markowitz (1952) and it addresses the problem of investment choice, which derives to asset allocation and portfolio construction (Markowitz, 1952:77).

In his article, “Portfolio selection”, Markowitz (1952) presented a quantitative approach for the portfolio selection problem which was named the modern portfolio theory (MPT). From there, a feasible solution for portfolio selection and construction was derived (Markowitz, 1952:78). The MPT is a mathematical formulation of the concept of diversification in investing (Wepener, 2014:5). The MPT provides a solid theoretical foundation for portfolio construction and a starting point for the development of other portfolio construction theories such as the separation theory by Tobin (1958) and the capital asset pricing model (CAPM) by Sharpe (1964).

Before the development of Markowitz’s portfolio selection techniques, investors constructed their portfolios by assessing rewards generated by individual assets (Darko, 2012:6). From the study of modern finance, Markowitz (1952:78) recommends that investors consider the overall portfolio risk-reward characteristics instead of constructing portfolios based on attractive risk-reward characteristics of individual securities. It is not the risk of an individual asset that matters when investing but rather the individual asset’s risk in relation to all the other assets included in a portfolio (Rubinstein, 2002:1042). It is beneficial for investors and industry professional to understand how to use the Markowitz theory to construct optimum portfolios that can suit their desired levels of return as well as their risk tolerance.

According to Markowitz (1952), the basic motive behind portfolio construction is dispersion of risk. In his theory, Markowitz emphasises that the fundamental and critical issue in investing is portfolio selection as well as constructing a portfolio satisfactory to the investor (Mangram, 2013:61). Markowitz (1952) further demonstrates that, under certain conditions portfolio selection can be reduced to balancing the expected portfolio return and reducing the portfolio variance. The core concept in portfolio selection is portfolio diversification which entails combining different classes of assets (Mangram, 2013:61). Diversification allows investors to spread the overall portfolio risk or minimise the portfolio unsystematic risk through holding a portfolio of assets that have different reactions to negative market volatilities (Lombard, 2015:8). Diversification can be done across asset classes, geographically as well as across different companies (Godi & Sibindi, 2014:490).

The rest of the chapter is organised as follows: section 1.2 gives an overview of the JSE. section 1.3 outlines the problem statement. section 1.4 outlines the aim of the study. The research questions are presented in section 1.5. The research objectives are outlined in section 1.6. section 1.7 discusses the rationale behind the study. Section 1.8 outlines the formulated hypothesis while section 1.9 delimits the study. Section 1.10 concludes the chapter by presenting the structure of the dissertation.

1.2 Overview of the Johannesburg Securities Exchange

The JSE was formed on 8 November 1887 after the discovery of gold in Witwatersrand to raise the ample desired capital to invest in the mining sector (Smith, Jefferis & Ryoo, 2002:477). The initial trading on the JSE started in a small tent which was later improved and upgraded to an automated electronic trading system in the early 1990s. By then it was named Johannesburg Stock Exchange as only shares were being traded, but later changed its name to Johannesburg Securities Exchange in 2000 (JSE, 2017). To date, the JSE is among the world’s top twenty largest stock markets with a market capitalisation $1,007 billion at the end of 2013. In Africa, the JSE is the biggest exchange with over 800 securities listed.

Among the developments of the JSE is its agreement with the London Stock Exchange in 2001, which enabled a cross-dealing between the two exchanges (JSE, 2014). Consequently, the JSE trading system was replaced with that of the LSE. This was followed by the acquisition of the Bond Exchange of South Africa (BESA) in 2009 (JSE, 2014). The BESA was rebranded as the JSE debt market which resulted in the inclusion of South African government and corporate bonds as well as interest rate derivatives to the JSE offering (JSE, 2014). In 2012, the JSE together with other exchanges founded the United Nations Sustainable Stock Exchanges as an initiative to explore and impact how exchanges can operate with stakeholders such as investors, regulators and companies so as to create more conducive capital markets. In 2014, the JSE was re-branded to show its identity as a modern African market place that links investors to growth opportunities in both global and SA market and the JSE logo was changed following the re-branding (JSE, 2017).

1.3 Problem statement

South African investors seek to maximise returns by investing on the JSE. According to Nofsinger (2016:1), if investors hope to benefit from modern investing and have more money in future, they must invest in stocks. Stocks can bring opportunities to investors, such as building wealth, growing passive income as well as accumulation of capital as stocks allow investors to earn potentially satisfactory future returns (Nofsinger, 2016:1). However, with a vast number of securities trading daily on the JSE, it becomes difficult for investors to make choices and effectively pick stocks that could yield expected maximum returns for their desired levels of risk. The challenging problem for investors is choosing a combination of stocks that yield a maximum return for a certain level of risk or conversely a set of portfolios that will minimise the risk level for a certain expected level of return (Markowitz, 1952:77).

With the high volatility of the SA market, efficient portfolio optimisation becomes a critical subject for both investors and fund managers. Portfolios constructed based on the underpinnings of the MPT have been found to perform better than their benchmarks internationally (Giri, 2016:84). As much as portfolio optimisation have been widely studied in developing economies, African stock markets should be given a distinct attention, especially South Africa

The South African economy is highly volatile as evinced by the South African Volatility Index (SAVI) for the period from 2007 to 2016 which ranged from as low as 12 to 60 (Cairns, 2016). The SA rand which used to be very strong in the 1970s is also very volatile. As at 1994, the rate stood at R3.55 to the dollar and has been depreciating ever since. The rand reached an all-time low of R16.84 to the dollar in January 2016 (Trading economics, 2016) which can be linked to an increase in the country's current account deficit, low savings and low GDP growth. Again, the abrupt cabinet reshuffling for example the removal of the Minister of Finance (Nhlanhla Nene) in December 2015 might also have contributed to the present depreciation of the rand to the dollar, as there were mixed views about his removal. The xenophobic attacks in South Africa might also be a deterrent to investors. On the other hand, the JSE is the biggest of 22 African security exchanges and is among the top 20 in the world (JSE, 2017). More than 800 securities with different risk-return characteristics trade daily on the JSE equity market (JSE, 2018). As a leading security exchange in Africa, it is ideal to investigate the MPT on the JSE.

With the uniqueness, volatility and instability of the SA economy, a question can be asked on how effective it can be to optimize a portfolio using sector indices. A very few studies on the SA market have been found that deal especially with how portfolios constructed based on MPT using sector indices could perform. With the progression of the markets driven by technology and changing investor sentiments, it will be ideal to investigate the traditional MPT based on the mean-variance optimisation (MVO) technique to see its practicability on the JSE more than 60 years since its commencement. Based on the JSE tradable sector, can an investor that applies Markowitz's mean-variance MPT achieve maximum returns?

1.4 Aim

The aim of the study was to determine whether an investor could apply the traditional MPT theory efficiently in modern times to construct an optimum portfolio based on the SA tradable sector index to achieve maximum returns. The efficiency of the constructed tradable sector index portfolio will be determined by its risk-return characteristics as compared to the JSE ALSI.

1.5 Research questions

In order to guide this study, the following research questions were addressed:

1. Does the JSE tradable sector index (INDI, FINI, RESI) represent an optimal portfolio?
2. Can the MPT model transform the JSE tradable sector index (INDI, FINI, RESI) into a mean variance optimum portfolio?
3. Can the MPT model capture diversification benefits using the JSE tradable sector index (INDI, FINI, RESI)?
4. Does the optimally constructed portfolio using the Markowitz model consistently out-perform the JSE ALSI?

1.6 Research objectives

In order to guide this study, the following research objectives were satisfied:

1. to establish whether the JSE tradable sector index (INDI, FINI, RESI) represents an efficient portfolio;
2. to establish whether global mean variance portfolios could be constructed using the JSE tradable sector index (INDI, FINI, RESI);
3. to determine whether there are any diversification benefits associated with selecting a portfolio constructed from the JSE tradable sector index (INDI, FINI, RESI); and
4. to determine whether the global mean variance portfolios constructed using the JSE tradable sector index (INDI, FINI, RESI) consistently outperform the JSE ALSI.

1.7 Justification of the study

The opening of international investment opportunities on the JSE has placed returns offered by the JSE ALSI into a larger basket of opportunities for investors, allowing investors to question the efficiency of returns in the SA market (Roopanand, 2001:1-2). The questionability of the market returns on the JSE emanates from the fact that the beta based on the JSE will be biased resulting from the international investment opportunities. However, employing the SA sector indices will explain the inherent risk better than the general JSE since selective sectors are used within the SA context.

The rationale for carrying out this research lies in the documented findings of Du Plessis and Ward (2009) that the optimally constructed JSE mean variance portfolios (MVPs) can outperform the market. Studies conducted by Mwamba and Suteni (2010) and Brouwer (2015) also presented evidence that portfolios constructed based on the mean variance model outperformed their benchmarks. A recent study by Oladele and Bradfield (2016) on the JSE using seven different low volatility portfolio construction techniques confirmed the findings of Du Plessis and Ward (2009), Mwamba and Suteni (2010) and Brouwer (2015), as performance of the MVP outperformed the JSE ALSI. The techniques used by Oladele and Bradfield (2016) included the equally-weighted, the equal-weighted low beta, the low volatility single index model, the equal risk contribution, the naïve risk parity, the maximum diversification portfolio and the MVP. From the documented performance of the MVPs by Du Plessis and Ward (2009), Mwamba and Suteni (2010) and Brouwer (2015) and Oladele and Bradfield (2016) among others, it can be deduced that MVPs are better performers than their benchmarks.

With more than five decades elapsed since the invention of the MPT, the significance of this study was therefore to determine the application of the MPT by examining the performance and efficiency of the portfolio constructed based on the underpinnings of the MPT using the three major JSE tradable sector indexes. A comparison was made with the JSE ALSI as a benchmark. The global market crush in 2008 affected the JSE stock performances (Venter, 2011:65–66), hence, the need to carry out this study. The outcome of this study will furnish investors with the knowledge on portfolio selection and construction strategy using the MPT based on the minimum variance (MV) technique. This will enable investors to make sound decisions when considering investing in sector indices on the JSE.

A 2017 report by Consilium Capital (Coetzee, 2017) focusing on the SA local index performance for the past 10 years documented that the $INDI_{25}$ index outperformed major global markets and even the S&P500 by 143%. The $FINI_{15}$, index gained 34.4% in its overall performance while the SA-listed property retained 87.9% of its value (Coetzee, 2017). However, the $RESI_{20}$ index lost 59.9% of its overall performance over the past decade (Coetzee, 2017). With such a performance trend of the local indices, an investor holding a well-diversified local index portfolio will tend to profit from diversification benefits. Portfolio diversification is the backbone of Markowitz's (1952) MPT, and this study sought to apply the MPT on the JSE tradable sector to construct an efficient portfolio and evaluate whether the constructed portfolio would outperform the JSE ALSI as a benchmark. The study used hypothetical constraints to represent investor risk tolerance and preferences.

This present study deviated from the previous studies on the JSE (Roopanand, 2001; Du Plessis & Ward, 2009; Mwamba & Suteni, 2010; Brouwer, 2015; and Oladele & Bradfield, 2016), in that while previous studies focused on component indices, individual securities, property industry, ETFs and mutual funds, the present study employed the JSE tradable sector indices. South Africa sector indices are a representation of all the JSE listed instruments, classified according to their sector categories, which are Resources (RESI), Financials (FINI) and Industrials (INDI), (JSE, 2018). Employing the JSE tradable sector indices enabled a comprehensive examination of the performance of JSE constituents in a portfolio, since a well representative sample was used. On the other hand, the present study focused on the three defined different market volatility periods in South Africa namely before, during and after the global financial crisis period, to determine the consistency of mean variance portfolios against the chosen benchmark.

1.8 Delimitations of the study

The study employed the MPT theory, investigating its applicability on the JSE tradable sector index as well as examining whether MPT is still as practically significant as it was more than six decades ago. Financial models that were later invented, such as the CAPM and the separation theorem were based and built upon the MPT incorporating new findings (Du Plessis & Ward, 2009:39). According to Fabozzi, Gupta and Markowitz (2002:20), the application of the MPT has been spreading from the 20th to the 21st century. The authors predicted that the MPT popularity would not fade away anytime in the near or even distant future; hence, the MPT can even occupy a permanent position in the world of finance.

This study was limited to the three tradable sector indices which comprise of 50 non-fixed risky securities from which a portfolio was constructed. A risk-free asset was not included which disqualified the CAPM and the separation theory since they extend the MPT by including a riskless asset in the portfolio (Du Plessis & Ward, 2009:39). The JSE sector indices can clearly explain the inherent risk-return characteristics of the SA market without bias since the indices derive their values from replicating the performance of their respective SA sectors (Yu, 2008:2).

1.9 Structure of the dissertation

The rest of the dissertation is structured as follows:

Chapter 2: Literature review

This chapter opens by discussing the basic theory of the Markowitz model. Portfolio theories that were built on the MPT framework are discussed as well. The chapter reflects studies on the application of MPT, which were done in developed and developing markets.

Chapter 3: Research methodology

The chapter discusses the methodological issues, the data employed, as well as the formulation of the MVP framework that was utilised. The data consist of daily closing share prices of the main three tradable sector indices of the JSE as well as for the JSE ALSI, which was used as a benchmark. The steps that were followed in constructing the portfolios are also discussed in this chapter.

Chapter 4: Research findings and discussion

This chapter presents descriptive statistics and empirical findings on the construction of efficient portfolios. The Markowitz MVP framework was employed to construct the portfolios based on different constraints.

Chapter 5: Conclusion

The final chapter concludes the study. It presents a summary of findings and ends by proffering recommendations and suggestions for further research.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter discusses the theoretical underpinnings of modern finance portfolio theories. A comparison of the theories was done to elucidate how the theories link and complement each other. Empirical studies on the application of the MPT in developed economies, in developing economies as well as on the JSE are also discussed. Furthermore, specific to the JSE, studies on the performance of sector-based portfolios are discussed. Finally, studies on diversification benefits due to application of the MPT are discussed briefly. The rest of the chapter is organised as follows: section 2.1 gives an overview of the modern portfolio theories. Section 2.2 discusses the efficient market hypothesis. Section 3.3 outlines the empirical literature review conducted, starting with developed then developing economies. Finally, section 2.4 concludes the chapter by outlining a summary of the whole chapter.

2.2 Modern portfolio theories

The MPT is a passive portfolio management approach, constituting three portfolio theories, which are commonly based on the portfolio risk-return profile for portfolio selection and construction (Garaba, 2005:9; Vukovic & Bjerknes, 2017:14). The three theories are the mean-variance analysis (MVA) by Markowitz (1952), the CAPM independently developed by Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966) as well as the arbitrage pricing theory (APT) by Ross (1976). The portfolio optimisation ground-breaking research by Markowitz (1952), qualified him to be the father of modern portfolio theory (Darko, 2012:6). Therefore, researchers cannot tackle portfolio optimisation without mentioning the work by Markowitz as it is the foundation of the subject. In this regard, Markowitz's MVA is explained as the root theory of MPT, followed by Tobin's (1958) separation theorem, which was proposed and built upon the works of Markowitz. The CAPM, which was introduced and expanded to cater for the shortcomings of the previous theories, is also discussed. Finally, the APT, which is the final block of the MPT, is discussed briefly. Empirical studies on portfolio optimisation conducted in developed and developing countries as well as on the JSE are discussed. Finally, several empirical studies conducted on diversification benefits in different markets are discussed.

2.2.1 MPT theory

Markowitz (1952:77) proposed the MPT as the root of portfolio optimisation. Before the development of MPT, the risks - rewards valuation of the portfolio was done by analysing individual securities independently (Panja, 2014:62). Investors would consider the security risk – return characteristics in an ad hoc fashion (Kolm, Tütüncü & Fabozzi, 2014:356). The standard pattern for investing was to construct a portfolio by first identifying the securities that hold great probabilities for gain with the low risk chances. In this regard, investors would think that bank stocks have good risk – return characteristics and construct an entire portfolio using only the bank stocks. By doing so, the investors will be omitting the effect of unsystematic risk which can be eliminated by diversification (Brown, 2012:200; Marx, Mpofu, De Beer, Nortje & Van de Venter, 2010:36). With the vastness of securities available on stock markets, it can be tedious and challenging for investors to pick stocks individually that would yield the investors' desired outcomes.

The MPT theory hypothesises on diversification benefits (Fabozzi *et al.*, 2002:8). By formalising the concept of diversification, Markowitz (1952:78) proposes that investors should consider portfolios based on their collective risk - return characteristics rather than focusing on individual securities without considering how they will perform collectively as a portfolio (Brown, 2015:24). The collective performance of assets in a portfolio can be estimated by utilising the historical returns of the individual assets, the standard deviation and their covariance to calculate the portfolio risk and return. Since the return and risk (mean and variance) relationship are being considered, the model was also referred to as the "mean- variance portfolio model", (Joshipura & Joshipura, 2015:140). Markowitz was the first to show clearly how portfolio variance can be reduced as a result of diversification (Olsen, 2014:9).

Diversification is a method of reducing portfolio risk by apportioning investment among various financial instruments, industries or asset classes (Fragkiskos, 2014:9-12). The level of portfolio risk can be reduced due to diversification (Markowitz, 1952:87-89). There are two types of risk, systematic and unsystematic risk (Rutterford & Sotiropoulos, 2016:2). Systematic risk is macro in nature and is related to an economy as a whole, for example, interest rates and inflation (Rutterford & Sotiropoulos, 2016:2). Investors cannot do anything to lessen systematic risk. Unsystematic risk, on the other hand, is firm - specific and is also known as diversifiable risk (Fragkiskos, 2014:9). Diversification can be used to eliminate or reduce unsystematic risk (Markowitz, 1952:89). MPT suggests that, as the number of securities in a portfolio increase, the level of portfolio risk will be decreasing (Yahaya, Abubakar & Garba, 2011:102). This can be diagrammatically presented in Figure 1 below:

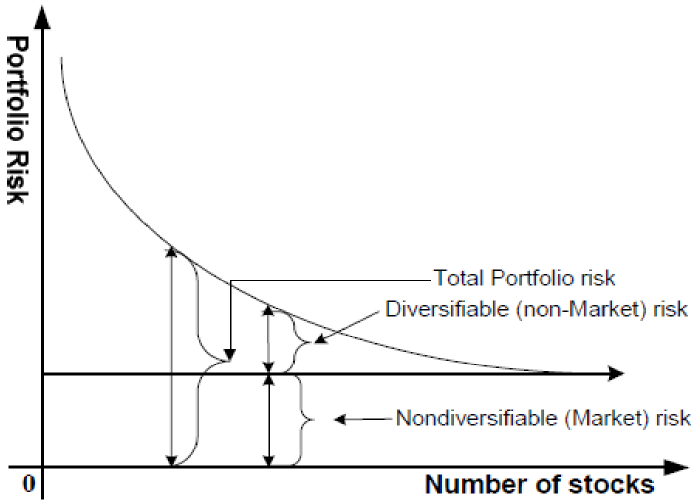

Figure 2.1: Systematic and Unsystematic risk

Source: Yahaya *et al*., (2011:104)

For diversification to be effective, the investment vehicles or securities must have a different reaction to certain market events, which Markowitz termed “the correlation”. Superior diversification benefits can be obtained by selecting assets from different industries and asset classes that are uncorrelated (Popina & Martyniuk, 2016:162). Different asset classes such as bonds and stocks react differently to hostile negative market events. As a result, the sensitivity of the entire portfolio will be reduced as the unpleasant movement in one asset class will be offset by a favourable movement in the other class or industry. The more uncorrelated the stocks are, the less the portfolio risk (Popina & Martyniuk, 2016:162).

The law of large numbers states that an investor can diversify among many several assets at the same time maximising returns whereby the actual return of the portfolio will be almost the same as the expected return (Markowitz 1952:79). In other words, the rule says there is a portfolio which gives a maximum return at the same time having a minimum variance (Markowitz, 1952:79). However, Markowitz (1952) disregarded the rule based on the fact that the portfolio with the maximum return is not necessarily the one with the lowest variance. In addition, diversification does not eliminate portfolio risk exclusively since there is always systematic risk which cannot be diversified away (Markowitz, 1952:79).

2.2.1.1 Expected return measurement

Markowitz (1952:78-80) suggests that the value of a security is best evaluated by its mean, variance, and its correlation to other securities in a portfolio. Within the infinite number of possible alternatives that an investor has to construct a portfolio, balancing the risk and return features of the portfolio can yield optimal results. Portfolio return refers to the anticipated earnings generated from the invested securities or assets (Markowitz, 1999:5). Its computation comprises finding the weighted average return of securities included in a portfolio by multiplying individual securities by their respective weights (Kisaka *et al*., 2015:9). Brown (2012:9-10) specifies a formula for calculating portfolio returns as follows:

$$E(rp) = \sum_{i=1}^{n} w_i \, E(r_i) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; Equation\ 1$$

where:

$\sum_{i=1}^{n} w_i$	=	1.0;
N	=	the number of securities;
w_i	=	the proportion of the funds invested in security *i*;
r_i, r_P	=	the return on *i*th security and portfolio *p*; and
$E(r_p)$	=	the expected portfolio returns.

2.2.1.2 Variance and covariance calculation

The portfolio variance (risk) is a measure of how returns of a set of securities constituting a portfolio fluctuate and deviate from the expected rate of return (Markowitz, 1999:5). In other words, it is the chance of unfavourable events happening. To calculate the portfolio variance, standard deviations and correlations of each individual security in a portfolio are used. On the other hand, covariance is a measure of how the assets in a portfolio can move in relation to each other (Markowitz, 1959:96-101). According to Chen, Chung, Ho and Hsu (2010:5), portfolio variance can be expressed as below:

$$\sigma_\rho^2 = (w_A\,\sigma_A)^2 + (w_B\sigma_B)^2 + (w_3\sigma_3)^2 + 2w_Aw_B\rho_{AB}\sigma_A\sigma_B +$$
$$2w_Aw_C\rho_{AC}\sigma_A\sigma_C + 2w_Bw_C\rho_{BC}\sigma_B\sigma_C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Equation\ 2$$

Where

$\rho_{AB},\ \rho_{AC},\ \rho_{BC}$ = correlation coefficient between the returns on assets AB, AC and BC.

$\sigma_A\sigma_B\sigma_C$ = standard deviations of returns of assets A, B and C

$W_AW_BW_C$ = the weight of each asset

Using the computing power technology, the magnitude of all feasible portfolios can be derived by a critical line as depicted in Figure 2.2. The set of portfolios constructed in this optimal manner conform to what Markowitz (1952:82) called the "efficient combinations frontier" (the critical line), which is a hyperbolic line that optimum portfolios lies considering the rationality of investors. The efficient frontier is a graph constructed with expected return on y-axis while the risk is on the x-axis (see Figure 2.2). The most efficient portfolio is one that gives the highest return for a given level of portfolio risk. Any point above the efficient frontier is unattainable and portfolios below the frontier are inefficient and would require rebalancing of asset classes for it to move closer to the efficient frontier. Markowitz concluded that an investor should select the optimum portfolio, which is the one that lies on or the one that is tangent to the efficient frontier (Engels, 2004:12). This is graphically presented below:

Y

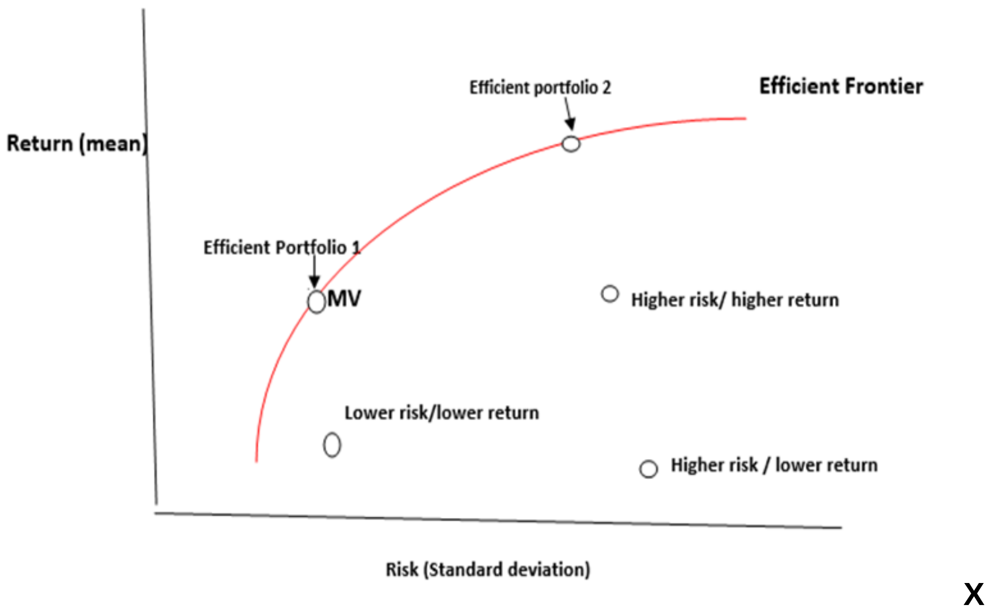

Figure 2.2: Minimum variance portfolio

Source: Ayodeji and Ingram (2015:44)

For a portfolio to be tangent to the efficient frontier, there is need for portfolio weighting and balancing (Markowitz 1952: 82-87). An investor cannot just pick one asset and expect it to be tangent to the efficient frontier. Several assets should be picked and combined in certain weights to form portfolios that will be examined using the model to determine whether the portfolios can be tangential to the efficient line (Markowitz 1952: 82-87).

By optimising portfolios, the idea is to come up with optimal solutions corresponding to certain constraints depending on the investor's risk - return tastes and preferences (Ayodeji & Ingram 2015:43). The basic assumption is that when investors are given a set of investments with the same reward but different risks, they will choose the minimum risk asset (Rutterford & Sotiropoulos, 2016:1). In other words, investors are assumed to be risk-averse. Risk-averse investors have minimal risk tolerance; hence they desire to hold portfolios with the least risk even if they expect not necessarily the highest levels of return. As the efficient frontier depicts optimum portfolios, such an investor will pick a portfolio with the least variance, which will be located on the lowest point of the efficient frontier (Ayodeji & Ingram 2015:44). The optimum portfolio clearly shows the proportion of capital to be invested in each stock and it signifies a maximum return for investors at a lowest possible variance (Ramanathan & Jahnavi, 2014:123). According to Ayodeji and Ingram (2015:39), such a portfolio is also called the minimum variance portfolio (MVP). This can be graphically presented as indicated in Figure 2.2.

An investor can only pick an asset depending on his or her risk profile. A high-risk preference investor, may pick efficient portfolio 2 (Figure 2.2), provided it is offering a commensurate higher return (Markowitz, 1952:85-87). As one moves further to the right along the efficient frontier, the graph in Figure 2.2 signifies a higher return associated with a higher risk. According to Markowitz (1952:87), all portfolios lying on the efficient frontier are efficient. It is however the investor's risk tolerance that will determine the optimal portfolio as determined by his or her risk preferences.

2.2.1.3 Assumptions of the mean-variance optimisation model

Markowitz (1952) assumed some factors underlying his model (Omisore, Yusuf & Christopher, 2011:22-23). These assumptions include:

1. markets are perfectly informationally efficient;
2. investors focus on the risk, return and portfolio correlations only when making their investment decisions;
3. assets' correlations are fixed and constant;
4. asset returns are normally distributed;
5. there are no transactional costs and taxes;
6. all investors have equal access to information at the same time; and
7. investors are price takers and they can borrow and lend money at a risk-free rate of interest.

2.2.1.4 Benefits of the mean-variance optimisation model

MVO model as the chief theory of portfolio optimisation outweighs other optimizing techniques due to its power in integrating portfolio objectives with constraints specified by individual investors (Lakhoo, 2016:9). A rapid processing of huge quantities of data is made possible using the MVO method, which is quite helpful especially for large financial institutions, which might instantaneously need to see the effect of new market information on portfolios. As an asset allocation tool, the MVO model has and still is being used worldwide with or without modifications at times and has had great success as a portfolio choice tool (Campbell & Viceira, 2002:2-6; Mendecka, 2006:8). Moreover, the simplicity of the MPT in mean-variance optimising makes it appealing to most investors and it can also be used when considering huge data sets (Contreras, *et al.*, 2016:4).

2.2.1.5 Limitations and criticism of the mean-variance optimisation model

The MVO model ignores some critical information about firms, for example the earnings, capital structure, investor sentiments as well as dividend yield, which might affect the performance of stocks (Lakhoo, 2016:10-11). The MVA basis for forecasting the future value of the securities was historical measurements, such as return, risk and correlation. Practically, historical values cannot accommodate new conditions which did not exist when the historical data were generated (Ayodeji & Ingram, 2015:40). Many investment professionals do not consider past security performances as good future predictors but rather choose to depend on macroeconomic views or the specific asset class views (Mendecka, 2006:11). Moreover, the MVO theory is postulated on the known future expected returns and risk while in practice such estimates are not always known and are subject to estimation error (Lakhoo, 2016:10-11). Considering only the favourable and high past performance of an asset is not a guarantee for a similar performance in the future (Ayodeji & Ingram, 2015:40).

One of the key assumptions of the theory is that all the markets are accurately efficient according to the efficient market theory (Omisore, *et al*., 2011:22–23). This however does not hold in all markets. Extensive research has been done on financial markets with results of inefficiency being documented (Omisore, *et al.*, 2011:22). Real markets have asymmetric information, insider information, as well as, some investors who are just more informed than others. On the JSE, studies by Afego (2015), Jefferis and Smith (2004), Thompson and Ward (1995), Van Heerden, *et al.*, (2013) among others documented mixed findings on the efficiency of the JSE.

The MVO model assumes that investors focus on the risk, return and portfolio correlations only when making their investment decisions and that all investors are rationale and risk-averse (Ayodeji & Ingram, 2015:43). However, practically investors tend to consider more factors such as the liquidity of the assets as well as other market behaviours. The rationality of all investors was criticised by behavioural finance as investors at times follow a herd behaviour (Bernales, Verousis, & Voukelatos, 2016:21). Some investors can willingly take higher risk investments even if these offer lower returns (Omisore *et al.,* 2011:23).

The MPT model assumes a market where there are no transactional costs and taxes (Omisore *et al*., 2011:23). However, in a real investment world situation, financial products are subject to fees as well as taxes. Costs, such as broker fees will alter the portfolio composition by deducting a certain percentage of the available funds no matter how small (Omisore *et al*., 2011:23).

In his portfolio optimisation model, Markowitz (1952) stipulated that when securities are randomly picked and combined proportionately into a portfolio, then the total risk will decline. However, Evans and Archer (1968) observed a diminishing effect in the risk reduction as more securities are added to the portfolio. They documented that the economic benefits of portfolio diversification are exhausted when a portfolio contains 10 or more stocks. Based on the findings of Evans and Archers (1968), it is suggested that the diversification benefits can be present up to a maximum of 10 to 15 stocks. On the other hand, Fisher and Lorie (1970) documented that using randomly selected assets, a portfolio of 32 assets could reduce the risk by 95% compared to a portfolio based on the entire New York Stock Exchange. Statman (1987) also confirmed a maximum number of 30 assets to yield maximum diversification benefits while Longin and Solnik (1995), Newbould and Poon (1993) and Tang (2004) individually documented that a portfolio with 8 to 20 constituents yield diversification benefits. On the other hand, Ivković, Clemens and Scott (2008) agreed on a 4 to 11 stock portfolio to reap diversification benefits, while Mbogo, Aduda and Mwangi (2017) approved a 16 to 20 stock portfolio size on the Kenyan stock market. The different authors used different methods of optimisation which included the MV method and came up with different results on the number of assets to effectively make up a portfolio. However, portfolio managers are advised not to create portfolios of too many assets since the diversification benefits will diminish and could cause superfluous diversification (Nwakanma & Gbanador, 2014:147).

2.2.1.6 Does MPT hold during financial crises and market downswings?

A financial crisis is a period when there is a sudden dramatic downward drop of the market because of domestic or external forces due to extreme manifestations between the financial sector and the real economy (Claessens & Kose, 2013:3-4). During such times, all the asset classes are likely to move in the same direction (Markowitz, Hebner & Brunson, 2009:4). Examples of financial crisis are that of 1929 to 1932, another on of 19 October 1987 as well as the recent 2008 one (Marumoagae, 2014:380-381). During financial crises, securities do not fall equally or at the same rate (Ali & Afzal, 2012:276). In the 2008 crisis, high beta securities fell at a high rate while the low beta securities had a relatively smaller downward move (Markowitz *et al*. 2009:4). As bonds are less risky, corporate bonds fell much less than equities. According to Markowitz *et al*. (2009:4) asset classes were moving in accordance with their historical betas during the 2008 crisis. The Standard & Poor's 500 (S&P500), which is a low-beta index fell by 36.8%while the higher-beta asset class index from the emerging markets dropped by 53.9% (Markowitz *et al*. 2009:4).

As much as literature says the MPT might have failed during the crisis, Markowitz *et al.* (2009:1) posit that MPT would be an effective tool to use in such times. That was justified using the simplified model of portfolio theory (SMPT) by Sharpe (1963). According to the SMPT, each security is correlated to another security since all securities are correlated to the market (Markowitz*, et al.,* 2009:1). The model introduced a new variable for computing return which is "an uncorrelated random idiosyncratic term" which is a source of periodic fluctuations in an asset. In other words, the idiosyncratic term of an asset is not correlated with any other asset; therefore, the idiosyncratic term of a portfolio will not be correlated to the market (Markowitz *et al*, 2009:2-3).

Due to the idiosyncratic term of securities being uncorrelated, diversification is made possible (Markowitz *et al*. 2009:3). However, during a crisis, the systematic risk tends to swamp even the idiosyncratic risk of the asset. In such a time, MPT could be of aid by its asset class diversification contribution (Markowitz *et al*. 2009:5). Mathematically, the MPT formulates the concept of diversification in investing in such a way that it hedges against market risk even if assets' returns are not negatively correlated (Omisore *et al.* 2011:21). The loss in value of a mixed class portfolio, for example a portfolio constituting bonds and equities could be less than that of a same asset class portfolio. As mentioned by Markowitz during his interview with Mark Kritzman in 2016, MPT investors can always balance their portfolios as a winning tool during market downswings. If a 60/40 portfolio is the perfect one for an investor, such investor should make sure that the portfolio is rebalanced in the event of the market going up or down to maintain the 60/40 mix. Markowitz emphasised that with proper rebalancing of a portfolio, the small investor would have won using MPT except for the brokerage fees (Kritzman & Markowitz, 2017).

2.2.2 Separation theorem

Tobin (1958) expanded on Markowitz's work based on an assumption that cash was a riskless asset in a theory which was termed separation theorem (French, 2003:62). Tobin believes that by adding a risk-free asset, such as cash or a government bond, to a portfolio, it is possible to outperform a risky portfolio in terms of both risk and return. By doing so, Tobin (1958) showed that a risk and return combination for an optimal portfolio consisting of risky and riskless assets would be a straight line which will be equivalent to the Markowitz's efficient frontier. The primary objective of the separation theorem was to provide an improved analysis of holding of cash (Markowitz,1999:9). Tobin (1958) named the new efficient frontier capital market line (Du Plessis & Ward, 2009:40). According to Tobin (1958), if investors are only concerned with risk and the level of return then the optimal portfolio will be identified somewhere along the capital market line (CML).

The CML is used in conjunction with the efficient frontier to show the rates of return for efficient portfolios constituting a risk-free asset and market portfolio. In other words, the CML shows a positive linear relationship between returns and portfolio betas. The point on which the efficient frontier touches the CML is called the tangency point and is the most efficient portfolio (Feldman & Reisman, 2003:9). This is illustrated diagrammatically in Figure 2.3 below:

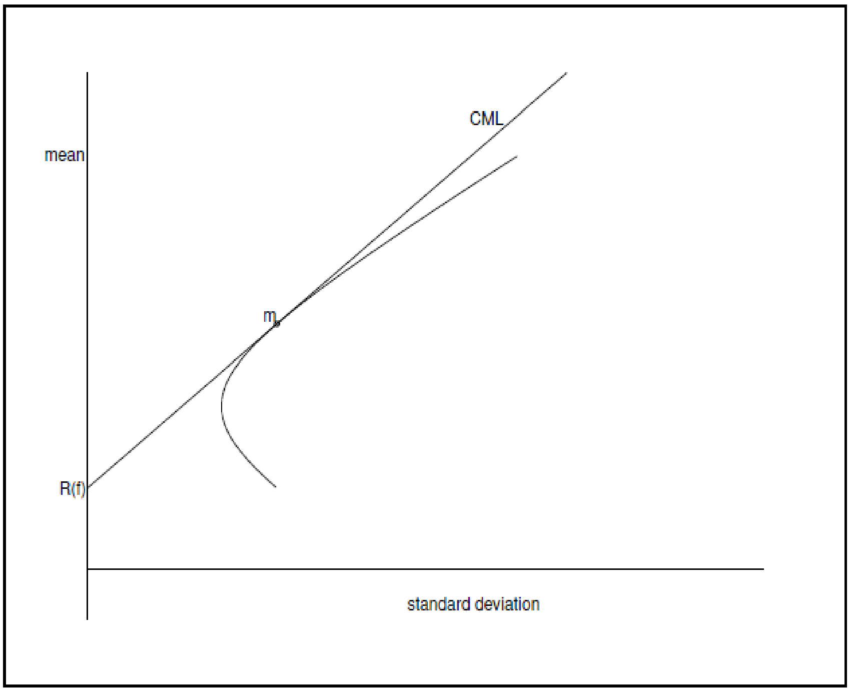

Figure 2.3 Tangency point (most efficient portfolio)

Source: Chen *et al.* (2010:17)

The point (m) as indicated in Figure 2.3 is the tangency point which represents the most efficient portfolio. On the other hand, R(f) represents the return of the risk free asset.

By combining a risk-free asset with a portfolio on the efficient frontier, a portfolio with superior risk-return profiles as compared to those on the efficient frontier can be constructed. Using the risk-free asset, investors can leverage their position by shorting the risk-free asset and using the proceeds to increase their investment in the super-efficient portfolio (Tobin, 1958:4-5).

The key assumption as proved by Tobin (1958) is that in the world there is only one safest asset, which is the risk-free asset. Tobin summary “don’t put all your eggs in one basket” clearly shows a need for diversification as well as inclusion of a risk-free asset within a portfolio, (Lian & Toften, 2015:19).

However, Campbell and Viciera (2002) criticise the separation theorem based on the notion that cash is not a riskless asset in the long run. The effect of interest rate and inflation affect the value of cash and cause a variance on its return (Markowitz, 1999:10). When the variance is measured using standard deviation, that will be a risk, hence, the portfolio choice of short term investors will end up being different from long term investors when using mean-variance precepts with an inclusion of cash as a riskless asset.

2.2.3 Capital asset pricing model (CAPM)

As an extension of the theories by Markowitz (1952) and Tobin (1958), another capital market theory was introduced and expanded independently by Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966). The CAPM was built upon the previous works of the MPT as a framework to describe the relationship between systematic risk and expected return of securities (Dempsey, 2013:2). The CAPM encapsulates that investors expect to attain a risk-free rate plus a market risk premium, multiplied by their market exposure which is the systematic risk. Systematic risk was quantified using beta according to the CAPM model. As an iteration of the MPT, CAPM assumes that investors hold fully diversified portfolios; hence, they hold portfolios in anticipation of being compensated for the systematic risk alone, which cannot be eliminated by diversification (Vladimirov, Stoilov & Stoilova, 2017:88). The CAPM, besides including a measure for the systematic risk, complements the MVA model by establishing a positive linear relationship between the risk and return of the asset and can be used as an asset and portfolio selection tool by industry professionals and investors, just like the MPT (Dzaja & Aljinovic, 2013:164-166).

While the MVA is a normative theory which states how investors should behave, the CAPM is a positive theory which shows how investors behave (Mangram, 2013:13). Since the introduction of the CAPM more than five decades ago, its accuracy in explaining asset returns has remained debatable. Although the CAPM was simple and rational, studies which were conducted to test the validity of the model revealed some restrictions and limitations of the study (Džaja and Aljinović, 2013:165). As much as the earlier studies (Black, Jensen & Scholes,1972; Fama & MacBeth,1973), agreed with the model, later studies (Michailidis, Tsopoglou, Papanastasiou & Mariola, 2006; Trifan, 2009; Choudhary & Choudhary, 2010; Sayeed, Khatun & Chowdhury, 2014) revealed contradicting results and commonly documented a limitation of the CAPM in sufficiently explaining asset returns.

2.2.4 The arbitrage pricing theory

In a bid to cater for the shortfalls of the CAPM, Ross (1976) introduced the arbitrage pricing model (APT). According to Gul and Khan (2013:86), the APT is a reformed version of the CAPM, which explains the expected return of the asset based on its sensitivity to fluctuations in both micro and macroeconomic factors. The core concept of the APT is that idiosyncratic risk can be diversified; hence, the equilibrium prices of securities will be approximately linearly related to their factor exposures (Bartram, Brown & Stulz, 2016:2-3). As stated by the APT, the return of an asset is explained as a linear function of various macro-economic factors, and not its idiosyncratic risk. Ideally, idiosyncratic risk is almost the same as unsystematic risk since it has little correlation with the market and it comprises all factors that affect an asset and not the entire market, such as the stock and its underlying company exposures at the microeconomic level (Bartram, *et al*., 2016:1-2).

As much as the CAPM is an instrumental tool in explaining market returns in modern finance, it lacks in some dimensions as it fails to explain the relationship between the portfolio stocks return and other non-company factors (Gul & Khan 2013:86). The APT model explains the relationship between a single asset and a portfolio under many different macroeconomic variables (Gul and Khan, 2013:86). On the other hand, the APT model assumes a no arbitrage situation; hence, any asset with a different price from the model is under or overpriced. In other words, if expected returns are not linearly related to the portfolio factor loadings (beta, macroeconomic factors, etc), then an arbitrage opportunity will arise (Nguyen, Stalin, Diagne, Aukea, Rootzen & Herbertsson, 2017: 11-14).

Ross (2017:3-5) stipulates that individual stock returns depend on expected as well as unanticipated events. In the case of expected events, investors incorporate them in the expected risk-return predictions. On the contrary, most returns realised are a result of unforeseen circumstances, hence arbitrage opportunities can be present since the future is unknown. However, as an assumption, the APT model emphasises that in an efficient market, arbitrage opportunities cannot be persistent as the opportunities will be exploited by investors and prices will correct automatically with time (Nguyen *et al*., 2017:14).

The combination of the MVA, the CAPM and the APT fulfilled the portfolio theory in modern finance thereby forming the MPT (Hodnett & Hsieh, 2012:860). Research has been done in several markets to show how the MPT model can be used for portfolio optimisation (Baker & Haugen, 2012; Blitz & Van Vliet, 2007; Haugen and Baker, 1991; Jagannathan & Ma, 2003). While the MVA as the root of portfolio optimization focuses on assessing the risk – reward relationship of assets to determine portfolio selection, the separation theory (CML) supplements by adding the risk-free rate to the analysis to alleviate the level of risk within a portfolio (Du Plessis & Ward, 2009:40). The CAPM complements the MPT theory by explaining the relationship between systematic risk and returns and the APT finishes this by explaining the portfolio returns in relation to multiple non-company factors such as macro-economic factors (Gul & Khan, 2013:86). All the theories (MVA, CAPM and APT) base on the underpinnings and assumptions of the efficient market hypothesis (EMH). However, for this study, only the MVA was employed for the determination of efficient portfolio construction and optimisation. A comparison of the three modern portfolio theories is reflected in Table 2.1.

Table 2.1: Comparison of the MPT models

Model	MVO	CAPM	APT
Originator(s)	Markowitz (1952)	Treynor (1961), Sharpe (1964), Linter (1965) and Mossin (1966) (independently)	Ross (1976)

Contributions	-The first model on portfolio theory which conceptualises on diversification	-An expansion of the MVO which introduced valuation of systematic risk -Was later modified by Sharpe (1964) to make it more practically easier and operational	-In addition to market risk, there are more variables to consider when pricing assets such as investor confidence, inflation and interest rates -The model is based on less restrictive assumptions
Risk measure	Standard deviation	Beta	Beta
Similarities/ Differences	-Single model for asset pricing -Based on the assumptions of EMH -Formula: $E(rp) = \sum_{i=1}^{n} w_i E(r_i)$	-Single model for asset pricing - Based on the assumptions of EMH -Formula: (r) = rf +b (rm - rf)	-Multi-factor model for asset pricing -Based on the assumptions of EMH though less restrictive -There is no arbitrage -Formula: (r)= = rf + b1 (factor 1) + b2 (factor 2) + b3 (factor 3)

2.3 The efficient market hypothesis

The MPT theoretically bases its principles on the underpinnings and assumptions of the efficient capital markets, which hypothesises that security prices reflect all the information available on the market (Hodnett & Hsieh, 2012:849). There are three types of market efficiency: strong form, semi-strong and the weak form of efficiency (Omisore *et al*., 2011:22). The weak form of efficiency states that security prices reflect all the past publicly available information. On the other hand, the semi-strong form of efficiency believes that security prices reflect all publicly available information and that the prices adjust revealing additional information in the market. Finally, the strong form of efficiency denotes hidden or insider information in the asset prices (Omisore *et al*., 2011:22).

For a market to be classified as efficient under the EMH, the security prices should incorporate all the available information (Omisore *et al*., 2011:22). All the information must be publicly available without any obstruction. In this modern technological era, the efficiency of markets is being improved due to advancement in technology as well as automation of stock exchanges (Njuguna, 2015:8). Technology has brought forth rapid developments in communication, high-speed Internet and mobile gadgets, which are now being used as efficient broadcasting systems to enhance information efficiency (Njuguna, 2015:8).

Several studies have focused on the efficiency of the JSE (Afego 2015; Jefferis & Smith, 2004; Thompson & Ward, 1995; Van Heerden, Rodrigues, Hockly, Lambert, Taljard & Phiri, 2013). In a bid to facilitate information flow, the JSE developed the Stock Exchange News Service (SENS) in 1997 as the only platform to release companies' information publicly in order to enhance transparency (Lombard, 2015:24).

2.4 Empirical literature review

The success of MVPs in outperforming benchmarks has been confirmed in numerous studies both for developed and developing economies (Baker & Haugen 2012; Blitz & Van Vliet 2007; Haugen & Baker, 1991; Jagannathan & Ma, 2003). The introduction of MVPs dates back to 1952 when the first block of MPT was originated by Markowitz (1952). As a result of the theoretic and practical effective performance of the MVPs, finance real-world practitioners are now investing more in low-volatility portfolios as compared to the high-risk portfolios (Baker & Haugen, 2012:3).

This literature review section of the study gives an overview of existing studies that have been done on the application and validity of the MPT in developed as well as developing markets. Studies on diversification benefits as a result of MPT application will also be discussed.

2.4.1 Empirical studies conducted in the developed markets

Several research studies have empirically tested the feasibility of Markowitz's MPT model as a portfolio optimisation strategy. Haugen and Baker (1991) were the pioneers of testing the model. The two researchers tested the MPT model on the United States (US) market based on the Wilshire 5000 (Haugen & Baker, 1991). The period of their study was 1972 to 1989. The authors constructed the minimum variance portfolios (MVPs) with a restriction of short selling at the beginning of each quarter. Sample covariance matrix computed over the trailing period of two years using monthly returns was used (Haugen & Baker, 1991). Furthermore, the authors observed that, portfolios constructed using Markowitz's model persistently outperformed the benchmark (Wilshire 5000) in terms of returns and volatility. The MVP constructed yielded a return, which was 22% higher and a risk 21% lower than the benchmark. This research work served as a basis for several later MVP studies.

Fifteen years later, Clarke, *et al.*, (2006) researched the performance of MVPs in the US equity market from 1968 to 2005. More recent portfolio construction techniques were used to construct MVPs using the 1000 largest US stocks. Clarke, *et al.*, (2006) used the S&P500 as a benchmark of the study. Confirming the study of Haugen and Baker (1991), Clarke, *et al.*, (2006) concluded that MVPs achieved an approximate 25% lower volatility than the benchmark without lowering returns. The performance of the constructed MVPs was tested using several covariance estimation methods and the conclusion confirmed the findings of Haugen and Baker (1991). For the 38 years studied, one can conclude that on average, performance of MVPs is consistent considering the time frame of the Haugen and Baker (1991) and Clarke *et al.* (2006) studies, which yielded almost the same results.

Many more studies such as Bower and Wentz (2005), Contreras, Lizama and Stein (2016), Širůček and Křen (2015), among others, have focused on investigating the application of the MPT model and the performance of low volatility portfolios on the US market. The studies on the US market concluded that the MPT can be effectively applied on the US market to optimise portfolios. Bower and Wentz (2005) further compared the Markowitz minimum variance (MV) model against the mean absolute deviation (MAD) and concluded that the strength of the two models were not significantly different in optimising portfolios. In addition, Contreras *et al.* (2016) confirmed a consistent outperformance of the market benchmark by the MVPs constructed except during the financial crisis period when the MVP trailed their benchmarks.

Kok, Giorgioni and Laws (2009) investigated the performance of Shariah-Compliant Indices (SCI) MVPs constructed using the MVO model on United Kingdom (UK) market. The FTSE100 was used as the benchmark against the MVP constructed which comprised of two SCI, two non-SCI and two sustainability indices (Kok, Giorgioni & Laws, 2009). The period of the study was 2001 to 2007. The authors concluded a reduction in the overall portfolio risk. Diversification benefits were therefore observed due to inclusion of the SCI on the portfolio. However, there was need to consider the correlation of the three selected indices first to yield better results in risk reduction as the less correlated the indices are, the more the diversification benefits.

Baker and Haugen (2012) analysed the performance of MVPs against high risk portfolios in 12 developing and 21 developed economies. The economies studied included the United Kingdom, Canada, Australia, the United States, Italy, Singapore, South Africa, Korea, Brazil and China among others. The study stretched from 1990 to 2011, a period of 10 years. Low-volatility portfolios were constructed for each economy and compared against high-risk portfolios. MVPs were concluded to outperform high-volatility portfolios (Baker & Haugen, 2012). That contradicted the basic pillar of finance which confirms that for a high-risk portfolio, a high return is expected (Markowitz, 1952:77). Investment practitioners therefore should analyse risk-return characteristics of portfolios before investing funds since high risk does not necessarily guarantee a higher return.

Rocha (2016) investigated the efficiency of the MPT minimum risk portfolios (MRPs) against the post-modern portfolio theory (PMPT) based on the European stock market. For a period of 18 years from January 1997 to December 2015, Rocha (2016) obtained stock data of 16 stocks from the Euro Stoxx 50 Index. The historical data were analysed, and daily returns for each stock were computed and then annualised. Efficient frontiers were derived after the average variance and covariance had been computed. The research confirmed that MPT and PMPT produced the same MRPs throughout the investment period.

From the studies carried out in developed markets, the MPT has been observed to be an effective tool for portfolio optimisation. The performance of the MVPs has also been noted to outperform their respective benchmarks (Baker & Haugen, 2012; Clarke, *et al.*, 2006; Haugen & Baker, 1991; Kok, Giorgioni & Laws, 2009; Rocha, 2016). It will be ideal to assess the success of the MVPs in developing markets to investigate if the same effect can be obtained. Developing markets tend to be more volatile than developed markets due to their political and economic conditions (Demirgüç-Kunt & Detragiache,1998:103). Specific to the SA market, volatility as measured by the South Africa Volatility Index (SAVI) for the period 2007 to 2016 ranged from as low as 12 to 60 (Cairns, 2016:1). Considering the high volatility of the SA market, due to political instability, the weakening rand and lately a series of downgrades, portfolio construction is highly important for investors to get good returns. Holding efficient optimally constructed and well-diversified portfolios might be of utmost importance to investors in such times, hence the need to assess whether the MPT could be applied effectively in the case of developing markets.

2.4.2 Empirical studies conducted in the developing markets

The applicability of the Markowitz model was examined and tested for the Nepalese Stock Exchange (Paudel & Koirala, 2006). The authors investigated the portfolio selection models using a sample of 30 stocks from the exchange. Using the data for 10 years (1997 to 2006), two stock portfolios were created using the MV method. For the period tested, the authors concluded that the MPT model was an effective tool in terms of optimal portfolio construction.

Gupta and Basu (2008) carried out a study on the Indian stock market over the period April 1997 to April 2007. Using the dynamic conditional correlation generalized autoregressive conditional heteroskedasticity (DCC GARCH) model, portfolio correlations for 10 industry sectors were estimated. A portfolio was then optimised, and a better performance was noted for the optimised portfolio (0.994) as compared to the benchmark (0.527) based on the Sharpe ratio (Gupta & Basu, 2009). Furthermore, the author noted that sector returns are normally influenced by the performance of the economy on the Indian Stock market. The study also found that investments in each sector react differently to market conditions and other factors. In that case, diversifying a portfolio based on sectors can be beneficial. Again, to obtain diversification benefits, correlation between the set of assets must be less than perfect so as to realise better portfolio performance.

Bausys (2009) conducted a study on the Baltic stock market using the euro-denominated market capitalisation weighted index OMX Baltic Benchmark (OMXBB) as the index for the analysis. The reason for using such an index was that the OMXBB captures all three Baltic stock markets (Estonian, Latvian, and Lithuanian). The OMXBBCAPPI (the OMX available as a weight-capped price index) was used as a benchmark against the constructed portfolio. The objective of the study was to show the effectiveness of the MVP in determining an optimum portfolio. The sample for the analysis only included the stocks that were listed in OMXBB in the years, 2001 to 2008 excluding the securities that had more than 10% of the required data missing (Bausys, 2009). The performance of the constructed MVP fluctuated during the period. The MVP was outperformed by its benchmark in a market uptrend from the year 2005 until when the market dropped in 2007. Bausys (2009) observed that the MVP performed better than its benchmark in bearish periods, demonstrating a superior performance during the 2007-2008 period of the global crisis and maintained its outperformance then.

Darko (2012) investigated portfolio optimisation using the MVP model on the Ghana Stock Exchange with a restriction in short-selling. The period of the study stretched from 2007 to 2010. Mean return, variance and covariance of the three selected indices

(ALSI, financial and non-financial) were computed for a period of 48 months. A positive correlation was noted among the indices although it was with a smaller percentage between the financial and the non-financial indices, which meant a difficulty in yielding diversification benefits. As a result, the study confirmed that for an optimum portfolio, an investor ought to allocate 83.44% of the funds in the non-financial index and 16.56% in the financial index to yield optimum returns.

The applicability of the MPT was investigated by Jiang (2013) on the Stock Exchange of Thailand (SET100). The examination period stretched from 2010 to 2011, a period of 486 days excluding non-trading days. The efficiency of the MPT was tested in an index (SET100) portfolio to determine the performance of MPT when a short-term investment horizon is considered. The top 30 outstanding companies from different sectors were selected and used to construct an MVP. Stocks were picked from sectors such as the agriculture and food sector, industrial business sector, resources and technology sector, property and construction industry, the service industry as well as from the finance sector (Jiang, 2013). An efficient set of portfolios was computed and constructed and the total return for the portfolio was found to be 32%. Tests were run using the Sharpe ratio, the Treynor ratio (see Hübner, 2005), the Jensen ratio (see Hübner, 2005) and the information ratio (see Goodwin, 1998) to determine the volatility of the portfolio. Jiang (2013) confirmed that the constructed efficient MVP outperformed the SET100 index by a significant margin in terms of both return and risk.

Thirimanna, Tilakartane, Mahakalanda and Pathirathne (2013) carried out a study on the Colombo Stock Exchange (Sri Lanka) to assess the performance of the MPT. The study stretched from 2009 to 2011. Two portfolio construction techniques were used, namely the MPT and the cointegration approach (see Wahab & Lashgari,1993:716-717). Sector portfolios were created and compared against each other to find a better strategy (Thirimanna, *et al.*, 2013). The portfolios were also compared against the benchmark index, the ASPI (i.e. the Sri Lanka Colombo All Share Price Index). The constructed portfolios were found to perform better than the market in terms of return. Considering portfolio selection, the MPT was observed to be the ideal technique on the Colombo Stock Exchange. However, in terms of the standard deviation, the market index (ASPI) was found to be better compared to the constructed portfolios. Ideally, the MVP is mainly for risk-averse investors, hence, it is supposed to be the minimum risk portfolio. Including uncorrelated assets, changing asset weightings as well as changing the number of assets in the portfolio could have reduced the MVP standard deviation (Markowitz, 1952:89).

Razak, Kamil and Elias (2014) applied the MPT model on the Malaysian market. The authors selected 50 out of the 290 equities listed in the FTSE Bursa Malaysia EMAS Index (FBMEMAS) from various sectors. The FBMEMAS comprises large and mid-cap stocks of the FTSE Bursa Malaysia 100 Index and the FTSE Bursa Malaysia Small Cap Index. Financial data were collected between January 2009 and June 2011, variables calculated and a Shariah-compliant MVP was constructed (Razak, *et al.*, 2014). The whole study period was sub-divided into five six-monthly periods to measure performance periodically, and the portfolio was rebalanced semi-annually to maintain the original risk-return features. The FBMEMAS Index was used as a benchmark. Razak *et al.* (2014) concluded that, for the entire period, the MVP outperformed the FBMEMAS. With the vastness of equities that are qualified to be Shariah-compliant nowadays, the MVP technique might be a valuable tool in portfolio selection and construction as investors will be able to assess risk-return characteristics of prospective equity investments thereby safeguarding their wealth (Razak, *et al.*, 2014).

Mbithi (2014) studied the Nairobi Securities Exchange (NSE) to determine the number of stock which could efficiently make up an optimum portfolio. By employing the MV framework, the author utilised data from the NSE from 2009 to 2013. Of the 60 NSE-listed companies, 43 were considered for the study (Mbithi, 2014) from which several portfolios of varied sizes were constructed by randomly selecting the stocks. The results from the study complemented the findings of Solnik (1974), Newbould and Poon (1993) and Tang (2004) who individually documented that portfolios with 8 to 20 constituents yield diversification benefits. As the number of securities increased, portfolio risk decreased, and the optimum portfolio was observed to have between 18 to 22 securities (Mbithi, 2014).

Baltes and Dragoe (2017) conducted a study on the Romanian Bucharest Stock Exchange (BVB). The period of the study stretched over six months, from May 2014 to October 2014. The authors took financial information from the three major companies of the Romanian Construction Sector listed on the BVB. The companies considered for the study were Impact Developer and Contractor (IMP), Transilvania Constructii (COTR) and Condmag (COMI). Average weekly returns based on the weekly closing stock prices were calculated and analysed. Baltes and Dragoe (2017) calculated the mean, covariance and correlation of the stocks following the original Markowitz (1952) formulation. Baltes and Dragoe (2017) found a positive correlation between IMP and COTR while a negative correlation was found between COTR and COMI. Moreover, the authors recommend a short selling of IMP stocks and conclude it profitable to invest in an efficient portfolio comprising of COTR and COMI only.

Portfolio optimisation using the Markowitz MV model was also investigated on the Nigerian Stock Exchange by Nnanwa, Urama and Ezepue (2016). The authors focused on the financial sector of the NSE for the period 2009 to 2014. Daily closing prices of 24 assets from the finance sector were utilised to calculate MVP variables. Four portfolios were then constructed using different asset weightings. Firstly, an equally weighted portfolio was constructed, and its performance compared to the market (Nnanwa, *et al*., 2016). The standard deviation of the equally weighted portfolio was observed to be better than the market; however, the return was very poor (0.00162%). Furthermore, another portfolio was then created, prioritising assets with better returns while other assets (such as Diamond Bank, Access, Fidelity Bank, etc) were even allocated 0%. The second portfolio had a higher standard deviation than the equally weighted portfolio (but lower than the market); however, the return was almost 52 times better (Nnanwa, *et al*., 2016). The third portfolio created offered a high return with a very high standard deviation, which was not recommended for risk-averse investors. In conclusion, the authors observed that an MVP constructed with efficient capital allocation guided by the MV framework could outperform the market. Correlation of the assets was also noted to be important for the reduction of portfolio risk.

Hamid (2016) studied the applicability of MPT using different market trends on the Indonesian market from 2005 to 2011. Within the study period, there was a bullish market (2006-2007), as well as a bearish period (2008-2009) as confirmed by Hamid (2016). Portfolio variables were calculated using data obtained from the LQ-45 index.

12 stocks were found to have positive returns during a bearish market trend while 16 stocks had positive returns in the bullish trend (Hamid, 2016). Two portfolios with different asset weightings were created for the two periods (bearish and bullish). In conclusion, the author confirmed that the MVP framework could be applied efficiently to construct an optimum portfolio on the Indonesian Stock Exchange. Hamid (2016) observed that the constructed portfolio in both periods outperformed that of the market benchmark (LQ-45 index) in terms of both risk and return. In a bullish market, the portfolio constituents were dominantly commodity stocks, while in the bearish market the constituent assets were the banking and manufacturing sector. Generally, the government will decrease interest rates in turbulent times (bearish). This will have an effect on the banking sector hence the good performance on such portfolio noted during the bearish market trend period.

A recent study to determine the efficiency of the MPT was done on the Karachi Stock Exchange (KSE) (Zaidi, 2017). The study stretched from 2002 to 2011. Stocks were picked from the KSE100, which was also used as the benchmark index. Several portfolios were created starting with a 32-stock portfolio based on the underpinnings of the MPT with a constraint of not short selling. The number of assets in the portfolio was further reduced and finally the tangency portfolio had nine stocks (Zaidi, 2017). Moreover, the risk-return characteristics of the constructed MVP outperformed the benchmark with the MVP return being 29.04% while that of the KSE100 index was 18% for the period considered. As noted in this study, making use of the MVP technique will create efficient portfolios and can even reduce the number of assets in a portfolio while maximising the overall returns.

Roopanand (2001) investigated the effectiveness of the MV model on the JSE using the ALSI based on component indices like the industrial and gold index locally as well as using the Dow Jones internationally. Using the period February 1983 to March 1999, historical annual returns for each index were calculated and the betas and covariances computed for a period of 182 months. The results of the study signified that a market portfolio emulating the ALSI only was mean variance inefficient (Roopanand, 2001). Furthermore, it was concluded that domestic returns by SA investors would be maximised by holding the Gold index, which was mean-variance efficient.

Garaba (2005) investigated the power of MPT as a security evaluation portfolio management technique compared to other traditional tools such as fundamental analysis (FA, see Abarbanell & Bushee, 1997), technical analysis (TA, see Lo, Mamaysky & Wang, 2000) and behavioural finance (BF, see Subrahmanyam, 2008) theory on the JSE. A survey was done using a sample of 110 out of the 322 asset management companies listed by the Financial Markets Directory (FMD) as at September 2003. From the questionnaire responses by asset managers, it was concluded that the MPT is not being employed mainly by asset managers for portfolio management and security evaluation. The reason for the low usage of MPT might be its heavy reliance on complex mathematics, which might be a challenge to asset managers. Moreover, MPT assumes no transaction costs; hence, it is difficult for asset managers to adjust the model to factor in transaction costs, taxation and other economic fundamentals (Omisore, Yusuf & Christopher, 2011). On the other hand, FA was regarded as the most significant portfolio management tool. However, Garaba (2005) recommended an integrated portfolio management strategy that incorporates both the traditional portfolio theory (FA and TA) as well as the MPT to enhance investor value and protection.

Another study to investigate the applicability of the MPT was carried out on the JSE by Du Plessis and Ward (2009). For the period January 1997 to December 2007, the authors analysed stocks and constructed four MVPs under different conditions. The authors then constructed a portfolio based on ex-ante returns for prediction of returns as well as to predict the covariance matrix. Periodic rebalancing was done on the optimal portfolios constructed and the results obtained compared against the benchmark used, which was the market (ALSI 40 index). For the period studied, the MVPs constructed outperformed the ALSI 40 even under the constrained conditions of no short-selling and/ or no more than 10% in any single security (Du Plessis & Ward, 2009).

Rodrigues (2010) studied the application of MPT focusing on the property market of South Africa. The study stretched from 1995 to 2006, utilising data from the Investment Property Databank (IPD) of South Africa. The research followed on Garaba (2005) in investigating the use of MPT by asset managers as a portfolio management technique, then proceeded to investigate how the model can be used as a strategic diversification tool using the SA property sector (Rodrigues, 2010). The results of the study showed that out of the 18 interviewees, only two were employing the MPT as a decision-making instrument for asset management. The results complemented the findings of Garaba (2005) who focused on the application of MPT on general asset management. Rodrigues (2010) recommended that investors can use the MPT for capital allocation and selection.

Mwamba and Suteni (2010) studied portfolio optimisation on the JSE using an alternative investment strategy with the MVP framework. The authors differentiated their model with the MVP based on the asset price movement and the return distribution. The authors used the log optimal growth, which maximises the long-term growth rate of the portfolio over a specified period. From November 1999 to October 2009, financial data of five randomly selected JSE assets from I-net Bridge were downloaded, processed and then utilised for portfolio construction. The results of the study emphasised that, the Markowitz mean variance model is a very important model in portfolio optimisation. While the MVA model is a buy and hold or one period strategy, the log-optimal strategy can be used when considering any short-run periods such as a year, a month or a week.

The MPT was also applied in South Africa by Du Plessis (2014) on the Industrial Development Corporation (IDC), the biggest SA financial institution which conceptualises generating Africa's sustainable economic growth. The main objective was to determine if the IDC is optimizing its capital allocation and to further determine which sectors should be invested more for the IDC to realize maximum returns. The results of the study showed that the current capital allocation strategy of the IDC was not optimised. The sectors being prioritised were electricity, gas, steam and water supply. From 2010 - 2014, 47.2% of capital was allocated to that sector which was only generating about 0.6% of the economic formal jobs in South Africa at the time.

By applying the MV theory, Du Plessis (2014) allocated a limit of 20% capital to each prioritised sector. A portfolio was optimised in such a way that maximised the IDC strategic objectives, which included creation of employment and increasing real growth output (Du Plessis, 2014). The portfolio created was designed to cater for other IDC objectives, which included support for entrepreneurs, support for B-BBEE, support for regional development, as well as promoting environmental sustainability (Du Plessis, 2014). Generally, Du Plessis (2014) concluded that the IDC could employ the MV framework efficiently as an allocation, decision making and optimising tool to attain favourable returns as well as to achieve both its long and short-term objectives.

Brouwer (2015), conducted a study using the MVO model to find the optimum portfolio of exchange traded funds (ETFs) on the JSE. For the period 2009 to 2013, JSE stock performance data were downloaded from the ETFSA and INET BFA websites then utilised. An analysis was done in a bid to determine the interrelatedness between the ETFs as a measure of diversification (Brouwer, 2015). Multiple optimisation runs were done with different risk - return combinations to draw up an efficient frontier of portfolio allocations. The performance of the optimised portfolio was evaluated for the year thereafter. The results of the study confirmed a positive performance of the optimised portfolio. However, the study period was limited to four years only, a period which might be too short to determine a long term performance of the constructed portfolio.

Oladele and Bradfield (2016) conducted a study on the JSE using seven different techniques to construct low-risk portfolios, including the MVP based on the nine FTSE/JSE sectors. From January 2003 to December 2013, portfolios were created using methodologies targeting low volatility, low beta, maximum diversification and low correlation among others. The techniques used by the authors included the equally-weighted, the equal-weighted low beta, the low volatility single index model, the equal risk contribution, the naïve risk parity (see Grundy & Malkiel, 1996), the maximum diversification portfolio and the MVP. Portfolios were then rebalanced annually and compared against the ALSI. As observed by Oladele and Bradfiel (2016), the low volatility portfolios created outperformed the JSE ALSI in terms of risk and returns.

The study by Contreras *et al.,* (2016) also included the JSE market. The authors studied the performance of mean-variance optimised (MVO) equity portfolios for retail investors, in 22 countries. For a period of 10 years, from 2005 to 2014, stock price data from the 22 markets were utilised and a back-test of MVO portfolio optimisation was conducted (Contreras *et al.,* 2016). The findings of the study confirmed that most MVO portfolios obtain a higher level of return than their respective benchmark indexes, and in many cases, do so without noticeable increases in their volatility. Furthermore, stability in the outperformance of the MVO portfolios was also noted on the JSE except for the year 2008. Limiting the study to equity portfolios only might have caused the underperformance of the constructed MVOs during 2008. As effective diversification involves different asset class investments, including more asset classes such as bonds and commodities, could have resulted in a different outcome in a period of a market downswing (Markowitz, 1952:89).

In both developed and developing markets, MVPs prove to perform better than their benchmarks. In terms of both return and volatility, the studies by Haugen and Baker (1991), Clarke *et al.* (2006), Bower and Wentz (2005), Gupta and Basu (2009), Paudel and Koirala (2006), Bausys (2009), Ahuja (2011), Darko (2012) as well as, Baltes and Dragoe (2015), among others, revealed a common result, namely that the MVPs are better performers than their benchmarks. Having compared the use of the MV model against MAD, Bower and Wentz (2005) also concluded that the MV method is effective for portfolio optimising. The efficiency of the MAD model was almost equal with the MV method, yielding an average return which was 0.0013% higher than that of the MV model. In the context of South Africa, Du Plessis (2014) found that an MVP framework can also be applied successfully outside financial markets.

With the evolution of financial markets driven by technology and evolving investor sentiments, the MVP framework have shown to be a strong and effective portfolio optimising tool in both developed and developing markets(Baker & Haugen 2012; Blitz & Van Vliet 2007; Haugen & Baker, 1991; Jagannathan & Ma, 2003, among others) . As developing economies are more volatile than developed markets due to economic and political distress, MVPs could benefit developing market investors as they proved to perform better than their benchmarks despite the market trend (Bausys, 2009; Darko, 2012; Gupta and Basu 2008; Mbithi, 2014; Razak, *et al.*, 2014; Roopanand, 2001; Thirimanna, *et al.*, 2013, among others). In an efficient market, where all the security information is publicly available, the diversification power of MVPs becomes key to winning, especially when the diversification is done effectively (Markowitz, 1952:88-89).

In South Africa, limited attention has been paid to constructing portfolios based on sector indices. Sector indices allow investors to hold a basket of different securities from different sectors of the economy. The three sectors employed in the present study were diverse enough; however, their correlation will be determined in Section 4.4 to assess whether there will be significant diversification benefits associated with a tradable sector index portfolio.

2.4.3 Consistency in performance of sector-based portfolios on the JSE

Limited studies have been done on the consistency in performance of sector-based or index MV portfolios on the JSE. Extensive studies have focused on investigating the performance of style-based portfolios, exchange-traded funds (ETFs), mutual funds and unit trusts (Devonport, 2014; Gilbertson & Vermaak, 1982; Mibiola, 2013; Oldham & Kroeger, 2005; Scher & Muller, 2005; Pillay, Muller & Ward, 2010; Runhaar, 2017; Wright, 2016). The JSE has more than 100 indices, the top being the ALSI, Top 40, INDI, RESI and FINI. Limited focus has been put on investigating the persistence in performance of such index portfolios against a benchmark on the JSE. Considering the evolving of financial markets, investors might be interested in knowing whether picking a tradable sector index portfolio could yield consistent performance results in both bullish and bearish markets.

As much as some sectors on the JSE are defensive during market downswings (for example pharmaceuticals, food industries and insurance), in times of financial crisis they might yield negative returns as observed during the 2008-2009 global financial crises (GFC) (Arguile, 2012: 2). Mostly defensive shares or portfolios yield a lower standard deviation than their benchmarks. One of the earlier studies on testing the consistency of sector-based low-volatility portfolios was carried out by Neu-Ner and Firer (1997). The authors tested consistency in performance of the JSE's defensive sectors (with a beta value less than 0.5) for the period 199 –1996. While the higher beta stocks (with an average beta of 1.1) yielded very high returns in the first 18 months of the study, the minimum risk shares experienced an increase in returns accompanied by a fall in risk (Neu-Ner & Firer, 1997). This could have been attributed by the superior performance of the economy at that point in time. During the last period of the study, the minimum beta shares yielded greater returns that the higher beta share portfolio, which confirmed a consistency in the performance of minimum risk shares as well as a superior overall return (Neu-Ner & Firer, 1997).

Arguile (2012) studied the performance of JSE 9 sectors (oil and gas, basic materials, industrials, consumer goods, health care, consumer services, telecommunications, financials, and technology) for a period from January 2000 to March 2009. The author employed the CAPM (representing the modern portfolio theories) and the GARCH models to determine the risk-return characteristics of the portfolios to pick the minimum risk portfolios. The sample period was sub-divided into two periods, which is the pre-global financial crisis (GFC) period and the GFC period. The JSE ALSI was used as the benchmark of the study. While sectors such as healthcare, consumer services and industrials maintained stability in both periods, financials and consumer goods sectors were observed to be less volatile than the market. The technology sector, telecommunications and oil and gas performed worse than the market (JSE ALSI), which might have been caused by the 2000 technology bubble. In conclusion, the healthcare sector (pharmaceuticals) proved to be consistent in both periods. Including such a stable security in a portfolio might be a necessity since in investment world, the future is always uncertain (Markowitz, Hebner & Brunson, 2009:11).

Marozva (2014) examined the performance of the JSE Socially Responsible Investments (SRI) index in relation to JSE ETFs. The period of the study spanned from 2004 to 2014. The CAPM (representing the modern portfolio theories) based on a single model was also used to estimate the performance of the SRI index. The performance of the SRI index and the ETFs was benchmarked against the JSE ALSI. The Sharpe ratio, Treynor ratio as well as the M-squared ratio (see Goetzmann, Ingersoll, Spiegel & Welch, 2002) were used to measure the performance of the SRI index against the ETFs. The SRI index was observed to underperform the ETFs as well as the benchmark during the bear market (Marozva, 2014). During a bullish market, the SRI index was observed neither to underperform nor to outperform the ETFs significantly. However, both the SRI index and the ETFs were observed to underperform the market as represented by the JSE ALSI.

2.4.4 Studies on diversification benefits using MPT

Portfolio diversification in both international and domestic context is one of the intensively studied subjects in the finance world (Gehringer, 2014:1-2). Fundamentally, diversifying a portfolio is critical as it adheres to the old-age saying, "not putting one's eggs in one basket" (Lian & Toften, 2015:19). Domestic portfolio diversification advantages have been documented by Markowitz (1952). These are more visible where asset correlations are negative. Both in bearish and bullish markets, diversification is necessary for investors to offset unfavourable security performances and avoid losing investment values (Markowitz, 1952:89).

The financial crises, economic imbalances and high volatility movements of asset prices have alarmed individual investors, advisors and investment professionals on how critical portfolio diversification can be. The future is always uncertain; hence, any investment planning should incorporate diversification which can be sectoral, across asset classes, or geographically across markets (Godi & Sibindi, 2014:490). Several studies have been done on diversification benefits using the MV framework in both developing and developed markets. A summary of some of the empirical studies on diversification benefits are shown in Table 2.2:

Considering the empirical evidence in the markets studied, portfolio optimisation using sector indices yielded diversification benefits by generating excess returns compared to their benchmarks. As noted in studies enumerated on developed as well as developing markets, the outperformance was due to the less than perfect correlation among the indices and securities used (Ahuja, 2015; Arouri, Nguyen & Pukthuanthong, 2014; Bang-Ariffin, Matemilola, Wahid & Abullah, 2017; Bhuyan, Kuhle, Ikromov & Chiemeke, 2014; Hopwood, 2015; Lee, Cheng & Chong, 2016; Matar, 2016;). Diversification benefits have also been observed to be present across sectors and in different asset classes from several different markets. The empirical evidence studied (Section 2.4.2 and 2.4.3) also showed the feasibility of Markowitz's MV model as a portfolio optimising tool. In addition, the study by Hopwood (2015) recommends investigation of sector-level portfolios to determine their reaction to certain market conditions in relation to the market.

Table 2:2 Empirical studies on diversification benefits

Author/ Authors	Market studied	Period studied	Results
Arouri, *et al.*, (2014)	US	1989-2010	-Mixed asset class diversification conducted -31 portfolios were constructed adding more asset classes such as oil, precious metals, currencies and real estate -The authors concluded that adding more asset classes to a portfolio is beneficial especially during market downswings
Bhuyan, *et al.*, (2014)	US	1997-2007	-A mixed asset class portfolio of real estate investment trusts (REITs), S&P500 and a 10-year treasury note was constructed -A positive correlation between REITS and stocks was observed -The authors concluded that the rate of return yielded by REITS to stocks was more important than their correlation. -Diversification was observed to be beneficial even with a positive correlation between assets
Ahuja (2015)	Karachi Stock Exchange	2007-2009	-An adverse relationship between portfolio size and risk was noted -Portfolio diversification was concluded to be beneficial on the Karachi Stock Exchange

Matar (2016)	Amman Stock Exchange	2005-2014	-A decrease in portfolio beta was noted as another security with a beta less than that of the existing portfolio was added -No effect of diversification was noted on systematic risk, confirming Markowitz's (1952) findings
Lee, *et al.*, (2016)	Malaysia Stock Exchange	2010-2014	-A negative relationship between portfolio size and risk was noted as unsystematic risk kept on decreasing with an increase in the number of stock in the portfolio -The authors concluded that diversification benefits could not be realised in a short period
Bang-Ariffin, *et al.,* (2017)	Malaysian Stock Exchange	1995-2013	-Diversification benefits were noted between the construction and property sector -A low positive correlation of 0.28 observed was concluded to bring diversification benefits
Hopwood (2015)	JSE	2002-2012	-Four style-based portfolios were constructed to investigate their conditional correlation with the market; hence, determine if they can yield diversification benefits in all market conditions (bearish and bullish) - All the portfolios constructed based on, small versus large, growth versus value, high versus low dividend yield and liquid versus illiquid styles were concluded to be highly correlated with the market during downswings. -In conclusion, diversifying using large firms, high-PE, high-DY and high-volume portfolios was observed to be beneficial in upswings while diversifying in small-firm low-PE, high-DY and low-volume portfolios was beneficial in downswings

2.5 Conclusion

This chapter discussed the theoretical underpinnings of the MPT, starting from the mean variance analysis by Markowitz (1952), to the separation theorem by Tobin (1958), CAPM by Treynor (1961), Sharpe (1964), Linter (1965) and Mossin (1966) independently as well as the APT by Ross (1976). The three main branches of the MPT have been compared and a discussion on how they complement each other followed. The effectiveness of the MPT during market downswings, such as market crush periods, was also discussed. The EMH as a supporting theory of MPT was also briefly discussed in the context of South Africa. In addition, empirical studies on the application of the MPT based on the mean-variance analysis in developed and developing markets have been considered leading to studies on consistency in performance of MVPs on the JSE being discussed. Since the MPT is based mainly on diversification, studies on diversification benefits due to application of the MPT have been discussed as well.

Chapter 3 considers the research methodology employed in this study. It will consider the construction of MVPs as well as the assessment of diversification benefits using the Markowitz MPT framework on the JSE.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

The purpose of this study was to apply the Markowitz MVP framework (mean-variance model) to construct an optimum portfolio using the tradable indices on the JSE. This chapter discusses the research design that was utilised, the methodological issues involved as well as the data employed. The mean-variance model being employed is also discussed. Finally, the possible biases and their remedies are explained.

3.2 Research design

There are two research design methods that can be used for research, namely qualitative and quantitative methods. A qualitative research method is utilised to obtain an in-depth understanding of the subject, while a quantitative design tests relationship and examines cause and effect relations on a subject (Merriam & Tisdell, 2015:15). An effective and appropriate research design and methods must be used in order to obtain a quality and reliable study result.

The Markowitz MVP model requires a calculation of variables and construction of asset portfolios for different risk-return combinations. Markowitz uses the arithmetic mean, the variance and the covariance as parameters for estimating security return and risk (Darko, 2012:23). The model is a form of quadratic problem, which made it ideal for this research to make use of quantitative research methods for an investigation of the applicability of the MVP model on the JSE.

A quantitative research design allows a systematic investigation, measuring, quantifying, testing of the hypotheses as well as establishment of the relationship between financial variables (Walliman, 2017:113). This assisted the researcher to find evidence in support of or to contradict whether the MVP model can be used to construct an efficient portfolio using the JSE tradable sector index. By making use of the quantitative approach, the researcher was also able to analyse a relationship between variables (Walliman, 2017:113-114), that is the correlation of securities and returns. The financial data were analysed and interpreted using Stata version 14 and the Microsoft Excel software package. A comparison was made with a chosen benchmark, namely the JSE ALSI.

3.2.1 Advantages of a quantitative research design

The quantitative approach permits for a comprehensive study of the subject and is ideal for studying subjects when huge quantities of data are involved at the same time, permitting generalisation of results (Creswell, 2002:153). Using quantitative research methods gives room for the research to be replicated, analysed and compared with similar studies. Information from various sources can be analysed through the use of mathematical models, summarised and compared against one another. More so, quantitative research is regarded as a way of obtaining the true facts about a subject. Because it uses standard means, predictions and controls are made possible and cause and effect relationships can be identified (Williams, 2007:66).

3.2.2 Disadvantages of a quantitative research design

The results obtained using a quantitative design are limited, as they provide numerical descriptions rather than detailed narrations with elaborations on how the result came into being (Williams, 2007:70). The quantitative research design does not relate to the views of each individual investor, as a qualitative approach might do. On its own, a quantitative research strategy does not address the complexity of a phenomenon and quite a large sample of the population must be studied for the results to be more accurate (Williams, 2007:70). Having weighed the advantages and disadvantages of the quantitative research design method, the researcher found the quantitative method to be the most ideal. Statistical inference was used to ensure that the data were a true representative of the whole population and that the results were statistically significant.

3.2.3 Inductive versus deductive approach

An inductive research approach is mainly associated with a qualitative method and involves, first, collecting data relevant to the subject of study, and then generating a new theory out of the study (Jebreen, 2012:162). On the other hand, in a deductive approach, a researcher employs, tests or investigates an existing theory or hypothesis (Gray, 2013:17). For this study, a theory by Markowitz (1952) was being employed and its application on the JSE investigated. A deductive approach was followed so as to investigate and draw conclusions in this study. Tests that were done as discussed in the literature review section showed that a deductive approach was employed, hence this study employed the same strategy.

3.3 Population / sample description and data sources

A population is an entire group of the subjects being studied. The target population of relevance to this study was all the tradable indices on the JSE. According to the JSE equity market statistics (JSE, 2018), there are more than 800 potential investments, of which approximately 300 are tradable indices and sub-indices on the JSE. However, not all the tradable indices are active on the JSE. The top JSE indices comprise of the Top 40, JSE ALSI, $INDI_{25}$, $RESI_{10}$ and $FINI_{15}$ indices.

The tradable sector indices form an integral part of the financial world on the JSE, as it constitutes assets from all the South African sectors. According to Yu (2008:22), those investment vehicles or assets derive their prices from other instruments and they trade intra-day on the JSE. The tradable sector indices give investors access to hold value of a number of companies from the same South African sector pooled together in one big basket. Investors can monitor these indices for decision making with respect to their portfolios.

A sample is a subset or a representation of the entire population with the same characteristics as the population (Walliman, 2017:94). Practically, it would not have been feasible to focus on the entire JSE with the vastness of securities trading daily, hence this study made use of the three JSE tradable sector indices, which comprise of 50 companies.

There are two types of sampling methods, namely probability and non-probability sampling. With probability sampling, each unit will stand an equal chance of being selected, while with non-probability sampling, selective units are chosen based on methods that do not give each unit an equal chance of being selected (Walliman, 2017:95). The sampling frame employed for this study was non-probability sampling. This was premised on a purposive sampling technique. With purposive sampling, a sample is picked based on a certain criterion depending on the qualities that the units to be studied possess (Tongco, 2007:152). For the purpose of this study, the sample was the three indices that constitute the JSE tradable sector, namely the $INDI_{25}$, $RESI_{10}$ and $FINI_{15}$ indices. The index codes for the 3 indices are J211, J210 and J212 respectively. All three indices had a base date of 1 February 2002.

The $INDI_{25}$, $RESI_{10}$ and $FINI_{15}$ indices were purposively selected as the sample of this study. The three indices represent different South African market sectors, which facilitates sector diversification onto the constructed portfolio. According to the Industry Classification Benchmark (ICB) of South Africa, the JSE ALSI was sub divided into three South African sectors, which are SA Resources, SA Financials and SA Industrials. SA Resources constitutes 12%, SA Financials 24% while SA Industrials constitutes 64% of the JSE ALSI (JS Exchange Regulatory Report, 2013). The three indices capture the most liquid, tradable instruments in their respective sectors.

Each sector index comprises of a number of different companies that are pooled together to maintain their values as one. The SA Resources sector constitutes the JSE listed companies that belong to the ICB Sectors of Oil & Gas Producers (0530) and Mining (1770). The second sector, which is SA Financials comprises of JSE-listed companies that belong to ICB Financials (8000). Finally, the SA Industrials sector comprises of all remaining companies, that is the JSE listed companies that do not belong to ICB Industry Financials (8000) or ICB Oil & Gas Producers (0530) and Mining (1770). The companies in each index are not static; they change daily to accommodate the top companies into their respective indices. The INDI index holds the daily top 25 companies from the industrial sector, the FINI index the daily top 15 from the financial sector while the RESI index holds the value of the daily top 10 companies from the resources sector. Table 3.2 below shows a list of some of the companies (and the company codes) that existed on the first day of this study (1 January 2007) and were also present on the last day of the study (31 December 2017).

Table 3.1: Names and codes of companies in the tradable sector indices from 2007 to 2017

INDI$_{25}$	**FINI$_{15}$**	**RESI$_{10}$**
Afrocentric Investment Corporate Ltd (ACT)	Capitec Bank Holding Ltd (CPI)	AECI Ltd (AFE)
Astral Foods Ltd (ARL)	Coronation Fund Managers Ltd (CML)	Anglo American Platinum Ltd (AMS)
Barloworld Ltd (BAW)	Hammerson PLC (HMN)	Anglo American PLC (AGL)
Cashton CTP Publishers and Printers (CAT)	Hosken Consolidated Investments Ltd (HCI)	Anglogold Ashanti Ltd (ANG)
Clicks Group Ltd (CLS)	Intu Properties (ITU)	Assore Ltd (ASR)
EOH Holdings Ltd (EOH)	Investec Plc (INP)	Harmony Gold Mining Company Ltd (HAR)
Grindrod Ltd (GND)	Liberty Holdings Ltd (LBH)	Northam Platinum Ltd (NHM)
Hudaco Industries Ltd (HDC)	MMI Holdings Ltd (MMI)	Omnia Holdings Ltd (OMN)
Lewis Group Ltd (LEW)	Nepi Rockcastle PLC (NRP)	Sappi Ltd (SAP)
MTN Group Ltd (MTN)	Peregrine Holdings Ltd (PGR)	
Murray and Roberts Holdings Ltd (MUR)	PSG Konsult Ltd (KST)	
Nampak Ltd (NPK)	Redefine Properties Ltd (RDF)	
Naspers Ltd (NPN)	Sanlam Ltd (SLM)	
Netcare Ltd (NTC)	Standard Bank Group Ltd (SBK)	
Oceana Group Ltd (OCE)	Texton Property Fund Ltd (TEX)	
RCL Foods Ltd (RCL)		
Reunert Ltd (RLO)		

Shoprite Holdings Ltd (SHP)		
Spur Corporation Ltd (SUR)		
Super Group Ltd (SPG)		
The Foschini Group Ltd (TFG)		
Tiger Brands Ltd (TBS)		
Tongaat Hulet Ltd (TON)		
Truworths International Ltd (TRU)		
Wilson Bayly Homes Ovcon Ltd (WBO)		

However, the constituents of the indices shown in Table 3.1 are not fixed. They keep on changing daily and the index will automatically pick the top companies to make up its constituents.

The period under consideration for the study was from 1 January 2007 to 31 December 2017 (a period of 132 months). This period was preferred because to the best knowledge of the researcher, no similar study focusing on the JSE was done in that period. In addition, according to the quarterly bulletin of SARB (2010), on average South Africa had an increase in foreign direct investment (FDI) prior to the proposed period (2002 to mid-2007, although it was stalemate in 2008 and decreased drastically in 2009), which might have affected sector index investment as well as returns for the proposed period. The selected period covers the economic volatilities, considering that it covers the 2007-2008 global financial crisis (GFC).

3.4 Data description

This study utilised data obtained from an online database, namely Bloomberg. The present study focused on the tradable sector indices in South Africa by assessing the efficiency of the Markowitz's MVP in optimising a portfolio using tradable sector indices on the JSE.

Using the Bloomberg financial database, the daily closing prices for each of the three indices were downloaded. The main information of interest was the daily closing price for each index, which was used to compute daily returns and volatilities. Considering the three tradable sector indices to under study, a total of 50 shares was included in the indices. Each of the three tradable sector indices had a base date of 24 June 2002.

The data collected from Bloomberg included the historical price records for the three indices, namely $INDI_{25}$, $FINI_{15}$ and $RESI_{10}$. The data were first filtered and cleaned to get rid of any irregularities. The study period was divided into three periods, starting with the pre GFC period which spanned from 1 January 2007 to 30 June 2007, the GFC and the post GFC period. The GFC period, spanning from 1 July 2007 to 31 August 2009; and the post GFC period which spanned from 1 September 2009 to 31

December 2017. Statistical analysis was done using Stata version 14 and Microsoft Excel software applications. Specifically, the following data points were collected:

- Index name
- Date
- Daily closing prices.

3.5 Methodological issues

From the empirical literature reviewed, a number of models have been used for optimising portfolios. These include mean-variance analysis, the mean absolute deviation model, conditional value at risk, naïve risky parity, minimax and the DCC GARCH model. The models originated from the Markowitz MVO model and they utilise several different risk measures, thereby creating a family of mean-risk models for portfolio optimisation (Mansini, Ogryczak & Speranza, 2014:518). Value-at- risk utilises an asymmetric risk measures and is based on a non-linear function, which might be complex to optimise (Brouwer, 2015:24).

Among all the mean-risk models, the MVO is the root theory and is based on simple underlying assumptions. The investor preferences in terms of the expected return and risk are explained in simple terms. In addition, the model only requires the investor expectations and covariances, which makes it easy to compute and use, even for individual investors. This study employed the standard Markowitz MVO in its original form without any modifications. Hypothetical constraints were used to represent investor risk tolerance and preferences. Mean-variance analysis is mainly ideal for passively managed portfolios. In other words, individual investors can use the framework for portfolio selection and construction without the help of portfolio managers (Contreras *et al*. 2016:1).

Among others, the research by Brouwer (2015) utilised the Markowitz's portfolio theory to optimise an MVP based on the JSE ETFs. The study focused on applying the model in its purest form diversifying among seven ETFs for the 2009-2013 study period. Brouwer (2015) created a model based on the Markowitz mean variance framework to optimise the portfolio and used an out of sample year period to evaluate the performance of the constructed portfolio. The model was effectively applied on the JSE ETF market, although the study period was limited to four years. ETFs passively track JSE indices such as the Top 40 and the INDI$_{25}$, hence indirectly the study by Brouwer applied the MVO model to the JSE indices (Brouwer, 2015:13). Investing in ETFs, although less costly, attracts appreciable fees practically. ETFs aim to track the performance of indices and benchmarks, which is not ideal for an investor aiming to beat a benchmark.

Extant studies, including that of Bausys (2009), Baker and Haugen (2012), Mbithi (2014) and Contreras *et al*., (2016), utilised the original MVO model, resulting in consistent findings irrespective of differences in time. Baltes and Dragoe (2017) successfully determined the optimal structure of the MVP based on three assets from the Romanian construction sector. The authors employed the standard MVO models without any modifications. However, a great number of the studies consulted focused largely on companies, ETFs and mutual funds, with less focus on sector indices. Following that, the researcher found it ideal to use indices and not individual companies to construct the optimum portfolio. A portfolio constructed based on the three indices is less cumbersome and costly, as it will track the movement of a number of companies as one in a basket, which will be much less costly than investing in individual companies. As a base for the methodology, the research by Bausys (2009) employed the MVO model focusing on sector indices, hence was the most ideal study to replicate methodologically.

The research by Bausys (2009) is one of the most comprehensive studies of MVP in the context of different sector indices (Baltic sectors). The study focused on the three indices of the Baltic market, namely Estonian, Latvian, and Lithuanian. In addition, the study by Bausys relied on an acknowledged previous study by Clarke *et al.,* (2006), which covered improved parameter estimation methods, its contribution having been published in dependable academic journals (Bausys, 2009:16). However, the model employed by Bausys (2009) requires some adaptation to suit the context of the JSE. Bausys (2009) used the adjusted version of Markowitz's MPT as the basis for research. However, in order to account for the specifics of the JSE tradable sector indices, this study aimed to apply the original version of the MPT based on the MVP model.

3.5.1 Data analysis

This study employed Microsoft Excel and Stata version 14 to run the estimations and calculate the portfolio variables for the computation of the MVP. For a period of 132 months (11 years), the daily downloaded stock prices for each index were arranged in ascending order based on dates. The average rate of return and standard deviation for each index were then estimated.

The study period was divided into three parts; the pre-GFC period, the GFC period and the post-GFC period. The GFC in South Africa occurred from July 2007 to August 2009 when the business cycle trough began (Venter, 2011:66). The GFC period hence, will start from 1 July 2007 to 31 August 2009. On the other hand, the pre-GFC period will start from 1 January 2007 to 30 June 2007. The remaining period starting from 1 September 2009 onwards is the post GFC period. For the three periods, portfolios were constructed with different constraints so as to test the performance of the MVPs as aligned to the four objectives of this study. The following constraints were considered;

- No inclusion of transaction costs
- Portfolios will be constructed with short selling allowed, and others with no short selling

3.5.2 Construction of an MVP

The first step in the construction of the MVP is to calculate the portfolio return. The portfolio return was calculated using Microsoft Excel based on the following equation:

$E_r=(X_{Iresi} * R_{RESI}) + (X_{iINDI} * R_{INDI}) + (X_{iFINI} * R_{FINI})$……………………………………Equation 3.1

Where:

Er: = the expected portfolio return

X_i: = the weight allocated in each of the three sectors

R: = the expected return of each sector calculated based on historical data

Random weights were used initially and were later changed as Excel Solver determined the optimal weights for the portfolio. The next step was calculating the portfolio standard deviation and covariance. The required values for the standard deviation and the covariance were calculated by applying the Excel STDEV and AVERAGE functions to the historic daily percentage return data. The total portfolio risk was obtained by the standard deviation of its time series returns. In this case, the 132–month annualised standard deviation for the daily returns was computed. After inputting the standard deviation variables in Excel Solver, a covariance matrix was then created using the relationship:

$$\sigma_p^2 = w^2_{RESI}\sigma^2_{RESI} + w^2_{INDI}\sigma^2_{INDI} + w^2_{FINI}\sigma^2_{FINI} + 2W_{RESI}W_{INDI}\sigma_{RESI}\sigma_{INDI}P_{RESIINDI} + 2W_{RESI}W_{FINI}\sigma_{RESI}\sigma_{FINI}P_{RESIFINI} + 2W_{INDI}W_{FINI}\sigma_{INDI}\sigma_{FINI}P_{INDIFINI} \ldots\ldots\ldots\ldots\text{Equation 3.2}$$

Where:

P = the correlation coefficient between the returns on assets A, B and C (representing INDI, FINI and RESI indices).

W = the weight of the asset

σ = the asset standard deviation

When calculating covariance, the objective was to determine how assets within a portfolio move together. When exposed to market volatilities, assets from different asset classes normally react differently, hence assessing how they move jointly will determine the effectiveness of combining them in a portfolio. The sample covariance between assets can be calculated using the formula below:

$$sample\ Cov\ (x, y) = \sum \left(. \frac{(Xi - x)(Yi - y)}{n - 1} \right) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots Equation\ 3.3$$

Where:

X = the independent variable (representing INDI, FINI or RESI indices)

Y = the dependent variable (representing INDI, FINI or RESI indices)

n = number of data points in the sample

x = the mean of the independent variable X

y = the mean of the dependent variable Y

Obtaining a covariance only is not enough to determine the strength on which assets jointly relate within a portfolio. To determine the strength of the relationship, it is necessary to calculate the correlation coefficient which ranges from -1 to +1 depending on the extent to which the assets are positively or negatively co-related (Popina & Martyniuk, 2016:162). The correlation can be obtained by dividing the covariance of the assets by the product of the standard deviation of the assets involved. Using Microsoft Excel, the covariance and the correlation can be calculated using the COVARIANCE.S and CORREL functions.

The objective for constructing the MVP is to optimise a portfolio, in other words minimising the variance while at the same time maximising the returns. The values obtained from Excel Solver were used to trace the efficient frontier, which is a parabolic line from which efficient portfolios lie. An XY scatter chart was selected from the Chart Wizard. The X axis depicted risk signified by standard deviation and the Y axis portfolio return. Data points were then connected by a smoothed line with columns corresponding to the portfolio risk (standard deviation) and portfolio return, and this conformed to the Markowitz's efficient frontier from which all the efficient portfolios lie.

3.5.3 Uncapped minimum variance portfolio

The major focus of the study was to construct an efficient MVP, hence portfolio 1 was an uncapped weight portfolio from which the exact weights were determined by the excel solver. This portfolio is referred to as global MVP Uncapped (GMVPU). The GMVPU addresses objectives 1 and 2, which are restated below:

1. to establish whether the JSE tradable sector index (INDI, FINI, RESI) represents an optimal portfolio
2. To establish whether global mean variance portfolios can be constructed using the JSE tradable sector index (INDI, FINI, RESI).

First, some random weights to be invested in each of the three indices were assigned based on the budget constraint. The total weights added up to 1, or 100%. This constraint was then formulated as a quadratic program and imposed into Excel Solver as:

$$X_{port} = X_{INDI} + X_{FINI} + X_{RESI} = 1 \dots\dots\dots\dots\dots\dots\dots\dots\dots Equation\ 3.4$$

Where:
X_{port}: = the portfolio weight
X_{INDI}: = the weight of the industrials sector
X_{FINI}: = the weight of the financial sector
X_{RESI}: = the weight of the resources sector

The MVP construction steps explained in 3.5.2 were followed. Having obtained the portfolio variables required, that is the weights, returns and standard deviations, Excel Solver will adjusted the weights until an optimum MVP was obtained.

3.5.4 Proportionately weighted mean variance portfolio

As stated by Kruger and Van Rensburg (2008:6), the results of diversification depend on the individual security weights of the portfolio of which equally weighting securities within a portfolio will effectively maximise diversification benefits. Diversification benefits are mostly achievable if stocks in a portfolio are negatively correlated (Popina & Martyniuk, 2016:162). Based on what Kruger and Van Rensburg (2008:6) documented, the second portfolio was constructed on an equally weighting principal.

However, for the portfolio to be a true representation of the JSE, the makeup of each of the indices involved had to be considered. As stated in the JSE 2013 Regulatory Report, SA Resources constitutes 12%, SA Financials have 24% while the SA

Industrials constitute 64% of the JSE ALSI. Therefore, to determine whether there are diversification benefits associated with investing in the three main tradable indices (INDI, FINI and RESI) on the JSE, each security in the portfolio will carry a weight proportionate to its holding on the JSE. Portfolio 2 addresses the issue of diversification as indicated in the third objective of the study which sought to determine whether there are any diversification benefits associated with selecting the tradable sector index constructed portfolio. The third objective is restated below:

3. To determine whether there are any diversification benefits associated with selecting a portfolio constructed from the JSE tradable sector index (INDI, FINI, RESI).

Portfolio 2 is referred to as proportionately weighted MVP (PWMVP). The first step was to construct a correlation matrix to assess how the three assets were correlated, using the Microsoft excel CORREL function. The MVP construction steps described in section 3.5.2 were repeated for this portfolio and the portfolio return was be calculated based on the formula below:

The fourth objective of this research sought to determine whether there was consistency in the performance of the constructed portfolio. In other words, if the GMVP outperforms the JSE ALSI, it had to be tested to determine whether the outperformance was consistent and statistically significant. In order to determine the performance of the constructed portfolios against the JSE ALSI as well as the statistical significance of the GMVPs performance, a t-test had to be run. The t-test was based on the following formulated hypothesis to be tested:

Hypothesis 1

H_0: The optimally constructed MVP using the JSE tradable sector index can consistently outperform the JSE ALSI

H_1: The optimally constructed MVP using the JSE tradable sector index cannot consistently outperform the JSE ALSI

To address Hypothesis 1, the performance of the constructed portfolios was compared against the JSE ALSI. An in-sample composite return graph for the three indices was plotted against the benchmark. Statistical inference was run in the form of a t-test to assess whether the performance of the constructed portfolios is statistically significant against the JSE ALSI.

To test for consistency in performance of the constructed MVPs, the in-sample risk-return characteristics of the constructed portfolios were analysed against the JSE ALSI. The 11-year study period was divided into three sub-periods as explained in Section 3.4, from which the performance of the constructed MVPs was assessed to determine whether they maintained the same trend.

3.6 Possible biases and their remedies

The inputs used in the MVO model were statistical estimates (typically created by analysing historical data), hence they could not be devoid of error. According to Chopra and Ziemba (2011:6), the sensitivity to errors in the estimates of a model can affect the input parameters even with a small margin which may result in large changes in the structure of the optimal portfolio. This inaccuracy might lead to over-investment in some asset classes and under-investment in others, hence distorts the results.

3.6.1 Survivorship bias

Survivorship bias is when a research study neglects to include some data aspects such as failing entities or non-existing companies that form part of the database, thereby influencing the outcome of the study conducted (Filip, 2014:47). In this sense, investors might end up making misguided decisions having been misled by the published financial data that might be biased. Companies might neglect to include poor-performing funds, hence biasing the statistical returns of such securities. As a result, investors will not achieve their anticipated returns having based their decisions on incomplete and misleading information. Survivorship bias is the reason why past returns should not be relied on totally when making investment decisions.

3.6.2 Time period bias

The research was based on an 11 - year period, which might make the results time period-specific. As a result, the study might be subject to a time period bias (DeFusco, McLeavey, Pinto, Anson & Runkle, 2015:162). In other words, the short period might not sufficiently test the efficiency of the indices in optimising portfolios. To remedy this bias, more time might be needed to ensure that the results are not period specific. On the other hand, a study that is conducted over a long period can also be subject to a time period bias due to structural and economic changes. These were taken into account in this study.

3.7 Conclusion

This chapter explained the methodological designs and methods employed for constructing efficient MVPs using the JSE tradable sector indices. The different constraints and limitations for the construction of the two portfolios were also described. Chapter 4 presents and discusses the findings of the study to determine whether the JSE tradable sector index can represent an optimal portfolio. In addition, Chapter 4 also discusses whether there are any diversification benefits associated with investing in the JSE tradable sector portfolio.

CHAPTER 4

RESEARCH FINDINGS AND DISCUSSION

4.1 Introduction

In this chapter, the author presents the data analysis and empirical findings on the construction of Markowitz's mean-variance portfolios, using the JSE tradable indices. Principally there were four questions being resolved. The first question was to determine whether the JSE tradable sector indices represent an optimal portfolio. The second question was to establish whether the MPT model could transform the tradable sector indices into a mean-variance optimum portfolio. The third question focused on assessing whether the MPT model could capture diversification benefits based on the JSE tradable sector indices. Finally, the last question was to determine whether the MVPs constructed could consistently out-perform the JSE ALSI. The author first restates the objectives of the study. Then the author presents descriptive statistics on the returns of the JSE tradable indices for the periods; before, during and after the 2007-2009 GFC. The author then develops the concept to present the empirical findings on the feasibility of construction of global minimum variance portfolios, using the JSE tradable sector indices for the periods before, during and after the GFC.

The rest of this chapter is organised as follows: in section 4.1 the author restates the research objectives. In section 4.2, the author explains the descriptive statistics employed. In section 4.3, the author describes the empirical findings on the performance of the Global MVP versus the JSE ALSI index, while in section 4.4, the author outlines the empirical findings on diversification. In section 4.5, the author discussed the empirical findings on the different constraints portfolios. Finally, in section 4.6, the author discusses the efficient frontier for the GMVPs constructed.

4.2 Restatement of research objectives

The primary aim of this study was to determine if an investor could apply the traditional MPT theory in modern times efficiently to construct an optimum portfolio, based on the SA tradable sector indices to achieve maximum returns.

The research aim was supported by four research objectives (see subsection 1.6) restated below:

- to establish whether the JSE tradable sector index (INDI, FINI, RESI) represents an optimal portfolio;
- to establish whether global mean-variance portfolios could be constructed using the JSE tradable sector index (INDI, FINI, RESI);
- to determine whether there are any diversification benefits associated with selecting a portfolio constructed from the JSE tradable sector index (INDI, FINI, RESI); and
- to determine whether the global mean-variance portfolios constructed using the JSE tradable sector index (INDI, FINI, RESI) consistently outperform the JSE ALSI.

4.3 Descriptive statistics

In this section, the author presents the descriptive statistics of the variables employed in this study. Five portfolios were constructed using different constraints. Two of the portfolios were constructed using Microsoft Excel, while three were constructed using Stata. The first step was to calculate the average annual returns and the standard deviation for the indices using Microsoft Excel. The average returns for the pre-GFC, the GFC, the post-GFC and the whole 11-year study period are illustrated in Tables 4.1 to 4.4.

Table 4.1: Average daily returns and standard deviations before the GFC period

	Return	Standard deviation
FINI	0.0167%	0.0117
RESI	0.1572%	0.0143
INDI	0.0698%	0.0087
JSE ALSI	0.1017%	0.0101

From the beginning of January 2007 up to end June 2007, the three indices performed as depicted in Table 4.1. The RESI index yielded super natural daily returns of 0.1572% with a standard deviation of 0.0143. The INDI index had an average daily return of 0.0698% while its standard deviation was 0.0087. Finally, the FINI index performed poorly, with a daily return of 0.0167% and a standard deviation of 0.0117. The market as represented by the JSE ALSI had an average return of 0.1017% with a standard deviation of 0.0101. It can therefore be concluded that during the pre-GFC period, the JSE tradable sector main indices performed better than the market in terms of risk as individual assets except for the FINI index which had the highest standard deviation. Only the RESI index outperformed the market in terms of the risk-return combination during the pre-GFC period.

Table 4.2: Average daily returns and standard deviations during the GFC period

	Return	Standard deviation
FINI	-0.0236%	0.0207
RESI	0.0087%	0.0274
INDI	0.0159%	0.0169
JSE ALSI	0.0152%	0.0183

The financial sector was affected the most during the 2008-2009 GFC, resulting in the FINI index experiencing a negative return of -0.0236% and a standard deviation of 0.0207 in the GFC period. The performance of the FINI index was below the JSE ALSI which had an average daily return of 0.0152% with a standard deviation of 0.0183 during that period. The INDI index had the highest rate of return (0.0159%) complemented with the lowest standard deviation (0.0169). Finally, the RESI index performed moderately in terms of its return. However, the RESI index had the highest standard deviation of 0.0274. If an investor had invested in the FINI index as an individual security, a great loss could have been incurred. Holding the three indices in a portfolio could make a difference.

Table 4.3: Average daily returns and standard deviations after the GFC period

	Return	**Standard deviation**
FINI	0.0079%	0.0116
RESI	0.0033%	0.0157
INDI	0.0717%	0.0099
JSE ALSI	0.0462%	0.0094

The performance of the FINI index improved during the post-GFC period. On the contrary, the RESI index performance in terms of return declined to 0.0033%, while it still maintained the highest standard deviation of 0.0157. The INDI index still maintained the lowest level of risk with a standard deviation of 0.0099 while it yielded the highest average rate of return of 0.0717%. Both the FINI index and the INDI index performed above the JSE ALSI which had a post-GFC risk-return combinations of 0.0462% and 0.0094.

Table 4.4: Average daily returns and standard deviations for the 11-year Period

	Return	Standard Deviation
FINI	0.0345%	0.0145
RESI	0.011%	0.0195
INDI	0.060%	0.0113
JSE ALSI	0.04	0.0122

For the whole period, the INDI index had the highest level of return of 0.060% while maintaining the lowest level of risk (0.0113). The FINI index performed moderately with a return of 0.0345% and a standard deviation of 0.0145. The RESI index tend to be the riskiest asset with a standard deviation of 0.0195 while it yielded a lowest return of 0.011% for the whole period. The trends of the three indices against the JSE ALSI index as a benchmark for the four periods can be shown on Figure 4.1 to 4.4 below.

From Figure 4.1 it can be seen that the RESI index initially performed below all the indices from the initial period until around mid-February 2007. The RESI index then outperformed the JSE ALSI from mid-February 2007 up to end of June 2007. The FINI index performed above all the indices, including the JSE ALSI from the 1st of January until 7 February 2007 when its performance started dropping. From then, both the FINI and INDI indices trailed the benchmark throughout the pre-GFC period.

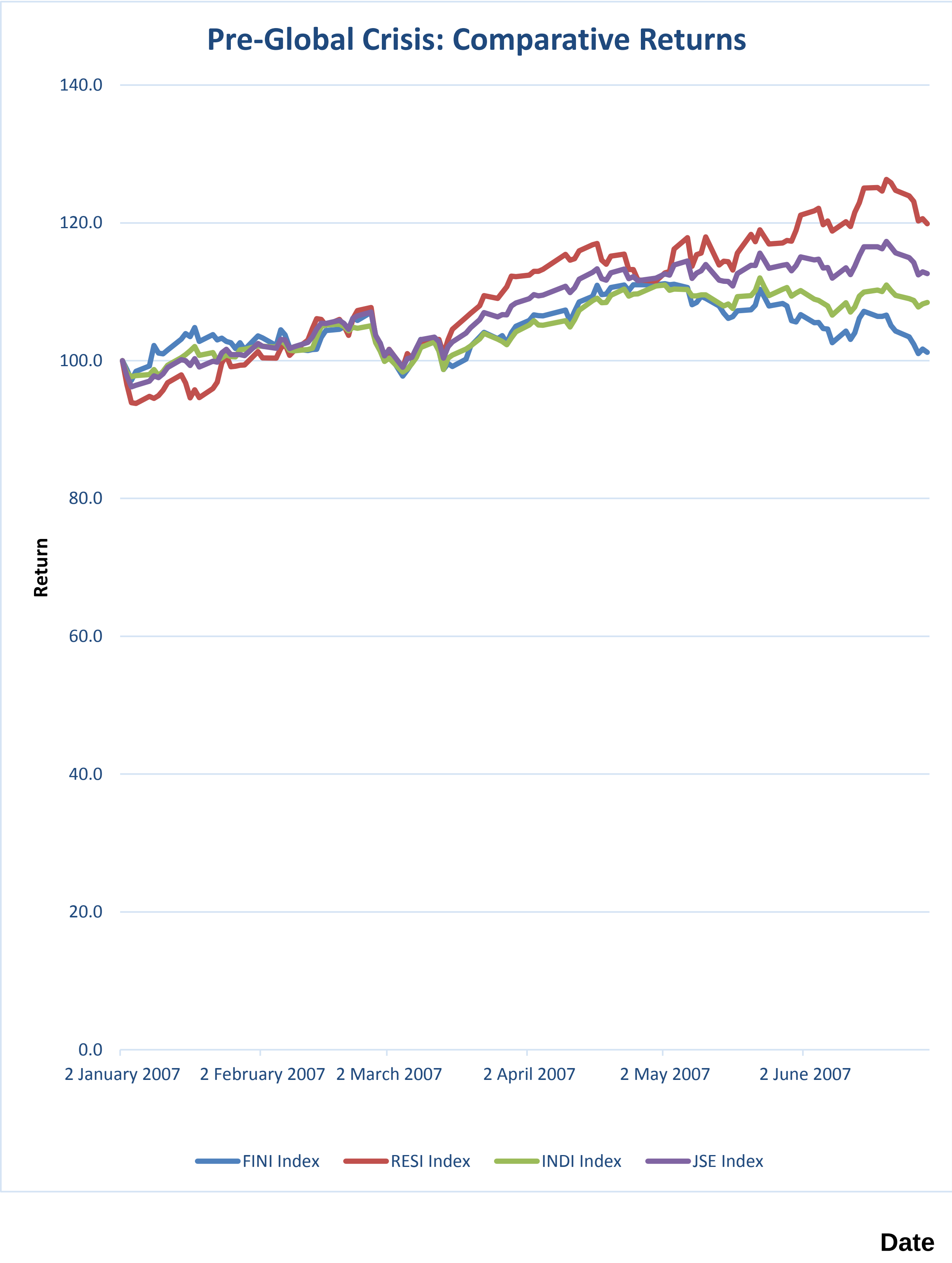

Figure 4.1: Comparative returns for the pre - GFC period

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

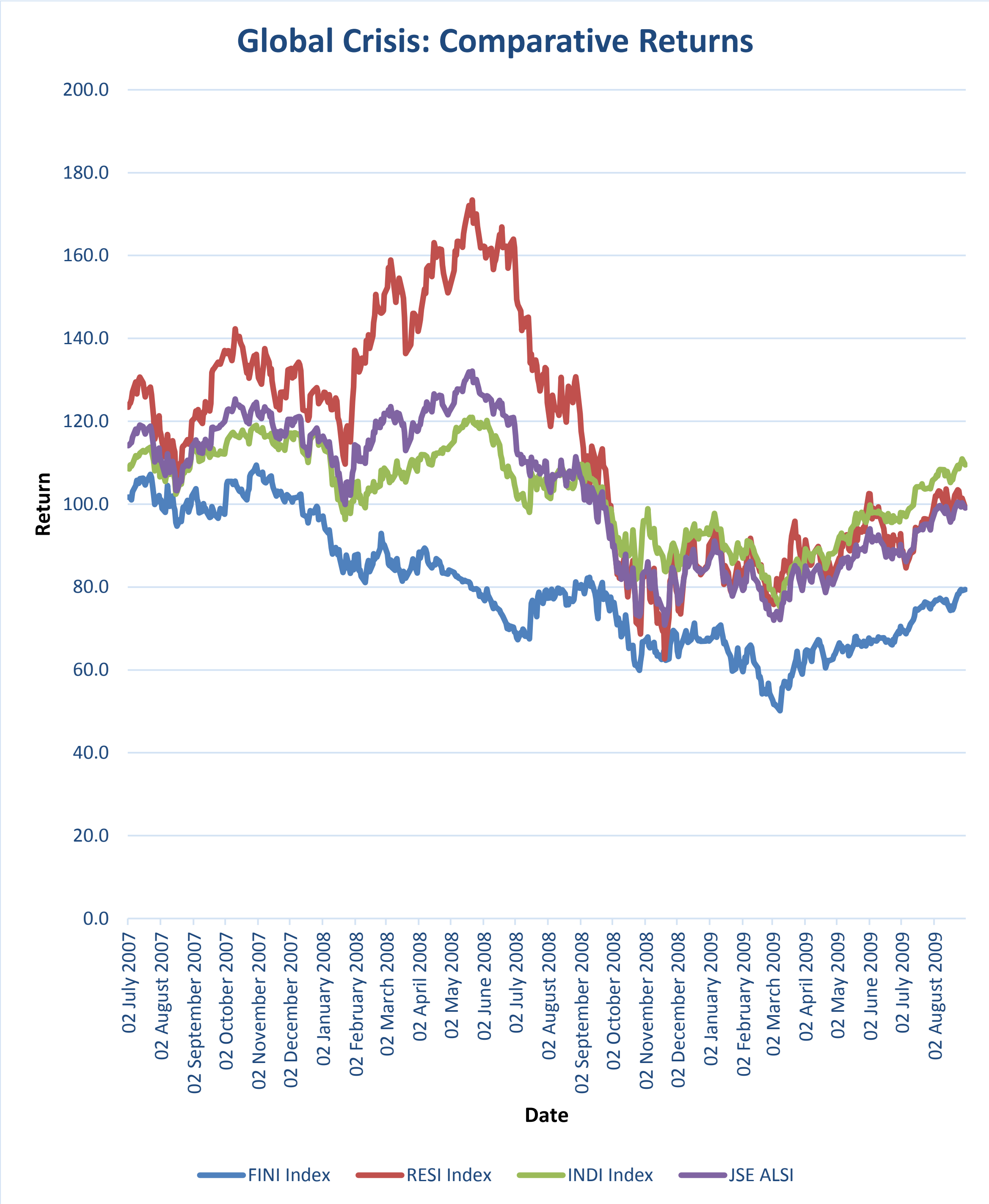

Figure 4.2: Comparative returns for the GFC period

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

During the GFC period, the JSE market was volatile, as can be noted in the movement of indices in Figure 4.2. The FINI index was the most affected to the extent that it trailed the benchmark throughout the period, having an average daily return of -0.0161% and a standard deviation of 0.0195. The RESI index outperformed the JSE ALSI up to end of October 2008, when its performance started dropping. On the other hand, the INDI index improved on its performance from October 2008, leading to an overperformance compared to all the indices, including the JSE ALSI, until the end of the GFC period.

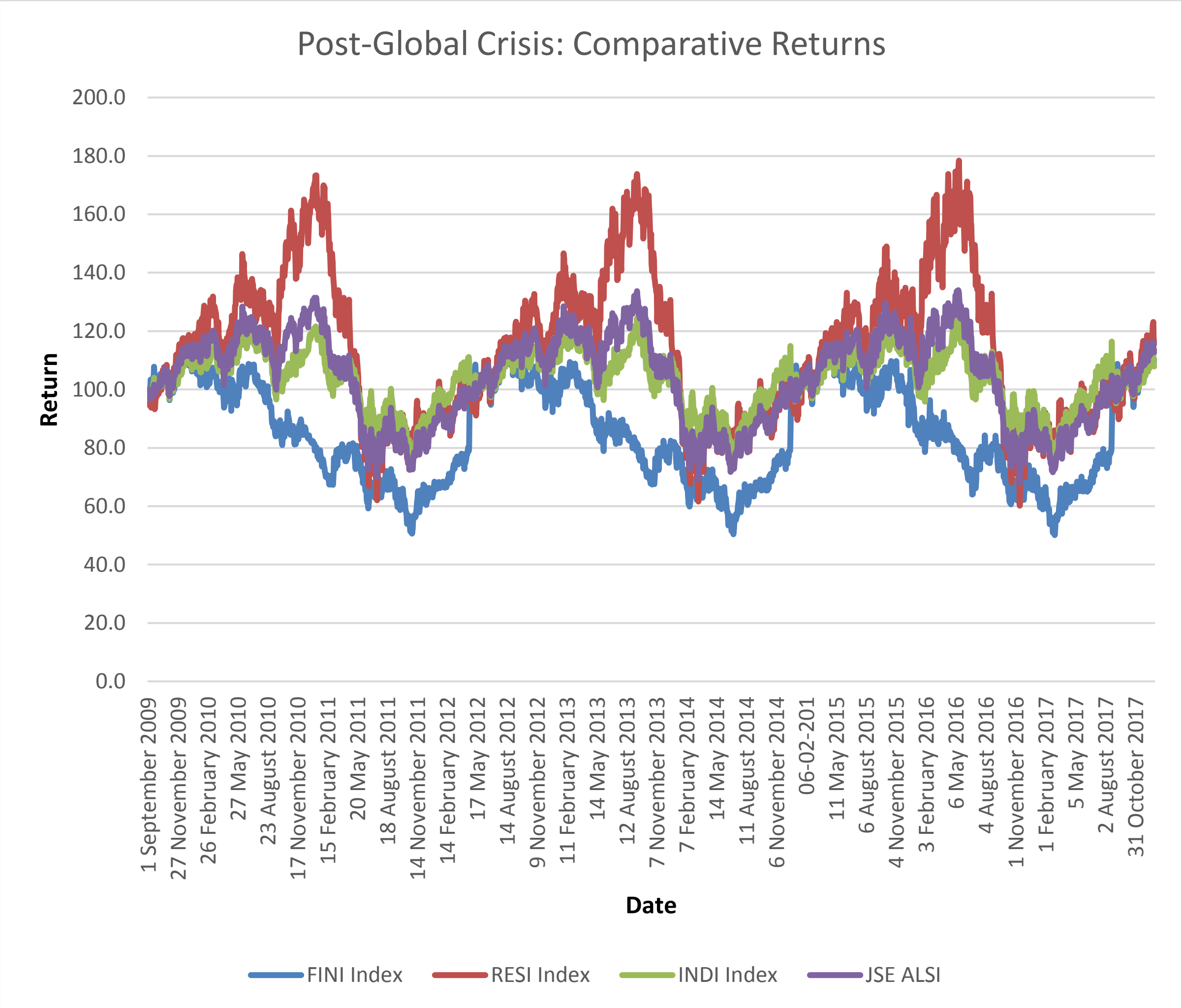

Figure 4.3: Comparative returns for the post-GFC period

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

The performance of the indices, as noted in Figure 4.3, followed the trend of the JSE ALSI. The RESI index improved its performance, though it was the most volatile with sharp increases and sharp falls in its performance. The RESI index could perform above all the indices at some points, but then a drastic fall would follow. Such volatility caused its post-GFC average return to be 0.0033%, accompanied by the highest standard deviation of 0.0157. On the other hand, the INDI index was a bit stable, performing more or less as a replica of the JSE ALSI. The FINI index underperformed compared to the benchmark, as measured by the JSE ALSI throughout the post-GFC period.

The expected return and standard deviation calculated variables were used to construct the MVP, following the Markowitz framework explained in the chapter on methodology. The three - asset portfolio optimisation problem was solved for portfolio 1 in Microsoft Excel, using Excel Solver with the no short - selling constraint. The first step was to assign some random weights for each index, so that the Solver could pick the optimum portfolios. Below are the results of the random-weight portfolios before Solver.

Table 4.5: Pre-GFC random-weighted portfolio

Pre-GFC period random-weighted portfolio	
Weights	
0.26	FINI
0.40	RESI
0.34	INDI
1	
Expected return	0.091%
Standard deviation	0.0101

A Pre-GFC random portfolio of 26% FINI index, 40% RESI index and 34% INDI index, could yield a daily average return of 0.091% and a standard deviation of 0.0101. A risk-averse investor would be interested in investing into a minimum risk portfolio at the same rate of return. The best risk-return combination was generated from the optimizing tool, which was the Solver.

Table 4.6: GFC random weighted portfolio

GFC period random-weighted portfolio		
Weights		
0.26	FINI	
0.40	RESI	
0.34	INDI	
1.00		
Expected return	0.0027%	
Standard deviation	0.0202	

The GFC period was associated with instability and all the indices tended to be more volatile than before. As a result, the random portfolio for the period yielded a return of 0.0027% on the expense of a higher standard deviation (0.0202) when compared to the pre- GFC period.

Table 4.7: Random weighted portfolio after the GFC

Post GFC period random-weighted portfolio		
Weights		
0.26	FINI	
0.40	RESI	
0.34	INDI	
1.00		
Expected return	0.0390%	
Standard deviation	0.0107	

By selecting random weights of 26% in the FINI index, 40% in the RESI index and 34% in the INDI index, it can be observed that the obtained random portfolio yielded a lower return (0.0390%) compared to the pre-GFC (0.091%), but higher than that of the GFC period (0.0027%). In terms of risk, the pre-GFC random-weight portfolio had the lowest standard deviation of 0.0101, followed by the post-GFC which had 0.0107, and then the GFC random-weighted portfolio which had a standard deviation of 0.0202. The overall performance of the random-weighted portfolio against the JSE ALSI as a benchmark, is presented in Figure 4.5 below.

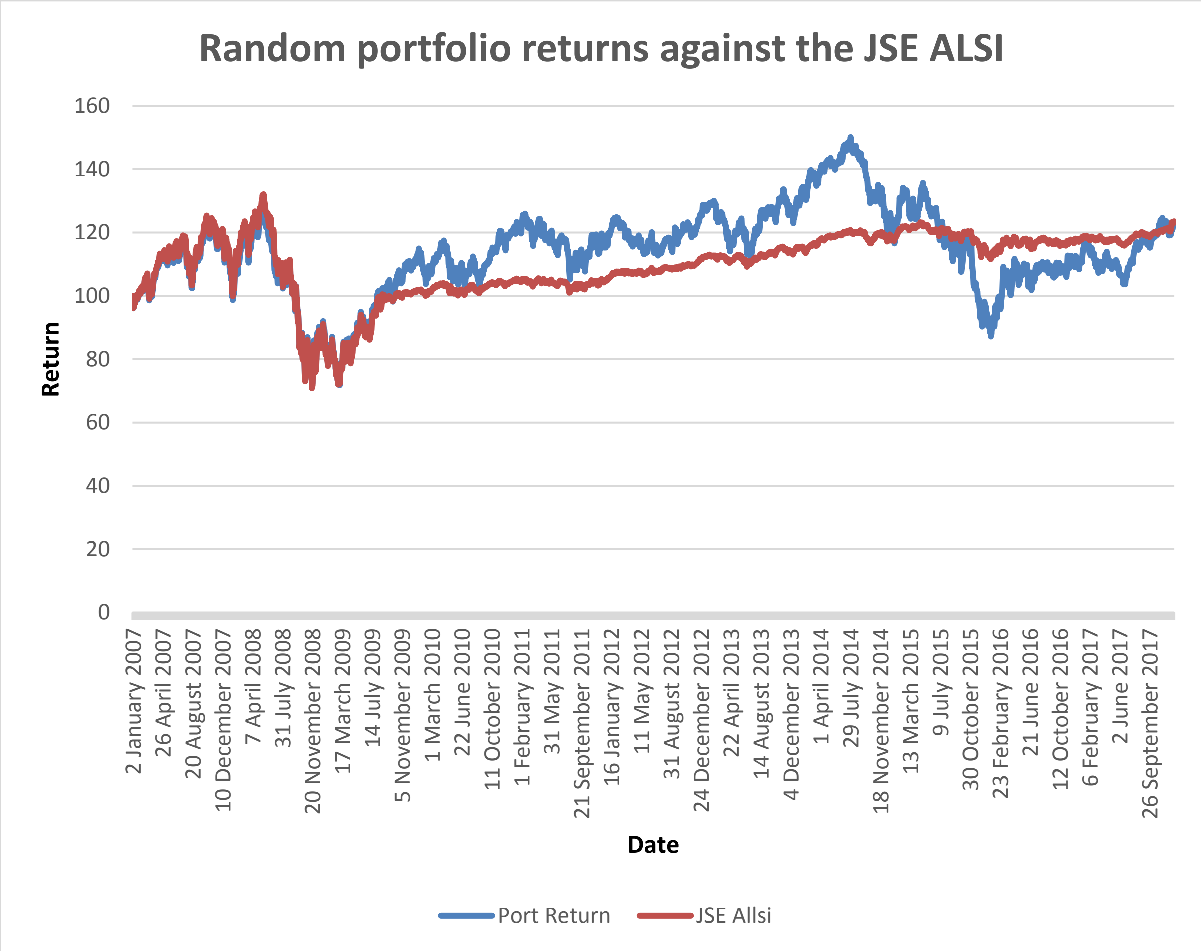

Figure 4.4 Random weight portfolio performance against the JSE ALSI

Source: researcher’s own compilation, JSE stock data obtained from Bloomberg

It can be noted that the random portfolio at some points performed below the JSE ALSI, especially during the GFC period. After the crisis, the portfolio performed better than the benchmark, but later trailed the benchmark from mid-August 2015 until the end of the period. On average, the random-weighted portfolio had a return of 0.0337% against a standard deviation of 0.0189, which is a worse performance compared to the JSE ALSI. The random-weighted portfolio could beat the market at some point by chance, without maintaining its performance consistently. Investors who pick such a portfolio can incur more risk with lower returns as this will be just a random move. From Figure 4.4, it can be concluded that optimising a portfolio is critical to keep the standard deviation as low as possible, while at the same time maximising returns.

To optimise a portfolio, the random weights were input into the Solver. Solver was constrained to minimise the standard deviation by changing the index weights. The Solver parameters are shown on Figure 4.5 below.

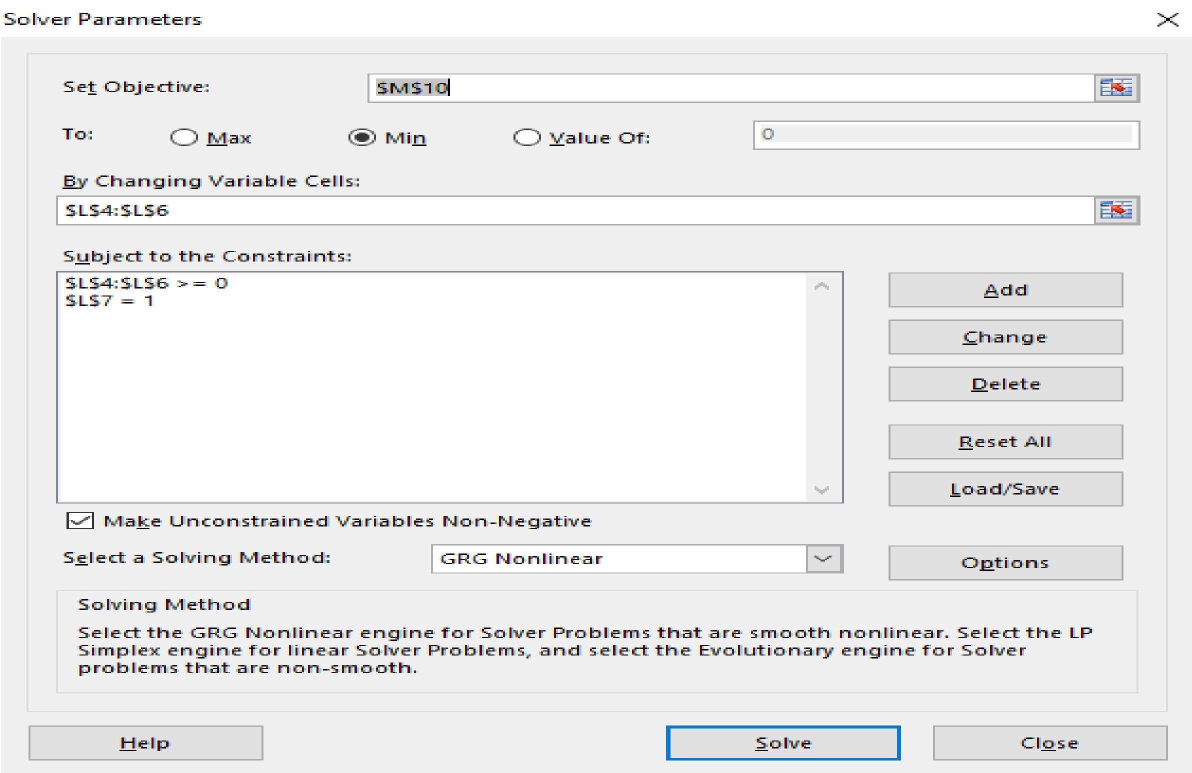

Figure 4.5: Solver parameters for optimising the portfolio

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

The set objective is the standard deviation cell (M10), which was minimised by changing the weights of the indices (L4-L6). To achieve the set objective, some constraints had to be set. The index weights were constrained to be equal to or greater than zero since the portfolio being constructed did not include short selling. The total portfolio weight was also constrained to be equal to 1 as indicated in Figure 4.5 above. After inputting all the constraints, Solver was instructed to solve the optimisation problem. The same process was repeated for all of the three periods.

The following results were produced by the Excel Solver for the uncapped weight global minimum variance portfolio (GMVPU).

Table 4.8: Portfolio variables for the GMVP Uncapped – before the GFC period

Mean	0.0698%
Standard deviation	0.0088
Weights	FINI = 0.00 RESI = 0.00 INDI = 100%

For a minimum variance portfolio during the pre-GFC period, an investor had to invest 100% of the capital into the INDI and nothing into the other two indices. Such a portfolio could yield an average daily return of 0.0698% with a standard deviation of 0.0088. However, holding such a portfolio might not be safe as it might not be diversified enough.

Table 4.9: Portfolio variables for the GMVP uncapped – during the GFC period

Mean	0.0131%
Standard deviation	0.0170
Weights	FINI = 0.073 RESI = 0.00 INDI = 0.927

From the portfolio weights tabulated in Table 4.9, it can be observed that the INDI index carries the biggest weight, followed by the FINI index, then the RESI index for the GFC period. The high volatility of the INDI led to its total exclusion from the minimum variance portfolio by Solver. The main objective of optimising a portfolio is to minimise volatility, hence excluding a more volatile asset despite its high return.

Table 4.10: Portfolio variables for the GMVP uncapped – after the GFC period

Mean	0.0644%
Standard deviation	0.0101
Weights	FINI = 0.21 RESI = 0.04 INDI = 0.75

In the post-GFC period, the RESI index which had been excluded from the portfolio during the GFC period was now included. The INDI index had the highest constituent percentage, followed by the FINI index, and then the RESI index. Including all the three assets yielded a return of 0.0644% with a standard deviation of 0.0101. Having combined the before, during and post-GFC periods, the MVP obtained yielded a return of 0.0556% with a standard deviation of 0.0157.

4.4 Empirical findings on the performance of the global MVP versus the JSE ALSI index

The constructed portfolio was then compared against the JSE ALSI for the three periods. The average return and standard deviation of the JSE ALSI is illustrated in Table 4.11.

Table 4.11: Average daily risk-return profile of the JSE ALSI index

	Average return	Standard deviation
Pre-GFC	0.1017%	0.0101
GFC	0.0152%	0.0183
Post-GFC	0.0462%	0.0094

When compared against the benchmark, the constructed GMVPU is observed (Table 4.8) to be under-performing during the pre-GFC period. The GMVPU yielded an average daily return of 0.0698% which is 0.0319% lower than the JSE ALSI (0.1017%). However, the constructed portfolio had a lower standard deviation of 0.0088 when compared to the 0.0101 of the JSE ALSI. Risk-averse investors could pick the GMVPU, as they will be mainly concerned about minimising risk.

During the GFC period, from July 2007 up to October 2008 the GMVPU started outperforming the JSE ALSI as depicted by its risk-return combinations in Table 4.9. The GMVPU daily average return of 0.0131% was higher than the JSE ALSI daily average return for the GFC period (0.0152%) In terms of risk, the GMVPU yielded a lower standard deviation of 0.0170 when compared to 0.0183 of the JSE ALSI.

After the GFC period, that is from November 2008 going forward, the GMVP maintained its performance, beating the JSE ALSI by a margin of 0.0182% (0.0644% against 0.0462% of the ALSI), in terms of return. The standard deviation for the GMVPU was 0.0101, which is slightly higher than that of the JSE ALSI (0.0094), hence the GMVPU maximised returns with a little or no significant altering of the level of risk. The risk-return combinations of the GMVPU against the JSE ALSI for the three periods were plotted on graphs and the following results were obtained:

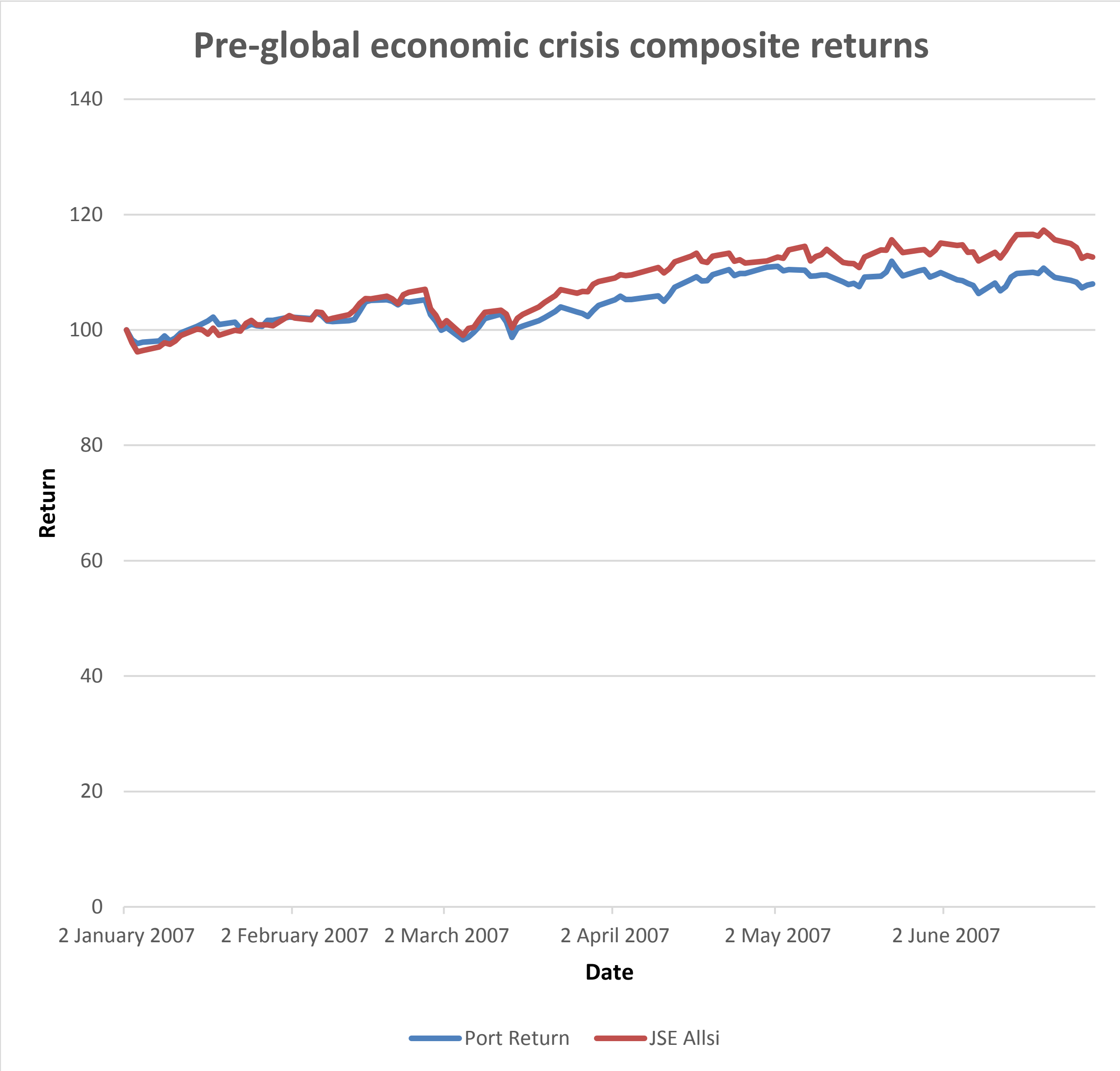

Figure 4.6: GMVPU return against JSE ALSI: pre-GFC period

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

From Figure 4.6, it can be noted that before the GFC period the JSE ALSI was outperforming the constructed MVP. From the beginning of the period up to mid-March 2007, the GMVPU imitated the performance of the market. The JSE ALSI improved on its performance yielding a return 0.0319% higher than the GMVP.

Figure 4.7: GMVPU return against JSE ALSI – GFC Period

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

During the bearish market period, investing 93% of the capital into the INDI index, 7% into FINI index and 0% into RESI index could yield an optimum portfolio trend as shown in Figure 4.7 above. On average, although the GMVP could underperform the market at some points, the GMVP have a relatively higher return than the JSE ALSI by a margin of 0.0077%.

After the GFC period, there was a pick-up in the market. The constructed portfolio was observed to improve on its performance as well. Compared to the benchmark, the graphical presentation of the constructed MVP is indicated in Figure 4.8.

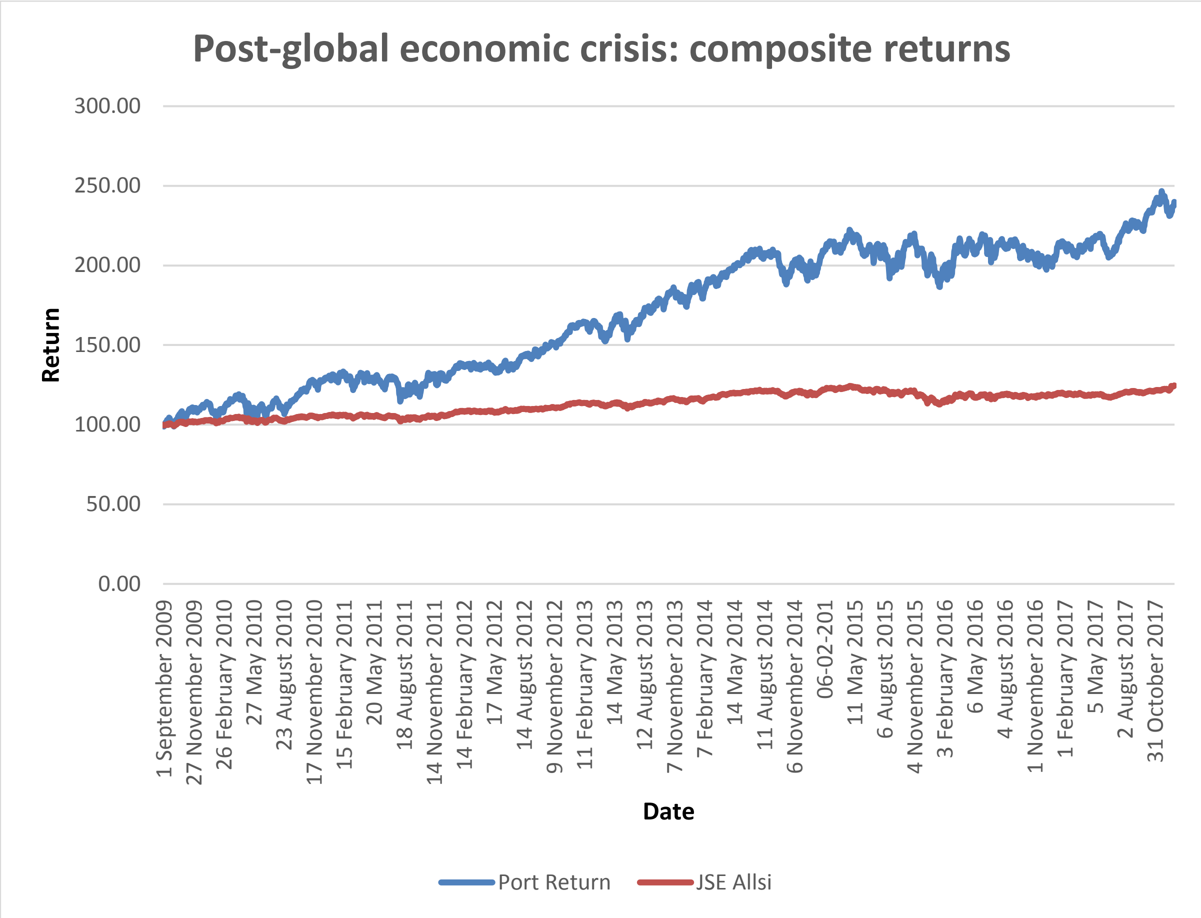

Figure 4.8: GMVPU returns against JSE ALSI – Post-GFC period

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

From Figure 4.8, it can be noted that the constructed GMVP is consistently outperforming the JSE ALSI index during the bull market period. Addressing objective 4 of the study, which sought to assess whether the GMVP could consistently outperform the benchmark, it can be concluded that the performance of the GMVP is consistent in its outperformance during a bullish market. The average return for the GMVP during the post-GFC period is 0.0644%, which is higher than the 0.0462% of the JSE ALSI.

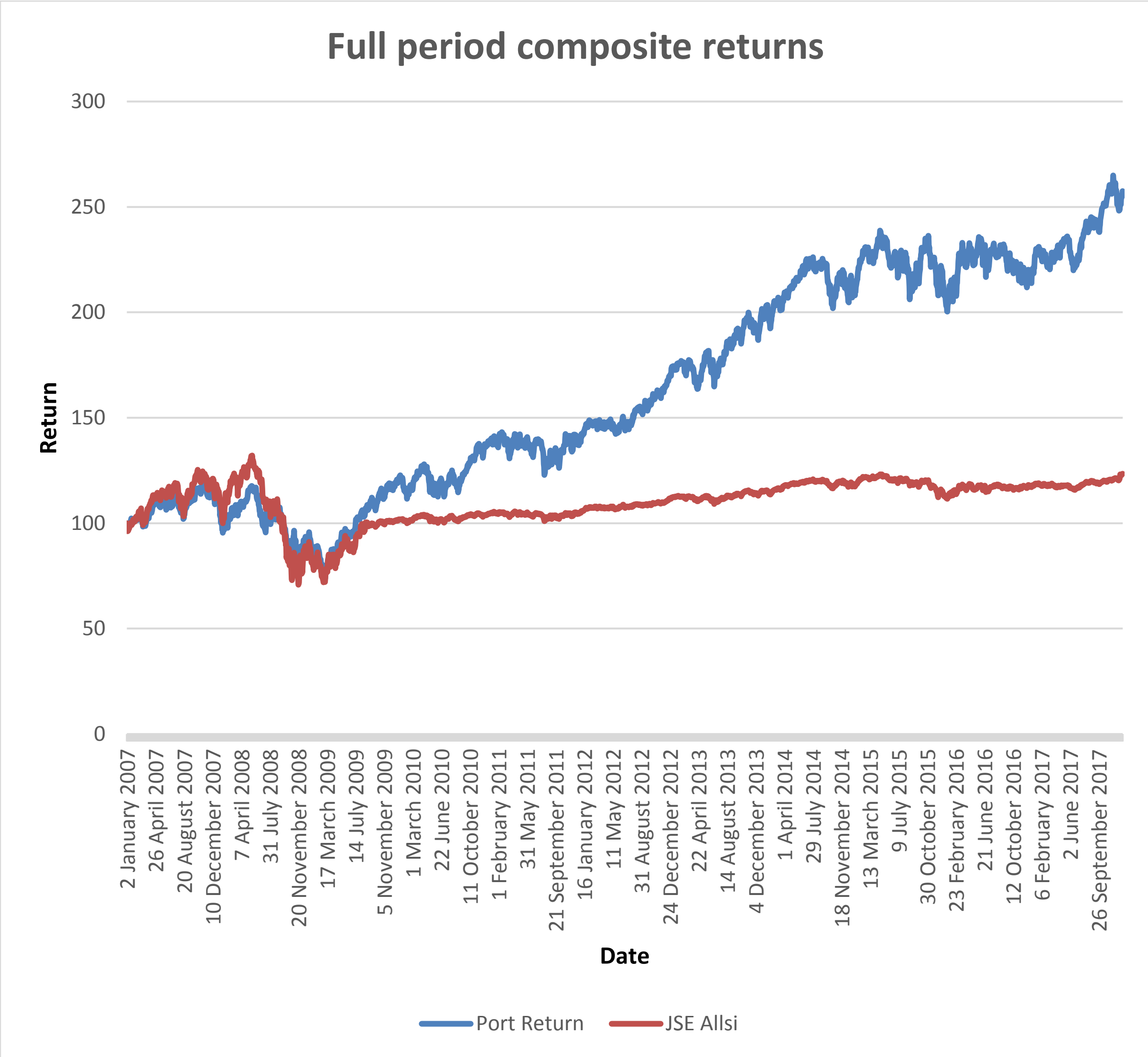

Figure 4.9: GMVPU return against JSE ALSI – 11-year period

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

The full period is a holistic indication of what transpired before the GFC, during the GFC and after the GFC. This signifies that the tradable indices on the JSE to a certain extent follow the movement style of the JSE ALSI. When the market goes down, the performance of the GMVPU also declines but with a lesser margin. Due to the power of sector diversification associated with investing in tradable indices, the GMVPU stays on top of the ALSI and performs better even during market downswings.

A t-test was run to determine if the results obtained from the performance of the GMVPU was statistically significant against the JSE ALSI in terms of both the return and the risk. The t-test was based on objective 4 of the study whose formulated hypotheses is restated as follows:

Hypothesis

H_0: The efficiently constructed MVP using the JSE tradable sector index can consistently outperform the JSE ALSI

H_1: The efficiently constructed MVP using the JSE tradable sector index cannot consistently outperform the JSE ALSI

The results obtained from the t-test are tabulated in Table 4.10 to 4.12 below.

4.12 T-test: two-sample assuming equal variances – Pre-GFC period

	GMVP	*JSE ALSI*
Mean	0.00102	0.00070
Variance	0.00010	0.00008
Observations	123	123
Pooled variance	0.00009	
Hypothesized mean difference	0	
df	244	
t Stat	0.26824	
P(T<=t) one-tail	0.39437	
t Critical one-tail	1.65112	
P(T<=t) two-tail	0.78873	
t Critical two-tail	1.96973	

4.13 T-test: two-sample assuming equal variances GFC period

	GMVP	**JSE ALSI**
Mean	0.00013	-0.00004
Variance	0.00028	0.00038
Observations	542	542
Pooled variance	0.00033	
Hypothesized mean difference	0	
df	1082	
t Stat	0.15880	
P(T<=t) one-tail	0.43693	
t Critical one-tail	1.64626	
P(T<=t) two-tail	0.87385	
t Critical two-tail	1.96216	

4.14 T-test: two-sample assuming equal variances – Post-GFC period

	MVP	JSE ALSI
Mean	0.00011	0.00046
Variance	0.00001	0.00009
Observations	2087	2087
Pooled variance	0.00005	
Hypothesized mean difference	0	
df	4172	
t Stat	-1.62097	
P(T<=t) one-tail	0.05255	
t Critical one-tail	1.64521	
P(T<=t) two-tail	0.10510	
t Critical two-tail	1.96053	

Alpha (level of significance) = 0.05

For the pre-GFC, the GFC and the post-GFC periods, the p-values for the two tail tests were 0.79, 0.87 and 0.11 respectively. The p-values obtained are more than the 0.05 alpha level of significance. As a result, the null hypothesis cannot be rejected based on the data used. In other words, for the period studied, the GMVP was consistently outperforming the JSE ALSI and the outperformance is statistically significant.

4.5 Empirical findings on diversification

The third research objective of the study was to determine whether there are any diversification benefits associated with investing on the JSE tradable sector. Diversification benefits are more where there is a negative correlation among the assets in a portfolio. The co-relationship of the three indices in the constructed portfolio is measured by the variance-covariance matrix. For the GMVPU, this can be presented in Tables 4.13 to 4.15 below.

Table 4.15: Variance–covariance and correlation matrix – pre-GFC period

Variance-covariance				Correlation			
Variance-covariance matrix				Correlation matrix			
	FINI	RESI	INDI		FINI	RESI	INDI
FINI	0.034	0.025	0.021	FINI	1		
RESI	0.025	0.049	0.019	RESI	0.6109	1	
INDI	0.021	0.019	0.019	INDI	0.8308	0.6372	1

A positive correlation was observed among all the three indices during the pre-GFC period. The FINI and RESI indices had a correlation coefficient of 0.61 while the RESI and INDI indices had 0.637. The FINI and the INDI indices had the highest correlation of 0.831. High positive correlations among assets will reduce the chances of maximising diversification benefits. However, the GMVP for the pre-GFC period have been noted to constitute of 100% investment in the INDI asset (portfolio 1 in Table 4.18), hence an investor will still enjoy the benefits of the optimised portfolio with no need to diversify.

Table 4.16: Variance–covariance and correlation matrix - GFC period

Variance-Co-variance

Variance-covariance matrix			
	FINI	**RESI**	**INDI**
FINI	0.108	0.080	0.068
RESI	0.080	0.221	0.080
INDI	0.068	0.080	0.072

Correlation

Correlation matrix			
	FINI	**RESI**	**INDI**
FINI	1		
RESI	0.52	1	
INDI	0.77	0.64	1

For the GFC period, a positive correlation of 0.52 was noted between the FINI and the RESI indices. The INDI and the RESI indices had a correlation coefficient of 0.64 while the FINI and INDI indices correlated with a margin of 0.78. All the three assets were found to be positively correlated making it difficult to reap maximum diversification benefits. However, since the indices were not perfectly positively correlated, pooling the three indices into a portfolio could fetch diversification benefits to a certain extent.

Table 4.17: Variance–covariance and correlation matrix – post-GFC period

Variance - Covariance

Variance-covariance matrix			
	FINI	**RESI**	**INDI**
FINI	0.034	0.023	0.022
RESI	0.023	0.064	0.022
INDI	0.022	0.022	0.025

Correlation

Correlation matrix			
	FINI	**RESI**	**INDI**
FINI	1		
RESI	0.47	1	
INDI	0.72	0.55	1

A positive correlation was also observed among all the three indices during the post-GFC period. The correlation coefficient between the FINI and the RESI indices was observed to be 0.47, while the RESI and the INDI indices had 0.55. Finally, the FINI and the INDI indices positively correlated with a coefficient of 0.72. Such a positive relationship among the assets means that the indices will be reacting almost in the same way to adverse market conditions. Despite the positive correlation observed among the three indices, the collective risk-return characteristics of the three assets are better off when compared to their individual, respective risk-return combinations. In other words, diversifying a portfolio among the three indices is beneficial, as it yielded better returns associated with lower standard deviations compared to the JSE ALSI.

4.6 Empirical findings on the different constraints portfolios

To assess the reliability of the MVPs in different scenarios, more portfolios were constructed, using different constraints. Four more portfolios were created which summed up the total number of constructed portfolios to be five including the GMVPU. Portfolios 1 (GMVPU) and two were constructed using Microsoft Excel, while 3, 4 and 5 were created using Stata. The values of the portfolios constructed using Stata were based on daily variables while the Microsoft Excel constructed portfolios were annualised. The following constraints were used:

- uncapped weight minimum variance portfolio (MVPU)
- proportionately weighted according to the tradable sector
- MVP – with short selling allowed,
- MVP – with no short selling which means FINI, INDI, RESI > 0
- MVP – no short selling with maximum weight restricted to 64%

The GMVPU and portfolio 4 had the same constraints. However, GMVPU was constructed using Microsoft Excel while portfolio 4 was constructed with Stata to determine whether the same results could be obtained using the two different optimising methods. For the GFC and the post-GFC period, the optimisation problem in Excel Solver and Stata produced the following results.

Table 4.18: Portfolio results for the five constructed portfolios before, during the GFC and post GFC periods

		Portfolio 1 GMVPU	Portfolio 2 proportionately weighted	Portfolio 3 short selling allowed	Portfolio 4 no short selling	Portfolio 5 max weight 64%
Pre-GFC period	Return	0.0698	0.0675%	0.0810%	0.0698%	0.0644%
	Standard deviation	0.0088	0.0094	0.0086	0.0088	0.0092
	Asset weights	FINI= 0.00 RESI= 0.00 INDI = 100%	FINI= 0.24 RESI=0.12 INDI =0.64	FINI = -0.197 RESI = 0.008 INDI = 1.189	FINI = 0.00 RESI = 0.00 INDI = 100%	FINI = 0.26 RESI = 0.1 INDI = 0.64
GFC period	Return	0.0130%	0.0056%	0.0128%	0.0125%	0.0021%
	Standard deviation	0.0170	0.0164	0.0168	0.0168	0.0172

	Asset weights	FINI= 0.07 RESI=0.00 INDI =0.93	FINI= 0.24 RESI=0.12 INDI =0.64	FINI= 0.09 RESI=-0.07 INDI =0.98	FINI =0.09 RESI = 0 INDI =0.91	FINI = 0.35 RESI=0.01 INDI= 0.64
Post-GFC period	Return	0.0643%	0.0587%	0.0635%	0.0635%	0.061%
	Standard deviation	0.0101	0.0101	0.00971	0.00971	0.00973
	Asset weights	FINI= 0.21 RESI=0.04 INDI =0.75	FINI= 0.24 RESI=0.12 INDI =0.64	FINI= 0.22 RESI=0.05 INDI = 0.73	FINI=0.22 RESI=0.05 INDI = 0.73	FINI = 0.29 RESI=0.07 INDI = 0.64

From Table 4.18 above, it can be concluded that despite employing different constraints, the MVO framework can be applied successfully to the JSE tradable sector. Investors can employ the model in different scenarios and constraints depending on their preferences.

The proportionately-weighted global mean-variance portfolio (PGMVP) as a true representation of the three indices (INDI, FINI and RESI) on the JSE, yielded a pre-GFC return of 0.0675%, which is 0.0342% less than the ALSI. In terms of risk, the PGMVP is 0.0007% less than ALSI during the pre-GFC period. During the GFC period, the PGMVP yielded an average daily return of 0.0056%, which was lower than the JSE ALSI, which had 0.0152%. However, the PGMVP outperformed the benchmark in terms of risk by a margin of 0.0019 during the GFC period. After the GFC period, the PGMVP generated a return of 0.0587 against 0.0462 of the ALSI. However, the risk of the PGMVP after the GFC period is 0.0007 higher than that of the JSE ALSI.

Portfolio 3 underperformed compared to the benchmark in terms of return during the pre-GFC period. Short selling was allowed for this portfolio. It then yielded a daily average return of 0.0810% while the JSE ALSI had 0.1017%. On the other hand, the standard deviation of portfolio 3 (0.0086) was lower than the JSE ALSI by a margin of 0.0015. During the GFC period, the constructed portfolio maintained its standard deviation (0.0168) against the JSE (0.183) but still underperformed in terms of returns. The average daily returns of portfolio 3 rose above the benchmark after the GFC period. The portfolio yielded a return of 0.0635% against the benchmark return of 0.0462%. In terms of the post-GFC standard deviation, portfolio 3 underperformed the benchmark by a margin of 0.0003.

Using daily variables to calculate the optimum portfolio, the most minimum variance portfolios were seen to be portfolios 1 and 4. Portfolio 3 included an opportunity for short selling while portfolio 4 restricted short sales. It is clear that the inclusion of short sales does not make a significant difference in the optimum portfolio as portfolio 3 and portfolio 4 yielded the same risk-return combinations for both the GFC and post-GFC periods. The exclusion of the INDI from the optimum portfolio adjusted the results, causing an insignificant difference between short-selling and non-short selling. As a result, it is advisable for investors to select portfolios with no short selling, as short-selling is a short-term strategy, which might involve high management costs. The Markowitz MVO model is a passive investment tool, hence selecting a non-short sale portfolio can be cost-effective and more ideal for long-term, risk-averse investors. It is therefore clear that investing in the three main tradable indices on the JSE yields diversification benefits and better performance when compared to the benchmark during especially in upward market trend periods.

4.7 The efficient frontier

Efficient frontiers were constructed using Stata for all the portfolios, while at the same time accommodating the different constraints. A set of portfolios were constructed to represent a variety to investors for the three periods. Efficient frontiers were constructed by using the continuously compounded daily returns to create 100 portfolios along the efficient frontier. Initially, a command for no short-selling was used (portfolios 1, 2, 4 and 5). The second command was for an efficient frontier allowing for short sales (portfolio 3). The efficient frontiers before, during and after GFC efficient frontiers are illustrated in Figure 4.10 to 4.15.

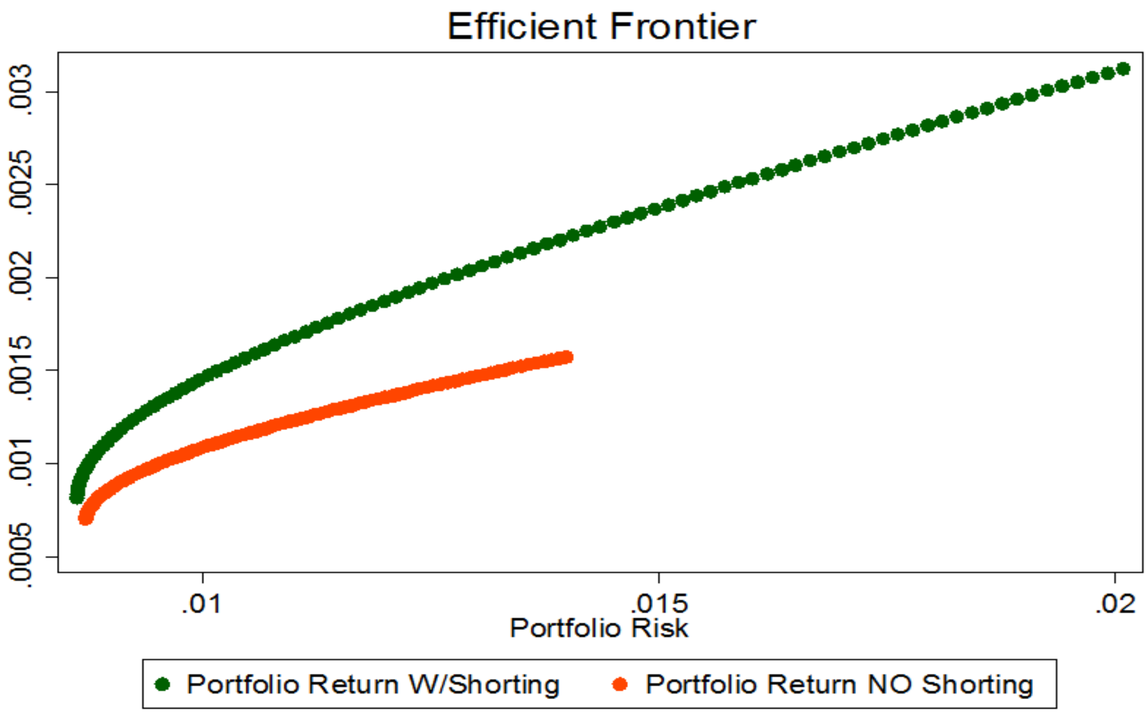

Figure 4.10: The pre-GFC efficient frontier (portfolios 1,2, 3 and 4)

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

Figure 4.10 represents efficient frontiers for portfolios 1, 2, 3 and 4. The green frontier represents portfolio 3 while the orange frontier is for portfolios without short selling (portfolios 1, 2 and 4). The portfolio risk-return combinations were joined by a hyperbolic line that corresponds to Markowitz's efficient frontier for the pre-GFC period. All the points on the efficient frontier represent efficient portfolios. Any point above the frontier is unattainable while any point below the frontier is inefficient. Investors can then choose any of the portfolios on the frontier, depending on their risk preferences. A risk-averse investor interested in short selling can choose the efficient minimum risk portfolio which is portfolio 3, with a standard deviation of 0.0086 and an average daily return of 0.0810. For such a portfolio, an investor must short 19.7% of FINI index and invest 118.9% into the INDI index. 0.8% of the capital should be invested into the RESI index. Portfolio 3 outperformed all the other portfolios though it underperformed the JSE ALSI.

For investors not interested in short-selling, the GMVPU and portfolio 4 will be ideal. The two portfolios have a daily average return of 0.0698% and a standard deviation of 0.0088. For both the portfolios, an investor should invest nothing into the RESI, and FINI indices. The available capital, which is 100% of the budget should be invested into the INDI index. A risk-loving investor can choose any portfolio on the far right of the frontier, depending on the risk tolerance and also considering whether it provides a commensurate return.

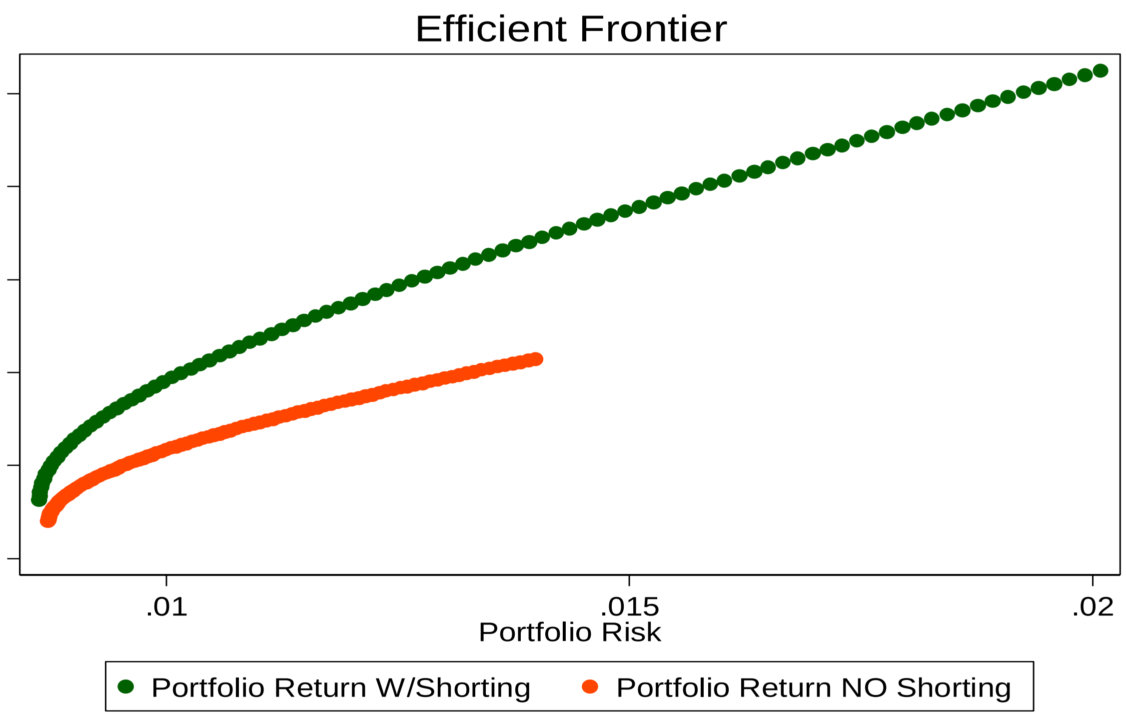

Figure 4.11: The pre-GFC efficient frontier (portfolio 5)

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

Having constrained the Stata gmvport module to allow a maximum of 64% in any index, an efficient frontier, as depicted in Figure 4.11 above was obtained for the pre-GFC period. A risk-averse investor could choose a portfolio with the lowest standard deviations, while risk lovers could pick portfolios with relatively high standard deviation provided they yield a commensurate rate of return. For a risk-averse investor, the targeted rate of return for the minimum variance portfolio 5, with no short selling, is 0.0644% with a standard deviation of 0.0092. An investor should put 64% of the capital into the INDI index, 26% to the FINI index and 1% to the RESI index to achieve the required portfolio outcome.

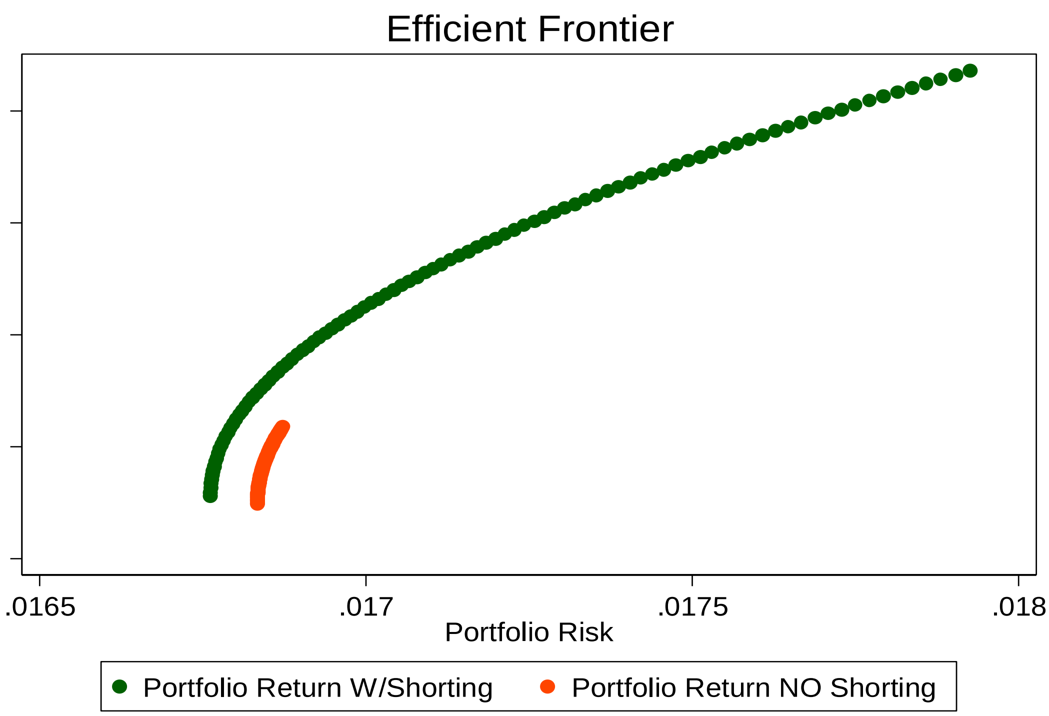

Figure 4.12: The GFC-efficient frontier (portfolios 1,2, 3 and 4)

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

During the GFC period, the most optimum MVPs were observed to be the GMVPU and portfolio 4. The two portfolios were observed to outperform the JSE ALSI in terms of standard deviation. The risk-averse investors can pick either of the two portfolios of which, for the GMVPU, 93% should be invested in the INDI index and 7% index in the FINI index. The GMVPU yielded a return of 0.0130% and a standard deviation of 0.0170. The RESI index was excluded for both the GMVPU and portfolio 4. On the other hand, 91% was allocated to the INDI index and 9% to the FINI index for portfolio 4 yielding a daily average return of 0.0125% and a standard deviation of 0.0168.

For portfolio 3 (with short selling), investors should allocate 98% to the INDI index, 9% to the FINI index and short sale 7% from the RESI index. Such a mix will yield a return of 0.0128 and a standard deviation of 0.0168. The short selling minimum variance portfolio has a lower return, compared to the GMVPU and portfolio 4. As a result, it is ideal for investors to take portfolios without short selling.

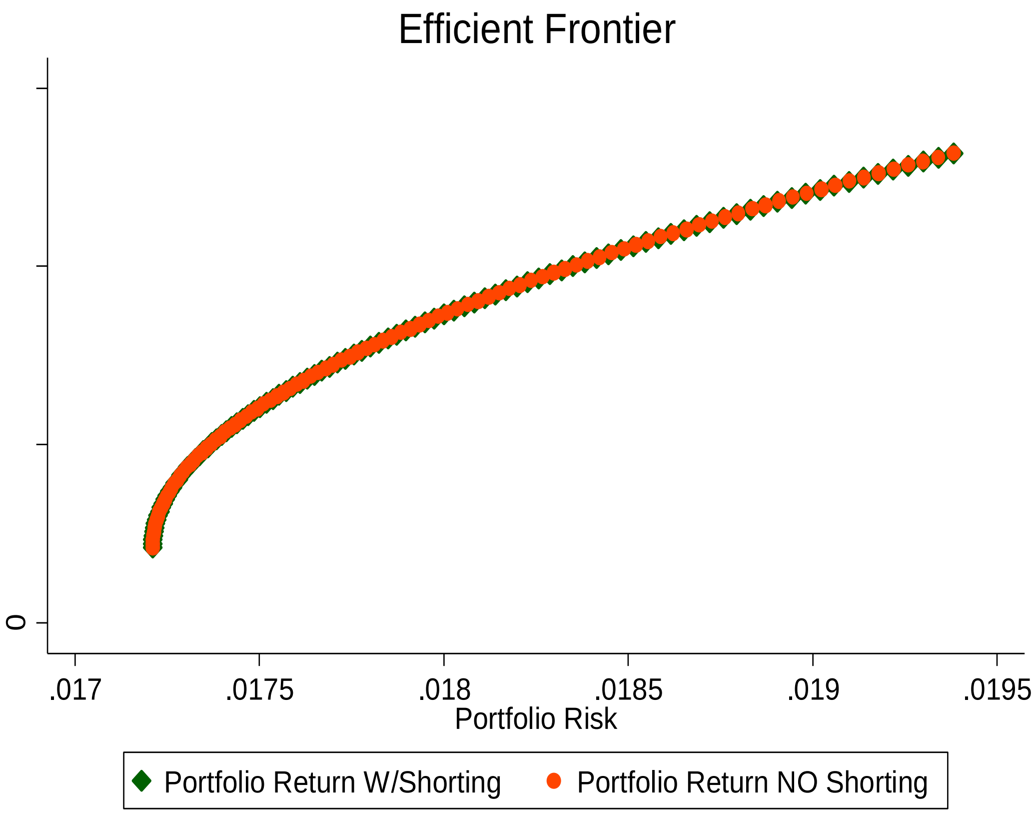

Figure 4.13: The GFC efficient frontier (portfolio 5)

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

As noted in Figure 4.13, the orange frontier overlaps the green frontier, signifying that the GMVP with short selling and the one without short selling had the same risk-return properties. The larger part of the capital (64%) was invested in the INDI index, while 35% was allocated to the FINI index. A small percentage of the budget (1%) was allocated to the RESI index. As defined by the optimising tool, no significant difference was noted between the performance of short selling and non-short selling portfolios.

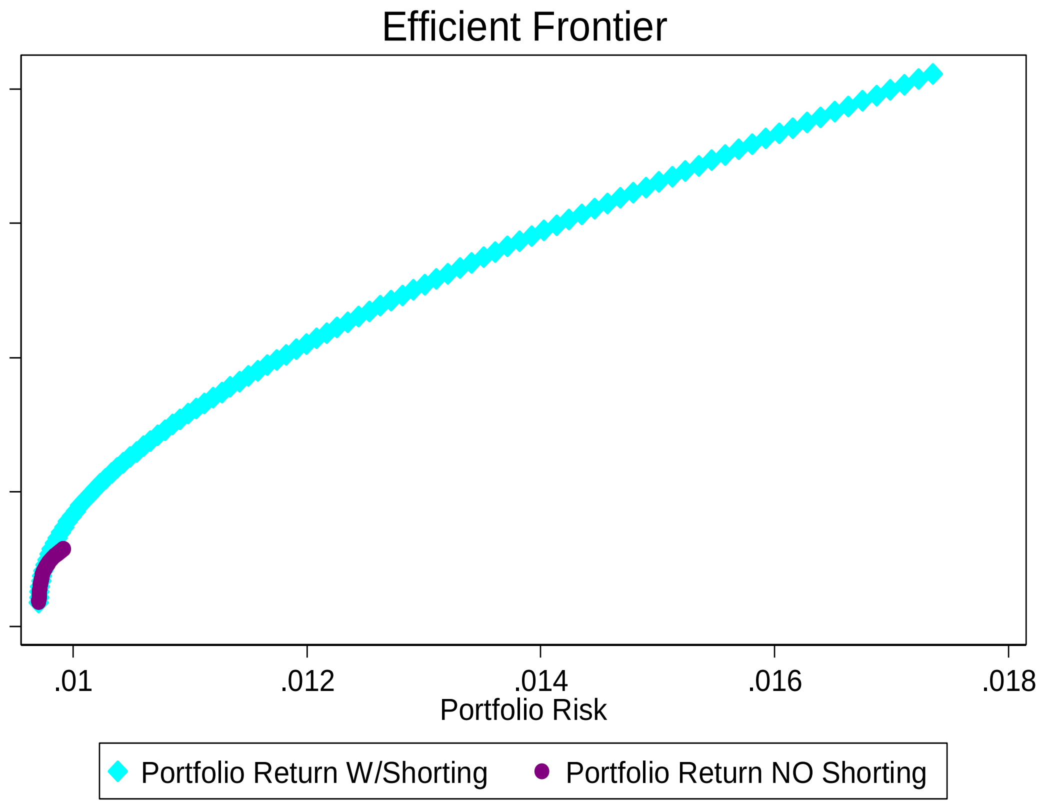

Figure 4.14: The post-GFC efficient frontier (Portfolios 1,2, 3 and 4)

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

Figure 4.14 represent the post-GFC frontiers for portfolios 1, 2, 3 and 4. The cyan frontier depicts 100 portfolios with short selling while the purple frontier represents the portfolios with no short selling during the post-GFC period. As noted in Figure 4.11, the purple frontier overlaps the green frontier signifying that the GMVP with short selling (portfolio 3) and the one without short selling (portfolio 4) had the same risk-return properties. That confirms that the GMVP obtained in Table 4.16 for portfolios 3 and 4 is optimum at a return of 0.0635% and a standard deviation of 0.00971.

There was no significant difference noted between the risk-return properties of the short-sale and no shorting GMVP. As a result, the two frontiers coincide, signifying that either of the selected portfolio would give the same risk-return combinations at any point. The difference lies on the management costs associated with the shorting strategy; hence, for risk-averse investors it could be ideal to pick portfolios with no shorting. All the MVPs outperformed the benchmark during the post-GFC period. However, portfolios 3 and 4 had the lowest standard deviation for the post-GFC period. As a result, it will be ideal for a non-short selling investor to pick portfolio 4. Alternatively, the investor can pick the GMVPU, which have a relatively higher average return though the standard deviation is slightly higher.

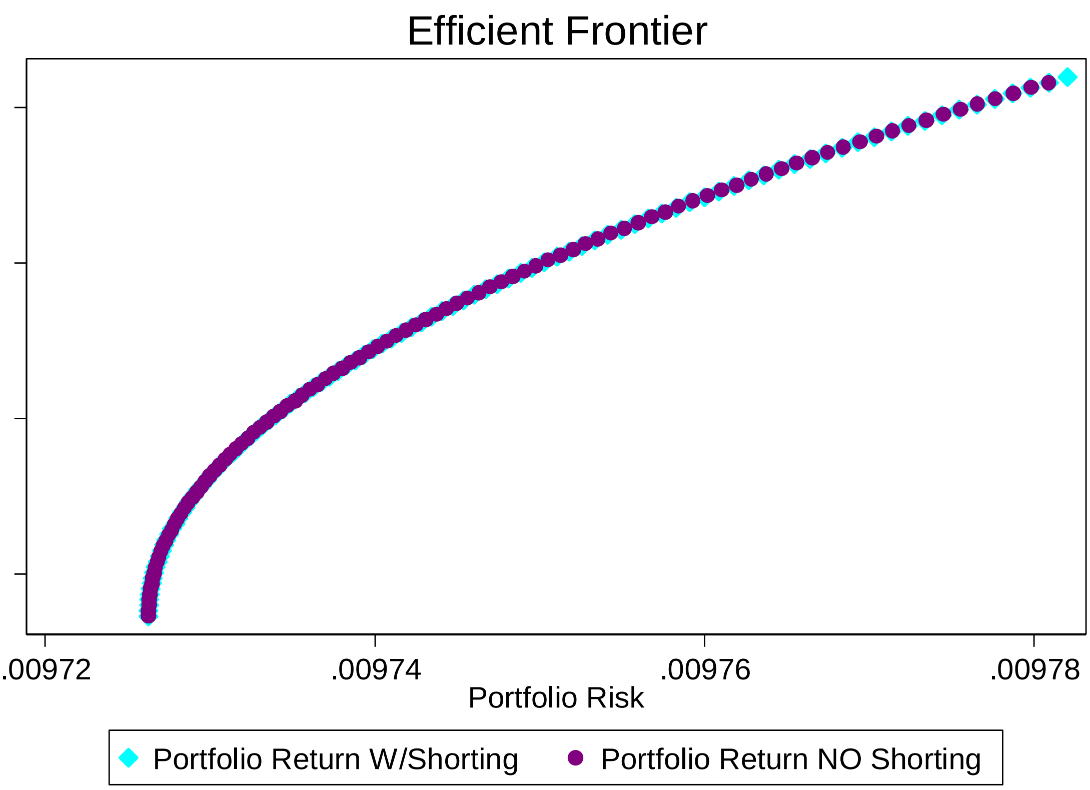

Figure 4.15 The Post GFC efficient frontier (portfolio 5)

Source: researcher's own compilation, JSE stock data obtained from Bloomberg

On this portfolio, 64% was constrained to be the maximum investment per each asset. For an optimum portfolio, 64% had to be invested in the INDI index, 29% in FINI index and 7% in RESI index. All the portfolios along the frontier are efficient; hence, an investor could choose any portfolio depending on his or her preferences for risk. However, not all the 100 portfolios are optimum. The most optimum portfolio is the one that minimise standard deviation while yielding a favourable level of return. For the post-GFC period, the GMVP yielded a return of 0.061% with a standard deviation of 0.0097.

This study tested the application of the MVO model to optimise a portfolio on the JSE using the tradable sector. Five portfolios with different constraints have been constructed successfully. All the MVPs constructed have proven to perform better than the JSE ALSI as a benchmark index. Sector diversification benefits have also been observed to be present as the portfolios outperformed the benchmark despite having a positive correlation among all the three indices.

Markowitz (1952) MVO model have proven to be successfully employable on the JSE based on the tradable sector indices. Following Markowitz (1952:78) the collective risk-return characteristics of the INDI, RESI and FINI were considered and compared against the JSE ALSI as a benchmark (Table 4.11 against Table 4.18). The total portfolio variance was noted to be reduced as a result of diversification. The present study hence substantiates what was noted by Markowitz (1952:89) and Yahaya, Abubakar & Garba, (2011:102), diversifying a portfolio decreases the level of portfolio risk.

According to Markowitz *et al*., (2009:1) MVO model could be an effective tool to employ during market downswings. In this present study, the MVO model have been examined during the GFC period. As a result, the model has been noted to be consistent in its ability to minimise risk as compared to the benchmark. In this present study, the MPT has also proven to be equally effective for portfolio optimisation in South Africa as it is in developing markets. Considering the high volatility of the SA market, due to political instability, the weakening rand and lately a series of downgrades, portfolio optimisation is highly important for investors to get good returns.

Several studies have assessed the efficiency of the MVO on the JSE employing individual securities, companies, mutual funds and ETFs. To the best knowledge of the researcher, a few studies have focused on portfolio optimisation based on tradable indices on the JSE. Du Plessis and Ward (2009) investigated the MVO based on individual stocks while Rodrigues (2010) focused on the SA property market. Brouwer (2015) focused on the JSE ETFs.

Oladele and Bradfield (2016) focused on the nine FTSE/JSE sectors to compare the low risk portfolio construction techniques including the MVO framework. Roopanand (2001) focused on JSE together with international indices, utilising the SA Industrial and Gold index. The Gold index was found to be mean variance-efficient while a portfolio emulating just the ALSI was found to be inefficient. This study is different from Roopanand (2001) as it focuses purely on SA indices to construct a portfolio, which is compared against the JSE ALSI.

This study confirms the findings of Brouwer (2015) on the performance of the Markowitz MVP on JSE. The difference of this study with Brouwer (2015) is that, the previous author focused on ETFs and the period of the study spanned only four years. On the other hand, the present study focused on the JSE tradable indices for a period of 10 years. As a result, it can be concluded that the MVP framework can be applied in any asset class successfully. In confirmation with the results of Brouwer (2015), the MVPs constructed in this study were observed to outperform the benchmark in terms of risk-return combination.

Contreras *et al* (2016) observed a consistency in the performance of the MVPs except for the year 2008 of their study. The study stretched for a 10-year period. Equity portfolios for retailers were utilised to assess the efficiency of the MVO framework. This present study documents findings that corroborate that of Contreras *et al* (2016) on the performance of the JSE. However, the MVPs were observed to outperform the JSE ALSI even during the GFC period. The difference of the two studies also lies in the fact that Contreras *et al* (2016) focused on equity retail portfolios while this present study utilised the JSE tradable indices. Despite the differences in securities employed, the same conclusion was reached about performance of the MVPs on the JSE after the GFC period.

4.7 Conclusion

This chapter discussed the analysis and the results obtained on constructing GMVPs using the JSE tradable sector. The optimum portfolios constructed were compared to the JSE ALSI. On average, the global minimum variance portfolios were observed to perform better than the JSE ALSI, even during the 2008-2009 financial crisis. The three indices employed for the GMVP construction signified the presence of diversification benefits despite the positive correlation among the indices. Prior studies using the same optimising framework were briefly outlined. The next chapter will present a summary of findings, conclusions and implications of the study.

CHAPTER 5

CONCLUSION

5.1 Introduction

In this concluding chapter, the author summarises the research findings and the contributions of the study. Principally, this research effort sought to establish whether the Markowitz portfolio optimisation framework could be applied by using the JSE tradable sector index. There were four central objectives underpinning this study (see section 1.6). The first objective was to establish whether the JSE tradable sector index represents an optimal portfolio. Secondly, the study sought to establish if global mean variance portfolios could be constructed using the JSE tradable sector index. Thirdly, this study examined if there were any diversification benefits associated with selecting a portfolio constructed from the JSE tradable index. Lastly, this study sought to establish whether global mean variance portfolios constructed using the JSE tradable sector indices could consistently outperform the returns of JSE ALSI index. The rest of the chapter is organised as follows: section 5.1 presents a summary of empirical findings, section 5.2 presents a summary of contributions of this study, and in section 5.3, the author discusses avenues for future research.

5.2 Summary of findings

An examination of portfolio optimisation was carried out on the JSE tradable sector indices. The Markowitz mean-variance optimisation framework was used as the base model for portfolio construction. The MVO model was applied to the tradable sector index, utilising the three main JSE tradable indices namely the $RESI_{25}$, $FINI_{15}$, and $INDI_{10}$ indices. Five portfolios were constructed using different underlying constraints. The JSE ALSI was used as a benchmark to assess the performance of the constructed portfolios.

The period of the study was divided into three parts, which is the pre-GFC, the GFC and the post-GFC periods. After constructing portfolios for each period, using different constraints and systems, it was observed that the MVPs performed better than the JSE ALSI, except during the pre-GFC period when the GMVPU trailed the benchmark. Two of the portfolios were constructed using Microsoft Excel while the other three were constructed using Stata. Both optimising software packages produced mutual the same results on the performance of the MVPs against the benchmark.

During the GFC period, the MVPs constructed outperformed the JSE ALSI in terms of minimising risk. The financial sector was severely affected by the crisis, hence the deficient performance of the FINI index which influenced the constructed GMVPs. The GMVPs were diversified among the three indices, which are in the same asset class, although they represent different JSE industries.

A positive correlation was noted among all the three indices which made it complex to reap maximum diversification benefits. However, the three indices were not perfectly positively correlated, hence pooling them into a portfolio prove to be beneficial in terms of their collective risk-return combinations against the benchmark.

The pre-GFC minimum variance portfolio (portfolios 1 and 4) yielded daily returns of 0.0698% with a standard deviation of 0.0088. For such a portfolio 100% of the capital had to be invested in the INDI index. No capital was to be invested in the FINI or RESI indices. It was therefore observed that the pre-GFC period was too short to construct a passive buy-and-hold portfolio based on the MVO framework. As a result, the pre-GFC GMVP was outperformed by the benchmark. The GMVP had an average annual return of 17.59% with a standard deviation of 0.14, while the JSE ALSI yielded a return of 25% with a standard deviation of 0.16.

During the market downswing period, the best mean-variance optimum portfolio was achieved by investing 0% into the RESI, 93% into the INDI and 7% of the budget into the FINI index. Such a portfolio could yield a mean of 0.0130% and a standard deviation of 0.0170. The average return of the JSE ALSI for the GFC period was 0.0152% with a standard deviation of 0.0183. In conclusion, the GMVP was observed to perform better than the benchmark in terms of the risk-return properties.

The optimum portfolio for the post-GFC period included the RESI into the basket. Investing 75% in INDI index, 4% in RESI index and 21% into the FINI index, optimised the post-GFC portfolio, yielding a daily average return of 0.0643% and a standard deviation of 0.0101. The performance of the GMVP was 4.6% higher than the benchmark in terms of return, with a standard deviation which was about 0.0007 higher than the JSE ALSI. As a result, it can be concluded that the GMVPs can produce better returns, with little or no effect on the risk of such a portfolio.

A t-test was run to determine whether the outperformance of the constructed portfolio was statistically significant. For all the periods the p-values were greater than 0.05, which means the GMVPs outperformed the benchmark with a statistically significant margin.

Including more asset classes such as bonds and commodities into the portfolio could have resulted in a different outcome during the GFC period. The indices used belong to the same asset; hence, they are positively correlated. Effective diversification can be achieved when different asset classes are used in a portfolio. A well-diversified portfolio can reduce risk, as a downswing in another asset class, for example in equities, can be offset by a rise in the bonds.

Annual or semi-annual rebalancing of the GMVPs could have resulted in an improved performance of the portfolios. Rebalancing portfolios have the effect of maintaining the initial optimum asset weights within the portfolio. However, the GMVPs constructed were observed to outperform the benchmark even without rebalancing the portfolio. In conclusion, the MVP was observed to be applicable on the JSE tradable indices.

5.3 Summary of contributions

This study employed the Markowitz MVO framework for optimisation of portfolios, using the JSE tradable sector indices. Prior studies have focused on different expanses of the JSE securities such as ETFs, mutual funds and even the property market. Roopanand (2001) investigates the effectiveness of the MV model, using JSE indices such as the Industrial and Gold index. However, the author included an international index, the Dow Jones. The present study is different in that it employs the tradable sector indices of the JSE, namely $RESI_{25}$, $FINI_{15}$, and $INDI_{10}$ indices as the focal point of the study, without any international assets so as purely to assess the performance of the local portfolios.

The study period stretched from January 2007 to December 2017, a period, which covered the market volatilities of the JSE. Analyses were carried out on data from before, during and after the GFC to investigate the performance of GMVPs. The different market phases indicated the efficiency of MVPs during different market trends comparatively. The study also observed that a global minimum variance portfolio constituting of the three main JSE tradable sector indices can outperform the benchmark consistently.

In corroboration with previous studies in developed and developing economies, the MVO have been noted to be instrumental and effective for portfolio optimisation on the JSE. This present study deviated from the previous studies on the JSE by focusing on the JSE tradable sector indices, which are a sound representation of the whole JSE. In other words, a comprehensive examination of the performance of the JSE constituents in a portfolio was done, considering that the tradable sector is made up of all the JSE securities categorised according to sectors into the indices. The present study also focused on the three defined different market volatility periods in South Africa namely before, during and after the global financial crisis period, to determine the consistency of mean variance portfolios against the chosen benchmark. The MVO model was examined and observed to be instrumental in efficiently optimising portfolios, signified by the GMVPs consistency in risk minimising better than the benchmark throughout the period. Investors and field practitioners can hence employ the model for portfolio construction since it is virtuous in risk minimising especially during market downswings.

5.4 Suggestions for future research

From the literature reviewed by the researcher, the MVO have been studied on developed and developing markets, inclusive of South Africa. In theory, the MVO framework proves to be an effective tool for asset selection and portfolio construction. The MVO model is still effective more than five decades after its introduction. The question remains whether the framework is practically being applied in the SA finance industry.

The pre-GFC period spanned from 1 January to 30 June 2007. Such a period was too short to construct a portfolio based on the passive buy and hold strategy. The study period could have stretched from an earlier date so that all the periods could be sizeable enough for meaningful analyses. As a result, there is a need to expand the pre-GFC period to assess the performance of the GMVPs against the benchmark.

Diversification is effective when different asset classes are included in portfolios. Including riskless assets such as bonds within a portfolio will have an effect of decreasing the total risk of a portfolio. The three tradable indices were purposively selected for the study. For future research, smaller indices and sub-indices on the JSE can be employed to assess their performance against the JSE ALSI.

The JSE is the biggest exchange in Africa. According to the 2017 database of the Financial Service Board registered financial service providers, there are more than 30 000 investment advisors and practitioners in South Africa. SA investment advisors daily create portfolios using different methods and techniques as part of their profession. Studies should be done on the practical use of the MVO model. The effectiveness of the MVO theory on the JSE should be tested in practice. There is a need to determine whether South African financial service providers are practically employing the MVO as a portfolio selection and construction tool. The study can be expanded to other African exchanges, both theoretically and practically as well

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Appendix A

More descriptive statistics of the variables under investigation

Pre GFC-period

Variable	FINI	INDI	RESI	ALSI
Minimum	-0.0311	-0.0235	-0.0350	-0.0251
Maximum	0.0222	0.0168	0.0289	0.0163
Mean	0.0017	0.0007	0.0016	0.0102
Standard deviation	0.0117	0.0087	0.0140	0.0099
Skewness	-0.268	-0.4724	- 0.6670	-0.7851
Kurtosis	3.1087	3.1119	3.2835	3.3741
Number of observations	123	123	123	123

GFC-period

Variable	FINI	INDI	RESI	ALSI
Minimum	-0.0499	-0.0382	-0.0822	-0.0461
Maximum	0.0553	0.0453	0.0829	0.0539
Mean	-0.0002	0.0002	0.0001	-0.00004
Standard deviation	0.0206	0.01687	0.02962	0.01960
Skewness	0.2676	0.2577	0.2392	0.0909
Kurtosis	3.9471	4.2337	5.0213	4.2025
Number of observations	542	542	542	542

Post GFC-period

Variable	FINI	INDI	RESI	ALSI
Minimum	-0.0289	-0.0264	-0.0392	-0.0267
Maximum	0.0294	0.0260	0.0427	0.0240
Mean	0.0051	0.0007	0.0000	0.0005
Standard deviation	0.0116	0.0099	0.0158	0.0095
Skewness	-0.2370	0.1250	0.0187	0.1460
Kurtosis	6.4870	4.6586	4.3150	4.3320
Number of observations	2088	2088	2088	2088

Appendix B

Turnitin Report

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DECLARATION

I hereby certify that the thesis by **SALLY HUNI** was properly language edited but without viewing the final version.

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