UNIVERSITI PUTRA MALAYSIA

LINEAR QUADRATIC REGULATOR WITH GENETIC ALGORITHM FOR FLEXIBLE STRUCTURES VIBRATION CONTROL

MOHAMMAD JAFARI

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By

MOHAMMAD JAFARI

Thesis Submitted to the School of Graduate Studies,
Universiti Putra Malaysia, in Fulfilment of the
Requirements for the Degree of Master of Science

January 2015

Dedicated to

My parents and My sister

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

LINEAR QUADRATIC REGULATOR WITH GENETIC ALGORITHM FOR FLEXIBLE STRUCTURES VIBRATION CONTROL

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Chairman: Professor Harijono Djojodihardjo, ScD, IPU

Faculty: Engineering

There has been tremendous growth in the study of active vibration suppression of flexible structures in aerospace and robotics applications. The mathematical modeling of flexible structure is usually complex; for the control of flexible structure one needs to design an algorithm which is related to mathematical model. This thesis is addressed to the comprehensive analysis for generic structure control that has not been adequately dealt with in the literature. The comprehensive account is given by way of a generic example with the solution of LQR problem using genetic algorithm for this structure. First, the dynamic analysis of a cantilever beam hinged with linear spring at the tip is studied analytically and numerically as a baseline. Then, a cantilever beam bonded with piezoelectric sensor and actuator is considered for the study of the vibration control. For this purpose, a flexible Euler–Bernoulli beam is analyzed using Hamiltonian mechanics. The free vibration problems of the beam structures are solved using analytical and finite element method. The analytical method can only be used for certain class of geometries particularly simple ones and the finite element method can be applied for more general cases. In addition the analytical method can be used for validation purposes. The first three major natural modes and frequencies for all these two methods have been verified by present study. In addition the results of these studies are compared to available and acceptable data for validation and assessment.

For dynamic problem, the state-space approach can be used to design the effective controller for convenience, accuracy and computational efficiency. In the workout examples, the first two modes are used to control. The effective vibration control is designed by resorting to two methods one is PID and the other is LQR. The PID, which is the most direct method, will be used as a reference in finding the better methods. The LQR is then utilized to obtain the better or eventually the best solution. The LQR is formulated by full–order state observer. These methods are elaborated and it was found that satisfactory answer can be obtained by using two modes of the beam. The work has demonstrated the effectiveness of LQR method judged from computation time and accuracy. In addition, to improve the LQR trial and error procedure, genetic algorithm

has been used to obtain the LQR weighting matrices. The method has been elaborated and the results obtained show better improvement than earlier trial and error method.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
Sebagai memenuhi keperluan untuk ijazah Master Sains

PENGATUR LINEAR KUADRATIK DENGAN ALGORITMA GENETIK UNTUK STRUKTUR FLEKSIBEL KAWALAN GETARAN

Oleh

MOHAMMAD JAFARI

Januari 2015

Pengerusi: Professor Harijono Djojodihardjo, ScD, IPU

Fakulti: Kejuruteraan

Terdapat pertumbuhan yang besar dalam kajian penindasan getaran aktif struktur fleksibel dalam aeroangkasa dan aplikasi robotik. Pemodelan matematik struktur fleksibel biasanya kompleks; bagi mengawal struktur fleksibel salah satu keperluan untuk mereka bentuk satu algoritma yang berkaitan dengan model matematik. Tesis ini ditujukan kepada analisis yang komprehensif untuk mengawal struktur generik yang belum ditangani dengan secukupnya dalam kesusasteraan. Akaun komprehensif diberikan melalui satu contoh generik dengan penyelesaian masalah LQR menggunakan algoritma genetik untuk struktur ini. Pertama, analisis dinamik rasuk julur berengsel dengan musim bunga linear di hujung dikaji secara analisis dan berangka sebagai garis asas. Kemudian, rasuk julur terikat dengan sensor piezoelektrik dan penggerak dianggap untuk kajian kawalan getaran. Untuk tujuan ini, yang fleksibel Euler-Bernoulli rasuk yang telah dijalankan dianalisis menggunakan mekanik Hamiltonian. Masalah getaran bebas daripada struktur rasuk diselesaikan dengan menggunakan kaedah analisis dan unsur terhingga. Kaedah analisis hanya boleh digunakan untuk kelas tertentu terutamanya geometri yang mudah dan kaedah unsur terhingga boleh digunakan untuk kes-kes yang lebih umum. Selain kaedah analisis boleh digunakan untuk tujuan pengesahan. Yang pertama tiga mod alam besar dan frekuensi untuk semua kedua-dua kaedah telah disahkan oleh kajian ini. Selain itu hasil kajian ini berbanding dengan yang ada dan diterima data untuk pengesahan dan penilaian.

Untuk masalah dinamik, pendekatan negeri-ruang boleh digunakan untuk mereka bentuk pengawal yang berkesan untuk kemudahan, ketepatan dan kecekapan pengkomputeran. Dalam contoh-contoh senaman, dua mod yang pertama digunakan untuk mengawal. Kawalan getaran berkesan direka dengan melakukan dua cara seseorang itu PID dan satu lagi adalah LQR. PID, yang merupakan kaedah yang paling langsung, akan digunakan sebagai rujukan dalam mencari kaedah yang lebih baik. LQR ini kemudiannya digunakan untuk mendapatkan yang lebih baik atau akhirnya penyelesaian terbaik. LQR ini dirumuskan secara sepenuh perintah negeri pemerhati. Kaedah-kaedah ini dihuraikan dan didapati bahawa jawapan yang memuaskan boleh diperolehi dengan menggunakan dua mod rasuk. Kerja-kerja ini telah menunjukkan keberkesanan kaedah LQR dinilai dari semasa pengiraan dan ketepatan. Di samping itu, untuk meningkatkan percubaan dan kesilapan prosedur LQR, algoritma genetik telah digunakan untuk mendapatkan matriks LQR pemberat. Kaedah ini telah dihuraikan secara terperinci dan keputusan yang diperolehi menunjukkan peningkatan lebih baik daripada percubaan awal dan kaedah kesilapan.

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I would like to give my heartfelt thanks to my parents and my sister for their sincere support and encouragement. My family deserves more thanks than I can ever possibly express.

I certify that a Thesis Examination Committee has met on January 27, 2015 to conduct the final examination of Mohammad Jafari on his thesis entitled “Linear Quadratic Regulator With Genetic Algorithm For Flexible Structures Vibration Control” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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LIST OF NOMENCLATURE

Latin Symbols	**Nomenclature**
A	Area of the cross section
b	Width
$[c]$	Damping Matrix
d_{pi}	piezoelectric strain constant
E	Elasticity Modulus
E_T	Total Energy
E^e	Electrical Feild
f	Force per unit of Length
g_{31}	Piezoelectric Voltage Constant
h	Thickness
I	Moment of inertia
J	LQR performance index
K	Global stiffness matrix
$[k]$	Stiffness matrix
l	Length
k_{sp}	Stiffness of spring
K_P, K_I, K_D	PID Gains
K_{lqr}	LQR Feedback Gain
M	Global Mass matrix
$[m]$	Mass matrix
P	Force
$\mathbf{Q}$ and $\mathbf{R}$	LQR weighting Matrices
T	Kinetic Energy
u	Displacement in x–axis
V	Voltage
v	Displacement in y–axis
W_{nc}	Non-Conservative Work
W	Displacement in z–axis

Greek Symbols	**Nomenclature**
α and β	Proportional Rayleigh Coefficients
ε_{ij}	Strain Component
ξ	Damping Coefficients
ξ_{pi}	Dielectric permittivity
Π	Potential or Strain Energy
ρ	Density
σ_{ij}	Stress Component
ω	Natural Frequency

LIST OF ABBREVIATIONS

ac	Actuator
bm	Beam
Eq.	Equation
elm	Element
FEM	Finite Element Method
GA	Genetic Algorithm
LQR	Linear Quadratic Regulator
PID	Proportional Integral Derivative
PVDF	Polyvinylidene Fluoride
PZT	Lead Zirconate Titanate
sn	Sensor

CHAPTER 1

INTRODUCTION

1.1 Research Background and Motivation

As the area of technology grows, products of engineering industry are going into smaller size, lighter weight, and more affordable cost (Meirovitch, 1990). In order to have more compact and inexpensive structures the outcome may be a more light and flexible structures. One of the important issues in flexible structures is to suppress the effect of the vibration. The vibration problem in structures may be generated by external disturbances or internal uncertainties such as frictions (Alkhatib and Golnaraghi, 2003). These matters can be observed in the area of mechanics such as spacecraft structures and robotic systems. Spacecraft and satellite structures often undergo vibrations and disturbances from the physical environment. Robotic systems contain flexible links with variations of loading, which causes the vibration problem.

Vibration control of light-weight structures is of great interest of many studies and investigations. The high cost of sending heavy masses and large volumes into space has prompted the wide utilization of light-weight structures in space applications, such as antennas, robot’s arms, solar panels. These kinds of structures are largely flexible, which results in lightly damped vibration, instability and fatigue. Two major approaches have been reported in the literatures for vibration control: passive and active (Korkmaz, 2011). In the passive method the damping of the structure is increased by using passive dampers or materials with significant viscoelasticity. This method can increase the total weight considerably and is best for high frequency modes. An active method integrates sensors and actuators with the flexible structure, operated by a control scheme. To suppress the adverse effect of vibration, sophisticated controller is required.

Active control approaches are widely reported in the literatures for the vibration control of structures (Alkhatib and Golnaraghi, 2003; Korkmaz, 2011). The active control approach makes use of actuators and sensors to find out some essential variables of the structure and suppress its vibration through minimizing the settling time and the maximum amplitude of the undesirable oscillation. This method requires a specific level of understanding about the dynamic behavior of continuous structures, such as beam or plate structure, via mathematical modelling. Selecting adequate sensor and actuator is an important issue in active vibration control. The conventional form of sensor and actuator, such as electro-hydraulic or electro-magnetic actuator, are not applicable to implement on the light-weight space structures. Thus, in recent years, a new form of sensor and actuator has been studied using smart materials, such as shape memory alloys and piezoelectric materials (Fuller *et al.*, 1996). The definition of smart material may be expressed as a material which adapts itself in response to environmental changes. Among smart materials, piezoelectric materials are widely studied in literatures, since they have many advantageous such as adequate accuracy in sensing and actuating, applicable in the wide frequency range of operations, applicable in distributed or discrete manner and available in different size, shape and arrangement (Moheimani, 2003).

The present work attempts to fill the gap in the literature through the formulation of a generic problem. It has taken into consideration the progress and state of the arts as elaborated in the literatures as well as examples elaborated by other researchers; the comprehensive solution method is considered to be unique.

1.2 Problem Statement

The vibration problem in light-weight flexible structures is one of the important issues in engineering design. These structures are very sensitive to their physical environment due to flexibility. This issue can generate fatigue or resonance in a structure, which is undesirable and cause of failure. Especially in the space structures, the need of saving mass at launch along with the necessity of large surfaces, for instance telecommunication antennas or solar arrays, can lead to highly flexible structures. This characteristic causes a serious challenge when a control must be applied in order to reorient the platform, or to compensate the effects of orbital disturbance. Indeed, flexibility brings undesirable oscillations that may cause resonance conditions when they interact with control actions (Gasbarri *et al.*, 2014). Hence, engineers try to avoid the vibration using control methods. In order to control the vibration, a dynamic modeling of the structure is required, where can be obtained using physical and mathematical laws and theories. The modeling of the structure can be sophisticated problem due to their geometry and application. A controller is designed based on the dynamic model of the structure. However, the development of a comprehensive analysis for control still needs further workout and proven techniques as a specific approach. Hence, Linear-Quadratic-Regulator controller with genetic algorithm for flexible structures vibration control is here elaborated as one of those techniques hence as a novel approaches.

Space and robot structures, as shown in Figure 1.1, can be simplified mostly in the form of beam and plate (Narayanan and Balamurugan, 2003). In this investigation, only beam theory is considered. From the fundamental beam theory Euler and Bernoulli developed one of the most practical and straightforward theories; however, as beam theory progresses, more sophisticated and accurate theories are developed like Rayleigh and Timoshenko beam theories (Rao, 2007). Euler-Bernoulli theory is applicable to long and thin span, for which plane sections can be assumed to remain plane and perpendicular to the beam axis, and shear stress and rotational inertia of the cross section can be neglected. Solar panel and antenna are very flexible and slender, so that Euler-Bernoulli beam theory can be considered. The equation of motion of a beam can be acquired using Newtonian mechanics, or analytical mechanics approaches such as Hamilton’s method and Lagrange method (Baruh, 1999). Hamiltonian mechanics is an elegant and convenient approach, since scalar equation of motion of the beam and boundary conditions can be obtained simultaneously.

Three investigations are considered in the present work. First one is the modal analysis of a beam structure hinged with a linear spring at the tip. The first study is developed in the second one which is the active vibration control of a flexible beam bonded with piezoelectric sensor and actuator using PID and LQR with observer controllers. The third study is improvement the LQR controller which discussed in the previous study. In third study, genetic algorithm is utilized to optimize the weighting matrices of LQR controller. These problems are organized in three chapters.

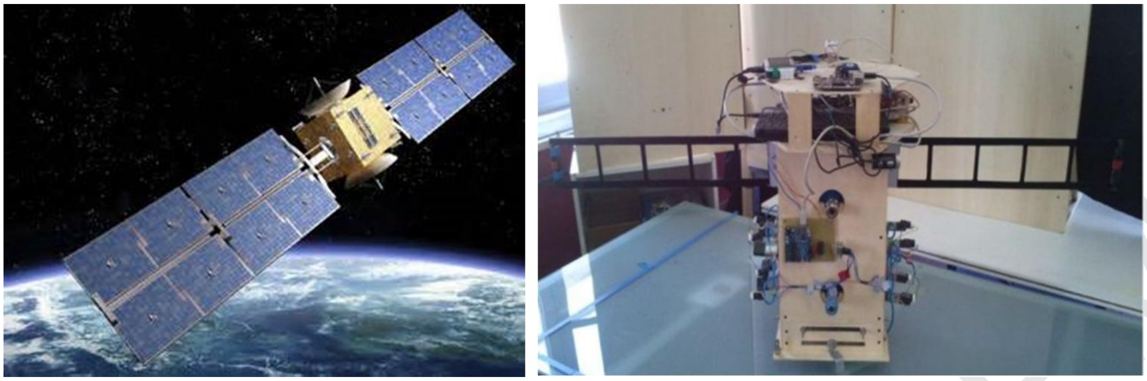

Figure 1.1. Solar Panel on a typical satellite (left) and an experimental model of Free-Floating Platform with two flexible appendages designed by Gasbarri *et al.* (2014) (right)

The first study considered is translated to a generic problem in the sense of its formulation, solution method and its potential for generating more complex one using the principle of superposition. Somewhat similar problem is reported in the literatures. This will be elaborated in the particular section in this work. However, these problems have not being elaborated in sufficient detail in the methods as well as solution approaches in the literature. The modal analysis of a cantilever beam with a linear spring hinged at the tip is investigated as generic problem. For this purpose Newtonian and Hamiltonian approach are utilized to give better insight and deliver workable procedure and example. Then, the natural frequencies and natural modes are acquired through analytical and numerical approaches.

The first study is extended to a smart beam in order to investigate the problem of vibration control of structure subjected to certain disturbances. The aim is to suppress the vibration of a flexible beam structure using piezoelectric material sensor and actuator through an efficient and straightforward controller. Several ways to control the vibration of flexible structures are reported in literatures (Alkhatib and Golnaraghi, 2003). One of the adequate and simple controllers is Proportional-Integral-Derivative (PID) controller, which is classified as classical control algorithm. PID controller minimizes the steady state error of the system (Ogata, 2010). Linear Quadratic Regulator (LQR) controller is another convenient method. LQR is expressed as optimal and modern controller, which is based on minimizing the performance index of a dynamic system (Lewis *et al.*, 2012). To develop a successful operation, it is hypothesized that most controllers have been developed for a finite number of natural modes where the controllability and observability conditions are met. Three different piezoelectric configurations are considered on a flexible beam for comparative study.

The weighting matrices of LQR control algorithm, Q and R, are usually determined with trial and error or experiment. However, a new approach is introduced to find the best weighting matrices, Q and R, subjected to the control performance using Genetic Algorithm optimization and search method. A simple piezoelectric configuration on the beam is considered in this study.

1.3 Research Objectives

The main goals of this thesis are to analyze the vibration of flexible beam structures and design an effective and simple controller to suppress the vibration of the smart beam. Hence, this thesis is separated into three main parts. First, the vibration of a cantilever beam with spring loading is analyzed as base line. Then this study is extended to a beam patched with piezoelectric layer as a smart structure in order to design and apply the control algorithm. Finally, an optimization method is utilized to find the best control performance for the Linear-Quadratic-Regulator control designs method. The major objectives of the thesis are listed as follows:

- To determine the basic vibrational characteristics (the natural frequencies and natural modes) of the controlled structure. For this purpose analytical (eg. methods of separation of variables) and finite element method are utilized. The later can be carried out through the use of Galerkin method in the weak formulation of the problem.

- To carry out optimization scheme for the preferred method (LQR), using heuristic and Genetic Algorithm methods.

- To assess the robustness of the control strategy carried out using the outlined approach for the generic structural system chosen and more involved ones.

1.4 Methodology

Following methods are used in order to achieve the objectives:

- To derive the characteristic equation of Beam with the translational spring hinged at tip as a baseline by using Hamilton's Principle and Newton's law

- To derive the beam patched with piezoelectric as a controllable structure by using Hamiltonian' Principle

- To solve the equation of motion of the system using analytical method and Finite Element method and validate the result with available theoretical and experimental studies

- To solve the Time response of the system using State-Space approach and use this method to design the controllers

- To utilize State-Space approach to design two controllers (Full-order state observer LQR and PID), so as to suppress the vibration of the beam based on state-space approach

- To construct the objective function based on full-order state observer LQR controller

- To find the best weighting matrices of LQR using genetic algorithm search and optimization method and compare it with heuristic method

1.5 Research Scope and Limitation

In the present work, the free vibration analysis of beam structures is investigated by assuming the beam can be modeled following the Euler-Bernoulli beam. This theory is desirable and acceptable for long and thin structures. A steel beam is used as a case study for the vibration analysis of cantilever beam hinged with transitional spring at the tip. An aluminum beam bonded with piezoelectric material is considered as case study for the vibration control of a light beam structure. Two common piezoelectric materials, PZT and PVDF, are used as both sensor and actuator. All beams are assumed to have uniform elastic modulus and cross section along the entire structure. Cantilever beams are considered in this dissertation since most of mechanical applications are clamped on one side. In the control part, first two major natural mode of the beam is hypothesized and utilized in design of controllers because other higher natural modes are insignificant amplitude in compare with first two modes in the beam structure. Proportional-Integral-Derivative control method is utilized as a baseline. Through many control algorithms, two straightforward controllers, PID and LQR with full-state observer, are utilized for controlling the vibration of the beam. Linear-quadratic-regulator is utilized as a primary feedback controller and Proportional-Integral-Derivative a secondary one. Full states are required for LQR feedback gain to determine the input of the system. For controlling the beam vibration by LQR control approach, all states variables for LQR feedback gain are not available to determine the input of the system. Since the system has just one output, which is the sensor voltage, an observer is required as an estimator to estimate all states variables. In order to obtain LQR weighting matrices, a systematic approach is utilized with heuristic and optimization method based on minimizing the settling time of the controller response. Using PID as a secondary control approach, the PID gains (K_P, K_I, and K_D) are determined with heuristic method for convenience. It should be noted, since the focus is on the complete and coherent LQR technique, that we are addressing, the similar elaboration on PID gains can be readily done but it is beyond the scope of the present thesis.

1.6 Organization of Thesis

A literature review including flexible structure dynamic, smart materials and vibration control is presented in Chapter Two. In Chapter Three, The dynamic analysis of a beam hinged with spring based on Euler-Bernoulli theory is described and free vibration solution is analyzed. A brief review of piezoelectric material and acquiring the governing equation of piezo-sensor and piezo-actuator are described. Then, the general equation of motion of a beam with piezoelectric layer is derived based on Euler-Bernoulli beam theory. The state-space representation of the beam is described in order to design the controller. PID and LQR with observer controllers are designed based on first two mode of the system. A closed loop equation of full-order state observer LQR controller is derived in order to use as objective function in the Genetic Algorithm optimization method. A brief review of Genetic Algorithm is also described. In Chapter Four, the numerical result of modal analysis, vibration control and optimization of LQR control are discussed using different case studies. The conclusion and future work are represented in Chapter Five. Deriving the equation of motion of a beam using Newton's second law is described in Appendix A. The Finite element method of the beam is comprehensively presented in Appendix B. The proofs of controllability, observability and *Riccati* equation are elaborated in Appendix C. The in-house finite element and genetic algorithm MATLAB® codes are given in Appendix D and Appendix E, respectively. Design of the controllers using SIMULINK® is demonstrated in Appendix F.

BIBLIOGRAPHY

Alam, M.S., & Tokhi, M. (2008). Hybrid fuzzy logic control with genetic optimisation for a single-link flexible manipulator. *Engineering Applications of Artificial Intelligence, 21*(6), 858-873.

Alkhatib, R., & Golnaraghi, M. (2003). Active structural vibration control: a review. *Shock and Vibration Digest, 35*(5), 367.

Ang, K., Wang, S., & Quek, S. (2002). Weighted energy linear quadratic regulator vibration control of piezoelectric composite plates. *Smart materials and structures, 11*(1), 98.

Antoulas, A.C. (2005). *Approximation of large-scale dynamical systems* (Vol. 6): Siam.

Azadi, M., Fazelzadeh, S., Eghtesad, M., & Azadi, E. (2011). Vibration suppression and adaptive-robust control of a smart flexible satellite with three axes maneuvering. *Acta Astronautica, 69*(5), 307-322.

Bailey, T., & Hubbard, J. (1985). Distributed piezoelectric-polymer active vibration control of a cantilever beam. *Journal of Guidance, Control, and Dynamics, 8*(5), 605-611.

Baruh, H. (1999). *Analytical dynamics*: WCB/McGraw-Hill Boston.

Basher, H.A. (2007). *Modeling and simulation of flexible robot manipulator with a prismatic joint.* Paper presented at the SoutheastCon, 2007. Proceedings. IEEE.

Beer, F., Johnston, E., & DeWolf, J. (2002). Mechanics of Materials, 2002: McGraw-Hill, New York.

Berthelot, J.-M. (1999). *Composite materials: mechanical behavior and structural analysis*: Springer.

Bhandari, D., Murthy, C., & Pal, S.K. (2012). Variance as a Stopping Criterion for Genetic Algorithms with Elitist Model. *Fundamenta Informaticae, 120*(2), 145-164.

Braghin, F., Cinquemani, S., & Resta, F. (2011). A model of magnetostrictive actuators for active vibration control. *Sensors and Actuators A: Physical, 165*(2), 342-350.

Chopra, A.K. (1995). *Dynamics of structures* (Vol. 3): Prentice Hall New Jersey.

Crawley, E.F., & De Luis, J. (1987). Use of piezoelectric actuators as elements of intelligent structures. *AIAA journal, 25*(10), 1373-1385.

Damjanovic, D. (1998). Ferroelectric, dielectric and piezoelectric properties of ferroelectric thin films and ceramics. *Reports on Progress in Physics, 61*(9), 1267.

Davies, A.J. (1980). The finite element method.

Davino, D., Giustiniani, A., & Visone, C. (2012). Smart magneto-elastic materials for passive damping applications. *International Journal of Applied Electromagnetics and Mechanics, 39*(1), 629-635.

Detwiler, D., Shen, M.-H., & Venkayya, V. (1995). Finite element analysis of laminated composite structures containing distributed piezoelectric actuators and sensors. *Finite Elements in Analysis and Design*, *20*(2), 87-100.

Djojodihardjo, H. (2013). Computational simulation for analysis and synthesis of impact resilient structure. *Acta Astronautica, 91*, 283-301.

Djojodihardjo, H. (2013). *Vibro-acoustic analysis of random vibration response of a flexible structure due to acoustic forcing*. Paper presented at the Space Technology & Systems Development, Beijing, China.

Eisenberger, M., & Abramovich, H. (1997). Shape control of non-symmetric piezolaminated composite beams. *Composite structures, 38*(1), 565-571.

Fei, J. (2005). *Active vibration control of flexible steel cantilever beam using piezoelectric actuators.* Paper presented at the System Theory, 2005. SSST'05. Proceedings of the Thirty-Seventh Southeastern Symposium on.

Fripp, M.L., & Hagood, N.W. (1995). *Comparsion of electrostrictive and piezoceramic actuation for vibration suppression.* Paper presented at the Smart Structures & Materials' 95.

Fuller, C.C., Elliott, S., & Nelson, P.A. (1996). *Active control of vibration*: Academic Press.

Gasbarri, P., Sabatini, M., Leonangeli, N., & Palmerini, G.B. (2014). Flexibility issues in discrete on–off actuated spacecraft: Numerical and experimental tests. *Acta Astronautica, 101*, 81-97.

Holland, J.H. (1992). Genetic algorithms. *Scientific american, 267*(1), 66-72.

Hong, S.Y. (1992). *Active vibration control of adaptive flexible structures using piezoelectric smart sensors and actuators.* (Doctoral dissertation), The Pennsylvania State University, PA 16801, United States.

Hong, Y.K., Park, H.-K., Lee, S.Q., Moon, K.S., Vanga, R.R., & Levy, M. (2004). *Design and performance of a self-sensing, self-actuating piezoelectric monomorph with interdigitated electrodes.* Paper presented at the Optics East.

Hutton, D.V. (2003). *Fundamentals of finite element analysis*: McGraw-Hill Science/Engineering/Math.

Ikuta, K., Aritomi, S., & Kabashima, T. (1992). *Tiny silent linear cybernetic actuator driven by piezoelectric device with electromagnetic clamp.* Paper presented at the Micro Electro Mechanical Systems, 1992, MEMS'92, Proceedings. An Investigation of Micro Structures, Sensors, Actuators, Machines and Robot. IEEE.

Jalili, N. (2010). Piezoelectric-based vibration control. *From Macro to Micro/Nano Scale Systems Springer*.

Jamula, M.T. (2012). *Passive/active vibration control of flexible structures.* (Master of Science), Northeastern University, Boston, MA, US.

Jaworski, J., & Dowell, E. (2008). Free vibration of a cantilevered beam with multiple steps: Comparison of several theoretical methods with experiment. *Journal of sound and vibration, 312*(4), 713-725.

Jovanović, M.M., Simonović, A.M., Zorić, N.D., Lukić, N.S., Stupar, S.N., & Ilić, S.S. (2013). Experimental studies on active vibration control of a smart composite beam using a PID controller. *Smart Materials and Structures, 22*(11), 115038.

Juang, J.-N. (1984). Optimal design of a passive vibration absorber for a truss beam. *Journal of Guidance, Control, and Dynamics, 7*(6), 733-739.

Khot, S., Yelve, N.P., Tomar, R., Desai, S., & Vittal, S. (2012). Active vibration control of cantilever beam by using PID based output feedback controller. *Journal of Vibration and Control, 18*(3), 366-372.

Korkmaz, S. (2011). A review of active structural control: challenges for engineering informatics. *Computers & Structures, 89*(23), 2113-2132.

Kreyszig, E. (2007). *Advanced engineering mathematics*: John Wiley & Sons.

Kusculuoglu, Z., Fallahi, B., & Royston, T. (2004). Finite element model of a beam with a piezoceramic patch actuator. *Journal of Sound and Vibration, 276*(1), 27-44.

Lallart, M., Cottinet, P.J., Guyomar, D., & Lebrun, L. (2012). Electrostrictive polymers for mechanical energy harvesting. *Journal of Polymer Science Part B: Polymer Physics, 50*(8), 523-535.

Le, S. (2009). *Active vibration control of a flexible beam.* (Master of Science), San Jose State University, San Jose, CA, US.

Lewis, F.L., Vrabie, D., & Syrmos, V.L. (2012). *Optimal control*: John Wiley & Sons.

Li, W.L. (2000). Free vibrations of beams with general boundary conditions. *Journal of Sound and Vibration, 237*(4), 709-725.

Lin, J., & Zheng, Y. (2012). Vibration suppression control of smart piezoelectric rotating truss structure by parallel neuro-fuzzy control with genetic algorithm tuning. *Journal of Sound and Vibration, 331*(16), 3677-3694.

Ma, K., & Ghasemi-Nejhad, M.N. (2005). Adaptive simultaneous precision positioning and vibration control of intelligent composite structures. *Journal of intelligent material systems and structures, 16*(2), 163-174.

Mason, W. (1950). *Piezoelectric Crystals and Their Applications to Ultrasonic*. New York: D. Van Nostrand Co.

Maurizi, M., Rossi, R., & Reyes, J. (1976). Vibration frequencies for a uniform beam with one end spring-hinged and subjected to a translational restraint at the other end. *Journal of Sound and Vibration, 48*(4), 565-568.

Meirovitch, L. (1980). *Computational methods in structural dynamics*: Springer.

Meirovitch, L. (1990). *Dynamics and control of structures*: John Wiley & Sons.

Meirovitch, L. (1997). *Principles and techniques of vibrations* (Vol. 1): Prentice Hall New Jersey.

Meitzler, A., Tiersten, H., Warner, A., Berlincourt, D., Couqin, G., & Welsh III, F. (1988). IEEE standard on piezoelectricity: Society.

Michalewicz, Z. (1996). *Genetic algorithms+ data structures= evolution programs*: springer.

Moheimani, S.R. (2003). A survey of recent innovations in vibration damping and control using shunted piezoelectric transducers. *Control Systems Technology, IEEE Transactions on, 11*(4), 482-494.

Narayanan, S., & Balamurugan, V. (2003). Finite element modelling of piezolaminated smart structures for active vibration control with distributed sensors and actuators. *Journal of sound and vibration, 262*(3), 529-562.

Newnham, R.E., & Ruschau, G.R. (1991). Smart electroceramics. *Journal of the American Ceramic Society, 74*(3), 463-480.

Nguyen, P., Sohn, J., & Choi, S. (2010). Position tracking control of a flexible beam using a piezoceramic actuator with a hysteretic compensator. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 224*(10), 2141-2153.

Obinata, G., & Anderson, B. (2001). *Model reduction for control system design*: Springer-Verlag New York, Inc.

Ogata, K. (2010). *Modern Control Engineering* (5th ed.). New Jersey, US: Pearson.

Otsuka, K., & Wayman, C.M. (1999). *Shape memory materials*: Cambridge university press.

Paget, C., Levin, K., & Delebarre, C. (2002). Actuation performance of embedded piezoceramic transducer in mechanically loaded composites. *Smart Materials and Structures, 11*(6), 886.

Park, C. (2003). Dynamics modelling of beams with shunted piezoelectric elements. *Journal of Sound and Vibration, 268*(1), 115-129.

Prasad, S.E., Waechter, D.F., Blacow, R.G., King, H.W., & Yaman, Y. (2005). *Application of piezoelectrics to smart structures.* Paper presented at the II ECCOMAS Thematic Conf. on Smart Structures and Materials, Lisbon, Portugal.

Purshouse, R.C., & Fleming, P.J. (2003). *Conflict, harmony, and independence: Relationships in evolutionary multi-criterion optimisation.* Paper presented at the Evolutionary multi-criterion optimization.

Qian, W., Liu, G., Chun, L., & Lam, K. (2003). Active vibration control of composite laminated cylindrical shells via surface-bonded magnetostrictive layers. *Smart materials and structures, 12*(6), 889.

Qiu, Z.-c., Shi, M.-l., Wang, B., & Xie, Z.-w. (2012). Genetic algorithm based active vibration control for a moving flexible smart beam driven by a pneumatic rod cylinder. *Journal of Sound and Vibration, 331*(10), 2233-2256.

Rao, S.S. (2007). *Vibration of continuous systems*: John Wiley & Sons.

Reeves, C.R., & Rowe, J.E. (2003). *Genetic algorithms: principles and perspectives: a guide to GA theory* (Vol. 20): Springer Science & Business Media.

Rogers, C.A. (1993). Intelligent material systems-the dawn of a new materials age. *Journal o f Intelligent Material System and Structures, Vol. 4.*, 4-12.

Rossit, C., & Laura, P. (2001). Free vibrations of a cantilever beam with a spring–mass system attached to the free end. *Ocean Engineering, 28*(7), 933-939.

Russ, R., Ma, K., & Ghasemi-Nejhad, M.N. (2006). *A finite element analysis approach with integrated PID control for simultaneous precision positioning and vibration suppression of smart structures.* Paper presented at the Smart Structures and Materials.

Saad, M.S., Jamaluddin, H., & Darus, I.Z.M. (2013). Active vibration control of a flexible beam using system identification and controller tuning by evolutionary algorithm. *Journal of Vibration and Control*, 1077546313505635.

Singh, M.P., & Moreschi, L.M. (2002). Optimal placement of dampers for passive response control. *Earthquake engineering & structural dynamics, 31*(4), 955-976.

Spier, C., Bruch, J., Sloss, J., Adali, S., & Sadek, I. (2010). Analytic and finite element solutions for active displacement feedback control using PZT patches. *Journal of Vibration and Control, 16*(3), 323-342.

Stavroulakis, G.E., Foutsitzi, G., Hadjigeorgiou, E., Marinova, D., & Baniotopoulos, C. (2005). Design and robust optimal control of smart beams with application on vibrations suppression. *Advances in Engineering Software, 36*(11), 806-813.

Vasques, C., & Dias Rodrigues, J. (2006). Active vibration control of smart piezoelectric beams: Comparison of classical and optimal feedback control strategies. *Computers & structures, 84*(22), 1402-1414.

Wang, S., Quek, S., & Ang, K. (2001). Vibration control of smart piezoelectric composite plates. *Smart materials and Structures, 10*(4), 637.

Wang, Z.L., & Kang, Z.C. (1998). *Functional and smart materials: structural evolution and structure analysis*. New York, US: Plenum Press.

Williams, J., Phillips, J., & Vaughn, G. (1999). Ultracompact LCD backlight inverters: A svelte beast cuts high voltage down to size. *Linear Technology Corp., Milpitas, Calif., Application Note, 81*.

Xu, S., & Koko, T. (2004). Finite element analysis and design of actively controlled piezoelectric smart structures. *Finite elements in analysis and design, 40*(3), 241-262.

Yalcintas, M., & Dai, H. (2004). Vibration suppression capabilities of magnetorheological materials based adaptive structures. *Smart Materials and Structures, 13*(1), 1.

Yan, Y., & Yam, L. (2002). Online detection of crack damage in composite plates using embedded piezoelectric actuators/sensors and wavelet analysis. *Composite Structures, 58*(1), 29-38.

Yousefi-Koma, A. (1997). *Active vibration control of smart structures using piezoelements.* (Doctoral dissertation), Carleton University, Ottawa, Canada.

Zabihollah, A., Sedagahti, R., & Ganesan, R. (2007). Active vibration suppression of smart laminated beams using layerwise theory and an optimal control strategy. *Smart Materials and structures, 16*(6), 2190.

Zhong, J. (2006). PID controller tuning: A short tutorial. *class lesson), Purdue University*.

Zhou, X. (2002). *Modeling of piezoceramics and piezoelectric laminates addressing complete coupling and hysteresis behavior*. (PhD Dissertation), Arizona State University, Arizona, US.

Zorić, N.D., Simonović, A.M., Mitrović, Z.S., & Stupar, S.N. (2012). Optimal vibration control of smart composite beams with optimal size and location of piezoelectric sensing and actuation. *Journal of Intelligent Material Systems and Structures*, 1045389X12463465.