Personalization of Future Urban Mobility

by

Xiang Song

M.S. Management Research, Massachusetts Institute of Technology, 2016

M.S. Transportation, Massachusetts Institute of Technology, 2013

B. Eng. Civil Engineering, Zhejiang University, 2011

Submitted to the Department of Civil and Environmental Engineering

in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Transportation

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2018

Signature redacted

Signature of Author ....

Department of Civil and Environmental Engineering

August 17, 2018

Certified by .... Signature redacted ....

Moshe E. Ben-Akiva

Edmund K. Turner Professor of Civil and Environmental Engineering

Thesis Supervisor

Certified by .... Signature redacted ....

Bilge Atasoy

Assistant Professor of Maritime Transport and Technology

Delft University of Technology

Thesis Supervisor

Signature redacted

Accepted by ....

Heidi Nepf

Donald and Martha Harleman Professor of Civil and Environmental Engineering

Chair, Graduate Program Committee

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Abstract

In the past few years, we have been experiencing rapid growth of new mobility solutions fueled by a myriad of innovations in technologies such as automated vehicles and in business models such as shared-ride services. The emerging mobility solutions are often required to be profitable, sustainable, and efficient while serving heterogeneous needs of mobility consumers. Given high-resolution consumer mobility behavior collected from smartphones and other GPS-enabled devices, the operational management strategies for future urban mobility can be personalized and serve for various system objectives.

This thesis focuses on the personalization of future urban mobility through the personalized menu optimization model. The model built upon individual consumer's choice behavior generates a personalized menu for app-based mobility solutions. It integrates behavioral modeling of consumer mobility choice with optimization objectives. Individual choice behavior is modeled through logit mixture and the parameters are estimated with a hierarchical Bayes (HB) procedure. In this thesis, we first present an enhancement to HB procedure with alternative priors for covariance matrix estimation in order to improve the estimation performance. We also evaluate the benefits of personalization through a Boston case study based on real travel survey data. In addition, we present a sequential personalized menu optimization algorithm that addresses trade-off between exploration (learn uncertain demand of menus) and exploitation (offer the best menu based on current knowledge). We illustrate the benefits of exploration under different conditions including different types of heterogeneity.

Thesis Supervisor: Moshe E. Ben-Akiva

Title: Edmund K. Turner Professor of Civil and Environmental Engineering

Thesis Supervisor: Bilge Atasoy

Title: Assistant Professor of Maritime Transport and Technology, Delft University of Technology

Acknowledgement

During the past two and half years in transportation doctoral program, I have been very fortunate to have received help and support from numerous people.

First, I would like to thank my advisor, Professor Moshe Ben-Akiva for his great mentorship and insights throughout my research and study in transportation. His guidance and support since my first year at MIT was invaluable to me.

I would also like to thank my co-advisor Professor Bilge Atasoy who have helped me tremendously throughout my doctoral study.

I thank my thesis committee members Professor Carolina Osorio and Professor Christopher Zegras for their valuable suggestions given during and after thesis committee meetings which have helped me improve my thesis research.

The Tripod project supported by ARPA-E funded my research assistantship to complete most of my thesis research. I would also thank CEE department to support me with the scholarship during my last summer at MIT, and assistance I received from Kiley and Max from CEE department when going through the thesis defense process.

I also thank peer transportation doctoral students Haizheng, Li, Zhan who will also graduate this summer for their help I received when preparing my thesis and the defense, and thank ITS lab members Arun, Kyungsoo, Mazen and Yundi for their suggestions on presentation.

I thank all the ITS lab members, assistants and research affiliates at MIT and at Singapore for their company and support.

Finally, I thank my family, particularly my parents and my wife Xiaoyu Zhang for their constant and unconditional love and support during my whole graduate life at MIT.

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Chapter 1
Introduction and Motivation

In the past decade, numerous urban passenger transportation options have been emerging to serve the needs of travelers and society. Shared-vehicle is becoming one of the most common ways of traveling and it keeps growing. Shared-vehicles including taxis and cars operated by ride-sharing companies such as Uber accounted for 4% of global miles traveled in 2015 but by 2030, Morgan Stanley estimates that number could reach 26% (Morgan Stanley Research, 2016).

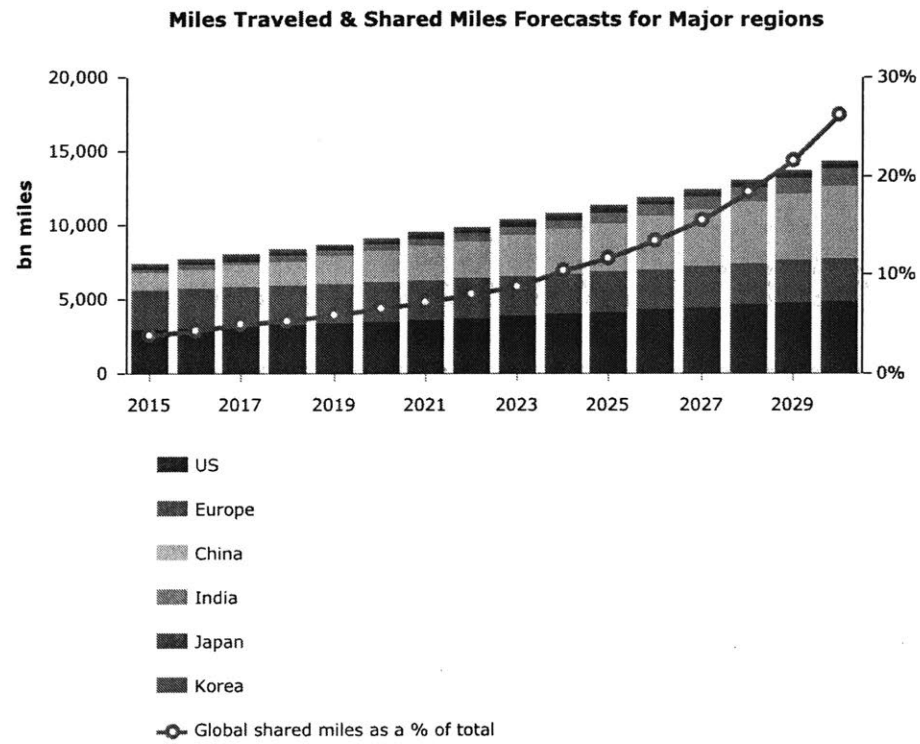

Fig. 1.1 Shared miles forecasts

While vehicle sharing can improve the vehicle utilization to 50%-60% of its full potential (Morgan Stanley Research, 2016), automated vehicles on the other hand can remove the human bottleneck and further improve the efficiency of travel. Many current vehicles have Level 1 and 2 technologies such as cruise control, hazard warning and automated parallel parking; some companies like Waymo and Uber have announced plans to begin testing a driverless taxi service (Litman, 2018).

With new transportation options arising, we have also observed the shift in language from transportation to mobility which represents a shift in thinking about how a transportation system should be designed and managed. Mobility is a user-centric concept that recognizes that urban transportation products and services must be responsive to the needs, habits, and preferences of travelers and society (Center for Automotive Research, 2016). Those technologies and business models for mobility solutions not only improve the operational efficiency of vehicles but also aim to be more responsive to mobility consumers' needs.

Most of these new mobility solutions are based on smartphone apps. The growing accessibility of the Internet and penetration rate of smartphones (see Fig. 1.2) have further accelerated the growth of these app-based mobility services. Highly penetrated smartphones not only provide platforms for app-based mobility solutions but also enable high-resolution mobility behavioral data collection at individual level through smartphone-based travel survey tools such as FMS (Future Mobility Sensing, Cottrill et al., 2013) which can be utilized for improving the design and operations of these mobility solutions.

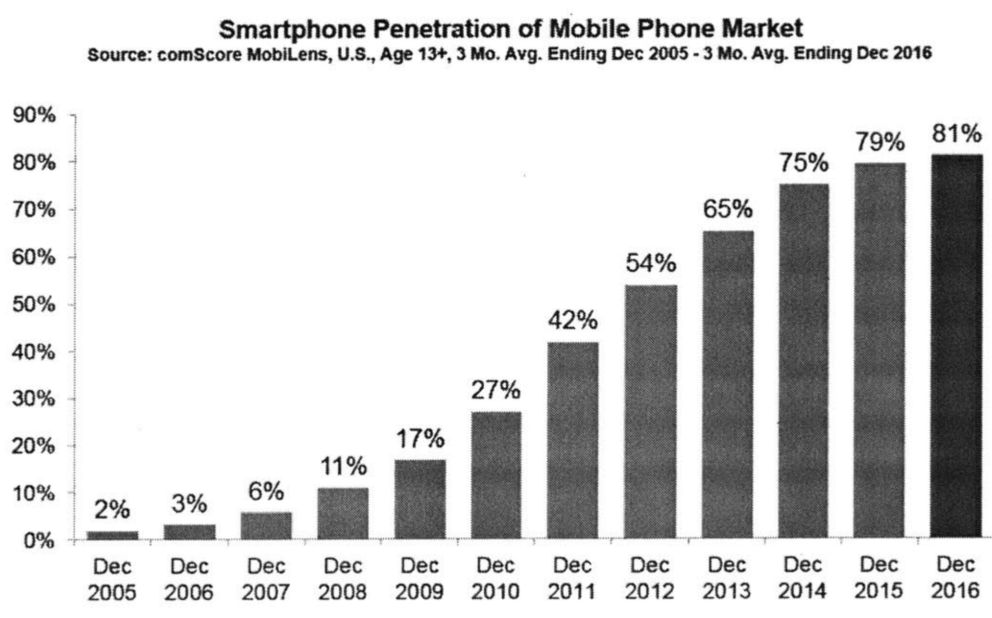

Fig. 1.2 Smartphone penetration of mobile phone market

As innovations in urban mobility are emerging, there has been a great amount of literature about these innovations. Some researchers have been focusing on behavioral experiments for new mobility solutions including intention to use, willingness to pay for ownership and adoption timing (Payre et al., 2014; Schoettle and Sivak 2014; Schoettle and Sivak 2015; Kyriakidis et al., 2015; Bansal and Kockelman, 2016). Some researchers have been developing and evaluating control strategies of the new mobility systems (Spieser et al., 2014; Zhang et al., 2015; Zhang and Pavone, 2016).

The existing literature either focuses on system operation (supply side) or consumer behavior (demand side). However, few have been studying future urban mobility solutions based on the needs from both consumer and system perspectives. The proposed operational strategies are somewhat separated from consumer behavior and are not personalized to serve heterogeneous consumers.

For all the consumer-centric industries such as urban mobility and retail, how to better serve the individual consumer is always one of the most important questions. If a consumer dislikes most of the travel options shown on the mobility app, they would likely opt out and choose other mobility solutions. Moreover, the probability that the consumer will visit the app in the future will decrease quickly. As almost all the interactions between consumer and mobility system happen through the app's user interface, an optimal control of what to show for an individual consumer, especially the menu of different travel options is the key for achieving both operational objectives and personalization.

Fig. 1.3 shows an example of an app-based urban mobility solution. The application is a trip planner where a consumer inputs origin, destination, and departure time as a request. A menu will be presented to this consumer which is a list of different travel options with attributes such as departure/arrival time. The menus are often constrained in size in these apps. Consumer can choose an option on the menu or opt out (leave the app and take other travel options not on the menu).

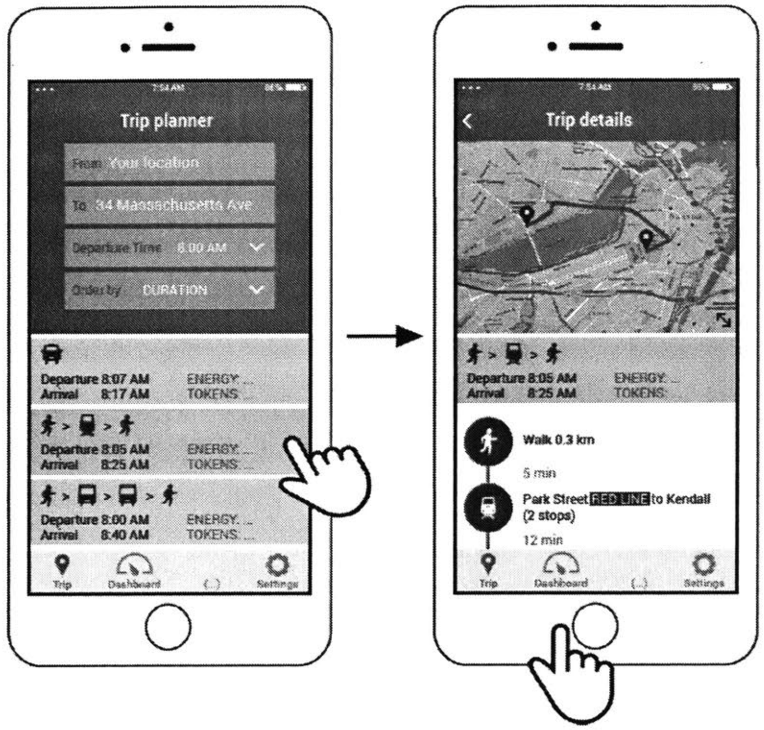

Fig. 1.3 App-based mobility solution

In this thesis, we propose a personalized menu optimization model that achieves both personalization and efficiency for future urban mobility. With consumer mobility behavioral data from either stated or revealed preferences sources or both, we can estimate logit mixture models for consumer mobility choice behavior. The personalized menu optimization model is built upon the individual's choice probabilities of travel options from the logit mixture model and maximizes expected revenue (or other objectives) through the selection of travel options to be presented on the menu.

The thesis focuses on improving the following aspects in the context of personalized recommendation in transportation, which can also be used in general in other relevant research fields:

- First, as the recommendation is highly associated with prediction accuracy of the choice model, we propose to use alternative priors suggested by statisticians instead of the inverse Wishart (IW) prior used in textbooks in economics and commercial software in market research for covariance matrix in logit mixture. It's the first time that the issue of using IW for logit mixture has been identified in transportation, economics and marketing fields, and those alternative priors are applied for logit mixture. Through experiments using synthetic and real data, we show that using alternative priors for logit mixture empirically outperform using inverse Wishart in terms of the estimation accuracy.

- Second, we propose a novel personalized menu optimization (PMO) model based on personalized estimates instead of non-personalized estimates used in existing choice-based optimization literature. The PMO utilizes more sophisticated choice models including logit mixture with inter- and intra-consumer heterogeneity compared to existing models. We show that the proposed method outperforms the method using non-personalized estimates as well as classical content-based recommendation methods, and the performance gap between PMO and content-based is larger when intra-consumer heterogeneity is incorporated.
- Third, we present a novel multi-armed bandit (MAB) approach for sequential personalized menu optimization problem which integrates a classical bandit learning algorithm and hierarchical Bayes estimates from logit mixture. It incorporates exploration that extends the second study where only exploitation is considered and extends the existing MAB algorithms where choice behavior is modeled as independent Beta distribution or multinomial logit. It is shown through numerical experiments that the approach outperforms the classical bandit learning algorithm and outperforms personalized menu optimization under disturbance when intra-consumer heterogeneity is taken into account.

The remainder of the thesis is organized as follows. In Chapter 2, we review relevant studies about personalized menu optimization. In Chapter 3, we introduce the logit mixture and hierarchical Bayesian procedure that estimates logit mixture. In addition, we propose an enhancement using alternative priors instead of inverse Wishart that improves the estimation accuracy, which will in turn improve the performance of personalized menu optimization. In Chapter 4, we evaluate the personalized menu optimization through a Boston case study in the context of a Smart Mobility system called Tripod. In Chapter 5, we focus on the sequential personalized menu optimization through multi-armed bandit approach and evaluated the need of exploration under different conditions. In Chapter 6, we summarize the findings in this thesis and discuss the future research directions.

Chapter 2

Literature Review

In this chapter, we review the relevant studies particularly about personalized recommendation in app-based Smart Mobility systems. Smart Mobility systems are emerging to provide innovative app-based mobility solutions that can deal with real-time requests from travelers with heterogeneous travel behavior. Companies including Uber, Lyft, and Zipcar have been successful in addressing traveler's needs and therefore have been attracting many travelers every day (Rayle et al., 2015). The recommendations from Smart Mobility systems need to be personalized and optimized in order to attract mobility consumers and operate efficiently, which can be achieved with personalized menu optimization. In this chapter, we will review relevant studies including travel behavioral modeling of individuals, existing personalized recommendation in general and in transportation, sequential personalized recommendation in general and in transportation, as well as relevant methodologies from other fields such as marketing, operations management and machine learning.

2.1 Travel behavioral modeling

In this section, we review relevant studies about travel behavioral modeling which is used by the personalized menu optimization model. Predicting individuals' travel choice behavior is one of the most important aspects of personalization in the transportation field. This is often conducted through different types of discrete choice models such as multinomial logit, nested logit, and logit mixture. In this thesis, we focus on logit mixture which gives estimates of individual specific taste parameters. The personalized menu optimization with logit mixture is based on an individual's preferences. For choice models such as multinomial logit, we may still call them personalization only when the prediction is utilizing individual specific attributes and such attributes are included in the utility function of the choice models. However, multinomial logit does not give us individual specific preference knowledge.

Logit mixture is increasingly used by transportation researchers and beyond for its benefits, one of which is the improved estimation performance (Hess and Train, 2011). There are various ways to estimate parameter values for logit mixture such as simulated maximum likelihood (SML) and hierarchical Bayes (HB). The HB procedure has been popular since its first application to marketing problems in the early 1990s (Allenby and Rossi, 2003) and it has several advantages over the classical SML procedure. The HB method has better estimation properties, such as consistency and efficiency, and it does not require maximization (Train, 2009). It has been applied in commercial software such as Sawtooth (Orme, 2009) and described in well-known economics and marketing books (Train, 2009; Rossi et al., 2012). The earliest impact of the HB procedure is in choice-based conjoint (CBC) analysis in marketing (Allenby and Lenk, 1994; McCulloch and Rossi, 1994; Allenby and Ginter, 1995), and marketing researchers have developed theories and applications of HB with CBC data (Moore et al., 1998; Liechty et al., 2008; Ku et al., 2017). Additionally, various studies extend the HB procedure to include decision heuristics such as screening rules (Gilbride and Allenby, 2003), advanced models that include both inter- and intra-personal heterogeneity (Becker et al., 2018), and tools for recommender systems based on HB (Danaf et al., 2018).

The estimation performance is obviously critical for personalization. A typical logit mixture model often uses inverse Wishart (IW) distribution as the noninformative prior for covariance matrix. However, based on knowledge from statisticians, we know that using IW prior would impose informativity when true values of standard deviations are small and can lead to the estimates for covariance matrix substantially deviating from true values (Alvarez et al., 2014), which in turn will impact the estimates of individual specific parameters. In this thesis, we propose to use alternative noninformative priors where no additional information is imposed for covariance matrix in HB method and show that using alternative priors would reach better estimation performance.

2.2 Personalized recommendation

2.2.1 Personalized recommendation in general

Recommender systems have been an important research field since the mid-1990s and have been applied in many fields including books, documents, images, movies, music, shopping and others

(Park et al., 2012). Many recommendation techniques have been developed including widely used collaborative filtering, content-based filtering, data mining techniques, knowledge-based, and context-aware methods (Ricci et al., 2015).

Most existing studies of personalized recommendation are based on classical collaborative filtering or content-based recommendation algorithms. Collaborative filtering predicts interest of the user by collecting preferences from similar other users. For example, if customer A bought books X, Y, Z and customer B bought books X and Y, the system will recommend customer B book Z. Content-based recommendation is based on a description of the item and user's profile. For example, a new book can be recommended based on its topic and user's historical choice of this topic.

Liu et al. (2011) presented a hybrid approach to recommend blog articles on mobile devices, which integrates personalized prediction of popularity of topic clusters, item-based collaborative filtering and choice history. Zhuang et al. (2011) presented a context-aware and personalized recommendation for mobile applications. It utilizes contexts including geo-locations and user choice history based on the idea of collaborative filtering to enhance the recommendation performance. Wang et al. (2015) presented a personalized recommendation for social network by taking advantage of data from sensor-rich smartphones. It discovers lifestyles of users from user-centric sensor data and recommend friends to users if someone has a similar lifestyle. Chen et al. (2017) presented a content-based personalized recommendation integrated with unsupervised textual embedding where content of items is embedded into latent feature space and showed its effectiveness with online news feed data.

The above studies are built upon classical recommendation algorithms. However, there are limitations with classical recommendation algorithms such as collaborative filtering and content-based recommendation algorithms. Collaborative filtering requires lots of information about each user and each item to make an accurate prediction, which is a cold-start problem. It's not suitable for the case study in Chapter 4 where choice history with user and with travel options is limited. Content-based recommendation techniques need very little information to start, which could be used for the case study in Chapter 4. However, it may be limited when it does not have enough information for each user to distinguish an item that a user likes from an item that a user dislikes (Pazzani and Billsus, 2007).

2.2.2 Personalized recommendation in transportation

There are limited studies about personalized recommendation in app-based transportation systems. Some transportation apps would simply recommend alternatives with shortest travel time and distance, which ignores the user's profile. Most recommendation techniques rely on both alternatives' attributes and users' preference profiles which are content-based recommendation algorithms. Nakamura et al. (2014) presented a personalized bus route recommender system based on user context and transaction histories, which is shown to be effective through a case study using Japanese data.

There are some recommender systems that enhance the classical recommendation algorithms based on real-time collected traffic data or stated knowledge from users. Tsai and Chung (2012) presented a recommender system for theme park tourists. The proposed recommender system collects tourist visiting sequences and time lengths through a Radio-Frequency Identification (RFID) system and generates proper route suggestions for visitors considering current facility queuing situations identified by RFID system. Liu et al. (2014) proposed a recommender system for self-drive tourists that is based on real-time traffic data. Instead of RFID, their system used a vehicle to vehicle communication system (V2VCS) as RFID requires huge infrastructure investments. Su et al. (2014) enhanced traditional route recommendation with crowdsourcing knowledge from human workers and showed that crowds' knowledge has improved the recommendation quality. Bajaj et al. (2015) presented a personalized route recommendation to improve convenience stated by users. In the following case study in Chapter 4, the system does not utilize data except for a user's choice history.

All the systems reviewed above do not model consumer choice behavior and are based on classical recommendation algorithms. In the next section, we present recommender systems based on discrete choice models.

2.2.3 Personalized menu optimization

With rich individual data becoming available in the Big Data era, discrete choice models have received interest in recommender systems due to their ability to predict individual choices (Chaptini, 2005; Polydoropoulou and Lambrou, 2012; Jiang et al., 2014). Jiang et al. (2014) used a multi-level nested multinomial logit model to measure users' preferences towards an entire recommendation list. The recommendation problem was formulated as a nonlinear binary integer programming problem. The authors pointed out that discrete choice models could incorporate product diversity in the proposed recommendation, which differed from traditional recommender systems. Choice models being able to well account for consumer heterogeneity is crucial for recommender systems. There exist various recommendation techniques that account explicitly for heterogeneity in user preferences (Allenby and Ginter, 1995; Rossi et al., 1996; Ansari et al., 2000).

In this thesis, we propose personalized menu optimization, which integrates travel behavioral modeling and optimization models. It often uses estimation results from hierarchical Bayesian (HB) procedure for logit mixture. Logit mixture model captures the individual-level taste heterogeneity and HB estimation provide estimates of individual-level taste parameters which can be used for computing choice probability of alternatives or consumer-surplus presented by log-sum (Ben-Akiva, 1973).

Personalized menu optimization is adapted from assortment optimization which is an arising topic in operations management that is becoming popular in many practical settings such as retailing and online advertising (Desir et al., 2015). The goal of assortment optimization is to select a subset of items to offer from a universe of substitutable items in order to maximize the expected revenue when consumers exhibit a random choice behavior. Different discrete choice models are often used to model the random choice behaviors of consumers including multinomial logit, nested logit, and logit mixture (Davis et al., 2013; Davis et al., 2014; Feldman and Topaloglu, 2015). The reader is referred to Kok et al. (2008) for more details of assortment optimization literature and industry practice. The assortment optimization model is flexible and can be the underlying control mechanism for different operational management systems by formulating different optimization models. It often assumes that the system knows consumers' utilities for product or option in the choice set from previous data collection. Then it maximizes the expected "revenue" as a function of choice probabilities that are controlled by the decision variable whether the option is in the assortment or not. "Revenue" can be monetary revenue or other beneficial performance measures.

The first application of assortment optimization methodology in the transportation field has been seen in FMOD (Flexible Mobility on Demand), which allocates flexible levels of services such as taxi and shared-taxi with the same type of vehicles to individual travelers (Atasoy et al., 2015a; Atasoy et al., 2015b). In FMOD, the personalized menu optimization selects a subset of travel options, including taxi, shared-taxi and mini-bus, to be presented on the menu in order to maximize the expected revenue or consumer surplus or both (Atasoy et al., 2015a; Atasoy et al., 2015b). FMOD uses multinomial logit as the underlying choice model.

In this thesis, we focus on cases where the underlying choice models are more complicated including logit mixture with inter-consumer (and intra-consumer heterogeneity). This has been applied in the context of Tripod. Tripod is an app-based smart mobility system that incentivizes travelers based on energy savings in order to increase the utilization of more energy efficient options (Azevedo et al., 2018). Travelers make trip requests at the Tripod app, and the user level optimization (User Optimization) generates personalized menus as a list of travel options including mode, departure time, route, energy usage and travel incentives in the form of tokens to incentivize the user for green travel options.

The personalized menu optimization is novel as it uses a continuous logit mixture with inter- and intra-consumer heterogeneity as the underlying choice model while in FMOD multinomial logit is used and in Feldman and Topaloglu (2015) a discrete logit mixture was used which assumed that individuals belong to the same segment sharing the same taste parameters. Both choice models did not use estimates for individual-specific taste parameters. The particular benefit of using logit mixture is its ability to capture individual-level taste preferences. Therefore, in Chapter 4, we evaluated the personalized menu optimization using logit mixture against the benchmark methods such as using non-personalized estimates from logit mixture and content-based recommendation algorithms to show the benefits of personalization. We use a Boston case study to show that the benefits hold with real data and a large sample.

2.2.4 Sequential personalized menu optimization

In previously reviewed studies, the personalized menu optimization is exogeneous with preference updates. It uses the expected rewards as the objective function to maximize and doesn't explore beyond the current expectation. In real life, we need to deal with sequential personalized menu optimization problem where we need to continuously update our belief of uncertain preferences. For example, we might like to recommend products towards which we know little about users' preferences in order to have better understanding of associated users' preferences. Such a problem can be formulated as a multi-armed bandit (MAB) problem.

The multi-armed bandit approach deals with the trade-off between exploration (learning consumer uncertain preferences of some alternatives) and exploitation (offer best alternatives based on current belief) where a recommendation decision is endogenous with preference updates. There are different types of multi-armed bandit problems including stochastic, adversarial, and Markovian depending on the assumed nature of reward process (Bubeck and Cesa-Bianchi, 2012). MAB problems usually do not have exact solutions except for some special cases (Scott, 2010) and many researchers have proposed different solution algorithms to different types of MAB problems including the upper confidence bound (UCB) algorithm in the stochastic case, the exponential-weight algorithm for exploration and exploitation (Exp3) algorithm in the adversarial case, and the so-called Gittins indices in the Markovian or Bayesian case (Bubeck and Cesa-Bianchi, 2012). A typical MAB problem can be stated as follows (Gittins et al., 1979): there are N arms, each having an unknown success probability of emitting a unit reward. The success probabilities of the arms are assumed to be independent of each other. Many policies have been proposed under independent-arm assumptions (Lai and Robbins, 1985; Auer et al., 2002). Related with personalized menu optimization, the arm is the offered menu which is a list of alternatives and the success means an alternative on the menu is being chosen by the user. In this thesis, we focus on the case where the menu size is one and therefore the arms are independent. If menu size is greater than one, the success probability of one arm/menu will depend on utility of multiple alternatives which means its reward is dependent on some of the other arms which have the same alternatives on the menu. It is a combinatorial bandit problem where existing techniques such as UCB do not work directly on these functions (Chen et al., 2017). We leave this more complicated case for future study.

There are a few heuristics for various MAB problems including:

- First explore then exploit, which has been adopted by Rusmevichientong et al. (2010) and Saure and Zeevi (2013) to solve dynamic assortment optimization problems.

- Epsilon-greedy: with epsilon probability, choose a random arm to explore, otherwise exploit.
- Gittins index: compute a Gittins index for each arm and choose the arm with highest index. More details can be found in Gittins (1979). One issue with Gittins index is that it is only optimal with geometric discounting (Berry and Fristedt, 1985).
- Randomized probability matching (RPM): randomly choose an arm with the probability that this arm is the best. A well-known special case of RPM is Thompson sampling (TS, Thompson, 1933). Chapelle and Li (2011) showed by empirical experiments that Thompson sampling performs better than UCB algorithms.
- Upper confidence bound (UCB): choose an arm with the highest upper confidence bound. It has many variants such as Bayes-UCB due to various ways to construct upper confidence bound and has been applied in many fields including personalized recommendation in news articles (Li et al., 2010) and digital coupons (Song, 2016).

For stochastic and adversarial bandit, the reader is referred to Bubeck and Cesa-Bianchi (2012). For Markovian bandit or Bayesian bandit, the book by Gittins et al. (2011) is the main reference.

Most existing literature in the MAB field does not deal with discrete choice models but often assumes choice behavior follows simple Beta distributions (Song, 2016). In operations management, there exists literature proposing online policy depending on a priori knowledge of length of horizon (Rusmevichientong et al., 2010; Saure and Zeevi, 2013) such as "explore first and exploit later" policy. In the MAB paradigm, Agrawal et al. (2017a, b) proposed an adapted Thompson sampling method and a UCB method which can deal with multinomial logit choice model but relies on specific exploration phases.

These methods are not suitable for a sequential personalized menu optimization setting where logit mixture is the underlying choice model and is much more complicated than multinomial logit. In this thesis, we focus on proposing a method which adapts the classical UCB algorithm by utilizing the HB estimator for logit mixture of inter- and intra- consumer heterogeneity.

In transportation, there are a few studies about MAB problems which focus on different types of sequential decision-making problems. Chancelier et al. (2007) have modeled route choice as a one-armed bandit problem (choice between a random and a safe route) under different information regimes. They showed that risk neutral individuals tend to select a risky route while risk-averse individuals choose safe route more frequently. Chancelier et al. (2009) further showed that when utility function is more concave, an individual has more chances to select a safe arm in a one-armed bandit problem. Ramosa et al. (2018) model the route choice problem as a multiagent reinforcement learning scenario. They analyzed how travel information provided from a mobile navigation app would impact the agent route choice decision using epsilon-greedy strategy that minimizes difference between chosen route and best route. Estes and Ball (2017) used the multi-armed bandit approach for a ground delay program that balances the demand and supply in terms of airport capacity.

2.3 Summary

In summary, this thesis makes various contributions to the field of transportation and methodologically to choice-based personalized recommendation and sequential choice-based personalized recommendation when compared to the existing literature.

- First, we propose to use alternative prior distributions suggested by statisticians for covariance matrix in logit mixture model instead of classical inverse Wishart (IW) distribution used in textbooks and commercial software for discrete choice models. Although these alternative priors exist in statistics literature, they have not been introduced and applied with discrete choice models before. It's the first time that the issue of using IW for logit mixture has been identified in transportation, economics and marketing fields, and those alternative priors are applied for logit mixture. We empirically analyze their superior estimation accuracy against the benchmark IW prior with synthetic and real data.
  In addition, we give guidance for researchers and practitioners who use logit mixture about when and which prior to use after comparing among alternative priors in terms of estimation accuracy and computational time.

- Second, we propose a novel choice-based personalized recommendation model which utilizes the logit normal mixture as the underlying choice model that captures individual specific taste preferences. The proposed model uses logit mixture with inter- and intra-consumer heterogeneity where previously only non-personalized choice models such as multinomial logit have been used.
  By using a Boston case study, we illustrate that the personalized menu optimization using individual specific preference estimates performs better than benchmark methods including non-personalized menu optimization model and two content-based recommendation algorithms. The benefits of the proposed model are more salient when there is more choice data and when the true choice model is more complicated with inter- and intra-consumer heterogeneity.
- Third, we present a novel sequential personalized menu optimization algorithm which integrates the idea of the classical UCB algorithm that deals with exploration and exploitation into the personalized menu optimization based on HB estimates of individual preferences. This algorithm extends the previous literature where user choice behavior is not modeled by logit mixture and extends the second study where we only consider exploitation. We illustrate that the proposed algorithm performs better than classical UCB algorithm and show that the benefits of exploration under disturbance against benchmark personalized menu optimization when we have inter-and intra-consumer heterogeneity.

Chapter 3

Logit Mixture and Hierarchical Bayes Procedure

3.1 Logit mixture

With the choice data collected through stated preference surveys and revealed preferences from smartphone apps or other resources, we can estimate a discrete choice model that captures an individual consumer's preferences. One of the key discrete choice models we apply in our methodology is called logit mixture model (also known as *mixed logit*) (Ben-Akiva et al., 2016).

Logit mixture models have many advantages, including improved estimation performance, which make them widely used by researchers and practitioners to represent random taste heterogeneity across consumers (Hess and Train, 2011). As reviewed in Chapter 2, the hierarchical Bayes (HB) procedure for estimating logit mixture has been popular and widely used in transportation, economics and marketing for various applications. The HB procedure, called Allenby-Train procedure (Ben-Akiva et al., 2016), has been applied in commercial software such as Sawtooth (Orme, 2009) and described in well-known economics and marketing books (Train, 2009; Rossi et al., 2012).

A typical logit mixture model with a linear utility specification is given as follows:

$$U_{jn} = V_{jn} + \epsilon_{jn} = x_{jn}\zeta_n + \epsilon_{jn} \tag{3.1}$$

where U_{jn} denotes individual n's utility for alternative j, x_{jn} denotes the attributes of alternative j available to individual n, ϵ_{jn} is i.i.d. Gumbel distributed. ζ_n is the random parameter vector representing taste of individual n, which follows a multivariate normal distribution with mean μ and covariance Ω represented by Equation (3.2).

$$\zeta_n \sim \mathcal{N}(\mu, \Omega) \tag{3.1}$$

The likelihood function for individual n given parameters μ, Ω is given by Equation (3.3).

$$P(d_n|\mu,\Omega) = \int_{\zeta_n}\left[\prod_{j=0}^{J_n} P_{jn}(\zeta_n)^{d_{jn}}\right] f(\zeta_n|\mu,\Omega)d\zeta_n \tag{3.2}$$

where d_{jn} is a binary variable such that it is 1 if individual n chooses alternative j and 0 otherwise. J_n denotes the number of alternatives in the choice set of individual n (opt-out alternative is represented by $j = 0$). d_n is the vector of choices over all alternatives for individual n and f denotes the normal probability density function.

The probability of individual n choosing alternative j given individual parameter vector ζ_n is shown in Equation (3.4).

$$P_{jn}(\zeta_n) = \frac{\exp\left(V_{jn}(\zeta_n)\right)}{\sum_{j'=0}^{J_n}\exp\left(V_{j'n}(\zeta_n)\right)} \tag{3.3}$$

3.2 Hierarchical Bayes procedure

In well-known textbooks in economics and marketing (Train, 2009; Rossi et al., 2012) and commercial software Sawtooth (Orme, 2009), a hierarchical Bayes (HB) procedure called Allenby-Train procedure (Ben-Akiva et al., 2016) is used for estimating the logit mixture with inter-consumer heterogeneity. The HB procedure treats the individual level coefficients as unknown parameters in the Bayesian estimation procedure. Therefore, we have three sets of parameters and the joint posterior distribution for all three sets of parameters is given as follows:

$$K(\mu,\Omega,\zeta_n\ \forall n|\mathrm{d_n}\ \forall n) \propto \left\{\prod_{n=1}^{N}\left[\left[\prod_{j=0}^{J_n} P_{jn}(\zeta_n)^{d_{jn}}\right]\mathcal{N}(\zeta_n|\mu,\Omega)\right]\right\} k(\mu)k(\Omega) \tag{3.4}$$

where the prior distributions are given as

$$k(\mu) \sim \mathcal{N}(\mu_0, A) \tag{3.5}$$

$$k(\Omega) \sim IW(v, \Sigma) \tag{3.6}$$

where μ_0 can be any value (e.g., zeros or flat logit model estimates); A is a diagonal matrix with large elements (e.g., $100I_T$) where T is the number of unknown population mean parameters; v is the degrees of freedom parameter, Σ is the scale matrix where statisticians set v to be $T+1$ and Σ to be I_T (a $T \times T$ identity matrix) (Barnard et al., 2000; O'Malley and Zaslavsky, 2005; Alvarez et al., 2014). Train (2009) suggested using $IW(T, TI_T)$. Rossi et al. (2012) suggested using $IW(T+3, (T+3)I_T)$ and $IW(T+4, (T+4)I_T)$. The reason why IW prior is used is because it's a conjugate prior for covariance matrix with multivariate normal likelihood, which means the posterior is also an IW distribution. Conjugacy enables Gibbs sampling update of population-level mean and covariance matrix in Markov Chain Monte Carlo (MCMC) which is computationally efficient. In the remainder of this paper, we will use $IW(T+1, I_T)$ and we will show by simulation data analysis that all these alternative specifications have issues.

Given Equation (3.5), we can obtain estimates of these parameters through various methods including random walk Monte Carlo (Gibbs sampling and Metropolis Hasting) which is the most common MCMC method, Hamiltonian Monte Carlo (HMC) which is also known as Hybrid Monte Carlo (Neal, 2011) and is also a MCMC method, and Variational Bayes (Jordan et al., 1999; Braun and McAuliffe, 2010). In current textbooks and commercial software, the Allenby-Train procedure for logit mixture often uses a random walk Monte Carlo method and constructs three Gibbs sampling steps for three sets of parameters μ, Ω, and $\zeta_n\ \forall n$.

Step 1 draw. $\mu | \Omega, \zeta_n$ for all n;

Step 2 draw. $\Omega | \mu, \zeta_n$ for all n;

Step 3 draw. $\zeta_n | \mu, \Omega$.

Steps 1 and 2 are conjugate normal updates with unknown (known) mean and known (unknown) variance. Step 3 follows a Metropolis Hasting (MH) algorithm. More details about this Bayesian procedure for logit mixture can be found in Ben-Akiva et al. (2016), Train (2009) and Rossi et al. (2012). In Becker et al. (2018), the Allenby-Train procedure for logit mixture with inter-consumer heterogeneity is extended for logit mixture with inter- and intra-consumer heterogeneity. More details of the extended Allenby-Train procedure for logit mixture with inter- and intra-consumer heterogeneity can also be found in Appendix A.

3.3 Enhancement to hierarchical Bayes procedure

3.3.1 Inverse Wishart distribution

The common HB procedure uses IW prior for covariance matrix estimation due to its conjugacy to covariance matrix with multivariate normal likelihood. Conjugacy gives convenience for estimation through Gibbs sampling and therefore we have previously described the Allenby-Train procedure. The IW distribution with degrees of freedom v and scale matrix Σ is shown as follows:

$$\Omega \sim IW(v, \Sigma) = \frac{\left|\Sigma^{\frac{v}{2}}\right|}{2^{\frac{vT}{2}} \Gamma_T(\frac{v}{2})} |\Omega|^{-\frac{v+T+1}{2}} e^{-\frac{1}{2}tr(\Sigma\Omega^{-1})} \qquad (3.7)$$

where tr() stands for trace of a matrix and Γ_T stands for the multivariate Gamma function.

In order to understand the impact of imposing an IW prior for covariance matrix, we need to know the properties of the IW distribution. First, the uncertainty for all variance parameters is controlled by a single degree of freedom parameter and thus provides no flexibility to incorporate different amounts of prior knowledge to different variance components (Gelman et al., 2003; Alvarez et al., 2014). Second, Gelman (2006) suggests that the marginal distribution for the variance has low density in a region near zero. Third, Tokuda et al. (2011) suggest that there is a priori dependence between variances and correlations when using IW prior. Given such properties of IW, Alvarez et al. (2014) identify through a simulation study that the estimates of variances will be inflated and the estimates of correlations will be deflated when the true relevant variance is small.

As suggested by Tokuda et al. (2011), the analytical property of IW distribution is difficult to obtain and researchers often use simulation analysis to understand its property. Therefore, we carry out simulation-based analysis to show the properties of an IW distribution that may lead to biased estimates. This simulation analysis adapts the R codes provided by Matt

Simpson (2012) on his website. Specifically, we draw 10,000 samples from $IW(3, I_2)$. Fig. 3.1 shows the histograms of standard deviations of the first and second components in covariance matrices from IW. Fig. 3.2 shows the variance of the first component vs. the correlation in covariance matrices from IW. The horizontal axis is shown in powers of 10 for better visualization.

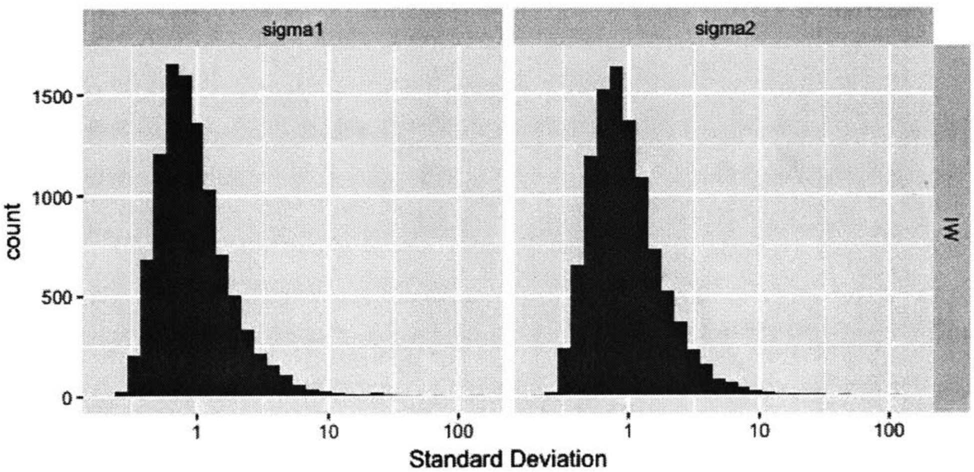

Fig. 3.1 Standard deviations of the first vs. second component from IW

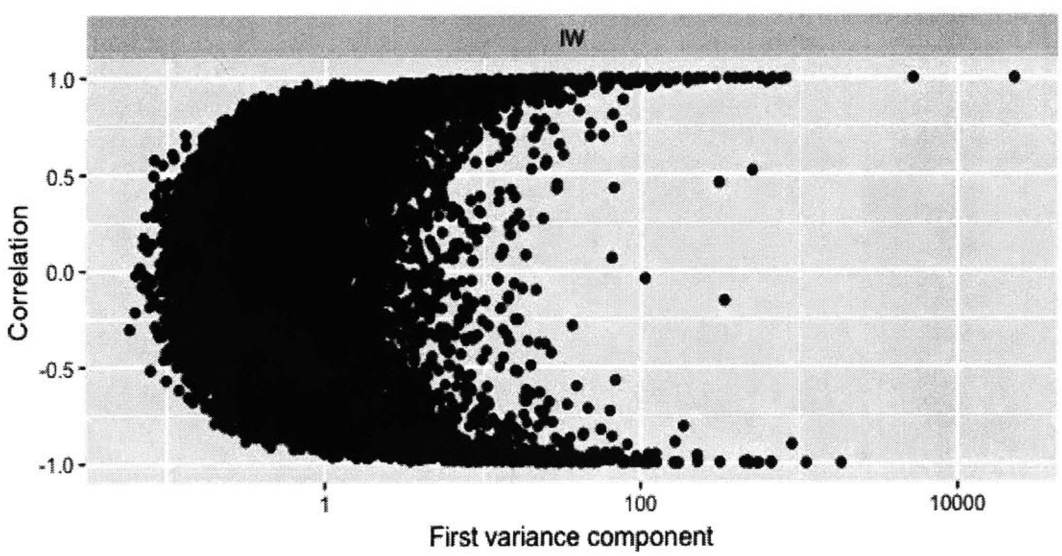

Fig. 3.2 Variance of the first component vs. correlation from IW

Fig. 3.1 indicates that IW has a very narrow range of values for the standard deviations. We see few draws below 0.1. This may cause inflated posterior estimates of standard deviations when true standard deviations are small. From Fig. 3.2, we observe strong dependence between correlation and variance where larger variances are associated with extreme correlations and small variances are associated with correlations near zero. Such dependence may cause the estimates of correlations to be biased towards zero when the true standard deviations are small. In a later section, we use similar plots to compare the alternative priors with IW. We also illustrate the issues with using IW prior within HB procedures for logit mixture through Monte Carlo simulation in a later section.

3.3.2 Alternative priors for covariance matrix

Ben-Akiva et al. (2016) note issues of inflated variance when applying the HB procedure. To address this problem, they divide the data by ten so the covariance matrix is relatively large and, therefore, issues when the true value of variance is small can be avoided. Rescaling the data is also suggested by Alvarez et al. (2014). We review three alternative priors for covariance matrix including a separation strategy (BMM), a scaled inverse Wishart (SIW), and a hierarchical inverse Wishart (HIW) in this section and show their properties through simulation.

BMM is proposed by Barnard et al. (2000) where standard deviations and correlations are modeled independently and later combined to form a prior on the covariance matrix

$$\Omega = \Lambda \mathrm{R} \Lambda \tag{3.8}$$

where Λ is a diagonal matrix with the i^{th} element λ_{i}

$$\log(\lambda_{\mathrm{i}}) \sim \mathrm{N}(\mathrm{b}_{\mathrm{i}}, \theta_{\mathrm{i}}^2) \tag{3.9}$$

and R is the correlation matrix of a covariance matrix that follows inverse Wishart distribution

$$\mathrm{R} = \Delta \mathrm{Q} \Delta \tag{3.10}$$

where Δ is a diagonal matrix with i^{th} element $\mathrm{Q}_{\mathrm{ii}}^{-\frac{1}{2}}$ and $\mathrm{Q} \sim IW(v, \Sigma)$. Under BMM specification, the variance component is more flexible compared to IW and is independent from correlation by construction. Correlations are marginally uniformly distributed when degrees of freedom are $T + 1$.

SIW is proposed by based O'Malley and Zaslavsky (2005) and is similar to BMM.

$$\Omega = \Lambda R \Lambda \tag{3.11}$$

where Λ is a diagonal matrix with the i^{th} element λ_i

$$\log(\lambda_i) \sim N(b_i, \theta_i^2) \tag{3.12}$$

and $R \sim IW(v, \Sigma)$. Under SIW specification, the standard deviation is the product of a log-normal and square root of a scaled inverse chi-square which provides more flexibility compared to IW. The correlations are also marginally uniformly distributed with degrees of freedom to be $T + 1$.

A recent alternative is HIW proposed by Huang and Wand (2013). The covariance matrix Ω is given by

$$\Omega \sim IW(v + d - 1, 2v\Lambda) \tag{13}$$

where Λ is a diagonal matrix with i^{th} element $1/\lambda_i$ which follows inverse Gamma distribution

$$\lambda_i \sim IG(\frac{1}{2}, \frac{1}{\theta_i^2}) \tag{3.14}$$

The corresponding density function of inverse Gamma such that $x \sim IG(\alpha, \beta)$ means that $p(x) \propto x^{-\alpha-1} e^{-\frac{\beta}{x}},\ x > 0$. As can be observed from Equations (3.14) and (3.15), HIW has more flexibility in terms of standard deviations. Setting of $v = 2$ ensures uniform distribution for correlations. Similar approaches with HIW were also proposed by Daniels and Kass (1999) and Bouriga and Feron (2013).

These alternative priors have been applied in other models such as random effects regression models. To our knowledge, none of them--except for HIW in Becker (2016)--has been applied in HB for logit mixture estimation. In order to illustrate the properties of these alternative priors, we draw 10,000 samples from these alternative priors as well as IW, and plot the histograms of standard deviations in Fig. 3.3 and scatter plots of variance of first component versus correlation in Fig. 3.4 to illustrate their properties. Following the similar simulation study by Alvarez et al. (2014), θ_i's in HIW are set to 1.04 and θ_i's in BMM and SIW are set to 1. b_i's in BMM and SIW are set to zero. Σ's are set to identity matrix.

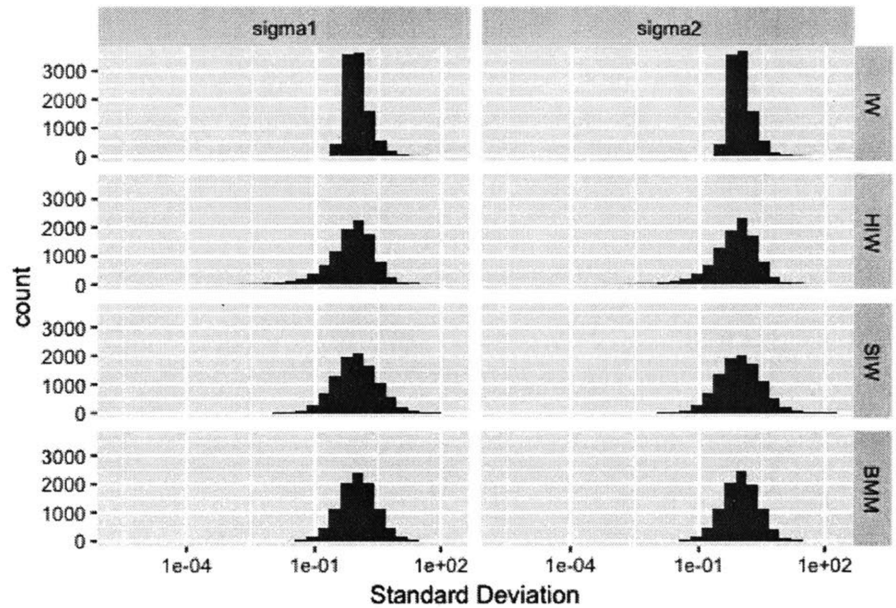

Fig. 3.3 Standard deviations of the first vs. the second component from alternative priors

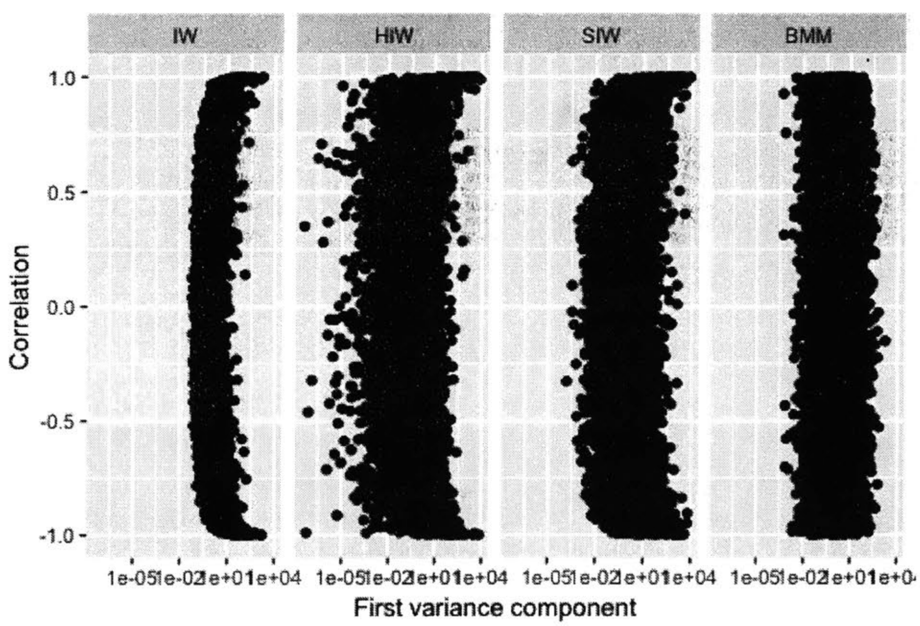

Fig. 3.4 Variance of the first component vs. correlation from alternative priors

As we observe from Fig. 3.3, histograms for HIW, SIW and BMM are similar except that HIW and SIW have slightly fatter tails. We can control the non-informativity of standard deviations through θ_i's. From Fig. 3.4, we observe that BMM has no dependence due to its construction. HIW and SIW have slight dependence among correlations and variances, but their dependence is not as strong as IW.

One important property for HIW is that it's conditional conjugate to covariance matrix with multivariate normal likelihood. Therefore, it's easier for us to extend the Allenby-Train procedure to estimate logit mixture with HIW prior. For SIW and BMM which are more complicated and require lots of coding of Metropolis Hastings, we don't use random walk Monte Carlo for estimation. Instead, we use R interface of Stan which only requires specifying model for estimation, where No-U-Turn-Sampler (NUTS) is the default estimation engine in Stan. NUTS is an adapted HMC method which can reach stationary distribution with fewer number of iterations than random walk Monte Carlo and does not require hand-tuning parameters as in HMC. More details of NUTS and Stan software can be found in Hoffman and Gelman (2014) and Carpenter et al. (2016) respectively. The Stan scripts can be found in Appendix B.

3.4 Case Study

3.4.1 Case study: choice of grapes

We use a Monte Carlo simulation study to illustrate issues in using IW in HB procedures for logit mixture as well as the advantages of using alternative priors. Alternative priors are evaluated in terms of estimation accuracy and computational time.

In this simulation study, respondents are presented with hypothetical choice situations for a choice of grapes. Bunches of grapes have attributes of price, sweetness, and crispness. Respondents are presented eight different choice scenarios and they may choose one out of three bunches or choose the no purchase option in each of those scenarios. The attributes of grapes include price, sweetness and crispness, where price is drawn uniformly from \$1.00 to \$4.00 and sweetness (tartness) and crispness (softness) are drawn with probability 0.5.

Assume the true utility is in the willingness to pay form. We define the utility of consumer n for alternative j (j=1,2,3) as follows:

$$U_{jn} \equiv I_n - P_{jn} + S_{jn}\beta_{Sn} + C_{jn}\beta_{Cn} + \alpha_n \varepsilon_{jn} \tag{3.15}$$

where I_n is the disposable income which cancels out in the utility maximization, ε_{jn} are i.i.d. Gumbel distributed, and $\zeta_n = (\beta_{Sn}, \beta_{Cn}, \log(\alpha_n))$ are the individual level preference parameters. The individual level parameters follow multivariate normal distribution with population level mean and population level covariance matrix. Two synthetic populations are generated with sample sizes 2,000 and 16,000 respectively. Given these data, we use HB procedures with IW and alternative priors to estimate these parameters. More details about the choice of grapes study can be found in Ben-Akiva et al. (2016).

With the NUTS method, the Markov chains reach stationary distributions with few iterations. We use the first 500 iterations as burn-in and the next 4,000 iterations for estimation. The burn-in means those iterations are discarded and not used for estimation since the Markov chains are not stationary distributions. All the Markov chains are stationary distributions according to the Heidelberger and Welch test (Heidelberger and Welch, 1983). The model estimation is run on a machine with a 64-bit 3.6 GHz Intel Core i7 CPU with 8 GB of memory.

Table 3.1 provides the true values together with sample values in Monte Carlo simulation and estimates of population mean/covariance and correlation with IW prior of 2000 individual data. As we described in the previous section, literature has suggested different IW specifications from $IW(T+1, I_T)$ including $IW(T, TI_T)$, $IW(T+3, (T+3)I_T)$ and $IW(T+4, (T+4)I_T)$. We denote the latter three as IW2, IW3 and IW4.

Table 3.1 Estimates of 2000 individuals with IW prior

Mean	TRUE	Sample	IW1	IW2	IW3	IW4
$\log(\alpha_n)$	-0.5	-0.4998	-0.5161	-0.5248	-0.5297	-0.5317
β_S	1	1.0052	1.0196	1.0119	1.0084	1.0093
β_C	0.9	0.8985	0.8603	0.8558	0.8503	0.8488
Std.Dev	TRUE	Sample	IW1	IW2	IW3	IW4
$\log(\alpha_n)$	0.3	0.3007	0.3379	0.3660	0.3892	0.3944

β_S	0.4	0.4021	0.4222	0.4494	0.4661	0.4727
β_C	0.3	0.2989	0.3310	0.3720	0.4014	0.4109
Correlation	TRUE	Sample	IW1	IW2	IW3	IW4
$\log(\alpha_n)$, β_S	0.8	0.8082	0.6605	0.5243	0.4423	0.4253
$\log(\alpha_n)$, β_C	0.96	0.9608	0.5405	0.4103	0.3335	0.3223
β_S, β_C	0.6	0.6132	0.4602	0.2955	0.2115	0.1839
Covariance	TRUE	Sample	IW	IW2	IW3	IW4
$\log(\alpha_n)$, β_S	0.096	0.0977	0.0942	0.0862	0.0802	0.0793
$\log(\alpha_n)$, β_C	0.0864	0.0863	0.0604	0.0559	0.0521	0.0522
β_S, β_C	0.072	0.0737	0.0636	0.0494	0.0396	0.0357

From Table 3.1, we notice that when the true values of standard deviations are small, the relevant estimates including standard deviations, correlations, and covariance terms deviate substantially from the true values. Specifically, the standard deviations are inflated and correlations are deflated. Covariances are also deflated as deflation in correlations is stronger than inflation in standard deviations. The results in Table 3.1 are consistent with findings from our previous simulation and literature. Since the issues with IW are common across alternative specifications, we use the first specification in the remainder of the paper.

The issues with IW may be also associated with other phenomena including amount of data in the likelihood, true correlations, and number of parameters. Further investigation with different simulation setups indicates that amount of data in terms of larger sample size and more choice situations could mitigate the issue as it is shown with the 16,000 sample size in latter section. However, the issue of IW still exists even at the 16,000 sample size with 16 choice situations or 24,000 sample sizes with 8 choice situations. We also observed that lower correlation could mitigate the issues of IW which is consistent with the literature highlighting the dependence between correlation and standard deviations for IW distribution. But the issue still exists with low correlation at 0.2. We didn't find an obvious pattern regarding the number of parameters.

Following the simulation study by Alvarez et al. (2014), the degrees of freedom are set to $T + 1$ in IW/SIW/BMM and v is set to 2 in HIW. θ_i's in HIW are set to $\sqrt{1000}$ and θ_i's in BMM and SIW are set to 100. b_i's in BMM and SIW are set to zero. Σ's are set to identity matrix in IW/SIW/BMM. The purpose of setting values of θ_i's is to make them noninformative. Therefore, we make the standard deviations large. Alternatively, for SIW and BMM, we might use uniform distributions to replace the lognormal distribution. We tried uniform distribution from 0 to 100 and the estimates with the 2,000 sample size are very close to standard SIW and BMM specifications shown in Table 3.2. The relative absolute errors of estimates of mean, standard deviations, correlations are mostly within 5% with a maximum of 12%.

In addition to using the NUTS method for estimation, we also implemented random walk Monte Carlo (Gibbs sampling as in Allenby-Train procedure) using IW prior and extending the current Allenby-Train procedure with using HIW prior as well. We didn't implement SIW and BMM with random walk Monte Carlo here because they are too complicated. With the random walk Monte Carlo method, IW-AT reach stationary distributions fast and we use the first 1,000 iterations for burn-in and keep the last 4,000 iterations for estimation. However, HIW-AT take many iterations to reach stationary distributions and we use 100,000 iterations and 150,000 iterations for burn-in for 2,000 and 16,000 sample sizes. Readers can refer to Hoffman and Gelman (2014) about the superior property of NUTS against classical random walk Monte Carlo.

Table 3.2 shows additional results from the HB procedure with HIW/SIW/BMM, from which we can compare the estimation accuracy and computational time. In the parentheses underneath the estimates, we show the corresponding absolute percentage errors with respective to the sample value (values are in %).

Table 3.2 Estimation results with sample size 2,000

Mean	TRUE	Sample	IW-AT	IW-NUTS	HIW-AT	HIW-NUTS	SIW-NUTS	BMM-NUTS

$\log(\alpha_n)$	-0.5	-0.4998	-0.5215 [4.35]	-0.5161 [3.27]	-0.5088 [1.80]	-0.5052 [1.09]	-0.5059 [1.23]	-0.5046 [0.95]
β_S	1	1.0052	1.0051 [0.01]	1.0196 [1.43]	1.0236 [1.83]	1.0240 [1.87]	1.0245 [1.92]	1.0245 [1.92]
β_C	0.9	0.8985	0.8616 [4.10]	0.8603 [4.25]	0.8798 [2.08]	0.8698 [3.19]	0.8704 [3.12]	0.8692 [3.26]
Std.Dev	TRUE	Sample	IW-AT	IW-NUTS	HIW-AT	HIW-NUTS	SIW-NUTS	BMM-NUTS
$\log(\alpha_n)$	0.3	0.3007	0.3570 [18.73]	0.3379 [12.38]	0.3053 [1.54]	0.3011 [0.13]	0.3014 [0.22]	0.3020 [0.43]
β_S	0.4	0.4021	0.4505 [12.04]	0.4222 [4.99]	0.4005 [0.39]	0.3940 [2.01]	0.3933 [2.19]	0.3841 [4.48]
β_C	0.3	0.2989	0.3799 [26.42]	0.3310 [10.74]	0.2502 [16.28]	0.2636 [11.79]	0.2630 [12.03]	0.2618 [12.40]
Correlation	TRUE	Sample	IW-AT	IW-NUTS	HIW-AT	HIW-NUTS	SIW-NUTS	BMM-NUTS
$\log(\alpha_n), \beta_S$	0.8	0.8082	0.5298 [34.44]	0.6605 [18.27]	0.9002 [11.38]	0.8788 [8.73]	0.8839 [9.37]	0.8819 [9.12]
$\log(\alpha_n), \beta_C$	0.96	0.9608	0.4109 [57.23]	0.5405 [43.75]	0.8411 [12.46]	0.8436 [12.20]	0.8402 [12.55]	0.8563 [10.87]
β_S, β_C	0.6	0.6132	0.2978 [51.43]	0.4602 [24.95]	0.8096 [32.02]	0.7970 [29.98]	0.8056 [31.37]	0.8693 [41.77]

Covariance	TRUE	Sample	IW-AT	IW-NUTS	HIW-AT	HIW-NUTS	SIW-NUTS	BMM-NUTS
$\log(\alpha_n), \beta_S$	0.096	0.0977	0.0852 [12.77]	0.0942 [3.54]	0.1101 [12.68]	0.1042 [6.69]	0.1046 [7.02]	0.1021 [4.55]
$\log(\alpha_n), \beta_C$	0.0864	0.0863	0.0554 [35.76]	0.0604 [30.06]	0.0643 [25.54]	0.0668 [22.55]	0.0663 [23.14]	0.0675 [21.82]
β_S, β_C	0.072	0.0737	0.0507 [31.21]	0.0636 [13.66]	0.0811 [10.10]	0.0823 [11.70]	0.0826 [12.10]	0.0868 [17.72]
Time (hr)			0.02	1.9	0.3	14.9	50.2	29.6

First, we observe that the estimations of standard deviations from HIW/SIW/BMM are better than IW in general except for β_C. Second, for correlations, we observe that estimates from HIW/SIW/BMM are better than IW except for β_S, β_C. Third, for covariances, the estimates from alternative priors are better for β_S, β_C and $\log(\alpha_n), \beta_C$ especially β_S, β_C and the estimates deviations are similar for $\log(\alpha_n), \beta_S$. In summary, with relatively small sample size, the alternative priors perform better than IW. There is no clear evidence that any one among HIW/SIW/BMM is better than others. According to literature, BMM seems to be the best alternative. The estimation results using the random walk Monte Carlo (IW and HIW) are similar to those using NUTS. However, the computational time is significantly shorter. Even for HIW-AT which uses 100,000 iterations for burn-in, it takes less than half an hour to run compared to 15 hours using NUTS. The reason is that NUTS takes a sophisticated Hamiltonian Monte Carlo for each iteration of each parameter update which is more time-consuming. However, NUTS requires fewer iterations to reach a stationary distribution as we use 500 burn-in iterations against HIW-AT uses more than 100,000 iterations. More details about property of NUTS versus conventional random walk Monte Carlo can be found in Hoffman and Gelman (2014).

In addition to estimation accuracy, we also observe that the computational times for IW, HIW, SIW, and BMM are 1.9, 14.9, 50.2 and 29.6 hours respectively. As alternative priors have more parameters, they are computationally more demanding than IW. Among alternative priors, HIW is the most efficient in terms of computational time. Note that the presented computational times are based on our current Stan codes and alternative software or optimized codes might have different values.

To further support the comparison in terms of computational time, we check the trace plots of the parameters to see when the Markov chains reach stationary distributions. According to the Heidelberg Welch test and visual inspection of trace plots, those chains reach stationary distributions quickly. Fig. 3.5 shows a trace plot of covariance of $\log(\alpha_n)$, β_S under SIW after dropping the first 500 iterations of burn-in periods under NUTS. As we can observe from the plot, the stationary distribution is reached. According to Raftery and Lewis (1996), close to 4,000 iterations are required to achieve reasonable accuracy concerning the 2.5th (and 97.5th) percentiles. So we compare the computational time for 4000 iterations instead of comparing the computational time that the Markov chain needs to reach stationary distribution.

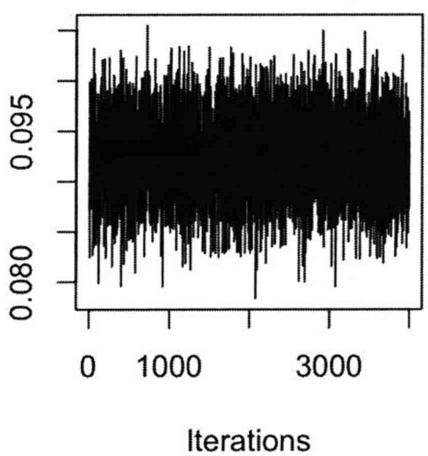

Fig. 3.5 Trace plot of covariance of $\log(\alpha_n)$, β_S under SIW

Table 3.3 presents the results of IW and alternative priors with a sample size of 16,000. Larger samples would improve the estimation performance and take a longer computational time.

Table 3.3 Estimation results with sample size 16,000

Mean	TRUE	Sample	IW-AT	IW-NUTS	HIW-AT	HIW-NUTS	SIW-NUTS	BMM-NUTS
$\log(\alpha_n)$	-0.5	-0.5024	-0.5114 [1.80]	-0.5080 [1.12]	-0.4989 [0.69]	-0.5021 [0.06]	-0.5016 [0.15]	-0.5019 [0.09]
β_S	1	0.9977	0.9818 [1.60]	0.9835 [1.43]	0.9847 [1.30]	0.9852 [1.26]	0.9852 [1.25]	0.9854 [1.23]
β_C	0.9	0.8976	0.9008 [0.36]	0.9043 [0.75]	0.9049 [0.81]	0.9087 [1.24]	0.9088 [1.25]	0.9086 [1.22]
Std.Dev	**TRUE**	**Sample**	**IW-AT**	**IW-NUTS**	**HIW-AT**	**HIW-NUTS**	**SIW-NUTS**	**BMM-NUTS**
$\log(\alpha_n)$	0.3	0.3011	0.3287 [9.18]	0.3167 [5.18]	0.3037 [0.87]	0.3004 [0.24]	0.3017 [0.20]	0.3014 [0.08]
β_S	0.4	0.4002	0.4289 [7.17]	0.4204 [5.05]	0.4020 [0.46]	0.4105 [2.58]	0.4010 [0.21]	0.4050 [1.20]
β_C	0.3	0.3013	0.3479 [15.47]	0.3289 [9.17]	0.2937 [2.51]	0.3042 [0.95]	0.2963 [1.66]	0.3030 [0.57]
Correlation	**TRUE**	**Sample**	**IW-AT**	**IW-NUTS**	**HIW-AT**	**HIW-NUTS**	**SIW-NUTS**	**BMM-NUTS**
$\log(\alpha_n), \beta_S$	0.8	0.8007	0.6153 [23.15]	0.6751 [15.68]	0.7275 [9.14]	0.7460 [6.83]	0.7633 [4.67]	0.7553 [5.67]
$\log(\alpha_n), \beta_C$	0.96	0.9604	0.6768 [29.53]	0.7638 [20.47]	0.9643 [0.40]	0.9433 [1.78]	0.9409 [2.03]	0.9382 [2.32]

β_S, β_C	0.6	0.6022	0.4103 [31.86]	0.4771 [20.77]	0.6918 [14.87]	0.5910 [1.87]	0.6723 [11.65]	0.6197 [2.90]
Covariance	TRUE	Sample	IW-AT	IW-NUTS	HIW-AT	HIW-NUTS	SIW-NUTS	BMM-NUTS
$\log(\alpha_n), \beta_S$	0.096	0.0965	0.0868 [10.09]	0.0899 [6.86]	0.0888 [7.95]	0.0919 [4.73]	0.0923 [4.37]	0.0921 [4.55]
$\log(\alpha_n), \beta_C$	0.0864	0.0871	0.0774 [11.13]	0.0796 [8.66]	0.0860 [1.23]	0.0861 [1.13]	0.0841 [3.49]	0.0856 [1.67]
β_S, β_C	0.072	0.0726	0.0612 [15.66]	0.0659 [9.28]	0.0817 [12.52]	0.0735 [1.28]	0.0795 [9.46]	0.0759 [4.61]
Time (hr)			0.2	51.9	6.6	90.5	262.8	218.3

From Table 3.2 and Table 3.3, we observe that IW using AT or NUTS with a 16,000 sample size perform slightly better than with 2000. The inflation in standard deviations and deflation in correlations are reduced but are still substantial. However, the estimates of parameters including standard deviations, correlations, and covariances from alternative priors including HIW-AT are much closer to the sample values than those from IW. Among three alternative priors, we cannot conclude which one is better. In addition, HIW takes less than half of the computational time by SIW and BMM.

Therefore, our findings from simulation study suggest that alternative priors can resolve the issues with IW when true values of standard deviations are small and HIW is the best among these alternative priors looking at the trade-off between estimation accuracy and computational efficiency.

3.4.2 Case study: Swissmetro

In the previous section, we identified the issues with the IW prior including inflated standard deviations and deflated correlations as well as the benefits with the HIW prior using Monte Carlo data. In order to further illustrate the issue with IW and benefits with the HIW, we compare the estimation results with IW and HIW from real stated preferences survey data.

The survey collects respondents' stated preferences of a new intercity passenger transportation mode called Swissmetro against the usual transport modes such as car and train. More details of the data can be found in Bierlaire et al. (2001). There were 1,192 respondents and each respondent made choices in nine choice situations. In each choice situation, the respondents were presented with three mode alternatives of Swissmetro (SM), train, and car. The utility functions for consumer n are in willingness-to-pay form and include alternative specific constants for SM and car, travel time and travel cost shown as Equations (3.17-3.19).

$$V_{Train,n} = (-\frac{Cost_{Train}}{factor} - \exp(\beta_{Time,n}) * \frac{Time_{Train}}{factor})/\exp(\beta_{Scale,n}) \tag{3.17}$$

$$V_{SM,n} = (ASC_{SM,n} - \frac{Cost_{SM}}{factor} - \exp(\beta_{Time,n}) * \frac{Time_{SM}}{factor})/\exp(\beta_{Scale,n}) \tag{3.18}$$

$$V_{Car,n} = (ASC_{Car,n} - \frac{Cost_{Car}}{factor} - \exp(\beta_{Time,n}) * \frac{Time_{Car}}{factor})/\exp(\beta_{Scale,n}) \tag{3.19}$$

In order to compare the IW and HIW, we scaled the data in utility function, i.e., divided travel time and travel cost by a scaling factor. Under different scaling factors, the magnitudes of some estimates including standard deviations might be different. We use scaling factors of 100 or 1,000 and the estimates are shown in Table 3.4. The estimation is conducted through NUTS. As an example, IW100 denotes estimation using IW prior and scale being 100.

Table 3.4 Estimation of different scaled Swissmetro data with IW and HIW

Mean	IW100	HIW100	IW1000	HIW1000
ASC_SM	0.1615	0.1307	0.0814	0.0139
ASC_Car	0.4129	0.3939	0.1527	0.0407

B_Scale	-1.6490	-1.6867	-2.7707	-3.9736
B_Time	0.0275	0.0196	1.0166	0.0243
StdDev	IW100	HIW100	IW1000	HIW1000
ASC_SM	0.6373	0.5918	0.2656	0.0591
ASC_Car	1.1882	1.1418	0.3687	0.1173
B_Scale	0.8734	0.9074	0.6914	0.9033
B_Time	1.1788	1.1844	1.0283	1.1984
Correlation	IW100	HIW100	IW1000	HIW1000
ASC_SM, ASC_Car	0.0721	0.0365	0.0426	0.0448
ASC_SM, B_Scale	-0.0108	-0.0287	-0.0407	-0.0490
ASC_SM, B_Time	-0.1578	-0.1679	-0.1181	-0.1498
ASC_Car, B_Scale	0.2744	0.3302	-0.0674	0.3366
ASC_Car, B_Time	0.6963	0.7033	0.4720	0.7127
B_Scale, B_Time	0.3313	0.3747	0.0753	0.4002

As we can observe from Table 3.4, the estimates at 100 scale with IW and HIW are similar where the standard deviations are close to 1. The estimates at 1,000 scale with IW deviate substantially from estimates with HIW where standard deviations of ASC_SM and ASC_Car are small. Specifically, estimates of standard deviations of ASC_SM and ASC_Car from IW are larger than those from HIW and estimates of correlations of ASC_Car and B_Scale, B_Scale and B_Time from IW are smaller than those from HIW. The computational time for each run is around 35 minutes and there is no substantial difference between IW and HIW in terms of computational time. The findings from Swissmetro data indicating inflated standard deviations and deflated correlations are consistent with previous findings in simulation.

3.5 Conclusion

In this chapter, we illustrate the issues with using IW in discrete choice models (logit mixture) as prior for the covariance matrix in the hierarchical Bayes procedure which has not been identified before in transportation, economics, and marketing. Specifically, the estimates of standard deviations inflate and the estimates of correlations deflate when the true values of standard deviations are small. We review three alternative priors proposed in the literature, HIW, SIW and BMM, to address the issues with IW and implement them in Stan (NUTS) and conventional MCMC (Allenby-Train procedure) to enhance the HB procedure for logit mixture.

The estimation accuracy achieved by alternative priors is better than IW by Monte Carlo simulation. However, HIW is much more efficient in terms of computational time than SIW and BMM which makes it the best alternative to IW in HB procedure for logit mixture. Real data case study using stated preferences data of Swissmetro further supports the issues of using IW prior and the benefits of HIW.

Based on the findings from this chapter, we suggest that when there is doubt about the use of IW, one should use HIW in the estimation procedure since it is a more accurate and still a computationally efficient alternative prior. Empirically in the choice of grapes example, we found issues of deflated correlations when all the standard deviations are set to 0.6. When precise knowledge is needed for a covariance matrix, one could use a separation strategy such as BMM in the estimation procedure.

Chapter 4

Personalized Menu Optimization

This chapter first introduces the personalized menu optimization model and its alternative formulations. Then we evaluate the personalized menu optimization using logit mixture against benchmark methods: content-based recommendation algorithms and non-personalized menu optimization that uses non-personalized choice model estimates. The evaluations are carried out through a Boston case study, using real data from a revealed preferences survey of greater Boston area travelers.

4.1 Personalized menu optimization overview

The personalized menu optimization consists of a travel behavioral model and a choice-based optimization model. Typically, a logit mixture model is used due to its capability of capturing individual-level taste preferences. In other words, we need differentiation of individuals in order to provide personalized menus to them. In this section, we will introduce a standard model for personalized menu optimization, some of its variants, and its solution methods.

First, we introduce the model with standard revenue maximization objective. Let p_j be the revenue associated with alternative j ($j = 1, \dots, NC$). It can be monetary revenue or other measures which are beneficial for the system operators, like energy savings in the context of Tripod. *NC* denotes total number of alternatives in the full choice set. Let *M* be the size of menu which is a list of travel alternatives to be presented on a smartphone app. It is common among recommender systems for mobile apps to have a size constraint to avoid information overload (Liu et al., 2011; Zhuang et al., 2011). According to knowledge from consumer psychology, the consumers face a two-stage process. They will not consider all the alternatives presented to them. They will first screen and then consider more seriously a much smaller set of alternatives (Hauser and Wernerfelt, 1990).

Let v_{nj} be the exponential utility of alternative j for consumer n. Note that if we use a multinomial logit model which is common in existing literature as in FMOD (Atasoy et al., 2015a; Atasoy et al., 2015b) and assortment optimization (Davis et al., 2013), the utilities are the same across different consumers if there are no individual-specific attributes in the utility function, i.e., we can omit index n in such case. In preference space, v_{nj} can be in general represented by $v_{nj} = \zeta_n z_j$. In willingness-to-pay space, we scale the travel cost coefficient to be -1. We can use lognormal distribution instead of normal distribution for coefficients that are known to be positive or negative in order to control the sign. Example of willingness-to-pay space with lognormal coefficients can be found in Section 4.2.

The personalized menu optimization will choose an M-size subset (menu) out of NC available alternatives for consumer n in order to maximize the expected revenue provided by the menu. The model is given as follows:

$$\max_{x_j, j=1,\dots,NC} \sum_{j=1}^{NC} p_j * \frac{x_j v_{nj}}{\sum_{j'=1}^{NC} x_{j'} v_{nj'} + v_{opt}} \tag{4.1}$$

subject to

$$\sum_{j=1}^{NC} x_j \leq M \tag{4.2}$$

$$x_j \in \{0,1\}, \forall j \in \{1, \dots, NC\} \tag{4.3}$$

where the objective function is a function of the binary decision variables through the choice probability; v_{opt} denotes the exponential utility of opt-out alternative (not choosing anything on the menu). The formula $\frac{x_j v_{nj}}{\sum_{j'=1}^{NC} x_{j'} v_{nj'} + v_{opt}}$ denotes the choice probability of alternative j for consumer n. The choice probability is a variable of the model as it includes the binary decision variables; $x_j = 1$ if alternative j is on the menu, $x_j = 0$ otherwise. Therefore, choice probability for alternative j will be 0 if is not presented on the menu.

The model (4.1-4.3) is complicated as the objective function is nonlinear with binary decision variables. When NC is small, this problem is easy to solve by enumerating and comparing objectives of all the feasible solutions. We have 16 travel options in total in the case study and it could involve enumerating more than 10,000 different solutions and picking the best among them. When *NC* grows large, the total number of solutions quickly becomes large due to its combinatorial nature. The model (4.1-4.3) has been applied in transportation context such as FMOD (Atasoy et al., 2015a; Atasoy et al., 2015b). When p_j is one, the objective of this model is to maximize the expected hit rate (probability that one of the offered alternatives is chosen) in recommender system literature.

This model (4.1-4.3) can be reformulated by replacing decision variables with new decision variables. We omit index *n* in the following equations. Particularly, we introduce w_j to be the choice probability of alternative *j*, i.e., $\frac{x_j v_{nj}}{\sum_{j'=1}^{NC} x_{j'} v_{nj'} + v_{opt}}$. Let w_0 denote the choice probability of opt-out alternative, i.e., $\frac{v_{opt}}{\sum_{j'=1}^{NC} x_{j'} v_{nj'} + v_{opt}}$. The reformulated model is presented in equations (4.4-4.7).

$$\max_{w_j, j=1,\dots,NC} \sum_{j=1}^{NC} p_j w_j \tag{4.4}$$

subject to

$$\sum_{j=1}^{NC} w_j + w_0 = 1 \tag{4.5}$$

$$\sum_{j=1}^{NC} w_j / v_j \le M \tag{4.6}$$

$$0 \le \frac{w_j}{v_j} \le w_0, \forall j \in \{1, \dots, NC\} \tag{4.7}$$

where the objective is still to maximize the expected revenue, constraint (4.5) denotes that the sum of choice probabilities equals one, and constraints (4.6) and (4.7) are based on substituting new decision variable and correspond to the menu size constraint. The new model is a common linear programming problem and can be solved by any linear programming solvers.

As we discussed in Chapter 2, the HB estimator is often used for logit mixture due to its improved estimation performance. When the HB estimator is used, instead of point estimate of ζ_n, we obtain a sequence of MCMC draws of ζ_n. Let v_{jn}^{ns} be the exponential utility of alternative j for consumer n using ns-th draw where $ns \in \{1, \dots, NS\}$. The objective of maximizing revenue can be rewritten as equation (4.8) with the same menu size constraint.

$$\max_{x_j, j\in\{1,\dots,NC\}} \sum_{j=1}^{NC} x_j \left(\frac{1}{NS} \sum_{ns=1}^{NS} \frac{p_j v_{jn}^{ns}}{\sum_{j'=1}^{NC} x_{j'} v_{jn}^{ns} + v_{opt}} \right) \tag{4.8}$$

This problem is difficult to solve with a nonlinear objective function with binary decision variables. The previous solution methodology cannot be applied here because there is an additional summation over number of draws. As a result, introducing new decision variables as in previous solution method would not lead to linear objective function. An exact solution is difficult to obtain and may not be efficient in terms of computational time.

If we assume that $v_{opt} = \sum_{j'=1}^{NC} (1 - x_{j'}) v_{jn}^{ns}$, i.e., the opt-out alternative represents all other alternatives that are not on the menu, then the objective becomes linear as in equation (4.9) which makes the problem easy to solve. This assumption is valid when the scale parameter for alternatives on the menu is the same as the scale parameter for alternatives off the menu. In other words, for the same alternative on or not on the menu, its choice probability remains the same. It would be close to reality when the size of the full choice set is small and users are aware of all the alternatives. This is a strong assumption especially when the choice set is large. It is an approximation to the model (4.8).

$$\max_{x_j, j\in\{1,\dots,NC\}} \sum_{j=1}^{NC} x_j \left(\frac{1}{NS} \sum_{ns=1}^{NS} \frac{p_j v_{jn}^{ns}}{\sum_{j'=1}^{NC} v_{jn}^{ns}} \right) \tag{4.9}$$

Particularly, we can simply sort the NC alternatives by $\sum_{ns=1}^{NS} \frac{p_j v_{jn}^{ns}}{\sum_{j'=1}^{NC} v_{jn}^{ns}}$ and find the top M alternatives which will be the solution to problem (4.9). In Section 4.2, we present a case study using this model with the same opt-out assumption as the size of full choice set is no greater than 16.

Besides revenue maximization or hit rate maximization objectives, transportation service providers may want to maximize the consumer's satisfaction. Here we present an alternative objective function that is to maximize the expected maximum utility of the options in the menu which represents the consumer surplus. Specifically, we use the log-sum formula to denote this objective function (Ben-Akiva, 1973). The marginal utility of income is omitted in equation (4.10) since it doesn't affect the optimization result.

$$\max_{x_j, j\in\{1,\dots,NC\}} \frac{1}{NS}\sum_{ns=1}^{NS}\left[log\sum_{j=1}^{NC} x_j v_{jn}^{ns}\right] \tag{4.10}$$

As this nonlinear objective function cannot be transformed into linear form and its decision variables are binary, it's difficult to solve problem (4.10) exactly. However, if the variations across different draws are not large, the exact solution may be the same or very similar to the solution to a simpler problem that moves average across draws inside the log-sum formula, i.e., $log\sum_{j=1}^{NC} x_j(\frac{1}{NS}\sum_{ns=1}^{NS} v_{jn}^{ns})$. The simpler problem is again easy to solve by finding the top M alternatives in terms of average utility $\frac{1}{NS}\sum_{ns=1}^{NS} v_{jn}^{ns}$. We focus on hit rate maximization in this thesis and do not use this model for the case study in Section 4.2.

Recommender systems require updating individual preferences continuously and in real-time. However, for logit mixture with individual and population level coefficients, re-estimating all the coefficients frequently using the Allenby-Train HB procedure might not be feasible due to computational limitations. In order to update individual preferences in real time, two interacting and repeated steps are applied: the offline and the online procedures.

The offline estimation procedure updates individual as well as population level parameters for all individuals. Periodically (e.g., every week), data are pooled and all coefficients (μ, Ω, and ζ_n) are updated to reflect the effects of all choices made by all individuals since the last update. This is done by applying the full HB procedure and obtaining draws from all posterior distributions.

The online estimation procedure updates individuals' preferences in real time as they make repeated choices. The individual specific parameters (ζ_n) are updated after every choice using the same MH procedure (i.e., step 3 of the Allenby-Train procedure), assuming that the population parameters μ and Ω are fixed. This update is computationally efficient, and it is done for the specific individual who has made a new choice. Danaf et al. (2018) showed that this method yields results that are close to those obtained from the full HB procedure. Therefore, the online update is important and efficient for improving the individual level parameters that will be used for the next choice of the user.

We evaluate the proposed personalized menu optimization against benchmark methods using non-personalized choice model estimates and content-based recommendation algorithms in the context of a trip planner in the following section.

4.2 Evaluation of personalized menu optimization

In this section, we will use data from the Massachusetts Travel Survey to evaluate the performance of personalized menu optimization against benchmark methods. First, we will describe those benchmark methods. Then we will introduce the setup of the case study and finally conduct evaluation experiments.

4.2.1 Benchmark methods

In this section, we describe the benchmark methods that we will use to evaluate against PMO that was introduced in previous section. Those methods include a non-personalized menu optimization (*NP*) and two content-based recommendation algorithms (*CB1* and *CB2*). When the system generates menus for trip choice X+1, we've conducted hierarchical Bayes estimation using historical data for the first X trip choices. The estimation results will be used by both PMO and NP.

- ***PMO***: For personalized menu optimization, it uses conditional posterior draws of individual (or menu-specific) coefficients to compute the utility of each alternative as in model (4.8). The solution to the model is the offered menu for that individual.

- *NP*: For the non-personalized menu optimization, it uses unconditional posterior of individual level parameter which doesn't capture the individual-level taste preferences to plug into the model (4.8). The solution to the model is the offered menu for that individual.
- ***CB1***: For the content-based recommendation algorithms, we treat mode as the only description feature of the travel option. The standard content-based recommendation algorithm would recommend travel options based on the empirical probability of each mode being chosen based on individual-specific history (Pazzani and Billsus 2007). For example, for a given individual, if bike mode is chosen in 2 out of 4 previous scenarios, the empirical probability of bike for fifth choice scenario is 0.5. We break ties of the same empirical probability by shorter travel time (as we have the same travel cost among the same mode). We call this CB1 index or CB1 method. The solution is to select M travel options with highest CB1 index.
- ***CB2***: Since we know individuals prefer shorter travel time, we enhance the CB1 index by multiplying it with an exponential term with negative travel time in order to adjust CB1 with their attractiveness with respect to travel time, which we call CB2 index. The CB2 index for alternative j of user n at choice scenario t would be as follows.

  $$CB2_{jnt} = CB1_{jnt} * \exp\left(-\theta * TT_{jnt}\right) \tag{4.13}$$

  where $CB1_{jnt}$ denotes the corresponding CB1 index; θ is a positive coefficient that controls the impact of travel time of the alternative. In the case study, we vary θ among different values (between 0.1 and 1) and find $\theta = 1$ performs the best. So we use $\theta = 1$ in the later study. For CB2 method, we choose M alternatives with the highest CB2 index.

For trip choice X+1, we apply different recommendation methods based on choice history data from trip 1 to trip X and attributes of available alternatives for choice X+1. Each method provides its own menu for each user. In the experiments, we vary the menu size constraints from 2 to 10.

4.2.2 Boston case study overview

In this and the following section, the added value of using the proposed methodology is illustrated against its counterparts in a Boston case study using real data. The case study is built upon real travelers and their travel history provided by MTS (Massachusetts Travel Survey). MTS includes travel diaries for 33,000 individuals belonging to 15,000 households. The data was collected between June 2010 and November 2011. Individuals were asked to fill out all the activities they performed in a designated weekday (24 hours, Monday to Friday), and to provide the activity location, the transport mode used to arrive at this location, the arrival and departure times, and accompanying household or non-household members.

In Song et al. (2018a), the user population in the case study is constructed from MTS data with individuals who have made at least five tours in the specified time period, which is of sample size 246. In this thesis, we extend the user population by randomly drawing a 5,154-size sample made up of people who have made one or more trips during the time period. Table 4.1 shows the number of trips made by individuals in the sample. The first column denotes which trip it is for each individual. The second column tells us how many individuals have made this number of trips. For example, 4408 individuals have made at least two trips; 5,154 individuals made at least one trip.

Table 4.1 Number of trips in the sample

Trip index	Remaining individuals
1	5,154
2	4,408
3	4,074
4	3,401
5	3,047

6	2,613
7	2,347
8	1,952
9	1,733
10	1,414

Their travel choice sets for various choice scenarios are also constructed by using Google Map API with origin, destination, and departure time from MTS data. There are five travel modes considered in each individual's choice set including walk, bike, car, car-pool, and transit. The sizes of choice set vary from 4 to 16 representing different routes and modes (some modes are not always available). With this data generation procedure, we can construct a group of users whose travel history, full choice sets, and selected alternatives are known. Assume that they were users of an app-based trip planner like Tripod and all their trips were made through Tripod. Then we have a group of Tripod users whose travel history is known. They are 1,414 individuals having made at least 10 trips. In this dataset, most users prefer to travel by car (more than 80% of total trips). Few trips are made in non-driving modes including 6% in walk, 1% in bike and 12% in transit. In terms of the whole MTS population, most trips (around 72%) are by car or car-pool. Few trips are made in non-driving including 10% in walk. In general, the modal share between selected sample and population are similar.

Table 4.2 Proportion of travel modes

Modes	Count	Proportion
Walk	2,416	0.06
Bike	376	0.01
Car	25,980	0.60

Carpool	9,757	0.22
Transit	5,097	0.12

With the choice sets and revealed choices from MTS data, one can estimate a logit mixture model capturing individuals' preferences of travel choices. The utility function is in willingness-to-pay specification with preference parameters including alternative specific constants (ASC) for bike, car, car-pooling, and transit, coefficient for travel time and travel cost (See equation 4.11). All ASCs are normally distributed while travel time/cost coefficient is transformed to be log-normally distributed to ensure that travel time enters the utility with a negative sign as expected.

Utility function individual n choosing option j of mode m:

$$U_{nj} = (ASC_m - \exp(\zeta_{n,Traveltime})TravelTime_j - TravelCost_j)/\exp(\zeta_{n,TravelCost}) \qquad (4.11)$$

where ASC_m denotes the alternative specific constant for alternative j's mode m except for mode walk.

It is assumed that user travel choice behavior follows a logit mixture model and it is also assumed that the offline update of logit mixture estimates based on all the choice situations of all individuals in the sample are the "true" preference parameter estimates, which are used to calculate hit rate. The individual may choose one option on the menu or opt-out, i.e., reject the all the options on the menu. The hit rate is defined as the probability that the user will choose one option on the menu, which is also equal to 1 minus the opt-out probability.

In this case study, the opt-out utility is defined as the expected maximum utility of all the options that are not on the menu, which is given by the log-sum as equation (4.12).

$$\mathrm{P_{opt}} = \frac{\sum_{j=1}^{NC}(1 - x_j)v_{nj}}{\sum_{j'=1}^{NC} v_{nj'}} \qquad (4.12)$$

The objective is to maximize the expected hit rate. Following such opt-out assumption, the problem reduces to problem (4.9) which is easy to solve by a sorting algorithm. The performance evaluation focuses on hit probability.

In this case study, we first use logit mixture with inter-consumer heterogeneity as the underlying choice model, then show the results under inter-and intra-consumer heterogeneity. The estimation of both models is through MCMC. Particularly, the HB procedure for logit mixture with inter- and intra-consumer heterogeneity can be found in Appendix A.

4.2.3 Evaluation under inter-consumer heterogeneity

First, we show the comparison among four methods under logit mixture with inter-consumer heterogeneity in Fig. 4.1 and Fig. 4.2. Fig. 4.1 shows the comparison among personalized (PMO), non-personalized (NP), and content-based methods (CB1 and CB2) on fifth trip choice. The horizontal axis denotes different menu sizes. The primary vertical axis shows the hit rate. The blue, orange, gray, and yellow bars denote CB1, CB2, NP, and PMO hit rate respectively. It is observed from Fig. 4.1 that personalized menu optimization outperforms non-personalized and content-based methods in terms of hit rate under various menu sizes. For non-personalized versus personalized, the gaps are large when menu sizes are medium and decrease with increasing menu size. In addition, we can notice that both CB methods perform very similarly to each other so we just use CB to denote both methods in later discussion. We can also observe that CB in general performs better than NP.

For some individuals, their trip menu generation results are the same for personalized and benchmark method. Therefore, we plot three additional lines which focus on average difference in hits for those trips with different menu generations for PMO versus NP, PMO versus CB1, and PMO versus CB2. For PMO versus NP, we observed that the average difference tops at medium menu size and then decreases. When menu size is small, the top alternatives identified by PMO and NP are similar therefore the average difference in hit rate is small. When menu size is large, i.e., many alternatives are already included on the menu, it is likely that the majority of critical alternative are chosen by PMO and NP, and therefore the average difference in hit rate is small across difference comparisons we make. For PMO versus CB, we observed that the average differences in hit rate across different menus sizes are often small and decrease with increasing menu size. However, the average number of menus across different menu sizes that PMO is different from NP is 925 but this number for PMO versus CB is 1,942. In other words, CB is generating more distinct menus than NP but its differences with PMO are usually smaller.

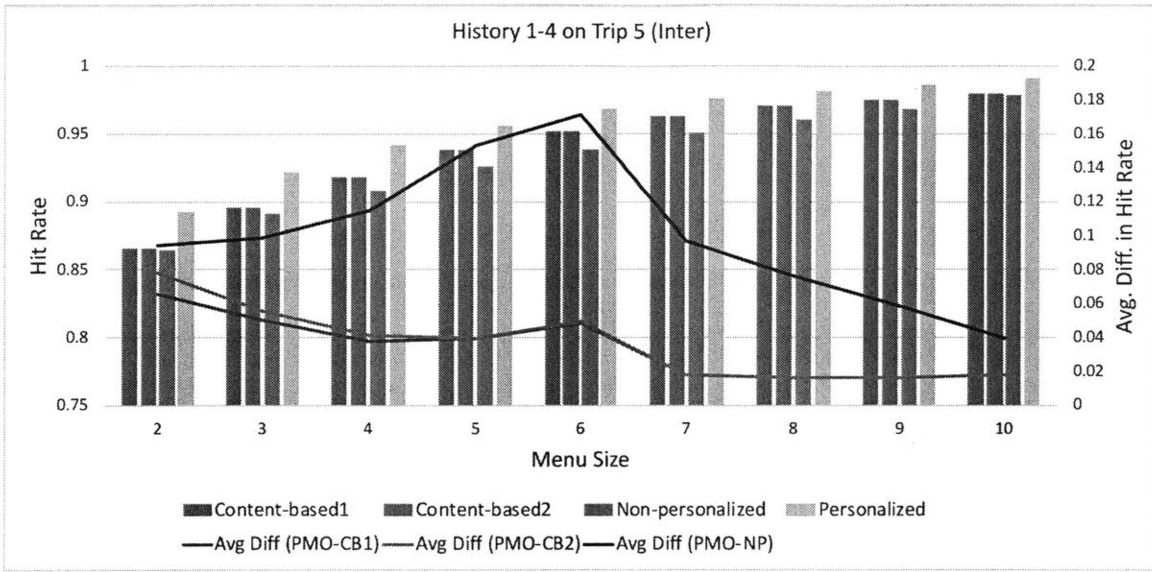

Fig. 4.1 Personalized versus non-personalized and content-based (test on trip 5) under inter-consumer heterogeneity

More trip data (Fig. 4.2) shows a similar pattern but the average difference (gap) is even larger for all benchmarks. This implies that the benefits of personalization are higher when there is more precise knowledge about choice probabilities as expected. It also implies that with less data, there might be "shrinkage effect" for hierarchical Bayesian estimation where individual level estimates are shrinking towards population-level parameters. For NP versus CB, CB still performs in general better than NP while the gap becomes smaller when menu size increases. The NP captures more critical options than CB when menu size is large.

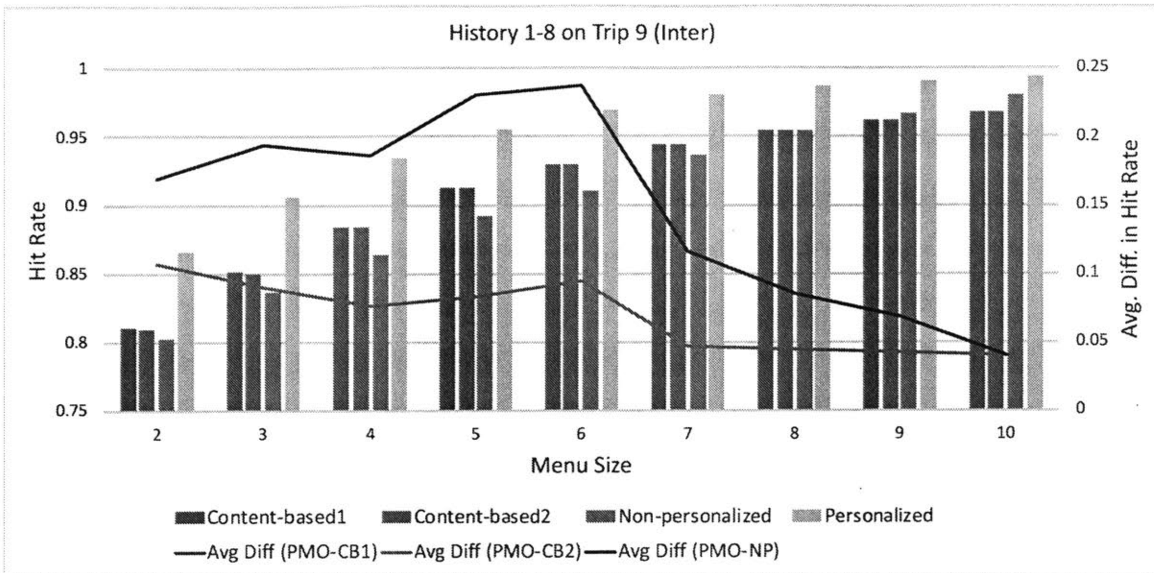

Fig 4.2 Personalized versus non-personalized and content-based (test on trip 9) under inter-consumer heterogeneity

4.2.4 Evaluation under inter- and intra-consumer heterogeneity

In addition to comparison under inter-consumer heterogeneity, we conduct comparison under inter- and intra-consumer heterogeneity and illustrate them in Fig. 4.3 and Fig. 4.4. Similarly, we can observe that PMO performs better than all the benchmark methods. In addition, by comparing corresponding results with inter-consumer heterogeneity, we can observe that the performance of both PMO and NP becomes better, which indicate the more sophisticated choice model is better at capturing user choice behavior. The gaps between PMO and NP, and PMO and CB methods quickly become smaller when menu size increases, which differs from the bell-shape pattern under inter-consumer heterogeneity, and means that the hit rates for top options are not as high as those under inter-only. Furthermore, NP performs better than CB methods with much smaller average difference in hit rate. Under more complicated choice behavior, CB methods performs worse than under inter-consumer heterogeneity. For CB methods, their performance is often very close, while CB2 performs better than CB1 when the menu size is small, which means weighting with respect to attractiveness of travel time helps improve the performance.

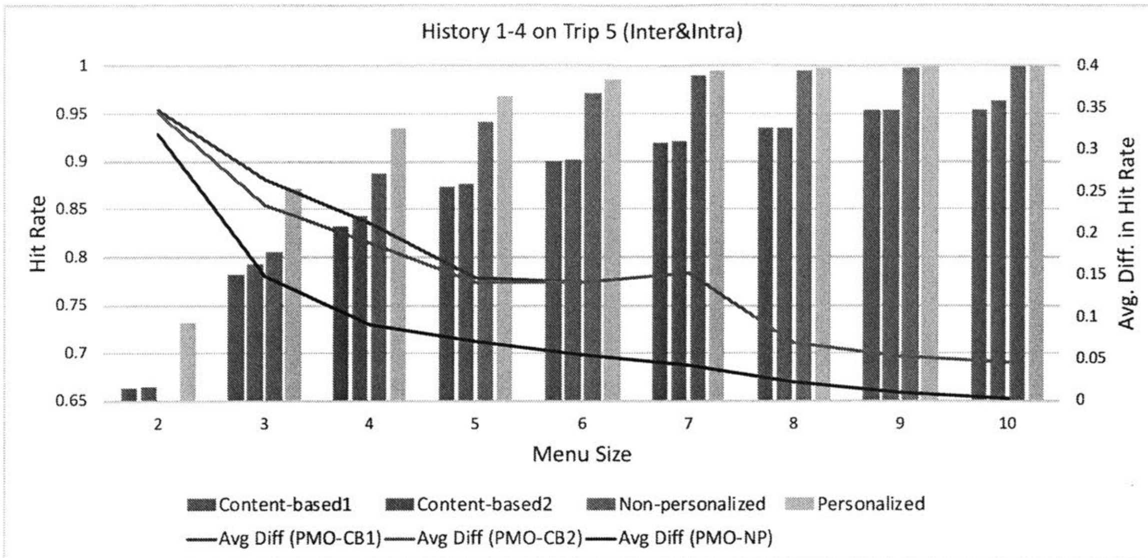

Fig. 4.3 Personalized versus non-personalized and content-based (test on trip 5) under inter- and intra-consumer heterogeneity

Fig. 4.4 shows the results tested on trip 9 under logit mixture with inter- and intra-consumer heterogeneity. Fig. 4.4 shows a similar pattern as Fig. 4.3 where both PMO and NP perform better than CB methods. The gap between PMO and NP is small and decreases as menu size increases. Note the line plots for PMO versus both CB methods are too close to distinguish in Fig. 4.4. In addition, Fig. 4.4 shows that with more choice data, the performance of PMO becomes better, which is consistent with the findings under inter-consumer heterogeneity.

Please note that the evaluation or calculation of hit rate for the previous two sections is based on assuming the "true" user behavior following logit mixture with inter-consumer heterogeneity and logit mixture with inter- and intra-consumer heterogeneity. In order to have a more meaningful comparison, we should use a consistent evaluation measure across inter-consumer heterogeneity and inter- and intra-consumer heterogeneity, such as whether the "true" choices of users for trip 5 or trip 9 have been chosen on the recommended menus or not.

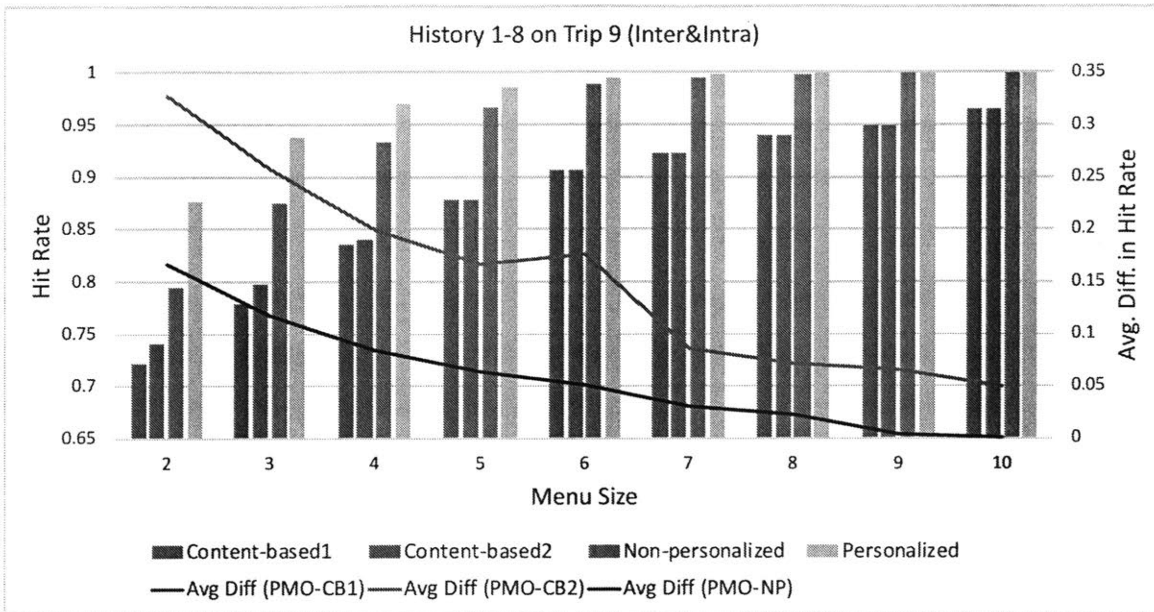

Fig. 4.4 Personalized versus non-personalized and content-based (test on trip 9) under inter- and intra-consumer heterogeneity

4.3 Conclusion

In this chapter, we proposed a novel personalized menu optimization using personalized estimates from logit mixture with inter- and intra-consumer heterogeneity and evaluated the proposed method against benchmark methods including non-personalized menu optimization that uses non-personalized estimates from choice model and content-based recommendation algorithms. Through a real-data case study, we illustrate that the personalized performs better than benchmark methods under inter- (and intra-) consumer heterogeneity.

Under inter-consumer heterogeneity, the gap between personalized and non-personalized is small when the menu size is small where few top alternatives are identified and when the menu size is close to size of full choice set where most critical alternatives are chosen on the menu. The content-based recommendation algorithms provide slightly better hit results than non-personalized, while it generates much more distinct menus than personalized. The benefits of personalization are more salient when more choice data are available, which is expected as more accurate estimates are obtained.

Under inter-and intra-consumer heterogeneity, both personalized and non-personalized methods have better performance, and the gap between them becomes smaller as menu size increases. However, content-based recommendation algorithms perform worse than under inter-consumer heterogeneity.

Chapter 5

Sequential Personalized Menu Optimization through Bandit Learning

5.1 Background

In previous studies, we illustrated the personalized menu optimization (PMO) model where a customer's choice behavior is captured by a known discrete choice model. However, in practice, the parameter value of the choice model is often not known and has to be learned. Previously, we applied a preference updater that is based on the HB estimator of logit mixture to provide PMO up-to-date estimates of the parameters of choice model. We didn't take into account learning the uncertain parameter values when making the recommendation decision. There might be cases where current estimates from the HB procedure indicate a car is the optimal alternative and PMO always offer a car alternative but actually a bus is the favorite alternative of the user. In such case, we need to go beyond exploit-only strategy which is to offer an "optimal" menu based on current estimates of the parameters but to explore other menus which may turn out to be optimal. Such problems that involve the trade-off between exploration and exploitation are often formulated as multi-armed bandit (MAB) problems. Most MAB problems do not have exact solutions except for some special cases such as classical application of the Gittins index (Scott, 2010). Many heuristics have been proposed to solve the problem as we have reviewed in Chapter 2.

In this chapter, we propose a novel method called *PMO-UCB* which is built upon the classical upper confidence bound (UCB) algorithm (Auer et al., 2002) and the previous PMO model. Particularly, we focus on a problem with menu size 1 in order to provide a proof-of-concept. The case with larger menu size would lead to a combinatorial bandit problem where the rewards of each arm are dependent on each other, where existing techniques such as UCB do not work (Chen et al., 2017) and which requires a more complicated solution method; therefore, it is not addressed in this thesis. In addition, we assume each alternative is a different mode in this thesis. Therefore, choosing alternative/mode X does not give preference information about alternative/mode Y.

The method is novel with respect to existing multi-armed bandit algorithms as its exploitation or expected reward is estimated by an HB estimator of logit mixture which differs from the simple empirical mean in classical UCB algorithm (Auer et al., 2002). Since we use *CB1* to denote empirical mean of historical mode choice, we use *CB1-UCB* to denote classical UCB algorithm in this chapter for consistency. We compared PMO against CB1 in Chapter 4. In this chapter, we will compare the proposed PMO-UCB against all the alternative methods including empirical mean (CB1), classical UCB algorithm (CB1-UCB) and exploit-only algorithm based on logit mixture estimates (PMO) under different conditions.

5.2 Problem and solutions

In this section, we present the problem formulation and solutions for the problem. Fig. 5.1 illustrates the decision process of the operator in a Tripod context. Assume *T* is the operational horizon length. At each time period, there are *N* consumers coming (in Fig. 5.1 we simplified *N*=1). The operator needs to decide which menu to offer (or in our case which alternative to offer) based on choice/menu history. After the operator offers the menu, the consumers need to decide whether to choose the alternative or opt out (choose nothing on the menu). After consumers made their choices, the operator needs to update the history, particularly the estimates of parameters of the choice model.

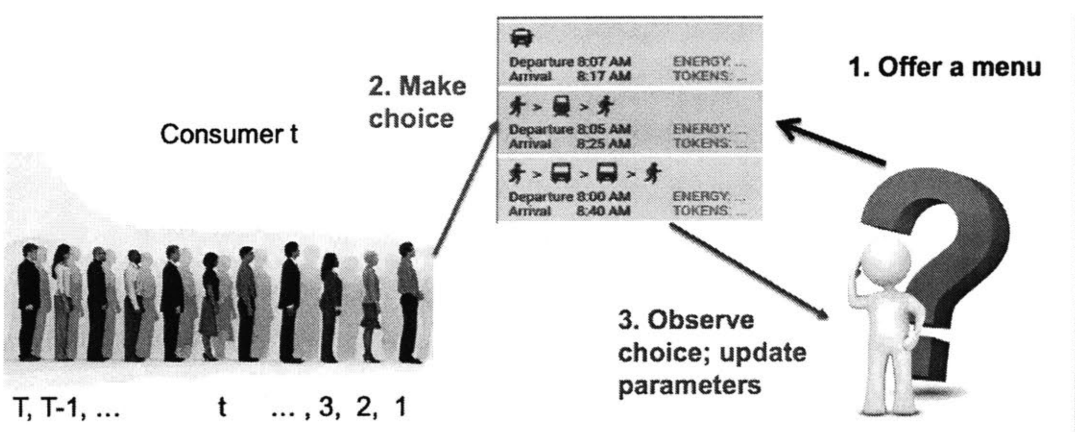

Fig. 5.1 Decision process

To further illustrate the problem, the objective of a single period is to maximize the hit probability by deciding which alternative to offer. Let P_{jnt} denote the choice probability of alternative *j* for consumer *n* at time *t*. Let $I(j, n, \tau)$ denote whether alternative *j* is offered to user *n* at time period τ. Since time period *t*, the operator needs to decide which alternative to be offered will maximize the total expected hit.

$$\max_{I(j,n,\tau),\forall \mathrm{j},\tau} \sum_{\tau=t}^{T} \sum_{n=1}^{N} P_{jn\tau} I(j, n, \tau) \tag{5.1}$$

Subject to

$$\sum_{j=1}^{NC} I(j, n, \tau) = 1, \forall n, \forall \tau \tag{5.2}$$

At time period *t*, actually operator just needs to decide on $I(j, n, t)$ based on all the choice history until time period *t-1*. Additionally, the choice probabilities in the future are estimated based on history including time period *t*. This problem does not have an exact solution.

In this study, we assume that the choice behavior follows logit mixture. Assume there is a clairvoyant who knows all the true parameter values. For logit mixture with inter- and intra-consumer heterogeneity, the choice probability of alternative j for user n at time period t is as follows.

$$P_{jnt}(\eta_{nt}) = \frac{\exp\left(\mathrm{u_{jnt}}(\eta_{\mathrm{nt}})\right)}{1 + \exp(\mathrm{u_{jnt}}(\eta_{\mathrm{nt}}))} \tag{5.3}$$

where $\mathrm{u_{jnt}}(\eta_{\mathrm{nt}})$ denotes the utility based on individual- and choice-situation -specific parameter η_{nt}.

$$\eta_{\mathrm{nt}} \sim \mathcal{N}(\zeta_n, \Omega_w) \tag{5.4}$$

$$\zeta_n \sim \mathcal{N}(\mu, \Omega_b) \tag{5.5}$$

where ζ_n denotes parameter mean vector for user n, Ω_w denotes covariance matrix of intra-consumer heterogeneity, μ denotes parameter mean vector of the sample, and Ω_b denotes covariance matrix of inter-consumer heterogeneity.

The estimation of η_{nt} can be done based on previous *t-1* time periods of choice historical data through the five-step HB procedure presented in Becker et al. (2018) and Appendix A. Since each time period has its own posterior estimates, we use $\eta_{nt}^{t-1,ns}$ denoting *ns*-th draw of *t-1*th estimation, which will be used for personalized menu optimization at time period *t*.

Let $r_{jnt} = P_j(\eta_{nt})$ denote the expected reward or "revenue" for the operator.

For the clairvoyant who knows all the true parameter values η_{nt}^*, the optimal menu for user *n* at time period *t* will be:

$$j_{nt}^* = argmax_j\, P_{jnt}(\eta_{nt}^*) \tag{5.6}$$

For the operator who has posterior estimates based on *t-1* choice history, the expected reward for menu *j* at time period *t* for user *n* would be:

$$\overline{r_{jnt}}(\eta_{nt}^{t-1}) = \frac{1}{NS} \sum_{ns=1}^{NS} \frac{\exp\left(u_{jnt}(\eta_{nt}^{t-1,ns})\right)}{1 + \exp(u_{jnt}(\eta_{nt}^{t-1,ns}))} \tag{5.7}$$

If we only consider exploitation, which is to obtain the maximum immediate revenue based on current knowledge, we choose to offer the menu as follows for user *n* at time period *t*, which is the personalized menu optimization (PMO) presented in Chapter 4.

$$j_{nt}^{\mathrm{PMO}} = argmax_j\, \overline{r_{jnt}}(\eta_{nt}^{t-1}) \tag{5.8}$$

However, since we use estimates of parameters that are uncertain, the offered menu may not be optimal. In addition, offering menu *j* will not give us information of alternative specific constants other than alternative *j*. We need to balance exploitation (offer the best menu based on current knowledge) and exploration (try other menus that may be optimal). Exploration will help us learn uncertain parameter values and will be beneficial for the objective of maximizing total number of hits across the whole operational horizon as in equation (5.1).

In order to balance the exploration and exploitation, we borrow the idea from one of the most widely used MAB heuristics, which is UCB. It uses the sum of empirical mean and confidence bonus to balance the exploration and exploitation.

The empirical mean based on choice history is as follows.

$$\overline{r_{jnt}} = \frac{1}{\sum_{\tau=1}^{t-1} I(j,n,\tau)} \sum_{\tau=1}^{t-1} r_{I(j,n,\tau)} \tag{5.9}$$

where we abuse the notation of r to also denote the realization of reward. If we choose alternatives just based on empirical mean, then it's consistent with the previous CB1 method as follows.

$$\mathrm{j}_{\mathrm{nt}}^{\mathrm{CB1}} = \arg\max_{j}\{\overline{r_{jnt}}\} \tag{5.10}$$

By adding a confidence bonus term, which presents uncertainty about this alternative, we offer a menu as follows for user n at time t.

$$\mathrm{j}_{\mathrm{nt}}^{\mathrm{CB1-UCB}} = \arg\max_{j}\{\overline{r_{jnt}} + \frac{1}{t-1}\sqrt{\frac{c\log(t)}{\sum_{\tau=1}^{t-1} I(j,n,\tau)}}\} \tag{5.11}$$

where the second term presents the "power" of exploration and constant c is a tuning parameter which controls the magnitude of exploration.

Given the hierarchical Bayes estimator for logit mixture, we can replace $\overline{r_{jnt}}$ with the estimated expected reward $\overline{r_{jnt}}(\eta_{nt}^{t-1})$ and call the heuristic PMO-UCB, which chooses the menu as follows.

$$\mathrm{j}_{\mathrm{nt}}^{\mathrm{PMO-UCB}} = \arg\max_{j}\{\overline{r_{jnt}}(\eta_{nt}^{t-1}) + \frac{1}{t-1}\sqrt{\frac{c\log(t)}{\sum_{\tau=1}^{t-1} I(j,n,\tau)}}\} \tag{5.12}$$

We can also think of the objective function as to get as close as possible to the optimal alternatives for each individual (closer to the clairvoyant). Particularly, we want to choose a solution method that minimizes the discrepancy between the optimal menus (by the clairvoyant) and offered menu by the solution or maximize the matching rate as in equation (5.13)

$$\max_{solution} \sum_{n=1}^{N} \frac{1\{j_{nt}^{*} = \mathrm{j}_{\mathrm{nt}}^{\mathrm{solution}}\}}{N} \tag{5.13}$$

5.3 Numerical experiments

In this section, we present numerical experiments under different conditions to evaluate performance among the solution methods including CB1, CB1-UCB, PMO, and PMO-UCB. We use 5 alternatives, and the utility of alternative j of user n at time period t is as follows.

$$\mathrm{u}_{\mathrm{jnt}}(\eta_{\mathrm{nt}}) = (\alpha_{\mathrm{jnt}} - \exp(\beta_{\mathrm{tt,n,t}})\,\mathrm{TT}_{\mathrm{j,n,t}} - \mathrm{TC}_{\mathrm{j,n,t}})/\exp(\beta_{\mathrm{tc,n,t}}) \tag{5.14}$$

where $\eta_{\mathrm{nt}} = (\alpha_{1nt}, \dots, \alpha_{(J-1)nt}, \beta_{tt,n,t}, \beta_{tc,n,t})$ denotes the menu-specific parameter vector of for user *n*; α_{jnt} denotes the alternative-specific constant (alternative *J* is normalized to zero); $\beta_{\mathrm{tt,n,t}}$ and $\beta_{\mathrm{tc,n,t}}$ denote the individual and menu-specific coefficients for travel time and travel cost. Index *t* denotes menu as each time period offers one menu.

In the first five periods, we play alternative t for all the individuals to warm up the system and obtain basic knowledge about alternatives. We construct a synthetic user sample by drawing N times from the multivariate normal distribution as the individual-level parameters. For the logit mixture with inter- and intra-consumer heterogeneity, we further draw the menu-specific parameters with individual-specific mean and covariance matrix for intra-consumer heterogeneity. At each time period, we offer one alternative for each user for different solution methods and compare whether the offered menu is the same as the optimal menu. Travel time and cost are drawn from Uniform [0,1] for every alternative *j*, user *n*, and time *t*.

5.3.1 Experiment under regular condition

In this section, we first compare among all the four methods. Two different sample mean vectors including (1, 3, 5, 7, 1, -1) and (0, 1, 2, 3, 1, -1) are used where the first four coefficients denote alternative-specific constants for alternatives one to four (alternative-specific constant for alternative five is normalized to zero), the last two coefficients are for travel time and travel cost as in equation (5.14). The covariances for inter- and intra- consumer heterogeneity are both diagonal matrix unless otherwise noted. The tuning parameter is set to be 2 unless otherwise noted.

We present plots where the y-axis denotes the matching rate (proportion of offered menus are optimal menus) and the x-axis denotes the time periods. Fig. 5.2 denotes the comparison under logit mixture with inter-only consumer heterogeneity. The left column denotes using p1=(1, 3, 5, 7, 1, -1) and right denotes p2=(0, 1, 2, 3, 1, -1). The top row denotes that variance equals diagonal matrix (I); the bottom row denotes that variance equals 100 times diagonal matrix (100I). In Fig. 5.2, we observe that performance of the proposed methods based on logit mixture estimates (PMO and PMO-UCB) is better than performance of classical methods. In addition, we can observe that exploration term does not help as PMO is slightly better than PMO-UCB and CB1 is slightly better than CB1-UCB. When variance becomes large, all four methods perform worse and are close to each other.

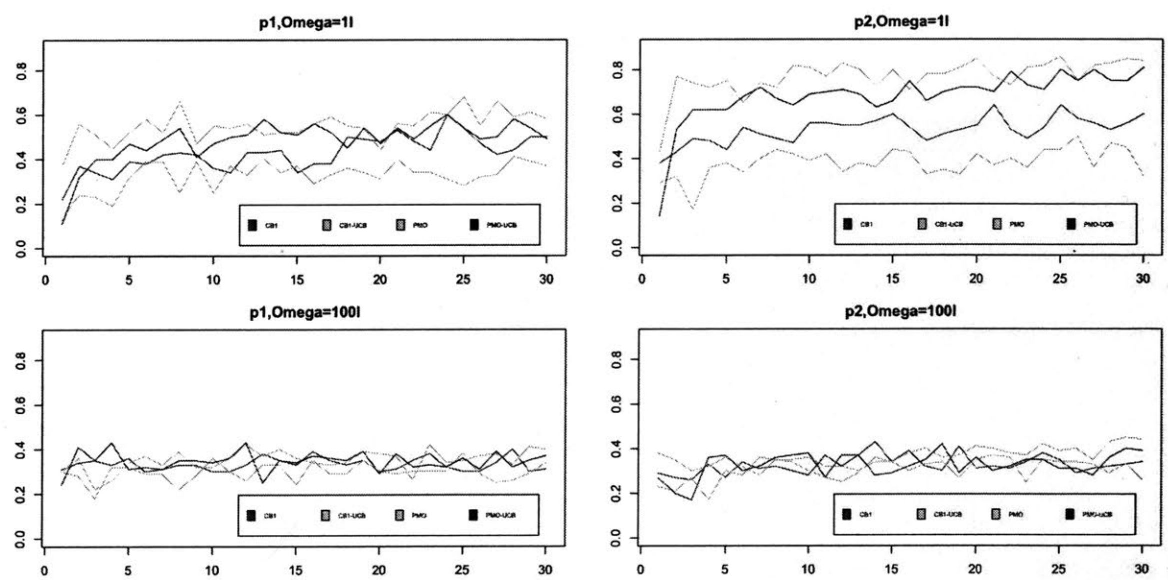

Fig. 5.2 Methods comparison under logit mixture with inter-consumer heterogeneity

Furthermore, we analyze the case where the true choice behavior follows logit mixture with inter- and intra-consumer heterogeneity. This means, for a given individual, his or her taste preferences are varying across time periods. It makes the preferences more difficult to learn. In Fig. 5.3, we observe consistent results that proposed methods based on logit mixture estimates are better in general under different true sample mean vectors. However, the gaps among all the methods are smaller than those under logit mixture with only inter-consumer heterogeneity. When variance is large, all the methods perform similarly worse.

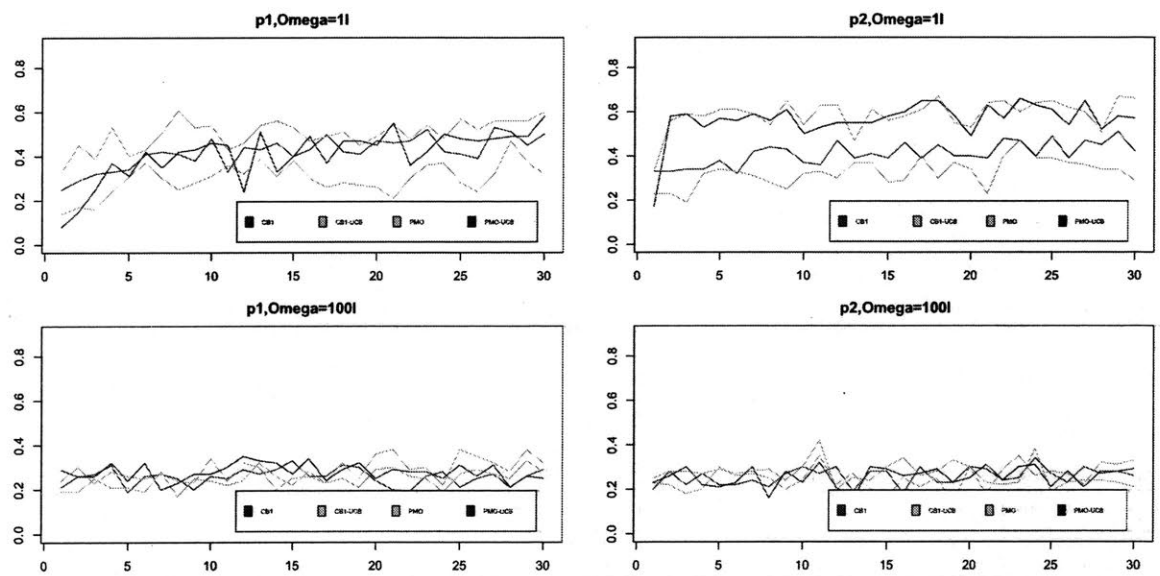

Fig. 5.3 Methods comparison under logit mixture with inter- and intra-consumer heterogeneity

We conclude that in general the proposed methods using logit mixture estimates (PMO and PMO-UCB) perform better than conventional benchmarks (CB1 and CB1-UCB). The gap between those methods becomes smaller under inter- and intra-consumer heterogeneity. When variance is large, all the methods perform worse. In addition, under regular conditions, the exploration does not help, which we will study further in the next section.

5.3.2 Benefits of exploration

As we have learned that PMO-UCB and PMO are the top two methods, we focus on analyzing those two in this section. Particularly, we compare PMO-UCB and PMO in order to evaluate the benefit of adding a confidence bonus term to explore beyond the expected value predicted by the HB estimator.

Fig. 5.4 illustrates the comparison between PMO-UCB and PMO under logit mixture. Here tuning parameter c equals 2. The sample mean vector used is (1, 3, 5, 7, 1, -1). The top two showcase where variance is identity matrix (I). The bottom two showcase where variance is large (100I). The left two show that the true choice model is logit mixture with inter-consumer heterogeneity. The right two show that the true choice model is logit mixture with inter-and intra-consumer heterogeneity.

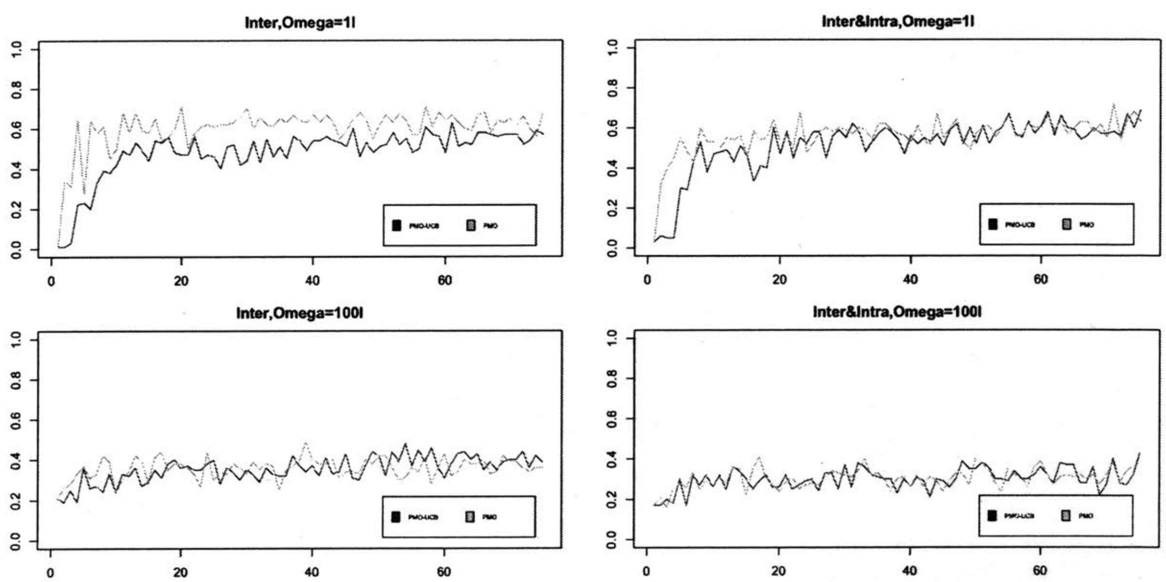

Fig. 5.4 Comparison between PMO-UCB and PMO

From Fig 5.4, we observe that under low variance, PMO is slightly better than PMO-UCB when true choice behavior follows a logit mixture with inter-consumer heterogeneity, which means without confidence bonus (c=0) would be the best case. One reason may be that the PMO has collected enough information about each alternative therefore there is no need to explore beyond estimated best alternatives. PMO-UCB's exploration makes it deviate more from clairvoyant (i.e., true values). When variance becomes large, the performance of both methods gets worse. With inter- and intra-consumer heterogeneity, the performance gap between PMO and PMO-UCB becomes smaller.

However, sometimes the best travel alternative may be under disturbance where for a certain period of time its attributes, e.g., travel time and travel cost, may be much worse than other alternatives. An exploit-only strategy, like PMO, might get trapped within suboptimal alternatives. In order to evaluate the benefits of exploration, we propose an alternative setting where the optimal alternative is under disturbance which prohibits offering it in an exploit-only strategy. Particularly, in the first BT time periods, we draw the travel time and travel cost of alternative 4 (which is the best alternative on average according to sample-level alternative specific constants) to be from Uniform [5,10]. Then as of time period BT+1, we start to draw time/cost from Uniform [0,1] as other alternatives. Fig. 5.5 illustrates the comparison under disturbance with logit mixture with only inter-consumer heterogeneity. The left shows where true variance matrix is 0.1I. The right shows where true variance is I.

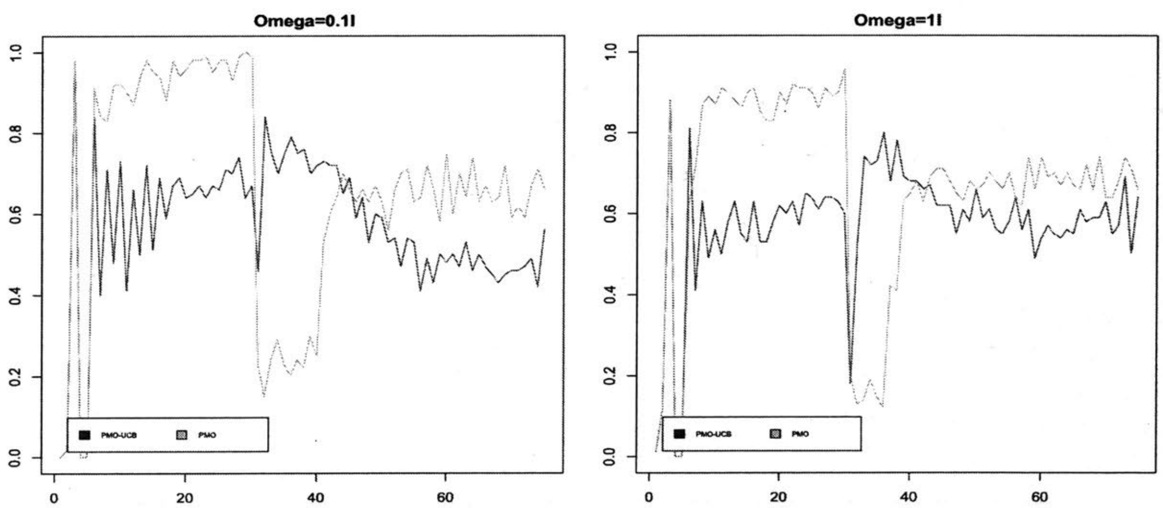

Fig. 5.5 Comparison between PMO-UCB and PMO under disturbance with logit mixture with inter-consumer heterogeneity, BT=30

During a disturbance, both methods rarely choose alternative 4 as it has significantly lower utility. When the disturbance is over, both methods have big drops in their matching rates as expected. For PMO-UCB, the drop is quickly recovered and it performs better than PMO for several periods. It takes more time periods for PMO to recover and eventually both methods reach similar matching rates though PMO performs slightly better. The recovery is easier for PMO when variance is a bit larger.

Furthermore, we consider cases where the true underlying choice model is logit mixture with inter- and intra-consumer heterogeneity. Fig. 5.6 illustrates the comparison under disturbance with logit mixture with inter- and intra-consumer heterogeneity. The left four showcase where true variance is 0.1I. The right four showcase where true variance is I. The four rows use different values of c as 0.5, 2, 5, and 10 respectively. The red plot denotes PMO-UCB and the blue plot denotes PMO.

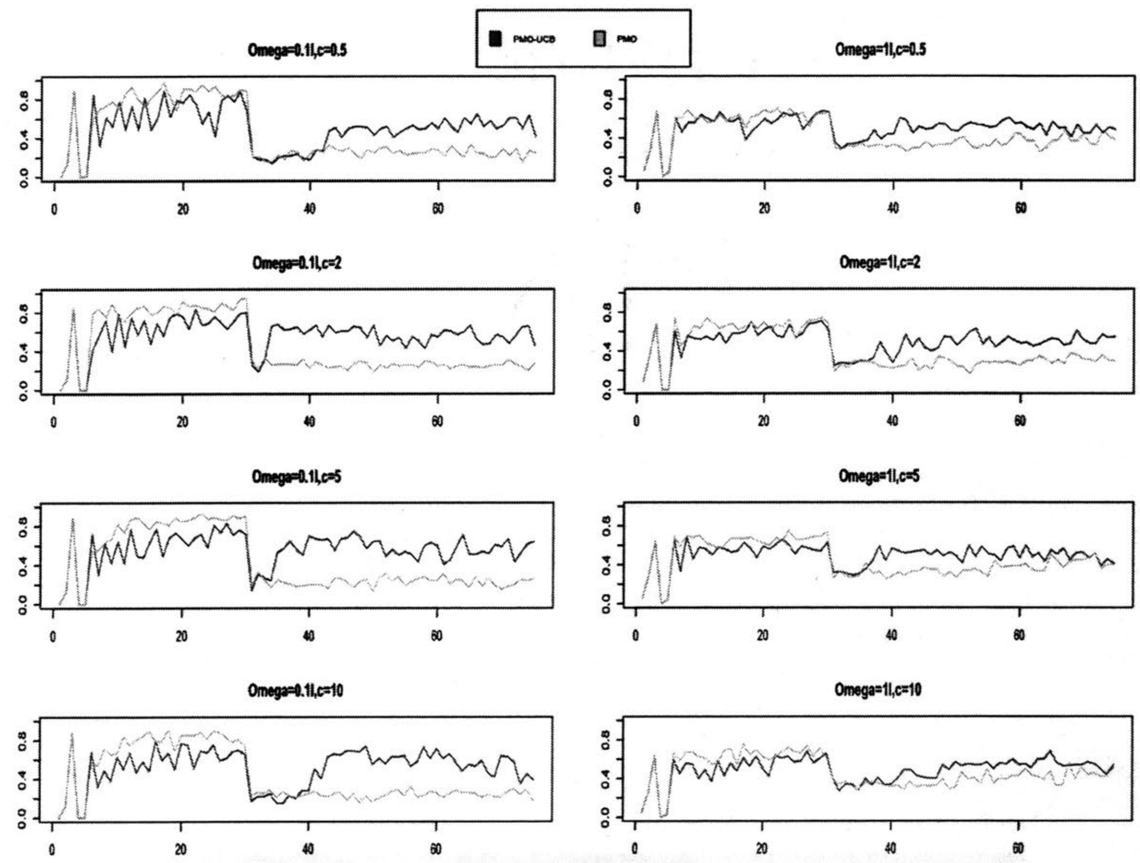

Fig. 5.6 Comparison between PMO-UCB and PMO under disturbance with logit mixture with inter-and intra-consumer heterogeneity, BT=30

Different from cases with logit mixture with only inter-consumer heterogeneity, PMO may get trapped with suboptimal alternatives when there is also intra-consumer heterogeneity and therefore PMO-UCB performs better. The reason behind this finding could be that under inter- and intra-consumer heterogeneity, it makes PMO more difficult to learn or get out of the suboptimum while PMO-UCB can still get out by exploration. The performance gap between

PMO and PMO-UCB also depends on the level of variance, i.e., lower variance has negative impact on performance of PMO.

When c=0, PMO-UCB reduces to PMO. The magnitude of c controls how much we want to explore beyond PMO results. A large value of c may explore too much and result in bad menus. Therefore, under disturbance with intra-consumer heterogeneity, there would be an optimal value of c. From Fig. 5.6, we can observe that under different variances, different values of c perform the best. Under variance of 0.1I, both c=2 and c=5 perform better than larger or smaller values of c. Under variance of I, c=2 performs the best. In real life, the optimal tuning parameter can be found through splitting user traffic and experimenting with different values of c to determine how large c is needed for the exploration.

5.4 Conclusion

In this chapter, we propose a novel method that adapts the classical UCB algorithm by using the HB estimates for logit mixture. The proposed algorithms (PMO-UCB and PMO) based on logit mixture estimates outperform the classical algorithms (CB1-UCB and CB1) under different parameter settings. The performance gap becomes smaller when the true choice model is logit mixture with inter- and intra-consumer heterogeneity. Under regular settings, both PMO and CB1 obtain good estimates, and adding exploration term explores beyond current good estimates, which makes algorithms with UCB perform worse than its counterparts. When intra-consumer heterogeneity is taken into account, the performance gap among all the methods becomes smaller as it becomes more difficult to capture user choice behavior.

Under an alternative setting where there is disturbance which prohibits system operators from offering optimal alternatives, the performance of PMO gets negatively impacted and when the true underlying model is logit mixture with inter- and intra-consumer heterogeneity, PMO performs worse than PMO-UCB, which indicates that more exploration is needed under disturbance. The magnitude of heterogeneity also has an impact on the relative performance; PMO performs worse when heterogeneity is low under disturbance.

In summary, when we believe the consumer heterogeneity among users is not high and intra-consumer heterogeneity exists, we propose to use PMO-UCB especially when there exists some disturbance for some alternatives. In other cases, PMO might perform better. In the future, we need to investigate cases where menu size is greater than one and therefore the rewards of different menus are correlated. It requires a different algorithm to deal with it and its combinatorial nature would make it computationally difficult to choose among many possible menus.

Chapter 6

Summary and Discussion

This thesis focuses on personalized menu optimization, which integrates travel behavior models and optimization. It can be used in many applications particularly app-based Smart Mobility systems (Atasoy et al., 2017a; Atasoy et al., 2017b; Song et al., 2017; Song et al., 2018a). The personalized menu optimization is often built upon hierarchical Bayes estimates of logit mixture and its performance is highly related to whether the choice model captures an individual's taste preferences or not.

This thesis presents three inter-related studies based on some published and working papers (Song et al., 2017; Becker et al., 2018; Danaf et al., 2018; Song et al., 2018a; Song et al., 2018b; Song et al., 2018c).

The first study focuses on improving the estimation performance of logit mixture by using alternative priors suggested in statistics literature instead of IW used by well-known books and commercial software in economics and marketing for covariance matrix in HB estimation procedure. The issue of IW is not known in transportation, economics and marketing field and those alternative priors have not been applied in discrete choice models (logit mixture) before. Both numerical and a real-data case study show that the enhancement by using alternative priors such as hierarchical inverse Wishart would improve the estimation performance.

The second study proposes a novel personalized menu optimization that uses personalized choice estimates from logit mixture. Particularly the PMO uses sophisticated choice model including logit mixture with inter- and intra-consumer heterogeneity while previous studies use non-personalized choice models such as multinomial logit. In a Boston case study, we show that the proposed model outperforms benchmark methods including non-personalized menu optimization and content-based recommendation algorithms.

The third study presents a novel multi-armed bandit algorithm for a sequential personalized menu optimization problem which integrates the classical bandit algorithm and personalized menu optimization using the hierarchical Bayes estimator for logit mixture. We extend the exploit-only solution from the previous study by incorporating exploration using this novel algorithm, which unified learning and optimization. Numerical experiments show that the proposed algorithm outperforms the classical benchmark and show the benefits of exploration under disturbance with intra-consumer heterogeneity.

There are several potential research directions that can be explored in the future. First, we would like to implement a logit mixture with alternative priors using different estimation methods such as variational Bayes to see if the computational time can be greatly reduced while maintaining the accuracy for these alternative priors. This is critical due to the real-time needs of personalized recommendation in the context of transportation and other fields. Second, we would like to evaluate the personalized menu optimization with stated preferences data as in Boston case study we construct the full choice set based on Google Map API which could be different from what the user was considering before making the trip. Third, we would like to explore exact solution algorithms for personalized menu optimization instead of using approximation as in this case study. Fourth, a resource-constrained problem can be formulated for sequential personalized menu optimization where each offered menu is associated with some resource that is globally constrained within operational horizon. For example, in a smart mobility system such as Tripod, we have a global token incentive constraint, which we didn't consider in this thesis. Basically, the system allocates tokens to different types of trips throughout the day. Due to global token budget constraint, the token allocation would be different under different remaining token budgets. Fifth, we can explore alternative solution methods for the third study including randomized probability matching. Sixth, we can extend the problem in the third study to be with larger menu size where we need to solve a problem with correlated outcomes.

As mentioned previously, the methodologies in this thesis are mostly motivated in a transportation context. However, the methodologies and findings can be used in various other fields where individuals are presented with alternatives. In order to achieve good personalization performance, we need to have data and appropriate model to identify the heterogeneous taste preferences among individuals. With learning of preferences and personalized menu optimization, we can improve the decision-making process when applied appropriately.

Appendix A. Hierarchical Bayes Procedure for Logit Mixture with Inter- and Intra-consumer Heterogeneity

Before turning to the specifics of the estimation procedure, it should be noted that the Hierarchical Inverse Wishart prior, introduced to Hierarchical Bayes for Logit Mixtures by Song et al. (2018), is omitted in the general description for the purpose of clarity. In addition, we present HB procedure for logit mixture with inter- and intra-consumer heterogeneity here. The HB procedure for inter-only consumer heterogeneity, Allenby-Train procedure, is a simplified version of this procedure without menu-specific coefficient and covariance that represents intra-consumer heterogeneity and can be found in Ben-Akiva et al. (2016). Here menu means choice scenario.

The joint posterior distribution that is the basis for the Gibbs Sampler is denoted in Eq. (1):

$$K(\mu, \zeta_n \, \forall n, \eta_{mn} \, \forall mn, \Omega_w, \Omega_b | d_n \forall n) \propto \prod_{n=1}^{N} \left[\prod_{m=1}^{M_n} \left[\prod_{j=1}^{J_{mn}} \left[P_j(\eta_{mn})^{d_{jmn}} \right] h(\eta_{mn} | \zeta_n, \Omega_w) \right] f(\zeta_n | \mu, \Omega_b) \right] k(\Omega_w) k(\mu) k(\Omega_b), \quad (1)$$

where:

$$\mathrm{k}(\mu) \sim \mathrm{N}(\mu_0, \mathrm{A}) \quad (2)$$

$$\mathrm{k}(\Omega_b) \sim \mathrm{IW}(\mathrm{T}, \mathrm{I_T}) \quad (3)$$

$$\mathrm{k}(\Omega_w) \sim \mathrm{IW}(\mathrm{T}, \mathrm{I_T}) \quad (4)$$

μ_0 represents the vector of means for the sample-level parameter's prior distribution and can be assigned arbitrary values, as A is a diagonal covariance matrix with diagonal values $a_{ii} \to \infty$, causing the prior to be diffuse. T depicts the number of unknown parameters, and I_T is the T-dimensional identity matrix.

Draws from the joint posterior are obtained by a five-layered Gibbs Sampler. In accordance with the concept of a Hierarchical Bayes estimator, the prior of the sample-level parameters is to be determined ex-ante and is updated with individual-level parameters. The density of each individual parameter in the sample-distribution serves again as the prior for each individual parameter. The data used to update the individual parameters consist of the menu parameters. Consistently, the density of the menu parameters in the distribution of the individual parameters is the prior for the menu-level parameters. Only the lowest level, the menu parameters, are updated using the likelihood of the collected data given the parameters.

Note that in the case of the Allenby-Train procedure the individual parameters are updated using the likelihood. Furthermore, a new layer for Ω_w , the covariance matrix accounting for intra-personal heterogeneity, is introduced. Subsequently, the current Gibbs Sampler iteration is denoted by superscript i. The assignment of starting values is discussed after the depiction of the procedure.

Step I - μ:

The conditional posterior of the sample-level parameter is proportional to right hand side of the term

$$K(\mu|\zeta_n\ \forall n, \eta_{mn}\ \forall mn, \Omega_w, \Omega_b) \propto f(\zeta_n \forall n|\mu, \Omega_b)k(\mu), \tag{5}$$

which refers to a Bayesian update of a multivariate normal distribution. Using the fact that $k(\mu)$ is diffuse, the conditional posterior can be simplified to $\mathcal{N}\left(\bar{\zeta}^{i-1}, \frac{\Omega_b^{i-1}}{N}\right)$, with $\bar{\zeta}^{i-1} = \frac{1}{N}\sum_n \zeta_n^{i-1}$. A draw from this multivariate normal distribution is obtained by

$$\mu^i = \bar{\zeta}^{i-1} + \Psi^{i-1}\omega, \tag{6}$$

where Ψ^{i-1} is the Cholesky factor of $\frac{\Omega_b^{i-1}}{N}$ and ω is a draw from the T-dimensional multivariate standard normal.

Step II- Ω_b:

The conditional posterior of Ω_b is shown on the right hand side of Eq. (7).

$$K(\Omega_b|\mu, \zeta_n\ \forall n, \eta_{mn}\ \forall mn, \Omega_w) \propto f(\zeta_n\ \forall n|\mu, \Omega_b)k(\Omega_b) \tag{7}$$

With the Inverse Wishart distribution being conjugate to the multivariate normal distribution, the closed form posterior is distributed Inverse Wishart with T+N degrees of freedom and scale matrix $TI+N\bar{V}_b$, where:

$$\bar{V}_b = \frac{1}{N}\sum_{n=1}^{N}\left(\zeta_n^{i-1} - \mu^i\right)\left(\zeta_n^{i-1} - \mu^i\right)' \tag{8}$$

A draw of Ω_b is obtained as:

$$\Omega_b^i = \left[\sum_{r=1}^{T+N}(\Gamma \upsilon_r)(\Gamma \upsilon_r)'\right]^{-1} \tag{9}$$

where υ_r is a draw from the T-dimensional standard normal distribution for $r = 1, \dots, T + N$, and Γ is the Cholesky factor of $[TI + N\bar{V}_b]^{-1}$.

Step III – Ω_w:

Drawing from the conditional posterior of the intra-personal covariance matrix Ω_w, see Eq. (10), is similar to the previous step, as it is also considered to be distributed Inverse-Wishart. For the sake of completeness, the step is again mathematically presented in detail. It should be pointed out that it was decided to weight each menu equivalently for the computation of Ω_w, as presented in Eq. (11). It is not regarded as appropriate to assign lower weights to menus of individuals of whom a lot of data are available.

$$K(\Omega_w|\mu, \zeta_n\ \forall n, \eta_{mn}\ \forall mn, \Omega_b) \propto h(\eta_{mn}\ \forall mn|\zeta_n\ \forall n, \Omega_w)k(\Omega_w) \tag{10}$$

The posterior's parameters are $T + M$ for the degrees of freedom and $TI_T + M\bar{V}_w$ for the scale matrix. M is the total number of menus in the data for all individuals, and:

$$\bar{V}_w = \frac{1}{M}\sum_{n=1}^{N}\sum_{m=1}^{M_n}\left(\eta_{mn}^{i-1} - \zeta_n^{i-1}\right)\left(\eta_{mn}^{i-1} - \zeta_n^{i-1}\right)' \tag{11}$$

After obtaining $T + M$ draws of a T-dimensional standard normal distribution, labeled υ_s, $s = 1, \dots, T + M$, the new draw of Ω_w is calculated as:

$$\Omega_w^i = \left[\sum_{s=1}^{T+\mathrm{M}} (\Gamma v_s)(\Gamma v_s)'\right]^{-1} \tag{12}$$

where Γ is the Cholesky factor of $[\mathrm{TI_T} + \mathrm{M}\overline{\mathrm{V}}_{\mathrm{w}}]^{-1}$.

Step IV – ζ_n:

The succeeding operations are repeated for each individual $n = 1, \dots, N$. Despite the numerous repetitions, the computational complexity is manageable, as the terms that require matrix inversion are identical among all individuals with the same number of menus. The individual specific conditional posterior is proportional to Eq. (13). The product of menu- and the individual-level parameter's distribution is multiplied over all menus of individual n

$$K(\zeta_n | \mu, \eta_{mn}\, \forall mn, \Omega_b, \Omega_w) \propto \prod_{m=1}^{M_n} h(\eta_{mn}\, | \zeta_n\, \forall n, \Omega_w) f(\zeta_n | \mu, \Omega_b)\ n \tag{13}$$

The conditional posterior distribution of ζ_n, is $\mathrm{N}(\overline{\zeta_n}, \Sigma_{\zeta n})$ where

$$\overline{\zeta_n} = \left([\Omega_b^i]^{-1} + M_n\ [\Omega_w^i]^{-1}\right)^{-1} \left([\Omega_b^i]^{-1} \mu_i + M_n [\Omega_w^i]^{-1} \frac{1}{M_n} \sum_{m=1}^{M_n} \eta_{mn}^{i-1}\right) \tag{14}$$

and

$$\Sigma_{\zeta n} = \left([\Omega_b^{i+1}]^{-1} + \mathrm{M_n}[\Omega_w^{i+1}]^{-1}\right)^{-1} \tag{15}$$

A draw form $\mathrm{N}(\overline{\zeta_n}, \Sigma_{\zeta n})$ is obtained by calculating $\zeta_n^i = \overline{\zeta_n} + \Psi_{\zeta_n} \omega$ where Ψ_{ζ_n} is the Cholesky factor of $\Sigma_{\zeta n}$ and ω is a draw from a T-dimensional standard normal.

Step V – η_{mn}:

The last step of the Gibbs Sampler is used to update the menu-level coefficients. The particular operation is executed for every menu $m = 1, \dots, M_n$ for every individual $n =$

$1, \ldots, N$. The numerator of the conditional posterior of a menu-level coefficient is given in Eq. (17).

$$K(\eta_{mn} | \mu, \zeta_n, \Omega_b, \Omega_w) \propto \prod_{j=0}^{J_{mn}} \left[P_j(\eta_{mn})^{d_{jmn}} \right] h(\eta_{mn}\ \forall mn | \zeta_n, \Omega_w), \tag{17}$$

$$n = 1, 2, \ldots, N, m = 1, 2, \ldots M$$

As the posterior does not possess a closed form, a draw of η_{mn}^{i} is obtained by the following Metropolis-Hastings step:

The trial draw $\tilde{\eta}_{mn}^{i}$ is obtained as depicted in Eq. (18):

$$\tilde{\eta}_{mn}^{i} = \eta_{mn}^{i-1} + \sqrt{\rho}\Lambda_{w}\upsilon, \tag{18}$$

where Λ_w is the Cholesky factor of Ω_w, υ are T independent variables from $N(0,1)$, and ρ is a parameter of the jumping distribution, adjusted continuously in every iteration. (Train, 2006) chooses to decrease (increase) ρ by 10% in case less (more) than 30% of the trial menu-level coefficients have been accepted. The trial draw $\tilde{\eta}_{mn}^{i}$ is accepted if:

$$u \leq \frac{\prod_{j=0}^{J_{mn}} \left[P_j\left(\tilde{\eta}_{mn}^{i}\right)^{d_{jmn}} \right] \mathrm{h}\left(\tilde{\eta}_{mn}^{i} | \zeta_n, \Omega_w\right)}{\prod_{j=0}^{J_{mn}} \left[P_j\left(\eta_{mn}^{i-1}\right)^{d_{jmn}} \right] \mathrm{h}\left(\eta_{mn}^{i-1} | \zeta_n, \Omega_w\right)} \tag{19}$$

where u is a draw from the standard uniform distribution.

Appendix B. Stan Scripts for Alternative Priors

Here we include the Stan scripts that we use for estimating logit mixture with different priors including IW, HIW, SIW and BMM for the choice of grapes study in Chapter 3. Note that we need to write R scripts to read data and put them into the consistent format as Stan reads it.

1. Stan code for IW

```
data {
int<lower=0> N; // Observations
int<lower=0> K; // Alternatives
int<lower=0> D; // Variables
int<lower=0> H; // Households
int<lower=0> id[N]; // ID variable
int z[N]; // Choice Indicator
vector[K] sweet[N]; // Sweet
vector[K] crisp[N]; // Crisp
vector[K] price[N]; //cost
cov_matrix[D] invR;
cov_matrix[D] mu_var_prior;
vector[D] mu_m_prior;
}
parameters {
vector[D] eta[H];
vector[D] etamu;
cov_matrix[D] etavar;
}
```

```
model {
real u[K];
 etamu ~ multi_normal(mu_m_prior, mu_var_prior);
 etavar ~ inv_wishart((D+1), invR);
for (h in 1:H)
        eta[h] ~ multi_normal(etamu,etavar);
for(n in 1:N) {
// Utilities
u[1] <- (-price[n,1]+ eta[id[n],2] .* sweet[n,1] + eta[id[n],3] .* crisp[n,1])/exp(eta[id[n],1]);
u[2] <- (-price[n,2]+ eta[id[n],2] .* sweet[n,2] + eta[id[n],3] .* crisp[n,2])/exp(eta[id[n],1]);
u[3] <- (-price[n,3]+ eta[id[n],2] .* sweet[n,3] + eta[id[n],3] .* crisp[n,3])/exp(eta[id[n],1]);
u[4] <- (-price[n,4]+ eta[id[n],2] .* sweet[n,4] + eta[id[n],3] .* crisp[n,4])/exp(eta[id[n],1]);
// Logit Log Likelihood
target+= u[z[n]] - log_sum_exp(u);
}
}
```

2. Stan code for HIW

```
data {
int<lower=0> N; // Observations
int<lower=0> K; // Alternatives
int<lower=0> D; // Variables
int<lower=0> H; // Households
```

```
int<lower=0> id[N]; // ID variable
int z[N]; // Choice Indicator
vector[K] sweet[N]; // Sweet
vector[K] crisp[N]; // Crisp
vector[K] price[N]; //cost
cov_matrix[D] invR;
cov_matrix[D] mu_var_prior;
vector[D] mu_m_prior;
//real<lower=0> A;
vector[D] A;
}
parameters {
vector[D] eta[H];
vector[D] etamu;
cov_matrix[D] etavar;
vector[D] a;
}
model {
real u[K];
etamu ~ multi_normal(mu_m_prior, mu_var_prior);
for (d in 1:D)
  a[d] ~gamma(0.5,1/A[d]^2);
etavar ~ inv_wishart((D+1), 2*2*diag_matrix(a));
for (h in 1:H)
```

```
    eta[h] ~ multi_normal(etamu,etavar);
for(n in 1:N) {
// Utilities
u[1] <- (-price[n,1]+ eta[id[n],2] .* sweet[n,1] + eta[id[n],3] .* crisp[n,1])/exp(eta[id[n],1]);
u[2] <- (-price[n,2]+ eta[id[n],2] .* sweet[n,2] + eta[id[n],3] .* crisp[n,2])/exp(eta[id[n],1]);
u[3] <- (-price[n,3]+ eta[id[n],2] .* sweet[n,3] + eta[id[n],3] .* crisp[n,3])/exp(eta[id[n],1]);
u[4] <- (-price[n,4]+ eta[id[n],2] .* sweet[n,4] + eta[id[n],3] .* crisp[n,4])/exp(eta[id[n],1]);
// Logit Log Likelihood
target+= u[z[n]] - log_sum_exp(u);
}
}
```

3. Stan code for SIW

```
data {
int<lower=0> N; // Observations
int<lower=0> K; // Alternatives
int<lower=0> D; // Variables
int<lower=0> H; // Households
int<lower=0> id[N]; // ID variable
int z[N]; // Choice Indicator
vector[K] sweet[N]; // Sweet
vector[K] crisp[N]; // Crisp
vector[K] price[N]; //cost
cov_matrix[D] invR;
```

```
cov_matrix[D] mu_var_prior;
vector[D] mu_m_prior;
vector[D] b;
vector[D] zeta;
cov_matrix[D] Lambda;
}
parameters {
vector[D] eta[H];
vector[D] etamu;
cov_matrix[D] Q;
vector[D] delta;
}
transformed parameters{
cov_matrix[D] etavar;
etavar=quad_form_diag(Q,delta);
}
model {
real u[K];
 etamu ~ multi_normal(mu_m_prior, mu_var_prior);
Q ~ inv_wishart((D+1),Lambda);
 for (i in 1:D)
   delta[i]~lognormal(b[i],zeta[i]);
for (h in 1:H)
        eta[h] ~ multi_normal(etamu,etavar);
```

```
for(n in 1:N) {
// Utilities
u[1] <- (-price[n,1]+ eta[id[n],2] .* sweet[n,1] + eta[id[n],3] .* crisp[n,1])/exp(eta[id[n],1]);
u[2] <- (-price[n,2]+ eta[id[n],2] .* sweet[n,2] + eta[id[n],3] .* crisp[n,2])/exp(eta[id[n],1]);
u[3] <- (-price[n,3]+ eta[id[n],2] .* sweet[n,3] + eta[id[n],3] .* crisp[n,3])/exp(eta[id[n],1]);
u[4] <- (-price[n,4]+ eta[id[n],2] .* sweet[n,4] + eta[id[n],3] .* crisp[n,4])/exp(eta[id[n],1]);
// Logit Log Likelihood
target+= u[z[n]] - log_sum_exp(u);
}
}
```

4. Stan code for BMM

```
data {
int<lower=0> N; // Observations
int<lower=0> K; // Alternatives
int<lower=0> D; // Variables
int<lower=0> H; // Households
int<lower=0> id[N]; // ID variable
int z[N]; // Choice Indicator
vector[K] sweet[N]; // Sweet
vector[K] crisp[N]; // Crisp
vector[K] price[N]; //cost
cov_matrix[D] invR;
cov_matrix[D] mu_var_prior;
```

```
vector[D] mu_m_prior;
vector[D] b;
vector[D] zeta;
cov_matrix[D] Lambda;
}
parameters {
vector[D] eta[H];
vector[D] etamu;
vector[D] delta;
cov_matrix[D] Q;
}
transformed parameters {
corr_matrix[D] R;
cov_matrix[D] QI;
cov_matrix[D] etavar;
  for (i in 1:D){
    for (j in 1:D){
      if (i==j) {
        QI[i,j]=sqrt(1/Q[i,i]);
      } else{
        QI[i,j]=0;
      }
    }
  }
```

```
R=QI*Q*QI;
etavar=quad_form_diag(R,delta);
}
model {
real u[K];
 etamu ~ multi_normal(mu_m_prior, mu_var_prior);
 for (i in 1:D)
    delta[i]~lognormal(b[i],zeta[i]);
Q~inv_wishart((D+1),Lambda);
for (h in 1:H)
         eta[h] ~ multi_normal(etamu,etavar);
for(n in 1:N) {
// Utilities
u[1] <- (-price[n,1]+ eta[id[n],2] .* sweet[n,1] + eta[id[n],3] .* crisp[n,1])/exp(eta[id[n],1]);
u[2] <- (-price[n,2]+ eta[id[n],2] .* sweet[n,2] + eta[id[n],3] .* crisp[n,2])/exp(eta[id[n],1]);
u[3] <- (-price[n,3]+ eta[id[n],2] .* sweet[n,3] + eta[id[n],3] .* crisp[n,3])/exp(eta[id[n],1]);
u[4] <- (-price[n,4]+ eta[id[n],2] .* sweet[n,4] + eta[id[n],3] .* crisp[n,4])/exp(eta[id[n],1]);
// Logit Log Likelihood
target+= u[z[n]] - log_sum_exp(u);
}
}
```

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