Listen to the market, hear the best policy decision, but don't always choose it.*

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2019-02-28

Abstract

Policymakers often consider policies with (a) uncertain social benefits *and* (b) uncertain impacts on the value of private assets; we characterize six ways (a) and (b) may be inter-related. Where investors have private information over (b), policymakers may attempt to learn this through the response of asset markets to *proposed* policies. However, where this information is concentrated, an informed trader may profitably hide his information and "manipulate" the market. We show that it is nonetheless generically optimal for policymakers to listen and respond to asset markets, but under specified conditions they must commit (e.g., through "political capital") to sometimes pursuing a policy even when the expected welfare effects are negative. Surprisingly, allowing traders to short-sell can make it *easier* for policymakers to induce truth-telling actions.

1 Introduction

In a variety of cases, a small set of private individuals may be better informed than policymakers about the effect of a potential policy. If these individuals buy and sell assets in publicly observable settings, policymakers might "listen to" these markets to decide whether to implement a particular policy.

We offer an example to motivate this idea. Suppose the Department of Energy (DOE) is considering spending $100 million on an R&D subsidy to spur innovation and increase profits in

*We thank Benjamin Bittschi, Holger Breinlich, Oliver Hart, David Hugh-Jones, Larry Samuelson, Santiago Oliveros, Motty Perry, Pedro Saffi, Jeroen van de Ven, Tianxi Wang, and various seminar participants for helpful tips. Mateusz Gatkowski offered helpful research assistance. Surajeet Chakravarty has participated in extensive conversations with us on the scope and limits of this paper, and possible extensions. The authors declare that they have no relevant or material financial interests that relate to the research described in this paper.

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the solar energy sector. The subsidy mainly targets a new photovoltaic cell being developed by a firm "Soylent" (SOY). However, only a few engineers working for the large investment firm Golden Sax (GS) have the expertise to judge whether this new cell is technically viable and thus profitable, or technically difficult and thus less likely to be profitable. Suppose the DOE announced that the funding had attained preliminary (but not yet final) institutional approval. If SOY immediately rose substantially, the DOE might attribute this to a GS purchase on the advice of their engineers, implying the project is likely viable, and decide to fund it. If, instead, SOY did not rise much, the DOE might see this as news that the project had a small chance of success, and decide not to fund it. However, this "naive listening" may give GS an incentive to purchase SOY even if its engineers advised against the project. A GS purchase, by boosting SOY greatly, would lead to DOE funding, further increasing SOY's expected returns and yielding a profit for GS. With such behavior by GS the SOY price reaction would be uninformative. However, we will show there is always a more sophisticated way of *listening* that will make the price informative.

These concerns are relevant to a variety of contexts, where there is private information about a policy's impact, and where a successful policy will increase the value of an asset.[1] A government may consider a trade agreement to only merit signing if it will bring a large enough profit for domestic industries (perhaps it involves a costly aid package for the partner country). We might consider a measure to protect intellectual property, or a policy subsidizing technical education, in the same vein: firms in the targeted industry may always benefit, but unless these benefits are large they may not justify the costs. There may also be private information about the impacts of policies whose success depends on significantly *reducing* rents in some industry, in order to increase consumer surplus or government revenue. For example, health-care reforms may aim to decrease rents in the insurance, drug, and hospital industries.[2] While *listening to the markets* may be helpful in each of these cases, this might be undermined by the possibility that investors will act to conceal their information (i.e., "manipulate" asset prices).

Our setup is broadly as follows. At a precise point in time, a Policymaker (PM) publicly considers whether to implement a specific new policy. We say that the policy is "good" when it will

[1] The policymaker's goal need not involve the firm's profit or the asset's value. An exogenous factor, e.g., the progress of technology or the potential supply of some natural resource, may happen to determine both whether a policy will be successful for the policymaker and whether a particular asset is profitable.

[2] For example, the USA's 2010 Affordable Care Act included an individual mandate and subsidies to purchase insurance as well as the establishment of "insurance exchanges" with regulations intended to reduce prices through encouraging competition and transparent pricing. It was widely assumed that that the mandate would have a side effect of boosting insurance company profits, while the exchanges and regulations were meant to counter this. Presumably policymakers hoped in net to reduce (or at least not increase) insurance company profits, but opponents of the bill argued that it would be a giveaway to the insurance industry.

increase the PM's welfare, and it is "bad" otherwise.[3] Next, an *Informed Investor* ($\mathcal{II}$) receives a *signal* of the policy's merit, and acts to maximize his profit. We model the $\mathcal{II}$ as a single investor, or, equivalently, a set of investors who coordinate and can thus drive the market.[4]

$\mathcal{II}$ may trade (with a passive *Market Maker*, henceforth "MM"). We assume that the $\mathcal{II}$ has no inherent interest in the asset's value, but may buy the asset, short sell it, or do nothing. After he trades one unit, the asset's price will change in response to the $\mathcal{II}$'s choice (we justify this reduced-form in the context of previous microstructure models in section 3.4); the PM observes this and decides whether or not to execute the policy.

For *listening* to always achieve the best policy, the interests of the PM and the relevant informed investor must be naturally aligned. For some parameters and conditions this will be the case, and $\mathcal{II}$ will reveal his private information through his profit-seeking behavior even knowing that the PM would certainly act on this information (here, the PM could just *ask* the $\mathcal{II}$ for his signal). However, this alignment condition need not be satisfied in general. For example, although (by consruction) the PM wants the policy to be executed if and only if it is *good*, an investor may *always* want the policy to be executed, or he may *never* want it to be executed.[5]

Nonetheless, the PM may still be able to extract information from $\mathcal{II}$ if she can make a binding commitment (or indirectly commit herself) that with a certain probability she will ignore the asset price and execute (or *not* execute) the policy anyways.[6] This changes $\mathcal{II}$'s incentives through two channels. First, it can reduce (or increase) the effect of the $\mathcal{II}$'s action on the probability the policy is executed, hence reducing or increasing the asset's relative expected value after each of $\mathcal{II}$'s actions. Second, if the probabilistic commitment is common knowledge, it will also affect the ex-ante expected returns to the asset, and hence increase or decrease the initial price, i.e., the price $\mathcal{II}$ must pay (or receive) for the asset.

Returning to our first example, a *sophisticated* DOE could build its institutional process in a way that makes it less responsive to the SOY price. Suppose the DOE had publicly committed its resources and reputation to this project in a way that, even if SOY rose little after the preliminary

[3] Obviously, the PM's incentives may or may not be aligned with the public interest; this central issue of public choice is out of the scope of this paper; future work may extend our model to explicitly derive and consider a rent-seeking PM's payoff function.

[4] We do not focus on the case where information is diffusely held. In such a case the informed investors might fail to coordinate on the equilibrium that collectively yields them higher profit given the PM's commitment. We return to this discussion in the final section.

[5] It is often more intuitive to consider the incentives of the affected firm or asset-holder, as these will only depend on the direct impact of the policy on the asset. However, the party with inside information need not be the initial asset-holder; furthermore the initial asset holders may be able sell off their holdings. As we show later, the incentives of an asset-holder and an informed investor may differ, but concerns about manipulation are present for each.

[6] (Boleslavsky et al., 2017) offer a similar insight; we discuss the key differences between our papers below.

approval, the DOE would, with some probability, feel compelled to fund the project even knowing it is likely a bad policy. For example, long before the preliminary approval the DOE might have hinted that the subsidy would be a central part of its new "Green Initiative"; or DOE might have begun signing contracts with firms involved in this technology. These prior actions may imply that with a certain probability, the DOE will fund the subsidy no matter what the market does. This would have boosted SOY even before the preliminary approval. The higher *initial* share price would decrease the expected profit for GS from buying and inducing the policy after the preliminary approval. This might deter GS from buying when its engineers advise against the project, but yet *not* deter GS from buying when its engineers endorse the project. This informative "separating behavior" is possible only because the funding adds *more* real value when the project is highly viable than when it is technically difficult, thus more profit for GS.

As in the above example, the mechanisms we propose are *indirect*: the PM does not directly pay the $\mathcal{II}$ as a function of his revealed signal and the outcome. Instead, she uses existing asset markets as a tool, and the efficiency of the mechanism depends on the structure and parameters of the environment. We justify this in section 7.

We present a simple model with binary policies, signals, and outcomes. We solve for six intuitive cases which map out the entire parameter space of incentive alignments, offering a taxonomy of these cases, and motivating them with real-world examples. Given ex-ante policy indifference we find that inducing truth-telling behavior and listening to markets is generically beneficial for the PM. We find an interesting asymmetry: where the PM is ex-ante indifferent, and where her commitment is common knowledge, she may commit to occasionally executing after a *bad* signal, but she always executes after a *good* signal.[7] We also find the surprising result that for a positive-measure parameter space, allowing the $\mathcal{II}$ to short-sell – giving $\mathcal{II}$ a greater set of options – may make implementation *easier*, thus increasing the PM's payoff.

Our work is largely a theoretical benchmark; we consider the case where the investor(s) have the maximum potential to manipulate a naive PM and we describe how the optimal mechanism involves randomization, and define the frontier of what the PM can achieve.

Our paper may be seen as a normative or prescriptive proposal. However, these considerations may also be reflected in current practices, and decision-makers are already taking prediction markets, as well as standard asset markets, into account (Arrow et al., 2008). There is anecdotal evidence of some policymaker "market-watching." Although PMs make policy announcements and float trial balloons, there is little evidence that they *explicitly* commit to tying policy to asset

[7] Note that this holds with when A_0 follows rational expectations, our maintained assumptions, but not for any arbitrary exogenous A_0. We discuss this further in section 5.2, offering intuition.

prices. However, as we discuss in the conclusion, legislative processes, policy trials, and "committing political capital" may also enable *listening* and informal conditional commitments, in essence a purification of the mixed/behavioral strategy.

Our paper proceeds as follows. Section 2 reviews the literature. We specify our formal model in section 3. In section 4 we give general lemmas and results. Section 5 considers alternative assumptions: allowing short-selling 5.1, an exogenous asset price 5.2, and multiple informed investors 5.3. In section 6 we discuss our model of trade and price formation in the context of the market's microstructure and existing trading institutions such as "dark pools". Section 7 presents additional academic and policy insights, considers real-world implementation, and offers suggestions for future extensions and empirical work.

2 Literature Review

Our work is related to two strands of literature: (i) papers on information and policy markets, typically assuming information is diffuse, and (ii) work on the policy-setting under macro-economic uncertainty, focusing on financial bailouts and implying a particular incentive alignment.

Several papers in the first strand have analyzed the relationship between the values of assets in conventional markets and the policy predictions of information markets, considering the implications of using such analysis to set policy.[8] A more extensive literature has performed "event studies" to measure the effect of policy announcements on conventional asset prices, considering the implications that can be drawn about the policies. A major concern is that if future policy itself reacts to the market's response to the policy announcement, the market response may be hard to interpret; this has been called the "circularity problem" (Bernanke and Mishkin, 1997; Sumner and Jackson, 2008).

To circumvent this circularity, others have proposed the use of "conditional prediction markets" (Hanson, 2013; Abramowicz, 2004). In such a market one asset takes a value, tied to some outcome, if a policy is executed, another asset takes a value if it is *not* executed, and otherwise trades are canceled. For example, Hahn and Tetlock (2003) consider assets whose values are tied to the level of GDP in the event (or non-event, for the second asset) of a carbon emissions cap, and consider what the difference in these asset prices reveal about the likely effect of such a cap.

[8] *Information markets* are markets which do not represent direct claims on tangible assets (Wolfers and Zitzewitz, 2006). Our analysis may also apply to corporate policy, in cases where the corporate management may know less about outcomes than the outside investors and speculators. Kau et al. (2008) find that, "on average, managers listen to the market: they are more likely to cancel investments when the market reacts unfavorably to the related announcement."

However, according to Hahn and Tetlock (2003), "[a] general concern is that information markets are susceptible to price manipulation by those with a vested interest in the policy decision". This suggests that the use of prediction markets, which tend to be thin and illiquid, may be limited.[9] When real assets such as firms and physical capital are strongly affected by a policy it may be difficult to make prediction markets large enough to deter manipulation. Furthermore, if the main impact of a policy is only known by a single individual or a coordinated group with the *ability to heavily invest or short sell*, bringing large numbers of uninformed traders into the prediction market will not deter manipulation.

It may be more effective for policymakers to learn from the movement of *real* large-scale asset markets and from real investment decisions when a policy is announced, explicitly recognizing the connection betwen the probability of executing the policy and the movement in the asset prices. Thus, policymakers could explicitly take into account the incentive for manipulation and use a mechanism to induce truth-telling behavior.

In a related context, Hanson and Oprea (2009) model a market microstructure with noise traders, potentially informed traders, a competitive market maker, and a thin prediction market. As in our model they have a single rational "manipulator"; however, in contrast to us they assume the manipulator has a specific preference over the market price or over "the beliefs of neutral observers influenced by the price." In other words, their manipulator is essentially a particular type of "noise trader" who has a specific goal unrelated to the asset's true value. In contrast, our $\mathcal{II}$ seeks only to make a profit off of the policy outcome he induces.[10]

In the second strand, the closest paper to ours is Boleslavsky et al. (2013) (henceforth "BTK"). Both our paper and BTK are broadly normative, considering a PM who "faces a trade-off between eliciting information from the asset market and using the information..." BTK characterize optimal intervention policy without commitment, and then with full committment (the equivalent of our optimal policy mechanisms involving commitments to probabilistic execution); however, BTK restrict this to implementations that make the $\mathcal{II}$ no worse off ("Pareto Improvements"). Both our work and BTK's consider indirect commitment strategies via political processes. Their "informed trader" has an endogenous (profit-seeking) motive to manipulate, as does our $\mathcal{II}$. The adjustment of asset prices to publicly-available information is also similar. However, our focus, assumptions, and hence our results differ substantially from theirs.

[9] Sumner and Jackson (2008) note that conditional and prediction markets are likely to be thin and hence unreliable, arguing government subsidies to trading are needed to combat this.

[10] Our model differs in other important ways. Our $\mathcal{II}$ has unique private information; they assume a large number of traders who can pay a cost to learn about the asset's true value and about the manipulator's preferred price. Another difference: we explicitly model the policy choice.

Their paper is about financial *bailouts*; they focus on a standard macroeconomic feedback story: if a sell-off triggers a corrective intervention, an informed investor will have less incentive to short sell on his negative information. The underlying economic situation (good or bad) is known only to informed traders. A bailout policy has known costs and is certain to bring the economy to a good state; it is worth executing if there is a high enough chance that the economy is bad (thus their "bad" economy loosely corresponds to our "good" policy). This implies that, for the informed trader, the policy has a (positive) impact on the asset's value in the bad-economy/good-policy state only. In this respect, our model is broader: we allow *all* possible co-relationships between a policy's value and its impact on assets.[11] Thus our work goes further to conceptually characterize the extent to which incentives are aligned across all constellations of parameters.

There are other important differences. BTK assume the policymaker (whom they call the "authority") does not want to intervene under the prior beliefs while we assume ex-ante neutrality. Our distinct setups lead to substantially different results. In particular, in their model without commitment "the policymaker never benefits from the ability to intervene". BTK's assumptions (particularly the policymaker's ex-ante preference not to intervene) ensure that the investor prefers to trade only up to the point that makes the PM indifferent to executing the policy. This contrasts strongly from our finding that in some contexts the optimal policy commitment achieves the first best, i.e., no commitment is required. (However, we find that for other alignments a commitment to probabilistic execution is necessary to implement information transmission.)

Siemroth (2017) considers a more general alignment of policymaker preference and asset values; as do we. He goes beyond this to consider a richer set of informative signals going to (possibly multiple) informed investors. He assumes competitive equilibrium pricing, rather than an explicit market microstructure and strategic traders. He derives general conditions for a "fully revealing rational expectations equilibrium", in which the investors' trades reveal all of the private information, while the PM acts optimally on this information. This is equivalent to our "first-best". It is characterized by the invertibility of the expected asset value function, a function of the traders' information, and the direct and indirect reaction to this. His intuition resembles ours in some ways, considering the "alignment" of policy maker and trader preferences. However, he does not allow the PM commit to probabilistic execution; his conclusion asks: "if the optimal policy reaction function leads to a self-defeating prophecy, when can commitment to another reaction function be superior, because it supports information revelation?" Our model helps answer this question. We show that

[11] In the context of our model, their setup puts them on the knife-edge between our case (i) and (vi), i.e., between a "treat" and a "paternalist" policy. However, the former ('treat') seems more intuitive – in the real-world, with a non-binary economic state, a bailout may typically be expected to increase relevant asset values, even if the bailout is not needed.

full revelation is always possible when the PM can commit to probabilistic execution. I.e., this expands the (PM-welfare-improving) implementation to the entire set of parameters (excepting a measure zero-set). In his terms, a commitment to probabilistic execution makes pricing function invertible for any parameters.

2.1 Empirical work and examples

There are several recent cases in which policymakers seem to have listened to the market, or where others have suggested that they should have done so.[12]

During the debate over the US Affordable Care Act, Milani (2010) tracked the stock returns of health insurance companies against a prediction market security whose payoff was tied to the inclusion of the "public option" in the bill. He concluded "the results reveal the market expectation of a negative effect of the public option on the value of health insurance companies ... of around 13%, but it does not support more calamitous scenarios." Friedman (2009) performed event studies on pharmaceutical firms' share prices as they introduced new drugs, comparing the implied profitability of (low versus high Medicare share) drugs before and after the introduction of the Medicare Part D prescription drug benefit. He used this to impute that the bill would lead to $205 billion in additional drug company profits.

Wolfers and Zitzewitz (2006) presented evidence, in the context of the Iraq war in the mid-2000's, that spot and futures market oil prices moved in line with a prediction market for a security that paid off if Saddam Hussein were removed from power by a certain date. They used this to estimate the distribution of investors' beliefs for the impact of the war on the economy, imputing "a substantial probability of an extremely adverse outcome."

As Wolfers and Zitzewitz (ibid) argue, the above sort of evidence could be used to "better understand the consequences of a prospective policy decision ... [and] to inform decision-making in real time." In light of the above evidence, the public option might have been scrapped, the drug benefit repealed or reformed, the Iraq war reconsidered, and the CAFTA agreement reinforced or cancelled (depending on whether PMs thought the gains were large enough).[13]

Poland's 2011-2012 experience seems to be a relatively clear-cut example. In November 2011,

[12] See also Boleslavsky et al. (2013), who offer several examples of cases where market activity influences both government and corporate policy.

[13] The Pentagon also attempted to use markets to predict geopolitical risks such as terrorism. The Defense Advanced Research Projects Agency proposed introducing a policy market in 2003. This project was cancelled, allegedly in light of concerns that bad actors might themselves invest, commit terrorist acts, and profit from this (Hanson, 2005). This would be a distinct form of manipulation from our discussion; such concerns involve *physical actions* taken in order to manipulate *markets*; we are considering *financial* transactions and investments made in order to gain profit from influencing *policy*, which affects asset values. We assume that the PM does not derive welfare from the investments; if these "investments" are acts of terrorism this is obviously the PM's concern.

Polish PM Donald Tusk announced a new minerals tax. Share prices of KGHM (Poland's sole copper producer) fell by roughly 25% in the two days following the announcement. On January 3, 2012, the Finance Ministry lowered this proposed tax rate after negative reactions from the Economy and Treasury ministries and from the company, which argued the tax would make output at one of its three mines inviable. Economy Minister Waldemar Pawlak explicitly mentioned the share price reaction in his criticism.[14]

3 Model

In this section we present our setup and derive incentive compatibility constraints and a complete set of results when an informed investor chooses between buying and not buying the asset. We will also present the setup and asset market allowing short selling, and we derive a feasible welfare improving (but perhaps not optimal) mechanism in which $\mathcal{II}$ can buy, do nothing, or short sell. As we note, most of our qualitative results hold for both setups. All proofs are in the Appendix unless noted otherwise.

We first define our notation. The state is denoted by $s \in \{G, B\}$, i.e., the *good* and the *bad* state (reflecting the welfare consequences of a policy, formalized below). There is an informed investor "$\mathcal{II}$", who receives a perfect signal of the true state.[15]

We will refer to the informed investor who has learned the state is s by $\mathcal{II}(s)$.

3.1 Timing

1. The PM commits to probabilities of execution $q(\hat{s}) \in (0, 1)$ as a function of the signal to be revealed $\hat{s} \in \{G, B\}$. These commitments are publicly observed or deduced by all parties.

2. $[t = 0]$ The initial asset price A_0 is formed by the unconditional expectation of the asset's final value A_1, taking the PM's policy commitment into account.

3. Nature chooses the state of the world, $s \in \{G, B\}$, with probabilities P_G and $P_B = 1 - P_G$, respectively.

[14] Pawlak: "If the tax had been more clearly presented during the prime minister's expose, there would be less unrest and less fluctuation in KGHM's share value. Now we have to prepare [the new tax] properly so that it benefits the state but doesn't kill KGHM" (http://www.wbj.pl/article-57461-deputy-pm-criticizes-copper-tax.html, accessed on 1 Jan. 2014)

[15] Our previous working paper made a distinction between states and signals as a step towards generality and realism; however, these results are qualitatively identical and yielded no additional insights. Note also that while we restrict the set of states/signalsto be two elements, we could easily extend the current setup into many states and many possible signals, with similar results.

4. The $\mathcal{II}$ learns s and chooses $i \in \{b, nb\}$ or $i \in \{b, nb, sh\}$ (depending on the environment we consider), where b, nb, and sh represent buying one unit, doing nothing, or short selling one unit, respectively. The asset price, hence the $\mathcal{II}$'s *action* becomes observable to the PM, sending a signal $\hat{s}$ to the PM.

5. The PM executes the policy with the pre-committed probability $q(\hat{s})$, where $\hat{s} = s$ in a truth telling equilibrium. We denote the realized policy choice as $p \in \{e, ne\}$ for execution and non-execution, respectively.

6. $[t = 1]$ Payoffs are realized, with the asset paying $A_1(p, s)$ to it's owner.

Consider stages 1 and 2. As the probability that a good or a bad policy is chosen—and the effects of these policies on asset prices—is common knowledge, all parties can infer the incentive-compatible probabilities of execution. For the third stage, our preferred real-world interpretation is that the PM chooses the details of the policy through an unpredictable process. Once these details are announced they constitute a signal of the policy's character and merit that only $\mathcal{II}$ can interpret.

Note that committing to probabilities $\langle q(G) = 1, q(B) > 0\rangle$ is equivalent to a commitment that with probability $q(B)$ she will ignore the asset price and execute the policy. Similarly $\langle q(G) < 1, q(B) = 0\rangle$ is equivalent to a commitment that with probability $1 - q(G)$ she will ignore the asset price and not execute the policy. (In each case, with the remaining probability she will choose her optimal ex-post action in light of the signal.)

3.2 Policymaker's objective function

The PM either executes a policy (denoted $p = e$) or does not (denoted $p = ne$). Her payoff with the policy choice p and the state s is $W(p, s)$. Note that the investor's decision is not directly in the PM's objective function. We assume the PM wants to execute the policy if and only if the true state is G; i.e., G represents the "good news" about the policy. Without loss of generality, we normalize the welfare of non-execution, in either state, to zero. In considering real-world examples, the $W(G)$ and $W(B)$ terms below can be thought of as the change in the PM's welfare from execution *relative to non-execution* in the good and bad states, respectively.[16]

Assumption 1 ***(i)*** $W(G) > W(ne) = 0 > W(B)$.

[16] In setting probabilities of policy execution the PM decision is only affected by the impact of execution *relative to non-execution* in the good and bad state, and the probability of each state. Neither the baseline welfare after *non-execution* in either state, nor the difference in these will be material. (Previous versions of our paper did not use this normalization).

For a given $\langle q(G), q(B) \rangle$, (implicitly assuming the PM has deduced the true signal) her expected payoff is:

$$\Omega(q(G), q(B)) := P_G W(G) + P_B W(B) \tag{1}$$

As long as the $\mathcal{II}$'s incentive compatibility constraints (described in sections 3.4.1 and 3.4.2) are satisfied (implying $\mathcal{II}$ reveals the true state) the PM wants to maximize probability $q(G)$ and minimize probability $q(B)$.

We assume that before learning the signal the PM is indifferent.[17]

Assumption 2 *The policymaker is ex-ante indifferent between policies* $\langle q(G) = 1, q(B) = 1 \rangle$ *and* $\langle q(G) = 0, q(B) = 0 \rangle$, *i.e.,*

$$W(G)P_G + W(B)P_B = 0.$$

This indifference assumption implies that, holding the $\mathcal{II}$'s behavior constant, changes to $q(B)$ and $q(G)$ have opposite direct effects on PM's welfare (in proportion to the underlying probability of each state P_B and P_G).

3.3 Asset market and the informed investor's payoff

The asset's fundamental value, $A_1(p, s)$, represents the discounted stream of future earnings from the asset; this will depend on the policy decision and on the state. After these become common knowledge (at $t = 1$), the asset's price will equal its fundamental value.[18]

Determination of A_0: We assume A_0 is based on expected outcomes for the correctly anticipated probabilities of execution, i.e., $A_0 = A_0(q(G), q(B))$.[19] This endogeneity (i.e., rational expectations) is both reasonable and relevant. If *alternatively,* the initial asset price A_0 were exogenous, then holding the asset could be profitable on average given the announced $\langle q(G), q(B) \rangle$. Note that because we assume A_0 reflects rational expectations, taking into account the $\mathcal{II}$'s optimisation, this becomes a zero-sum game wrt the expected profits of those who buy/sell the asset. If $\mathcal{II}$ makes a profit on average, the people he trades with must make a loss.

[17] This is somewhat general; it could also apply to the *margin* of a policy over which the PM is indifferent. Alternately, suppose the PM has an ex-ante preference, but the preference is not too extreme, so the signal still matters. If the PM was ex-ante inclined to *not execute*, a commitment to $q(B > 0)$ would be costly. Here "listening" will only be worth doing if the welfare gain from the truthful signal exceeds the implementation cost.

[18] We assume that no earnings accrue from the asset until after time $t = 1$; this is without loss of generality.

[19] The results would be equivalent if we alternatively allowed these probabilities to *not* be correctly anticipated but assumed they were publicly announced before A_0 is set.

We can imagine several justifications for A_0 being unaffected by the policy commitments— see footnote—however, these do not seem reasonable or relevant to our empirical examples.[20] We briefly discuss the model with an exogenous A_0 in section 5.2; further details are available by request.

3.4 Price formation

We assume that at least some small amount of the asset is publicly traded at its expected value—*unconditional* on state at $t = 0$ and uninformed by the proposed trade—but after this trade the price nonetheless fully adjusts to reflect the information, and this is publicly observed. This is our strongest assumption. We return to this in section 6, considering models with noise traders, gradual revelation of information through dynamic trading, and the empirical phenomenon of "dark pools'. Of course, the *magnitude* of the amount that can be traded before price adjusts will not affect our results qualitatively; thus, we normalize it to *one unit.*

We further assume that the signal and the state are *specific* to the policy, i.e., if the policy is *not* executed, then the asset's value does not depend on the state:

Assumption 3

$$A_1(ne) := A_1(p = ne, s = G) = A_1(p = ne, s = B).$$

Thus $A_1(ne)$ is invariant to the state, i.e.,

$A_1(ne, s) = A_1(ne)$ for any $s \in \{G, B\}$.

We next derive the initial asset price:

$$\begin{aligned} A_0(q(G), q(B)) =& P_G q(G) A_1(e, G) + P_B q(B) A_1(e, B) \\ &+ [P_G(1 - q(G)) + P_B(1 - q(B))] A_1(ne) \\ =& P_G q(G)[A_1(e, G) - A_1(ne)] + P_B q(B)[A_1(e, B) - A_1(ne)] + A_1(ne) \\ =& q(G) P_G \Delta A_1(G) + q(B) P_B \Delta A_1(B) + A_1(ne), \end{aligned} \tag{2}$$

where $\Delta A_1(s) := A_1(e|s) - A_1(ne)$ is the expected benefit of execution (relative to non-execution) to an asset-holder given state s.

[20] Two possible justifications: (i) If only the $\mathcal{II}$ could profit from holding the asset at $t = 1$ (e.g., through his own production process) and he holds all the bargaining power, then A_0 would be priced at cost, regardless of the q's. However, this would imply that the PM could only identify the signal if she could identify *who* the $\mathcal{II}$ was *in advance* and identify his precise choice, a difficult proposition. (ii) If the policy was considered a zero-probability event, and the policy as well as the q functions were announced to the $\mathcal{II}$'s before being publicly announced, or the $\mathcal{II}$'s could react to this information before the UTs, then A_0 might also be unaffected by the q's; it also seems unlikely that the PM could orchestrate this. Furthermore, neither of these scenarios seem to reflect the empirical cases we describe.

$\mathcal{II}$**'s payoffs:** $\mathcal{II}$'s payoff is his net return from buying, doing nothing, or short selling the asset. As noted above, we assume that he can buy the first unit at price A_0 before his action is detected. Holding policy constant at $p \in \{e, ne\}$, his payoffs from buying, short selling, and doing nothing are, respectively:

$$V(p, s, i = b) = A_1(p, s) - A_0, \tag{3}$$

$$V(p, s, i = sh) = A_0 - A_1(p, s) = -V(p, s, i = b), \tag{4}$$

$$V(p, s, i = nb) = 0. \tag{5}$$

E.g., if he buys at time $t = 0$, setting $i = b$, he pays A_0 and earns the asset's fundamental value A_1 at $t = 1$.[21] We next define $\mathcal{II}$'s expected payoff from buying when he learns the state is s and he leads the PM to believe the state was $\hat{s}$ (henceforth, "his action reports $\hat{s}$").

$$\begin{aligned}\mathbb{E}V(i = b|s, \hat{s}) = &\sum_{s=G,B} [q(\hat{s})V(e, s, b) + (1 - q(\hat{s}))V(ne, s, b)] \\ &= q(\hat{s})[V(e, s) - V(ne)] + V(ne) = q(\hat{s})[A_1(e, s) - A_1(ne)] + A_1(ne) - A_0(q(G), q(B)) \\ &= q(\hat{s})\Delta A_1(s) + A_1(ne) - A_0(q(G), q(B)). \end{aligned} \tag{6}$$

In other words, when the $\mathcal{II}$ learns the state is s and his action reports $\hat{s}$, his expected payoff from buying sums $q(\hat{s})\Delta A_1(s)$—the expected increase in asset value given the true and reported signals— plus the baseline value under non-execution $A_1(ne)$, less the cost of buying the asset: $A_0(q(G), q(B))$. Similarly, in state s when his action reports $\hat{s}$, $\mathcal{II}$'s profit from short selling is

$$\mathbb{E}V(i = sh|s, \hat{s}) = -q(\hat{s})\Delta A_1(s) - A_1(ne) + A_0(q(G), q(B)) = -\mathbb{E}V(i = b|s, \hat{s}). \tag{7}$$

Expanding $A(ne) - A_0(q(G), q(B))$, equations (6) and (7) become:

$$\mathbb{E}V(i = b|s, \hat{s}) = q(\hat{s})\Delta A_1(s) - [q(G)P_G\Delta A_1(G) + q(B)P_B\Delta A_1(B)], \tag{8}$$

$$\mathbb{E}V(i = sh|s, \hat{s}) = -q(\hat{s})\Delta A_1(s) + [q(G)P_G\Delta A_1(G) + q(B)P_B\Delta A_1(B)]. \tag{9}$$

The first term on the right hand side of (8) and (9) is the expected gain or loss in asset value from probabilistic execution given the true and reported signals. The bracketed term is the *ex-ante*

[21] Similarly, if he short sells at time $t = 0$ (i.e., setting $i = sh$), he gets paid A_0 and then must buy the asset at $t = 1$. Note that the payoff from short selling is (holding the policy constant) the negative of the payoff from buying. If he does nothing he neither pays nor receives anything at $t = 0$ and neither owns nor owes the asset at time $t = 1$, hence the zero payoff.

expectation of this gain or loss, i.e., $A_0 - A_1(ne)$.

3.4.1 Incentive compatibility constraints: shorting not allowed

For $\mathcal{II}$'s behavior to be truth-telling, he must prefer to take action $i \in \{b, nb\}$ in the good state and prefer to take the other action in a bad state. There are two alternative sets of IC constraints: the PM may interpret $i = b$ and $i = nb$ as (i) the investor having received signals G and B respectively, or (ii) the investor having received signals B and G respectively.

Below, constraint (10) ensures that $\mathcal{II}$ prefers to buy in the good state, and (11) ensures he prefers not to buy in the bad state. I.e., the first constraint deters $\mathcal{II}(G)$ from mimicking $\mathcal{II}(B)$, and the second deters $\mathcal{II}(B)$ from mimicking $\mathcal{II}(G)$. The second set is stated similarly.

The 1st set: Buy only in state G

$$q(G)\Delta A_1(G) - [q(G)P_G\Delta A_1(G) + q(B)P_B\Delta A_1(B)] \geq 0$$
$$\Leftrightarrow q(G)\Delta A_1(G) \geq q(B)\Delta A_1(B) \tag{10}$$
$$0 \geq q(G)\Delta A_1(B) - [q(G)P_G\Delta A_1(G) + q(B)P_B\Delta A_1(B)]$$
$$\Leftrightarrow P_G q(G)\Delta A_1(G) \geq (q(G) - P_B q(B))\Delta A_1(B) \tag{11}$$

The 2nd set: Buy only in state B

$$q(B)\Delta A_1(B) \geq q(G)\Delta A_1(G) \tag{12}$$
$$P_B q(B)\Delta A_1(B) \geq (q(B) - P_G q(G))\Delta A_1(G) \tag{13}$$

Remark 1: We do not allow the PM to set direct payments after any combination of revealed signal and realized state. However, $A_0(q(G), q(B))$ plays a role similar to a direct payment in the standard mechanism design model with transferable utility.[22]

Remark 2: Our paper can also be compared with the literature on costly signaling models following Spence (1973): $\mathcal{II}$ sends a costly signal by buying/not buying (or short-selling) the asset, and upon observing $\mathcal{II}$'s action, the PM chooses execution/non-execution. There are two important differences. Most obviously, the PM commits to the probability of execution, $(q(G), q(B))$;

[22] As shown in the $\mathcal{II}$'s payoff (e.g., $q(G)\Delta(A) + A_1(ne) - A_0(q(G), q(B))$), the choice of randomized policy execution $\langle q(G), q(B)\rangle$ changes the amount the $\mathcal{II}$ has to pay/receive (i.e., $A_0(q(G), q(B))$). More specifically, if $\mathcal{II}$ chooses to buy, his net payoff is $q(G)\Delta(A) + A_1(ne) - A_0(q(G), q(B))$, and he has to pay $A_0(q(G), q(B))$. However, unlike the direct payment in the standard mechanism design literature, the monetary allocation $A_0(q(G), q(B))$ depends on the allocation of a non-money commodity (i.e., the randomized allocation $\langle q(G), q(B)\rangle$).

except where incentives are naturally aligned, the standard Spencian signaling will not be effective. Furthermore, the strategy $q(s)$ after observing signal s also influences $A_0(q(G), q(B))$, i.e, the equilibrium play determines the payoffs (including out-of-equilibrium payoffs). (See online appendix 4.1.1 for further discussion of this.)

3.4.2 Incentive compatibility constraints: buy/short implementation

In an alternative environment where short selling is allowed (which we return to in section 5), the PM may prefer a "buy/short" implementation, interpreting $i = b$ and $i = sh$ (i) as the investor having received signal G and B respectively, or (ii) B and G respectively.[23] Again, there are two sets of IC constraints.

The 1st set: Buy if and only if the signal is G, otherwise short sell.

$$q(G)\Delta A_1(G) - \mathbb{E}(\Delta A_1|q(G), q(B)) \geq -q(B)\Delta A_1(G) + \mathbb{E}(\Delta A_1|q(G), q(B)) \tag{14}$$

$$-q(B)\Delta A_1(B) + \mathbb{E}(\Delta A_1|q(G), q(B)) \geq q(G)\Delta A_1(B) - \mathbb{E}(\Delta A_1|q(G), q(B)) \tag{15}$$

where $\mathbb{E}(\Delta A_1|q(G), q(B)) := q(G)P_G\Delta A_1(G) + q(B)P_B\Delta A_1(B)$.

The 2nd set: Buy if and only if the signal is B, otherwise short sell.

$$-q(G)\Delta A_1(G) + \mathbb{E}(\Delta A_1|q(G), q(B)) \geq q(B)\Delta A_1(G) - \mathbb{E}(\Delta A_1|q(G), q(B)) \tag{16}$$

$$q(B)\Delta A_1(B) - \mathbb{E}(\Delta A_1|q(G), q(B)) \geq -q(G)\Delta A_1(B) + \mathbb{E}(\Delta A_1|q(G), q(B)) \tag{17}$$

4 Analysis

4.1 General results

With the aforementioned commitment $\langle q(G), q(B)\rangle$, the PM's problem is:[24]

$$\max_{0\leq q(\cdot)\leq 1} P_G q(G)W(G) + P_B q(B)W(B)$$

$$s.t. \;\; \text{set 1 [(10) and (11)] or set 2 [(12) and (13)].} \tag{18}$$

[23] We impose that $\mathcal{II}$ cannot choose "do nothing". However, we verify in online appendix 1.4. (https://davidreinstein.files.wordpress.com/2010/07/online-appendix.pdf) that under the implementation we derive, $\mathcal{II}$ will not want to "do nothing".

[24] These refers to the constraints *without* allowing short-selling. We offer a complete characterization for the no-short-selling case only. However, all of the following propositions and corollaries carry over to the case allowing short-selling.

Definition 1 (First best policy) *$\langle q(G)=1, q(B)=0\rangle$ is the first best policy.*

Definition 2 (Blind policy) *$\langle q(G), q(B)\rangle$ with $q(G)=q(B)$ is a blind policy.*

Definition 3 (Incentive-constrained optimal policy) *An incentive-constrained optimal policy solves the PM's maximization problem (18).*

Considering equation (1), the first best policy trivially maximizes the PM's welfare since $W(G) > 0$ and $W(B) < 0$. However, it may not be incentive compatible.

In contrast, a PM who employs a *blind policy* does not "listen to markets" – the signal does not alter her probability of executing, i.e., $q(G) = q(B)$. This policy is always incentive compatible (i.e., it leads $\mathcal{II}$ to take a distinct action in each state). This is intuitive. If the PM ignores the signal, she implements with the same probability whether or not the $\mathcal{II}$ buys the asset, and everyone knows this. $\mathcal{II}$ will thus buy in the state where the policy's impact on the asset is more positive (or less negative) than it would be in the other state. Thus he will take a distinct action for each state, revealing the state (excepting the case where the state is uninformative about the policy's impact, the knife-edge where $\Delta A_1(B) = \Delta A_1(G)$).

Proposition 4 *Any blind policy is incentive compatible (i.e., it satisfies one of the sets of incentive compatibility constraints). This holds whether or not short selling is allowed.*

The blind policy represents a lower-bound on the welfare that the PM can achieve. Note that the PM would never choose to execute the policy more frequently conditional on a bad signal than conditional on a good signal, as this would make her worse off than ignoring the signal.

Lemma 5 *The incentive-constrained optimal $\langle q(G), q(B)\rangle$ satisfies $q(G) \geq q(B)$, whether or not short selling is allowed.*

We show that $q(G) = 1$ for any implementation that does strictly better then a blind policy.[25]. Note that this proposition depends critically on the assumption of *endogenous* A_0; as we demonstrate in section 5.2, if A_0 were exogenous, this proposition would not hold.

Moreover, we show later that generically, the optimal policy *is* superior to a blind policy.

Proposition 6 *Any policy that yields higher welfare for the PM than a blind policy satisfies $1 = q(G) \geq q(B)$, whether or not short selling is allowed.*

[25] This result is consistent with the asymmetry of the problem; the PM "naturally" wants $q(B) = 0$ and $q(G) = 1$

Given that $q(G) = 1$, this commitment is equivalent to a commitment that with probability $q(B)$ she will ignore the asset price and execute the policy (and with the remaining probability she will choose her action in light of the signal).

As $q(G) = 1$, we focus on $q(B)$ in for the characterizing incentive-constrained efficiency. With two constraints for each set and a single choice variable $q(B)$, it is trivial that at most one constraint binds generically.

Corollary 7 *For a given set, at most one of the two incentive compatibility constraints will bind (whether or not short selling is allowed).*

The first set of inequalities (buy only if good) are summarized below, followed by the second set:

$$q(G)\Delta A_1(G) - \mathbb{E}(\Delta A_1|q(G), q(B)) \geq 0 \geq q(G)\Delta A_1(B) - \mathbb{E}(\Delta A_1|q(G), q(B)), \quad (19)$$

$$q(B)\Delta A_1(G) - \mathbb{E}(\Delta A_1|q(G), q(B)) \leq 0 \leq q(B)\Delta A_1(B) - \mathbb{E}(\Delta A_1|q(G), q(B)). \quad (20)$$

The set $\langle q(G), q(B)\rangle$ satisfying inequalities (19) is non-empty only if $\Delta A_1(G) \geq \Delta A_1(B)$; the set satisfying (20) is non-empty only if $\Delta A_1(G) \leq \Delta A_1(B)$.[26] From this, we derive the following proposition (computations in the appendix; further intuition in section 4.2.1).

Proposition 8 *The PM induces Set 1 ("buy only if good") if $\Delta A_1(G) > \Delta A_1(B)$ and Set 2 ("buy only if bad") if $\Delta A_1(G) < \Delta A_1(B)$, both where short selling is prohibited and in the buy/short implementation.* (For each case, it is impossible to implement the alternate set.)

We give further intuition for this in supplement 3.1. In essence, the PM needs to get the $\mathcal{II}$ to prefer to buy in one state but not in the other, knowing that the probability of execution depends only on the $\mathcal{II}$'s action and not the state, and the initial price does not depend on the state. Thus the remaining element in the $\mathcal{II}$'s profit, the *change in the asset's value* from execution must be more positive in the state in which he is induced to buy the asset.

Finally, we show that the PM always does better by listening to the market.

Proposition 9 *The incentive-constrained efficient policy is generically superior to blind policies, whether or not short selling is allowed.*

[26] Since $\Delta A_1(G) \geq \Delta A_1(B)$ and $\Delta A_1(B) \geq \Delta A_1(G)$ are obviously exclusive (excepting the knife-edge case) and exhaustive, only one set can be implemented generically, and the relative size of $\Delta A_1(G)$ and $\Delta A_1(B)$ fully determines which set of incentive compatibility constraints are used. The same holds when short-selling is allowed and the PM implements a mechanism getting the investor to choose $i \in \{b, sh\}$ after a good signal and choose the other element of $\{b, sh\}$ after a bad signal.

The proof of this proposition (in the appendix) shows that generically, for incentive-constrained efficiency, $q(B) < 1$. This is then combined with Proposition 6 to show that generically $1 = q(G) > q(B)$, which is superior to a blind policy. In other words, listening is generically better then not listening. Thus ex-ante indifference implies that the PM is willing to make commitment $\langle q(G), q(B)\rangle$ in order to learn the state (noting that where $\langle q(G) = 1, q(B) = 0\rangle$ is incentive-compatible, commitment is unneccesary).

4.2 Full characterization: Six cases

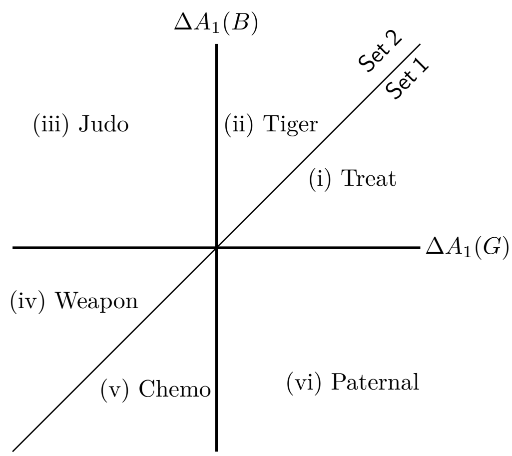

Figure 1: Taxonomy of incentive alignment between PM and an *asset-owner*

4.2.1 Taxonomy and examples

We allow the impact of the policy on the asset's value to have *any* possible relationship with the policy's merit (the state). Considering the direction and relative size of the impact (of the policy on the asset's value) in the good versus the bad state gives us six cases, depicted in figure 1.

I.e., we characterize six possible relationships between the PM's and the *asset owner's* incentives. Although it is the $\mathcal{II}$ and not the asset owner whose incentives matter, the impact on the *asset's value* is more easily interpreted and applied to the real world.

In case (i) the asset's value increases whenever the policy is executed, and it increases *more* under the good state. Thus the policy represents a *treat* for the asset-owner; execution will always increase the asset's value, but from the PM's perspective, it is only worth "buying this treat" where the signal suggests it will be *very* beneficial to the asset holder. The "Soylent" R&D example from the introduction fits this case. Alternately, this can describe a trade policy involving costly concessions or membership dues that is only worth executing if it boosts a particular export or service sector by a sufficient amount (e.g., see Breinlich, 2011).[27] A macroeconomic stimulus or a bank or international bailout may have similar properties. [28]

In case (ii) the asset's value increases when the policy is executed, but less so under the good state. This could describe a benefit program such as the USA's Medicare part D that was expected to be somewhat profitable for the drug industry, but to also yield other public benefits. However, depending on the true market structure and true prospects for innovation, the drug industry might be able to use this to reap excess profits at the expense of consumers (see Friedman, 2009). We refer to this as a "tiger", reflecting this potential for opportunism: you may be able to harness a tiger, but if it escapes its leash it eats at your expense.

In case (iii) the asset's value increases when policy is executed under the bad state, and decreases when it is executed under the good state. Here the interaction between the policymaker and the industry is largely zero-sum. The policy may be a tax reform or regulation intended to be harsh and punitive. However, it may backfire, perhaps if the firm finds loopholes, and may actually increase profits (hence the term "Judo").

In case (iv) the asset's value decreases whenever the policy is executed, but it decreases more under the good state. This policy is a "weapon" worth using only if it is fierce enough. It is intended/expected to severely reduce profits; perhaps these profits are seen as monopoly or monopsony rents, and thus reducing them may increase consumer surplus. There may be some cost to administering this policy. It may require a severe regulatory burden so it will only be worth doing if it has a major "trust busting" effect.[29]

In case (v) the asset's value decreases when the policy is executed, and more so under the bad state. This may reflect a tax increase or increased regulatory burden on industry, but one that is not intended to be excessively burdensome; for example, the Polish mining tax (section 2.1). This also may reflect a stricter price cap for a regulated industry such as a utility; the government wants

[27] Breinlich (2011) examined stock market reactions to the 1989 Canada-United States Free Trade Agreement (CAFTA). He found that "increases in the likelihood of ratification led to stock market gains of exporting firms relative to non-exporters", and used this to impute increased "expected per-period profits of exporters by around 6-7% relative to non-exporters." Another prominent example: the UK's consideration of paying to preserve the "passporting" rights of London financial firms.

[28] Extending the model, suppose the $\mathcal{II}$ is a large bond speculator, the PM is the European Union, and the policy is a package guaranteeing bonds against default, requiring austerity measures, and giving loans and aid to Greece. The default risk and the effectiveness of the policy are both uncertain. The bond holders (and Greek leaders) might prefer the EU to provide the maximal aid, but it may not be worth the cost to the EU. The EU could "announce consideration" of a policy, implying a certain conditional probability of execution, and see how the markets react. The *direction* of the likely effect is known (bonds will increase in value and yields will decline) but the *magnitude* of the effect will determine whether to execute the policy. The bailout environment in BTK resembles this: there an asset-holder benefits from a bailout in the "policy is good" state, but neither suffers nor benefits when policy is executed in the other state.

[29] Advocates of this policy may argue that it will reduce "excess profiteering" by monopolists and oligopolists. Opponents may argue it will have little effect on rents, as the oligopolists will find ways to evade it, yet it will lead to large bureaucratic costs and negative unintended consequences for consumers.

to limit profits but not to bankrupt the firm(s). Like *chemotherapy*, this policy is expected to do some damage, but it is only successful if it does not harm the patient (or asset) too much.

In case (vi) the asset's value increases when policy is executed under the good state, and decreases when it is executed under the bad state. This policy may be directly designed to benefit the industry, such as a change in regulations meant to deter destructive competition, or allow coordination on an industry-standard. This may apply to patent reform, or to a complicated change in trade agreements or in immigration law, or a cap on executive pay.[30] We call this a *paternalist* policy because at best it helps an industry achieve higher profits than they could achieve alone, but at worst it represents a misguided government overreach that hurts the private sector.

Returning to the optimal policy problem, Figure 2 describes the binding constraints in each region, with reference to the incentive compatibility constraints from section 3.4.1. (Note that $\emptyset$ means that no constraint is binding, i.e., the PM achieves the first best).

For a *paternalistic* policy the results are intuitive. Both the PM and the asset-holder want a *good* paternalistic policy to be implemented, and neither want a bad one to be implemented; thus, an asset-holder would want to truthfully reveal the signal to the PM. In this case (but not generally) the $\mathcal{II}$'s optimizing behavior will also reveal the true signal *even if the PM always listens*, setting $P_B = 0$ and $P_G = 1$. (Note that this does not hold when short-selling is allowed in the paternalistic case, as we note later.)

However, the governing constraints do not fully line up with our taxonomy, as the $\mathcal{II}$ need not be the asset-holder. In some cases the asset-holder's incentives are aligned with the PM's, but the $\mathcal{II}$'s are not (or vice-versa).

The *Chemo* case is less immediately intuitive: how can the PM get an $\mathcal{II}$ to buy in the good state, knowing that this will increase the probability of policy execution, and knowing that policy decreases the asset's value?

The answer is that the PM commits to still executing sometimes in the bad state (after "not buying"). As executing in the bad state reduces the asset's value *more* than executing in the good state, the $\mathcal{II}$ revealing that "the state is good" can *increase* the asset's value relative to the prior expectation. Yes, revealing this raises the probability the policy is executed (which is "bad" for the asset). However, through another channel it is "good news" for the asset: now we know the worst outcome for the asset (policy executed *and* bad state) will not occur. Thus, if PM commits to a

[30] E.g., Breinlich (2011) examined stock market reactions to the 1989 Canada-United States Free Trade Agreement (CAFTA). He found that "increases in the likelihood of ratification led to stock market gains of exporting firms relative to non-exporters", and used this to impute increased "expected per-period profits of exporters by around 6-7% relative to non-exporters."

Case	Exogenous condition	Induce "Buy iff"	Binding constraint	Optimal $q(B)$
(i) "Treat"	$\Delta A_1(G) > \Delta A_1(B) > 0$	G	$\emptyset$ or eq. (11) $(B \to G)$	$\max(0, \frac{1}{P_G}(1 - P_B \frac{\Delta A_1(G}{\Delta A_1(B}$
(ii) "Tiger"	$\Delta A_1(B) > \Delta A_1(G) > 0$	B	eq. (12) $(B \to G)$	$\frac{\Delta A_1(G)}{\Delta A_1(B)}$
(iii) "Judo"	$\Delta A_1(B) > 0 > \Delta A_1(G)$	B	eq. (13) $(G \to B)$	$\frac{P_B}{1-P_G \frac{\Delta A_1(B)}{\Delta A_1(G)}}$
(iv) "Weapon"	$0 > \Delta A_1(B) > \Delta A_1(G)$	B	eq. (13) $(G \to B)$	$\frac{P_B}{1-P_G \frac{\Delta A_1(B)}{\Delta A_1(G)}}$
(v) "Chemo"	$0 > \Delta A_1(G) > \Delta A_1(B)$	G	eq. (10) $(G \to B)$	$\frac{\Delta A_1(G)}{\Delta A_1(B)}$
(vi) "Paternal"	$\Delta A_1(G) > 0 > \Delta A_1(B)$	G	$\emptyset$	0

Notes: Short-selling is forbidden by assumption. $q(G) = 1$ is optimal in all cases. "$B \to G$" refers to the binding IC constraint for type B, and "$G \to B$" the binding IC constraint for type G.

Table 1: Six cases, no short-selling: Constraints and optimal mechanisms

high enough $q(B)$ the net effect of the above on the asset's expected value is positive, which will induce the $\mathcal{II}$ to "buy if good".[31]

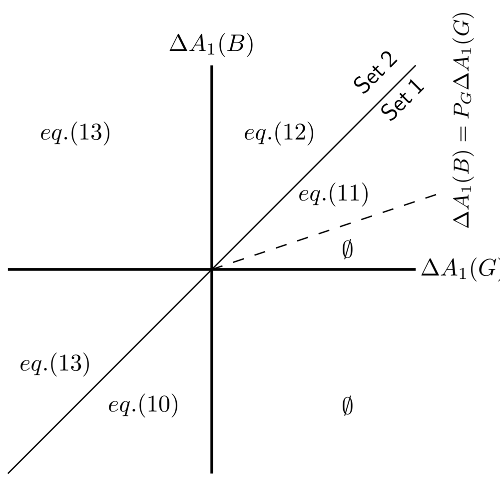

Figure 2: Binding constraints without short-selling

Binding constraints and results on optimal policy mechanisms are listed in table 1 and summarized in figure 2 (and also figure 3).

Note in particular that for each case, as $\Delta A_1(G) \to \Delta A_1(B)$, $q(B) \to 1$, leaving no gain to the PM. The more similar the policy's impact on the asset under either state, the more difficult it is for the PM to get the $\mathcal{II}$ to act distinctly under each signal. Further comparative statics can be trivially derived from the results in table 1. E.g., in the *Judo* case, as the bad state becomes more likely ($P_B \to 1$) the optimal $q(B)$ also moves closer to one; policy is almost always executed.

[31] You might be waylaid by the thought "won't this require executing in the bad state more than in the good state"? This need not be the case: consider that the asset's value may decrease *far more* after execution in the bad versus good state, and the PM doesn't care directly about the asset's value.

5 Comparison to alternative assumptions

5.1 Comparison: main model versus buy/short implementation

In many real-world settings investors may be able to short sell an asset without a large transaction cost. With short selling, there are potentially six ways of implementing truth-telling behavior: through $\mathcal{II}$ choosing any of "buy, do nothing, or short sell" after a good signal, and choosing any *other* action after a bad signal. Each implementation requires that four IC constraints be satisfied, as after each signal, one action must be preferred over the other two. For tractability we focus on the "buy/short" implementations; IC constraints for these were given in section 3.4.2.

In this implementation, the binding constraint, and thus the feasible implementation, depends on whether $P_G > P_B$. Again, an "asymmetric asset gain" may be traded off against the "relative information advantage" of inducing the less-likely PM choice. All results for the buy/short implementation are derived in the online appendix 1.2.[32]

We summarize the optimal policies for each of the six cases, separately for $P_G > P_B$ and $P_G < P_B$. (Since $q(G) = 1$ for all of our derived implementations it is left out of the graphs.)

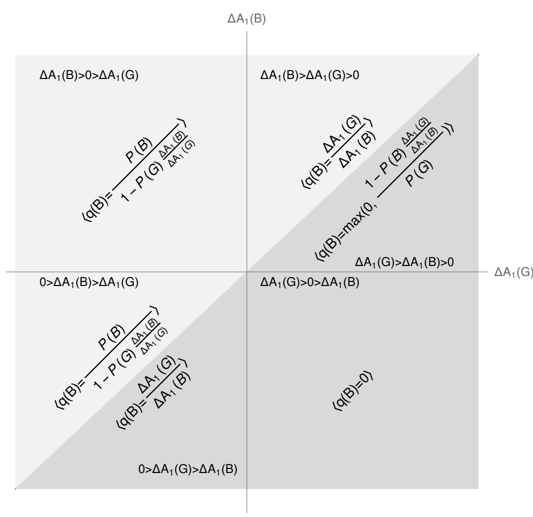

Figure 3: Without short-selling

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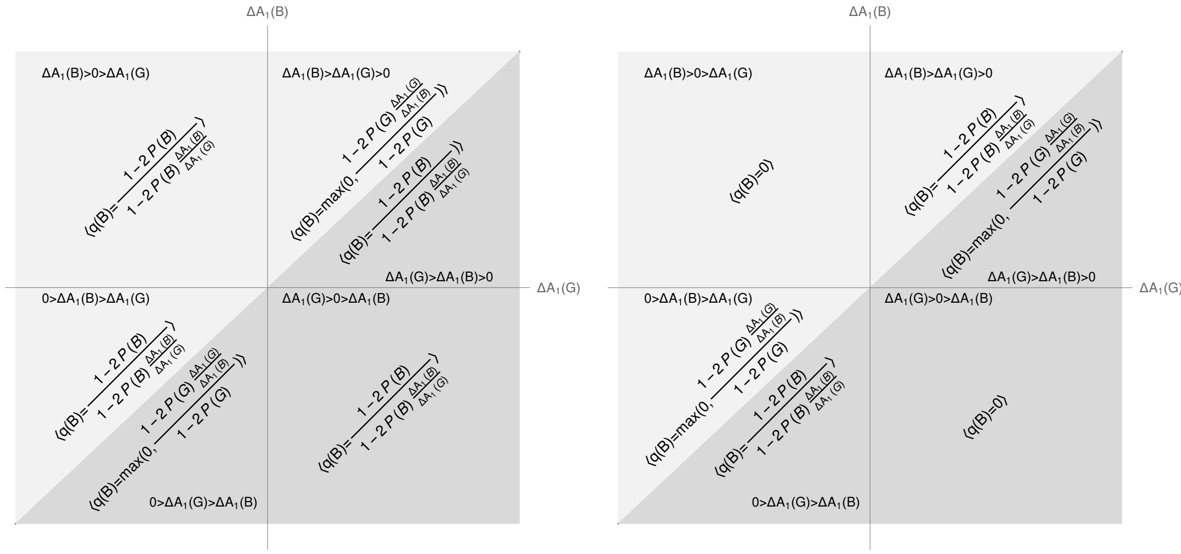
[32] https://davidreinstein.files.wordpress.com/2010/07/online-appendix.pdf
[33] Interactive diagrams can be seen at `http://tosoil.byus.net/3d/test.html`



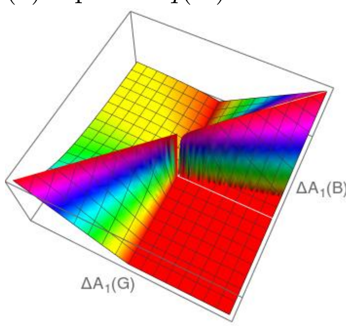
$P_G > \frac{1}{2} > P_B$ $\quad$ $P_G < \frac{1}{2} < P_B$

Figure 4: Allowing short-selling

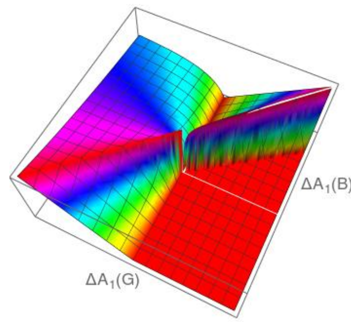
Intuition might suggest that restricting the tools available to the informed investor, e.g., by forbidding short sale, would make it harder for him to “manipulate” the policy outcome and conversely, easier to get him to reveal his private information. However, allowing $\mathcal{II}$ to short sell may *lower* the incentive compatible $q(B)$ and thus improve the PM’s welfare. There are two potentially countervailing effects. Consider case (v), “chemo”, where the implementation involves “buy if $s = G$.” A potentially profitable short-selling opportunity means $\mathcal{II}$ may not prefer to buy after a good signal. On the other hand, allowing short-selling may make it *easier* to dissuade him from buying after a bad signal. Although allowing short sale provides a further tool for $\mathcal{II}$’s deviation, it also can provide him more benefit when he is truthful. Short sale makes both the right-hand side and the left hand side of each incentive compatibility constraint larger; hence, it may become easier (or harder) to enforce truth-telling.

It is easy to see that allowing short selling improves welfare in the following cases. Comparing figures 3 and 4, the buy/short implementation sets $q(B) = 0$ if $\Delta A_1(G) < 0 < \Delta A_1(B)$ and $P_G < \frac{1}{2} < P_B$, while $q(B) > 0$ in the main model. The same holds for $P_G > \frac{1}{2} > P_B$, $\Delta A_1(B) < \Delta A_1(G) < 0$, $1 - 2P_G\frac{\Delta A_1(G)}{\Delta A_1(B)} < 0$. Similarly, for $P_G > \frac{1}{2} > P_B$, $0 < \Delta A_1(G) < \Delta A_1(B)$, $1 - 2P_G\frac{\Delta A_1(G)}{\Delta A_1(B)} < 0$. Figure 5 depicts the optimal policy for specific values of P_G and P_B, allowing a visual comparison of $q(B)$ in the two environments (with or without short-selling).

(a) Optimal $q(B)$ without short-selling

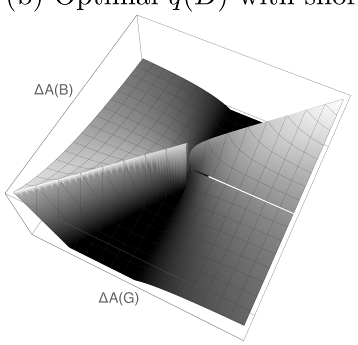


$P_G = \frac{3}{4} > P_B = \frac{1}{4}$

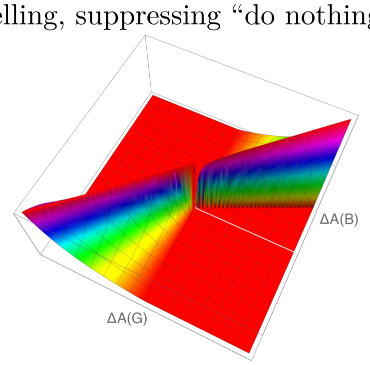


$P_G = \frac{1}{4} < P_B = \frac{3}{4}$

(b) Optimal $q(B)$ with short-selling, suppressing "do nothing"



$P_G = \frac{3}{4} > P_B = \frac{1}{4}$



$P_G = \frac{1}{4} < P_B = \frac{3}{4}$

Figure 5: Example: Optimal $q(B)$

We have demonstrated that for some parameters, allowing short selling helps the PM.[34]

5.2 Comparison: main model versus model with exogenous A_0

Proposition 6 (stating that the PM may set $q(B) > 0$ but will always set $q(G) = 1$) depended on the endogeneity of A_0 in $(q(G), q(B))$. We now show that $q(B) = 0$ and $q(G) < 1$ can be optimal if A_0 is *exogenous*. We offer a single example (the full case-by-case analysis is available by request). For brevity, we assume short-selling is not allowed. We consider an example resembling case (i) of our main model, letting the $\mathcal{II}$'s payoffs satisfy:

$$V(e, G) > V(e, B) > 0 > V(ne, s), \tag{21}$$

where $V(p, s) := P_s A_1(p, s, i = b) - A_0$. I.e., for this case an $\mathcal{II}$ who buys gets a profit when a good policy is executed, a lower profit when a bad policy is executed, and sustains a loss when the policy is not executed. Inequalities (21) can be derived from the assumption of $\Delta A_1(G) > \Delta A_1(B) > 0$ and $A_0 > 0$.

Condition (21) implies that $\mathcal{II}(G)$ prefers *buying* to *doing nothing*, and $\mathcal{II}(B)$ prefers *doing nothing* to *buying*. This requires the following incentive compatibility constraints:

$$q(G)(V(e, s, b)) + (1 - q(G))V(ne, s, b) \geq 0, \tag{22}$$

$$0 \geq q(G)V(e, s, b) + (1 - q(G))V(ne, s, b). \tag{23}$$

These constraints are simplified into:

$$q(G)[V(e, G) - V(ne, G)] \geq -V(ne, G) \text{ and } -V(ne, B) \geq q(G)[V(e, B) - V(ne, B)]. \tag{24}$$

Given that $q(B)$ has no effect on these IC constraints (as it does not affect A_0), the PM sets $q(B) = 0$. Note that the same intuition does not go through in the main model with *endogenous* A_0; setting $q(B) > 0$ in our preferred model will affect the asset's initial price and thus the $\mathcal{II}$'s relative incentives to buy or "not buy".

On the other hand $q(G)$ cannot be either 0 or 1: if $q(G) = 1$, then the investor with signal $s = B$ always succeeds in deceiving the policymaker, and if $q(G) = 0$, then the policymaker never executes the policy, which implies an inefficiency since the policymaker ignores valuable information. Thus $q(G)$ must be a number between 0 and 1. This demonstrates that with a fixed initial asset price

[34] As we have only solved for the buy/short implementation, we cannot determine whether for some parameter values, the PM becomes worse off by allowing short selling.

A_0, the PM sets $\langle q(B) = 0, q(G) < 1 \rangle$ for some ranges of parameters.

5.3 Multiple informed investors

In the real-world, there may be *several* investors with relevant private information. The mechanisms discussed above could be similarly implemented if the PM set rules such as, for case 1, "execute if and only if the price increase reflects the majority of informed investors buying the asset". In general, if the other $\mathcal{II}$'s all take the action that reports the true signal, no $\mathcal{II}$ prefers to deviate, *whether or not his deviation would change the policy.* No $\mathcal{II}$ gains from the policy itself, only from his investment profits; thus the identical IC constraints should guarantee this.

However, there might be a coordination failure among the $\mathcal{II}$s: they might be in an equilibrium that yields each of them lower payoffs, given the PM's announced mechanism. E.g., in case (i), the PM might set probabilities of execution such than any $\mathcal{II}$ gets non-negative profit from buying and inducing execution after a good signal, and a non-positive profit from buying and inducing execution after a bad signal. Nonetheless, all of the $\mathcal{II}$'s might choose "do nothing" after a good signal, inducing non-execution. In such a case no $\mathcal{II}$ would have an incentive to deviate to "buy", as this would not affect the PM's decision and thus the deviation would yield negative profits.

6 Microstructure, justification of trade and pricing

As mentioned in section 3.4, our above model of trade and price formation requires further justification. We allowed the $\mathcal{II}$ to buy (or short) the asset at the uninformed price $A_0(\alpha(G), \alpha(B))$, yet no one else in the economy would have a motive to trade with him at this price. Furthermore, we did not explain why there is trading or how the initial price is observed.

For intuition, suppose, reasonably, that there are multiple market makers, or at least a competitive fringe. Suppose $\mathcal{II}$ offered to buy from a MM. At this point, this MM would recognize the (true) signal, which would be ultimately passed to the PM, leading to the predictable policy reaction.[35] Because of this, MM would then be unwilling to sell for any price below $\tilde{p} := \alpha(\hat{s})\Delta A_1(s) + A_1(ne)$, and no other MM could do better by selling at a lower price; the price would be expected to rise to $\tilde{p}$. This would leave no room for the $\mathcal{II}$ to profit from any trade after any signal.

BTK's microstructure makes the polar opposite assumptions to our previous assumptions. In BTK, in any period, nature draws either a noise (or "liquidity") trader, or an informed trader (of the good or bad-state type). In effect, the trader (in particular the $\mathcal{II}$) must announce the

[35] With multiple-market makers, MM-A anticipates that even if he does not sell to $\mathcal{II}$, MM-B would do so, leading to the public signal.

total amount that he would like to buy or sell before he can exchange any amount. The market maker will offer him a price that represents the expected value of the asset, taking into account *all* the information conveyed by the trade he requests. In BTK, the noise trades follow a massless distribution over the entire real line.

Our above pricing/trading assumption could be *literally* justified by a very simple institutional structure. Suppose a market-maker must post a set price, and must be willing to buy or sell one (normalized) unit at this price. She can adjust the price freely after each trade. Such an institutional structure might be imposed by the government, or it might arise naturally due to menu costs or search costs. However, if $\mathcal{II}$ is the only trader, MM will, on average, lose money on these first units traded. On the other hand, if we introduce additional *noise traders* as well as a continuous trade (resembling BTK), then $\mathcal{II}$'s trades will send only a *probablistic* signal to the PM. Unlike in BTK, as our PM is ex-ante indifferent *any* trade that is even marginally informative should sway the policy in one direction or another. The $\mathcal{II}$ will consider his trade's impact both on the per-unit price he pays and on policy-setting. The PM's mechanism-design problem becomes complicated; we save this for future work.

The BTK assumptions are not universal in the microstructure literature. The prominent Kyle model (1985) justifies an intermediate case. Here, both the fundamental value (observed by an $\mathcal{II}$) and the distribution of net trades from noise traders follow a normal distribution. Kyle's market maker only observes the sum of the $\mathcal{II}$'s and noise traders' simultaneous demand in each sequential auction, and sets a price that brings her zero expected profits. This leads to a price that incorporates half of the private information in each auction. The $\mathcal{II}$ acts as an intertemporal monopolist and chooses a trade to maximize the product of quantity and the markup above the fundamental value; i.e., he takes some profits during this adjustment process. At the limit of continuous sequential auctions, "all private information is incorporated into prices by the end of trading." For our purposes, Kyle's key insight is that "the strategic exercise of monopoly power by an insider is in no way inconsistent with prices being set efficiently in the semi-strong sense"; this sketches a second justification for our previous assumption that the informed investor can trade a number of units (normalized to one) at the uninformed price.

Finally, "trading at a fixed price" seems to be common in the real world. Large financial actors such as institutional investors often trade in "dark pools" which offer them a fixed price, and delay the release of information on their trades. According to **?**Zhu (2014):

> ...dark pools... derive execution prices from lit venues. For example, a typical, classical form of dark pools matches customer orders at prices derived from lit venue[s],

such as the midpoint of the national best bid and offer (NBBO) or the volume-weighted average price (VWAP).

Dark pools became an important part of stock market trade since 2007's full implementation of Regulation National Market System (Securities and Exchange Commission 2005). There is a consensus that dark pools take up a substantial share of equities trading.[36]

7 Policy insights and implementation, synthesis

We have presented a simple binary-policy framework, intuitively depicting a complete set of relationships between private signals, asset values, and PM's welfare, and solving for the optimal policy mechanism, giving some intuition for the comparative statics. We found that listening is generically strictly welfare improving, and that the optimal mechanism involves sometimes/never executing after a *bad* signal, but always executes after a *good* signal. We compared our main model's results to the "buy/short" implementation, showing that allowing short-selling can *improve* the PM's welfare. We considered the robustness of our model to allowing multiple informed investors, and demonstrated the sensitivity of our results to our assumption that the initial asset prices are endogenously determined in light of the policy commitments.

Considering all three setups yields an additional insight (not presented above). Consider case (vi), the *Paternalist* policy. The first best is achieved in our main model, where shorting is not allowed.[37] This agrees with an intuitive notion of "aligned incentives": both the government and an *asset holder* want the policy executed if and only if it is *good*. If the A_0 were exogenous, as discussed in section 5.2, the first-best alignment would hold for *any* P_G. $\mathcal{II}$ would care only about the policy's impact on the *asset's fundamental value*; he would always want to buy after a good signal, inducing the "good" policy and making a profit on the asset, and would never want to buy after a bad signal. On the other hand, with an endogenous A_0, if the good signal is the more common one, shorting yields a greater "information advantage" than does buying, implying $q(B) > 0$ may be required to deter shorting after a good signal. A similar insight holds for case (iii), i.e., wherever the expected *direction* of the policy impact on the asset depends on the signal. A caution to policymakers: even if ex-post incentives are aligned, the incentive to buy or short sell an asset may not be.

For policies where voters do not have a strong issue identification, policy performance, rather

[36] The SEC reported in 2010 that 32 (narrowly defined) dark pools take up 7.9% of US total equity trading volume; and Tabb Group estimates 12%.

[37] If shorting is allowed (section 5.1) the PM attains the first-best if the good state less likely than the bad state.

than immediate public opinion, is what matters most.[38] Thus, after floating policy "trial balloons" politicians may listen to both polls and markets.[39] The ability to listen and conditionally commit, perhaps imprecisely, may be embodied in the political system. In a system with several branches of government, the framers of a constitution could either allow an executive (President or Premier) to execute policy unilaterally, or require her to put a bill to the legislative branch.

While an executive cannot precisely set the probability of passage after good or bad market news, she can make the initial bill's language more or less palatable. For example, President Obama could have first submitted his health care bill mandating broad birth-control coverage, which presumably would have made it unlikely to pass. If it failed, but the market's reaction was favorable, he could then have submitted a similar bill without the birth-control provision. Constitutional and procedural rules determine the length of time a bill is considered, whether its sponsor can introduce unrelated "riders," and how much it can be adjusted throughout the process. A long deliberation permitting amendments and reconsiderations may allow legislators more time and flexibility to listen to the market; "fast-tracking" will limit this. The bill's precise timeline can also affect the conditional probabilities of execution. If the market's signal about the policy is revealed with a known hazard rate, the date that a bill is scheduled *for a* vote will determine the probability that the (good or bad) signal will have been revealed.

Adding legislator-specific favors and "pork", or unfavorable "poison" to a bill will also can directly affect the conditional probabilities. Suppose that there is uncertainty about some legislator's preferences, while some legislators' preferences can be identified, e.g., known "swing Democrats" always support the bill after a good signal, oppose it after a bad signal. Suppose that offering pork for a legislator's district raises the probability that she votes for a bill after *any* signal – as long as she is not already certain to vote for (against) the bill. Here, offering pork for a swing Democrat will increase $q(B)$ without affecting $q(G)$.

Another such strategy is *committing political capital.* A government or party may come out strongly in support of a policy, putting their credibility on the line, and making it costly (but not impossible) to later vote against the policy if the market reveals a negative signal.[40] This might have *no effect* on the probability of execution after a *good* signal. The strength of the commitment

[38] This is likely to hold for technical "hard issues" (Carmines and Stimson, 1980). Fiorina (1978), among others found some evidence for "retrospective voting"; however, there is debate over its explanatory power (see e.g., Fiorina et al., 2003).

[39] Listening to the market is not equivalent to a referendum; the commitments we describe allow the PM to use the market to *extract private information* about the potential results of policies. Unlike referendum voters, traders in the market are "voting" about what they think is profitable, but not necessarily voting for what benefits them as a private citizen.

[40] This cost could come from a loss of reputation for managerial expertise as in Prendergast and Stole (1996) or a simple voter dislike or distrust of inconsistency and "flip-flopping", perhaps signaling untrustworthiness.

will depend on the number of legislators asked to speak in favor of the policy. Suppose there is asymmetric information over legislators' true preferences, or there are other sources of randomness such as revealed public support. Thus, the likelihood that the bill is passed after a bad signal would increase in the number of legislators committed to the policy.

The commitment might also take the form of a policy trial, perhaps one with a high probability of a type-I or type-II error. The PM could commit to follow the results of the trial if they are strongly significant in one direction or the other, which may be a small fraction of the time. Here, the PM might not actually expect to learn from the trial itself; she cares more about how the market reacts to the *announcement* of the trial. Where the trial's results are *not* significant, she can follow the signals generated by $\mathcal{II}$'s behavior.

For more technical policies and technocratic policymakers commitments might involve *explicit* randomization.[41] However, we think it more likely will occur through the less precise methods just described, exploiting randomness inherent in the political system. We have presented suggestive evidence that legislators and executives are already taking the market's reaction into account. Still, even if policymakers do not *explicitly* consider introducing randomness, we present a framework for considering the use of market signals to adapt policy.

We chose to focus on an indirect mechanism because an *explicit direct* mechanism – paying investors for insider information – may not be feasible. Such "handouts" would likely be politically unpopular. It also may be difficult to identify *which* investors have the accurate information, or to offer "informants' incentives to reveal themselves.[42] Boleslavsky et al. (2017) model (a firm's) choice between using "markets and mechanisms" to elicit private information. They find an additional barrier to using a direct revelation mechanism: the presence of such a mechanism itself improves the trading payoffs of an informed agent). This may make it impossible to screen out uninformed agents, or at least make it preferable to rely on markets for information.

Our model could be extended in several ways. Future work might consider a policymaker who seeks to influence *investor* behavior. Alternately, one could model an investor with an *inherent* interest, e.g., who owns an asset affected by the policy which cannot be sold without costs. The

[41] Wolfers and Zitzewitz (2006) note that if there are several public and private signals of a policy's efficacy, the difference in a conditional asset's value may be difficult to interpret. E.g., the US may only execute a carbon cap if there is severe flooding of the Eastern seaboard; hence the conditional expectation of GDP in the event of a carbon cap may also reflect the expectation of the effect this flooding. Thus, introducing exogenous randomness might have an additional benefit: it may help improve the interpretation of market signals even *without* manipulation. This is scope for future work.

[42] The government might ask investors who claim to have inside information to pledge a large enough forfeit – in case they are proven wrong – to induce only truth-tellers to come forward. However, limited liability may make this impossible. Existing asset markets are already suited to deal with these commitment problems. Furthermore, if the government offered commitments to these "informants", it would still have to carefully monitor the informants' asset market positions, as outside investments could undermine their incentives truth telling.

most valuable extensions may be empirical. Economists should seek to identify and measure the *ways* in which particular asset values will be differentially affected by policies, and how this relates to the policies' welfare consequences. Where a connection is found, economists should measure the extent to which information is concentrated in the hands of potential "manipulators." Armed with this knowledge, policymakers could set up a "listening process", bearing in mind the implementation concerns we highlight. In particular, they may need to limit the extent to which good or bad news feeds directly into policy, incorporating literal or approximate policy commitments.

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8 Appendix

8.1 Proofs

Proof of Proposition 4 If $\Delta A_1(G) \geq \Delta A_1(B)$, or $\Delta A_1(G) \leq \Delta A_1(B)$, it is trivial that the first set, or the second set, respectively, of incentive compatibility constraints are satisfied with $q(G) = q(B)$. ∎

Proof of Lemma 5

Considering expression (1) in light of the PM's indifference (Assumption 2), $q(G) < q(B)$ implies that the sum of the last two terms in expression (1) is negative; while $q(G) = q(B)$ (i.e., *not listening to markets*) implies this sum is zero. Thus we must have $q(G) \geq q(B)$. ∎

Computations for proof of Proposition 8

Inequalities (14) and (15) and Inequalities (16) and (17) are written as follows with short selling allowed, for the mechanism considered:

$$[q(B) + q(G)]\Delta A_1(B) - 2\mathbb{E}(\Delta A_1) \geq 0 \geq [q(B) + q(G)]\Delta A_1(G) - 2\mathbb{E}(\Delta A_1) \quad (25)$$

$$[q(B) + q(G)]\Delta A_1(B) - 2\mathbb{E}(\Delta A_1) \leq 0 \leq [q(B) + q(G)]\Delta A_1(G) - 2\mathbb{E}(\Delta A_1) \quad (26)$$

The first set (the second set) is non-empty only if $\Delta A_1(B) \geq \Delta A_1(G)$ ($\Delta A_1(B) \leq \Delta A_1(G)$). Note that $\Delta A_1(G) \geq \Delta A_1(B)$ and $\Delta A_1(B) \geq \Delta A_1(G)$ are exclusive (excepting the knife-edge case) and exhaustive. By the same reason in the environment without short selling, the relative size of $\Delta A_1(G)$ and $\Delta A_1(B)$ fully determines which set of incentive compatibility constraints are used.

Proof of proposition 6

Given that $\mathcal{II}$'s payoff is a linear function of $q(G)$ and $q(B)$ without a constant term (see statements 10-13), incentive compatibility constraints are characterized as $Cq(G) \gtreqless q(B)$ for some constant term C (with or without short sale).

Suppose a constraint being considered is of the form $Cq(G) \geq q(B)$. If $C \geq 0$ then the first best $\langle q(G) = 1, q(B) = 0\rangle$ trivially satisfies this, implying that this constraint is non-binding. On the other hand, if $C < 0$, then only the policy $q(G) = q(B) = 0$ (a particular version of a blind policy) can implement truth-telling (but $C > 0$ must have been ruled out for some other reason, because we know that any blind policy yields truth-telling). Note also that $C > 1$ is impossible.

Thus, any *binding* IC constraint for an implementation yielding PM-welfare greater than a blind policy can be expressed as $Cq(G) \leq q(B)$.

Considering $Cq(G) \leq q(B)$, we see that $C \leq 1$ must hold, as $q(G) \geq q(B)$ is needed for welfare to exceed the blind policy (Lemma 5).[43]

If $C = 1$ we must have $q(G) = q(B) = 1$, again a blind policy.

If neither constraint binds then the PM sets the first-best policy $1 = q(G) > q(B) = 0$.

Considering a binding constraint where $C < 1$, if it were met with $Cq(G) = q(B) < 1$ the PM would not be optimizing: she could increase welfare while preserving this constraint by raising $q(G)$ some amount ϵ and raising $q(B)$ by a smaller amount $C\epsilon$. ∎

[43] Thus, unless $C \leq 1$, we have a contradiction ($Cq(B) \leq Cq(G) \leq q(B)$).

Proof of proposition 9: To prove this we show that generically, for incentive-constrained optimality, $q(B) < 1$. From Proposition 6, this implies generically $1 = q(G) > q(B)$, which is superior to a blind policy.

Generically, $\Delta A_1(G) \neq \Delta A_1(B)$. (i) Assume $\Delta A_1(G) > \Delta A_1(B)$. Suppose that $q(G) = q(B) = 1$. Then Set 1 becomes: $P_B\Big[\Delta A_1(G) - A_1(B)\Big] \geq 0$, $P_G\Big[\Delta A_1(B) - A_1(G)\Big] \leq 0$, whether short selling is allowed or not, as shown in the proof for Proposition 4. We can see that both of these two constraints hold with strict inequality, implying $q(B) < 1$ would also satisfy these constraints, and thus $q(B) = 1$ must be suboptimal. (ii) Assuming $\Delta A_1(G) < \Delta A_1(B)$, we can apply the same logic to the second set of incentive compatibility constraints to derive a contradiction. ∎

8.2 Deriving case-by-case solutions (from section 4.2)

Case (i), "A Treat", $\Delta A_1(G) > \Delta A_1(B) > 0$: As the PM wants to *minimize* $q(B)$, the only constraint that might bind is the second one. Note that the bracket term can be positive or negative depending on the sign of $\Delta A_1(B) - P_B\Delta A_1(G)$, as $\frac{\Delta A_1(B)-\Delta A_1(G)}{P_G} + \Delta A_1(G) = \frac{1}{P_G}(\Delta A_1(B) - (1 - P_G)\Delta A_1(G)) = \frac{1}{P_G}(\Delta A_1(B) - P_B\Delta A_1(G))$. If $\frac{1}{P_G}(\Delta A_1(B) - P_B\Delta A_1(G)) < 0$, the constraint does not bind, and the optimal solution is $< q(G) = 1, q(B) = 0 >$. On the other hand if $\frac{1}{P_G}(\Delta A_1(B) - P_B\Delta A_1(G)) > 0$, the constraint is binding; thus, the optimal solution is $\langle q(G) = 1, q(B) = \frac{1}{P_G}(1 - P_B\frac{\Delta A_1(G)}{\Delta A_1(B)})\rangle$.

Case (ii), "Tiger", $\Delta A_1(B) > \Delta A_1(G) > 0$: Since the PM wants to minimize $q(B)$, only the first inequality binds in Set 1: $\Delta A_1(B)q(B) \geq \Delta A_1(G) \geq [\frac{\Delta A_1(G)-\Delta A_1(B)}{P_B} + \Delta A_1(B)]q(B)$, i.e., the constraint for B not to mimic G. The solution is derived from this.

Case (iii), "Judo", $\Delta A_1(B) > 0 > \Delta A_1(G)$:

In Set 2: $\Delta A_1(B)q(B) \geq \Delta A_1(G) \geq [\frac{\Delta A_1(G)-\Delta A_1(B)}{P_B} + \Delta A_1(B)]q(B)$, the only constraint that might bind is the second one, which is re-written as $1 \leq [\frac{1-\frac{\Delta A_1(B)}{\Delta A_1(G)}}{P_B} + \frac{\Delta A_1(B)}{\Delta A_1(G)}]q(B)$. Note that the bracket term is simplified as $\frac{1}{P_B}[1 - \frac{\Delta A_1(B)}{\Delta A_1(G)} + P_B\frac{\Delta A_1(B)}{\Delta A_1(G)}] = \frac{1-P_G\frac{\Delta A_1(B)}{\Delta A_1(G)}}{P_B}$, which is larger than 1 since $P_B < 1$ and $1 - P_G\frac{\Delta A_1(B)}{\Delta A_1(G)} > 1$.

Case (iv), "Weapon", $0 > \Delta A_1(B) > \Delta A_1(G)$: Since the PM wants to minimize $q(B)$ in Set 2 ($[-\Delta A_1(B)]q(B) \leq [-\Delta A_1(G)] \leq [\frac{-\Delta A_1(G)+\Delta A_1(B)}{P_B} - \Delta A_1(B)]q(B)$), the second inequality binds, which is re-written as: $1 \leq [\frac{1-\frac{\Delta A_1(B)}{\Delta A_1(G)}}{P_B} + \frac{\Delta A_1(B)}{\Delta A_1(G)}]q(B)$. Note that the bracketed term is simplified as $\frac{1}{P_B}[1 - \frac{\Delta A_1(B)}{\Delta A_1(G)} + P_B\frac{\Delta A_1(B)}{\Delta A_1(G)}] = \frac{1-P_G\frac{\Delta A_1(B)}{\Delta A_1(G)}}{P_B}$, which is larger than 1 since $(1 - P_G\frac{\Delta A_1(B)}{\Delta A_1(G)}) > 1 - P_G = P_B$.

Case (v), "Chemotherapy", $0 > \Delta A_1(G) > \Delta A_1(B)$: We consider Set 1: $[-\Delta A_1(G)] \leq [-\Delta A_1(B)]q(B) \leq [\frac{-\Delta A_1(B)+\Delta A_1(G)}{P_G} - \Delta A_1(G)]$ where the last bracket term is positive. Since the PM wants to minimize $q(B)$, the first inequality binds, and the solution is simply derived.

Case (vi), "Paternalist", $\Delta A_1(G) > 0 > \Delta A_1(B)$: Consider Set 1: $\Delta A_1(G) \geq \Delta A_1(B)q(B) \geq [\frac{\Delta A_1(B)-\Delta A_1(G)}{P_G} + \Delta A_1(G)]$. To minimize $q(B)$, the only constraint that might bind is the second constraint. Dividing both sides by $\Delta A_1(B)$, we derive $q(B) \leq \left[\frac{1-\frac{\Delta A_1(G)}{\Delta A_1(B)}}{P_G} + \frac{\Delta A_1(G)}{\Delta A_1(B)}\right]$ since $\Delta A_1(B)$ is negative. Then $q(B) = 0$ satisfies the constraint.

Part I

Additional Comments (online supplement to paper)

Note: these comments will be provided as web links; they are supplements for the interested reader, but should not be considered essential to the paper

1 Comments: Literature Review

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Comparing models; our models and BTK

We provide a summary of the different models (some are solved, but the others are not). The models differ in the following parameters: some are critical, and the others not.

$$A(e,G), A(ne,G), A(e,B), A(ne,B), k(\cdot)$$

These exogenous terms are defined above, other than the function $k(\cdot)$.

To encompass each model, we generalize the price formation as follows. The trade t results in a price $p(t)$ set by the market maker and this price is strictly between the ex-ante price p_0 and the fully informed price $p^I(t)$, defined further below.

$$p(t) = p_0 + k(t)\left[p^I(t) - p_0\right] \tag{27}$$

or $p(t) = k(t)\, p^I(t) + (1 - k(t))p_0$. $k(t)$ thus represents the extent to which the price adjusts to the trade, which we assume to be increasing in the size of the trade.)If the trade comes from the $\mathcal{II}$ under a truth-telling mechanism, this represents the extent to which the price adjusts to the true state.) We take this as an exogenous parameter, but it is justified by a model of incremental trading; here this can be interpreted as the "speed with which the price reflects all the relevant information."

The relevant values (with some normalization) for different models are:

$$\begin{aligned} \text{BKT:} \quad & A(e,G) = A(e,B) = A(ne,B) = 0, A(ne,G) = -1, \\ & k(\cdot) \equiv 1 \\ \text{Our main model:} \quad & A(e,G) = \Delta A(G), A(e,B) = \Delta A(B), A(ne,B) = A(ne,G) = 0 \\ & k(t) = 0 \;\; \text{for } t \leq 1; 1 \;\; \text{otherwise} \end{aligned} \tag{28}$$

Where, for the BTK model, we re-define their "good" state as the state in which the PM prefers to execute the policy. In our model the PM's welfare function can have *any* relationship to the asset's value, but in BTK the PM always wants the outcome that coincides with the *high* asset value.

In summary, BKT consider the case that when the economy is strong, the equivalent of our state B, the intervention/execution doesn't do anything, but if it is poor (the equivalent of our state G), it raises the asset's value. Note that given BKT's assumed microstructure, the price updates instantly, i.e., $k(\cdot) \equiv 1$.

In our above model we assume that the state is only relevant to the asset's value in the event that policy is executed, i.e., the state is specifically relevant to the *impact of the policy*.

Our assumption that the investor can purchase one unit of the asset without being detected by the market implies $k(t) = 0$ for $t \leq 1$. The difference not mentioned above is the presence of uninformed investors in model O. However, the fact that $k(t) = 0$ for $t \leq 1$ implies that we do not need uninformed investor for the model since informed investors can "hide" behind $k(t) = 0$ up to $t = 1$,

2 Comments: Model

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2.1 Comments: Timing

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2.1.1 Basic story

How is it that the policy announcement, and consequent $\mathcal{II}$ trade/price adustment leads to a discrete adjustment which the PM can observe and react to? Shouldn't this all have been evolving gradually, and un-noticably?

We might broadly consider the following timing:

T0. This represents the period from the dawn of time until policy is "considered." The (infinite) set from which policies can be chosen is common knowledge; there is "incompleteness". [44] Everyone anticipates 'some sort of a policy' but at T1 the exact nature of the proposed policy is made public. The probability of signals β and γ that arise from a draw from this infinite set is also common knowledge. $q(\beta)$ and $q(\gamma)$ are chosen by the PM, and these are common knowledge (either through strategic reasoning, or because the PM has announced these $q(.)$'s). All the payoffs functions are common knowledge.

However, none of the investors, nor the PM, know the signal.

The initial asset price A_0 is set.

T1. This represents a point in time that all the actors can anticipate.

At T1, the considered policy is drawn from an infinite set and made public; the 'details' are revealed. The Informed Investor then gets a signal of the likely impact of this policy on the asset. II buys or not, at price A_0, and the price reacts to this. The new price (and/or II's decision) is common knowledge.

T2. The PM observes this and implements or not, following the q function. The true state s is revealed. The asset price again responds, moving to $A1$.

2.1.2 Equivalent story

An alternate story that yields an equivalent model for our purposes: An assumptions is that, before the policy is announced, invented, or discovered, the asset price equals the asset's fundamental value *if the policy is not implemented*, namely, $A_0(ne) := A_1(ne)$.

Again, the informed investor receives the signal only after the policy is announced. At this point, the informed investor can again choose to buy the asset at price A_0. Thus, his incentives are sas in our model.

[44] This incompleteness is needed to explain "why have informed investors not already determined the price before the policy is announced?"

The PM will observe an immediate jump from $A_1(ne)$ to$E(A_1|\sigma)$ with no stop at A_0. However, the information transmission is equivalent, as the because the PM can infer what A_0 would have been and thus infer the signal from the price $EA_1|\sigma$ (the price after the announced policy and q functions but before the execution decision and true state are known).

The first story has the desirable property that ex-ante beliefs are consistent in a Bayesian sense.

2.1.3 Incentive compatibility constraints: buy/short implementation

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3 Comments: Analysis

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3.1 Comments: General results

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Discussion of IC constraints and intuition

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Satisfying IC constraints requires a correctly-signed difference in the payoffs to an action between the two states, and a level condition—these payoffs must surround zero.

As "not buy" yields zero profit, the levels and differences in the $\mathcal{II}$'s profit after "not buy" is obviously not a concern. These are ignored. Thus our discussion focuses on the payoffs after "buy."

Reprising the IC constraints (without short sale), for set 1 and set 2, respectively, and reversing the order of the latter:

$$\underbrace{q(G)\Delta A_1(G)}_{\mathcal{II}(G)\text{ knows}} - \underbrace{\mathbb{E}(\Delta A_1|q(G),q(B))}_{\text{MM thinks}} \geq 0 \geq \underbrace{q(G)\Delta A_1(B)}_{\text{deviating }\mathcal{II}(B)\text{ knows}} - \underbrace{\mathbb{E}(\Delta A_1|q(G),q(B))}_{\text{MM thinks}}, \tag{29}$$

$$\underbrace{q(B)\Delta A_1(B)}_{\mathcal{II}(B)\text{ knows}} - \underbrace{\mathbb{E}(\Delta A_1|q(G),q(B))}_{\text{MM thinks}} \geq 0 \geq \underbrace{q(B)\Delta A_1(G)}_{\text{deviating }\mathcal{II}(G)\text{ knows}} - \underbrace{\mathbb{E}(\Delta A_1|q(B),q(B))}_{\text{MM thinks}}. \tag{30}$$

For each set, the "honest" $\mathcal{II}$ who buys (in the "expected to buy" state $s(b)$) must profit, and the deviating $\mathcal{II}$ who buys (in the "expected not to buy" state) must lose (weakly in both cases, of course). Differencing out the common $A_1(ne)$ term, the profit can be seen as the difference between what $\mathcal{II}$ knows the asset's expected value will be (after his action) and what an outsider (such as the MM) unconditionally expects it to be. I.e., the $\mathcal{II}$'s profit is based on his information advantage.

The latter "MM thinks" term, which impacts A_0, is the same in all cases: changing it will either increase or decrease the profit from buying. Its effect on profit is the same for both types, as it simply represents the initial price paid.[45] For a given set, let $s(b)$ and $s(nb)$ represent the states in which the PM is inducing "buy" and "not buy", respectively. For each set the chosen $q(s(b))$ affects both of the "$\mathcal{II}$ knows" terms as well as the "MM thinks" term on each side. In contrast, the PM's choice for $q(s(nb))$ affects only the "MM thinks" term. The PM's probability of execution in the state where she is trying to induce "not buy" doesn't change the *final* value of the asset in a relevant way.

[45] Thus, it cannot be used to reverse the inequality in the payoffs to the honest and deviating types.

As we have shown, the PM can only implement actions where the impact of policy on the asset's final value is greater in the "Buy" state. Considering the above terms, as the initial price and probability of execution after an action is the same in both states, this implies that, given the chosen set/implementation, the payoff to honest behavior will *always* (weakly) exceed the payoff to deviation. Thus, we only care about the level condition, bringing up or down the payoffs for each choice.

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Why is implementation asymmetric? (proposition 6)

Restated in terms of state $s(i)$, "the state where choice i is induced"

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Why, with exogenous A_0, the PM sometimes needs to use one tool, and sometimes the other:

If A_0 cannot change, then raising $q(B)$ does nothing to dissuade investment in state B: If $\mathcal{II}$ buys he convinces the PM the state is *good*; so $q(B)$ has no impact. If $\mathcal{II}$ does not buy his payoff is 0 no matter what. On the other hand by reducing $q(g)$ she can scale down the profit from buying and inducing execution, boosting the asset's value. As this increase is naturally *lower* in the bad state, scaling it down can make it smaller than the (exogenous) asset price in the bad state, but larger in the good state. *This is demonstrated for one case in section 4.2 in the appendix.*

Returning to the main model (again, for the Treat case), why does the PM only use one tool?

To simplify the intuition, assume $P_G = P_B = 1/2$; as the PM is ex-ante indifferent, this implies that the welfare cost of raising $q(B)$ and lowering $q(G)$ are always equal (and constant).[46] Thus, whichever tool has a greater marginal effect (or *any effect in the right direction*) on relaxing the relevant IC constraint will be used. In the Treat case, the relevant constraint is "get bad type not to buy"). Thus it is a race between:

Note: this comment is still being crafted

Finally, we show that the PM always does better by listening to the market. ...

Discussion of chosen set and binding constraints in relation to figure 2.

We offer a simple intuitive explanation for (Proposition 8) "why the PM induces the informed investor to purchase only in the state in which policy has a more positive impact on the asset's value."

In order to get a distinct behavior in each state, two incentive compatibility constraints must be satisfied. In essence, this requires a *difference* and a *level* condition; there must be a greater payoff to buying in the "buy if" state (relative to the "don't buy" state), and these payoffs must surround zero (one negative, the other positive).[47]

Equation 8 separates this into two terms. The bracketed term represents the asset's price (and its unconditional expected value). This term does not depend on the *actual* state, so it cannot be used to drive a wedge between the relevant payoffs.

The other term—$q(\hat{s})\Delta A_1(s)$—is the expected increase in the asset's value *conditional on* the state s and on the signal $\hat{s}$ that II sends through his action. This is the product of the probability

[46] If we don't restrict P_G, the following is scaled by the relative probability of each state.

[47] Technically, there is a similar concern regarding the relative payoffs to "not buy" in each state. However, those payoffs are zero by assumption, so we only have to worry about the relative payoffs to "buy". In model allowing short-sale, this of course gets more complicated.

of execution after the signal sent ($q(\hat{s})$) and the impact of policy on the asset's value in the true state s. As a particular action sends the same signal in each state, the probabilities of execution are the same. Thus, the difference in payoffs monotonically increases in the difference in the impact of policy on the asset in each state, $\Delta A_1(s_b) - \Delta A_1(s_{nb})$, where s_b (s_{nb}) is the state in which the PM induces (not) buying.

Thus, a necessary condition for implementation: "the policy must have a more beneficial impact on the asset in the state where the II buys."

We can consolidate this intuition as follows. When an investor buys the asset, he ends up reaping its final value A_1. While it is true that the *informed* investor's choice also influences policy, the impact of (not) investing on the policy chosen is the same no matter the true state. As noted, the price paid is also naturally the same in each state.

Thus, the only way to get informed investor to want to buy the asset in one state and not in the other state is for there to be a *greater reward* in the state he chooses to buy it. As the impact on the policy is the same both states and the initial prices are the same in each state, the only way this can occur is for the asset's value to be more positively affected by the policy in the state where the PM induces "Buy".

The intuition is more straightforward for the case where the PM induces the investor to buy in the *good* state. For the other case, where the policy has a more positive impact on the asset in the *bad* state, the PM may induce "buy only if bad." $\mathcal{II}$ buys the asset in the bad state, which gets the PM to execute the policy with a *lower* probability then she otherwise would have. (Recall, we need not worry about relative payoffs from the "not buy" action.) The payoffs from buying in the truly bad state must be made to exceed the payoffs from buying in the truly good state. Again, in both states, buying sends the same signal to the PM, yielding the same probability of execution, and the initial prices are the same. What we need here is that the impact of policy on the asset is more positive in the bad state than in the good state, which exactly characterizes this case.

4 Online Appendix

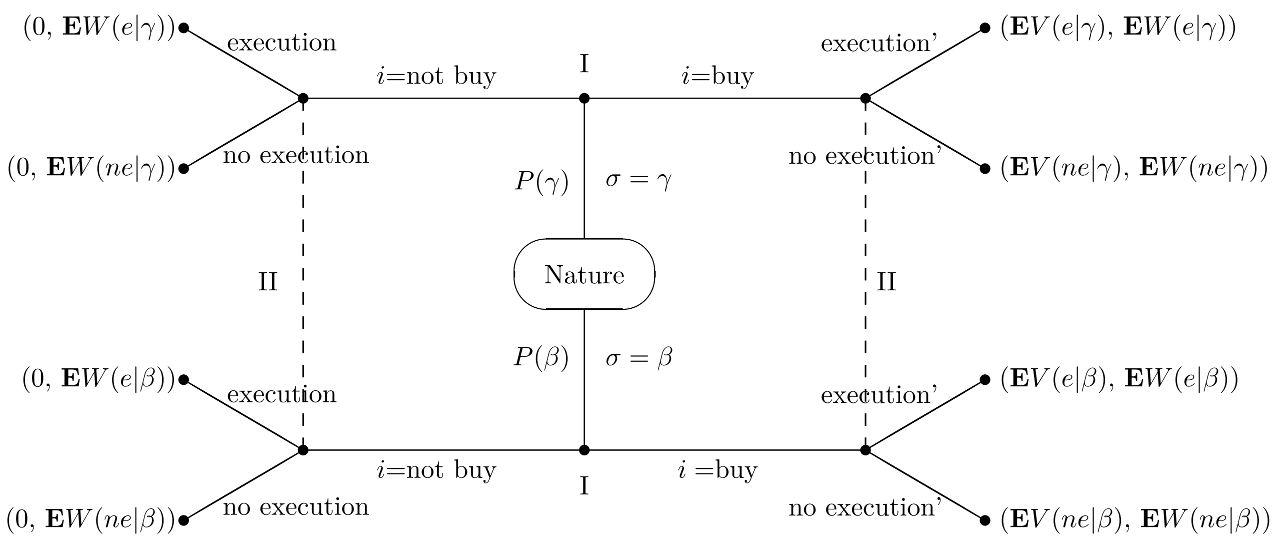
4.1 Without commitment

The gain from being able to commit might not be as large as we have computed, if there is a non-zero gain from listening without commitment. (We now rule this out). The game without commitment resembles a costly signaling game. Outside of the "first best" regions, the equilibrium in this game will probably involve both the $\mathcal{II}$ and PM playing mixed strategies. If the PM is playing a fully mixed strategy then he must get zero gains, as he is indifferent between executing and not executing by assumption. If he is partially mixing (mixing after one action but not the other), he might get a positive gain; this can be ruled out by a simple proof.

4.1.1 Value of commitment

We previously considered an environment in which the policymaker can commit to policy $\langle q(\gamma), q(\beta)\rangle$. In this section, we investigate the value associated with the commitment. To be more precise, we calculate the additional welfare that the PM can achieve by the commitment.

Let us normalize the policymaker's payoff by setting $\sum_{s,\sigma} P(\sigma, s)W(ne, s) = 0$. Then, without the commitment technology of the PM, the $\mathcal{II}$ and the PM play the following game.



4.1.2 $\Delta A_1(\gamma) > \Delta A_1(\beta) > 0$

For the case where $\Delta A_1(\gamma) > \Delta A_1(\beta) > 0$, it is easy to verify that there are only two kinds of pooling equilibria. Because of the ex-ante indifference, there are many equilibria, but we can classify all the equilibria into two. The first equilibrium is such that the investor always chooses $i = buy$ whether the signal is γ or β, and the second equilibrium is such that the investor always chooses $i = \textit{not buy}$ whether the signal is γ or β. We can show that the only equilibrium surviving the Intuitive Criterion of Cho and Kreps is such that the investor always chooses $i = buy$ whether the signal is γ or β, the policymaker chooses *execution* upon the observation of *buy* and *no execution* upon the observation of *not buy*.

With the unique Bayesian Nash equilibrium surviving Intuitive Criterion, we see that the benefit is in fact only $\sum_{s,\sigma} P(\sigma, s) W(ne, s)$. On the other hand, the welfare in section **??** is strictly positive:

$$P(\gamma)q^*(\gamma)\Delta\Delta W(\gamma) + P(\beta)q^*(\beta)\Delta W(\beta)$$

where

$$\left\langle q^*(\gamma) = 1, q^*(\beta) = \max\left(0, \frac{1}{P(\gamma)}\left(1 - P(\beta)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\right)\right)\right\rangle.$$

Thus the additional welfare is:

$$P(\beta)[-\Delta W(\beta)]\left(1 - \max\left(0, \frac{1}{P(\gamma)}\left(1 - P(\beta)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\right)\right)\right).$$

Note the following.
– As $\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}$ decreases to 1, the additional welfare converges to 0.
– As $P(\gamma)$ increases to 1 (or as $P(\beta)$ decreases to 0), the additional welfare converges to 0.

4.2 Derivation for case of fixed A_0, no short-selling, condition (21)

The first set of incentive compatibility constraints (24) is equivalent to the following under condition (21):

$$q(\gamma) \geq \frac{-\mathbb{E}\,V(ne|\gamma)}{\mathbb{E}\,V(e|\gamma) - \mathbb{E}\,V(ne|\gamma)} \quad \text{and} \quad q(\gamma) \leq \frac{-\mathbb{E}\,V(ne|\beta)}{\mathbb{E}\,V(e|\beta) - \mathbb{E}\,V(ne|\beta)}.$$

Moreover we get the following with condition (21):

$$\frac{-\mathbb{E}\,V(ne|\gamma)}{\mathbb{E}\,V(e|\gamma) - \mathbb{E}\,V(ne|\gamma)} < \frac{-\mathbb{E}\,V(ne|\beta)}{\mathbb{E}\,V(e|\beta) - \mathbb{E}\,V(ne|\beta)}.$$

Thus, the two constraints become

$$\frac{-\mathbb{E}\,V(ne|\gamma)}{\mathbb{E}\,V(e|\gamma) - \mathbb{E}\,V(ne|\gamma)} \leq q(\gamma) \leq \frac{-\mathbb{E}\,V(ne|\beta)}{\mathbb{E}\,V(e|\beta) - \mathbb{E}\,V(ne|\beta)}.$$

The policymaker wants to increase $q(\gamma)$ as long as the two incentive compatibility constraints are satisfied. Thus, we conclude:

$$q(\gamma) = \frac{-\mathbb{E}\,V(ne|\beta)}{\mathbb{E}\,V(e|\beta) - \mathbb{E}\,V(ne|\beta)} = \frac{1}{\frac{\mathbb{E}\,V(e|\beta)}{-\mathbb{E}\,V(ne|\beta)} + 1}.$$

Note that this value is between 0 and 1 under condition 21.

4.3 [Online] Model with short selling

With short selling, there are potentially six distinct ways of implementing truth telling behavior: this can occur through $\mathcal{II}$ choosing any of "buy, do nothing, or short sell" after a good signal, and choosing any *other* action after a bad signal. Each of these implementations requires that four IC constraints be satisfied, as after each signal, one action must be preferred above the other two actions. For tractability we focus on two potential implementations: "buy if γ and short sell if β, and vice-versa".

Set 1 (Buy if and only if the signal is γ):

Here, the PM interprets $i = b$ as $\mathcal{II}$ having received γ. The IC constraints when short selling is allowed are:

$$q(\gamma)\Delta A_1(\gamma) + A_1(ne) - A_0(q(\gamma), q(\beta)) \geq -q(\beta)\Delta A_1(\gamma) - A_1(ne) + A_0(q(\gamma), q(\beta)), \qquad (IC_{11})$$
$$-q(\beta)\Delta A_1(\beta) - A_1(ne) + A_0(q(\gamma), q(\beta)) \geq q(\gamma)\Delta A_1(\beta) + A_1(ne) - A_0(q(\gamma), q(\beta)). \qquad (IC_{12})$$

The first constraint ensures the $\mathcal{II}$ prefers to buy when he gets signal γ; the second constraint ensures he prefers to short sell when he gets signal β.

We show in section 4.5 that choosing *doing nothing* is never superior to choosing *buy* in the current context.

We rearrange IC_{11} for intuition (the intuition for IC_{12} is similar):

$$\begin{aligned}&\underbrace{q(\gamma)\Delta A_1(\gamma)}_{(i)} - \underbrace{(q(\gamma)\Delta A_1(\gamma)P(\gamma) + q(\beta)\Delta A_1(\beta)P(\beta))}_{(ii)}\\ &\geq \underbrace{-q(\beta)\Delta A_1(\gamma)}_{(i)} + \underbrace{(q(\gamma)\Delta A_1(\gamma)P(\gamma) + q(\beta)\Delta A_1(\beta)P(\beta))}_{(ii)}\end{aligned}$$

On each side of the above inequality, (i) represents the $\mathcal{II}$'s expectation of the gain (or loss) in the asset's value given the signal he is sending, and (ii) is the ex-ante expected gain (or loss) in the asset's value.

Note that if short-selling is not allowed, the right-hand side will be zero.

Inequalities IC_{11} and IC_{12} can also be written as:

$$\Delta A_1(\gamma)\Big[q(\beta)\Big(1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) + q(\gamma)(1 - 2P(\gamma)))\Big] \geq 0, \qquad (IC'_{11})$$
$$\Delta A_1(\beta)\Big[q(\gamma)\Big(1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) + q(\beta)(1 - 2P(\beta)))\Big] \leq 0. \qquad (IC'_{12})$$

Set 2 (Buy if and only if the signal is β):

Here, the PM interprets $i = sh$ as $\mathcal{II}$ having received γ. The incentive compatibility constraints when short sale is allowed are:

$$-q(\gamma)\Delta A_1(\gamma) - A_1(ne) + A_0(q(\gamma), q(\beta)) \geq q(\beta)\Delta A_1(\gamma) + A_1(ne) - A_0(q(\gamma), q(\beta)), \qquad (IC_{21})$$
$$q(\beta)\Delta A_1(\beta) + A_1(ne) - A_0(q(\gamma), q(\beta)) \geq -q(\gamma)\Delta A_1(\beta) - A_1(ne) + A_0(q(\gamma), q(\beta)), \qquad (IC_{22})$$

The first constraint ensures that $\mathcal{II}(\gamma)$ prefers buying to short selling; the second constraint ensures that $\mathcal{II}(\beta)$ prefers short selling to buying. Without short-selling, the left (right) side of the first (second) inequality is zero.

Again, we show in section 4.5 that choosing *do nothing* is never superior to choosing *buy* in the current context.

Sets 1 and 2 are precisely the converse of each other.

$$\underbrace{[q(\gamma) + q(\beta)]\Delta A_1(\gamma)}_{(a)} \underbrace{-2\mathbb{E}(\Delta A_1)}_{(b)} \geq 0 \geq \underbrace{[q(\gamma) + q(\beta)]\Delta A_1(\beta)}_{(a)} \underbrace{-2\mathbb{E}(\Delta A_1)}_{(b)}, \qquad (31)$$

where $\mathbb{E}(\Delta A_1) := A_0 - A_1(ne)$.

4.3.1 Further simplification of the two sets of incentive compatibility constraints

With $q(\gamma) = 1$, we can rewrite the second constraint of Set 1 as:

$$\begin{aligned} & P(\gamma)\Delta A_1(\gamma) \geq \Delta A_1(\beta) - q(\beta)P(\beta)\Delta A_1(\gamma) \\ \Leftrightarrow \quad & q(\beta)P(\beta)\Delta A_1(\beta) \geq (\Delta A_1(\beta) - P(\gamma)\Delta A_1(\gamma)) = (\Delta A_1(\beta) - (1 - P(\beta))\Delta A_1(\gamma)) \\ & q(\beta)\Delta A_1(\beta) \geq \Big(\frac{\Delta A_1(\beta) - \Delta A_1(\gamma)}{P(\beta)} + \Delta A_1(\gamma)\Big) \end{aligned}$$

Thus Set 1 is combined into:

$$\Delta A_1(\gamma) \geq \Delta A_1(\beta)q(\beta) \geq \Big[\frac{\Delta A_1(\beta) - \Delta A_1(\gamma)}{P(\gamma)} + \Delta A_1(\gamma)\Big], \tag{32}$$

and Set 2 is similarly combined into

$$\Delta A_1(\beta)q(\beta) \geq \Delta A_1(\gamma) \geq \Big[\frac{\Delta A_1(\gamma) - \Delta A_1(\beta)}{P(\beta)} + \Delta A_1(\beta)\Big]q(\beta). \tag{33}$$

4.4 [Online] Derivations, allowing short-selling, cases 1-6

Case (i), "A Treat": $\Delta A_1(\gamma) > \Delta A_1(\beta) > 0$, *i.e., the asset's value increases when the policy is executed, more so with the good signal.*

Proposition 8 implies that Set 1 is relevant, and Proposition 6 implies $q(\gamma) = 1$. Since $\Delta A_1(\gamma) > 0$ and $\Delta A_1(\beta) > 0$, we can rewrite IC'_{11} and IC'_{12} substituting these out:

$$q(\beta)\Big(1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - (1 - 2P(\beta)) \geq 0, \quad \Big(1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1 - 2P(\gamma)) \leq 0.$$

[Condition 1: $P(\gamma) = \frac{1}{2} = P(\beta)$] Then the two constraints become:

$$q(\beta)\Big(1 - \frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) \geq 0, \quad \Big(1 - \frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) \leq 0.$$

Thus we can reduce $q(\beta)$ to zero, and the incentive-constrained optimal policy achieves the first best, $\langle q(\gamma) = 1, q(\beta) = 0\rangle$.

[Condition 2: $P(\gamma) > \frac{1}{2} > P(\beta)$] Under this condition the above is simplified into:

$$q(\beta)\underbrace{\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)}}_{(a)} \geq 1, \quad \underbrace{\frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}}_{(b)} \geq q(\beta)$$

Both (a) and (b) are larger than 1. Since the PM wants to decrease $q(\beta)$ as much as she can, the only relevant (i.e., binding) constraint is the first one. Thus we conclude:

$$\text{For } P(\gamma) > 1/2 > P(\beta), \ \Big\langle q(\gamma) = 1, q(\beta) = \frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Big\rangle.$$

[**Condition 3:** $P(\gamma) < \frac{1}{2} < P(\beta)$] Then the ICs are simplified into:

$$q(\beta)\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)} \leq 1, \quad \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)} \leq q(\beta)$$

The first constraint is irrelevant irrespective of the sign of $\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}$ as the PM wants to minimize $q(\beta)$ (note that $q(\beta) = 0$ trivially satisfies the first constraint). If $\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}$ is negative, then the second constraint is also irrelevant; the optimal solution is $\langle q(\gamma) = 1, q(\beta) = 0\rangle$. If $\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}$ is positive, it is smaller than 1. Then the second constraint is relevant, i.e., the second constraint binds, and we derive the optimal solution $\left\langle q(\gamma) = 1, q(\beta) = \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}\right\rangle$.

To summarize, the optimal policy interprets the $\mathcal{II}$'s buying as signal γ and his not buying as signal β. The optimal incentive-constrained optimal policy is:

$$\text{For } P(\gamma) < 1/2 < P(\beta), \quad \left\langle q(\gamma) = 1, q(\beta) = \max\left(0, \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}\right)\right\rangle,$$
$$\text{for } P(\gamma) > 1/2 > P(\beta), \quad \left\langle q(\gamma) = 1, q(\beta) = \frac{1-2P(\beta)}{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\right\rangle,$$
$$\text{and for } P(\gamma) = 1/2 = P(\beta), \quad \langle q(\gamma) = 1, q(\beta) = 0\rangle, \text{i.e., the first best.}$$

Note that $P(\gamma) = 1/2 = P(\beta)$ implies the first best, $\langle q(\gamma) = 1, q(\beta) = 0\rangle$ for all cases, so we will not mention it further.
Solution: The PM will use Set 1, i.e., will induce the $\mathcal{II}$ to short sell under the good signal, and buy under the bad signal. She will do this by setting:

$$\left\langle\ q(\gamma) = 1, q(\beta) = \frac{1-2P(\beta)}{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\right\rangle \text{ if } P(\gamma) < 1/2 < P(\beta),$$

$$\text{and setting} \left\langle\ q(\gamma) = 1, q(\beta) = \max\left(0, \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}\right)\right\rangle \text{ if } P(\gamma) > 1/2 > P(\beta).$$

Remark 3 [Binding constraints and signal probabilities]: Consider why the second constraint (IC_{12}) does not bind where $P(\gamma) > P(\beta)$. Suppose these probabilities, and suppose the policy is at the first-best $\langle q(\gamma) = 1, q(\beta) = 0\rangle$. Here, an $\mathcal{II}(\beta)$ type will want to short sell and deter execution. The UTs believe that the policy is executed "most of the time"; thus deterring execution yields the larger information advantage or "surprise". $\mathcal{II}(\beta)$ can profit from this information advantage in proportion to $\Delta A_1(\gamma)$, as when he short-sells the UTs compensate him for their predicted execution after the *good* signal.

In contrast, if $\mathcal{II}(\beta)$ were to buy and induce execution, this would bring only the smaller $\Delta A_1(\beta)$ in ex-post profit while to buy the asset he would have to compensate the UTs for their expectation of asset gains from execution under the good signal, paying them $P(\gamma)\Delta A_1(\gamma)$.

Now consider why (IC_{12}) *may* bind where $P(\gamma) < P(\beta)$. Here if the policy were first-best the UTs would believe it would be executed less then half of the time; thus *inducing* execution yields the larger surprise, an information advantage of $1 - P(\gamma)$. On the other hand, the (ex-post) gain from inducing execution here is only proportional to $\Delta A_1(\beta)$ but he must pay the UT's for the asset in proportion to $\Delta A_1(\gamma)$; the "asymmetric asset gain" hurts him here. In contrast, by short-selling and deterring execution he is inducing a smaller surprise but will not have to pay for the asymmetric asset gain. These two effects go in the opposite direction, and where the "larger surprise" advantage outweighs the "asymmetric asset gain" cost – i.e., where i.e., where $2P(\gamma) < \frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}$ – then IC_{12} will bind.

4.4.1 Case ii: $\Delta A_1(\beta) > \Delta A_1(\gamma) > 0$

Proposition 8 implies that Set 2 is the relevant one. Also $q(\gamma) = 1$ from Proposition 6. Since $\Delta A_1(\beta) > \Delta A_1(\gamma) > 0$, we can simplify the two incentive constraints into:

$$q(\beta)\Big(1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - (1 - 2P(\beta)) \le 0, \quad \Big(1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1 - 2P(\gamma)) \ge 0$$

[Case 1: $P(\gamma) > \frac{1}{2} > P(\beta)$**]** Then the above is simplified into:

$$q(\beta)\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)} \le 1, \quad \underbrace{\frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}}_{(b)} \le q(\beta)$$

Clearly, the first constraint is not relevant as the PM wants to minimize $q(\beta)$. If $(b) > 0$, then the solution is $< q(\gamma) = 1, q(\beta) = \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)} >$. If $(b) \le 0$, then the second constraint is also irrelevant by the same reason; thus the solution is $< q(\gamma) = 1, q(\beta) = 0 >$. In summary, the solution is:

$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \max\Big(0, \frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}\Big)\Big\rangle$$

[Case 2: $P(\gamma) < \frac{1}{2} < P(\beta)$**]**. The incentive compatibility constraints are:

$$q(\beta)\underbrace{\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)}}_{(a)} \ge 1, \quad 1 \ge \underbrace{\frac{1 - 2P(\gamma)}{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}}_{(b)} q(\beta)$$

Then the second constraint is irrelevant as PM wants to minimize $q(\beta)$. Note $(a) > 1$, so the solution is:

$$\text{For } P(\gamma) < 1/2 < P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Big\rangle.$$

The contrast between cases (i) and (ii) offers real-world lessons. The choice of which type of behavior to try to induce depends on the relative gains to the asset under the good or bad policy. If the asset does better when the policy is *bad*, policymakers may want to get informed investors to buy only if the policy is bad, promising to execute the policy with some minimum probability.

4.4.2 Case iii: $\Delta A_1(\beta) > 0 > \Delta A_1(\gamma)$

Proposition 8 implies that Set 2 is the relevant one. Also $q(\gamma) = 1$ from Proposition 6. Since $\Delta A_1(\beta) > 0 > \Delta A_1(\gamma)$, we can simplify these into:

$$q(\beta)\Big(1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - (1 - 2P(\beta)) \geq 0, \quad \Big(1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1 - 2P(\gamma)) \geq 0$$

[Case 1: $P(\gamma) > \frac{1}{2} > P(\beta)$] The above is simplified into:

$$q(\beta)\underbrace{\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)}}_{(a)} \geq 1, \quad \underbrace{\frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}}_{(b)} \leq q(\beta)$$

Note that $(a) > 1$ and $(b) < 0$; thus, the second constraint is irrelevant, and the result follows.

$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Big\rangle.$$

[Case 2: $P(\gamma) < \frac{1}{2} < P(\beta)$] The incentive compatibility constraints are:

$$q(\beta)\underbrace{\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)}}_{(a)<0} \leq 1, \quad 1 \geq \underbrace{\frac{1 - 2P(\gamma)}{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}}_{(b)>0} q(\beta)$$

Note that since $(a) < 0$ and $(b) > 0$ hold, both of the constraints are irrelevant, so the solution is:

$$\text{For } P(\gamma) < 1/2 < P(\beta),\ \langle q(\gamma) = 1, q(\beta) = 0\rangle.$$

Solution: The PM will use Set 2 and induce the $\mathcal{II}$ to short sell under the good signal, and buy under the bad signal, by setting:

$$\text{For } P(\gamma) < 1/2 < P(\beta),\ \langle q(\gamma) = 1, q(\beta) = 0\rangle.$$
$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Big\rangle.$$

4.4.3 Case iv: $0 > \Delta A_1(\beta) > \Delta A_1(\gamma)$

Proposition 8 implies that Set 2 is the relevant one. Also $q(\gamma) = 1$ from Proposition 6. Since $0 > \Delta A_1(\beta) > \Delta A_1(\gamma)$, the constraints are simplified into:

$$q(\beta)\Big(1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - (1 - 2P(\beta)) \geq 0, \quad \Big(1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1 - 2P(\gamma)) \leq 0$$

[Case 1: $P(\gamma) > \frac{1}{2} > P(\beta)$] The above can be simplified into:

$$q(\beta)\underbrace{\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)}}_{(a)} \geq 1, \quad \underbrace{\frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}}_{(b)} \geq q(\beta)$$

Note that $(a) > 1$ and $(b) > 1$; thus, the second constraint is irrelevant, and the result follows.
[Case 2: $P(\gamma) < \frac{1}{2} < P(\beta)$] then the IC constraints can be written as:

$$q(\beta) \underbrace{\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)}}_{(a)<1} \leq 1, \quad \underbrace{\frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}}_{(b)<1} \leq q(\beta)$$

Since $(a) < 1$, the first constraint is irrelevant. If $(b) < 0$, the second constraint is also irrelevant; and the optimal solution is $< q(\gamma) = 1, q(\beta) = 0 >$. If $0 < (b) < 1$, then the optimal solution is $< q(\gamma) = 1, q(\beta) = \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)} >$. So the result follows.
Solution: The PM will use Set 2 and induce the $\mathcal{II}$ to short sell under the good signal and buy under the bad signal, by setting:

$$\text{For } P(\gamma) > 1/2 > P(\beta), \ \Big\langle q(\gamma) = 1, q(\beta) = \frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}} \Big\rangle,$$
$$\text{and for } P(\gamma) < 1/2 < P(\beta), \ \Big\langle q(\gamma) = 1, q(\beta) = \max\Big(0, \frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}\Big) \Big\rangle$$

4.4.4 Case v: $0 > \Delta A_1(\gamma) > \Delta A_1(\beta)$

Proposition 8 implies that Set 1 is the relevant one. Also $q(\gamma) = 1$ from Proposition 6.
Set 1 is:

$$\Delta A_1(\gamma)\Big[q(\beta)\Big(1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - q(\gamma)(1 - 2P(\beta))\Big] \geq 0,$$
$$\Delta A_1(\beta)\Big[q(\gamma)\Big(1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1 - 2P(\gamma))\Big] \leq 0.$$

Since $0 > \Delta A_1(\gamma) > \Delta A_1(\beta)$, we can simplify the above into:

$$q(\beta)\Big(1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - (1 - 2P(\beta)) \leq 0, \quad \Big(1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1 - 2P(\gamma)) \geq 0$$

[Case 1: $P(\gamma) > \frac{1}{2} > P(\beta)$] Then the above is simplified into:

$$q(\beta) \underbrace{\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)}}_{(a)} \leq 1, \quad \underbrace{\frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}}_{(b)} \leq q(\beta)$$

Clearly the first constraint is irrelevant as the PM wants to minimize $q(\beta)$.

If $(b) \in (0, 1)$, then the optimal solution is $< q(\gamma) = 1, q(\beta) = \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)} >$. If $(b) < 0$, then the second constraint is also irrelevant; thus, the optimal solution is $< q(\gamma) = 1, q(\beta) = 0 >$. In summary, the optimal solution is:

$$\text{For } P(\gamma) > 1/2 > P(\beta), \ \Big\langle q(\gamma) = 1, q(\beta) = \max\Big(0, \frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}\Big) \Big\rangle.$$

[Case 2: $P(\gamma) < \frac{1}{2} < P(\beta)$**]** The incentive compatibility constraints are:

$$q(\beta)\underbrace{\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}}_{(a)>1} \geq 1, \quad \underbrace{\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}}_{(b)>1} \geq q(\beta)$$

Note that $(a) > 1$ and $(b) > 1$. Thus, the second constraint is irrelevant, and the result follows.
Solution: The PM will use Set 1 and induce the $\mathcal{II}$ to buy under the good signal and short sell under the bad signal, by setting:

$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \max\Big(0, \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}\Big)\Big\rangle, \text{and}$$
$$\text{for } P(\gamma) < 1/2 < P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \frac{1-2P(\beta)}{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Big\rangle.$$

4.4.5 Case vi: $\Delta A_1(\gamma) > 0 > \Delta A_1(\beta)$

Proposition 8 implies that Set 1 is the relevant one. Also $q(\gamma) = 1$ from Proposition 6. Since $\Delta A_1(\gamma) > 0 > \Delta A_1(\beta)$, we can simplify these into:

$$q(\beta)\Big(1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - (1-2P(\beta)) \geq 0, \quad \Big(1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1-2P(\gamma)) \geq 0$$

[Case 1: $P(\gamma) > \frac{1}{2} > P(\beta)$**]** Then the above is simplified into:

$$q(\beta)\underbrace{\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}}_{(a)} \geq 1, \quad \underbrace{\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}}_{(b)} \leq q(\beta)$$

Note that $(a) > 1$ and $(b) < 0$, so that the second constraint is irrelevant, and the result follows from the first constraint.
[Case 2: $P(\gamma) < \frac{1}{2} < P(\beta)$**]** The incentive compatibility constraints are:

$$q(\beta)\underbrace{\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}}_{(a)<0} \leq 1, \quad \underbrace{\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}}_{(b)>1} \geq q(\beta)$$

Note that $(a) < 0$ and $(b) > 1$. Thus, the first constraint is irrelevant, and the result follows.
Solution: The PM will induce the $\mathcal{II}$ to buy under the good signal and short sell under the bad signal, by setting:

$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \frac{1-2P(\beta)}{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Big)\Big\rangle.$$
$$\text{For } P(\gamma) < 1/2 < P(\beta),\ \langle q(\gamma) = 1, q(\beta) = 0\rangle.$$

4.5 Short selling or buying is superior to *not buying*

Consider the case of $\Delta A_1(\gamma) > \Delta A_1(\beta)$. $\mathcal{II}(\gamma)$'s payoff when he buys and $\mathcal{II}(\beta)$'s payoff when he short sells are, respectively:

$$q(\gamma)\Delta A_1(\gamma) - (P(\gamma)q(\gamma)\Delta A(\gamma) + P(\beta)q(\beta)\Delta A(\beta)) = P(\beta)[q(\gamma)\Delta A_1(\gamma) - q(\beta)\Delta A_1(\beta)], \text{ and}$$
$$-q(\beta)\Delta A_1(\beta) + (P(\gamma)q(\gamma)\Delta A(\gamma) + P(\beta)q(\beta)\Delta A(\beta)) = P(\gamma)[q(\gamma)\Delta A_1(\gamma) - q(\beta)\Delta A_1(\beta)]$$

As shown in Lemma 5, $q(\gamma) \geq q(\beta)$ must be the case. (Note that the proof for Lemma 5 depends only on the PM's welfare function.)

For case (i) in which $\Delta A_1(\gamma) > \Delta A_1(\beta) > 0$, both of the above payoffs are positive since $q(\gamma) \geq q(\beta)$. The same holds for case (vi) in which $\Delta A_1(\gamma) > 0 > \Delta A_1(\beta)$. For case (v) in which $0 > \Delta A_1(\gamma) > \Delta A_1(\beta)$, suppose $P(\gamma) > P(\beta)$. Plugging the optimal $q(\beta)$ and $q(\gamma) = 1$ into $q(\gamma)\Delta A_1(\gamma) - q(\beta)\Delta A_1(\beta)$, we derive:

$$\Delta A_1(\gamma) - \frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}\Delta A_1(\beta) = \frac{\Delta A_1(\gamma) - \Delta A_1(\beta)}{1 - 2P(\gamma)} > 0.$$

On the other hand, suppose $P(\gamma) < P(\beta)$. Plugging the optimal $q(\beta)$ and $q(\gamma) = 1$ into $q(\gamma)\Delta A_1(\gamma) - q(\beta)\Delta A_1(\beta)$, we derive:

$$\Delta A_1(\gamma) - \frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Delta A_1(\beta) = \frac{\Delta A_1(\gamma) - \Delta A_1(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}} > 0.$$

Thus, we have shown that the two payoffs are non-negative when $\Delta A_1(\gamma) > \Delta A_1(\beta)$.

Similarly, consider the case of $\Delta A_1(\gamma) < \Delta A_1(\beta)$. $\mathcal{II}(\gamma)$'s payoff when he short sells and $\mathcal{II}(\beta)$'s payoff when he buys are, respectively:

$$-q(\gamma)\Delta A_1(\gamma) + (P(\gamma)q(\gamma)\Delta A(\gamma) + P(\beta)q(\beta)\Delta A(\beta)) = P(\gamma)[q(\beta)\Delta A_1(\beta) - q(\gamma)\Delta A_1(\gamma)], \text{ and}$$
$$q(\beta)\Delta A_1(\beta) - (P(\gamma)q(\gamma)\Delta A(\gamma) + P(\beta)q(\beta)\Delta A(\beta)) = P(\beta)[q(\beta)\Delta A_1(\beta) - q(\gamma)\Delta A_1(\gamma)]$$

For case (ii) in which $\Delta A_1(\beta) > \Delta A_1(\gamma) > 0$, suppose $P(\gamma) > P(\beta)$. Plugging the optimal $q(\beta)$ and $q(\gamma) = 1$ into $[q(\beta)\Delta A_1(\beta) - q(\gamma)\Delta A_1(\gamma)]$, we derive

$$\frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}\Delta A_1(\beta) - \Delta A_1(\gamma) = \frac{\Delta A_1(\beta) - \Delta A_1(\gamma)}{1 - 2P(\gamma)} > 0.$$

On the other hand, suppose $P(\gamma) < P(\beta)$. Plugging the optimal $q(\beta)$ and $q(\gamma) = 1$ into $[q(\beta)\Delta A_1(\beta) - q(\gamma)\Delta A_1(\gamma)]$, we derive

$$\frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Delta A_1(\beta) - \Delta A_1(\gamma) = \frac{\Delta A_1(\beta) - \Delta A_1(\gamma)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}} > 0.$$

For case (iii) in which $\Delta A_1(\beta) > 0 > \Delta A_1(\gamma)$, the two payoffs are trivially non-negative. The same holds for case (iv) in which $0 > \Delta A_1(\beta) > \Delta A_1(\gamma)$.

Thus we have shown that the payoffs are also non-negative when $\Delta A_1(\gamma) < \Delta A_1(\beta)$.