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# Stochastic multi-period multi-product multi-objective Aggregate Production Planning model in multi-echelon supply chain 

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#### Abstract

In this paper a multi-period multi-product multi-objective aggregate production planning (APP) model is proposed for an uncertain multi-echelon supply chain considering financial risk, customer satisfaction, and human resource training. Three conflictive objective functions and several sets of real constraints are considered concurrently in the proposed APP model. Some parameters of the proposed model are assumed to be uncertain and handled through a two-stage stochastic programming (TSSP) approach. The proposed TSSP is solved using three multi-objective solution procedures, i.e., the goal attainment technique, the modified $\varepsilon$-constraint method, and STEM method. The whole procedure is applied in an automotive resin and oil supply chain as a real case study wherein the efficacy and applicability of the proposed approaches are illustrated in comparison with existing experimental production planning method.


Key words: Uncertain aggregate production planning, Supply chain management, Automotive Industry.

## 1. Introduction

Aggregate production planning (APP) is lowresolution and high-level plan over a medium or long period of time (Leung et al., 2003). The APP problems are addressed in the scope of tactical and operational levels in supply chains. Several methods and approaches including heuristics, mathematical models, and experimental methods were proposed to handle APPs (Mirzapour Al-eHashem, 2012). APPs usually involve several type of uncertainties. Stochastic programming is a general way of incorporating probabilistic uncertainty into optimization problems.

In this paper, a two-stage stochastic programming model is proposed to deal with a new multi-product multi-period multi-objective aggregate production planning (APP) problem in a supply chain in presence of uncertainty. The proposed problem is modeled using multi-objective mixed-integer mathematical programming. Three objective functions including minimizing expected total costs of supply chain, maximizing expected customer satisfaction level, and minimizing expected supply chain downside risk are considered concurrently. Several constraints such as the work force levels, available time, inventory levels, quantity of production, machine capacity, and quantity of raw material purchased, quantity of

[^0]products sold, backordering level, and financial risk of supply chain are also considered. Some parameters of the proposed model such as demand values, supply capacities, transportation costs, and shortage costs are assumed to be uncertain and handled through a two-stage stochastic programming (TSSP) approach. Then, three solution procedures, i.e., (I) goal attainment technique, (II) modified $\varepsilon$-constraint method, and (III) STEM method, are proposed to solve the proposed TSSP, distinctively. The proposed approaches are applied on an automotive paint supply chain as a real case study. The efficacy and applicability of the proposed approach is illustrated in the case study.

The main contributions of this study are to: (I) Proposing a new multi-product multi-period multiobjective APP problem in multi-echelon supply chain through mixed-integer multi-objective mathematical programming; (II) Developing a two-stage stochastic programming approach to solve the multi-echelon supply chain problem considering the supply chain downside risk, (III) Adapting three multi-objective solution procedures, including goal attainment method, modified $\varepsilon$-constraint technique, and STEM method, to solve the problem, (IV) Applying the proposed problem and solution procedures in a real case study, and (V) Comparing the results of the three solution procedures.

The next parts of this paper are organized as follows. In Section 2, the literature of past works is presented. The proposed mathematical model for multi-objective multi-period multi-product aggregate production planning in supply chain is developed in Section 3. The solution procedures are also presented in Section 3. The real case study and results are presented in Section 4. Section 5 is allocated to summarize the conclusion remarks and the recommendations for future research.

## 2. Literature of Past Works

In general, APP is defined as one of the major production planning categories (Giannoccaro and Pontrandolfo, 2001; Mula et al., 2006). Since classic model proposed by Holt et al., (1955) and Holt et al., (1961) the APP problem has been studied extensively (Leung and Wu, 2004). Nam and Logendran, (1992) classified APP models. Gunasekaran et al., (1998) developed a mathematical model to determine the optimum lot-sizes for a set of products and the capacity required to produce them in a multi-stage production system.

Leung et al. (2003) addressed the problem of aggregate production planning (APP) for a multinational lingerie company in Hong Kong. The multi-site production planning problem considered the production loading plans among manufacturing factories subject to certain restrictions, such as production capacity, workforce level, storage space and resource conditions of the factories. Leung et al. (2003) developed a multi-objective model to solve the associated production planning problem, in which the profit was maximized but production penalties resulting from going over/under quotas and the change in workforce level were minimized.

Sha and Che, (2006) proposed a novel multi-phase mathematical approach for the design of a complex supply chain network. The proposed approach was based on the genetic algorithm (GA), the analytical hierarchy process (AHP), and the multi-attribute utility theory (MAUT). Kogan and Portugal, (2006) focused on the control decisions in the area of multiperiod, aggregate production planning. The goal was to minimize the expected total costs including productivity, overtime as well as over- and underproduction costs.

Liang (2007) developed an interactive possibilistic linear programming ( $i$-PLP) approach to solve multiproduct and multi-time period APP problems with multiple imprecise objectives and cost coefficients by triangular possibility distributions in uncertain environments. The imprecise multi-objective APP model designed tried to minimise total production costs and changes in work-force level with reference to imprecise demand, cost coefficients, available resources and capacity. Rizk et al., (2008) proposed a tight mixed integer programming model for integrated planning of production and distribution in the network. They also proposed a sequential solution approach, based on the independent, but synchronized, solutions of the production and distribution sub-problems.

Techawiboonwong and Yenradee (2010) presented the aggregate production planning for multiple product types where the worker resource could be transferred among the production lines. A mathematical model was formulated in spreadsheet format. Then the spreadsheet-solver technique was used as a tool to solve the model. A real situation of a manufacturing company was selected as a case study. The actual data was used to test and validate the proposed model.

Sakallı et al. (2010) discussed a possibilistic aggregate production planning (APP) model for blending problem in a brass factory. The main problem was about computing optimal amounts of raw materials for the total production of several types of brass in a planning period. The model basically had a multi-blend model formulation in which demand quantities, percentages of the ingredient in some raw materials, cost coefficients, minimum and maximum procurement amounts were all imprecise and had triangular possibility distributions. A mathematical model and a solution algorithm were proposed to solve the model.

Mirzapour Al-e-hashem et al., (2011) solved the multisite, multi-period and multi-product APP problem under uncertainty for a supply chain consisting of multiple-supplier, multiple-producer and multiplecustomer. They also considered the costs related to supply chain and demands as the uncertain parameters. Mirzapour Al-e-hashem et al., (2012) developed a multi-site, multi-period, multi-product, and multiobjective robust APP with regard to conflicts among total costs of supply chain, customer service level, and productivity of workers during medium-term planning horizon in an uncertain environment.

Corominas et al., (2012) discussed the joint aggregate planning of a production system with manufacturing new units and remanufacturing. Karmarkar and Rajaram, (2012) discussed a competition version of APP model with capacity constraints. Toptal et al., (2012) proposed manufacturer's planning problem to schedule order production and transportation to respective destinations. Han et al., (2013) proposed a linear programming model for a hybrid remanufacturing and manufacturing system for production planning problems with deterministic returns. Wang and Zheng, (2013) developed a responsive and flexible production planning system to cope with uncertain manufacturing factors.

Fortunately, the APP models can deal with details of real-world problems while often efficient algorithms are proposed in order to solve them. As identified by many researchers (Bushuev, 2014), the APP cost function is convex and piecewise. Bushuev, (2014) proposed a new convex optimization approach for solving the APP problem. Yan et al., (2014) modeled an integrated optimization production planning and scheduling problem through a non-linear mixed integer programming formulation. Yan et al., (2014) developed an iterative genetic algorithm to solve the problem.

Several researches have been dedicated to the field of production planning, logistics and supply chain in recent years. Khalili-Damghani and Shahrokh (2014) proposed a multi-period multi-objective multiproduct aggregate production planning problem. Three objective functions, including minimizing total cost, maximizing customer services level, and maximizing the quality of end product, were considered, simultaneously. Several constraints were also considered by Khalili-Damghani and Shahrokh (2014). The proposed problem was solved using Fuzzy Goal Programming (FGP) approach (KhaliliDamghani and Shahrokh, 2014). Khalili-Damghani et al., (2017) proposed a customized genetic algorithm to solve multi-period cross docking truck scheduling problem. Tavana et al., (2017) compared drone shipping versus truck delivery in a cross-docking system with multiple fleets and products. Tahmasebi et al. (2017) developed a model for the problem of location-routing in post offices. Tahmasebi et al. (2017) defined a Bi-Objective Location-Routing Problem for Locating Town Post Office and Routing Parcels. The problem was modeled through mixedinteger mathematical programming. Hafezolkotob et al., (2016) proposed a multi-objective multiperiod multi-product supply chain network problem. The problem was modeled using a multi-objective mixed integer mathematical programming. The objectives were maximizing the total profit of logistics, maximizing service level, and minimizing inconsistency of operations. Khalili-Damghani and Ghasemi (2016) proposed an uncertain decentralized decision making approach through coordination mechanism for a multi-product supply chain planning problem. Rezaeisaray et al., (2016) proposed a hybrid multi-criteria decision making approach based on decision making trial and evaluation (DEMATLE), fuzzy analytic network process (FANP) and ordinal/ cardinal data envelopment analysis (DEA) model to select ousourcing suppliers. Khalili-Damghani and Tajik Khaveh (2015) proposed a logistic planning and design problem in a multi-echelon supply chain consisting of suppliers, manufacturers, and distribution centers. A multi-objective mixed integer mathematical programming model for both decreasing several logistics costs and increasing the service level in supply chain was proposed. KhaliliDamghani et al. (2015) proposed a new bi-objective mixed integer mathematical programming to reduce the total cost of the supply chain and to balance the workload of distribution centers while the due dates of delivery of perishable product were met, concurrently. Hafezolkotob and Khalili-Damghani (2015) proposed a multi-objective multi-period
supply chain design and planning problem. The problem tried to minimise logistic costs and maximise service level in a three-echelon multi-product supply chain considering back orders. The layers of chain included suppliers, manufacturers and distribution centres. The parts of logistic costs were discussed and modelled while service level was also interpreted as low level of backorder and shortening the delivery time of products to customers. Khalili-Damghani et al., (2014) proposed a fuzzy bi-objective mixedinteger programming method for solving supply chain network design problems under ambiguous and vague conditions. Khalili- Damghani and Naderi (2014) proposed a mathematical location-routing model of repair centres and ammunition depots in order to support soldiers in civil wars.

The real world problems usually include some types of uncertainty. Stochastic Programming (SP) is a well-known approach to handle uncertainties in optimization_problems. In a two-stage stochastic optimization approach the uncertain model parameters are considered random variables with an associated probability distribution and the decision variables are classified into two stages. The firststage variables correspond to those decisions that need to be made "here-and-now", prior to the realization of the uncertainty. The second-stage or recourse variables correspond to those decisions made after the uncertainty is revealed and are usually referred to as "wait-and-see" decisions. After the first-stage decisions are taken and the random events realized, the second-stage decisions are made subject to the restrictions imposed by the second-stage problem. Due to the stochastic nature of the performance associated with the second-stage decisions, the objective function consists of the sum of the first-stage performance measure and the expected second-stage performance (Barbarosoglu and Arda, 2004; Guillen et al., 2005; Azaron et al., 2008). Uncertainty can be handled through different paradigms. In stochastic programming approach, two different methodologies can be applied. The first methodology is the distribution-based approach (Petkov and Maranas, 1998). The second methodology is the scenario-based approach. In this approach the uncertainty is described by a set of discrete scenarios. Each scenario is associated with a probability level (Poojari et al., 2008). The advantage of this methodology is that there is no limitation for the number of considered uncertain parameters (Mirzapour Al-e-hashem et al., 2012).

Stochastic programming can be a useful choice for modeling Supply Chain Planning (SCP) with uncertain parameters. Kira et al., (1997) formulated a hierarchical production planning (HPP) model under uncertain demand. A stochastic linear programming model (SLP) was proposed to better reflect the reality. Dupačová, (2002) discussed the applications of stochastic programming. Leung et al., (2006) presented a stochastic programming approach for multi-site aggregate production planning with uncertain demand data. Leung and Wu , (2004) developed a robust optimization model and applied it to the stochastic aggregate production planning. Leung et al., (2006) addressed the production planning problem with additional constraints, such as production plant preference selection. They proposed a stochastic programming approach to determine optimal medium-term production loading plans under an uncertain environment. Leung et al., (2007) developed a robust optimization model to solve production planning problems for perishable products in an uncertain environment in which the setup costs, production costs, labour costs, inventory costs, and workforce costs were minimized. Karabuk, (2008) developed a stochastic programming model in order to address the yarn production planning problem. The proposed model explicitly included uncertainty in the form of discrete demand scenarios. Azaron et al., (2008) proposed a two-stage stochastic model, in order to take into account, the effects of the uncertainty in the production scenario for multiobjective supply chain design. They used the goal attainment technique to solve the multi-objective problem. In this method, the preferred solution is sensitive to the goal vector and the weighting vector given by the decision-maker; the same as the goal programming technique. The main drawback of the goal attainment technique to solve the problem is that the preferred solution extremely depends on the goals and weights. To overcome this drawback, Azaron et al., (2008) developed a multi-objective stochastic programming model to design robust supply chain configuration networks. They used STEM and SWT methods, which are two main interactive multi-objective techniques, to solve the multi-objective model. Mirzapour Al-e-hashem et al., (2012) presented a multi-objective model to deal with a multi-period multi-product multi-site APP problem under uncertainty and used an efficient algorithm that is a combination of a modified $\varepsilon$-constraint method and genetic algorithm to solve their problem. Mirzapour Al-e-hashem et al., (2011) developed a multi-objective two stage stochastic programming model to deal with a multi-period
multi-product multi-site production-distribution planning problem and applied a hybrid algorithm that is a combination of Monte Carlo sampling method, modified $\varepsilon$-constraint method and L-shaped method.

Kazemi-Zanjani et al., (2011) investigated multiperiod, multi-product (MPMP) production planning in a manufacturing environment with non-homogeneous raw materials, and consequently random process yields. A two-stage stochastic program with recourse was proposed to address the problem. Abdelaziz, (2012) presented various solution approaches for multi-objective stochastic problems. Mirzapour Al-e-hashem et al., (2013) developed a stochastic APP approach in a green supply chain. Kazemi-Zanjani et al., (2013) proposed a stochastic multi-period, multi-product sawmill production planning problem. The proposed model considered two important issues: (i) randomness in yield and in demand; and (ii) set-up constraints. Uncertainties were modelled by a scenario tree in a multi-stage environment.

Due to literature of past works, there is no unique research addressing the multi-period multi-product multi-objective APP in an uncertain multi-echelon supply chain considering financial risk, customer satisfaction and training. In this paper, we are going to address this problem and apply it to a real case study.

## 3. Stochastic Aggregate Production planning problem in Supply Chain

In this section the proposed stochastic aggregate production planning problem is developed. First the problem and assumptions are defined. The multiobjective aggregate production planning is proposed. Then, the classic two-stage stochastic programming model is revisited and the stochastic aggregate production planning is developed.

### 3.1. Problem definition and assumptions

The proposed multi-period, multi-objective, multiproduct APP problem in supply chains is described based on the following assumptions. It is notable that the assumptions are formed based on real-case study of the research. In this problem, we are going to an aggregate production plan for a three-echelon supply chain incorporating supplier, manufacturers,
and customers. This plan will be involved in determination of purchase, production, and delivery decision simultaneously.

- There are $S$ supplier, one manufacturing plant, and $C$ customers in the supply chain.
- Planning is accomplished in a horizon consists of T time periods $(t=1, \ldots, T)$.
- Batch production system with the capability of producing several kinds of the product is considered.
- The producer can produce $I$ types of products to response market demand.
- Demand can be either satisfied or backordered.
- No subcontracting is allowed for products.
- Two working shifts are considered in a day ( $q \in\{1,2\}$ ). Regular time production ( $q=1$ ) and overtime production $(q=2)$.
- The holding cost of inventory of products are predetermined and known in advance.
- Several skill levels ( $k$-levels) are considered for workforce.
- During the planning horizon, training courses are accomplished and the skill level of workforce is improved.
- There are also several (i.e., $l=1,2, \ldots, L$ ) types of training courses. The first type of training $(l=1)$ enhances the workers from level $k=1$ to level $k=2$. The second type of training $(l=2)$ enhances the workers from level $k=2$ to level $k=3$ and so on.
- Salary of workers is independent of unit production cost.
- The safety stock is considered in the quantity of production in each period.
- According to demand of market, hiring and firing of manpower are eligible with restricted limits.
- The nominal capacity of production is usually decreased by a fixed failure percentage into actual capacity.
- If an unexpected failure occurs during a shift the repair process is accomplished in the next shift.
- In each period of planning, the shortage of production is recovered by overtime production.
- Due to inflation and low holding costs, keeping finished products is economic.
- The transportation costs between factory and each customer's location are known in advance.
- Three objective functions are considered as expected total costs, expected customer satisfaction level, and expected supply chain downside risk.
- Future economic scenarios (i.e., the nature status) will fit into one of four possible scenarios; (I) Boom, (II) Good, (III) Fair, and (IV) Poor, with associated probabilities of $p_{1}, p_{2}, p_{3}$ and $p_{4}$, respectively.
- The uncertainty of parameters in real case study are modeled using two-stage stochastic programming.
- Stochastic Programming methods usually do not provide any control on the solution's variability throughout the different scenarios. Therefore, the downside risk (or the risk of loss) is considered as a risk measure and incorporated into the twostage stochastic programming model (Azaron et al., 2008; Azaron et al., 2010).
- Goal Attainment technique, Modified $\varepsilon$-constraint method, and STEM method used to solve the problem.


### 3.2. Notations: Parameters, Indices, and Decision Variables

Notations used in the proposed multi-objective mathematical programming are summarized in following tables. The notations of objective functions are presented in Table 1.

It is notable that $Z_{2}$ and $Z_{3}$ which are qualitative objective functions are measured through Equations (18)-(20). Table 2 shows the notation of sets and associated indices.

Table 3 present the notations used for parameters and decision variables are illustrated in Table 4.

Table 1. Notation of Objective Functions.

| Objective <br> function | Definition |
| :--- | :--- |
| $Z_{1}$ | Expected Total Costs of Supply Chain |
| $Z_{2}$ | Expected Customer Satisfaction Level |
| $Z_{3}$ | Expected Supply Chain Downside Risk |

Table 2. Sets and Indices.

| Notation | Definition |
| :--- | :--- |
| $T$ | Number of periods in the planning horizon; <br> $t=1, \ldots, T$ |
| $I$ | Number of product types; $i=1, \ldots, I$ |
| $M$ | Raw material type; $m=1, \ldots, M$ |
| $Q$ | Types of shifts $\mathrm{q} \in\{1,2\} .(\mathrm{q}=1 ;$ regular time, <br> and q=2; overtime $)$ |
| $K$ | Skill levels of workers; $k=1,2, \ldots, K$ |
| $L$ | Types of training; $l=1,2, \ldots, L$ |
| $S$ | Suppliers; $s=1,2, \ldots, S$ |
| $C$ | Customers; $c=1,2, \ldots, C$ |
| $J$ | Number of objective Functions; $j=1,2,3$. |
| $N$ | Types of scenarios; $n=1,2, \ldots, N$ |

### 3.3. Framework of Two-Stage Stochastic programming Model

The general form of the two-stage stochastic programming model is briefly revisited (Dantzig, 1955; Kall and Wallace, 2003; Ruszczynski and Shapiro; 2003):

$$
\begin{equation*}
\operatorname{Min} c_{1}^{T} x+E_{\omega}\{\varphi(x, \omega)\} \tag{1}
\end{equation*}
$$

Subject to:
$A x=b$
$x \in R_{+}^{n_{1}}$

Where
$\varphi(x, \omega)=\operatorname{Min} q^{T} y$
Subject to:

$$
\begin{equation*}
T(\omega) x+W y=h(\omega) \tag{5}
\end{equation*}
$$

$y \in R_{+}^{n_{2}}$
Where $E_{\omega}\{\cdot\}$ is the expected value function, $\omega$ is the random vector, $h(\omega)$, and $T(\omega)$ represent a particular sample from a multivariate probability space $\Omega$. Equations (1)-(3) and equations (4)-(6) refer to the first and second stages, respectively. The vector of $x$ is first-stage decision variables. The optimal value of $x$ is not conditional on the realization of the uncertain parameters. The parameter $c_{1}$ is the vector of cost coefficients at the first stage. $A$ is the first-stage coefficient matrix and $b$ is the corresponding righthand side vectors. Also, the vector of $y$ is secondstage (recourse) decision variables, $q$ is the vector

Table 3. Notations for Parameters.

| Parameter | Definition | Unit |
| :---: | :---: | :---: |
| $\mathrm{CO}_{\text {it }}$ | Production cost per hour for product $i$ in shift $q$ in period $t$ | dollar |
| $A_{i t}$ | Process time of product $i$ in period $t$ | hour |
| $\underline{H R}$ | Minimum workforce available | man/hour |
| $C L_{k t}$ | Cost workforce of level $k$ in period $t$ | dollar |
| $\mathrm{CH}_{\text {kt }}$ | Hiring cost workforce of level $k$ in period $t$ | dollar |
| $C F_{k t}$ | Firing cost workforce of level $k$ in period $t$ | dollar |
| $C T_{l t}$ | Training cost of type $l$ in period $t$ | dollar |
| $\alpha_{t}$ | Fraction of the workforce variation allowed in period $t$ | man/hour |
| $M T_{i t}$ | Required machine hours to produce unit of product $i$ in period $t$ | machine/hour |
| $\varepsilon_{t}$ | Percentage of machine capacity that is lost due to interruption in period $t$ | machine/hour |
| $\mu_{t}$ | Percentage of machine capacity that is lost due to repairs in period $t$ | machine/hour |
| $M C_{i q t}$ | Maximum of machine capacity that is available for product $i$ in shift $q$ in period $t$ | machine/hour |
| $A T_{i q t}$ | Available regular time of product type $i$ in both shifts in period $t$ | hour |
| $\gamma_{i m}$ | Units of raw material type $m$ required to produce one unit of product $i$ | kilo-gram |
| SSR ${ }_{m}$ | Safety stock of raw material type $m$ | kilo-gram |
| $C R_{m t}$ | Holding cost for raw material type $m$ in period $t$ in factory | dollar |
| $V R_{m t}$ | Available capacity of factory for storage of raw material type $m$ in period $t$ | kilo-gram |
| $C P_{i t}$ | Holding cost of unit of product $i$ in period $t$ in factory | dollar |
| $V P_{i t}$ | Available capacity of factory for storage of finished-product $i$ in period $t$ | kilo-gram |
| $S S P_{i}$ | Safety stock of product $i$ | kilo-gram |
| $D U_{\text {ic }}$ | Due date of product $i$ in customer location $c$ | day |
| LTC ${ }_{c}$ | Lead time required for shipping products from factory to customer center $c$ | dat |
| $L T S_{s}$ | Lead time required for shipping raw material from supplier $s$ to factory | day |
| $T 1_{s t}$ | Transportation cost from supplier $s$ to factory in period $t$ | dollar |
| $T 2_{\text {ict }}^{n}$ | Transportation cost from factory to customer center $c$ in period $t$ in scenario $n$ | dollar |
| $\delta$ | Percentage of machines capacity that is available for overtime. | machine/hour |
| OL | Percentage of workforce that is available for overtime. | man/hour |
| $\tau$ | Minimum percentage of workers that is available for training. | man/hour |
| $T C_{n}$ | Total cost when the scenario $n$ is realized | dollar |
| $\Omega_{n}$ | Available budget in scenario $n$ | dollar |
| $p_{n}$ | Occurrence probability of scenario $n$ | - |
| $D E_{\text {ict }}^{n}$ | Demand of product $i$ in customer center $c$ in period $t$ in scenario $n$ | quantity |
| $\theta_{i t}$ | Allowable shortage of product $i$ in period $t$ | quantity |
| $\zeta_{i}$ | Allowable sale of product $i$ | quantity |
| $\pi_{m}$ | Allowable purchase of raw material $m$ | kilo-gram |
| $C B_{\text {ict }}^{n}$ | Backordering cost of product $i$ in period $t$ in customer center $c$ in scenario $n$ | dollar |
| $C M_{\text {smt }}$ | Cost of raw material $m$ purchased from supplier $s$ in period $t$ | dollar |

Table 4. Notation for Decision Variables.

| Decision variables | Definition | Unit |
| :--- | :--- | :--- |
| $X_{i q t}$ | Number of product $i$ produced in shift $q$ of period $t$ | quantity |
| $X L_{k t}$ | Number of available workers of level $k$ in period $t$ | $\mathrm{man} / \mathrm{hour}$ |
| $X H_{k t}$ | Number of hired workers of level $k$ in period $t$ | $\mathrm{man} /$ hour |
| $X F_{k t}$ | Number of fired workers of level $k$ in period $t$ | $\mathrm{man} / \mathrm{hour}$ |
| $X T_{l t}$ | Number of workers trained course level $l$ in period $t$ | man $/ \mathrm{hour}$ |
| $X R_{m t}$ | Inventory level of raw material type $m$ at the end of period $t$ in factory | kilo-gram |
| $X M_{s m t}$ | Number of units of raw material $m$ purchased from supplier $s$ to factory | kilo-gram |
| $X P_{i t}$ | Inventory level of finished-product $i$ in period $t$ in factory | quantity |
| $D R_{n}$ | Supply chain downside risk of scenario $n$ | - |
| $D_{n}$ | Delivery time of scenario $n$ | day |
| $Y_{i c t}^{n}$ | Number of units of product $i$ produced by factory for customer $c$ in period $t$ in scenario $n$ | quantity |
| $B_{i c t}^{n}$ | Backorder level of product $i$ in period $t$ in customer location $c$ in scenario $n$ | quantity |

of cost (recourse) coefficient for the second stage, $W$ is the second-stage (recourse) coefficient matrix, and $h(\omega)$ is the corresponding right-hand side vector. $T(\omega)$ is the matrix that ties the two stages together. In the second-stage model, the random constraint defined in (5), $h(\omega)-T(\omega) x$, is the goal constraint. The violation of this constraint are allowed, but the associated penalty cost, $q^{T} y$, will influence the choice of $x$. The function $\varphi(x, \omega)$ is the recourse penalty cost or second-stage value function, and the notation $E_{\omega}\{\varphi(x, \omega)\}$ denotes the expected value of recourse penalty cost (second-stage value function) with respect to the random vector $\omega$.

Assuming the distribution of $\omega$ is discrete, i.e. the random parameter takes one of a finite set of values (scenarios) $\left\{\omega_{1}, \ldots, \omega_{n}\right\}$ having probabilities $\left\{p_{1}, \ldots, p_{n}\right\}$, the two-stage model can be re-formulated as follows (Ahmed, 2010):

$$
\begin{equation*}
\operatorname{Min} \sum p_{n}\left(c_{1}^{T} x_{n}+q_{n}^{T} y_{n}\right) \tag{7}
\end{equation*}
$$

Subject to:
$A x_{n}=b \quad \forall n=1, \ldots, N$
$T_{n}(\omega) x_{n}+W_{n} y_{n}=h_{n}(\omega), \quad \forall n=1, \ldots, N$
$x_{n} \in R_{+}^{n_{1}} \quad \forall n=1, \ldots, N$
$y_{n} \in R_{+}^{n_{2}} \quad \forall n=1, \ldots, N$
$x_{1}=x_{2}=\ldots=x_{n}$

Note that duplicates of the first-stage variable have been presented for each scenario. The last constraint, known as the non-anticipatively constraint, guarantees that the first-stage variables are identical across the different scenarios.

### 3.4. Multi-objective Stochastic APP Model formulations

### 3.4.1. Objective functions

Three objective functions are considered for the proposed model of this research as follows:

### 3.4.1.1. Expected Total Costs of Supply Chain

$\operatorname{Min} Z_{1}=\sum_{n \in N} p_{n} T C_{n}$

The objective function (13) is total expected cost of supply chain. The total costs of supply chain includes eleven terms as follow: (I) production costs per unit, (II) holding costs of products, (III) holding costs of raw materials, (IV) The cost of purchased materials, (V) costs of salary of workers, (VI) costs of hiring, (VII) costs of firing, (VIII) costs of training, (IX) transportation costs of raw materials, (X) transportation costs of end-products, and (XI) backordering costs, respectively. The sum of these eleven terms is called as total cost $\left(T C_{n}\right)$ when the scenario $n$ is realized and defined in (14).

$$
\begin{align*}
& T C_{n}=\sum_{i \in I} \sum_{q \in\{1,2} \sum_{t \in T} C O_{i q t} X_{i q t}+\sum_{i \in I} \sum_{t \in T} C P_{i t} X P_{i t}+ \\
& +\sum_{m \in M} \sum_{t \in T} C R_{m t} X R_{m t}+\sum_{s \in S} \sum_{m \in M} \sum_{t \in T} C M_{s m t} X M_{s m t}+ \\
& +\sum_{k \in K} \sum_{t \in T} C L_{k t} X L_{k t}+\sum_{k \in K} \sum_{k \in T} C H_{k t} X H_{k t}+ \\
& +\sum_{k \in K} \sum_{t \in T} C F_{k t} X F_{k t}+\sum_{l \in L} \sum_{t \in T} C T_{l t} X T_{l t}+\sum_{s \in S} \sum_{m \in M} \sum_{t \in T} T 1_{s t} X M_{s m t}+ \\
& +\sum_{i \in I} \sum_{c \in C} \sum_{t \in T} T 2_{i c t}^{n} Y_{i c t}^{n}+\sum_{i \in I} \sum_{c \in C} \sum_{t \in T} C B_{i c t}^{n} B_{i c t}^{n}, \quad \forall n \tag{14}
\end{align*}
$$

Based on two-stage stochastic programming models, it can be classified into first and second-stage variables as follows:

$$
\begin{equation*}
T C=\left(\sum_{n \in N} p_{n} T C_{n}\right) x+\sum_{n \in N} p_{n} q_{n}^{T} y_{n} \tag{15}
\end{equation*}
$$

Equation (15) includes two terms as: (I) the term "( $\left.\sum_{n \in N} p_{n} c_{n}\right) x$ " is The First-Stage Variables (FSv), which is represented by equation (16), (II) the $\sum_{\text {term " } n \in N} p_{n} q_{n}^{T} y_{n}$ " is The Second-Stage Variables (SSv) that is represented by equation (17),

$$
\begin{align*}
F S v & =\sum_{i \in I} \sum_{q \in\{1,2\}} \sum_{t \in T} C O_{i q t} X_{i q t}+\sum_{i \in I} \sum_{t \in T} C P_{i t} X P_{i t}+ \\
& +\sum_{m \in M} \sum_{t \in T} C R_{m t} X R_{m t}+\sum_{s} \sum_{m} \sum_{t} C M_{s m t} X M_{s m t} \\
& +\sum_{k \in K} \sum_{t \in T} C L_{k t} X L_{k t}+\sum_{k \in K} \sum_{t \in T} C H_{k t} X H_{k t} \\
& +\sum_{k \in K} \sum_{t \in T} C F_{k t} X F_{k t}+\sum_{l \in L} \sum_{t \in T} C T_{l t} X T_{l t} \\
& +\sum_{s} \sum_{m} \sum_{t} T 1_{s t} X M_{s m t} \tag{16}
\end{align*}
$$

Expected Customer Satisfaction Level

$$
\begin{align*}
S S v & =\sum_{n \in N} p_{n}\left(\sum_{i \in I} \sum_{c \in C} \sum_{t \in T} T 2_{i c t}^{n} Y_{i c t}^{n}\right)+ \\
& +\sum_{n \in N} p_{n}\left(\sum_{i \in I} \sum_{c \in C} \sum_{t \in T} C B_{i c t}^{n} B_{i c t}^{n}\right), \quad \forall n \tag{17}
\end{align*}
$$

$\operatorname{Min} Z_{2}=\sum_{n} p_{n} D_{n}$

Where,
$D_{n}=\left|\sum_{i \in I} \sum_{c \in C} \sum_{t \in T} A_{i t} Y_{i c t}^{n}-D U_{i c}\right|, \forall n$

In this research, on time delivery is considered as a measure to assess the customer satisfaction level. This objective function minimizes the difference between delivery time and the due date for all type of products. As the probabilities of different scenarios are considered in objective function (18), so it calculates the expected customer satisfaction level. Delivery of product to customers earlier than due date, which we called "earliness", is not suitable while delivery of product to customers later then due date, which is called "tardiness", is not also desirable. So, the equation (19) minimizes both earliness and tardiness, simultaneously.

This goal includes two terms as: (I) actual delivery time of product $i$ to customer $c$, and (II) due date of delivery of products $i$ to customer $c$ according to contract. Where $p_{n}$ represent the occurrence probability of the $n$-th scenario.

### 3.4.1.2. Expected Supply Chain Downside Risk

$\operatorname{Min} Z_{3}=\sum_{n} p_{n} D R_{n}$

The objective function (20) minimizes the downside risk or the expected total loss. Where $p_{n}$ and $D R_{n}$ represent the probability of occurrence of the $n$-th scenario and the downside risk of scenario $n$, respectively.

### 3.4.2. Constraints

The workforce level constraints are considered using (21)-(25).

$$
\begin{align*}
& \underline{H R} \leq \sum_{k \in K} X L_{k t}, \quad \forall t  \tag{21}\\
& X L_{k t}=X L_{k(t-1)}+X H_{k t}-X F_{k t}+X T_{l t} \\
& \forall k, \forall t, t>1  \tag{22}\\
& X H_{k t}+X F_{k t} \leq \alpha_{(t-1)} \cdot X L_{k(t-1)}, \forall k, \forall t, t>1 \tag{23}
\end{align*}
$$

$$
\begin{equation*}
\tau \cdot \underline{H R} \leq \sum_{t \in T} X T_{l t}, \quad \forall l \tag{24}
\end{equation*}
$$

$$
\begin{equation*}
\sum_{l \in L} X T_{l t} \leq X L_{k(t-1)}, \quad \forall k, \forall t, t>1 \tag{25}
\end{equation*}
$$

Set of constraints (21), which are written for all periods of planning, assures that a minimum number of workers should be utilized in a period of planning. Set of constraints (22) is a balance equation for workforce level and ensures that the available workforce with skill level $k$ in a certain period are equal to the workforce with the same skill level $k$ in previous period plus the change of workforce level in the current period. Set of constraint (23) assures that the change in workforce level in each period of planning does not exceed a predetermined proportion of workers in each period. Set of constraints (24) assures that in all periods of planning, the trained workforce should be greater than or equal to a predetermined percentage of the minimum available workers during all periods. Set of constraint (25) assures that the number of workforces with skill level $k$ who are trained for upper skill levels in current period cannot exceed the available workforce with skill level $k$ in previous period.

The available time limit of working shifts is presented using constraints (26)-(27).

$$
\begin{align*}
& \sum_{i \in I} A_{i t} X_{i q t} \leq \sum_{i \in I} \sum_{k \in K} A T_{i q t} \cdot X L_{k t}, \forall t, q=1  \tag{26}\\
& \sum_{i \in I} A_{i t} X_{i q t} \leq \sum_{i \in I} \sum_{k \in K} A T_{i q t} . O L . X L_{k t}, \forall t, q=2 \tag{27}
\end{align*}
$$

Set of constraints (26)-(27) assures that the required production time for all periods of planning and in each working shifts are less than or equal to available regular production time and overtime, respectively.

The inventory level situations are demonstrated using constraints (28)-(29). The limitations of inventory level of raw material and the production limitations in each period of planning are presented using constraints (30)-(31).

$$
\begin{align*}
& \sum_{i \in I} X P_{i i t}=\sum_{i \in l} X P_{i(t-1)}+\sum_{i \in l} \sum_{q \in \mid 1,2)} X_{i q t}-\sum_{i \in l} \sum_{c \in C} Y_{i c\left(t-L T C_{c}\right)}^{n}, \\
& \forall n, \forall t, t>1  \tag{28}\\
& X R_{m t}=X R_{m(t-1)}+\sum_{s \in S} X M_{s m\left(t-L T X_{s}\right)}-\sum_{i \in I} \sum_{q \in[1,2\}} X_{i q(t-1)} \cdot \gamma_{i m}, \\
& \forall m, \forall t, t>1  \tag{29}\\
& S S R_{m} \leq X R_{m t}, \quad \forall m, \forall t  \tag{30}\\
& S S P_{i} \leq \sum_{q \in \mathbb{Q}} X_{i q q}, \quad \forall i, \forall t \tag{31}
\end{align*}
$$

Set of constraints (28), which are written for all products, assures that the amount of inventory of finished products in period $t$ in factory is equal to the amount of inventory of finished products in period $t-1$ in factory plus the amount of produced finishedgoods in period $t$ in both working shifts minus the amount of all products produced by factory for all customers without considering the lead time. Set of constraint (29) assures the balance of raw materials. Set of constraints (30) assures the satisfaction of safety stock of raw materials. Set of constraints (31), which is written for all product types and all periods of planning, guarantee the satisfaction of safety stock of finished-products in working shifts. The capacity of machines for each planning periods for both working shifts are presented using constrains (32)(33).

$$
\begin{align*}
& \sum_{i \in I} M T_{i t} \cdot X_{i q t} \leq \sum_{i \in I} M C_{i q t}-\varepsilon_{t} \cdot \sum_{i \in I} M C_{i q t}, \\
& \forall t, q=1  \tag{32}\\
& \sum_{i \in I} M T_{i t} \cdot X_{i q t} \leq \delta \cdot \sum_{i \in I} M C_{i q t}-\mu_{t} \cdot \delta \cdot \sum_{i \in I} M C_{i q t}, \\
& \forall t, q=2 \tag{33}
\end{align*}
$$

Set of constraints (32)-(33) assures the satisfaction of the maximum available capacity of machines in regular time and overtime, respectively. The limitation of number of units of raw material $m$ purchased from all suppliers to factory in each period of planning are presented using constraint (34).
$\pi_{m} \cdot S S R_{m} \leq \sum_{s \in S} X M_{s m m}, \forall m, \forall t$

Set of constraints (34), assures that the number of units of raw material $m$ purchased from all suppliers are more than or equal to the certain percentage of the safety stock of raw material $m$. This determines the lower limit of purchase of raw material $m$. Generally, the purchase orders of raw materials are seasonal. For instance, Equation (34) is written for $t=3,6$ in a planning horizon consists of six periods.

For each scenario, the maximum amount of all products that provided by factory for all customers is presented using constraints (35). For each scenario, the minimum amount of product $i$ that provided through factory for customer $c$ in the period $t$ is presented using constraints (36).

$$
\begin{align*}
& \sum_{i \in I} \sum_{c \in C} \sum_{t \in T} Y_{i c\left(t-L T C_{c}\right)}^{n} \leq \sum_{i \in I} \sum_{c \in C} \sum_{t \in T}\left(D E_{i c t}^{n}+B_{i c(t-1)}^{n}\right), \forall n  \tag{35}\\
& \zeta_{i} \cdot D E_{i c t}^{n} \leq Y_{i c t}^{n}, \quad \forall n, \forall i, \forall c, \forall t \tag{36}
\end{align*}
$$

Set of constraints (35) assures that the amount of all products produced by factory for all customers without considering the lead time in each scenario, is less than or equal to the amount of backorder of all products for all customers in all previous periods and the demand of all products for all customers in all periods. Set of constraints (36) assures that the amount of sales or the number of units of product $i$ produced by factory for customer $c$ in period $t$ in scenario $n$, are more than or equal to the certain percentage of the demand which determines the lower limit of sales. The backorders are accepted and the associated constraints are presented as (37)-(38).
$\sum_{i \in I} B_{i c t}^{n} \leq \sum_{i \in I} \theta_{i t} D E_{i c t}^{n}, \forall n, \forall c, \forall t, t \neq T$
$B_{i c t}^{n}=0, \quad \forall n, \forall i, \forall c, \forall t=T$
Set of constraints (37) assures that the backorder level at the end of period $t$ cannot exceed the certain percentage of the demand. This determines the upper limit of shortage. Set of constraints (38) assures that there is no possibility for backordering at the last of planning period. The financial risk of supply chain is presented using constraint (39).
$T C_{n}-\Omega_{n} \leq D R_{n}, \quad \forall n$

Where total costs of supply chain $\left(T C_{n}\right)$ was defined in (14). Also $\Omega_{n}$ and $D R_{n}$ are available budget of scenario $n$ and the supply chain downside risk for
scenario $n$, respectively. Non-negativity of decision variables are presented in (40)-(41).

$$
\begin{align*}
& X_{i q t}, X R_{m t}, X M_{s m t}, X P_{i t}, B_{i c t}^{n}, Y_{i c t}^{n} \geq 0, \\
& \forall n, \forall i, \forall q, \forall t, \forall c, \forall s, \forall m \tag{40}
\end{align*}
$$

$X L_{k t}, X H_{k t}, X F_{k t}, X T_{l t} \geq 0$,
$\forall k, \forall l, \forall t$

## 4. Solution Procedures

Many real-world optimization problems are involved with more than a single objective function. In Multi-Objective Mathematical Programming (MOMP), there is no single optimal solution that simultaneously optimizes all the objective functions. Therefore, the decision makers are looking for the non-dominated solutions, instead of a single optimal solution. The methods for solving MOMP problems can be classified into three major categories: the a priori methods, the a posteriori methods and the interactive methods (Hwang and Masud, 1980; Masud and Hwang, 1980; Miettinen, 1998; Mavrotas, 2009). The strengths and weaknesses of MOMP methods are presented in Table 5.

Table 5. The Strengths and Weaknesses of the methods for solving MOMP problems

| Methods | Advantages | Shortcomings |
| :--- | :--- | :--- |
| A-Priori | Easy, low <br> computational <br> burden, available <br> software | Need <br> unrealistically <br> precise <br> information, <br> need extensive <br> sensitivity analysis |
| A-Posteriori | The expression of <br> preference follows <br> the optimization <br> phase, can produce <br> subsets of efficient <br> solutions | Computational <br> burden, not widely <br> available software |
| Interactive | Iterative, the decision <br> maker guides the <br> search, the decision <br> maker "learns" about <br> the problem | Need extensive <br> interaction with the <br> decision maker, <br> the decision maker <br> decides based on <br> samples |

In this paper, three MOMP methods, called: (I) Goal Attainment method, (II) Modified $\varepsilon$-constraint method, and (III) STEM method, are used to
solve the problem (13)-(41). The Goal Attainment method is one of the a priori methods, the modified $\varepsilon$-constraint is a posteriori method and presents a "comprehensive framework" to obtain the optimal Pareto solutions for the multi-objective optimization problem (Haimes et al., 1971; Mirzapour Al-ehashem et al., 2012). The STEM method is an interactive method which presents "good" solutions and the relative importance of the objectives for decision maker.

### 4.1. Goal Attainment Method

The goal attainment which was introduced by Gembicki and Haimes, (1975) is presented here:

## Minimize $v$

Subject to:

$$
\begin{align*}
& v \in R^{+}, \quad x \in \psi  \tag{43}\\
& F_{j}(x)-w_{j} v \leq F_{j}^{*} \quad j=1, \ldots, n \tag{44}
\end{align*}
$$

Where, $\psi$ is the feasible region, the term $v$ is the attainment element, the weigh vector, $w_{i}$, is used to express the relative importance of objectives $j, F_{j}^{*}$ is the goal of objective function $j$, and $F_{i}(x)$ the objective function $j$. The problem (13)-(41) is transformed into the optimization problem (45)-(47) using the goal attainment approach stated in (42)-(44):

Minimize $v$

Subject to:
Constraints (21)-(41)
$Z_{j}-w_{j} v \leq b_{j}, \quad \forall j=1,2,3$
where, $Z_{j}, j=1,2,3$ are objective functions, respectively. Also, are goals of objective functions, and are weights of objective functions, respectively. The solution is sensitive to value of goals and weights. The weights relate the relative under-attainment of the goals and a smaller $w$ is associated with the more important objectives. Using goal attainment method, the best solution can be determined by the nearest Pareto-Optimal solution from $b_{i}, j=1,2,3$. The weights are generally normalized so that
$\sum_{j=1}^{3} w_{j}=1$

### 4.2. Modified $\varepsilon$-constraint Method

The Modified $\varepsilon$-constraint method which was first introduced by Haimes et al., (1971) is also adapted to solve the proposed multi-objective problem.

Step 1. Select one of the objective functions as the main objective function and convert other objective functions into constraint. Then create the payoff table by the individual optimization of each objective functions separately. Find the range of each objective function.

Step 2. Determine the grid points. Then, divide the range of each objective function to $\lambda$ equal intervals using $\lambda$-1 intermediate equidistant grid points; that are used to vary parametrically the right-hand side of objective function.

Step 3. Solve the resultant model for each value of right-hand side.

So an MOMP with minimization objective functions is converted to model (48)-(50) using $\varepsilon$-constraint method:

$$
\begin{equation*}
\varphi=\operatorname{Min}\left\{Z_{j}(x)-\theta \sum_{u \neq j} \frac{s_{u}}{r_{u}}\right\} \tag{48}
\end{equation*}
$$

Subject to:

$$
\begin{align*}
& Z_{u}(x)+s_{u}=\varepsilon_{u}, \quad \forall u \neq j  \tag{49}\\
& x \in X, s_{u} \in R^{+} \tag{50}
\end{align*}
$$

Where $\theta$ is a small number usually between 10-6 and 10-3 $s_{u}$ is a slack variable, $r_{u}$ is the range of objective function $Z_{u}, \varepsilon_{u}$ is right-hand side which is parametrically determine in it associated range, and $X$ is the feasible region of the primary model.

The first objective function (i.e., expected total costs of supply chain) is selected as the main objective function and the other objective functions (i.e., expected customer satisfaction level, and expected supply chain downside risk) are converted into constraint. Therefore the following optimization model (51)-(54) is considered to solve the problem (13)-(41).

$$
\begin{equation*}
\varphi=\left\{\operatorname{Min} Z_{1}-\theta\left(\frac{s_{2}}{r_{2}}+\frac{s_{3}}{r_{3}}\right)\right\} \tag{51}
\end{equation*}
$$

Subject to:
Constraints (21)-(41)
$Z_{u}+s_{u}=\boldsymbol{\varepsilon}_{u}, \quad \forall u=2,3$
$s_{2}, s_{3} \in R^{+}$
Where, $r_{2}$ and $r_{3}$ are the ranges of $Z_{2}$ and $Z_{3}$, respectively. We used Pay-off Table to obtain the range of each objective function.

### 4.3. STEM Method

STEM method which was first proposed by Benayoun et al., 1971 to interactively solve an MOMP, is revisited here briefly.

Step 1. Construct a pay-off table using single objective optimization problems. Therefore $Z_{j}^{*}, j=1, \ldots, n$ are the optimal objective value of single objective models (55)-(56). Minimize $Z_{j}(x), j=1, \ldots, n$.

Subject to
$x \in \psi$ (Feasible region of problem)
Step 2. At the $\ell$-th cycle, the optimum solution of the model (57)-(60) is sought, which is the nearest to the ideal solution $Z_{j}^{*}$ :

Minimize $\gamma$
Subject to:
$\left(Z_{j}(x)-Z_{j}^{*}\right) \rho_{j} \leq \gamma, \quad j=1, \ldots, n$
$x \in X^{\ell}$
$\gamma \geq 0$
Where $X^{\ell}$ includes $\psi$ plus any constraint added in the previous $\ell-1$ cycles, $\rho_{j}$ gives the relative importance of the distances to the optimal solution, and $\gamma$ is the associated mini-max variable. If $f_{j}^{\text {max }}$ and $f_{j}^{\text {min }}$ be the maximum and minimum values of objective $j$; then $\rho_{j}$ can be determined using (61).
$\rho_{j}=\frac{\frac{Z_{j}^{\max }-Z_{j}^{\min }}{Z_{m a x}^{\max }}}{\sum_{j=1}^{n} \frac{Z_{j}^{\max }-Z_{j}^{\min }}{Z_{j}^{\max }}}, j=1, \ldots, n$
Step 3. The compromise solution $X^{\ell}$ is presented to the decision maker (DM). If some of the objectives are satisfactory and others are not, the DM relaxes a satisfactory objective $Z_{j}^{\ell}$ enough to allow an
improvement of the unsatisfactory objectives, called the set $u$, in the next iterative cycle (Azaron et al., 2008). The DM gives $\Delta Z_{j}$ as the amount of acceptable relaxation. For the next cycle the feasible region is modified using (62).

$$
x^{l+1}=\left\{\begin{array}{c}
x^{l}  \tag{62}\\
Z_{j}(x) \leq Z_{j}\left(x^{\prime}\right)+\Delta Z_{j}, \quad j \in u \\
Z_{j}(x) \leq Z_{j}\left(x^{l}\right), \quad j=1, \ldots, n, j \notin u
\end{array}\right.
$$

The weight $\rho_{j}, j=1, \ldots, n, j \notin u$ is set to zero and the calculation phase of cycle $\ell+1$ begins.

The application of STEM method to solve the problem (13)-(41) results in model (63)-(68).

Minimize $\gamma$
Subject to:
Constrains (21)-(41)
$\left(Z_{j}-Z_{j}^{*}\right) \rho_{j} \leq \gamma, \quad \forall j=1,2,3$
$\rho_{j}=\frac{\beta_{j}}{\boldsymbol{\beta}_{u}+\beta_{u}}, \quad \forall j=1,2,3, \quad \forall u \neq j$
$\beta_{j}=\frac{Z_{j}^{\max }-Z_{j}^{\min }}{Z_{j}^{\max }}, \forall j=1,2,3$
$\gamma \geq 0$
$Z_{1}^{*}, Z_{2}^{*}$, and $Z_{3}^{*}$ are achieved using pay-off table. In order to gather relative importance of objective function the procedure by Barzilai, 1997 is used.

## 5. Model Implementation

The proposed model (13)-(41) is coded and implemented in LINGO software. We used the Goal Attainment technique (45)-(47), Modified $\varepsilon$-constraint (51)-(54) and STEM methods (63)-(68) to solve the proposed model (13)-(41), separately. Results of the three methods are compared. Moreover, the experimental model that was used in the factory was also considered.

### 5.1. A real-world industrial case study

In order to illustrate the applicability and efficacy of proposed methodology, the proposed model is
applied in Teiph-Saipa Company. Teiph-Saipa Paint and Resin Industries Company was established in 1967 under the license from Denmark Dyrup and under the name of Dyrup Iran. With around fifty years' experience in manufacturing different kinds of paint an resin for automotive industry, industrial and construction paints and coatings, and having the world's most recent technologies and formulations, this company is working as one of the subsidiaries of Saipa Group. The products of Teiph-Saipa Company are mainly distributed throughout Iran and middleeast. This company owns two customer centers located in two different cities. The main customer of this company is SAIPA as the second largest Iranian automotive manufacturer. Raw materials are supplied from three suppliers located in Italia, Korea, and Germany. Recently, the company has faced with several issues and problems such as decrease of customer satisfaction levels, financial problems, and high total costs. We are going to apply the proposed models in Teiph-Saipa Company. The following assumptions are considered for planning in Teiph-Saipa Company.

The planning horizon consists of six months. There are three suppliers and two customers. There are two family groups of products. Aggregate unit of production is Ton. Demand can be either satisfied or backordered. Two working shifts are considered in a day. Regular production time is 8 hours per shift and overtime production is approximately 3 hours per shift. To produce the products, 24 types of raw materials are required. Four skill levels are considered for workers as Low ( $k=1$ ), Medium $(k=2)$, Good ( $k=3$ ) and High ( $k=4$ ). There are 3 types of training ( $l=1,2,3$.). Repairs are done just in shift 2 (i.e., overtime). Since the filters of reservoirs should occasionally be replaced, inevitable stops are usually occurred during shift 1 (regular times). If the demand of one period is higher than production capacity in regular times and on hand inventory levels also unable to satisfy this demand, the production is continued in overtime. In this planning horizon, the planned purchase order of raw material $m$ from suppliers have been organized for $t=3$ and $t=6$. The holding cost of inventory is low during the planning horizon. Five operators are working in each site. Maximum available budget is 696970 (Dollar) over the planning horizon.

Based on historical data of sale department, the future economic scenarios is estimated as; (I) Boom, (II) Good, (III) Fair, and (IV) Poor, with associated probabilities of $0.40,0.30,0.20$ and 0.10 , respectively.

In this case, allowable purchase $\left(\pi_{m}\right)$ for all raw materials and allowable sale ( $\zeta_{i}$ ) for all products are constant values 0.001 and 0.01 , respectively. Also, other deterministic parameters are set as $\delta=0.125$, $\tau=0.3, \mathrm{OL}=0.5$, and $\mathrm{HR}=10$. According to the different economic scenarios, the available budgets for each scenario are shown in Table 6.

Table 6. Available Budgets data (dollar).

|  | Scenario |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Parameter | Boom; $n=1$ | Good; $n=2$ | Fair; $n=3$ | Poor; $n=4$ |
| $\Omega_{n}$ | 696970 | 666700 | 636400 | 606000 |

The Backordering costs for different products in each customer center are shown in Table 7.

The Transportation costs from factory to each customer center and demand data are shown in Table 8.

### 5.2. Computational Results

In this sub-section, the results of the application of the proposed model and solution procedures are presented for the case study. After the data collection, essential information are summarized and reported

Table 7. Backordering costs data; $\mathrm{CB}_{i c t}^{n}$ (Dollar/unit).

|  |  |  | Period |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Item | Customer | Scenario | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 1 | Boom | 272.7 | 272.7 | 278.8 | 227 | 454.5 | 281.8 |
|  |  | Good | 257.6 | 257.6 | 263.6 | 212 | 439 | 266.6 |
|  |  | Fair | 242.4 | 242.4 | 248.5 | 197 | 424 | 251.5 |
|  |  | Poor | 227 | 227 | 233 | 182 | 409 | 236 |
|  | 2 | Boom | 182 | 182 | 172.7 | 227 | 69.7 | 169.7 |
|  |  | Good | 167 | 167 | 157.5 | 212 | 54.5 | 154.5 |
|  |  | Fair | 151.5 | 151.5 | 142 | 197 | 39 | 139 |
|  |  | Poor | 136 | 136 | 127 | 182 | 24 | 124 |
| 2 | 1 | Boom | 303 | 203 | 69.7 | 151.5 | 100 | 109 |
|  |  | Good | 288 | 188 | 54.5 | 136 | 84.8 | 94 |
|  |  | Fair | 272.7 | 172.7 | 39.5 | 121 | 69.7 | 78.7 |
|  |  | Poor | 257.6 | 157.5 | 24 | 106 | 54.5 | 63.6 |
|  |  | Boom | 69.7 | 100 | 233 | 151.5 | 203 | 194 |
|  |  | Good | 54.5 | 84.8 | 218 | 136 | 187.8 | 178.7 |
|  |  | Fair | 39.4 | 69.7 | 203 | 121 | 172.7 | 163.6 |
|  |  | Poor | 24 | 54.5 | 187.8 | 106 | 157.5 | 148.5 |

Table 8. Transportation costs; $\mathrm{T}_{i c t}^{n}$ (Dollar/unit), and Demand data; $\mathrm{DE}_{i c t}^{n}$ (ton).

| Item | Customer | Scenario | T2 ${ }_{\text {ict }}^{n}$ Period |  |  |  |  |  | $\mathrm{DE}_{\text {ict }}^{n}$ Period |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 1 | Boom | 151.5 | 454.5 | 454.5 | 545 | 454.5 | 606 | 6 | 6 | 8 | 11 | 9 | 10 |
|  |  | Good | 121 | 424 | 424 | 515 | 424 | 575.7 | 5 | 5 | 7 | 10 | 8 | 9 |
|  |  | Fair | 91 | 394 | 394 | 484.8 | 394 | 545 | 4 | 4 | 6 | 9 | 7 | 8 |
|  |  | Poor | 60.6 | 363.6 | 363.6 | 454.5 | 363.6 | 515 | 3 | 3 | 5 | 8 | 6 | 7 |
|  | 2 | Boom | 212 | 424 | 424 | 636 | 212 | 848 | 4 | 4 | 5 | 11 | 3 | 6 |
|  |  | Good | 181.8 | 394 | 394 | 606 | 181.8 | 818 | 3 | 3 | 4 | 10 | 2 | 5 |
|  |  | Fair | 151.5 | 363.6 | 363.6 | 575.7 | 151.5 | 787.8 | 2 | 2 | 3 | 9 | 1 | 4 |
|  |  | Poor | 121 | 333 | 333 | 545 | 121 | 757.6 | 1 | 1 | 2 | 8 | 0 | 3 |
| 2 | 1 | Boom | 151.5 | 454.5 | 454.5 | 545 | 454.5 | 606 | 4 | 10 | 3 | 7 | 3 | 4 |
|  |  | Good | 121 | 424 | 424 | 515 | 424 | 575.7 | 3 | 9 | 2 | 6 | 2 | 3 |
|  |  | Fair | 91 | 394 | 394 | 484.8 | 394 | 545 | 2 | 8 | 1 | 5 | 1 | 2 |
|  |  | Poor | 60.6 | 363.6 | 363.6 | 454.5 | $1.2 \times 10^{7}$ | 515 | 1 | 7 | 0 | 4 | 0 | 1 |
|  | 2 | Boom | 212 | 424 | 424 | 636 | 212 | 848 | 3 | 5 | 7 | 7 | 4 | 7 |
|  |  | Good | 181.8 | 394 | 394 | 606 | 181.8 | 818 | 2 | 4 | 6 | 6 | 3 | 6 |
|  |  | Fair | 151.5 | 363.6 | 363.6 | 575.7 | 151.5 | 787.8 | 1 | 3 | 5 | 5 | 2 | 5 |
|  |  | Poor | 121 | 333 | 333 | 545 | 121 | 757.6 | 0 | 2 | 4 | 4 | 1 | 4 |

in the form of Tables 6-8. Then, proposed model is developed using these data. Therefore, a multiobjective stochastic mathematical model is formed and is solved using three solution approaches, separately. Also, the amount of the First-Stage and the Second-Stage variables of the two-stage stochastic programming model was determined using three solution approaches. The proposed model was analyzed by all three methods (Goal Attainment, Modified $\varepsilon$-constraint, and STEM) in four different scenarios (Boom, Good, Fair and Poor) and its results were discussed. Finally, stochastic programming model are analyzed using the results of the best solution procedure and the results are presented. Also, the methods are prioritized due to the results and the most effective method is selected to solve the problem.

LINGO software is used to code and solve the proposed stochastic APP problem. For all three methods, the first objective function is considered as the main objective function. The preference of DM on priority of objective functions are asked in form of weights of goals: $w_{1}=0.66, w_{2}=0.14$, and $w_{3}=0.20$. In the beginning, we make the pay-off table, which is shown in Table 9. According to payoff table, minimum and maximum values for objectives are determined as $Z_{1} \in[576367.27,697000]$, $Z_{2} \in[0,1428]$, and $Z_{3} \in[0,1515.15]$. The total run
time and iteration of stochastic mathematical model are $1146(\mathrm{~s})$ and 47077, respectively.

The results of Goal Attainment technique (GA), modified $\varepsilon$-constraint method ( $\mathrm{M} \varepsilon-C$ ), and STEM method are presented in Table 10, Table 11, and Table 12, respectively.

The amount of the First-Stage variables (FSv) and Second-Stage variables (SSv) of the two-stage stochastic programming model for the APP problem are shown in Table 13. The second-stage variables in Goal Attainment method have the lowest amount in comparison with other methods.

The total cost (Z1), the deviation of delivery time and due dates (Z2), and the downside risk (Z3) of all scenarios for all solution methods are presented in Table 14. Also, the total computational time and iterations to solve the problem are shown in Table 14.

Table 9. The payoff table of stochastic programming problem.

| Optimal solution | Z 1 (dollar) | Z 2 (hours) | Z ( dollar) |
| :--- | :---: | :---: | :---: |
| $Z_{1}^{*}$ | 576367.27 | 718.4 | 1515.15 |
| $Z_{2}^{*}$ | 633167.57 | 0 | 151.28 |
| $Z_{3}^{*}$ | 697000 | 1428 | 0 |
| Goal value (Gv) | 576367.27 | 0 | 0 |

Table 10. The results of Goal Attainment technique (GA).

| Weights |  |  | Objective Values |  |  | $\gamma$ | CPU time (second) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $w_{1}$ | $w_{2}$ | $w_{3}$ | $Z_{1}$ | $Z_{2}$ | $Z_{3}$ |  |  |
| 0.66 | 0.14 | 0.20 | 576367.27 | 288.21 | $2.03 \mathrm{e}-2$ | 3353.67 | 1053 |

Table 11. The results of modified $\varepsilon$-constraint method (M $\varepsilon-C$ )

| Right-hand sides |  | Slack Variables |  | Objective Values |  |  | $\theta$ | $\varphi$ | CPU time <br> (second) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\varepsilon_{2}$ | $\varepsilon_{3}$ | s2 | s3 | $Z_{1}$ | $Z_{2}$ | $Z_{3}$ |  |  |  |
| 1428 | $4.1123 \times 10^{-8}$ | 1312.83 | 0 | 576367.27 | 115.16 | $4.11 \times 10^{-8}$ | $10^{-6}$ | 578551.81 | 1049 |

Table 12. The results of STEM method.

| Relative importance |  |  | Objective Values |  |  | $\gamma$ | CPU time (second) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\rho 1$ | $\rho 2$ | p3 | $Z_{1}$ | $Z_{2}$ | $Z_{3}$ |  |  |
| 0.08 | 0.46 | 0.46 | 576367.27 | 288.21 | $4.29 \times 10^{-10}$ | 188.95 | 1063 |

Table 13. First and Second-Stage variables of stochastic programming.

| Methods | FSv | SSv |
| :--- | :---: | :---: |
| Goal Attainment | 575697.88 | 699.34 |
| Modified $\varepsilon$-constraint | 575697.88 | 2853.94 |
| STEM | 575697.88 | 699.37 |

Table 14. The results of solution procedures for stochastic programming model.

|  |  | Method |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Scenarios | Goal Attainment | Modified $\varepsilon$-constraint | STEM |
| Z1 | Boom | 575990.90 | 576430.30 | 576430.30 |
|  | Good | 575898.48 | 580733.03 | 576365.75 |
|  | Fair | 575819.39 | 579582.72 | 576304.54 |
|  | Poor | 575752.72 | 578432.42 | 576244.54 |
| Z2 | Boom | 115.16 | 115.16 | 115.16 |
|  | Good | 86.47 | 0 | 86.47 |
|  | Fair | 57.69 | 0 | 57.69 |
|  | Poor | 28.88 | 0 | 28.88 |
| Z3 | Boom | $4.94 \times 10^{-2}$ | 0 | 0 |
|  | Good | $1.16 \times 10^{-3}$ | $5.60 \times 10^{-8}$ | $9.87 \times 10^{-10}$ |
|  | Fair | $4.64 \times 10^{-4}$ | $9.54 \times 10^{-8}$ | $7.14 \times 10^{-10}$ |
|  | Poor | $9.28 \times 10^{-4}$ | $5.22 \times 10^{-8}$ | $7.24 \times 10^{-10}$ |
| Run Time (s) |  | 1053 | 1049 | 1063 |
| Iterations |  | 39414 | 39380 | 41079 |

Since the problem has multiple objectives, the appropriate solutions are selected by decision makers regarding to the different scenarios and the importance of each goal in desired scenario, the amount of objective functions in each method and each scenario are shown in Table 14. It can be concluded from Table 14 that the results of STEM method and Modified $\varepsilon$-constraint method during Boom scenario is the same, approximately. But in other scenarios, the results for each method are different. So, decision makers select the effective solution with regard to his/her attitude toward nature states. Also, the run time, total number of iterations, total number of variables, and total number of iterations are approximately equal for all solution approaches. The resultant model size for each solution approach is presented in Table 15.

### 5.3. Comparing the results

We compare the results of proposed methods for all scenarios in Figure 1. With regard to the Future economic scenarios and the results that obtained in Table 14 and Figure 1, decisions are made as following scenarios based on attitude of decision maker toward status of the nature.

In Figure 1, the objective function values of solution approaches for different scenarios are presented, separately. According to the Figure 1, for all scenarios; (I) the best solution for minimizing the expected total cost of supply chain is the Goal Attainment method, (II) the best solution for minimizing the deviation of delivery time and due dates is the Modified $\varepsilon$-constraint method, and (III) the best solutions for minimizing the supply chain downside risk are Modified $\varepsilon$-constraint and STEM methods. Therefore, adaptation between the results of solutions and company's strategic is very important for decision makers.

The best solutions found using all approaches are presented compared with existing empirical model (EM) and goal value (GV) in Table 16. They are also depicted in Figure 2.

In Table 16, the results of all solution procedures are compared. Since the stochastic APP problem is a multi-objective problem with conflicting objectives, the results of the value of objective functions for each method are also proposed. As previously

Table 15. Model size for the case study problem.

|  |  | Goal Attainment | Modified $\varepsilon$-constraint | STEM |
| :---: | :---: | :---: | :---: | :---: |
| Number of Variables | Total | 907 | 929 | 907 |
|  | Nonlinear | 188 | 184 | 190 |
|  | Integers | 114 | 114 | 114 |
| Number of Non-zeros | Total | 6873 | 8133 | 7735 |
|  | Nonlinear | 474 | 468 | 474 |
| Number of Constraints | Total | 1125 | 1327 | 1124 |
|  | Nonlinear | 13 | 7 | 10 |

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Figure 1. Comparing the objective function values for different scenarios.
stated, the objective value of each solution approach is different; (I) In Goal Attainment method, the attainment element value (the term $v$ ) is 3353.67, (II) The value of objective function in Modified $\varepsilon$-constraint method ( $\varphi$ ) is 578551.81, (III) The value of the associated mini-max variable ( $\gamma$ ) in the STEM method is 188.95 . These are shown in column "objective value" in Table 16. The value of Z1, Z2, and Z 3 are the objective functions of original supply chain network design problem. It can be concluded form content of Table 16 that the achieved solution
by proposed methods dominates the empirical model currently are used in the company.

As mentioned, the proposed methods, empirical model, and goal value are also compared in Figure 2. Figure 2 shows that the total costs of supply chain, the deviation of delivery time and due dates, and the supply chain downside risk are all better in the solutions proposed by the Goal Attainment (GA), Modified $\varepsilon$-constraint ( $\mathrm{M} \varepsilon-C$ ), and STEM methods. Given the Figure 2, for the first objective (total costs of supply chain) the results of Goal Attainment (GA)

Table 16. General Comparison of Results.

| Methods | Solution Procedure | Objective Functions |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Objective value | Z | Z | $\mathrm{Z}_{3}$ |
| Proposed Model | Goal Attainment (GA) | 3353.67 | 576367.27 | 288.21 | $2.03 \mathrm{e}-2$ |
|  | Modified $\varepsilon$-constraint ( $\mathrm{M} \varepsilon-C$ ) | 578551.81 | 578551.81 | 115.16 | $4.11 \mathrm{e}-8$ |
|  | STEM | 188.95 | 576367.27 | 288.21 | $4.29 \mathrm{e}-10$ |
| The Empirical Model (EM) |  |  | 668585.75 | 409.26 | 987 |
| Goal value (GV) |  |  | 576367.27 | 0 | 0 |



Figure 2. Comparing the achieved objective functions.
and STEM methods are recommended. For the second objective (deviation of delivery time and due dates) the results of Modified $\varepsilon$-constraint ( $\mathrm{M} \varepsilon-C$ ) method are suggested. Also, for the third objective (supply chain downside risk) all methods have good results, but we propose STEM, $\mathrm{M} \varepsilon-C$, and Goal Attainment (GA), respectively due to increasing order of third objective function reported in Table 16.

The solution approaches are prioritized due to the obtained results for multi-objective stochastic programming model in Table 17. For each objective functions, methods are prioritized. The STEM method provides a production plan with the lowest total production cost, and lowest down-side risk among the other approaches. Also, the best satisfaction level is achieved by Modified $\varepsilon$-constraint method.

In order to achieve an appropriate solution approach, the decision-maker needs to see this range of outcomes, to be able to trade-off one goal against the other in terms of the results. So, by solving the proposed stochastic APP problem, it is concluded that the relationship between total costs of supply chain, deviation of delivery time and due dates, and supply chain downside risk is not clear and it is not possible to easily define a utility function. That is why three multi-objective approaches (i.e., GA, $\mathrm{M} \varepsilon-C$, and STEM) were adopted to this stochastic APP problem. Indeed, these methods were selected from three major categories (the a priori, the a posteriori, and the interactive methods) to better understand the problem and to be able to cover the broader area of solutions for decision makers.

It seems that by increasing scenarios, the results of STEM and Modified $\varepsilon$-constraint methods are to be similar, approximately. Also, by increasing the goal for each objective, a wider space for other objectives can be achieved. It is notable that although several solution procedures were applied on the proposed problem and several solutions were achieved, but the final decision is highly related to the attitude of decision maker toward state of the nature.

### 5.4. Research Findings

Optimistic Manager. An optimistic decision maker expects for the Boom scenario. Under the Boom scenario; and based on relative importance of objective functions in our case study ( $w_{1}=0.66$, $w_{2}=0.14$, and $w_{3}=0.20$ ) the total cost of supply chain has the best value in the proposed Goal Attainment method. So, the production plan of the supply
chain is aligned with the outputs of goal attainment method in this case. It is notable that, under the Boom scenario, the results of the second objective function, for all methods are similar. The third objective function of goal attainment method, under the Boom scenario, is also tolerable in comparison with the other methods. Finally, it can be conclude that, under Boom scenario, the goal attainment method is preferred.

Neutral Manager. A neutral decision maker gives expects for the Fair scenario. Again, under the Fair scenario; and based on relative importance of objective functions in our case study ( $w_{1}=0.66$, $w_{2}=0.14$, and $w_{3}=0.20$ ) the total cost of supply chain has the best value in the proposed Goal Attainment method. Although, it is notable that, for the second objective function under the aforementioned situation, the modified Modified $\varepsilon$-constraint method presents the best results. It is notable that, under the Fair scenario, the results of the third objective function, for all methods are approximately similar. Finally, it can be concluded that, under Fair scenario, and considering the great relative importance of first objective function, the goal attainment suggests the most suitable production plan.

Pessimistic Manager. A neutral decision maker gives expects for the Poor scenario. Again, under the Poor scenario; and based on relative importance of objective functions in our case study ( $w_{1}=0.66$, $w_{2}=0.14$, and $w_{3}=0.20$ ) the total cost of supply chain has the best value in the proposed Goal Attainment method. Although, it is notable that, for the second objective function under the aforementioned situation, again the modified Modified $\varepsilon$-constraint method presents the best results. It is notable that, under the Poor scenario, the results of the third objective function, for all methods are approximately similar. Finally, it can be concluded that, under Poor scenario, and considering the great relative importance of first objective function, the goal attainment suggests the most suitable production plan again.

The following points are achieved based on implementation of proposed approach in real case study.

All production plans, provided by proposed methods of this study, dominate the existing experimental method of planning the case study.

No hiring and firing is suggested for the next six periods.

The overtime-shift is just used when the regular time cannot satisfy the demands, or handling the training programs.

The model suggests training for some periods of planning.

The cross-functional quality teams are formed in order to learn new techniques.

The performance and experiments of workers are shared during implementation phase.

The employee engagement with the new production plan is measured frequently to feedback the success of plan.

All workers are trained based on their performance records in order to enhance the learning curves which influences the production time and quality.

The customer satisfaction level is measured frequently through a survey questionnaire about quality of delivery, quality of product, and time of delivery.

The above issues will help the execution team to check whether the plan is implemented correctly or a deviation is occurred.

The main results include: (1) the huge costs of hiring and firing were reduced through a proper plan proposed by the approach of this study, (2) although the training courses imposed costs to company, but its positive effects on the timely delivery of products and learning curves of workers was illustrated, (3) the total cost of the system was increasingly reduced in comparison with the experimental production plan, (4) the satisfaction degree of customers was increased amazingly, (4) the down-side risk of supply chain was decreased through proposed approach.

## 6. Conclusions Remarks

In this paper a new aggregate production planning problem in supply chain considering financial risk, customer satisfaction, and training was proposed. The expected total cost as the main objective function, the customer satisfaction level, and the downside risk of supply chain were considered as objective function, simultaneously. Several constraints regarding workforce level, inventory level, production capacity, backorder, and risk were also considered. The proposed problem was modeled using mixed integer multi-objective mathematical model. Then, a new multi-objective stochastic optimization approach was developed to handle this problem. The uncertainty of real-world problem was represented by a set of discrete scenarios with given probability of occurrence. A priori method, a posteriori method, and an interactive method were developed and adapted to solve the proposed multiobjective stochastic mathematical model.

A real case study of color and resin Company called Teiph-Saipa, which produced several products was selected as a practical environment in order to test the suitability and applicability of proposed model and solution approaches. The proposed models were coded in LINGO software. The results of three methods were compared and analyzed. The results of all approaches were compared with those of experimental method in this case study. All methods dominate the existing experimental method. Moreover, some decision making scenarios based on decision maker's attitude toward uncertainty in nature was developed and discussed. The proposed model of this study can be improved by adding constraints on skill level of workers and their learning curves. The outsourcing option can be considered in the modelling procedure, since many companies may outsource some of the operations. The effects of inflation and discounts may be considered the longterm planning. The failure of facilities and machines, and the rate of products returned from customers can also be considered in future research works.

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