Predicting Intra-Day Load Profiles Under Time-Of-Use Tariffs Using Smart Meter Data

Kiguchi Y[a], Heo Y[b], Weeks M[c], Choudhary R[a]

[a] *Department of Engineering, University of Cambridge, UK*
[b] *School of Civil, Environmental, and Architectural Engineering, Korea University, Korea*
[c] *Faculty of Economics and Clare College, University of Cambridge, UK*

Abstract

The installation of smart meters enabling electricity load to be measured with half-hourly granularity provides an innovative demand-side management opportunity that is likely to be advantageous for both utility companies and customers. Time-of-use tariffs are widely considered to be the most promising solution for optimising energy consumption in the residential sector. Although there exists a large body of research on demand response in electricity pricing, a practical framework to *forecast* user adaptation under different Time-of-use tariffs has not been fully developed. The novelty of this work is to provide the first top-down statistical modelling of residential customer demand response following the adoption of a Time-of-use tariff and report the model's accuracy and the feature importance. The importance of statistical moments to capture various lifestyle constraints based on smart meter data, which enables this model to be agnostic about household characteristics, is discussed. 646 households in Ireland during pre/post-intervention of Time-of-use tariff is used for validation. The value of Mean Absolute Percentage Error in forecasting average load for a group of households with the Random Forest method investigated is 2.05% for the weekday and 1.48% for the weekday peak time.

Keywords: Time-of-use pricing, Demand-side management, Smart meters, Electricity consumption modelling, Load shifting, Residential electricity demand

List of Tables

List of Figures

Nomenclature

t	Continuous Time index in a day
τ	Discrete Time slot index in a day
$\mathbf{c} = \{c_{d,\tau}\}$	Matrix which represents the consumption at time stamp τ of d^{th} day
C_τ	Consumption at time stamp τ over days
$\bar{\mathbf{C}} = \{\bar{C}_\tau\}$	Average half-hourly consumption on previous tariff
$\tilde{\mathbf{C}} = \{\tilde{C}_\tau\}$	Average half-hourly consumption on a new tariff
$\hat{\mathbf{C}} = \{\hat{C}_\tau\}$	Predicted average daily consumption on a new tariff
$\bar{\mathbf{C}}^{avg}$	Average $\bar{\mathbf{C}}$ in the group
$\tilde{\mathbf{C}}^{avg}$	Average $\tilde{\mathbf{C}}$ in the group
$\hat{\mathbf{C}}^{avg}$	Average $\hat{\mathbf{C}}$ in the group
$\mu_n(c_\tau)$	The nth statistic moment of consumption at time stamp τ
R_{peak}	Average consumption reduction over peak time (Wh)
$\mathrm{R}_{peak}(W)$	Average power reduction over peak time (W)
MAPE	Mean Absolute Percentage Error
MAPE_g	MAPE in the group
$\mathrm{MAPE}_{g,peak}$	MAPE in the group over peak time
$\mathrm{APE}_{g,peak}(W)$	Absolute Percentage Error in the group over peak time (W)
LR	Linear Regression
DT	Decision Tree
NN	Neural Network

1. Introduction

Residential energy sectors worldwide are facing the emerging development of smart meters combined with better techniques for streaming and processing large volumes of metering data into useful information. The ongoing roll-out of smart meters provides a clearer impetus for increasing policy support of demand-side management (DSM) solutions than ever before [1]. Time-of-use (TOU) tariffs, also known as time-dependent pricing, are a DSM solution wherein the price of electricity varies depending upon the time of the day and the day of the week. The tariff structure is designed to yield potential price savings for the end-user, and targets peak electricity load shifting so as to constrain the electricity load on a given sub-station. Studies have shown that TOU tariffs can be particularly effective in the residential sector, as they offer a more certain financial incentive to customers than other more complex price-based DSM programs such as real time pricing [2].

Alongside smart meter installation, by 2020 TOU tariffs will become available in most of the EU, United States, Japan, and Australia [3]. The success of TOU tariffs as a DSM solution depends upon consumers changing the timing of their energy demand based on a given tariff structure. Recently, large-scale longitudinal studies have been conducted to evaluate the behavioural change of residential customers pre- and post-intervention of a TOU tariff. The Customer-Led Network Revolution (CLNR) project in UK [4] confirmed load shifting from peak to off-peak periods throughout a two-year trial with 576 households. Torriti [5] monitored 1446 households in Northern Italy and showed that while TOU tariffs result in a significant level of load shifting in the morning, the evening peaks do not change and overall consumption is in fact increased by 13.69%. Faruqui and Malxo [6] examined 15 pilot projects that showed TOU rates induce a drop in peak demand that ranges between 3% and 6%. Wang and Li [7] reported that potential peak reduction in the residential sector is much smaller than that of the commercial and industrial sector based on Federal Energy Regulatory Commission survey [8]. On the other hand, Faruqui et al. [9] concluded that residential customers are more price responsive than small business customers. They also examined estimates of the price elasticity of demand across 42 different TOU studies, and found a positive relationship between peak to off-peak price ratio and peak reduction. Gils [10] confirmed that household consumers hold a large potential load reduction in most European countries.

To date a large scale implementation of TOU tariffs has not taken place. In recent years, researchers have developed modelling frameworks which can allow utility companies to exploit smart meter data in order to predict potential load shifts when designing TOU tariff structures. These modelling frameworks generally follow one of the three distinct approaches: econometric models with an emphasis on estimating price-elasticity, bottom-up dis-aggregation of household consumption according to electrical appliances and their time of use, and top-down statistical models.

A classical approach estimates price-elasticities of demand distinguishing between the elasticity of demand due to price changes of the good itself (own-price elasticity) and of other goods (cross-price elasticity). The own-price elasticity provides an estimate of the percentage change in usage during a particular period (i.e. day or billing period) that results from a 1% change in price during that period. The cross-price elasticity provides estimates of the elasticity of substitution between peak, mid-peak and off-peak periods. For example, Kirschen et al. [11] modelled consumer behaviour using a matrix of own-price and cross-price elasticities and showed the effect of market structure on the elasticity of the demand for electricity. Goel et al. [12] modelled customer response using the matrix of own/cross-elasticities, based on the assumption of constant elasticities. Venkatesan et al. [13] emphasised the importance of distinguishing between different consumer types considering different scenarios and levels of consumer rationality. Recently, Katz et al. [14] employed a similar approach to evaluate load-shift incentives for household demand response, comparing the effects of hourly pricing and a simple rebate scheme. An advantage of these approaches follows from the assumption that price-elasticities of demand are scale free, and under certain assumptions, are applicable out of sample. However, most of these studies have been conducted prior to the roll-out of smart meters. The integration of this classical approach with newly available energy big data is a new challenge in this field.

An alternative approach considers the potential for load shifting at the level of individual appliances, utilising information on the dis-aggregation of total household consumption according to electrical appliances (or activities). The advantage of this approach is the ability to identify the primary cause of load variation by associating load-shifting with appliances. For example, Armstrong et al. [15] computed consumption profiles for different types of activities based on publicly available data on energy use. Gottwalt et al. [16] applied this to evaluate the capability of residential load shifting when smart appliance and TOU tariffs are applied. Shao et al.[17] proposed a physics-based residential load model at the appliance level based on controllable load such as space cooling/heating, water heater, clothes dryer, and electric vehicles for demand response modelling. McKenna et al. [18] also constructed a bottom-up demand response model, combining multiple models such as hot water demand model and thermal appliances. Xu et al.[19] explicitly acknowledged variability across consumer response by applying a shifting boundary to limit the maximum loaf-shift in certain groups of customers.

A disadvantage of this approach is the requirement of ex-ante identification of demographic or appliance variables or the installation of additional sensors to record activities that influence consumption. This is difficult to access without dedicated and costly studies and must be continuously updated as household appliances and activities change. This point is also emphasised by Armel et al. [20] recommending a 1-minute to 1-second data frequency to infer the usage of key appliances.

The last framework is a top-down approach based on the use of statistical models using consumption data. A top-down approach usually treats the load at an aggregated level and does not distinguish energy consumption due to individual consumer or any appliances (see Swan and Ugursal [21]). The strengths of a number of top-down models is the emphasis on historical energy consumption which is indicative of the expected pace of change with regards to energy consumption. This approach is used widely by utility companies to forecast future energy demand. With the continued fall in computation costs, non-linear techniques such as Decision Trees (DT), Artificial Neural Networks (ANN), and Support Vector Machines(SVM) have been used for medium-term electric load forecasting (see Hahn et al. [22] and Hernandez et al [23]). However, the reliance on historical data can have a number of drawbacks given that there is no in-built capability to model discontinuous user behaviour such as weather changes, introduction of new appliances, and an adoption to a TOU tariff. Therefore applying a top-down approach for a problem of forecasting consumer response to TOU tariffs is a challenging issue.

This paper presents a new top-down framework for predicting load profiles following the introduction of a TOU tariff, using demographics and historical electricity usage in the pre-intervention period. The influence of a TOU tariff introduction is usually evaluated with a time-horizon of one month to one year, comparing electricity consumption during pre/post introduction. In terms of the time-horizon, this work falls into the category of medium-term load forecasting. This paper emphasises that this forecasting problem differs from standard medium-term forecasting models given the need to account for consumer behaviour adaptation to a TOU tariff.

The accuracy of a prediction model plays a vital role in making important decisions on the operation and planning of a power utility system [24]. The measure most frequently used to assess the accuracy of a medium-term load forecasting model is Mean Absolute Percentage Error (MAPE) [22]. However, current studies in TOU prediction do not report the accuracy of their models in a standard form. Without such a standardised measure of accuracy, it is difficult to compare the performance of the competing TOU models.

Although there exists a large body of research on demand response in electricity pricing, a practical framework to forecast user adaptation under different TOU tariffs has not fully developed. The novelty of this work is to provide the first data driven modelling of residential customer demand response following the adoption of a TOU tariff. In particular, this study evaluates the importance of lifestyle constraints which are constructed using statistical moments based on historical usage. The question as to the relative importance of demographic information and historical load profiles in the context of forecasting the impact of TOU on demand response, is of considerable interest to both companies and policymakers.

The key contributions of this paper are summarised below.

1. The first top-down statistical model designed to *forecast* residential customer demand response following the adoption of a TOU tariff, and evaluate its predictive performance accuracy using MAPE.
2. The first model to explicitly include lifestyle constraints influencing user adaptation to a TOU tariff.

The remainder of this paper is organised as follows. Section 2 introduces the dataset used to develop and test the modelling framework. In Section 3, the dataset and a list of relevant features for model development are described. Relevant statistical techniques and the accuracy of measurements are discussed in the Section 4. In Section 5, a number of summary measures demonstrate the predictive accuracy, comparing with other studies. The final Section concludes the paper with limitations and future works. This paper focuses exclusively upon *active* demand response due to behaviour adaptation. As such it is assumed that no automated energy storage is present in the sample households.

2. Data

The dataset used in this study is taken from the Electricity Smart Metering Customer Behaviour Trial carried out by the Irish Commission for Energy Regulation (CER) [25]. This dataset consists of half hourly observations for a total of 4232 households, with a benchmark period of approximately 6 months and a trial period of one year. During the trial period households were randomly allocated to one of the four TOU tariffs (TOU-A, TOU-B, TOU-C and TOU-D) along with billing information as an incentive for load shifting. CER also collected demographic information via questionnaires to research participants, such as: gender, socioeconomic classification, age group, income level, list of appliances, internet access, number of other residents, housing type, employment status, owner/tenant, education level etc.

Four major demographic features are used in this research: gender, age group, social class, and number of other residents. In this study, the subset of 646 TOU tariff participants (households) with the same level of DSM stimuli (bimonthly bill, energy usage statement and electricity monitor) are used. Similarly, the subset of 929 households who are not assigned to any TOU tariffs are used as a *control group*.

It is possible that a weather effect might impact the differences in load profiles across the pre/post observation periods. In this regard the utilisation of a *control group*, who by definition are no different from the TOU group (apart from facing a constant flat tariff), can be used to isolate any confounding effect of this type.

In order to further rule out the impact of other changes, such as a change in consumption behaviour on our estimate of the impact of the introduction of TOU tariffs, a subset of the data around the TOU introduction on 1st January 2010 is used. Specifically, two periods of one month each are considered; December 2009, and January 2019. These make it possible to analyse the pre/post-intervention effect. By limiting the period to just two months (one month pre/post the introduction of TOU tariffs), any seasonality effect should be minimised, making it easier to isolate the demand response due to the introduction of the TOU tariff.

The structure of TOU tariffs and the flat-rate tariff are presented in Table 1. TOU tariffs A, B, C and D have three different rates within a given weekday, and two different rates within a non-weekday (weekend and bank holidays). Peak ratios are calculated by taking the ratio of the peak time rate of weekday and non-weekday respectively to the original flat tariff rate 14.10. Household assigned to the *control group* remain on the flat tariff during the post-intervention period.

Tariff	Flat (Control)	TOU-A	TOU-B	TOU-C	TOU-D
Off-peak	14.10	12.00	11.00	10.00	9.00
Mid-peak	14.10	14.00	13.50	13.00	12.50
Peak	14.10	20.00	26.00	32.00	38.00
Weekday peak ratio	1.00	1.42	1.84	2.27	2.70
Non-weekday peak ratio	1.00	0.99	0.96	0.92	0.89

Table 1: Flat-rate tariff and the four TOU tariffs are used in the CER study. The values are cents per kWh. Off-peak (23:00-08:00), Mid-peak (weekdays 08:00-17:00, 19:00-23:00, non-weekdays 17:00-19:00), Peak (17:00-19:00).

3. Modelling Framework

An estimate of the impact of the TOU tariffs is obtained by comparing, for each household, the historical load profile generated under a flat tariff (pre-intervention) and the load profile under the TOU tariff (post-intervention). Although this is of interest in itself, companies and regulators will only observe historical load profiles at the point in which TOU tariffs are actually introduced into the market. In this sense the key objects that are of interest are the forecasts of the load profiles once households are offered TOU tariffs, particularly in the peak periods for weekdays, where it matters most to both consumers and energy suppliers.

Figure 1 summarises the key steps of the proposed modelling. Two types of input data (historical smart meter data, and demographic data) are initially prepared. The smart meter data over 62 days (December 2009 and January 2010) recorded at half-hour intervals is split into weekday and non-weekday subsets as the structure of TOU tariffs is different in the given dataset.

Once data is prepared, the statistical model is created, starting from the feature extraction. Given 646 households on one of the four TOU tariffs with 48 half-hourly electricity consumption, which are aggregated over the periods, there are 31008 data samples= 646×48 half-hour electricity consumption as input samples for the models. A description of a data sample for one household is given in Table 4. The outputs from the statistical model are validated using cross-validation. More details are explained in the Section 4.1.

The output generated from the model is an intra-day predicted load curves for each household assigned to a TOU tariff. This represents average consumption for each of the half-hour intervals averaged over the prediction period. By utilising error metrics such as MAPE with the unseen test data, model performance is evaluated (see 4.2).

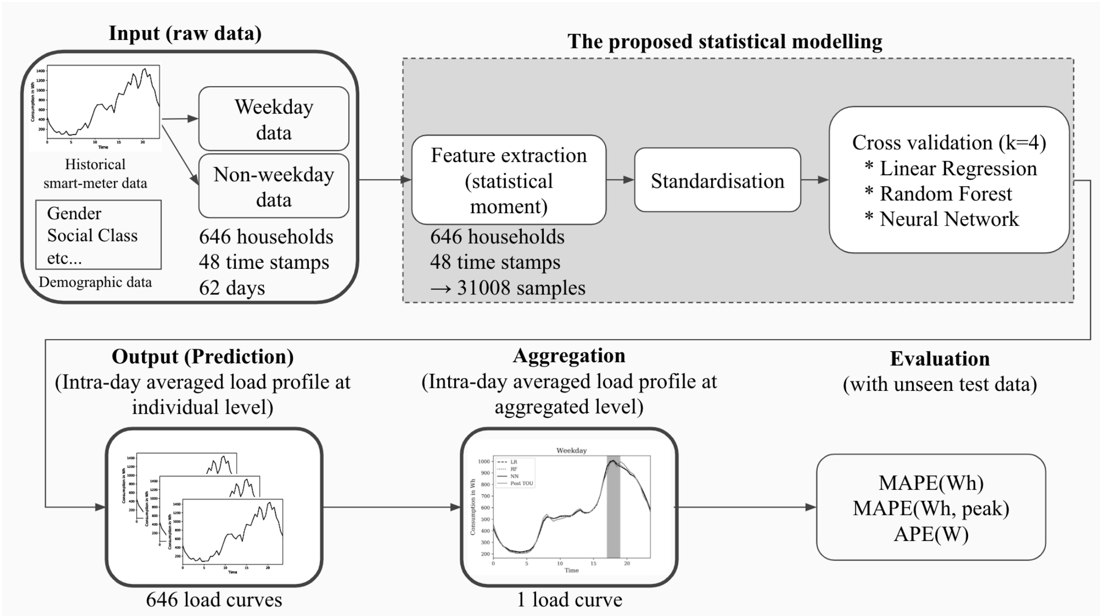

Figure 1: Each step of the proposed model from the input to the performance evaluation.

3.1. Historical consumption

At time t a load history, say $c_t, c_{t-1}, c_{t-2}, \ldots$, is observed, where a given element, say c_j, denotes a meter reading recording the total energy consumed in a given time interval (ie. half-hour). This time series of meter readings is then collected in a $D \times n$ matrix $\mathbf{c} = \{c_{d,\tau}\}$, where $\tau = 1, ..., n$ indexes the time stamps for the d^{th} day. C_τ is a $D \times 1$ vector that contains the readings for the τ^{th} time stamp collected over D days. If $\mathbf{c}$ is averaged along the columns then $\bar{\mathbf{C}} = \{\bar{C}_\tau\}$ is a $n \times 1$ vector, representing the average intra-day shape of the electricity demand curve under a flat tariff. $\tilde{\mathbf{C}}$ denotes a comparable object for a TOU tariff, where prices vary over the τ intervals. The consumption level during the τ^{th} interval is considered as a random variable C_τ, with $\mathbf{C} = \{C_\tau\}$ a $N \times 1$ random vector.

In Figure 2, two plots extracted from the dataset are presented. The first presents the half-hourly consumption corresponding to five consecutive weekdays of electricity usage for a given customer; each dot represents the consumption measured in a given half-hour. The second plot shows the historical load profile, $\bar{\mathbf{C}}$ ($D = 5$), representing the average half-hour consumption over the same five weekdays.

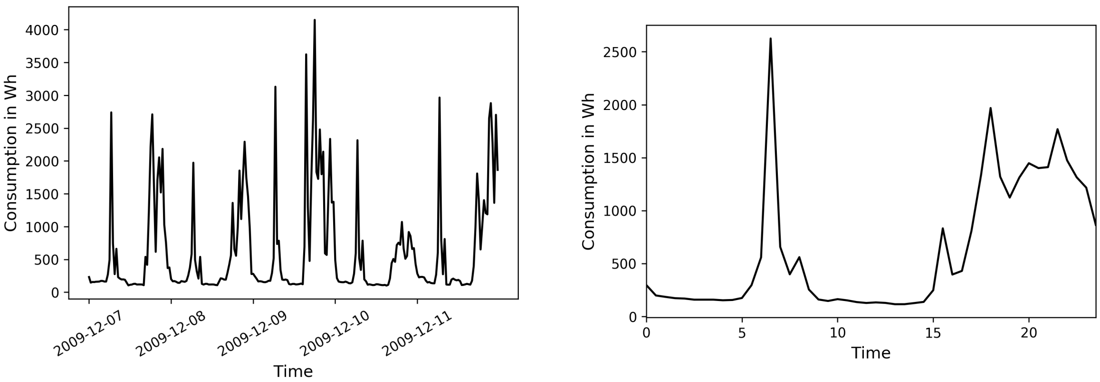

Figure 2: Left: half-hourly electricity consumption for five consecutive days for a particular user. Right: Average intra-day electricity demand curve over the same five weekdays.

3.2. Statistical description of lifestyle

Making inference on consumer lifestyle patterns using smart meter data is a critical component of energy efficiency programs and other services. Recently, Beckel et al. [26] inferred household characteristics from electricity consumption data by using statistical models with a large number of features such as average consumption during different times of the day, features related to temporal dynamics, and the first ten principal components [27]. However, the feature construction processes could be improved accordingly. First, the importance of each feature to the model performance is not examined. Second, some popular metrics in statistics, such as the moments of a distribution, which are frequently used to capture the shape of a distribution (see the study by William et al. [28]), are not used. An advantage in the use of statistical moments is that it compresses all the information contained in the data into a very small number of expressions. This study utilises the first four moments: mean, variance, skewness and kurtosis, measuring their importance for the model performance, and comparing against other demographic characteristics.

For each time stamp τ, the nth moment is derived as

$$\mu_n(c_\tau) = \sum_{d=1}^{D} (c_{d,\tau} - \bar{C}_\tau)^n p_\tau(c_{d,\tau}) \tag{1}$$

where $p_\tau(c)$ is the probability of having consumption c at the time stamp τ. In the context of the residential electricity consumption, each moment indicates a particular aspect of consumer behaviour. In this study, the time stamp τ represents a half-hourly period between midnight ($\tau = 0$) and 11:30pm ($\tau = 47$).

Figure 3 illustrates average consumption $\bar{C}_\tau$ of a single household at four different time stamps ($\tau = 0, 12, 24, 36$) during weekdays at the pre-intervention period (December 2009), and Table 2 presents the values for the moments. $\bar{C}_\tau$ is highest at 18:00, and lowest at midnight 0:00. A high variance $\mu_2(c_\tau)$, indicates that consumption at time stamp τ is relatively unpredictable. As expected, in this example consumption is more variable at 12:00 and 18:00 than during the time interval 0:00−6:00. Skewness $\mu_3(c_\tau)$ is a measure of the lopsidedness of the distribution. A distribution having a longer tail on the right will have a positive skewness. This household has a high value of $\mu_3(c_\tau)$ at 12:00 (see Table 2), reflecting occasional energy-intensive activities at midday such as cooking and laundry. Finally, $\mu_4(c_\tau)$ kurtosis, is a measure of the heaviness of the tail of the distribution, compared to the normal distribution of the same variance is presented. The values at 0:00 and 6:00 are much lower than during the day so that the night-time electricity consumption pattern (most likely sleeping based on low consumption) is consistent. In summary, these features of load profiles provide valuable information about household behaviour based on their own past consumption.

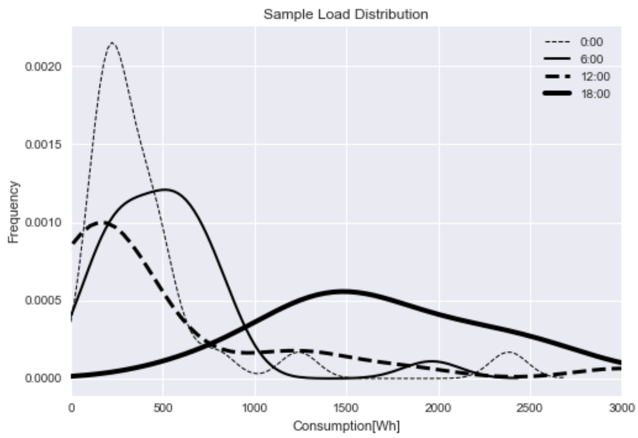

Figure 3: Consumption distribution of a single household at different time stamp τ ($\tau = 0, 12, 24, 36$) during weekdays at the pre-intervention period (December 2009).

Time	0:00	6:00	12:00	18:00
$\bar{C}_\tau$	445	527	689	1870
$\mu_2(c_\tau)$ $[\times 10^5]$	2.30	1.43	10.6	6.72
$\mu_3(c_\tau)$ $[\times 10^8]$	3.32	1.20	24.0	6.51
$\mu_4(c_\tau)$ $[\times 10^{11}]$	6.34	1.91	80.0	18.6

Table 2: Moments of the consumption distribution (in Wh) of a single household at different time stamp τ ($\tau = 0, 12, 24, 36$) during weekdays at the pre-intervention period (December 2009).

3.3. List of Variables

Table 3 summarises the variables used in this analysis. The target variable[1] $\tilde{C}_\tau$ denotes average consumption in Wh under a TOU tariff at time stamp τ, a half-hourly period between midnight ($\tau = 0$) and 11:30pm ($\tau = 47$). Variables $\bar{C}_\tau$, $\mu_2(c_\tau)$, $\mu_3(c_\tau)$, and $\mu_4(c_\tau)$ are used for training and are therefore calculated over the training period (December 2009); $\tilde{C}_\tau$ is averaged over the evaluation period (January 2010).

The variables used for model development are categorised into three feature groups (FG)s depending on the nature of the parameters. The first FG includes the first four statistical moment. Each variable $\bar{C}_\tau$, $\mu_2(c_\tau)$, $\mu_3(c_\tau)$ and $\mu_4(c_\tau)$ are standardised by subtracting the mean from each feature and dividing by its standard deviation. This technique has been confirmed effective in feature selection by Jennifer and Carla [29]. The second FG includes tariff information including rates under the flat and the TOU tariffs, and the peak ratio. The third FG has the four demographic features, and the definitions of the each feature are described in Table 3.

Group	Variable	**Number of possible values**	**Description**
	τ	48	Represents a half-hourly period between midnight (0) and 11:30pm (47)
1	$\bar{C}_\tau$	Continuous	Mean
1	$\mu_2(c_\tau)$	Continuous	Variance
1	$\mu_3(c_\tau)$	Continuous	Skewness
1	$\mu_4(c_\tau)$	Continuous	Kurtosis
2	Flat price	Continuous	Tariff rate given a specific time under flat tariff
2	TOU price	Continuous	Tariff rate given a specific time under TOU tariff
2	Peak ratio	Continuous	A ratio of the peak time rate to the average rate.
3	Age group	5	0:26-35, 1:36-45, 2:46-55, 3:56-65, 4:65+
3	Gender	2	0:Female, 1:Male
3	Socioeconomic classification	6	AB, C1, C2, DE, F, Refused
3	Other living residents	3	0:Only adults, 1:Adults and children, 2:None
	$\tilde{C}_\tau$	Continuous	Average consumption in Wh under a TOU tariff (target variable).

Table 3: List of variables utilised in this analysis.

Table 4 illustrates that the form of a data sample for a given household comprises the variables described in Table 3. Note that $\bar{C}_\tau$, $\mu_2(c_\tau)$, $\mu_3(c_\tau)$ and $\mu_4(c_\tau)$ are standardised and $\tilde{C}_\tau$ is not.

τ	$\bar{C}_\tau$	$\mu_2(c_\tau)$	$\mu_3(c_\tau)$	$\mu_4(c_\tau)$	Flat price	TOU price
13	0.615	0.483	0.078	-0.015	14.1	12.0

Peak ratio	Age group	Gender	Socioeconomic classification	Other living residents	$\tilde{C}_\tau$
1.67	4	0	C2	2	801

Table 4: One sample data of a household.

[1] The target has 48 values since this model is interested in forecasting intra-day load profile at every half-hour point averaging over days.

3.4. Performance Metrics

Cappers et al. [30] used two different performance metrics to evaluate demand response performance: the one compares the actual load reduction to what was initially subscribed to a demand response program, and the other one estimates the customer's actual demand response load curtailment compared to their peak demand. Similarly in physics, *Energy* and *Power* are two major metrics for quantifying the status of electricity: *Energy* is the product of power and time (measured in Watt-hours), and *Power* is the flow of energy at any one time and is measured in Watts (W). Therefore, to indicate the peak reduction in *Energy* and *Power*, the following two metrics R_{peak} and $\mathrm{R}_{peak}(W)$ are used for this paper respectively:

$$\mathrm{R}_{peak} = \frac{1}{n_{peak}} \sum_{\tau \in Peak} \frac{\bar{C}^{avg}_{\tau} - \tilde{C}^{avg}_{\tau}}{\bar{C}^{avg}_{\tau}}, \tag{2}$$

$$\mathrm{R}_{peak}(W) = \frac{\max_{\tau \in Peak} \bar{C}^{avg}_{\tau} - \max_{\tau \in Peak} \tilde{C}^{avg}_{\tau}}{\max_{\tau \in Peak} \bar{C}^{avg}_{\tau}} \tag{3}$$

where $\tilde{C}^{avg}_{\tau} = \frac{1}{M} \sum_{m=1}^{M} \tilde{C}^{(m)}_{\tau}$, M denotes the number of households, $\tilde{C}^{avg}_{\tau}$ is the observed average consumption. n_{peak} is the number of intervals that correspond to peak time ($n_{peak} = 4$).

3.5. Preliminary Analysis

A peak reduction effect could be influenced by a number of factors such as weather conditions, and changes of occupancy behaviour. Limiting the observed period to two months (one month each for pre/post TOU tariff intervention) minimises these external influences. We also compare the load profiles of the *control group* during who remain on the flat tariff in the two periods, to evaluate the potential of these factors to confound our estimate of the impact of the introduction of TOU tariffs on average load profiles.

Figure 4 present the average consumption profiles of TOU and the *control group* during the pre/post intervention period; Table 5 summarises the peak reduction of each group relative to the benchmark period. The electricity consumption of the *control group* remains unchanged during weekdays, and shows a slight increase during the weekends, whereas all TOU groups show the significant peak reduction during the peak time.

An independent t-test is conducted to compare the differences in the consumption between the pre and post observation periods for TOU and the *control group*. As a result, for the TOU group, there was a significant difference in the peak consumption during the pre/post intervention periods ($\mathrm{p}_{\text{value}} = 9.9 \times 10^{-6}$), while there wasn't for the *control group* ($\mathrm{p}_{\text{value}} = 0.91$).

This result concludes that there are no external factors that brings the peak reduction during the observation periods, and the TOU tariff price signals is considered to be the sole factor to realise the peak reduction.

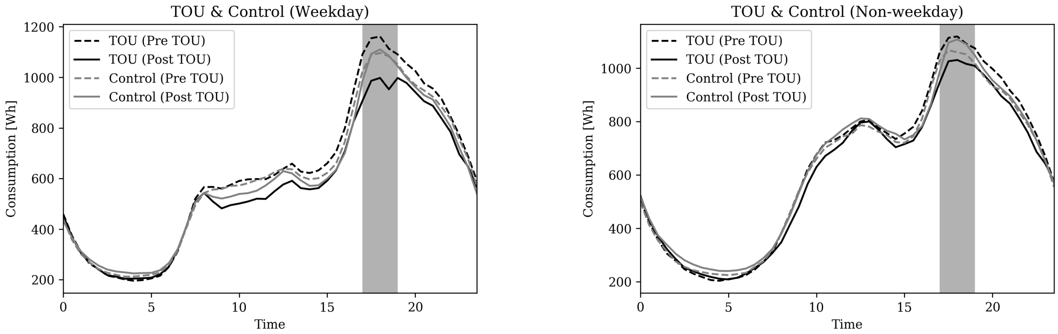

Figure 4: Comparison of average consumption profiles between the TOU group and the *control group*

The extent of load shifting for the TOU tariff participants is generally consistent with the relative magnitude of incentives given by the peak ratios. Customers assigned to TOU tariff D, who are given the highest incentive for load shifting, reveal the largest demand response with a 16.77% energy reduction and 15.79% power reduction during the weekdays. During the non-weekdays the reduction is smaller for all tariffs, and consistent with the relative low peak ratio.

Date type	Metric	Control	TOU-A	TOU-B	TOU-C	TOU-D
Weekday	R_{peak}	-0.3 %	12.41 %	12.10 %	12.46 %	16.77 %
Weekday	$R_{peak}(W)$	-1.3 %	13.43 %	13.13 %	12.72 %	15.79 %
Non-weekday	R_{peak}	-3.4 %	6.31 %	10.76 %	6.59 %	7.11 %
Non-weekday	$R_{peak}(W)$	-3.8 %	7.60 %	12.25 %	8.62 %	7.67 %
	Number of households	929	230	93	233	90

Table 5: Overall mean of peak-load reduction.

Figure 5 observes that the distribution of peak reduction is widely spread at the individual level across the four tariffs; 265 out of 646 households (41.0%) actually increased peak consumption despite the penalised peak rate. This indicates that individual load consumption does not necessarily react to a TOU tariff, although the aggregated load over the same TOU tariff group reacts more rationally to minimise the energy bill (Table 5 shows that all tariff groups achieved both *Energy* and *Power* reduction at the aggregated level). Therefore the impact of TOU tariff should be examined at the aggregated scale.

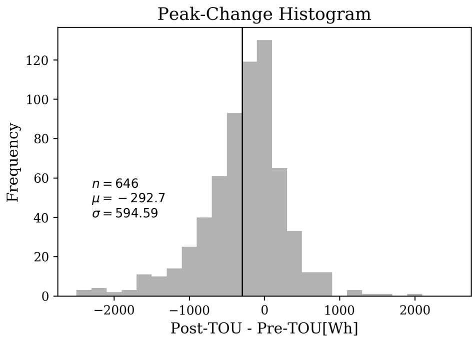

Figure 5: Peak-change at the individual level after an introduction to a TOU tariff.

4. Modelling Techniques

This paper utilises three common predictive modelling techniques for load forecasting: LR, ANN and DT. Predictive modelling seeks to locate rules for predicting the values of one or more variables in a data set (outputs) from the values of other variables in the data set (inputs). These inputs and outputs are the energy consumption data described in Section 2.

The popularity of the LR model may be attributed to the simplicity and interpretability of model parameters. Traditionally this approach has been the most popular modelling technique for utility companies in predicting energy consumption. Ranjan and Jain [31] demonstrated the application of linear regression models of energy consumption for different seasons in Delhi. Similarly, Al-Garni et al. [32] analysed in Eastern Saudi Arabia, and Tsuo and Yau [33] did in Hong Kong.

ANNs somewhat mimic the learning process of a human brain. Instead of complex rules and mathematical routines, ANNs are able to learn the key information patterns within a multidimensional information domain. Kalogirou and Bojic [34] explained the two key advantages of ANNs in the context of energy prediction. First, ANNs operate like a "black box" model, requiring no information about the system, such as functional form. Instead, in keeping with a machine learning approach, the ANN learns the relationship between the input parameters and the controlled and uncontrolled variables by studying historical data. Another advantage is their ability to handle large and complex systems with many interrelated parameters. The success of ANNs is based, in part, on an ability to ignore input data that are of minimal significance and concentrate instead on the more important inputs.

DT is a non-parametric supervised machine learning method which partitions the data into "leaves" defined by covariates in order to estimate the individual outcomes. DT is constructed by recursively splitting the data in order to minimise the mean square error of estimated outcomes. The method can then be used to predict the value of a target variable utilising simple decision rules learned from the data features. The algorithm used in this paper is CART (classification and regression trees) [35].

Model selection, which in the case of decision trees is the partition that defines the tree, and estimation are carried out on the training data with the goal of minimising expected mean squared error in the "holdout" or "test" data. In some cases the selection and estimation of a model also requires a choice of value for one or more tuning parameters. The model conducts a grid search over the maximum depth, whose effectiveness to avoid over-fitting the model to the training dataset has been shown by Safavian and Landgrebe [36].

In this analysis 22944 samples, each of which denotes a datum of a single household at a given time stamp, are generated from 48 half-hour data samples of 646 households as training data (see Section 4.1). These samples, combined with the 12 features outlined in Table 3, are used to predict the target variable $\tilde{C}_\tau$. An example of a tree is illustrated with the setting of maximum depth of 2 in Figure 6.[2] The samples in the top node are partitioned using recursive binary splitting to generate the prediction in the bottom node. Features such as $X[1]$ (average consumption $\bar{C}$) are used for the binary split. In this analysis, the optimal maximum depth based upon minimum MAPE, is 30 over the range 10 to 50.

As noted by Strobl and Zeileis[37], single trees can be unstable such that small changes in the training data can lead to very different models or trees. This can be corrected by constructing a large number of DT at training time and outputting the class that is the mean prediction of the individual trees; this technique is called Random Forest (RF) [38]. Hence, this paper uses RF instead of DT. Another common way of preventing this is cross-validation as discussed by Sterlin and Patrick[39]. This will be explained in the following section.

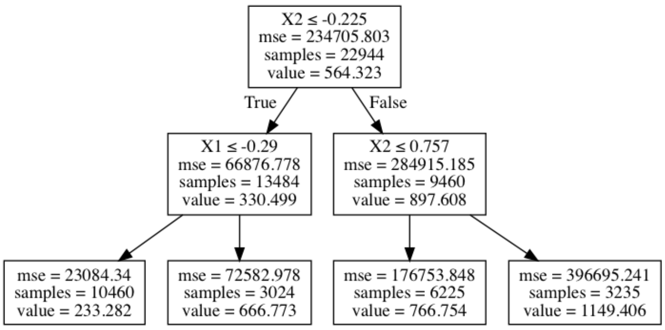

Figure 6: An example decision tree with the setting of maximum depth of 2.

4.1. Cross validation

As standard in statistical modelling, k-fold cross validation, whose success in accuracy estimation has been reported by Ron [40], is used to check the accuracy of model outputs by splitting the data across all tariffs into four equal subsets: three for training and one for testing. All data samples are split into four groups so that each subset volume is

[2] A larger number for maximum tree depth generates more granular predictive values.

almost equal for each tariff (see Figure 7).

The cross validation process is repeated four times ($k = 4$), so each subset of the data is used once for validation purposes. As a result, 646 load curves are predicted from the four folds. These results are then averaged to give a single estimation for each time period.

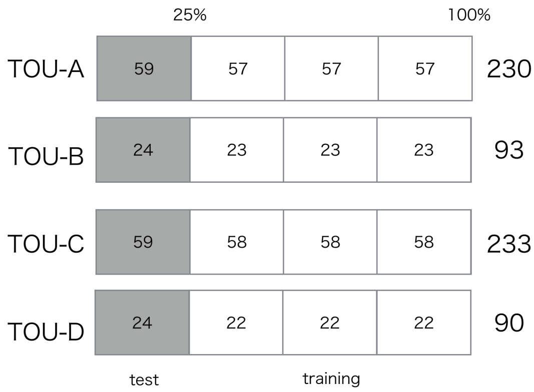

Figure 7: Test and training samples for k-fold cross validation ($k = 4$).

4.2. Evaluation measures

The model accuracy is evaluated using Mean Absolute Percentage Error (MAPE), the measure most frequently used to assess the performance of a model in the field of a medium-term load forecasting such as the study by Hahn et al.[22]. The main objective is to minimise the MAPE over household groups, and particularly during the peak time. Two MAPE metrics are used:

$$\text{MAPE}_g^i = \frac{1}{n}\sum_{\tau=1}^{n}\left|\frac{\tilde{C}_\tau^{avg} - \hat{C}_\tau^{avg}}{\tilde{C}_\tau^{avg}}\right|, \tag{4}$$

$$\text{MAPE}_{g,peak}^i = \frac{1}{n_{peak}}\sum_{\tau\in Peak}\left|\frac{\tilde{C}_\tau^{avg} - \hat{C}_\tau^{avg}}{\tilde{C}_\tau^{avg}}\right|, \tag{5}$$

where $\tilde{C}_\tau^{avg} = \frac{1}{M}\sum_{m=1}^{M}\tilde{C}_\tau^{(m)}$, M denotes the number of households, i indexes the cross validation fold, m indexes households, $\tilde{C}_\tau^{avg}$ is the observed average consumption, and $\hat{C}_\tau^{avg}$ is the prediction made with the proposed model. n is the number of half-hour intervals and n_{peak} is the number of intervals that correspond to the peak time ($n = 48$, $n_{peak} = 4$).

In addition to MAPE, Absolute Percentage Error (APE) is used to measure the model's power(W) prediction. APE is calculated as,

$$\text{APE}_{g,peak}^i(W) = \left|\frac{\max_{\tau\in Peak}\tilde{C}_\tau^{avg} - \max_{\tau\in Peak}\hat{C}_\tau^{avg}}{\max_{\tau\in Peak}\tilde{C}_\tau^{avg}}\right|. \tag{6}$$

Given $k = 4$ cross validation, the average across the folds is taken as the final MAPE_g, $\text{MAPE}_{g,peak}$ and $\text{APE}_{g,peak}$.

5. Results

Table 6 presents estimates of MAPE for the three different techniques (LR, NN and RF). The RF model outperforms LR and NN for both values of MAPE, especially in terms of $\text{MAPE}_{g,peak}$ during the weekdays, which is the main performance indicator. In forecasting average load for a group of households RF model yields a MAPE value of 2.05% for the weekday and 1.48% for the weekday peak time.

Date type	Model	LR	RF	NN
Weekday	MAPE_g	2.95%	2.05%	3.12%
Weekday	$\text{MAPE}_{g,peak}$	1.78%	1.48%	1.94%
Weekday	$\text{APE}_{g,peak}(W)$	0.60%	0.13%	0.58%
Non-weekday	MAPE_g	7.65%	2.66%	6.15%
Non-weekday	$\text{MAPE}_{g,peak}$	0.77%	1.61%	0.25%
Non-weekday	$\text{APE}_{g,peak}(W)$	0.64%	1.69%	3.0×10^{-4}%

Table 6: Comparison of the three statistical models on test data

In using MAPE as a standardised measure of model performance the proposed model compares favourably relative to existing medium-term forecast studies with a similar forecasting time horizon. Pedregal and Trapero [41] demonstrated a comparable finding with a MAPE varying between 5% to over 10% based upon medium-term(12 week ahead) hourly electricity forecasting at an aggregated level. Al-Hamadi and Soliman [42] presented a model to forecast weekly average intra-day load profile with a time-horizon of a year, delivering the result that the MAPE is 3.8%. Given that this forecasting problem includes the demand response following the introduction of a TOU tariff, this model incorporates additional complexity compared to other existing medium-term load forecasting studies. In this respect the accuracy of the proposed model is competitive in medium-term load forecasting models, and should be of practical use for decision making to assess the medium-term impact of load adjustment to TOU.

Figure 8 presents the intra-day load profiles across the different models. As a reference point, the line of the actual post-intervention load profile (labelled as 'Post-TOU', averaged load curve in January 2010) is also given. All three models forecast the peak reduction closely.

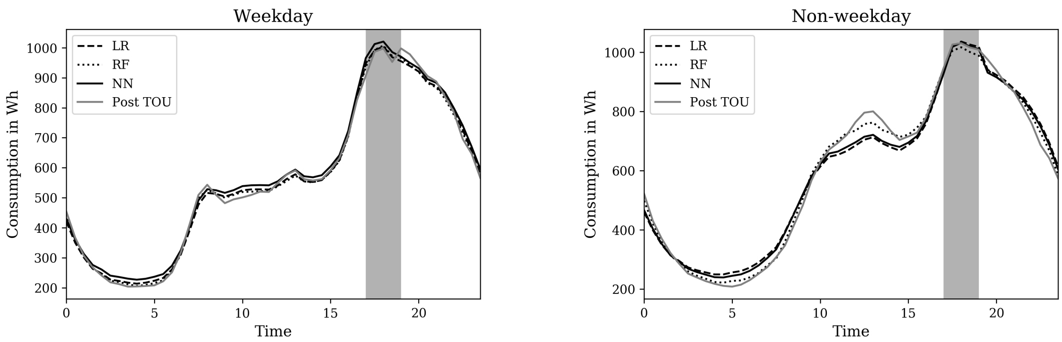

Figure 8: Comparison of post- TOU load predictions using three different statistical techniques.

Figure 9 shows the prediction errors between post-intervention load profile and each prediction. LR and NN have most of their errors at the border of the peak time where abrupt behavioural changes have been observed. The results consistently favour RF as the preferred modelling technique.

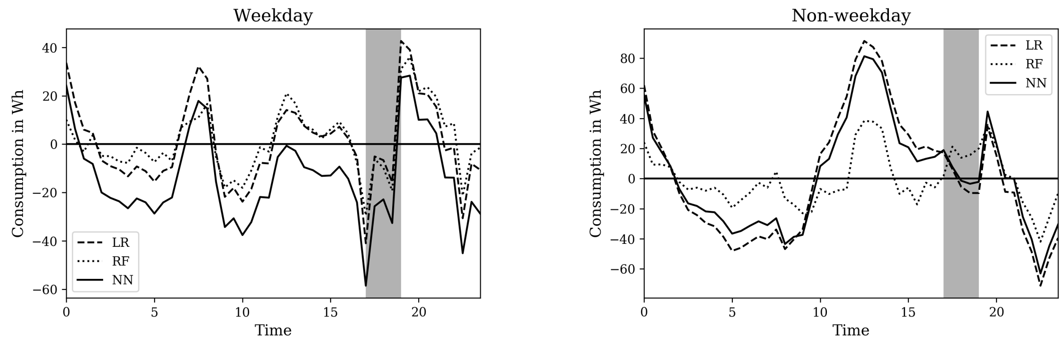

Figure 9: Prediction error in Wh between actual load profile subsequent to an introduction of TOU tariffs, and predicted load profiles for three different statistical techniques.

5.1. Feature Importance

DT model results provide clear information on the importance of significant factors for prediction based on Gini coefficient [43]. In regression analysis, its value is calculated as

$$G = \sum_{i=1}^{n} p_i(1 - p_i), \tag{7}$$

where n represents the number of total "leaves" and p_i is the ratio of the ith leave.

The importance of a feature is computed as the (normalised) total reduction of the criterion that is attributable to that feature. Table 7 reports this importance of the RF model. The higher the rating, the more important the feature. Every time a split of a node is made on variable m the Gini impurity criterion for the two descendent nodes is less than the parent node. Summing the decreases in the Gini measure for each individual variable over all trees in the forest gives a variable importance that is often very consistent with the permutation importance measure [44].

The paper finds that the average consumption feature is the most important predictor. It is also noteworthy that the most relevant features are statistical features of consumption, suggesting the tree strongly uses these features to forecast the load shifting. Although the feature TOU price, and peak ratio does not have an influential effect in this model, it is important to remember that the window of the peak period of four TOU tariffs is fixed under this trial, so that the effectiveness of these two features might be underestimated. Therefore, further trials with different windows of peak periods is needed to examine the importance of these parameters.

Feature	Importance
average	66.77%
kurtosis	9.87%
time	8.61%
skewness	5.79%
variance	5.73%
TOU price	0.90%
other living residence	0.65%
age group	0.62%
social class	0.59%
peak ratio	0.27%
gender	0.19%

Table 7: Features importance according to RF. These variables are summarised in table 3

An additional important finding is that none of the demographic features generate a significant contribution to the predictive capacity. Eliminating these features with low importance values could improve the model performance. This method of feature selection has been utilised by a number of studies. Granitto et al. [45], for instance, has introduced random forest recursive feature elimination to determine small subsets of features with high discrimination levels on chemical dataset. Uriarte and Andrés [46] has also applied this technique for gene selection.

Table 8 shows the RF model performance with/without demographic variables. It should be noted that the absence of demographic information does not lead to a deterioration in model performance, and the model even works better. A recent study of electric behavioural analyses conducted by O'Neil and Weeks [47] similarly observed the similar effect. This finding removes the extra cost for energy companies and analysts since the collection of demographic data is costly, with a limited ability to increase the predictive ability of a given model.

Date type	Model	With demographic	Without demographic
Weekday	MAPE_g	2.05%	1.80%
Weekday	$\text{MAPE}_{g,peak}$	1.48%	1.11%
Weekday	$\text{APE}_{g,peak}(W)$	0.14%	0.05%

Table 8: Comparison of the RF model performance with/without demographic features

5.2. Prediction of Intra-day Load Profile

The actual and predicted intra-day load profile generated by the RF model are presented in Figure 10. The results demonstrate that for the peak periods the model successfully captures the behavioural change for all tariffs.

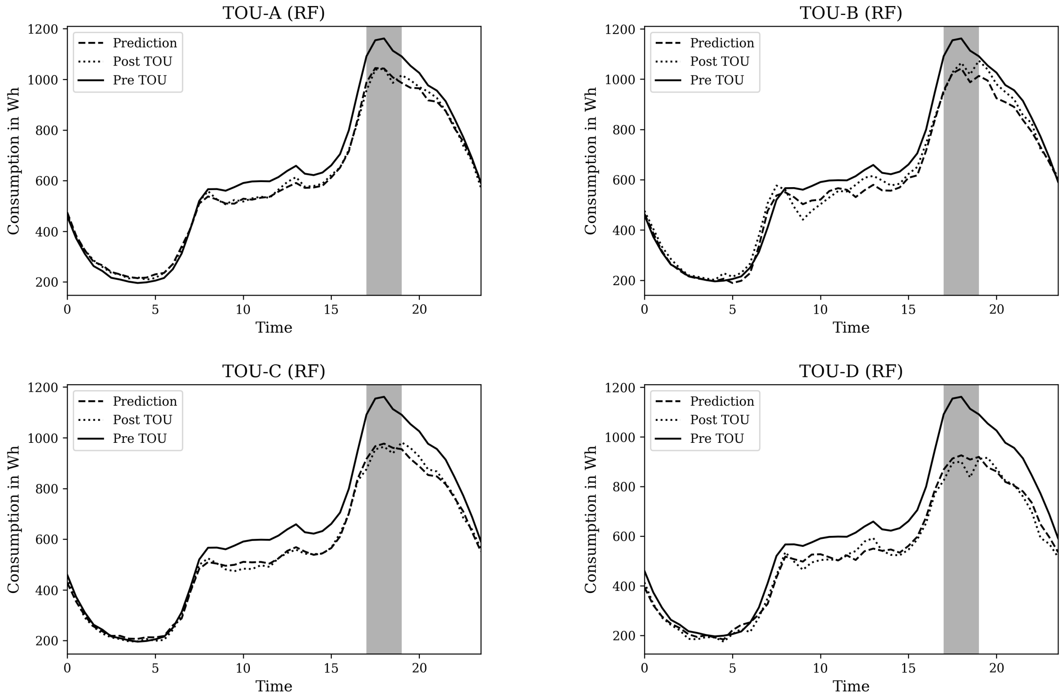

Figure 10: 'Pre-TOU' and 'Post-TOU' actual load under the flat-rate tariff and a TOU tariff respectively. 'Prediction' is predicted load curve by this model.

The difference between 'Post-TOU' predictions and actual load are demonstrated in Figure 11 for each group. These lines are more irregular than the similar analysis in Figure 9, since users are divided into the four tariff groups, where the number of households in each group relatively small. Each line shows zigzag patterns around the zero line, and no significant over/underestimation in any particular time periods or any groups has been observed. The common phenomenon observed across the four groups is the negative spike at the beginning of the peak period, and positive spike at the end of the peak period. This indicates customer demand response is not as immediate as the model predictions, with around a 30 minutes time lag prior to customer adoption. This characteristic is not captured by the proposed model, and is the source of the most significant errors in the modelling performance.

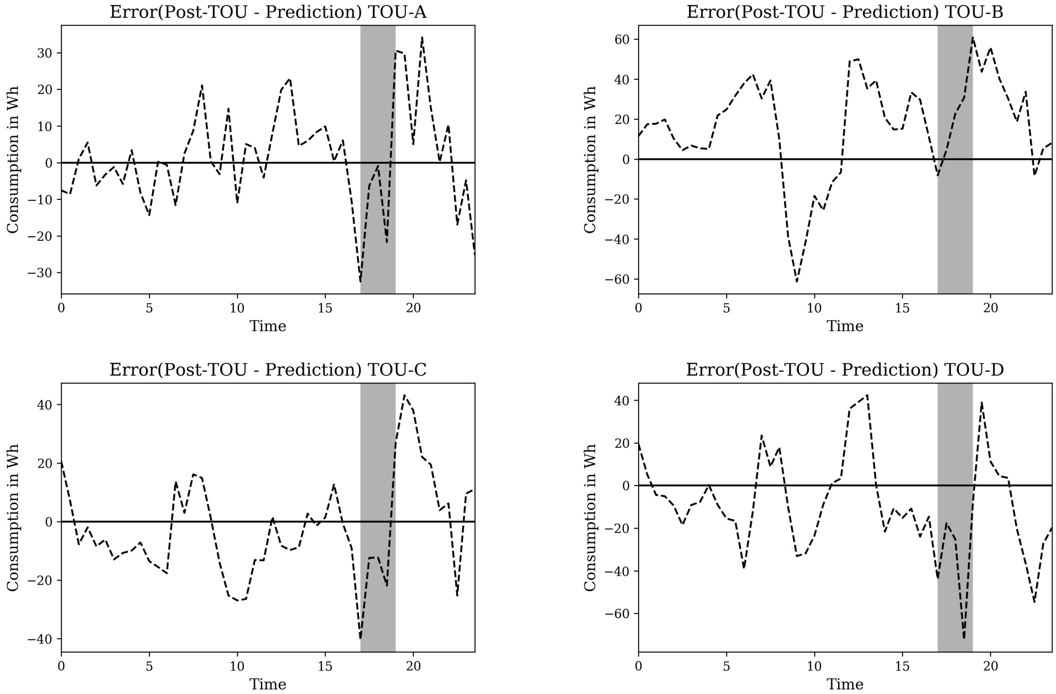

Figure 11: Prediction error in Wh between actual load profile subsequent to an introduction of TOU tariffs, and predicted load profiles for the DT model.

6. Conclusion and Future Works

The model developed in this study can be used to *forecast* the impact of the introduction of a Time-of-use tariff. The novelty of this study lies in explicitly accounting for consumer variability by extracting key features from past data. By incorporating lifestyle constraints, measured by a number of functions of historical load, the proposed model is able to predict the full intra-day load profiles with low MAPE. The MAPE value in forecasting average load for a group of households with the best model RF is 2.05% for the weekday and 1.48% for the weekday peak time. Random Forest is the preferred modelling technique based upon a comparison with Neural Networks and Linear Regression. By comparing the model's accuracy against a number of important studies of medium-term load forecasting at an aggregate level, the paper demonstrates that the model can be of practical use for decision making to assess the medium-term impact of load adjustment to a TOU tariff introduction.

The key findings of this paper can be summarised as follows. First, we show that top-down statistical modelling of historical smart meter data can be used to forecast the effectiveness of a TOU tariff. This can help energy companies to design TOU tariffs and optimise energy sourcing strategy accordingly. Second, the paper demonstrates it is possible to infer key features from the data that capture lifestyle constraints at an individual level, and determine the shape of an aggregate load profile; no ex-ante data on demographics is required to run this model to generate this competitive accuracy. This removes the additional cost collecting demographic data, unlocking further value of the metering infrastructure without requiring any changes to the smart meters that have already been deployed.

Since the proposed model is trained and evaluated based upon smart meter trial data from Ireland, some extra work to improve external validity is still required if an energy company seeks to apply this model to their customer base. However, to the best of authors' knowledge, this is the first study that reports the model performance in MAPE, and the effectiveness of each feature for the load forecasting following the adoption of a TOU tariff.

The main limitation of this paper is forecasting accuracy at the level of the individual level. Individual load forecasting is not the main interest of energy companies for design of TOU tariffs and optimisation of energy sourcing. However, the likelihood of peak load reduction following the introduction of a TOU tariff across individuals is important for optimising their marketing scope. Also, it is crucial to understand what the energy bill looks like with some modification of their daily life patterns for a household. Some types of households are not suited to such kind of treatment since their life patterns have limited flexibility to adjust the peak load accordingly. Regulators have shown interest in protecting vulnerable customers from the widespread roll-out of TOU tariffs. This paper does not examine the causes or likelihood of peak shifting at the individual level, hence as a future work, further investigations are expected for individual analysis.

Another limitation and future work is the data collection for this TOU dataset. To the author's best knowledge, CER dataset is the most comprehensive one available to academic use for this purpose, however, the trial data, collected in 2009 and 2010, might be outdated. For example, the availability of price/incentive based programmes offered to the residential sector has since increased. Therefore, up-to-date consumption dataset which observes pre/post intervention under various price/incentive based demand response programme is vital for further investigation of a TOU analysis.

Acknowledgements

This research has been funded by EPCO Inc., Japan. Riku Arakawa, Nobuto Yoshimura, Dimitry Foures, Giuseppe Vettigli, and Paul Monroe have contributed to the discussions and modelling.

References

[1] J. Torriti, M. G. Hassan, M. Leach, Demand response experience in europe: Policies, programmes and implementation, Energy 35 (4) (2010) 1575–1583.

[2] S. J. Darby, I. Pisica, Focus on electricity tariffs: experience and exploration of different charging schemes.

[3] ABI Research, Smart metering devices and services update, https://www.abiresearch.com/marketresearch/product/1018337smartmeteringdevicesandservicesupdate/ (2015).

[4] R. Wardle, C. Barteczko-Hibbert, D. Miller, E. Sidebotham, Initial load profiles from clnr intervention trials, Customer-Led Network Revolution.

[5] J. Torriti, Price-based demand side management: Assessing the impacts of time-of-use tariffs on residential electricity demand and peak shifting in northern italy, Energy 44 (1) (2012) 576–583.

[6] A. Faruqui, S. Sergici, Household response to dynamic pricing of electricity: a survey of 15 experiments, Journal of regulatory Economics 38 (2) (2010) 193–225.

[7] Y. Wang, L. Li, Time-of-use electricity pricing for industrial customers: A survey of us utilities, Applied Energy 149 (2015) 89–103.

[8] Federal Energy Regulatory Commission, U.s. federal energy regulatory commission. assessment of demand response and advanced metering staff report 2012, https://www.ferc.gov/legal/staff-reports/12-20-12-demand-response.pdf (2014).

[9] A. Faruqui, S. Sergici, N. Lessem, D. Mountain, Impact measurement of tariff changes when experimentation is not an option—a case study of ontario, canada, Energy Economics 52 (2015) 39–48.

[10] H. C. Gils, Assessment of the theoretical demand response potential in europe, Energy 67 (2014) 1–18.

[11] D. S. Kirschen, G. Strbac, P. Cumperayot, D. de Paiva Mendes, Factoring the elasticity of demand in electricity prices, IEEE Transactions on Power Systems 15 (2) (2000) 612–617.

[12] L. Goel, Q. Wu, P. Wang, Reliability enhancement of a deregulated power system considering demand response, in: Power Engineering Society General Meeting, 2006. IEEE, IEEE, pp. 6–pp.

[13] N. Venkatesan, J. Solanki, S. K. Solanki, Residential demand response model and impact on voltage profile and losses of an electric distribution network, Applied energy 96 (2012) 84–91.

[14] J. Katz, F. M. Andersen, P. E. Morthorst, Load-shift incentives for household demand response: Evaluation of hourly dynamic pricing and rebate schemes in a wind-based electricity system, Energy 115 (2016) 1602–1616.

[15] M. M. Armstrong, M. C. Swinton, H. Ribberink, I. Beausoleil-Morrison, J. Millette, Synthetically derived profiles for representing occupant-driven electric loads in canadian housing, Journal of Building Performance Simulation 2 (1) (2009) 15–30.

[16] S. Gottwalt, W. Ketter, C. Block, J. Collins, C. Weinhardt, Demand side management—a simulation of household behavior under variable prices, Energy policy 39 (12) (2011) 8163–8174.

[17] S. Shao, M. Pipattanasomporn, S. Rahman, Development of physical-based demand response-enabled residential load models, IEEE Transactions on power systems 28 (2) (2013) 607–614.

[18] K. McKenna, A. Keane, et al., Residential load modeling of price-based demand response for network impact studies., IEEE Trans. Smart Grid 7 (5) (2016) 2285–2294.

[19] F. Y. Xu, T. Zhang, L. L. Lai, H. Zhou, Shifting boundary for price-based residential demand response and applications, Applied Energy 146 (2015) 353–370.

[20] K. C. Armel, A. Gupta, G. Shrimali, A. Albert, Is disaggregation the holy grail of energy efficiency? the case of electricity, Energy Policy 52 (2013) 213–234.

[21] L. G. Swan, V. I. Ugursal, Modeling of end-use energy consumption in the residential sector: A review of modeling techniques, Renewable and sustainable energy reviews 13 (8) (2009) 1819–1835.

[22] H. Hahn, S. Meyer-Nieberg, S. Pickl, Electric load forecasting methods: Tools for decision making, European journal of operational research 199 (3) (2009) 902–907.

[23] L. Hernandez, C. Baladron, J. M. Aguiar, B. Carro, A. J. Sanchez-Esguevillas, J. Lloret, J. Massana, A survey on electric power demand forecasting: future trends in smart grids, microgrids and smart buildings, IEEE Communications Surveys & Tutorials 16 (3) (2014) 1460–1495.

[24] H. S. Hippert, C. E. Pedreira, R. C. Souza, Neural networks for short-term load forecasting: A review and evaluation, IEEE Transactions on power systems 16 (1) (2001) 44–55.

[25] Commission for Energy Regulation, Electricity smart metering customer behaviour trials findings report, Tech. rep., Dublin (2011).

[26] C. Beckel, L. Sadamori, T. Staake, S. Santini, Revealing household characteristics from smart meter data, Energy 78 (2014) 397–410.

[27] S. Wold, K. Esbensen, P. Geladi, Principal component analysis, Chemometrics and intelligent laboratory systems 2 (1-3) (1987) 37–52.

[28] W. H. Press, B. P. Flannery, S. A. Teukolsky, W. T. Vetterling, Numerical recipes in c: the art of scientific programming, Section 10 (1992) 408–412.

[29] J. G. Dy, C. E. Brodley, Feature selection for unsupervised learning, Journal of machine learning research 5 (Aug) (2004) 845–889.

[30] P. Cappers, C. Goldman, D. Kathan, Demand response in us electricity markets: Empirical evidence, Energy 35 (4) (2010) 1526–1535.

[31] M. Ranjan, V. Jain, Modelling of electrical energy consumption in delhi, Energy 24 (4) (1999) 351–361.

[32] A. Z. Al-Garni, S. M. Zubair, J. S. Nizami, A regression model for electric-energy-consumption forecasting in eastern saudi arabia, Energy 19 (10) (1994) 1043–1049.

[33] G. K. Tso, K. K. Yau, A study of domestic energy usage patterns in hong kong, Energy 28 (15) (2003) 1671–1682.

[34] S. A. Kalogirou, M. Bojic, Artificial neural networks for the prediction of the energy consumption of a passive solar building, Energy 25 (5) (2000) 479–491.

[35] J. R. Quinlan, C4. 5: programs for machine learning, Elsevier, 2014.

[36] S. R. Safavian, D. Landgrebe, A survey of decision tree classifier methodology, IEEE transactions on systems, man, and cybernetics 21 (3) (1991) 660–674.

[37] C. Strobl, A. Zeileis, Danger: High power!–exploring the statistical properties of a test for random forest variable importance.

[38] J. Friedman, T. Hastie, R. Tibshirani, The elements of statistical learning, Vol. 1, Springer series in statistics New York, NY, USA:, 2001.

[39] P. Sterlin, Overfitting prevention with cross-validation, Supervised Machine Learning Report 83.

[40] R. Kohavi, et al., A study of cross-validation and bootstrap for accuracy estimation and model selection, in: Ijcai, Vol. 14, Montreal, Canada, 1995, pp. 1137–1145.

[41] D. J. Pedregal, J. R. Trapero, Mid-term hourly electricity forecasting based on a multi-rate approach, Energy Conversion and Management 51 (1) (2010) 105–111.

[42] H. Al-Hamadi, S. Soliman, Long-term/mid-term electric load forecasting based on short-term correlation and annual growth, Electric power systems research 74 (3) (2005) 353–361.

[43] L. Breiman, Classification and regression trees, Routledge, 2017.

[44] L. Breiman, A. Cutler, Random forests-classification description, Department of Statistics, Berkeley 2.

[45] P. M. Granitto, C. Furlanello, F. Biasioli, F. Gasperi, Recursive feature elimination with random forest for ptr-ms analysis of agroindustrial products, Chemometrics and Intelligent Laboratory Systems 83 (2) (2006) 83–90.

[46] R. Díaz-Uriarte, S. A. De Andres, Gene selection and classification of microarray data using random forest, BMC bioinformatics 7 (1) (2006) 3.

[47] E. O'Neill, M. Weeks, Causal tree estimation of heterogeneous household response to time-of-use electricity pricing schemes, arXiv preprint arXiv:1810.09179.