Real Collars as Alternative Incentives for Subsidizing Energy Facilities

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Abstract

We suggest that real collars may be an acceptable incentive for encouraging development of low (or no) carbon energy generating facilities as an alternative for high feed-in-tariffs. Composed of pairs of put and call American perpetuity options, feasible collars perform a similar role as subsidies by raising the value of an energy generator. We provide novel analytical solutions for real collars and partial collars, plus floor and ceiling partial derivatives. The "gains/losses" of the energy generator as perceived parameter values change are compared to those of the government providing the collar, and floor or ceiling only, viewing the arrangement as a real option game between principal and agent. A volatility increase first increases, then decreases the "gains" of the generator.

1 Introduction

We present a collar option as a suitable policy device for a government (GOV) providing a subsidy for a private energy generator (GEN) by guaranteeing a floor in the face of adverse circumstances, and simultaneously capturing abnormally high returns when the circumstances are sufficiently favorable. The analysis of collars adopts a real option formulation because the guarantee on the downside and bonus compensation for the government on the upside are expressible as real options. Using an American perpetuity model, a minimum price guarantee (with a high ceiling) can create a value equivalent to a high feed-in-tariff which has often been used in the UK.

The National Audit Office (2016, 2017) states that the Hinkley Point C Arrangement (HPCA) involves a subsidy to the state supported EDF and CGN entities of a present value of some 30 billion pounds, as the difference between the feed-in-tariff (92.5 pounds per MWhr in 2012 prices) and the current UK electricity price as of March 2016. Possible benefits for the U.K. government in the HPCA are reductions in the feed-in-tariff for electricity if construction costs are below expected levels, and a payment to the GOV if eventually the project IRR exceeds specified levels. The National Audit Office (2017) does not investigate the incentives for the GEN to control construction costs, or to operate just short of the level that triggers an upside call option[1], or to reduce the project volatility by hedging or issuing electricity price risk sharing debt instruments. An alternative collar with a guaranteed electricity price at or below the current UK electricity price could provide equivalent value for the nuclear GEN[2].

There is a vast literature on subsidies used to promote certain types of energy facilities, which Couture and Gagnon (2009) separate into market independent (feed-in-tariffs) and market dependent categories. One market independent subsidy cited is the Spanish 2007 "variable premium" that involves a type of floor and cap, where the highest premium (over the market electricity price) is paid when the electricity price is low, and zero when the price exceeds a

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[1] The Hinkley Point C agreement, expected to operate for 60 years, specifies that the GOV (on behalf of UK electricity customers) may recover some of the construction costs less than the expected amounts, and obtain a payment from the GEN if the IRR on the project turns out to be more than 11.4%. The concessionaire is owned by Électricité de France (EDF) and China General Nuclear Power Group (CGN).

[2] A case study showing that the approximate value of the current FiT for HPCA might be replicated by a suitable floor and ceiling is available from the authors.

ceiling (so the GEN receives all of the higher price, without a premium). González (2009) provides some detail on these premium collars, which were (€cents/kWh) 25.4-34.4 for solar, 7.3-8.5 for small on-shore wind, and 15.4-16.6 for energy crops in 2007. Meira et al. (2008) illustrates how this system worked for wind power during 2006. Brandão and Saraiva (2008) and Shan et al. (2010) propose that government guarantees for infrastructure projects should involve a European collar. Fernandes et al. (2015) suggest a collar-type insurance for wind power in Brazil, where the generator has promised to supply power even during times where there is little wind. Except for European collars, these authors do not propose analytical solutions for these long-life collar arrangements.

There are some analytical studies for perpetual subsidies (Boomsma and Linnerud, 2015, Adkins and Paxson, 2015), and for perpetual floors/ceilings. Takashima et al. (2010) design a private-public partnership (PPP) deal involving government debt participation that incorporates a floor on the future maximum loss level where the investor has the right to sell back the project whenever adverse conditions emerge. Armada et al. (2012) make an analytical comparison of various subsidy policies including minimum revenue guarantees.

Several authors focus on the conflict between a principal GOV and agent concessionaire GEN implicit in contracts. Chevalier-Roignant et al. (2011) and Azevedo and Paxson (2014) survey many real option game problems between principal and agent. Scandizzo and Ventura (2010) focus on "calculating a baseline to organize a concession contract…to measure the balance of power between the public and the private party".

Our contribution consists of analytical models for a post-investment (ACTIVE) collar, so the costs and benefits to the GEN and GOV can be clearly identified, initially and as the parameter values evolve over time. Also, it is easy to see what initial parameter values the GEN and GOV are likely to over (under) estimate or emphasize, and what basic incentives are evident for the two parties to a subsidized energy facility arrangement. The basic game theory applicable to a principal and an agent is that the incentives for the agent should be allied to the objectives of the principal, and that the principal monitors periodically the performance of the agent to see whether those objectives are being met.

This paper is organized in the following way. In the next section, we formulate the collar model analytically, along with floor only and ceiling only models. In section 3, we show the analytical

partial derivatives of the calls and puts with respect to changing the floor/ceiling, perhaps useful for both parties in negotiating a satisfactory arrangement. In section 4, further insights are gained from performing a numerical sensitivity analysis. Section 5 presents some of the more interesting aspects of "who wins, who loses, why" between the GEN and GOV as parameter values change. The versatility of the analytical representation is demonstrated in section 6 through an extension to partial collars. Section 7 is a conclusion.

2 Fundamental Models

For a firm in a monopolistic situation confronting a sole source of uncertainty due to output price[3] variability, and ignoring operating costs and taxes, the operation of an energy generator depends primarily (or solely) on the electricity price evolution, which is specified by the geometric Brownian motion process[4]:

$$\mathrm{d}P = \alpha P \mathrm{d}t + \sigma P \mathrm{d}W \tag{1}$$

where α denotes the expected price risk-neutral drift, σ the price volatility, and $\mathrm{d}W$ an increment of the standard Wiener process. Using contingent claims analysis, the option to operate the project $F(P)$ follows the risk-neutral valuation relationship:

$$\tfrac{1}{2}\sigma^2 P^2 \frac{\partial^2 F}{\partial P^2} + (r - \delta) P \frac{\partial F}{\partial P} - rF = 0 \tag{2}$$

where $r > \alpha$ denotes the risk-free interest rate and $\delta = r - \alpha$ thee convenience yield or the rate of return shortfall. The generic solution to (2) is:

$$F(P) = A_1 P^{\beta_1} + A_2 P^{\beta_2} \tag{3}$$

where A_1, A_2 are to be determined generic constants for calls and puts, and β_1, β_2 are, respectively, the positive and negative roots of the fundamental equation, which are given by:

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[3] This assumption is perhaps more valid for nuclear power, which operates at a constant baseload except for some planned (and also emergency) outages. This model can easily be altered to involve quantity (Q) uncertainty where R=P*Q. However, our analysis assumes Q is ignored, but the original construction cost is expressed in terms of unit capacity for Q.

[4] Tests for whether gBm process is appropriate for UK monthly baseload electricity prices are available from the authors, some similar to those in Dockendorf and Paxson (2013).

$$\beta_1, \beta_2 = \left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right) \pm \sqrt{\left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{4}$$

In (3), if $A_2 = 0$ then F is a continuously increasing function of P and represents an American perpetual call option, Samuelson (1965), while if $A_1 = 0$ then F is a decreasing function and represents a put option, Merton (1973) and Merton (1990).

2.1 Real Collar Option for an ACTIVE Project

A collar option is designed to confine the output price for an active project to a tailored range, by restricting its value to lie between a floor P_L and a cap P_H. Whenever the price trajectory falls below the floor, the received output price is assigned the value P_L, and whenever it exceeds the cap, it is assigned the value P_H. By restricting the price to this range, the firm benefits from receiving a price that never falls below P_L and obtains protection against adverse price movements, whilst at the same time, it is being forced never to receive a price exceeding P_H, and so sacrifices the upside potential. For an active project, the revenue accruing to the firm is given by $\pi_C(P) = \min\{\max\{P_L, P\}, P_H\} \times Q$ (we assume Q=1) and its value V_C is described by the risk-neutral valuation relationship:

$$\tfrac{1}{2}\sigma^2 P^2 \frac{\partial^2 V_C}{\partial P^2} + (r-\delta)P\frac{\partial V_C}{\partial P} - rV_C + \pi_C(P) = 0. \tag{5}$$

The valuation of a with-collar active project is conceived over three mutually exclusive exhaustive regimes, I, II and III, specified on the P line, each with its own distinct valuation function. Regimes I, II and III are defined by $P \le P_L$, $P_L < P < P_H$ and $P_H \le P$, respectively. Over Regime I, the firm is granted a more attractive fixed price P_L compared with the variable price P, but also possesses a call-style option to switch to the more favorable Regime II as soon as P exceeds P_L. This switch option increases in value with P and has the generic form AP^{β_1}, where A denotes a to be determined generic coefficient. Over Regime III, the firm is not only

obliged to accept the less attractive fixed price P_H instead of P but also has to sell a put-style option to switch to the less favorable Regime II as soon as P falls below P_H. This switch option decreases in value with P and has the generic form AP^{β_2}. Over Regime II, the firm receives the variable price P, possesses a put-style option to switch to the more favorable Regime I as soon as P falls to P_L, but sells a call-style option to switch to the less favorable Regime III as soon as P attains P_H. The various switch options are displayed in Table 1, where A denotes a generic coefficient.

Table 1: The Various Switch Options

From – To	Option Type	Value	Sign of A
I – II	Call	AP^{β_1}	+
II – I	Put	AP^{β_2}	+
II – III	Call	AP^{β_1}	-
III – II	Put	AP^{β_2}	-

If the subscript C denotes the with-collar arrangement, then after paying the investment cost, the valuation function for the firm owning the ACTIVE project is:

$$V_C(P) = \begin{cases} \dfrac{P_L}{r} + A_{C11}P^{\beta_1} & \text{for } P \leq P_L \\[2mm] \dfrac{P}{\delta} + A_{C21}P^{\beta_1} + A_{C22}P^{\beta_2} & \text{for } P_L < P < P_H \\[2mm] \dfrac{P_H}{r} + A_{C32}P^{\beta_2} & \text{for } P_H \leq P. \end{cases} \tag{6}$$

In (6), the first numerical subscript for a coefficient denotes the regime $\{1 = I, 2 = II, 3 = III\}$, while the second denotes a call if 1 or a put if 2. The coefficients A_{C11}, A_{C22} are expected to be positive because the firm owns the options and a switch is beneficial. In contrast, A_{C21}, A_{C32} are

expected to be negative because the firm is writing the options and is being penalized by the switch. The real collar is composed of a pair of both call and put options. The first pair facilitates switching back and forth between Regime I and II, which results in an advantage for the GEN, while the second pair facilitates switching back and forth between Regime II and III, which results in a disadvantage for the firm. The real collar design differs from the typical European collar, which only involves buying and selling two distinct options.

A switch in either direction between Regime I and II occurs when $P = P_L$. It is optimal provided the value-matching relationship:

$$\frac{P_L}{r} + A_{C11}P^{\beta_1} = \frac{P}{\delta} + A_{C21}P^{\beta_1} + A_{C22}P^{\beta_2} \tag{7}$$

and its smooth-pasting condition[5] expressed as:

$$\beta_1 A_{C11}P^{\beta_1} = \frac{P}{\delta} + \beta_1 A_{C21}P^{\beta_1} + \beta_2 A_{C22}P^{\beta_2} \tag{8}$$

both hold when evaluated at $P = P_L$. Similarly, a switch in either direction between Regime II and III occurs when $P = P_H$. It is optimal provided the value-matching relationship:

$$\frac{P}{\delta} + A_{C21}P^{\beta_1} + A_{C22}P^{\beta_2} = \frac{P_H}{r} + A_{C32}P^{\beta_2} \tag{9}$$

and its smooth-pasting condition expressed as:

$$\frac{P}{\delta} + \beta_1 A_{C21}P^{\beta_1} + \beta_2 A_{C22}P^{\beta_2} = \beta_2 A_{C32}P^{\beta_2} \tag{10}$$

both hold when evaluated at $P = P_H$. A novel expression for the option coefficients is:

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[5] Obtained through multiplying both sides of the first derivative by P.

$$A_{C11} = \left[\frac{P_H}{P_H^{\beta_1}} - \frac{P_L}{P_L^{\beta_1}} \right] \times \frac{(r\beta_2 - r - \delta\beta_2)}{(\beta_1 - \beta_2) r\delta} > 0, \quad A_{C21} = \frac{P_H (r\beta_2 - r - \delta\beta_2)}{P_H^{\beta_1} (\beta_1 - \beta_2) r\delta} < 0,$$

$$A_{C22} = \frac{-P_L (r\beta_1 - r - \delta\beta_1)}{P_L^{\beta_2} (\beta_1 - \beta_2) r\delta} > 0, \quad A_{C32} = \left[\frac{P_H}{P_H^{\beta_2}} - \frac{P_L}{P_L^{\beta_2}} \right] \times \frac{(r\beta_1 - r - \delta\beta_1)}{(\beta_1 - \beta_2) r\delta} < 0. \tag{11}$$

The signs of the four option coefficients comply with expectations. Other findings can also be derived. The coefficient A_{C22} for the option to switch from Regime II to I, which depends on only P_L and not on P_H, increases in size with P_L. This switch option becomes more valuable for the firm as the floor level increases. Similarly, the coefficient A_{C21} for the option to switch from Regime II to III, which depends on only P_H and not on P_L, decreases in magnitude with P_H. This switch option becomes less valuable for the government as the cap level increases. The coefficients A_{C11} and A_{C32} for the switch option from Regime I to II and from Regime III to II, respectively, depend on both P_L and P_H.

2.2 Floor and Cap Options

The analogous results for the floor only and the cap (ceiling) only are shown below.

Price Floor Model

We use the additional subscript f to indicate a model with only a floor. From (6) the active project valuation function becomes:

$$V_{Cf}(P) = \begin{cases} \dfrac{P_L}{r} + A_{Cf11} P^{\beta_1} & \text{for } P \leq P_L \\[2mm] \dfrac{P}{\delta} + A_{Cf22} P^{\beta_2} & \text{for } P_L < P, \end{cases} \tag{12}$$

with:

$$A_{Cf11} = \frac{-P_L (r\beta_2 - r - \delta\beta_2)}{P_L^{\beta_1} (\beta_1 - \beta_2) r\delta} \geq 0, \quad A_{Cf22} = \frac{-P_L (r\beta_1 - r - \delta\beta_1)}{P_L^{\beta_2} (\beta_1 - \beta_2) r\delta} \geq 0. \tag{13}$$

A high floor is favorable for the firm, and so represents a government granted subsidy, Armada et al. (2012).

Price Cap Model

We use the additional subscript c to indicate a model with only a cap. From (6) the active project valuation function becomes:

$$V_{Cc}(P) = \begin{cases} \dfrac{P}{\delta} + A_{Cc21}P^{\beta_1} & \text{for } P < P_H \\[2mm] \dfrac{P_H}{r} + A_{Cc32}P^{\beta_2} & \text{for } P_H \leq P, \end{cases} \qquad (14)$$

with:

$$A_{Cc21} = \frac{P_H}{P_H^{\beta_1}}\frac{(r\beta_2 - r - \delta\beta_2)}{(\beta_1 - \beta_2)r\delta} \leq 0, \quad A_{Cc32} = \frac{P_H}{P_H^{\beta_2}}\frac{(r\beta_1 - r - \delta\beta_1)}{(\beta_1 - \beta_2)r\delta} \leq 0. \qquad (15)$$

A ceiling is less desirable for the GEN than an operation without a cap, and consequently it is imposed by, for example, a government intent on offering a subsidy but shielding electricity customers against escalating prices.

3 Collar and Floor and Ceiling Partial Derivatives

Calculating the partial derivatives of the option coefficients with respect to changes in the floor or ceiling is novel, and perhaps enables each contracting party to view clearly the impact of different floor and/or ceiling levels on the real collar arrangement. The more traditional deltas (partial derivatives of the option coefficients and value of the floor or ceiling), and gammas (second derivative of the option coefficients and value of the floor or ceiling) are used to show that indeed the original ODE is solved, see Table 2.

The first derivatives of the ACTIVE option coefficients with respect to changes in P are as follows:

$$\Delta V_C(P) = \begin{cases} \beta_1 A_{C11}P^{\beta_1-1} & \text{for } P \leq P_L \\[2mm] \dfrac{1}{\delta} + \beta_1 A_{C21}P^{\beta_1-1} + \beta_2 A_{C22}P^{\beta_2-1} & \text{for } P_L < P < P_H \\[2mm] \beta_2 A_{C32}P^{\beta_2-1} & \text{for } P_H \leq P. \end{cases} \qquad (16)$$

The first derivatives of the ACTIVE option coefficients with respect to changes in P_L are:

$$A_{C11} = \left[-(1-\beta_1)P_L^{-\beta_1} \right] \times \frac{(r\beta_2 - r - \delta\beta_2)}{(\beta_1 - \beta_2)r\delta},$$

$$A_{C22} = \left[-(1-\beta_2)P_L^{-\beta_2} \right] \times \frac{(r\beta_1 - r - \delta\beta_1)}{(\beta_1 - \beta_2)r\delta}, \tag{17}$$

$$A_{C32} = \left[-(1-\beta_2)P_L^{-\beta_2} \right] \times \frac{(r\beta_1 - r - \delta\beta_1)}{(\beta_1 - \beta_2)r\delta}.$$

The first derivatives of the ACTIVE option coefficients with respect to changes in P_H are:

$$A_{C11} = \left[(1-\beta_1)P_H^{-\beta_1} \right] \times \frac{(r\beta_2 - r - \delta\beta_2)}{(\beta_1 - \beta_2)r\delta},$$

$$A_{C21} = \left[(1-\beta_1)P_H^{-\beta_1} \right] \times \frac{(r\beta_1 - r - \delta\beta_1)}{(\beta_1 - \beta_2)r\delta}, \tag{18}$$

$$A_{C32} = \left[(1-\beta_2)P_H^{-\beta_2} \right] \times \frac{(r\beta_1 - r - \delta\beta_1)}{(\beta_1 - \beta_2)r\delta}.$$

One reason for calculating the collar, and floor and ceiling "deltas", is to provide an analytical framework for viewing the price exposure of a generator with a collar, compared to without a collar, and for displaying the significance of a proposed floor or ceiling on the value of option elements in any GOV and GEN arrangement.

4. Numerical Illustrations

Although the analytical results reveal some interesting properties, further insights into model behavior are obtainable from numerical evaluations. The base case parameter values are Q=1, σ=25%, r=4%, and δ=4%. The evaluated power parameters for these values are $\beta_1 = 1.7369$ and $\beta_2 = -0.7369$ from (4). The basic spreadsheet is Table 2, with the base case parameter values, which show that the option coefficient signs are as expected and that the ODE (5) is solved using the calculated delta (16) and gamma.

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In Table 3 the evaluated switch option coefficients, $A_{C11}, A_{C21}, A_{C22}, A_{C32}$ are shown in Panels A-D, respectively, for various floor and cap levels. The floor levels are chosen to vary between a minimum $P_L = 0$ and a maximum $P_L = 4$, and the cap levels between a minimum $P_H = 10$, and a maximum $P_H = \infty$. As expected, all the four coefficients adopt the correct sign, A_{C21} is independent of P_L and A_{C22} of P_H, while A_{C11}, A_{C32} depend on both. Further, A_{C11}, the coefficient for the option to switch from Regime I to II, decreases with P_L but increases with P_H, since for any feasible Regime I price level, the switch option is more valuable for lower P_L levels because of the time value of money and that the price level is closer to P_L, and for higher P_H levels because less is being sacrificed. Similarly, A_{C32}, the negative coefficient for the option to switch from Regime III to II increases in magnitude with P_H because of the time value of money and decreases with P_L because less is being sacrificed. Finally, A_{C21}, the negative coefficient for the option to switch from Regime II to III decreases in magnitude with P_H because less is being sacrificed at higher P_H levels, while A_{C22}, the coefficient for the option to switch from Regime II to I increases with P_L because more is being gained for higher P_L levels. Note that the coefficients for the price floor are also available from Table 3 in the rows where $P_H = \infty$, while those for the price cap model are available from the columns where $P_L = 0$.

Tables 2 & 3 about here

The cost of the subsidy can be neutralized and the collar made "costless" by suitably engineering its floor and cap levels. For the ACTIVE GEN, a "costless collar" might be obtained equating the written call and protective put $A_{C21}P^{\beta_1}, A_{C22}P^{\beta_2}$ for $P_L < P < P_H$. For instance, for base case parameter values when P=6, P_L=4, P_H=15.6, $-A_{C21}P^{\beta_1} = A_{C22}P^{\beta_2}$. Some illustrative "costless" P_L and P_H pairs are presented in Table 4. The pairs are inversely related, as expected, since for the collar to remain "costless", any increase in the floor and reduction in downside risk has to be compensated by an additional sacrifice of the upside potential.

Table 4 about here

4.1 Changes in Price

Figure 1 shows that the collar value VC (6) exceeds the PV, left hand terms of (6) when P is low, VC and VC PV are equal when P=6, since the floor and ceiling are designed as costless collars at P=6 and the other base case parameter values, VC<VC PV when P>6 as the option coefficients A_{C21} and A_{C22} decrease as P approaches and then exceeds P_H.

*** *Figure 1 about here* ***

4.2 Changes in Volatility

Figure 2 shows that the VC first slightly exceeds VC PV as price volatility increases from nearly zero, but then falls short of the VC PV as volatility increases beyond 25%, which is the base case where $A_{C21}=-A_{C22}$, a costless collar. The GEN has an incentive to hedge against increased volatility, but should welcome volatility slightly lower than 25%.

*** *Figure 2 about here* ***

4.3 Changes in Interest Rate

Figure 3 shows that the VC collar is somewhat less valuable as the interest rate increases past the base case rate of 4%. The held put option becomes less valuable at high interest rates, and the negative written call option value much more negative as r>4%.

*** *Figure 3 about here* ***

4.4 Changes in Yield

Figure 4 shows that the PV is significantly affected by changes in the yield, since between the floor and ceiling cash flows are valued dividing by the yield. From a costless collar when δ=4%, the written call option increases much more than the held put option decreases when δ falls, but the PV of P increases, so in theory the GEN should favor yield decreases. Perhaps the interpretation that δ represents the convenience yield derived from the term structure of electricity futures prices implies that δ is exogeneous, so the GEN observes rather than influences δ. Alternatively, perhaps this figure shows one of the limitations of the basic model.

*** *Figure 4 about here* ***

5. Risk Sharing: Who Wins, Who Loses, Why?

In the principal-agent problem (GOV-GEN) and risk-sharing aspects of collar and floor or ceiling only arrangements, who wins, and who loses, as parameter values change are likely to be the indicators for GEN versus GOV incentives after the initial transaction. Initially, it is assumed that the GEN pays the "fair value" of the facility arrangement to the GOV, and the construction cost has already been paid by the GOV. In the base case, we also assume that the GOV has offered the GEN a "costless" real collar arrangement, where the value of a call written by the GEN to the GOV on upside prices higher than the ceiling P_H and a put written by the GOV to the GEN on downside prices lower than a floor P_L are equal and initially $P_L<P<P_H$. It is assumed that both the GEN and the GOV agree on the initial and subsequent perceived parameter values, which is perhaps a heroic assumption. Disagreements and different views would produce different results and interpretations[6]. The effect of changes in the parameter values are *zero-sum games,* where the GEN gain/loss is equal to the GOV loss/gain, so that the GEN plus GOV is zero. Generally, the figures below confirm the previous figures on sensitivity, but are stated in an alternative format.

We show here six examples of GEN versus GOV results, as parameter values change. In Figure 5, from an initial "costless" collar if $P_L=4$ and $P_H =15.6$ and $P=6$, if P increases, the GEN gains and GOV loses relatively in a *zero-sum game*.

*** Figure 5 about here ***

In Figure 6a, from an initial volatility of 25% when P=6, if volatility increases, in a *zero-sum game* (where the benefits and costs change, but remain equal to each other with opposite signs) the GEN suffers from the negative call increasing more than the put, and GOV benefits. If volatility decreases from 25%, GEN benefits from the negative call decreasing less than the put, but when the volatility is close to zero, neither is of any value, thereby reverting to a costless collar.

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[6] If the floor and ceiling are based on observable monthly average electricity prices, conceivably an independent third party could verify the price barriers as specified in any legal arrangement. However, different views of expected future price volatilities and convenience yields over several years could lead to both parties simultaneously justifying gains or losses using these real option models, hardly a zero-sum game.

These volatility (vega) patterns are dependent on the level of P relative to P_L and P_H. When P=2, given the basic collar arrangement, the GOV gains when A_{C11} falls as volatility decreases below 25% as shown in Figure 6b. The opposite occurs when P=18, given the basic collar arrangement, the GOV gains when $-A_{C32}$ falls as volatility increases above 25% as shown in Figure 6c. As in Figure 2 the GEN is typically hurt by high volatility, which is not intuitive.

*** *Figures 6a, 6b and 6c about here* ***

In Figure 7, GEN loses if the interest rate rises above 4%, and the GOV gains, showing perhaps that in the current low interest rate environment, previous collar arrangements are not necessarily good for investors, unless they have issued fixed rate long term debt without mandatory redemptions. It is hard to imagine a GOV benefitting or losing from a change in interest rates, since the basic interest rate is an economic policy instrument, and most GOVs have a mix of short and long-term debt, including pension and medical expense obligations.

*** *Figure 7 about here* ***

In Figure 8, suppose that δ represents the convenience yield derived from electricity futures, so a low δ implies a nearly flat term structure of future prices. If current futures forecast future prices, that is hardly beneficial for the GEN who benefits from higher P especially when $P_L<P<P_H$, even though the effect on increasing the PV of the price maybe beneficial for the GEN. The so-called asset yield, dividend, or convenience yield, or return "shortfall" is a difficult concept to interpret in most applications, illustrated in this case.

*** *Figure 8 about here* ***

It is interesting to compare collar arrangements with different floors and ceilings, and with floor only or ceiling only arrangements. Figure 9 shows the risk sharing collar arrangements between GEN and GOV as a function of various levels of the floor. It is natural that the GEN benefits (and the GOV suffers) from higher floors, in a *zero-sum game*.

*** *Figures 9 and 10 about here* ***

Equally dramatic in a *zero-sum game* is the effect of changes in price volatility on risk sharing when there is only a floor, or alternatively only a ceiling. Figure 10 shows the risk allocation with a floor only as a function of P volatility, but with the GEN benefit increasing with volatility

up to a certain point (about 45%), when thereafter volatility increases result in a decline of the GEN benefit, in contrast to Figures 6a and 6b for a collar.

6 Partial Collars

We now consider an additional illustrative case to investigate whether the findings for the plain vanilla collar formulation concerning the nature of the option coefficients extend to more complicated collar designs. We increase the number of regimes and formulate the shared revenue for the outer regimes of the collar to depend on a proportion (less than 100%) of the revenue under the floors and over the ceilings. Our findings for this extension demonstrate that an analytical solution is obtainable despite the increase in complexity. Some of the sensitivities to changes in parameter values are similar to the previous collar model, but some are surprising. The notation we use in this section are specific, except that β_1 and β_2 are specified by (4).

Shaoul et al. (2012) report that for a U.K. rail franchise agreement, investors are reimbursed for 50% of any revenue shortfall below 98% of forecast and 80% below 96%, but suffer a claw-back of 50% of revenue exceeding 102%, equivalent to partial puts and calls. The Hinkley Point C arrangement specifies that if the project IRR exceeds 11.4% in nominal terms, the gain is shared 30:70 between the GOV and the GEN, and 60:40 if the IRR exceeds 13.5% in nominal terms and 11.5% in real terms.

We suppose there is a symmetrical arrangement with two downside risk sharing and two upside risk sharing arrangements. The actual price for an output is denoted by P. For the purpose of determining the price to be received by the GEN, the agreement with the government divides the price schedule into 5 distinct exhaustive regimes. The four junctions for neighboring regimes occur at $P = P_{LL}$, where P_{LL} represents the lowest limit, at $P = P_L$ where P_L is the lower limit, at $P = P_H$ where P_H is the higher limit, and at $P = P_{HH}$ where P_{HH} is the highest limit. Under Regime I with $P < P_{LL}$, the "price received" by the GEN is the actual price P plus a proportion $1 - w_{LL}$ of the shortfall below P_{LL} and a proportion $1 - w_L$ of the difference (P$_L$-P$_{LL}$). Under Regime II with $P_{LL} \leq P < P_L$, the price received is P plus a proportion $1 - w_L$ of the shortfall from P$_L$, where $0 \leq w_{LL} \leq w_L \leq 1$. Under Regime III with $P_L \leq P < P_H$, the price received is P,

and under Regime IV with $P_H \le P < P_{HH}$, the price received is P less a proportion $1 - w_H$ of $(P_H - P)$. Under Regime V with $P \ge P_{HH}$, the price received is P less a proportion $1 - w_{HH}$ of (P-P$_{HH}$) and less a proportion $1 - w_H$ of (P$_{HH}$-P$_H$), where $0 \le w_{HH} < w_H \le 1$. In the absence of any fixed costs and taxation, the regime value is determined not only from the price schedule but also from the presence of any switch options.

For each regime, if there exist opportunities for switching to an higher or lower neighboring regime, then these are represented by options, a call-style option for upward switching and a put-style option for downward switching, so both Regime II , III and IV are characterized by both call and put options, while Regime I by a call and Regime V by a put. Also, a switch producing a price advantage is represented by a positive option value coefficient, while that for a price disadvantage by a negative coefficient. The specification and associated price values for each of the five regimes are listed in Table 5.

Table 5 Regime Specification and Price Schedule

Regime	Specification	Value
I	$P < P_{LL}$	$V_I(P) = A_{11}P^{\beta_1}$ $+ \dfrac{w_{LL}P}{\delta} + \dfrac{(1-w_L)P_L}{r} + \dfrac{(w_L - w_{LL})P_{LL}}{r}$
II	$P_{LL} \le P < P_L$	$V_{II}(P) = A_{21}P^{\beta_1} + A_{22}P^{\beta_2}$ $+ \dfrac{w_L P}{\delta} + \dfrac{(1-w_L)P_L}{r}$
III	$P_L \le P < P_H$	$V_{III}(P) = A_{31}P^{\beta_1} + A_{32}P^{\beta_2}$ $+ \dfrac{P}{\delta}$
IV	$P_H \le P < P_{HH}$	$V_{IV}(P) = A_{41}P^{\beta_1} + A_{42}P^{\beta_2}$ $+ \dfrac{w_H P}{\delta} + \dfrac{(1-w_H)P_H}{r}$

V	$P_{HH} \le P$	$V_V(P) = A_{52}P^{\beta_2}$ $+ \dfrac{w_{HH}P}{\delta} + \dfrac{(1-w_H)P_H}{r} + \dfrac{(w_H - w_{HH})P_{HH}}{r}$

The eight unknown switch option coefficients, $A_{11}, A_{21}, A_{22}, A_{31}, A_{32}, A_{41}, A_{42}, A_{52}$, are determined from the value matching relationships and associated smooth pasting conditions. The value matching relationships, defined at each of the 4 junctions of neighboring regimes are:

$$\left[V_{II}(P) - V_I(P) \right]\Big|_{P=P_{LL}} = 0 \tag{19}$$

$$\left[V_{III}(P) - V_{II}(P) \right]\Big|_{P=P_L} = 0 \tag{20}$$

$$\left[V_{IV}(P) - V_{III}(P) \right]\Big|_{P=P_H} = 0 \tag{21}$$

$$\left[V_V(P) - V_{IV}(P) \right]\Big|_{P=P_{HH}} = 0 \tag{22}$$

Equations (19)-(22) together with the 4 associated smooth pasting conditions are sufficient to solve for the unknowns. The resulting solutions together with their signs are presented in Table 6. The coefficients having a positive value indicate that the corresponding switch options are owned by the GEN and contribute to their value, whilst those having a negative sign are sold or written. Table 6

Solutions and Conditions for the Switch Option Coefficients Partial Collar Model

Coefficient Solution	**Condition**
$A_{22} = \dfrac{(w_L - w_{LL})P_{LL}(r\beta_1 - r - \delta\beta_1)}{P_{LL}^{\beta_2}(\beta_1 - \beta_2)r\delta}$	$A_{22} \ge 0$
$A_{31} = -\dfrac{(1-w_H)P_H(r\beta_2 - r - \beta_2)}{P_H^{\beta_1}(\beta_1 - \beta_2)r\delta}$	$A_{31} \le 0$
$A_{21} = A_{31} + \dfrac{(1-w_L)P_L(r\beta_2 - r - \delta\beta_2)}{P_L^{\beta_1}(\beta_1 - \beta_2)r\delta}$	$A_{21} \ge 0$
$A_{32} = A_{22} + \dfrac{(1-w_L)P_H(r\beta_1 - r - \delta\beta_1)}{P_H^{\beta_2}(\beta_1 - \beta_2)r\delta}$	$A_{32} \ge 0$

$$A_{11} = A_{21} + A_{22}\frac{P_{LL}^{\beta_2}}{P_{LL}^{\beta_1}} + \frac{(w_L - w_{LL})P_{LL}(r-\delta)}{P_{LL}^{\beta_1} r\delta}$$	$A_{11} \geq 0$
$$A_{41} = A_{31}\frac{P_H^{\beta_1}}{P_H^{\beta_2}} + A_{32} + \frac{(1-w_H)P_H(r-\delta)}{r\delta P_H^{\beta_2}}$$ $$A_{42} = A_{31}\frac{P_H^{\beta_1}}{P_H^{\beta_2}} + A_{32} + \frac{(1-w_H)P_H(r-\delta)}{r\delta P_H^{\beta_2}} - A_{41}\frac{P_H^{\beta_1}}{P_H^{\beta_2}}$$ $$A_{52} = A_{41}\frac{P_{HH}^{\beta_1}}{P_{HH}^{\beta_2}} + A_{42} + \frac{(w_H - w_{HH})P_{HH}(r-\delta)}{r\delta P_{HH}^{\beta_2}}$$	<0

The ACTIVE partial collars have somewhat different sensitivities to changes in P and P volatility than the previous "full" collars[7]. Partial collars with the parameter values the same as Table 2, except the layers, designed so the VC value is about the same (VC=138) as the basic result, are floors 3.5,4, and ceilings 10, 10.5, and the risk sharing is .25, .5 on the downside and .5, .25 on the upside. Figure 11 shows a decreasing sensitivity to increases in P, which is intuitive compared to 100% risk sharing below the floor and above the ceiling. [While not exactly comparable due to the different ceiling, the value of a partial collar increases 96% and a full collar 76% as P goes from 6 to 18.]

The sensitivity of partial collar to changes in P volatility ("vegas") are slightly different, with the VC partial collar also decreasing as volatility increases, as Figure 12 shows with P=6 but with P=2 and P=18 below the figure.

***Figures 11 and 12 about here ***

How does the particular collar setting affect the exposure of a GEN to changes in prices, which is a critical risk consideration? Without a collar, the price exposure (delta) is of course simply 1/δ, or the LHS term of (6).   With a collar, the price exposure is the middle row of (16). When P=6, Figure 13 shows that the ACTIVE project with a collar becomes less sensitive to changes in P as the floor increases, but the floor delta from the middle row of (17) becomes more important as the floor increases.

***Figure 13 about here ***

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[7] A spreadsheet for the partial collars is available from the authors.

7   Conclusion

In a mainly analytical way, the properties of a plain vanilla collar, made up of a floor and cap, are investigated for an active asset using a real option formulation. The collar is composed of pairs of American perpetuity put and call options that confine a focal variable, such as revenue, price or volume, to a designated field specified by the floor and cap.

The role of the cap is to mitigate the cost to the government of guaranteeing a floor, and thereby inhibits the spread of any allegations of being over-generous. Governments can even create a "costless" collar by selecting a floor to induce investment and a cap that neutralizes the additional value it creates.

We provide novel analytical solutions for collars (11), floors only (13), ceilings only (15), floor (17) and ceiling deltas (18), and partial collars (Tables 5 and 6). In negotiating the original terms, it is critical for the GEN to focus on how the project value is sensitive to input changes, shown in Figures 1 to 4. Another use of these analytical expressions is to identify clearly the gains and losses for a principal (GOV) and agent (GEN) participating in an energy generating facility as parameter values change, shown in Figures 5 to 9. Different real option games are envisioned, where changes in some of the parameter values after an initial deal result in a *zero-sum game*. This basic framework may be useful in viewing the intended consequences of different appropriate arrangements, and in identifying incentives for the agent in operating a generating facility.

The primary numerical finding is that the GEN and GOV have contrasting "vegas" that is sensitivities to changes in perceived future price volatility, with generally the GEN hurt by high volatilities after the initial "fair" deal (Figures 2 and 6 a,b,c). Layered collars, and less than 100% risk sharing under the floors and over the ceilings expose the GEN less to changes in price (Figure 11) and price volatilities (Figure 12), which is intuitive. The precise exposure depends on the floor and ceiling levels relative to the current price (Figure 13), and on the degree of risk sharing (downside and upside proportions for different layers).

There are several implicit assumptions behind our analytical framework. (i) The arrangements are perpetual American call or put options, and a perpetual series of cash flows, viewed in continuous time. Real arrangements may not perpetual, so both the options and the cash flows

would have to be reformatted as perpetuals less forward start options[8], especially for shorter-term arrangements. This may not be a significant problem for 60 year arrangements when discount rates are high. (ii) Parameter values such as interest rates, yield, price volatility, price floors and ceilings are considered constant or deterministic. Relaxing some of these assumptions is an interesting extension. (iii) Sometimes arrangements specify that the concession termination is based on a specified achieved internal rate of return, or cumulative net present value, or accumulated net cash flows. We do not focus on negotiated exit prices or for GEN or GOV determined exit timing. (iv) GEN facilities and arrangements ignore GOV creditability issues and unexpected failures or physical disasters. GEN arrangements are envisioned as enforceable, without credit or default risk for either party, and investments are irrevocable, immediate, and terms cannot be re-negotiated over time. (v) The framework models are viewed in continuous time whereas prices, minimum and maximum revenue compensations and payments are likely to occur in discrete time, which would have to be specified (and verified) in any practical arrangement. (vi) We do not allow for operating costs that are not already deducted from the net price or net revenues, or for periodic maintenance requirements. (vii) The revenue stream ignores other possible real options such as project cancellation, downsizing, renegotiation, expansion and resale, dynamic pricing for times of usage, and extensions into other activities such as further arrangements with electricity customers, or retail activities for motorway operators. (viii) While the GEN arrangement cited herein concerns an electricity generating facility, other arrangements such as transportation, building and operating hospitals and educational establishments may not have clear objectives such as sharing revenue risks and benefits. Suitably designed optional elements may incorporate some of the same, or conceivably completely different objectives. Most of these issues present interesting aspects for future research, maybe starting with this basic framework.

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[8] See Adkins et al. (2018) for a solution to this problem.

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Figure 1

Sensitivity of Collar to Changes in P

P	0	2	4	6	8	10	12	14	16	18	20
VC	100.0000003	107.68	125.60	150.00	174.84	197.77	218.05	235.44	249.84	261.50	271.10
VC PV	100.0000000	100.00	100.00	150.00	200.00	250.00	300.00	350.00	390.06	390.06	390.06
AC11*P^β1	**0.00**	**7.68**	25.60	51.77	85.32	125.72	172.55	225.53	284.40	348.97	419.04
AC21*P^β1	0.00	-4.45	**-14.83**	**-29.98**	**-49.42**	**-72.81**	**-99.94**	-130.62	-164.72	-202.11	-242.70
AC22*P^β2	99547	67.37	**40.42**	**29.98**	**24.25**	**20.58**	**17.99**	16.06	14.55	13.34	12.35
AC32*P^β2	-959191	-649.14	-389.49	-288.89	-233.70	-198.27	-173.34	**-154.72**	**-140.22**	**-128.57**	**-118.96**

VC is from EQ 6, based on an initial arrangement of the base case in Table 2, except that P_L=4 and P_H=15.6 (a costless collar when P=6) where the option coefficient A_{C11} holds when $P<P_L$, and A_{C32} when $P>P_H$, otherwise A_{C21} and A_{C22}, compared to the PV (6) without the option coefficients.

Figure 2

Collar Value as Function of Volatility

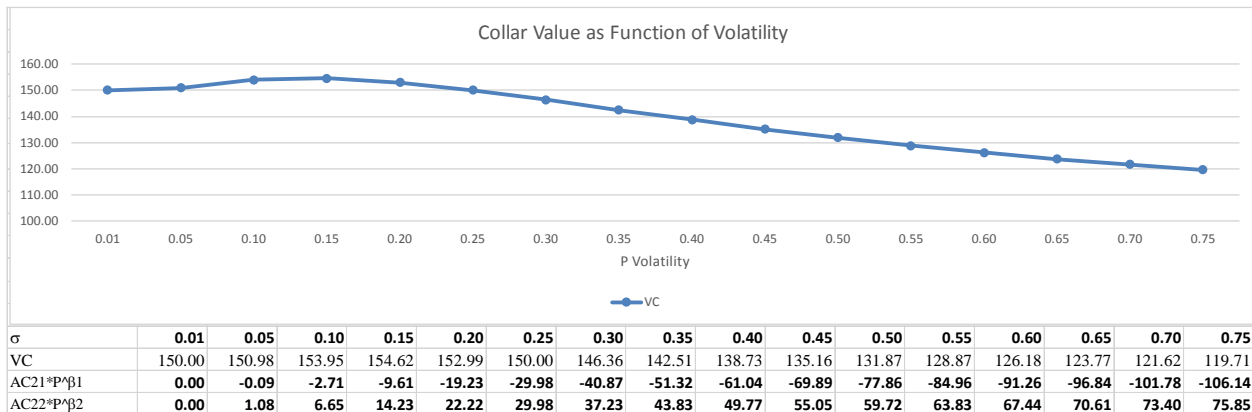| σ	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
VC	150.00	150.98	153.95	154.62	152.99	150.00	146.36	142.51	138.73	135.16	131.87	128.87	126.18	123.77	121.62	119.71
AC21*P^β1	0.00	-0.09	-2.71	-9.61	-19.23	-29.98	-40.87	-51.32	-61.04	-69.89	-77.86	-84.96	-91.26	-96.84	-101.78	-106.14
AC22*P^β2	0.00	1.08	6.65	14.23	22.22	29.98	37.23	43.83	49.77	55.05	59.72	63.83	67.44	70.61	73.40	75.85

VC (6) is based on an initial arrangement of the base case in Table 2, except for varying volatility, if P_L=4 and P_H=15.6 (a costless collar when P=6). The option coefficients $-A_{C21}$ =A_{C22} only when P=6, but the negative call value increases at a faster rate than the put option coefficient as volatility increases past 25%, so the collar arrangement is lower than the PV without the option coefficients at higher levels of volatility.

Figure 3

AC21*P^β1	-29.98	-36.74	-43.46	-49.93	-56.02	-61.68	-66.88	-71.63	-75.97
AC22*P^β2	29.98	17.26	10.36	6.42	4.09	2.67	1.78	1.21	0.84
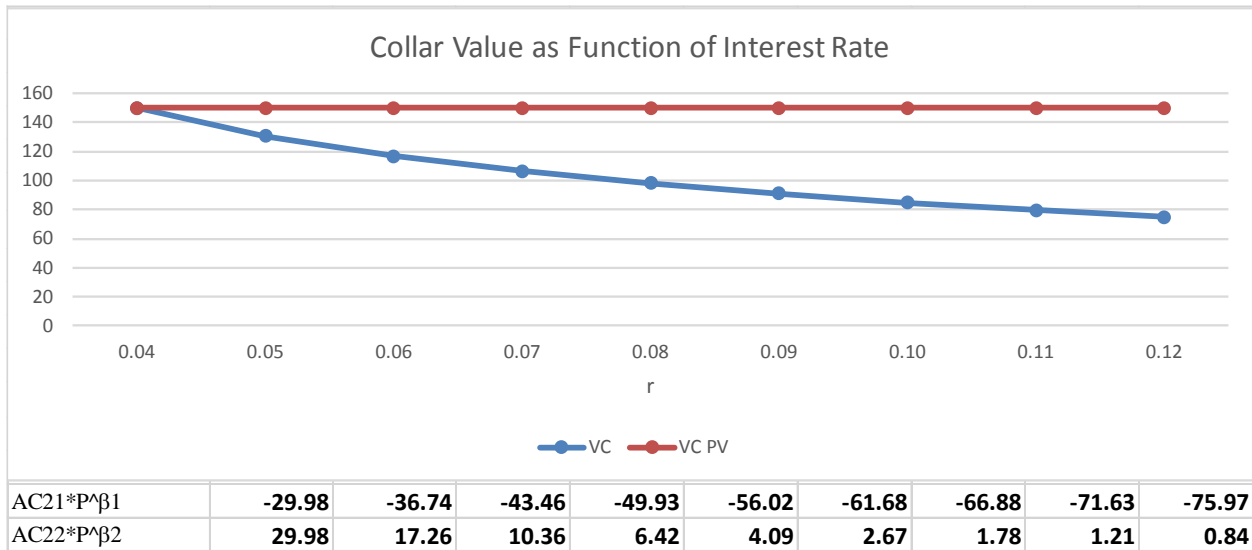
VC (6) is based on an initial arrangement of the base case in Table 2, except for varying the interest rate, when $P_L=4$ and $P_H=15.6$ (a costless collar) with P=6. The option coefficients A_{C21} and A_{C22} hold when $P_L<P<P_H$. The collar arrangement is compared to the PV without the option coefficients. The PV is not affected by changes in the interest rate because the cash flows are uncertain, and so divided by the yield.

Figure 4

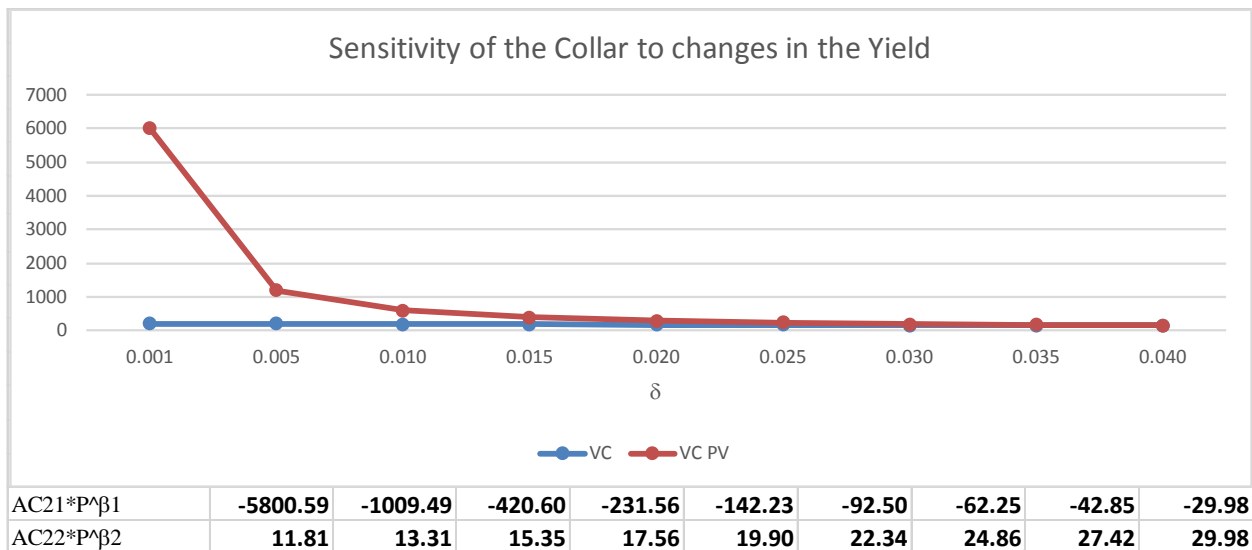|									
AC21*P^β1	-5800.59	-1009.49	-420.60	-231.56	-142.23	-92.50	-62.25	-42.85	-29.98
AC22*P^β2	11.81	13.31	15.35	17.56	19.90	22.34	24.86	27.42	29.98

The VC (6) is based on an initial arrangement of the base case in Table 2, except for varying the yield, while $P_L=4$ and $P_H=15.6$ (a costless collar) with P=6. The option coefficients A_{C21} and A_{C22} hold when $P_L<P<P_H$. The collar arrangement is compared to the PV without the option coefficients.

Figure 5

Risk Sharing as Function of P

Legend: GOV Profit — GEN + GOV — GEN Profit
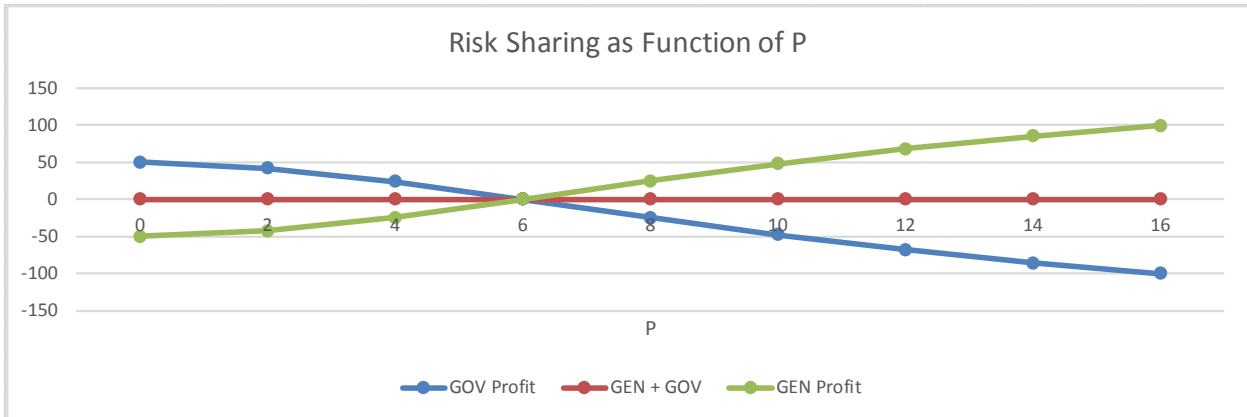
Assumes							
At P=0, the GEN would have lost -150 without the Min P guarantee and RO, but instead loses -50.							
At P=16, the GEN would have gained 400-150=250 without the ceiling ceded to the GOV, but instead gains 100.							
Assumes the GOV has ceded control over a valuable monopoly, so GOV profit deducts the PV at all P (customers lose).							
ACTIVE facility is sold to GEN at fair value P/δ when P=6 and							
Government (GOV) guarantees a minimum P of 4 and receives all P over 15.60.							
At P=6 and the other parameter values, -call=put for a "costless collar",							
so the combined "profit" over the fair value of the GEN and GOV is 0.							
Ignores the initial construction cost K of the facility, which could equal the VC if GEN demands no excess profits.							

Figure 6a
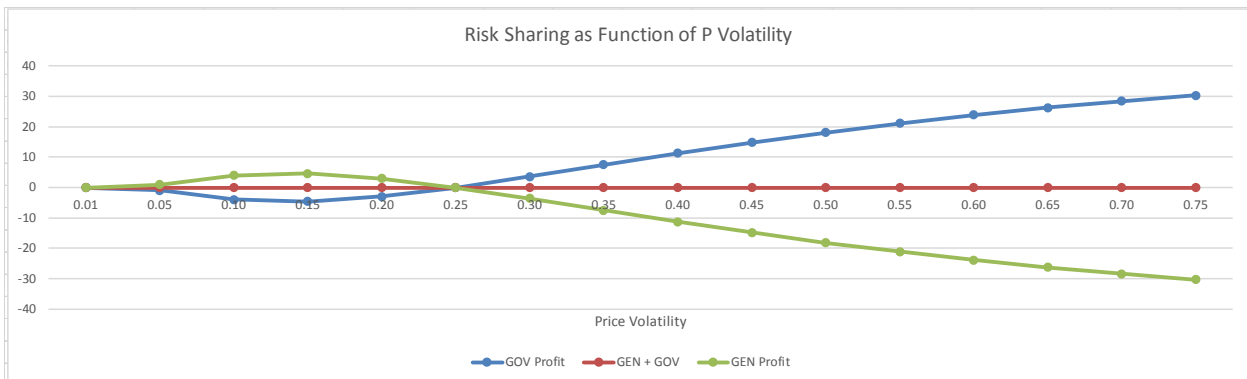
Risk Sharing as Function of P Volatility

Legend: GOV Profit — GEN + GOV — GEN Profit

Interpretation							
At σ=.01, the call and put for both the GEN and GOV would have been of little value, when P=6.							
At σ=.75, the call would be worth 106.14 for the GOV (and -106.4 for the GEN), while the put would be worth -75.85 for the GOV.							
So with these values, the GEN would welcome P volatility below 25%, and the GOV benefit from higher volatility.							
ACTIVE facility is sold to GEN at fair value P/δ when							
Government (GOV) guarantees a minimum P of 4 and receives all P over 15.60.							
At P=6 and the other parameter values, -call=put for a "costless collar",							
so the combined "profit" over the fair value of the GEN and GOV is 0.							

Figure 6b

Volatility Effect on Risk Sharing (Ac_{11}) when P=2
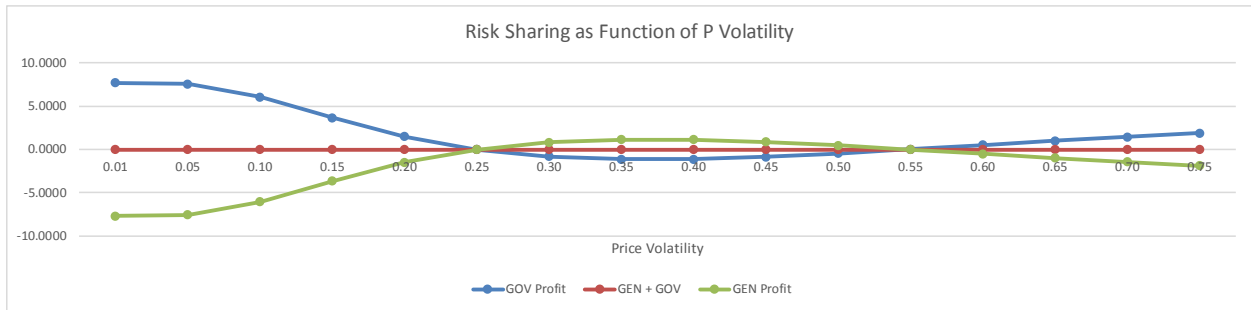
Risk Sharing as Function of P Volatility

Figure 6c

Volatility Effect on Risk Sharing (A_{c32}) when P=18

Risk Sharing as Function of P Volatility

Figure 7

Risk Sharing as Function of Interest Rate

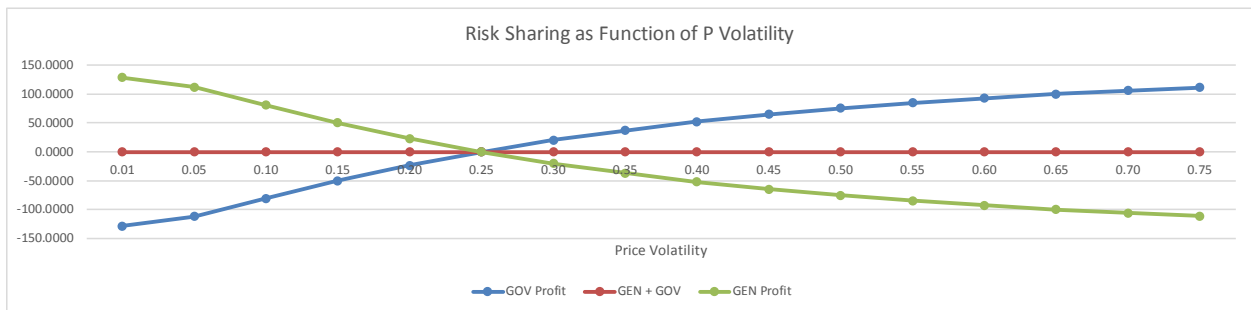| Assumes the GOV has ceded control over a valuable monopoly, so GOV profit deducts the PV when P=6.					
ACTIVE facility is sold to GEN at fair value P/δ when P=6 and					
Government (GOV) guarantees a minimum P of 4 and receives all P over 15.60, interest rate is 4%.					
At P=6 and the other parameter values, -CALL=PUT for a "costless collar",					
so the combined "profit" over the fair value of the GEN and GOV is 0.					

Figure 8

Risk Sharing as Function of Yield

Assumes the GOV has ceded control over a valuable monopoly, so GOV profit deducts the PV when P=6.					
ACTIVE facility is sold to GEN at fair value P/δ when P=6 and					
Government (GOV) guarantees a minimum P of 4 and receives all P over 15.60, interest rate is 4%.					
At P=6 and the other parameter values, -CALL=PUT for a "costless collar",					
so the combined "profit" over the fair value of the GEN and GOV is 0.					

Figure 9

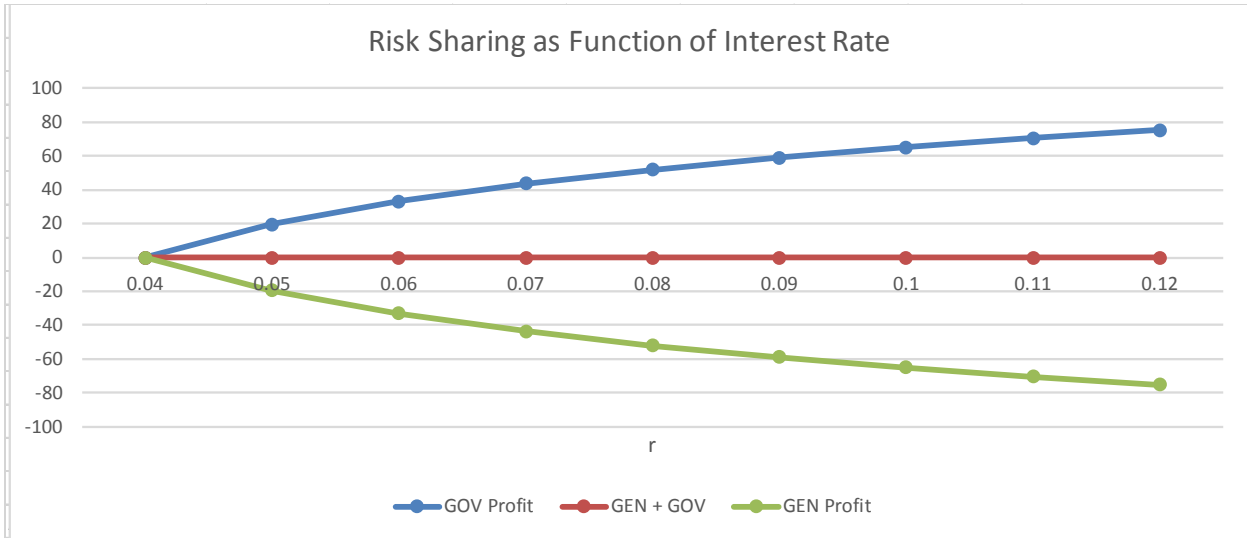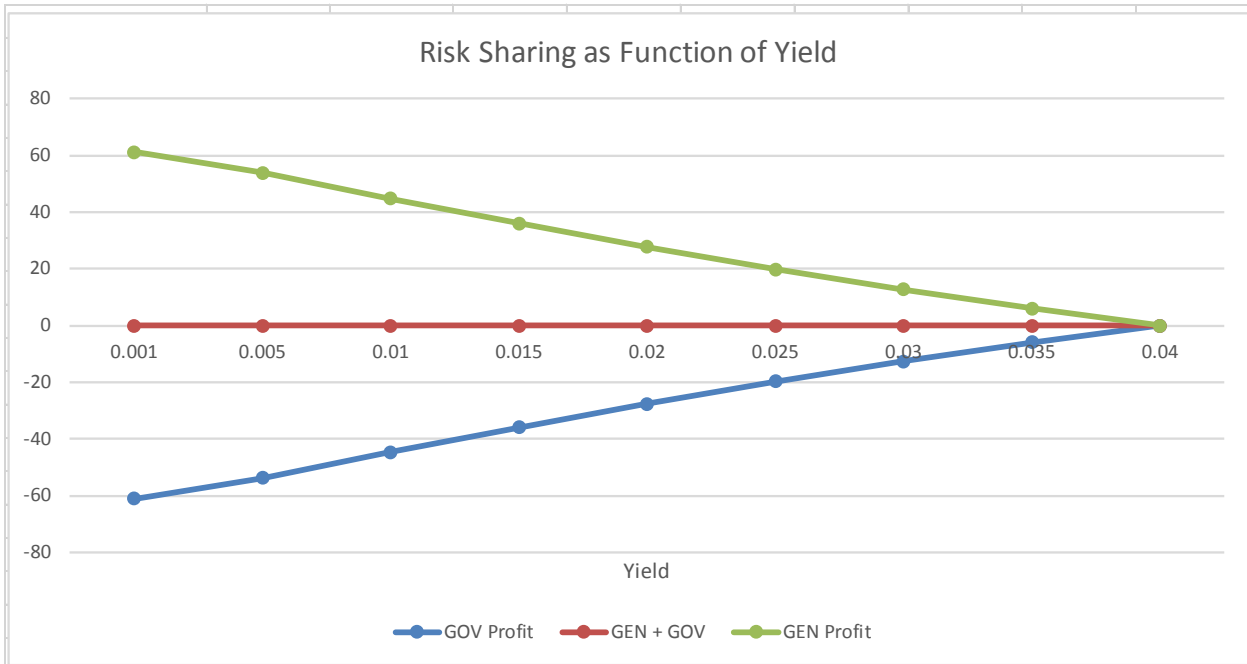## Risk Sharing as function of Floor
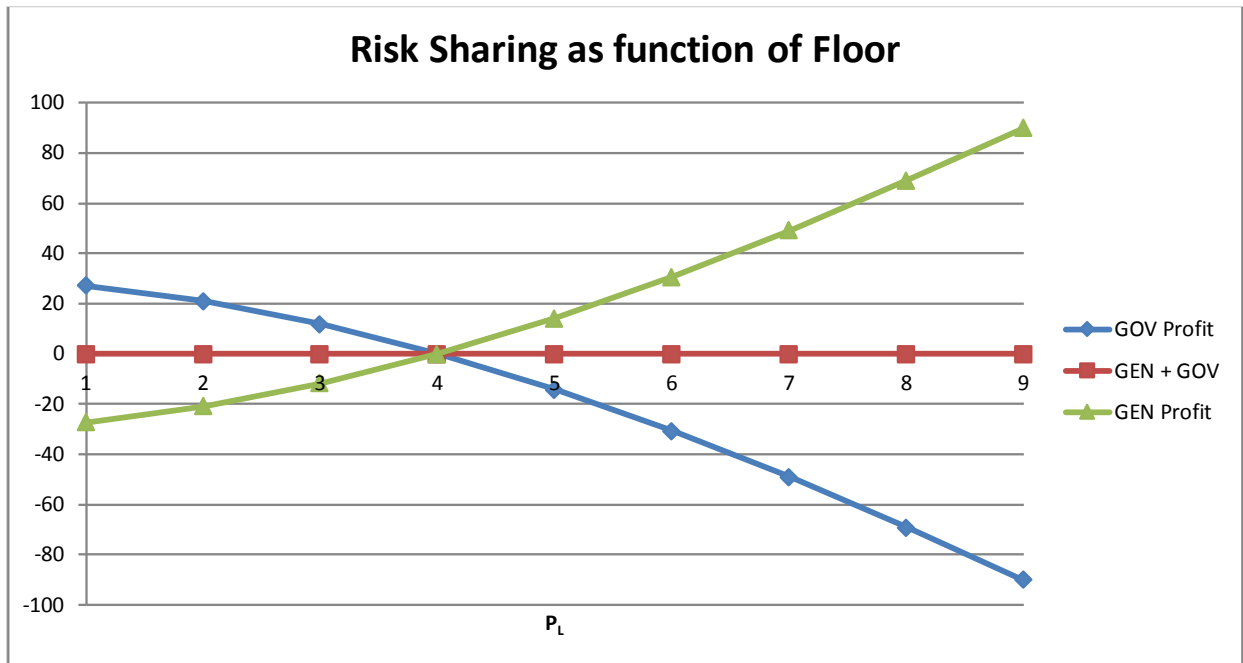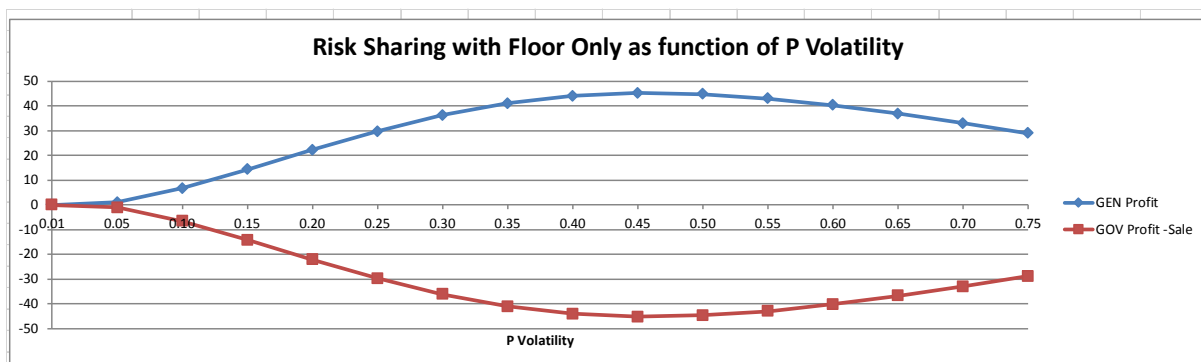

Legend: GOV Profit, GEN + GOV, GEN Profit

Assumes the GOV has ceded control over a valuable monopoly, so GOV profit deducts the PV when P=6.
ACTIVE facility is sold to GEN at fair value P/δ when P=6 and δ is 4%. Base case is
GOV guarantees a minimum R of 4 and receives all R over 15.60, interest rate is 4%.
At P=6 and the other parameter values, -CALL=PUT for a "costless collar",
so the combined "profit" over the fair value of the GEN and GOV is 0.

Figure 10

Risk Sharing with Floor Only as function of P Volatility

Legend: GEN Profit, GOV Profit -Sale

Interpretation
At σ=.01, the CALL and PUT for both the GEN and GOV would have been of little value, when P=6 when there is a floor only.
At σ=.75, the CALL would be worth 46.93 for the GOV (and -46.93 for the GEN), while the PUT would be worth -75.85 for the GOV.
So with these values, the GEN would welcome P volatility especially around 45%.
Assumes the monopoly over which the GOV cedes control is of no value to the GOV.
ACTIVE facility is sold to GEN at fair value P/δ when P=6.

Figure 11

P	2	4	6	8	10	12	14	16	18
VC	**40.26**	**101.91**	**138.24**	**173.03**	**205.68**	**223.71**	**240.37**	**256.12**	**271.23**
Regime I	**40.26**	51.90	63.14	74.03	84.60	94.85	104.81	114.49	123.89
Regime II	64.62	**101.91**	139.12	178.27	219.66	263.33	309.25	357.37	407.63
Regime III	67.24	101.91	**138.24**	**173.03**	**205.68**	236.08	264.20	290.08	313.77
Regime IV	6.21	88.66	135.85	172.96	205.68	236.05	265.00	293.02	320.39
Regime V	54.72	125.33	160.35	185.20	205.63	**223.71**	**240.37**	**256.12**	**271.23**
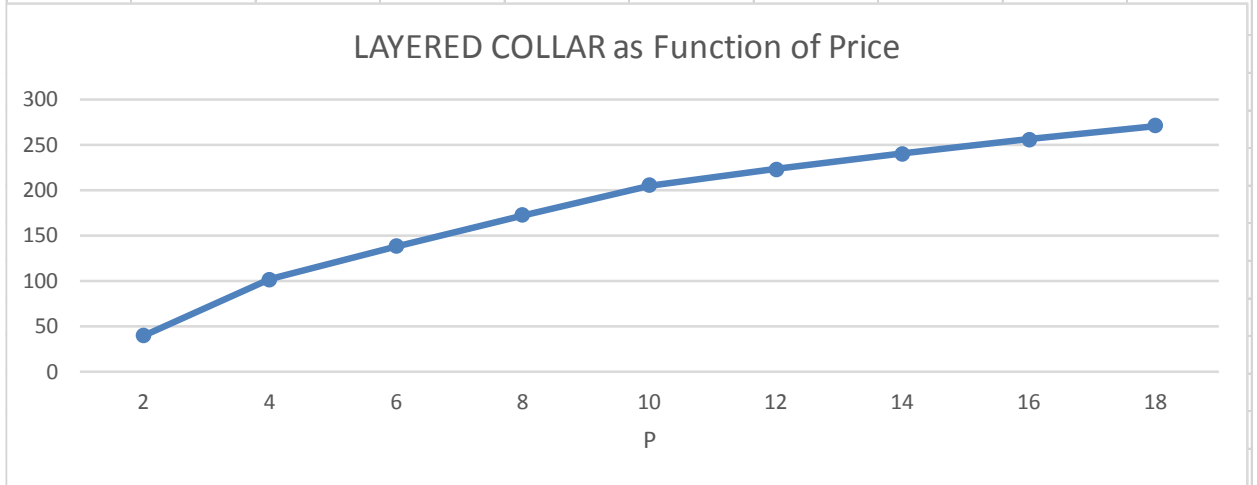
LAYERED COLLAR as Function of Price

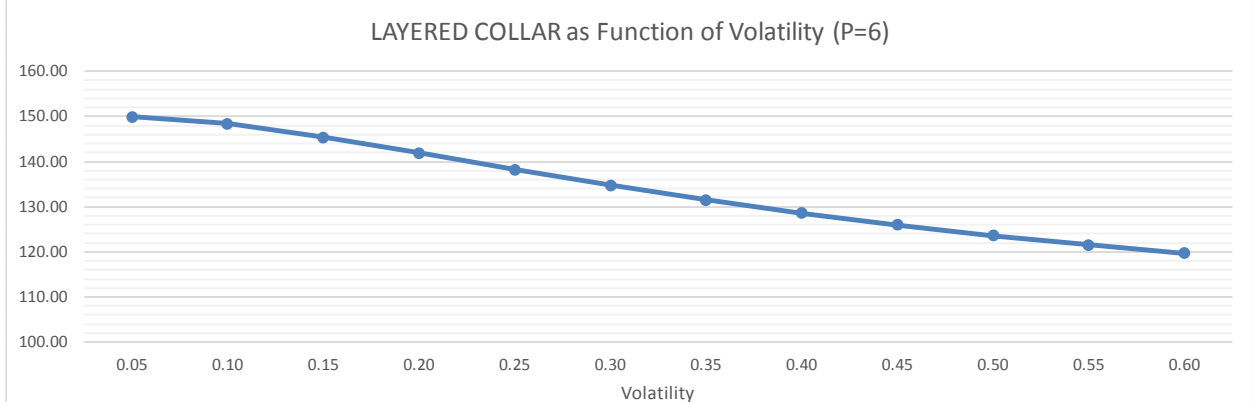Using Table 2 parameter values, except that floors are 3.5, 4 and ceilings 10, 10.5, risk sharing is .25, .5 on the downside and .5, .25 on the upside, calculated from Table 5 and 6 equations. Note that the layered collar values are based on escalating regimes I when P=2, II when P=4, III when P=6 to 10, V when P is 12+.

Figure 12

σ	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
VC P=6	**149.95**	**148.38**	**145.39**	**141.86**	**138.24**	**134.76**	**131.54**	**128.60**	**125.97**	**123.63**	**121.56**	**119.74**

LAYERED COLLAR as Function of Volatility (P=6)

VC P=2	40.62	40.79	40.93	40.74	40.26	39.57	38.77	37.94	37.12	36.34	35.61	34.95
VC P=18	299.47	294.76	287.25	279.11	271.23	263.93	257.34	251.47	246.28	241.72	237.72	234.22

Using Table 2 parameter values, except that floors are 3.5, 4 and ceilings 10, 10.5, risk sharing is .25, .5 on the downside and .5, .25 on the upside, calculated from Table 5 and 6 equations. The layered collar values are based on regime III when P=6, but regime I when P=2, and V when P=18 are also shown.

Figure 13

PL	1	1.5	2	2.5	3	3.5	4	4.5	5
VC Δ	12.62	12.28	11.85	11.33	10.72	10.03	9.27	8.44	7.53
$\Delta AC22*P^{\wedge}\beta2$	17.55	23.67	29.25	34.48	39.44	44.19	48.76	53.18	57.47

Collar Δ and Floor Δ as Function of Floor
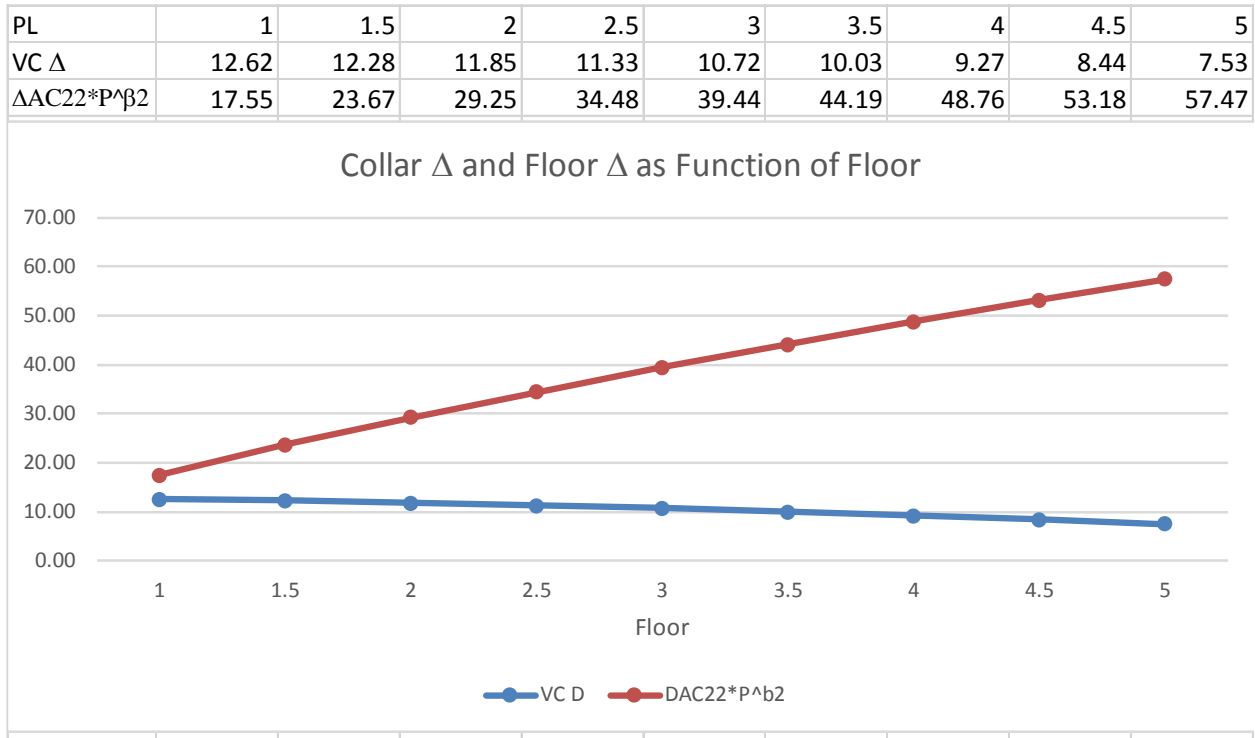
VC Δ is the middle row of (16), showing that the ACTIVE project with a collar when P=6 becomes less sensitive to changes in P as the floor increases, but the floor Δ from (17) becomes more important as the floor increases.

Table 2

Basic Inputs and Outputs for COLLAR ARRANGEMENT

	A	B	C	D
1			**ACTIVE GEN WITH COLLAR**	
2	**INPUT**			EQ
3	P	6.00		
4	K	100.00		
5	σ	0.25		
6	r	0.04		
7	δ	0.04		
8	P_L	**4**		
9	P_H	**10**		
10	**OUTPUT**			
11	VC	**138.3688**		6
12	VC PV	**150.0000**	**IF(B3<B8,B8/B6,IF(B3>B9,B9/B6,B3/B7))**	6
13	P/δ	150.0000	B3/B7	
14	$β_1$	1.7369	0.5-(B6-B7)/(B5^2)+SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
15	β2	-0.7369	0.5-(B6-B7)/(B5^2)-SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
16	AC11*P^β1	**40.1361**	**B21*(B3^B14)**	
17	AC21*P^β1	**-41.6129**	**B22*(B3^B14)**	
18	AC22*P^β2	**29.9818**	**B23*(B3^B15)**	
19	AC32*P^β2	**-117.2670**	**B24*(B3^B15)**	
20	VC	138.3688	B12+B17+B18	
21	AC11	1.7862	(B9/(B9^B14)-B8/(B8^B14))*(B25/B27)	11
22	AC21	-1.8520	(B9/(B9^B14))*(B25/B27)	11
23	AC22	112.2797	(-B8/(B8^B15))*(B26/B27)	11
24	AC32	-439.16	(B9/(B9^B15)-B8/(B8^B15))*(B26/B27)	11
25	[ ]	-0.0400	(B6*B15-B6-B7*B15)	11
26	( )	-0.0400	(B6*B14-B6-B7*B14)	11
27	{ }	0.0040	(B14-B15)*B6*B7	11
28	VC		IF(B3<B8,B8/B6+B21*(B3^B14),IF(B3>B9,B9/B6+B24*(B3^B15),B3/B7+B22*(B3^B14)+B23*(B3^B15)))	
29	ODE	0.0000	0.5*(B5^2)*(B3^2)*B31+(B6-B7)*B3*B30-B6*B11+MIN(MAX(B8,B3),B9)	5
30	VC Δ	9.2711	IF(B3<B8,B14*B21*(B3^(B14-1)),IF(B3>B9,B15*B24*(B3^(B15-1)),1/B7+B14*B22*(B3^(B14-1))+B15*B23*(B3^(B15-1))))	16
31	VC Γ	-0.4136	IF(B3<B8,B14*(B14-1)*B21*(B3^(B14-2)),IF(B3>B9,B15*(B15-1)*B24*(B3^(B15-2)),B14*(B14-1)*B22*(B3^(B14-2))+B15*(B15-1)*B23*(B3^(B15-2))))	
32	Floor Deltas			
33	ΔAC11*P^β1	-0.6703	-((1-B14)*(B8^(-B14))*(B25/B27))	17
34	ΔAC22*P^β2	48.7556	-(1-B15)*B8^(-B15)*(B26/B27)	17
35	ΔAC32*P^β2	48.7556	-(1-B15)*B8^(-B15)*(B26/B27)	17
36	Ceiling Deltas			
37	ΔAC11*P^β1	0.1365	(1-B14)*B9^(-B14)*(B25/B27)	18
38	ΔAC21*P^β1	0.1365	(1-B14)*(B9^(-B14)*(B25/B27))	18
39	ΔAC32*P^β2	-95.7809	(1-B15)*(B9^(-B15)*(B26/B27))	18

The COLLAR VC is from (6), power parameters from (4), the option coefficients from (11), the collar delta from (16), the floor deltas from (17) and the ceiling deltas from (18).

Table 3

Switch Option Coefficients for the With-Collar Model for Variations in Floor and Cap Levels

	Panel A: A_{C11}					Panel B: A_{C21}				
P_H	P_L=0	P_L=1	P_L=2	P_L=3	P_L=4	P_L=0	P_L=1	P_L=2	P_L=3	P_L=4
10	0.0000	8.2537	4.2116	2.6454	1.7862	-1.8520	-1.8520	-1.8520	-1.8520	-1.8520
20	0.0000	8.9944	4.9523	3.3862	2.5270	-1.1112	-1.1112	-1.1112	-1.1112	-1.1112
50	0.0000	9.5400	5.4979	3.9317	3.0726	-0.5656	-0.5656	-0.5656	-0.5656	-0.5656
100	0.0000	9.7663	5.7241	4.1580	3.2988	-0.3394	-0.3394	-0.3394	-0.3394	-0.3394
200	0.0000	9.9020	5.8599	4.2937	3.4346	-0.2036	-0.2036	-0.2036	-0.2036	-0.2036
500	0.0000	10.0020	5.9599	4.3937	3.5345	-0.1037	-0.1037	-0.1037	-0.1037	-0.1037
1000	0.0000	10.0435	6.0013	4.4352	3.5760	-0.0622	-0.0622	-0.0622	-0.0622	-0.0622
Infinity	0.0000	10.1057	6.0635	4.4974	3.6382	0.0000	0.0000	0.0000	0.0000	0.0000

	Panel C: A_{C22}					Panel D: A_{C32}				
P_H	P_L=0	P_L=1	P_L=2	P_L=3	P_L=4	P_L=0	P_L=1	P_L=2	P_L=3	P_L=4
10	0.000	10.106	33.685	68.123	112.280	-551	-541	-518	-483	-439
20	0.000	10.106	33.685	68.123	112.280	-1838	-1828	-1804	-1770	-1726
50	0.000	10.106	33.685	68.123	112.280	-9027	-9017	-8994	-8959	-8915
100	0.000	10.106	33.685	68.123	112.280	-30090	-30080	-30057	-30022	-29978
200	0.000	10.106	33.685	68.123	112.280	-100299	-100289	-100265	-100231	-100187
500	0.000	10.106	33.685	68.123	112.280	-492596	-492586	-492562	-492528	-492484
1000	0.000	10.106	33.685	68.123	112.280	-1641948	-1641938	-1641914	-1641880	-1641836
Infinity	0.000	10.106	33.685	68.123	112.280	Infinity	Infinity	Infinity	Infinity	Infinity

Using the baseline data, the coefficients $A_{C11}, A_{C21}, A_{C22}, A_{C32}$ are evaluated from (11) for the various indicated P_L and P_H values.

Table 4

Costless Collars

P_L	1	1.5	2	2.5	3	3.5	4	4.5	5
P_H	409.48	157.47	79.93	47.24	30.74	21.37	15.60	11.82	9.22
$AC21*P^{\beta 1}$	-2.70	-5.46	-8.99	-13.25	-18.19	-23.78	-29.98	-36.79	-44.18
$AC22*P^{\beta 2}$	2.70	5.46	8.99	13.25	18.19	23.78	29.98	36.79	44.18

Based on Table 2 base parameter values except for changing P_L, the P_H is calculated as that ceiling which results in the negative written call equal to the held put for Regime II, when P is between P_L and P_H.