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Research Article

Implications of $\delta_l^{\mathrm{CP}} \sim 270^\circ$ and $\theta_{23} \gtrsim 45^\circ$ for Texture Specific Lepton Mass Matrices and $0\nu\beta\beta$ Decay

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We study the phenomenological consequences of recent results from atmospheric and accelerator neutrino experiments, favoring normal neutrino mass ordering $m_1 < m_2 < m_3$, a near maximal lepton Dirac CP phase $\delta_l \sim 270^\circ$ along with $\theta_{23} \gtrsim 45^\circ$, for possible realization of natural structure in the lepton mass matrices characterized by $(M_{ij}) \sim O(\sqrt{m_i m_j})$ for $i, j = 1, 2, 3$. It is observed that deviations from parallel texture structures for M_l and M_ν are essential for realizing such structures. In particular, such hierarchical neutrino mass matrices are not supportive for a vanishing neutrino mass $m_{\nu_1} \to 0$ characterized by $\mathrm{Det} M_\nu \neq 0$ and predict $m_{\nu_1} \simeq (0.1\text{–}8.0)$ meV, $m_{\nu_2} \simeq (8.0\text{–}13.0)$ meV, $m_{\nu_3} \simeq (47.0\text{–}52.0)$ meV, $\Sigma \simeq (56.0\text{–}71.0)$ meV, and $\langle m_{ee} \rangle \simeq (0.01\text{–}10.0)$ meV, respectively, indicating that the task of observing a $0\nu\beta\beta$ decay may be rather challenging for near future experiments.

1. Introduction

One of the intriguing phenomena in particle physics is the origin of fermion masses which appear to span several orders of magnitudes starting with neutrinos to the top quark. The masses and flavor mixing schemes of quarks and leptons are significantly different with the quark sector exhibiting strong mass hierarchy, small mixing angles, and relatively heavier mass spectrum whereas the neutrinos are extremely light while two of their mixing angles are still large. In the current scenario, there is also a lack of consensus on the nature of neutrinos, that is, Dirac or Majorana along with doubts on the possible ordering of neutrino masses, namely, normal, that is, $m_1 < m_2 < m_3$ (NO), or inverted, that is, $m_3 < m_1 < m_2$ (IO). This nevertheless makes the task of constructing the fermion mass matrices nontrivial especially in the context of quark-lepton complementarity.

The confirmation of Higgs Boson by the ATLAS and CMS collaborations [1] completes the Standard Model (SM) of particle physics. Within this model, the quark mass terms in the Lagrangian are expressible as

$$-L_{\mathrm{mass}}^{\mathrm{quarks}} = \overline{q}_{uL} M_u q_{uR} + \overline{q}_{dL} M_d q_{dR} + \text{h.c.}, \quad (1)$$

where $q_{uL(R)}$ and $q_{dL(R)}$ are the left (right) handed quark fields and M_q are the quark mass matrices with u, d for the "up" type and "down" type quarks. The resulting weak charged current quark interactions are given by

$$-L_{\mathrm{cc}}^{\mathrm{quarks}} = \frac{g}{\sqrt{2}} \overline{(u\ \ c\ \ t)}_L \gamma^\mu V_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + \text{h.c.}, \quad (2)$$

where $V_{\mathrm{CKM}} = U_L^{u\dagger} U_L^d$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2, 3] or the quark mixing matrix measuring the nontrivial mismatch between the flavor and mass eigenstates of quarks; for example,

$$\begin{aligned} U_L^{u\dagger} M_u M_u^\dagger U_R^u &= \mathrm{Diag}\left(m_u^2,\ m_c^2,\ m_t^2\right), \\ U_L^{d\dagger} M_d M_d^\dagger U_R^d &= \mathrm{Diag}\left(m_d^2,\ m_s^2,\ m_b^2\right), \end{aligned} \quad (3)$$

where U^q are unitary matrices.

Interestingly, the quark masses as well as the elements of CKM matrix observe a hierarchical pattern: namely, $m_1 \ll m_2 \ll m_3$ and $(V_{ub}, V_{td}) < (V_{cb}, V_{ts}) < (V_{us}, V_{cd}) < (V_{ud}, V_{cs}, V_{tb})$. It is natural to expect this hierarchy to be embedded within the corresponding quark mass matrices: namely, (for $q = u, d$)

$$M_{11} < M_{12,21} \lesssim M_{13,31} < M_{22} < M_{23,32} < M_{33}, \qquad (4)$$

with $M_{22} \ll M_{33}$. Recent investigations [4] in this regard indicate that the current quark mixing data indeed permit the quark mass matrices to have such a natural and hierarchical structure provided $(M_{ij}) \sim O(\sqrt{m_i m_j})$ for $i, j = 1, 2, 3$, $i \neq j$ and $(M_{ii}) \sim O(m_i)$. Such hierarchical mass matrices have been referred to as *natural mass matrices* [5]. In particular, naturalness provides a rationale framework to correlate the observed fermion mass ratios, the corresponding mass matrices, and observed mixing angles. Specifically, for $(M_{13}) = (M_{31}) \neq 0$, the observed strong hierarchy among the quark masses and CKM elements gets naturally translated onto the structure of the corresponding mass matrices.

A concomitant of such naturalness in mass matrices is the absence of parallel texture structure for the "up" and "down" type quark mass matrices [4]. A *parallel* texture structure corresponds, for example, to the mass matrices M_u and M_d with texture zeros at identical positions in both the mass matrices. Hierarchical structures have fetched greater importance in the literature as these predict certain very simple yet compelling relations among the CKM elements and the quark mass ratios [4, 6–14].

However, the mass spectrum for leptons is quite distinguished from the quark sector, wherein the charged leptons masses are strongly hierarchical, that is, $m_e \ll m_\mu \ll m_\tau$, while at least two of the neutrinos are allowed to have the same order of mass. It should be interesting to investigate if naturalness can provide a unique explanation for the fermion mass matrices, corresponding observed mass spectra, and the mixing angles both for the quark and the lepton sectors.

Since the neutrinos are massless within the SM, one has to explore beyond the realms of SM to comprehend the origin of neutrino masses and observe neutrino oscillations phenomenon. A simplistic way to achieve this is to extend the SM theory by assuming neutrinos as Dirac-like particles. In this case, the neutrinos acquire mass through the Higgs mechanism in the similar way as quarks and charged leptons do within the SM, through a Dirac mass term: for example,

$$\begin{aligned} -L^{l}_{\text{mass}} &= \bar{l}_L M_l l_R + \text{h.c.}, \\ -2L^{\text{Dirac}}_{\text{mass}} &= \bar{\nu}_L M_{\nu_D} \nu_R + \text{h.c.}, \end{aligned} \qquad (5)$$

where M_l and M_{ν_D} represent the charged-lepton and Dirac neutrino mass matrix, respectively. Indeed, the current experiments have not ruled out such a possibility. In this context, it is also observed that highly suppressed Yukawa couplings for Dirac neutrinos can naturally be achieved using models with extra spatial dimensions [15, 16] or through radiative mechanisms [17–22]. However, such a possibility is perceived to be highly unlikely due to several orders of magnitude difference among m_α $(\alpha = e, \mu, \tau)$ and m_{ν_i} $(i = 1, 2, 3)$.

A rather convincing and natural explanation of neutrino masses can be obtained if neutrinos are assumed to be Majorana particles. This usually involves adding the lepton number (and flavor) violating Majorana mass terms for neutrinos in the Lagrangian; for example,

$$-2L^{\text{Majorana}}_{\text{mass}} = \bar{\nu}_L M_{\nu_L} \nu^c_R + \bar{\nu}^c_L M_{\nu_R} \nu_R, \qquad (6)$$

where M_{ν_L} and M_{ν_R} correspond, respectively, to the left and right-handed Majorana neutrino mass matrices and the latter usually has an extremely high mass scale. This facilitates generating the light neutrino masses through Type I or Type II seesaw mechanisms: namely,

$$\begin{aligned} M_\nu &= -M^T_{\nu_D} M^{-1}_{\nu_R} M_{\nu_D}, \\ M_\nu &= M_{\nu_L} - M^T_{\nu_D} M^{-1}_{\nu_R} M_{\nu_D}, \end{aligned} \qquad (7)$$

where M_ν is usually a complex symmetric matrix: for example,

$$M_\nu = \begin{pmatrix} e_\nu & a_\nu & f_\nu \\ a_\nu & d_\nu & b_\nu \\ f_\nu & b_\nu & c_\nu \end{pmatrix}. \qquad (8)$$

This allows writing the corresponding charged weak current term for leptons as

$$-L^{\text{leptons}}_{\text{cc}} = \frac{g}{\sqrt{2}} \overline{(\nu_e \;\; \nu_\mu \;\; \nu_\tau)}_L \gamma^\mu V \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L W^+_\mu + \text{h.c.}, \qquad (9)$$

where $V = V_{\text{PMNS}} = U^\dagger_{lL} U_{\nu_L}$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [23] or the neutrino mixing matrix and emerges through the diagonalization of the matrices M_l and M_ν: for example,

$$\begin{aligned} U^\dagger_{lL} M_l M^\dagger_l U_{lR} &= \text{Diag}\left(m^2_e,\; m^2_\mu,\; m^2_\tau\right), \\ U^\dagger_{\nu_L} M_\nu M^\dagger_\nu U_{\nu_R} &= \text{Diag}\left(m^2_{\nu_1},\; m^2_{\nu_2},\; m^2_{\nu_3}\right). \end{aligned} \qquad (10)$$

This mixing matrix relates the neutrino flavor states to the neutrino mass eigenstates through

$$\nu_{\alpha L} = \sum_{i=1,2,3} V_{\alpha i} \nu_{iL}. \qquad (11)$$

In the standard parametrization [24], the PMNS matrix is expressed as $V = U \cdot P_o$, where $P_o \equiv \text{Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ with ρ, σ being two Majorana CP violating phases and U can be parametrized in terms of three mixing angles θ_{12}, θ_{13}, and θ_{23} and one Dirac CP violating phase δ_l: namely,

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_l} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_l} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_l} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_l} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_l} & c_{23}c_{13} \end{pmatrix} \qquad (12)$$

with $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ for $ij = 12, 13, 23$. The neutrino oscillation experiments provide constraints on the three mixing angles θ_{12}, θ_{13}, and θ_{23} along with the two mass square differences: namely, $\delta m^2 = m_2^2 - m_1^2$ and $\Delta m^2 = \eta[m_3^2 - (m_1^2 + m_2^2)/2]$ with $\eta = +1$ for NO and $\eta = -1$ for IO cases.

In the current scenario, the global picture of neutrino oscillation parameters for NO at 3σ suggests [25]

$$\begin{aligned}\delta m^2 &= (6.99\text{–}8.18) \times 10^{-5}\ \mathrm{eV}^2,\\ \Delta m^2 &= (2.23\text{–}2.61) \times 10^{-3}\ \mathrm{eV}^2,\\ s_{12}^2 &= 0.259\text{–}0.359,\\ s_{13}^2 &= 0.0176\text{–}0.0295,\\ s_{23}^2 &= 0.374\text{–}0.626,\\ \delta_l\,(1\sigma) &= 201^\circ\text{–}239^\circ.\end{aligned} \tag{13}$$

Moreover, the above data does not seem to forbid $m_{\nu_1} = 0$ for NO or $m_{\nu_3} = 0$ for IO cases, the signatures for which are obtained through $\mathrm{Det}\, M_\nu = 0$. The Planck collaboration measurements of the cosmic microwave background [26] provide further insight into the sum of absolute neutrino masses: for example,

$$\Sigma = m_{\nu_1} + m_{\nu_2} + m_{\nu_3} < 0.23\ \mathrm{eV}. \tag{14}$$

More recent results from long-baseline accelerator neutrino experiments T2K [27] and NOνA [28] are indicative of a near maximal Dirac CP phase: that is,

$$\begin{aligned}\delta_l &\sim 270^\circ,\\ \theta_{23} &\gtrsim 45^\circ\end{aligned} \tag{15}$$

along with preference for the normal ordering (NO) of neutrino masses. These results are also supported by the preliminary results from the atmospheric neutrino experiment at Super-Kamiokande [28]. In addition, a statistical analysis of the cosmological data [29] also indicates preference for NO providing maximum likelihood for Majorana effective mass: that is,

$$\langle m_{ee} \rangle < 16\ \mathrm{meV} \tag{16}$$

in neutrinoless double beta decay at 1σ where

$$\langle m_{ee} \rangle = \left| e^{i\rho} \left| U_{e1}^2 \right| m_{\nu_1} + e^{i\sigma} \left| U_{e2}^2 \right| m_{\nu_2} + \left| U_{e3}^2 \right| m_{\nu_3} \right|. \tag{17}$$

As the mixing angles are related to the corresponding mass matrices, it therefore becomes desirable to study the implications of a combination of NO, $\delta_{\mathrm{CP}} \sim 270^\circ$ along with $\theta_{23} \geq 45^\circ$ for lepton mass matrices assuming quarks and lepton mass matrices have similar origins and investigate the conditions affecting the possibility of obtaining natural lepton mass matrices, synchronous with the quark sector. Nevertheless, from a top-down prospective, it should be more economical to have a common framework explaining the fermion masses and mixing for the quark and lepton sectors.

2. Lepton Mass Matrices

Phenomenologically, the problem of constructing the fermion mass matrices has always been a difficult task within the framework of Standard Model (SM) and its possible extensions, wherein the flavor structure of these matrices is usually not constrained by the gauge symmetry. As a result, the matrices M_l and M_ν remain arbitrary 3×3 complex matrices, thereby involving several free parameters as compared to the number of physical observables, namely, six lepton masses, three mixing angles, and one Dirac-like CP phase δ_l along with two Majorana phases ρ and σ.

In this regard, the "texture zero" ansatz initiated by Weinberg [30] and Fritzsch [31, 32] has been quite successful in explaining the fermion masses and mixing patterns [33–52]. However, one requires handling all possible texture structures on a case to case basis. In this context, a common framework allowing for the study of such possibilities is more desirable. This is addressed in the following section.

3. Constructing the PMNS Matrix

In order to reconstruct the PMNS matrix, one requires obtaining the diagonalizing transformations for the corresponding mass matrices. To start with, for $q = l, \nu$, we consider the following two possibilities of texture one-zero mass matrices, as

$$\begin{aligned}M_q &= \begin{pmatrix} e_q e^{i\psi_q} & a_q e^{i\alpha_q} & 0 \\ a_q e^{i\alpha_q} & d_q e^{i\omega_q} & b_q e^{i\beta_q} \\ 0 & b_q e^{i\beta_q} & c_q e^{i\gamma_q} \end{pmatrix},\\ M_q' &= \begin{pmatrix} 0 & a_q' e^{i\alpha_q} & f_q' e^{i\Delta_q} \\ a_q' e^{i\alpha_q} & d_q' e^{i\omega_q} & b_q' e^{i\beta_q} \\ f_q' e^{i\Delta_q} & b_q' e^{i\beta_q} & c_q' e^{i\gamma_q} \end{pmatrix},\end{aligned} \tag{18}$$

referred to as Type I and Type II structures, respectively, in the following text.

One may also consider these matrices to be Hermitian for Dirac neutrinos. Using standard procedures, it is not possible to obtain the exact diagonalizing transformations for the latter case. In order to avoid a large number of free parameters in these matrices, we assume that the phases are factorizable in these, requiring

$$\begin{aligned}\psi_q &= 2\alpha_q,\\ \omega_q &= 0,\\ \Delta_q &= \alpha_q + \beta_q,\\ \gamma_q &= 2\beta_q\end{aligned} \tag{19}$$

for symmetric M_q and M'_q and

$$\begin{aligned} \psi_q &= 0, \\ \omega_q &= 0, \\ \Delta_q &= \alpha_q + \beta_q, \\ \gamma_q &= 0 \end{aligned} \tag{20}$$

for Hermitian M_q and M'_q.

The diagonalization of M_q above is realized using

$$M_q^{\text{Diag}} = O_q^T \widetilde{M}_q O_q = \text{Diag}\left(m_1, -m_2, m_3\right), \tag{21}$$

with $1,2,3 = e,\mu,\tau$ for $q = l$ and $1,2,3 = \nu_1,\nu_2,\nu_3$ for $q = \nu$. Here

$$\begin{aligned} P_q &= \text{Diag}\left(e^{-i\alpha_q}, 1, e^{-i\kappa\beta_q}\right), \\ \widetilde{M}_q &= P_q M_q Q_q = \begin{pmatrix} e_q & a_q & 0 \\ a_q & d_q & b_q \\ 0 & b_q & c_q \end{pmatrix}, \end{aligned} \tag{22}$$

and $Q = P$ (symmetric case) and $Q = P^\dagger$ (Hermitian case). Considering e_q and d_q as free parameters, one can write [45]

$$O_q = \begin{pmatrix} \sqrt{\dfrac{(e_q+m_2)(m_3-e_q)(c_q-m_1)}{(c_q-e_q)(m_3-m_1)(m_2+m_1)}} & \sqrt{\dfrac{(m_1-e_q)(m_3-e_q)(c_q+m_2)}{(c_q-e_q)(m_3+m_2)(m_2+m_1)}} & \sqrt{\dfrac{(m_1-e_q)(e_q+m_2)(m_3-c_q)}{(c_q-e_q)(m_3+m_2)(m_3-m_1)}} \\ \sqrt{\dfrac{(m_1-e_q)(c_q-m_1)}{(m_3-m_1)(m_2+m_1)}} & -\sqrt{\dfrac{(e_q+m_2)(c_q+m_2)}{(m_3+m_2)(m_2+m_1)}} & \sqrt{\dfrac{(m_3-e_q)(m_3-c_q)}{(m_3+m_2)(m_3-m_1)}} \\ -\sqrt{\dfrac{(m_1-e_q)(m_3-c_q)(c_q+m_2)}{(c_q-e_q)(m_3-m_1)(m_2+m_1)}} & \sqrt{\dfrac{(e_q+m_2)(c_q-m_1)(m_3-c_q)}{(c_q-e_q)(m_3+m_2)(m_2+m_1)}} & \sqrt{\dfrac{(m_3-e_q)(c_q-m_1)(c_q+m_2)}{(c_q-e_q)(m_3+m_2)(m_3-m_1)}} \end{pmatrix} \tag{23}$$

such that

$$\begin{aligned} c_q &= m_1 - m_2 + m_3 - d_q - e_q, \\ a_q &= \sqrt{\frac{(m_1-e_q)(m_2+e_q)(m_3-e_q)}{(c_q-e_q)}}, \\ b_q &= \sqrt{\frac{(c_q-m_1)(m_3-c_q)(c_q+m_2)}{(c_q-e_q)}}, \\ m_1 &> e_q > -m_2, \\ (m_3 - m_2 - e_q) &> d_q > (m_1 - m_2 - e_q). \end{aligned} \tag{24}$$

The above constraints on the parameters e_q and d_q nevertheless allow hierarchical mass matrices: that is, $e_q < a_q < d_q < b_q < c_q$. Texture rotation from (13) and (31) positions in M_q to (11) position in M'_q is realized by rotating (11) element in M_q to (13) and (31) positions in M'_q through a unitary transformation R_q on M_q using

$$M_q \longrightarrow M'_q = R_q^T M_q R_q, \tag{25}$$

for symmetric mass matrices and

$$M_q \longrightarrow M'_q = R_q^\dagger M_q R_q, \tag{26}$$

for Hermitian case, where R_q is a complex rotation matrix in the 1–3-generation plane: for example,

$$R_q = \begin{pmatrix} \cos\eta_{13_q} & 0 & -e^{-i(\alpha_q-\kappa\beta_q)}\sin\eta_{13_q} \\ 0 & 1 & 0 \\ e^{i(\alpha_q-\kappa\beta_q)}\sin\eta_{13_q} & 0 & \cos\eta_{13_q} \end{pmatrix}, \tag{27}$$

where $\kappa = +1$ for symmetric matrices and $\kappa = -1$ for Hermitian matrices.

The condition of a texture zero rotation from $(13,31)$ positions in M_q to (11) position in M'_q requires

$$0 = e_q\cos^2\eta_{13_q} + c_q\sin^2\eta_{13_q}, \tag{28}$$

which can be translated to

$$\begin{aligned} \tan^2\eta_{13_q} &= -\frac{e_q}{c_q} \Longrightarrow \\ \tan\eta_{13_q} &= \tau_q\sqrt{-\frac{e_q}{c_q}}, \end{aligned} \tag{29}$$

where $\tau_q = \pm1$ and e_q is always negative. Note that the rotation angle η_{13_q} is not a free parameter and is completely fixed through e_q and c_q due to repositioning of texture zeros as a result of the rotation R_q. One can now relate the matrix elements in M_q' with the corresponding elements in M_q: for example,

$$a_q' = \left| a_q \cos\eta_{13_q} + \tau_q b_q \sin\eta_{13_q} \right|,$$
$$b_q' = \left| b_q \cos\eta_{13_q} - \tau_q a_q \sin\eta_{13_q} \right|,$$
$$c_q' = c_q \cos^2\eta_{13_q} + e_q \sin^2\eta_{13_q}, \quad (30)$$
$$d_q' = d_q,$$
$$f_q' = \left| \sqrt{-e_q c_q} \right|.$$

The texture rotation in 1–3-generation plane allows $d_q' = d_q$. Note that $f_q' \propto \sqrt{-e_q}$, while the other off-diagonal elements essentially get rescaled due to texture rotation. Furthermore, for $e_q \sim -m_1$, one expects $f_q' \sim O(\sqrt{m_1 m_3})$ allowing hierarchical structures in Type II possibility: namely, $a_l' < f_l' < d_l' < b_l' < c_l'$ along with $a_\nu' \sim f_\nu' \sim d_\nu' \lesssim b_\nu' \lesssim c_\nu'$ since $O(\sqrt{m_{\nu_1} m_{\nu_2}}) \sim O(\sqrt{m_{\nu_1} m_{\nu_3}}) \sim O(m_{\nu_2})$ are allowed by oscillation data. Henceforth, it is trivial to obtain the orthogonal transformation O_q' for M_q' (symmetric case) as

$$O_q' = P_q R_q^T P_q^\dagger O_q = \widetilde{R}_q^T O_q \quad (31)$$

and (Hermitian case) as

$$O_q' = P_q^\dagger R_q^\dagger P_q O_q = \widetilde{R}_q^T O_q \quad (32)$$

with

$$M_q'^{\text{Diag}} = O_q'^T \widetilde{M}_q' O_q' = M_q^{\text{Diag}}. \quad (33)$$

with $\widetilde{M}_q' = P_q M' Q_q$. Note that in the absence of texture rotation, $\widetilde{R}_q = I$ (unit matrix) for M_q while

$$\widetilde{R}_q = \begin{pmatrix} \cos\eta_{13_q} & 0 & -\sin\eta_{13_q} \\ 0 & 1 & 0 \\ \sin\eta_{13_q} & 0 & \cos\eta_{13_q} \end{pmatrix} \quad (34)$$

for M_q' signifying the corresponding effect of such rotation on real diagonalizing transformation O_q'. The resulting mixing matrix for M_q and/or M_q' may be constructed as

$$V = O_l^T \widetilde{R}_l P_l P_\nu^\dagger \widetilde{R}_\nu^T O_\nu. \quad (35)$$

Also $P_l P_\nu^\dagger = \text{Diag}(e^{-i\phi_1}, 1, e^{i\phi_2})$, $\phi_1 = \alpha_l - \alpha_\nu$, and $\phi_2 = \beta_\nu - \beta_l$ (symmetrics case) or $\phi_2 = \beta_l - \beta_\nu$ (Hermitian case). Note that a change in sign for a_q' and f_q' can always be accommodated in the redefinition of phases α_q and β_q which only appear implicitly in the PMNS matrix through ϕ_1 and ϕ_2. Considering the six lepton masses, ϕ_1, ϕ_2, d_q, and e_q as free parameters, one can reconstruct the unitary mixing matrix V using the above procedure and confront it with the current oscillation data. In lieu of this, we restrict our investigation to only texture four-zero mass matrices involving ten free parameters. Furthermore, the condition of naturalness forbids a texture zero at (33) matrix elements.

Recent works [53–56] in this regard suggest that there exist several viable texture structures of lepton mass matrices. Most of these investigations work in the flavor basis with diagonal charged-lepton mass matrix or enforce parallel texture structures for lepton mass matrices M_l and M_ν. In this letter, we investigate all possible structures for four-zero lepton mass matrices, both symmetric and/or Hermitian, assuming factorizable phases (for simplicity) in these. The resulting structures are summarized in Tables 1 and 2 wherein we enlist all texture five and four zeros in agreement with current data at 3σ. X_l and X_ν in the tables represent the position of texture zeros in the corresponding mass matrices. It is observed that the constraints of naturalness, near maximal δ_l, $s_{23}^2 \gtrsim 0.50$, and normal ordering for neutrino masses, taken together, greatly reduce the number of possible viable structures and only a few possibilities seem to survive the test. The possibility of a vanishing neutrino mass is also studied for these texture structures.

4. Fritzsch-Like Four Zeros

It has been observed [36, 47] that, in the absence of $\delta_l \sim 270^\circ$ constraint, the Fritzsch-like texture four-zero mass matrices are physically equivalent to the generic lepton mass matrices. Interestingly, these matrices can be obtained from the above structures using the assumption of $e_q = 0$ and $f_q' = 0$. In particular, $R_q = \widetilde{R}_q = I$, where I is a unit matrix, for this case. The predictions from these matrices and their experimental tests can be found in previous works. To start with, using (13), (35) and allowing free variations to the parameters m_{ν_1}, d_l, d_ν, ϕ_1, and ϕ_2, we first reconstruct the viable structures for $\widetilde{M}_l$ (in units of GeV) and $\widetilde{M}_\nu$ (in units of eV) for $d_\nu \sim m_{\nu_2}$ using the available oscillation data and obtain the following best-fits:

$$\widetilde{M}_l = \begin{pmatrix} 0 & 0.007-0.010 & 0 \\ 0.007\text{–}0.010 & 0\text{–}0.822 & 0.423\text{–}0.924 \\ 0 & 0.423\text{–}0.924 & 0.822\text{–}1.644 \end{pmatrix} \text{GeV},$$
$$\widetilde{M}_\nu = \begin{pmatrix} 0 & 0.0066\text{–}0.0104 & 0 \\ 0.0066\text{–}0.0104 & 0.0076\text{–}0.0115 & 0.0223\text{–}0.0260 \\ 0 & 0.0223\text{–}0.0260 & 0.0302\text{–}0.0383 \end{pmatrix} \text{eV}, \quad (36)$$

along with $\phi_1 = 0^\circ$–50°, 267°–360° and $\phi_2 = 180^\circ$–285°. The corresponding predictions for the absolute neutrino masses Σ and $\langle m_{ee}\rangle$ read $m_{\nu_1} = (2.96\text{–}6.70)$ meV, $m_{\nu_2} = (9.05\text{–}11.50)$ meV, $m_{\nu_3} = (47.7\text{–}51.9)$ meV, $\Sigma = 60.2\text{–}69.6$ meV, and $\langle m_{ee}\rangle = 0.008\text{–}9.00$ meV, respectively. In the context of agreement with $\delta_l \sim 270^\circ$ along with $\theta_{23} \gtrsim 45^\circ$, it is observed that naturalness is allowed in M_ν independent of s_{23} octant. This is depicted in Figure 1 where one observes that $d_\nu \lesssim m_{\nu_2}$ is still consistent with $s_{23}^2 \gtrsim 0.5$. However, one finds that the near maximal constraint of $\delta_l \simeq 270^\circ$ requires large deviation

Table 1: Viable texture five zeros in relation to $s_{23}^2 \gtrsim 0.5$, $\delta_l \sim 270^\circ$, naturalness, and $m_{\nu_1} = 0$.

Sr.	X_l	X_ν	(a) $s_{23}^2 \gtrsim 0.5$	(b) $\delta_l \sim 270^\circ$	(c) natural	(a + c)	(b + c)	Det $M_\nu = 0$
1	11, 22, 13, 31	11, 13, 31	√	×	√	√	×	×
2	11, 13, 31	11, 22, 13, 31	√	×	×	×	×	×
3	11, 13, 31, 23, 32	11, 13, 31	√	×	×	×	×	×
4	12, 21, 22, 13, 31	11, 13, 31	√	×	×	×	×	×
5	11, 13, 31, 23, 32	11, 12, 21	√	×	×	×	×	×
6	11, 22, 13, 31	11, 12, 21	√	×	√	√	×	×
7	12, 21, 22, 13, 31	11, 12, 21	√	×	√	√	×	×
8	11, 12, 21, 23, 32	11, 13, 31	√	×	×	×	×	×

Table 2: Viable texture four zeros in relation to $s_{23}^2 \gtrsim 0.5$, $\delta_l \sim 270^\circ$, naturalness, and Det $M_\nu = 0$.

Sr.	X_l	X_ν	(a) $s_{23}^2 \gtrsim 0.5$	(b) $\delta_l \sim 270^\circ$	(c) natural	(a + c)	(b + c)	Det $M_\nu = 0$
1	11, 13, 31	11, 13, 31	√	√	√	√	×	×
2	13, 31, 23, 32	11, 13, 31	√	√	×	×	×	×
3	**11, 22**	**11, 13, 31**	√	√	√	√	√	×
4	11, 13, 31	11, 22	√	√	√	×	×	√
5	13, 31	11, 22, 13, 31	√	√	√	×	√	×
6	11, 22, 13, 31	13, 31	√	×	√	√	×	√
7	13, 31	11, 13, 31, 23, 32	√	√	×	×	×	×
8	11, 13, 31, 23, 32	13, 31	√	×	×	×	×	√
9	11	11, 22, 13, 31	√	√	√	×	√	×
10	11, 22, 13, 31	11	√	×	√	√	×	√
11	11	11, 13, 31, 23, 32	√	√	×	×	×	×
12	11, 13, 31, 23, 32	11	√	×	×	×	×	×
13	**22, 13, 31**	**11, 13, 31**	√	√	√	√	√	×
14	**11, 12, 21**	**11, 13, 31**	√	√	√	√	√	×
15	11, 13, 31	11, 12, 21	√	√	√	√	×	×
16	13, 31, 23, 32	11, 12, 21	√	√	×	×	×	×
17	22, 13, 31	11, 12, 21	√	√	√	√	×	×
18	11, 12, 21	11, 22	√	√	×	×	×	√
19	**11, 22**	**11, 12, 21**	√	√	√	√	√	×
20	12, 21, 13, 31	11, 13, 31	√	√	×	×	×	×
21	12, 21, 13, 31	11, 22	√	√	×	×	×	×
22	22, 12, 21, 13, 31	13, 31	√	×	×	×	×	√
23	12, 21, 22, 13, 31	11	√	×	√	√	×	×
24	11, 23, 32	11, 13, 31	√	×	×	×	×	×
25	11, 12, 21, 23, 32	11	√	×	×	×	×	×
26	11, 12, 21, 23, 32	13, 31	√	×	√	√	×	×
27	13, 31	11, 12, 21, 23, 32	√	√	×	×	×	×
28	11	11, 12, 21, 23, 32	√	√	×	×	×	×

of M_l from a possible natural structure. In particular, we identify three vital sources for CP violation in these matrices: namely, the two nontrivial phases ϕ_1, ϕ_2 along with the free parameter d_l as elaborated in Figure 2, indicating $d_l > 0.6\,\text{GeV} \sim m_\tau/3 \gg m_\mu$ is required to obtain $\delta_l \simeq 270^\circ$. This also implies that Fritzsch-like texture five-zero matrices ($d_l = 0$) should be ruled out by $\delta_l \simeq 270^\circ$. Our study reveals this conclusion to hold true for all possible texture five-zero structures, all of which seem to be ruled out by a near maximal δ_l; see Table 1. This calls upon investigating alternate texture structures, which on the one hand account for near maximal δ_l and at the same time allow possible natural structures for M_l and M_ν (i.e., $M_{jk} \sim O(m_j m_k)$).

5. Natural Lepton Mass Matrices

In this context, $e_q \neq 0$ and/or $f'_q \neq 0$ in the corresponding mass matrices provide greater possibility of realizing naturalness in corresponding mass matrices as compared to the Fritzsch-like structures wherein interactions between

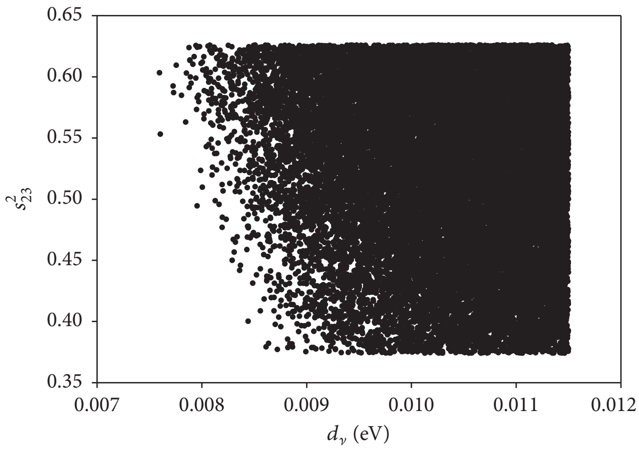

FIGURE 1: s_{23}^2 versus d_ν for Fritzsch-like four zeros.

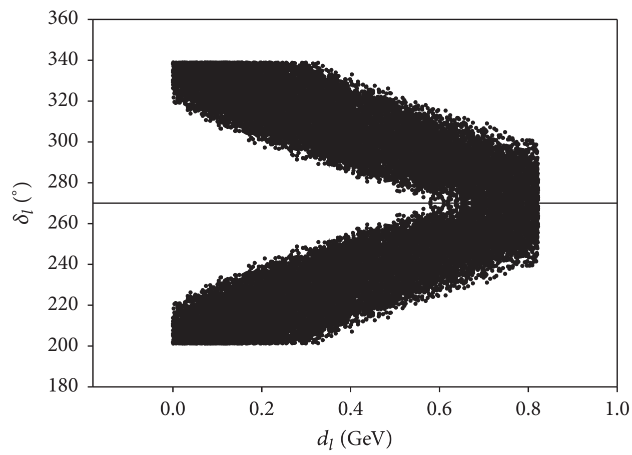

FIGURE 2: δ_l versus d_l for Fritzsch-like four zeros.

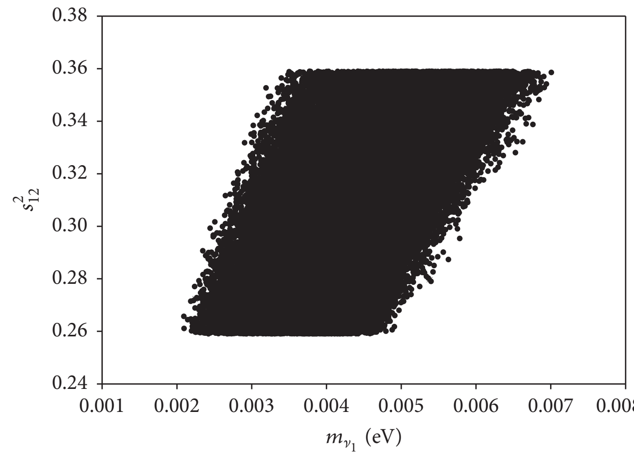

FIGURE 3: s_{12}^2 versus m_{ν_1} for Case A.

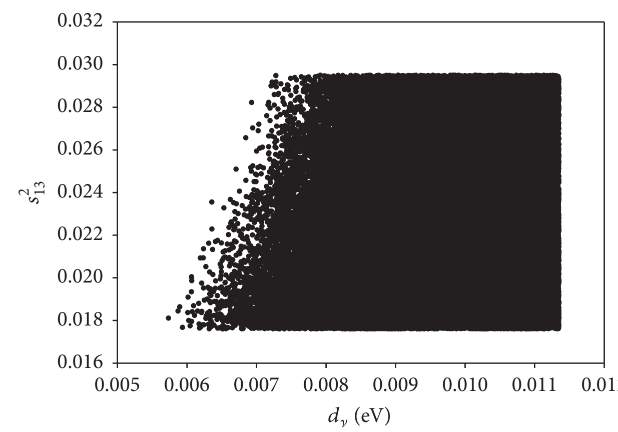

FIGURE 4: s_{13}^2 versus d_ν for Case A.

the first and third generations of leptons are suppressed due to texture zeros invoked at (11) and (13, 31) matrix elements. At least, for the quark sector, nonvanishing (13, 31) elements are observed to be crucial in effectuating the natural structures of corresponding mass matrices. A careful analysis of all possible texture four-zero structures reveals that only four possibilities for natural structures are allowed by recent data; see Table 2. We categorize these as Type I and Type II, based on the texture structure of M_ν.

5.1. Type I $M_\nu(11) = M_\nu(13, 31) = 0$

Case A ($M_l(22) = M_l(13, 31) = 0$). The viable best-fit structures of the lepton mass matrices are summarized below:

$$\widetilde{M}_l = \begin{pmatrix} -0.003\text{–}0 & 0.007\text{–}0.019 & 0 \\ 0.007\text{–}0.019 & 0 & 0.416\text{–}0.426 \\ 0 & 0.416\text{–}0.426 & 1.644\text{–}1.647 \end{pmatrix} \text{GeV},$$

$$\widetilde{M}_\nu = \begin{pmatrix} 0 & 0.0053\text{–}0.0106 & 0 \\ 0.0053\text{–}0.0106 & 0.0057\text{–}0.0123 & 0.0221\text{–}0.0272 \\ 0 & 0.0221\text{–}0.0272 & 0.0285\text{–}0.0394 \end{pmatrix} \text{eV}, \tag{37}$$

with $\phi_1 = 0^\circ$–340°, $\phi_2 = 98^\circ$–265° and $m_{\nu_1} = (1.99$–$7.01)$ meV, $m_{\nu_2} = (8.65$–$11.3)$ meV, $m_{\nu_3} = (47.7$–$51.9)$ meV, $\Sigma = (58.7$–$70.0)$ meV, and $\langle m_{ee}\rangle = (0.01$–$9.23)$ meV, respectively. Like the Fritzsch-like texture four zeros, $s_{12}^2 \propto m_{\nu_1}$ [36, 38, 47, 57] as depicted in Figure 3. However, the other two mixing angles are fixed by the free parameter d_ν illustrated in Figures 4 and 5. The latter also indicates that natural structure for M_ν is allowed independent of s_{23} octant, with $d_\nu \lesssim m_{\nu_2}$ also accounting for $s_{23}^2 > 0.5$. Finally, the parameter $e_l \ll m_\mu$ accounts for near maximal δ_l as shown in Figure 6. In particular, a small deviation of $\delta_l \to 270^\circ \pm 30^\circ$ provides greater agreement of $e_l \sim 5$ MeV with the notion of naturalness in the corresponding mass matrix.

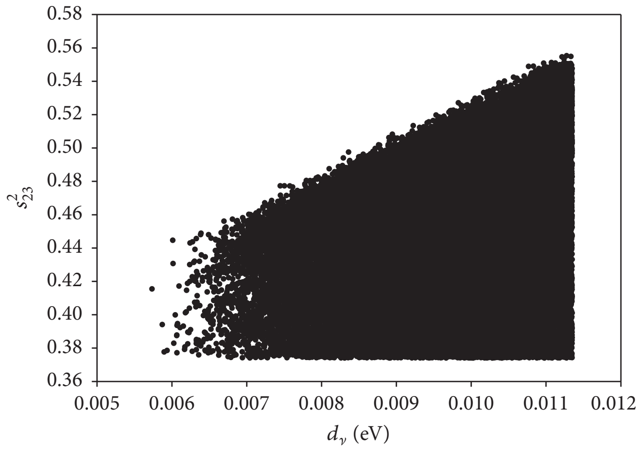

Figure 5: s_{23}^2 versus d_ν for Case A.

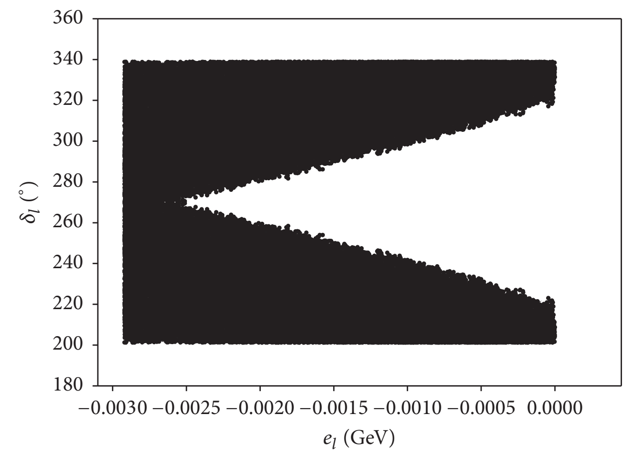

Figure 6: δ_l versus e_l for Case A.

Case B ($M_l(11) = M_l(22) = 0$). We obtain the following viable best-fit structures for these lepton mass matrices: namely,

$$\widetilde{M}_l' = \begin{pmatrix} 0 & 0.001\text{–}0.007 & 0.0003\text{–}0.089 \\ 0.001\text{–}0.007 & 0 & 0.413\text{–}0.423 \\ 0.0003\text{–}0.089 & 0.413\text{–}0.423 & 1.644 \end{pmatrix} \text{GeV},$$

$$\widetilde{M}_\nu = \begin{pmatrix} 0 & 0.0056\text{–}0.0111 & 0 \\ 0.0056\text{–}0.0111 & 0.0065\text{–}0.0116 & 0.0223\text{–}0.0266 \\ 0 & 0.0223\text{–}0.0266 & 0.0294\text{–}0.0390 \end{pmatrix} \text{eV}, \tag{38}$$

wherein $\phi_1 = 0^\circ\text{–}11^\circ, 251^\circ\text{–}360^\circ$, $\phi_2 = 89^\circ\text{–}268^\circ$ and $m_{\nu_1} = (2.26\text{–}7.53)$ meV, $m_{\nu_2} = (8.73\text{–}11.6)$ meV, $m_{\nu_3} = (47.7\text{–}52.0)$ meV, $\Sigma = (59.0\text{–}71.0)$ meV, and $\langle m_{ee}\rangle = (0.01\text{–}10.0)$ meV, respectively. m_{ν_1} dependence for s_{12}^2 remains the same as before while the other two mixing angles are fixed by the parameter d_ν. Furthermore, apart from phases ϕ_1 and ϕ_2, δ_l is now fixed by the parameter $f_l = \sqrt{-e_l c_l}$

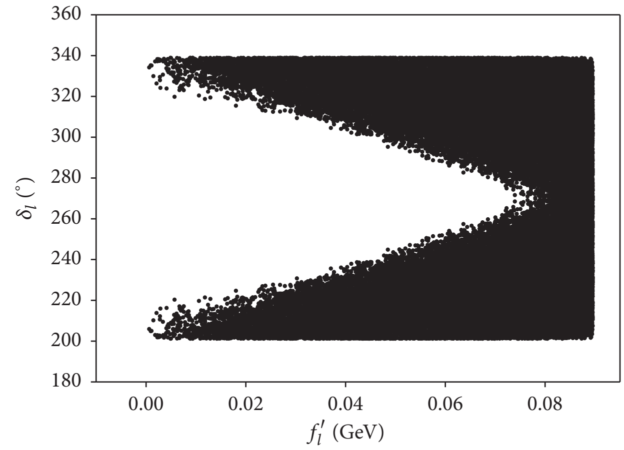

Figure 7: δ_l versus f_l' for Case B.

as shown in Figure 7. Naturalness in M_l' and M_ν seems to be in good agreement with $\delta_l \sim 270^\circ$ and $s_{23}^2 \gtrsim 0.5$ compatible with $f_l' \sim 0.075\,\text{GeV} \sim O(\sqrt{m_e m_\mu}) < m_\mu$ and $d_\nu \lesssim m_{\nu_2}$, respectively. A greater agreement with naturalness in M_l is achieved for $\delta_l \to 270^\circ \pm 30^\circ$ up to $f_l' \sim 0.05$ GeV.

Case C ($M_l(11) = M_l(12,21) = 0$). The viable best-fit structures so obtained for these lepton mass matrices are shown below:

$$\widetilde{M}_l' = \begin{pmatrix} 0 & 0 & 0.029\text{–}0.167 \\ 0 & 0.003\text{–}0.103 & 0.395\text{–}0.580 \\ 0.029\text{–}0.167 & 0.395\text{–}0.580 & 1.54\text{–}1.64 \end{pmatrix} \text{GeV},$$

$$\widetilde{M}_\nu = \begin{pmatrix} 0 & 0.0022\text{–}0.0113 & 0 \\ 0.0022\text{–}0.0113 & 0.0027\text{–}0.0117 & 0.0223\text{–}0.0277 \\ 0 & 0.0223\text{–}0.0277 & 0.0283\text{–}0.0399 \end{pmatrix} \text{eV}, \tag{39}$$

wherein $\phi_1 = 0^\circ\text{–}36^\circ, 175^\circ\text{–}360^\circ$, $\phi_2 = 97^\circ\text{–}265^\circ$ and $m_{\nu_1} = (0.4\text{–}7.8)$ meV, $m_{\nu_2} = (8.4\text{–}11.7)$ meV, $m_{\nu_3} = (47.6\text{–}52.0)$ meV, $\Sigma = (56.9\text{–}71.2)$ meV, and $\langle m_{ee}\rangle = (0.02\text{–}9.6)$ meV, respectively. It is noteworthy that the condition of texture zero at $M_l'(12,21)$, that is, $a_l' = 0$, fixes the parameter e_l and hence

$$f_l' = \sqrt{-e_l c_l} \tag{40}$$

through (30) with

$$e_l = -\frac{m_e m_\mu m_\tau}{d_l c_l}. \tag{41}$$

This results in only one free parameter $d_l = d_l'$ in M_l'. This is depicted in Figure 8. This parameter also determines the Dirac-like CP phase as shown in Figure 9. Other observations pertaining to the dependence of mixing angles remain the same as in previous cases. It is clear that naturalness in M_l' and M_ν is in good agreement with $\delta_l \sim 270^\circ$ and $s_{23}^2 \gtrsim 0.5$ compatible with $f_l' \sim 0.064\,\text{GeV} \sim O(\sqrt{m_e m_\mu}) < m_\mu$ and $d_\nu \lesssim m_{\nu_2}$, respectively.

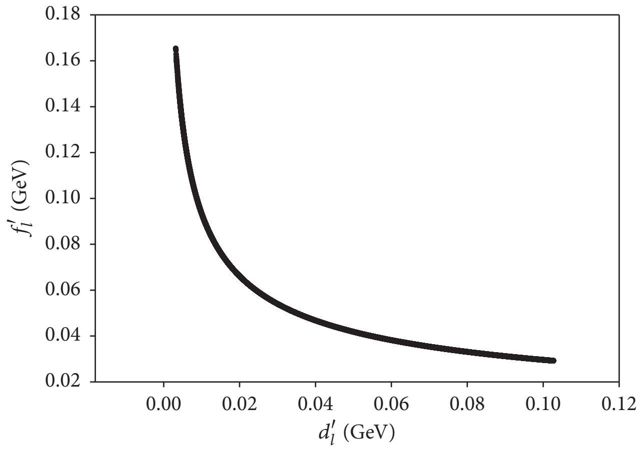

Figure 8: f_l' versus d_l' for Case C.

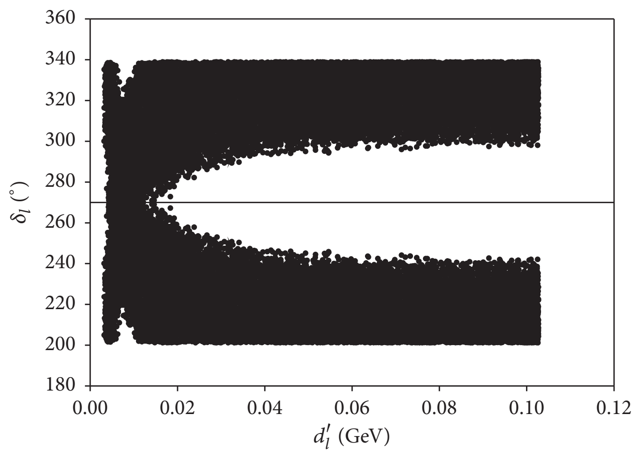

Figure 9: δ_l versus d_l' for Case C.

5.2. Type II $M_\nu(11) = M_\nu(12, 21) = 0$

Case D ($M_l(11) = M_\nu(22) = 0$). The best-fit values obtained for this possibility are summarized below:

$$\widetilde{M}_l' = \begin{pmatrix} 0 & 0.007\text{–}0.094 & 0.0004\text{–}0.220 \\ 0.007\text{–}0.095 & 0 & 0.348\text{–}0.423 \\ 0.0004\text{–}0.220 & 0.348\text{–}0.423 & 1.644 \end{pmatrix} \text{GeV},$$

$$\widetilde{M}_\nu = \begin{pmatrix} 0 & 0 & 0.006\text{–}0.019 \\ 0 & 0.0041\text{–}0.0120 & 0.0178\text{–}0.0269 \\ 0.006\text{–}0.019 & 0.0178\text{–}0.0269 & 0.0282\text{–}0.0393 \end{pmatrix} \text{eV}, \quad (42)$$

wherein $\phi_1 = 0^\circ\text{–}23^\circ, 256^\circ\text{–}360^\circ$, $\phi_2 = 98^\circ\text{–}261^\circ$ and $m_{\nu_1} = (1.1\text{–}7.9)$ meV, $m_{\nu_2} = (8.4\text{–}12.5)$ meV, $m_{\nu_3} = (47.6\text{–}51.9)$ meV, $\Sigma = (57.5\text{–}70.8)$ meV, and $\langle m_{ee}\rangle = (0.01\text{–}9.56)$ meV, respectively. It is observed that naturalness is in good agreement with $\delta_l \sim 270^\circ$ and $s_{23}^2 \gtrsim 0.5$ compatible with $|f_l'| \sim 0.088$ GeV $\sim O(\sqrt{m_e m_\mu}) < m_\mu$; see Figure 10 and $d_\nu \lesssim m_{\nu_2}$, respectively. Again a greater agreement with naturalness in M_l can be achieved for $\delta_l \to 270^\circ \pm 30^\circ$ up to $|f_l'| \sim 0.05$ GeV.

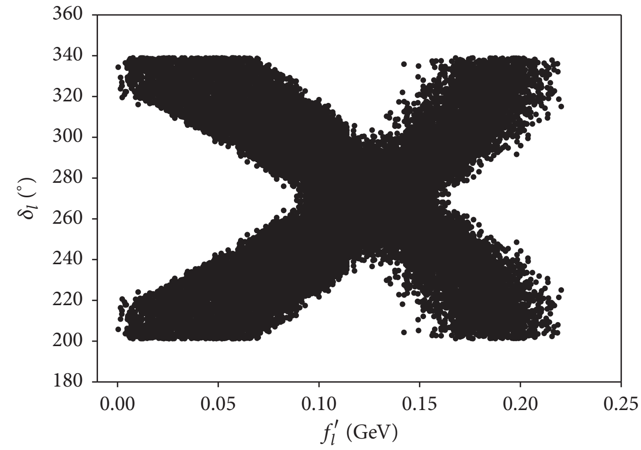

Figure 10: δ_l versus f_l' for Case D.

6. Conclusions

Assuming factorizable phases in lepton mass matrices, we show that natural mass matrices characterized by $(M_{ij}) \sim O(\sqrt{m_i m_j})$ for $i, j = 1, 2, 3, i \neq j$, and $(M_{ii}) \sim O(m_i)$ provide a reasonable explanation for the observed fermion masses and flavor mixing patterns in the quark as well as the lepton sectors. It is also observed that deviations from parallel texture structures for $M_{l,d}$ and $M_{\nu,u}$ are essential for establishing such natural structures. Such phenomenological textures have also been observed to be stable under the renormalization group running from the lightest right-handed neutrino mass scale to the electroweak scale [39, 40, 53, 58, 59].

Interestingly, naturalness in the lepton sector implies $s_{12} \propto O(\sqrt{m_{\nu_1}/m_{\nu_2}})$ and $s_{23}^2 \propto d_\nu/c_\nu$ or $s_{23} \propto O(\sqrt{m_{\nu_2}/m_{\nu_3}})$ such that the observed large values of these mixing angles are perhaps indicative of the possible realization of the neutrino mass ratios as obtained above: that is, $m_{\nu_1} \simeq (0.1\text{–}8.0)$ meV, $m_{\nu_2} \simeq (8.0\text{–}13.0)$ meV, $m_{\nu_3} \simeq (47.0\text{–}52.0)$ meV, $\Sigma \simeq (56.0\text{–}71.0)$ meV, and $\langle m_{ee}\rangle \simeq (0.01\text{–}10.0)$ meV, respectively. In particular, the possibility of a vanishing neutrino mass, that is, $m_{\nu_1} = 0$, is not supported by natural lepton matrices. From the point of view of $0\nu\beta\beta$ decays, these results seem to indicate that multiton scale detectors may be required to possibly observe signals for such processes.

Competing Interests

The author declares that they have no competing interests.

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