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# Intensity of singular stress field for three-dimensional butt joint to evaluate the adhesive strength

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**Abstract.** Adhesive joints are extensively used in various manufacturing processes in different industrial sectors because of its high fatigue resistance. Different materials properties cause the singular stress field, whose intensity is depending on the adhesive joint geometry. Our previous studies show that debonding strength can be expressed as a constant value of the critical intensity of singular stress field (ISSF) by using two-dimensional butt joint models. By considering real specimen geometry, in this paper, the ISSFs on the interface outer edges of three-dimensional butt joints are analysed by varying the adhesive thicknesses. A mesh-independent technique combined with three-dimensional finite element method (FEM) is shown to evaluate the ISSF. The ISSF distributions on the interface outer edges are analysed in comparison with the previous two-dimensional results. It is found that the critical ISSF considered 3D geometry is almost constant independent of the adhesive thickness.

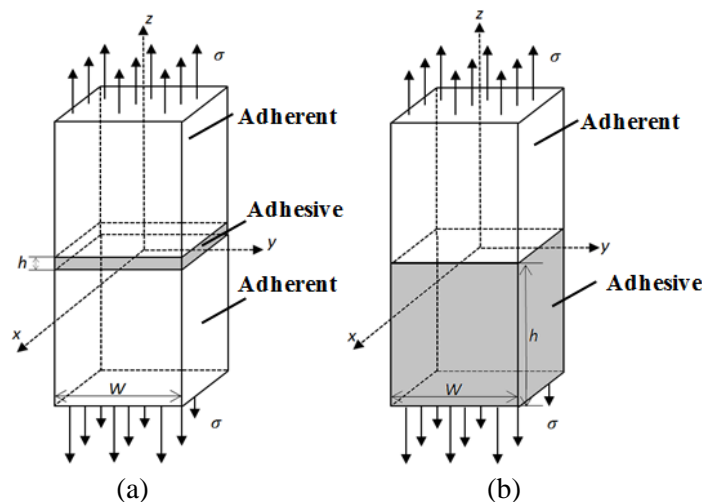
## 1. Introduction

Nowadays composite materials become one of the most important structural materials in various industrial fields to replace previous conventional joining technologies. However, different materials properties cause singular stress at the end of the interface, which may lead to failure of bonding portions in structures. Several problems on debonding strength have been studied recently [1-3]. By using butt and single lap joints, the study showed that the tensile strength of the butt joints decreased as adhesive thickness increases while the shear strength did not affected by the adhesive thickness of single lap joints [4]. The same result that related to adhesive strength and adhesive thickness has been observed [5-7]. On the other hand, the study of the effects of surface roughness, adhesive thickness and combined stresses on the adhesive strength for various metals bonded with epoxy resin has been also discussed [8].

Moreover, the experimental studies also justified that the result will be affected by the residual strain of the adhesive layer [9-12]. Suzuki [13] experimentally observed the specimens where S35C JIS medium carbon steel plates are bonded by epoxy resin and it is found that the adhesive strength decreases with increasing adhesive thickness  $h$  and tends to be a constant when  $h/W \geq 1$ , where  $W$  is the width of specimen. In addition, Akisanya and Meng [14] discussed the experimental adhesive strength for the butt joints with rectangular cross section. To explain those experimental results, Noda *et al* [15-16] proposed a useful method to calculate the intensity of singular stress field (ISSF) at the



adhesive dissimilar joint by focusing on the stresses at the edge calculated by finite element method (FEM). FEM can be used for many engineering applications conveniently [17-19]. They found that the ISSF at the interface corner of two-dimensional butt joint decreases with decreasing the adhesive thickness. Also, Noda *et al* [20] obtained the critical ISSF for Suzuki's experimental specimen by using two-dimensional butt joint models, and found that the adhesive strength  $\sigma_c$  can be evaluated as a constant critical ISSF ( $K_{\sigma c} = \text{const}$ ) for carbon steel/epoxy resin. In this paper, by considering real specimen 3D geometry, the ISSFs on the interface outer edges of 3D butt joints will be analysed by varying the adhesive thicknesses to confirm the validity of previous 2D modelling. The reference solution in figure 1(b) will be studied under arbitrary material combinations.



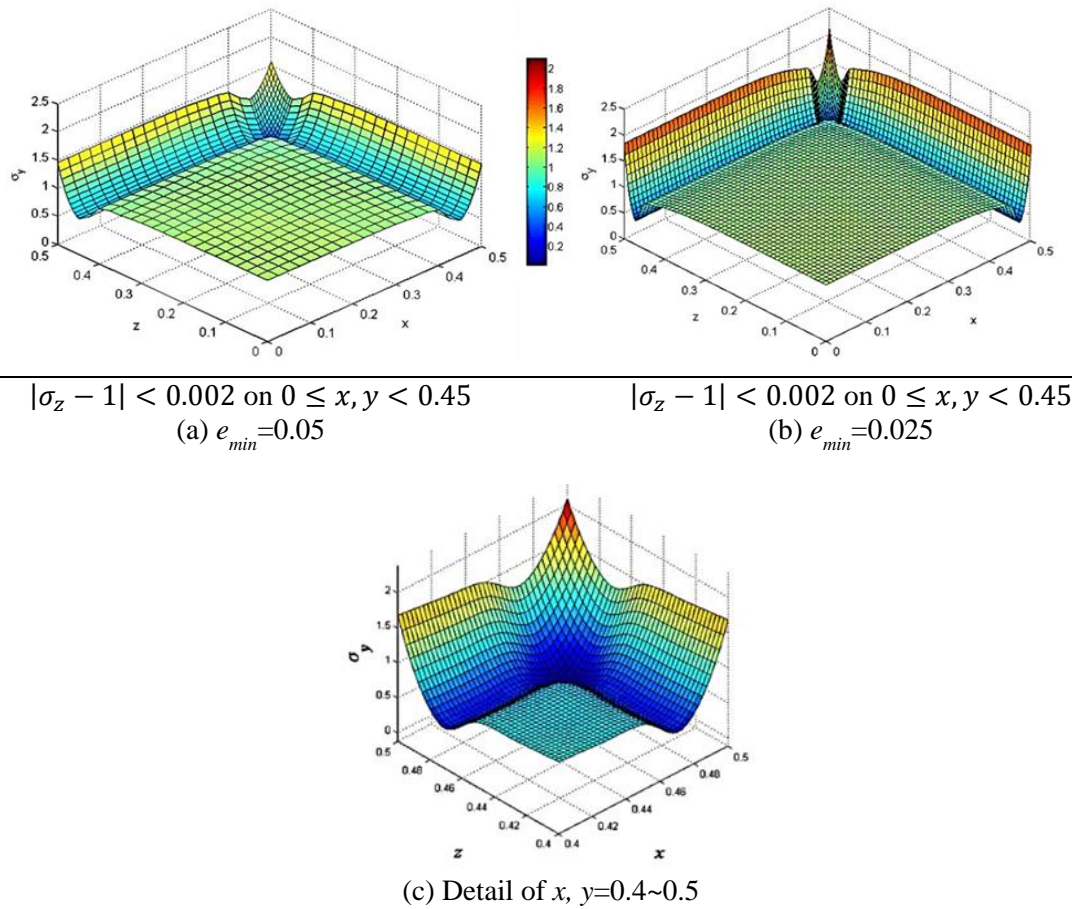
**Figure 1.** Three-dimensional butt joint when  $h/W = 0.01$  (a) and  $h/W = 1.0$  (b).

## 2. Three-dimensional mesh-independent technique to obtain ISSF

The most popular ISSF is known as the stress intensity factor for cracks. To obtain more general ISSF for evaluating interface strength, several analytical techniques were applied [21-28]. One of the most used numerical modelling techniques is the finite element method, which can be used for many engineering applications conveniently [29-35]. In order to obtain the ISSF, a mesh-independent technique was proposed for two-dimensional butt joints. The details are indicated in [15,16,20].

By considering the real specimen geometry, in this study, 3D butt joint as shown in figure 1(a) will be analysed. In order to apply the mesh-independent technique to 3D problem, a reference 3D solution is necessary. Therefore, 3D butt joint for  $h/W = 1.0$  as shown in figure 1(b) is analysed under arbitrary material combinations. Then, the adhesively bonded specimen used by Suzuki [13] is considered as comparison.

At this stage, the elastic properties for the materials are assumed as Young's modulus  $E=210$  GPa and Poisson's ratio  $\nu=0.3$  for S35C, and  $E=3.14$  GPa and  $\nu=0.37$  for epoxy resin [13]. Figure 2 shows that in the interior interface area  $0 \leq x, y < 0.45$  the accurate FEM stress is obtained as  $|\sigma_z - 1| < 0.002$  independent of the mesh size. However, the FEM stress values near the outer of interface sides are inaccurate since they are different depending on the mesh size due to the existence of the singular stress field along the outer interface side. Focusing on the outer interface side from  $y=0$  to  $y=W/2$  (see table 1), the constant FEM stress decreases first as shown in figure 2 and then increases rapidly when  $y$  is close to the outer interface corner. This is because another stronger singular stress field exists around the outer interface corner. In this paper, the ISSF distribution along the outer interface side in the three-dimensional joint will be focused.



**Figure 2.** FEM stress distribution  $\sigma_z$  of three-dimensional butt joint on  $z = \pm h/2$  when  $E_1 = 210\text{GPa}$ ,  $\nu_1 = 0.3$ ,  $E_2 = 3.14\text{GPa}$ ,  $\nu_2 = 0.37$ .

**Table 1.** FEM stress distributions for three-dimensional joint under tension obtained by different mesh sizes when  $E_1=210\text{ GPa}$ ,  $\nu_1=0.3$ ,  $E_2=3.14\text{ GPa}$  and  $\nu_2=0.37$ .

y	Minimum mesh size $e_{min}=1/3200$ around the edge			Minimum mesh size $e_{min}=1/12800$ around the edge		
	$\sigma_{y,h/W=0.01}^{3D}$	$\sigma_{y,h/W \geq 1}^{3D}$	$\frac{\sigma_{y,h/W=0.01}^{3D,FEM}}{\sigma_{y,h/W \geq 1}^{3D,FEM}}$	$\sigma_{y,h/W=0.01}^{3D}$	$\sigma_{y,h/W \geq 1}^{3D}$	$\frac{\sigma_{y,h/W=0.01}^{3D}}{\sigma_{y,h/W \geq 1}^{3D}}$
0.000	3.282	13.006	0.252	4.941	19.540	0.253
0.053	3.282	12.991	0.253	4.939	19.513	0.253
0.105	3.283	12.978	0.253	4.939	19.498	0.253
0.158	3.284	12.956	0.253	4.941	19.471	0.254
0.211	3.285	12.931	0.254	4.942	19.418	0.255
0.263	3.287	12.908	0.255	4.945	19.390	0.255
0.316	3.290	12.900	0.255	4.950	19.382	0.255
0.368	3.294	12.944	0.254	4.957	19.444	0.255
0.421	3.303	13.129	0.252	4.970	19.718	0.252
0.447	3.311	13.374	0.248	4.982	20.082	0.248
0.474	3.302	13.933	0.237	4.968	20.931	0.237
0.500	4.483	31.002	0.145	7.538	52.086	0.145

Table 1 shows the FEM stress distribution along the outer interface side in three-dimensional butt joint for  $h/W = 0.01$  and  $h/W \geq 1$ . It is seen that the FEM stress  $\sigma_{y,h/W=0.01}^{3D}$  and  $\sigma_{y,h/W \geq 1}^{3D}$  are inaccurate since they are different depending on the mesh size. However, the FEM stress ratio  $\sigma_{y,h/W=0.01}^{3D} / \sigma_{y,h/W \geq 1}^{3D}$  is very accurate since they are the same independent of the mesh size. Therefore, for example, by focusing on the middle point of the side, the ISSF ratio can be expressed as the FEM stress ratio as shown in equation (1) [15,16,20].

$$\frac{K_{\sigma}^{3D}}{K_{\sigma}^{REF}} = \frac{F_{\sigma}^{3D} \sigma_z^{\infty} W^{1-\lambda}}{F_{\sigma}^{REF} \sigma_z^{\infty} W^{1-\lambda}} = \frac{\lim_{r \rightarrow 0} [r^{1-\lambda} \times \sigma_z^{3D,Real}(r)]}{\lim_{r \rightarrow 0} [r^{1-\lambda} \times \sigma_z^{REF,Real}(r)]} = \lim_{r \rightarrow 0} \left[ \frac{r^{1-\lambda} \times \sigma_z^{3D,FEM}(r)}{r^{1-\lambda} \times \sigma_z^{REF,FEM}(r)} \right] = \frac{\sigma_z^{3D,FEM}(0)}{\sigma_z^{REF,FEM}(0)} \quad (1)$$

To discuss the ISSF distribution along the outer interface side, 3D butt joint in figure 1(b) is considered as the new reference solution in the next section under arbitrary material combination. The singularity index along the outer interface side can be obtained from equation (2).

$$\left[ \sin^2\left(\frac{\pi}{2}\lambda\right) - \lambda^2 \right]^2 \beta^2 + 2\lambda^2 \left[ \sin^2\left(\frac{\pi}{2}\lambda\right) - \lambda^2 \right] \alpha\beta + \lambda^2 [\lambda^2 - 1] \alpha^2 + \frac{\sin^2(\lambda\pi)}{4} = 0 \quad (2)$$

Here,  $\alpha$  and  $\beta$  are the Dundurs' composite parameters defined by the shearing modulus and Poisson's ratio as

$$\alpha = \frac{G_1(\kappa_2 + 1) - G_2(\kappa_1 + 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)}, \quad \beta = \frac{G_1(\kappa_2 - 1) - G_2(\kappa_1 - 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)} \quad (3)$$

where

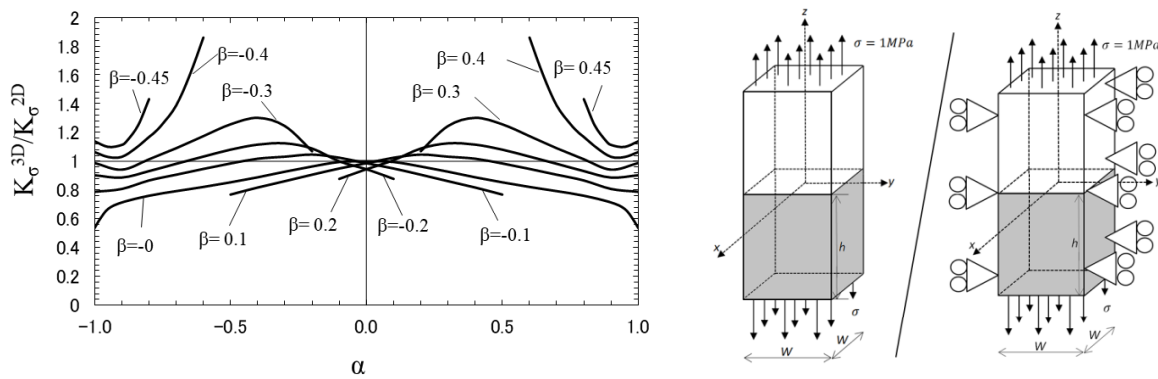
$$\kappa_j = \begin{cases} \frac{3 - \nu_j}{1 + \nu_j} & (\text{plane stress}) \\ 3 - 4\nu_j & (\text{plane strain}) \end{cases} \quad (j=1,2). \quad (4)$$

### 3. The ISSF of 3D reference solution under arbitrary material combinations

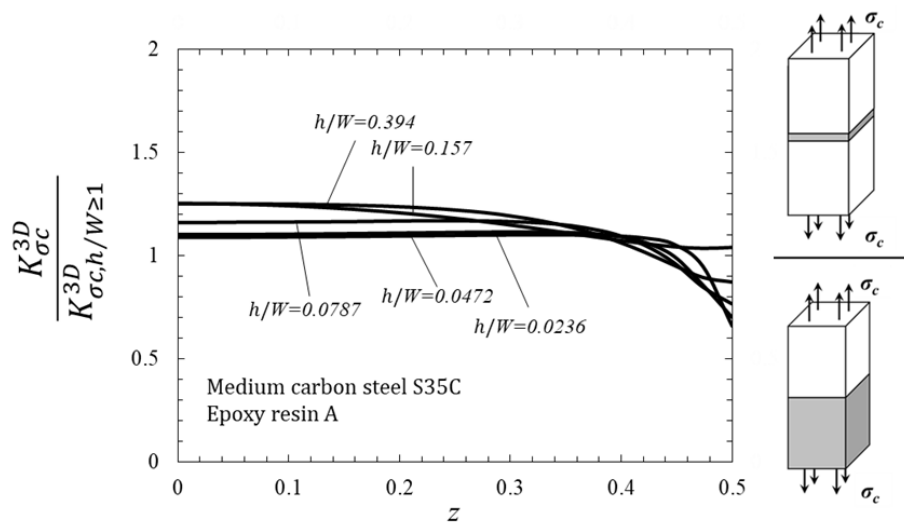
By using the similar mesh-independent technique described in section 2, the reference solution in figure 1(a) can be obtained. Figure 3 shows the ISSF ratio of the reference solution obtained from figures 1(b) and 1(b) with fixed displacement in the y-direction (plane strain 2D bonded plate). Here, 2D ISSF  $K_{\sigma}^{2D}$  was accurately obtained and indicated in [15,16,20]. In figure 3, the maximum ISSF ratio  $K_{\sigma}^{3D}(W/2) / K_{\sigma}^{2D}(W/2)$  is shown under fixed  $(\alpha, \beta)$  because  $(\alpha, \beta)$  cannot totally control 3D ISSF [19]. For  $-0.45 \leq \beta \leq 0.45$ , the ISSF ratio is in the range  $0.537 \leq K_{\sigma}^{3D}(W/2) / K_{\sigma}^{2D}(W/2) \leq 1.90$ .

### 4. The critical ISSF distribution and critical ISSF of three-dimensional butt joint

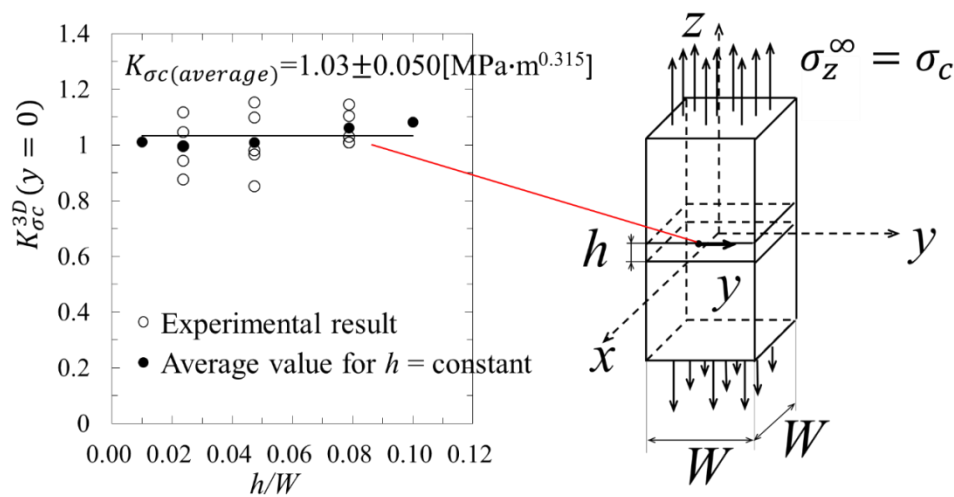
The critical ISSF distributions are obtained when the debonding occurs for Suzuki's specimen [13] where the adherents S35C are bonded by adhesive epoxy as shown in figure 1(a). Figure 4 shows the normalized critical ISSF distributions for various  $h$ . The critical ISSF distributions are nearly the same for different adhesive thickness  $h$ . Figure 5 shows the critical ISSF  $K_{\sigma_c}^{3D}$  focusing on the middle point  $y=0$  of the outer interface side. The experimental observation shows the debonding starts from the outer interface corner, but the debonding strength can be expressed as a constant critical ISSF  $K_{\sigma_c}^{3D}(0)$ . Figure 5 coincides with the results obtained by using 2D model [20].



**Figure 3.** The maximum values of  $K_{\sigma}^{3D}(W/2)/K_{\sigma}^{2D}(W/2)$  for various  $(\alpha, \beta)$ .



**Figure 4.** Critical ISSF distribution on the outer edge of the three-dimensional butt joint.



**Figure 5.** Constant critical ISSF focusing on  $(y=0)$ .

## 5. Conclusion

In this study, the three-dimensional butt joint has been analyzed in terms of the intensity of singular stress field. Without considering any defects and residual strain, the elastic and homogenous adhesive is assumed to evaluate the debonding strength. The FEM stress distributions of three-dimensional butt joint are obtained by using different mesh sizes. The results show that FEM stress ratio remains constant near the middle of the outer interface side which is independent of element size. Also, the critical constant ISSF  $K_{\sigma c}^{3D} = const$  can be used to express the adhesive strength by focusing on the middle point on the outer interface side. The validity of 2D modelling is confirmed since 3D results coincide with the results obtained by using 2D butt joint model [20].

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