



Θ -modifications on weak spaces

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Abstract. In this article, we want to study and investigate if it is possible to use the notions of weak structures to develop a new theory of θ - modifications in weak spaces and study their properties, finally we study some forms of weak continuity using this modifications.

1 Introduction

In [5], Császár and Makai Jr. introduced and studied the notions of $\delta_{\mu_1\mu_2}$ -open sets and $\theta_{\mu_1\mu_2}$ -open sets defined by two generalized topologies μ_1 and μ_2 on a nonempty set X and they proved that: $\delta_{\mu_1\mu_2}$ and $\theta_{\mu_1\mu_2}$ are generalized topologies on X and $\theta_{\mu_1\mu_2} \subseteq \delta_{\mu_1\mu_2} \subseteq \mu_1$. The notions of $(\theta_{w_1w_2}, \theta_{\sigma_1\sigma_2})$ -continuous was introduced and characterized by W. K. Min in [6], also introduced and characterized the notions of $(\delta_{w_1w_2}, \delta_{\sigma_1\sigma_2})$ -continuous on generalized topological

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spaces and $(\delta_{w_1 w_2}, \theta_{v_1 v_2})$ -continuous. W. K. Min in [7], introduced the notions of mixed weak $(\mu, \nu_1 \nu_2)$ -continuity between a generalized topology μ and two generalized topologies ν_1, ν_2 , also he introduced and characterized continuity in terms of mixed generalized $(\nu_1, \nu_2)'$ -semiopen sets, $(\nu_1, \nu_2)'$ -preopen sets, (ν_1, ν_2) -preopen sets [4], (ν_1, ν_2) - β '-open sets and $\theta(\nu_1, \nu_2)$ '-open sets [5]. Ugur Sengul in [12], using the δ and θ -modifications in bigeneralized topologies, introduced the notion of $(\delta_{\mu_1 \mu_2}, \theta_{\sigma_1 \sigma_2})$ -continuity between two Bi-GTSs. Also he characterized such continuity in terms of mixed generalized open sets: $\delta_{\mu_1 \mu_2}$ -open sets, $\theta_{\mu_1 \mu_2}$ -open sets. In this article, we want to study if it is possible, using weak structures to make a new theory related to θ -modifications of weak spaces and study some weak forms of continuity.

2 Preliminaries

Definition 1 [9] *Let X be a nonempty set. A subfamily w_X of the power set $P(X)$ is called a weak structure on X if it satisfies the following:*

1. $\emptyset \in w_X$ and $X \in w_X$.
2. For $U_1, U_2 \in w_X$, $U_1 \cap U_2 \in w_X$

The pair (X, w_X) is called a w -space on X . An element $U \in w_X$ is called w -open set and the complement of a w -open set is a w -closed set

Definition 2 [9] *Let (X, w_X) be a w -space. For a subset A of X ,*

1. *The w -closure of A is defined as $wC(A) = \bigcap \{F : A \subseteq F, X \setminus F \in w_X\}$.*
2. *The w -interior of A is defined as $wI(A) = \bigcup \{U : U \subseteq A, U \in w_X\}$.*

Theorem 1 [9] *Let (X, w_X) be a w -space on X and A, B subsets of X . Then the following hold:*

1. *If $A \subseteq B$, then $wI(A) \subseteq wI(B)$ and $wC(A) \subseteq wC(B)$.*
2. *$wI(wI(A)) = wI(A)$ and $wC(wC(A)) = wC(A)$.*
3. *$wC(X \setminus A) = X \setminus wI(A)$ and $wI(X \setminus A) = X \setminus wC(A)$.*
4. *$x \in wC(A)$ if and only if $U \cap A \neq \emptyset$, for all $U \in w_X$ with $x \in U$.*
5. *$x \in wI(A)$ if and only if there exists $U \in w_X$ with $x \in U$, such that $U \subseteq A$.*

6. If A is w -closed (resp. w -open), then $wC(A) = A$ (resp. $wI(A) = A$).

Theorem 2 [11] *Let (X, w_X) be a w -space on X and A, B subsets of X . Then the following hold:*

1. $wI(A \cap B) = wI(A) \cap wI(B)$.
2. $wC(A \cup B) = wC(A) \cup wC(B)$.

Theorem 3 *Let (X, w_X) be a w -space on X and A, B subsets of X . Then the following hold:*

1. $wI(A) \cup wI(B) \subseteq wI(A \cup B)$.
2. $wC(A \cap B) \subseteq wC(A) \cap wC(B)$.

3 Modification on weak structures

Throughout this paper if w_1, w_2 are two weak structures on a nonempty set X . Then (X, w_1, w_2) is called a biweak space. Recall that Császár, A. [3], showed that the δ and θ -modifications of topological spaces can be generalized for the case when the topology is replaced by the generalized topologies μ_1, μ_2 in the sense of [1]. W. K. Min [6], gave a characterization for $(\theta_{\mu_1\mu_2}, \theta_{\sigma_1\sigma_2})$ -continuity and introduce the concepts of $(\delta_{\mu_1\mu_2}, \delta_{\sigma_1\sigma_2})$ -continuity on generalized topological spaces and investigate the relationship between $(\delta_{\mu_1\mu_2}, \theta_{\sigma_1\sigma_2})$ -continuity, $(\theta_{\mu_1\mu_2}, \theta_{\sigma_1\sigma_2})$ -continuity and $(\delta_{\mu_1\mu_2}, \delta_{\sigma_1\sigma_2})$ -continuity. In our case, we want to study what happen when the generalized topologies are replaced by weak structures.

Definition 3 *Let (X, w_1, w_2) be a biweak space. A subset A of X is said to be $\Upsilon_{w_1w_2}$ -open (resp. $\Upsilon_{w_1w_2}$ -closed) if $A = w_1I(w_2C(A))$ (resp. $A = w_1C(w_2I(A))$).*

Example 1 *Let (X, w_1, w_2) be a biweak space, where $X = \{a, b, c\}$, $w_1 = \{\emptyset, X, \{a\}, \{b\}\}$ and $w_2 = \{\emptyset, X, \{a\}, \{c\}\}$.*

Observe that the set $A = \{b\}$ is $\Upsilon_{w_1w_2}$ -open, the set $B = \{c\}$ is $\Upsilon_{w_2w_1}$ -open and the set $C = \{a, b\}$ is not $\Upsilon_{w_2w_1}$ -open set.

Definition 4 *Let (X, w_1, w_2) be a biweak space.*

1. $A \in \theta_{w_1w_2}$ if and only if for each $x \in A$, there exists an $U \in w_1$ such that $x \in U \subseteq w_2C(U) \subseteq A$.

2. $A \in \delta_{w_1 w_2}$ if and only if $A \subset X$ and if $x \in A$, there exists a w_2 -closed set F such that $x \in w_1 I(F) \subseteq A$.

Example 2 In Example 1:

1. $\theta_{w_1 w_2} = \{\emptyset, X, \{b\}, \{a, b\}\}$,
2. $\delta_{w_1 w_2} = \{\emptyset, X, \{b\}, \{a, b\}\}$,
3. $\theta_{w_2 w_1} = \{\emptyset, X, \{c\}, \{a, c\}\}$,
4. $\delta_{w_2 w_1} = \{\emptyset, X, \{a, c\}, \{c\}\}$.

Example 3 Let (X, w_1, w_2) be a biweak space, where $X = \{a, b, c\}$, $w_1 = \{\emptyset, X, \{a\}, \{b\}\}$ and $w_2 = \{\emptyset, X, \{a\}, \{a, b\}, \{c\}\}$.

Observe that:

1. $\theta_{w_1 w_2} = \{\emptyset, X, \{b\}, \{a, b\}\}$,
2. $\delta_{w_1 w_2} = \{\emptyset, X, \{a, b\}, \{b\}\}$,
3. $\theta_{w_2 w_1} = \{\emptyset, X, \{c\}, \{a, c\}\}$,
4. $\delta_{w_2 w_1} = \{\emptyset, X, \{a, c\}, \{c\}\}$.

Example 4 Let (X, w_1, w_2) be a biweak space, where $X = \{a, b, c\}$, $w_1 = \{\emptyset, X, \{a\}, \{b\}, \{c\}\}$ and $w_2 = \{\emptyset, X, \{a\}, \{a, b\}, \{c\}\}$. Observe that:

1. $\theta_{w_1 w_2} = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$,
2. $\delta_{w_1 w_2} = \{\emptyset, X, \{c\}, \{a, b\}, \{b, c\}\}$,
3. $\theta_{w_2 w_1} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$,
4. $\delta_{w_2 w_1} = \{\emptyset, X, \{a, b\}, \{c\}\}$.

Example 5 Let $X = \{a, b, c\}$ with weak structures $w_1 = \{\emptyset, X, \{b\}\}$ and $w_2 = \{\emptyset, X, \{a\}\}$. Observe that:

1. $\theta_{w_1 w_2} = \{\emptyset, X\}$,
2. $\delta_{w_1 w_2} = \{\emptyset, X, \{b\}\}$,
3. $\theta_{w_2 w_1} = \{\emptyset, X\}$,

4. $\delta_{w_2w_1} = \{\emptyset, X, \{a\}\}$.

Remark 1 According with Example 4, $\delta_{w_1w_2}$ is not necessary a weak structures on X , then first of all, we have an answer. We can not doing similarly modification as [5], if we replace generalized topology by weak structure.

Theorem 4 Let (X, w_1, w_2) be a biweak space. The collection $\theta_{w_1w_2}$ is a strong generalized topology on X .

Proof. It is easy to see that: \emptyset and X belong to $\theta_{w_1w_2}$. Now consider $\{U_i : i \in I\}$ a collection of elements of $\theta_{w_1w_2}$ and $x \in \bigcup_{i \in I} U_i$, then for some $i \in I$, $x \in U_i$ and then there is $V_i \in w_1$, such that $x \in U_i \subseteq w_2C(V_i) \subseteq U_i \subseteq \bigcup_{i \in I} U_i$. It follows that $\bigcup_{i \in I} U_i \in \theta_{w_1w_2}$. □

Theorem 5 Let (X, w_1, w_2) be a biweak space. The collection $\theta_{w_1w_2}$ is a weak structure on X .

Proof. It is easy to see that: \emptyset and X belong to $\theta_{w_1w_2}$. Now consider U_1, U_2 two elements of $\theta_{w_1w_2}$ and $x \in U_1 \cap U_2$, then $x \in U_i$ for $i = 1, 2$. Then there exists $V_i \in w_i$ for $i = 1, 2$, such that $x \in V_i$ and $w_2C(V_i) \subseteq U_i$. It follows that $x \in V_1 \cap V_2$ and $w_2C(V_1) \cap w_2C(V_2) \subseteq U_1 \cap U_2$. But $V_1 \cap V_2 \in w_1$ and $V_1 \cap V_2 \subseteq w_2C(V_1 \cap V_2) \subseteq w_2C(V_1) \cap w_2C(V_2) \subseteq U_1 \cap U_2$. Hence $U_1 \cap U_2 \in \theta_{w_1w_2}$. □

Remark 2 Observe that if (X, w_1, w_2) is a biweak space, $\theta_{w_1w_2}$ is a topology on X .

Theorem 6 Let (X, w_1, w_2) be a biweak space. The collection $\delta_{w_1w_2}$ is a strong generalized topology on X .

Proof. It is easy to see that: \emptyset and X belong to $\delta_{w_1w_2}$. Consider $\{V_i : i \in I\}$ a collection of elements of $\delta_{w_1w_2}$ and $x \in \bigcup_{i \in I} V_i$, then for some $i \in I$, $x \in V_i$ and then there is w_2 -closed set F such that $x \in w_1I(F) \subseteq V_i$ and hence, $x \in w_1I(F) \subseteq V_i \subseteq \bigcup_{i \in I} V_i$. In consequence, $\bigcup_{i \in I} V_i \in \delta_{w_1w_2}$. □

Remark 3 According with Example 3, $\theta_{w_1w_2} \subsetneq w_1$ and by Example 4, $\theta_{w_1w_2} \subsetneq \delta_{w_1w_2}$ and $\delta_{w_1w_2} \subsetneq w_1$.

Remark 4 Let (X, w_1, w_2) be a biweak space. There are no relation between $\theta_{w_1w_2}$ and $\delta_{w_1w_2}$, see Examples 4 and 5.

Remark 5 If we start with a biweak space (X, w_1, w_2) . We obtain that $\theta_{w_1 w_2}$ is a topology on X , see Remark 2. $\delta_{w_1 w_2}$ is a strong generalized topology on X , see Theorem 6 and there are no relation between $\theta_{w_1 w_2}$ and $\delta_{w_1 w_2}$, see Examples 4 and 5.

Definition 5 $A \in \theta_{w_1 w_2}$ is called $\theta_{w_1 w_2}$ -open set and its complement is called $\theta_{w_1 w_2}$ -closed.

According with Definition 5, we define the $\theta_{w_1 w_2}$ -closure of a subset A of X , as follows:

Definition 6 Let (X, w_1, w_2) be a biweak space.

1. The $\theta_{w_1 w_2}$ -closure of A is defined as:

$$C\theta_{w_1 w_2}(A) = \bigcap \{F : A \subseteq F, F \text{ is } \theta_{w_1 w_2}\text{-closed set in } X\}.$$
2. The $\theta_{w_1 w_2}$ -interior of A is defined as:

$$I\theta_{w_1 w_2}(A) = \bigcup \{U : U \subseteq A, U \text{ is } \theta_{w_1 w_2}\text{-open set in } X\}.$$
3. $\gamma\theta_{w_1 w_2}(A) = \{x \in X : w_2 C(U) \cap A \neq \emptyset, \text{ for every } U \in w_1 \text{ containing } x\}.$

Example 6 In Example 2. The $C\theta_{w_1 w_2}(\emptyset) = \emptyset$, $C\theta_{w_1 w_2}(X) = X$, $C\theta_{w_1 w_2}(\{a\}) = \{a, c\}$, $C\theta_{w_1 w_2}(\{b\}) = X$, $C\theta_{w_1 w_2}(\{c\}) = \{c\}$, $C\theta_{w_1 w_2}(\{a, b\}) = X$, $C\theta_{w_1 w_2}(\{a, c\}) = \{a, c\}$, $C\theta_{w_1 w_2}(\{b, c\}) = X$.

Theorem 7 Let (X, w_1, w_2) and (X, v_1, v_2) be two biweak space and $A \subseteq X$. If $w_1 \subseteq v_1$ and $w_2 \subseteq v_2$. Then $\theta_{w_1 w_2} \subseteq \theta_{v_1 v_2}$

Proof. Let $A \in \theta_{w_1 w_2}$ and $x \in A$, then there exists an $U \in w_1$ such that $x \in U \subseteq w_2 C(U) \subseteq A$. Since $w_1 \subseteq v_1$, $U \in v_1$ and $v_2 C(U) \subseteq w_2 C(U) \subseteq A$. \square

Theorem 8 Let (X, w_1, w_2) be a biweak space and $A \subseteq X$. The following are true:

1. $A \subseteq \gamma\theta_{w_1 w_2}(A) \subseteq C\theta_{w_1 w_2}(A)$.
2. A is $\theta_{w_1 w_2}$ -closed if and only if $A = \gamma\theta_{w_1 w_2}(A)$.
3. $x \in I\theta_{w_1 w_2}(A)$ if and only if there exists a w_1 -open set U containing x such that $x \in U \subseteq w_2 C(U) \subseteq A$.
4. if A is w_2 -open, then $w_1 C(A) = \gamma\theta_{w_1 w_2}(A)$.

- Proof.** 1. Since $\theta_{w_1w_2}$ is a weak space, the result follows.
 2. If A is $\theta_{w_1w_2}$ -closed, then $A = C\theta_{w_1w_2}(A)$, Now using 1, the result follows.
 3. Is a consequence of Definition 6.
 4. Let $x \in w_1C(A)$ and U any w_1 -open set containing x , then $U \cap A \neq \emptyset$, follows that $w_2C(U) \cap A \neq \emptyset$ and then $w_1C(A) \subseteq \gamma\theta_{w_1w_2}(A)$. Now consider $x \in \gamma\theta_{w_1w_2}(A)$, then for each w_1 -open set U containing x , $w_2C(U) \cap A \neq \emptyset$, and then there exists an element $z \in w_2C(U) \cap A$, since A is w_2 -open, $z \in A$, therefore $x \in w_1C(A)$. \square

4 Modification on weak continuous functions

Definition 7 Let (X, w_1, w_2) and (Y, v_1, v_2) be two biweak spaces. A function $f : X \rightarrow Y$ is said to be $(\Theta_{w_1w_2}, \Theta_{v_1v_2})$ -continuous if for every $\Theta_{v_1v_2}$ -open set V , $f^{-1}(V)$ is $\Theta_{w_1w_2}$ -open.

Observe that if (X, w_1, w_2) and (Y, v_1, v_2) are two biweak spaces, $\Theta_{w_1w_2}$ and $\Theta_{v_1v_2}$ are topologies, then the notion of $(\Theta_{w_1w_2}, \Theta_{v_1v_2})$ -continuous functions is similar to the well known concept of continuous functions.

Example 7 In Example 4. Observe that:

1. $\theta_{w_1w_2} = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$,
2. $\theta_{w_2w_1} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$,

The identity function $f : X \rightarrow X$ is $(\Theta_{w_2w_1}, \Theta_{w_1w_2})$ -continuous but is not $(\Theta_{w_1w_2}, \Theta_{w_2w_1})$ -continuous.

Theorem 9 Let (X, w_1, w_2) and (Y, v_1, v_2) be two biweak spaces; let $f : X \rightarrow Y$. Then the following are equivalent:

1. f is $(\Theta_{w_1w_2}, \Theta_{v_1v_2})$ -continuous,
2. For each $x \in X$ and each $\Theta_{v_1v_2}$ -open set V containing $f(x)$, there exists a $\Theta_{w_1w_2}$ -open set U containing x such that $f(U) \subseteq V$.
3. For each $x \in X$ and each $\Theta_{v_1v_2}$ -open set V containing $f(x)$, there exists a w_1 -open set U containing x such that $f(w_2C(U)) \subseteq V$.

Proof. The proof follows applying definition. \square

Definition 8 Let (X, w_1) be a weak space and (Y, v_1, v_2) be a biweak space. A function $f : (X, w_1) \rightarrow (Y, v_1, v_2)$ is said to be faintly $(w_1, \Theta_{v_1 v_2})$ -continuous if for every $\Theta_{v_1 v_2}$ -open set U , $f^{-1}(U)$ is w_1 -open.

Example 8 Let (X, w_1, w_2) and (Y, v_1, v_2) be a biweak spaces, where $X = Y = \{a, b, c\}$, $w_1 = \{\emptyset, X, \{a\}, \{b\}, \{c\}\}$, $w_2 = \{\emptyset, X, \{a\}, \{a, b\}, \{c\}\}$, $v_1 = \{\emptyset, Y, \{a\}, \{b\}\}$ and $v_2 = \{\emptyset, Y, \{a\}, \{c\}\}$.

Observe that:

1. $\Theta_{w_1 w_2} = \{\emptyset, X, \{b\}, \{a, b\}\}$,
2. $\Theta_{w_2 w_1} = \{\emptyset, X, \{c\}, \{a, c\}\}$,
3. $\Theta_{v_1 v_2} = \{\emptyset, X, \{b\}, \{a, b\}\}$,
4. $\Theta_{v_2 v_1} = \{\emptyset, X, \{c\}, \{a, c\}\}$.

Consider a function $f : (X, w_2) \rightarrow (Y, v_1, v_2)$ defined as $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is faintly $(w_2, \Theta_{v_1 v_2})$ -continuous but is neither $(\Theta_{w_1 w_2}, \Theta_{v_1 v_2})$ -continuous nor $(\Theta_{w_2 w_1}, \Theta_{v_1 v_2})$ -continuous

Example 9 The function defined in Example 4 is not faintly $(w_1, \Theta_{w_1 w_2})$ -continuous

Remark 6 If (X, w_1, w_2) , (Y, v_1, v_2) are two biweak spaces and $f : (X, w_1, w_2) \rightarrow (Y, v_1, v_2)$ is a function. The concepts of $(\Theta_{w_1 w_2}, \Theta_{v_1 v_2})$ -continuous and faintly $(w_1, \Theta_{v_1 v_2})$ -continuous are independent.

Theorem 10 Let (X, w_1, w_2) and (Y, v_1, v_2) be two biweak spaces. If $f : (X, w_1, w_2) \rightarrow (Y, v_1, v_2)$ is $(\Theta_{w_1 w_2}, \Theta_{v_1 v_2})$ -continuous, then for every $\Theta_{v_1 v_2}$ -closed set F , $f^{-1}(F)$ is a $\Theta_{w_1 w_2}$ -closed set.

Proof. It follows by duality. □

Definition 9 Let (X, w_1) be a weak space and (Y, v_1, v_2) be a biweak space. A function $f : (X, w_1) \rightarrow (Y, v_1, v_2)$ is said to be mixed weakly $(w_1, v_1 v_2)$ -continuous at $x \in X$ if for every v_1 -open set V , containing $f(x)$, there exists a w_1 -open set U containing x such that $f(U) \subseteq v_2 C(V)$. Then f is mixed weakly $(w_1, v_1 v_2)$ -continuous if it is mixed weakly $(w_1, v_1 v_2)$ -continuous at every point $x \in X$.

Example 10 Let (X, w_1) be a weak space and (Y, v_1, v_2) be a biweak space, where $X = Y = \{a, b, c\}$ and weak structures: $w_1 = \{\emptyset, X, \{a\}, \{b\}\}$, $v_1 = \{\emptyset, X, \{b\}\}$ and $v_2 = \{\emptyset, X, \{a\}\}$. Consider $f : (X, w_1) \rightarrow (Y, v_1, v_2)$, defined as $f(a) = b$, $f(b) = c$, $f(c) = a$. Then f is mixed weakly (w_1, v_1v_2) -continuous.

Remark 7 Let (X, w) be a weak space and (Y, v_1, v_2) be a biweak space. If $v_1 = v_2$, then the notion of mixed weakly (w, v_1v_2) -continuous function is just the notion of weak weakly (w, v_1) -continuous functions, that is, for any v_1 -open set V , there exists a w_1 -open set U such that $f(U) \subseteq v_1C(V)$.

Theorem 11 Let $f : X \rightarrow Y$ be a function, w_1 a weak structure on a nonempty set X , and v_1, v_2 be two weak structures on a nonempty set Y . Then:

1. If f is mixed weakly (w_1, v_1v_2) -continuous, then $f(w_1C(A)) \subseteq \gamma\theta_{v_1v_2}(f(A))$ for every subset A of X .
2. If $f(w_1C(A)) \subseteq \gamma\theta_{v_1v_2}(f(A))$ for every subset A of X , then $w_1C(f^{-1}(v_2I(G))) \subseteq f^{-1}(v_1C(V))$ for every v_2 -open set V of Y .

Proof. 1. Consider $A \subseteq X$, $x \in w_1C(A)$ and V any v_1 -open set containing $f(x)$. By hypothesis f is mixed weakly (w_1, v_1v_2) -continuous, then there exists a w_1 -open set U containing x such that $f(U) \subseteq v_2C(V)$. Since $x \in w_1C(A)$ and U is a w_1 -open set U containing x , $A \cap U \neq \emptyset$. In consequence, $\emptyset \neq f(A) \cap f(U) \subseteq v_2C(V) \cap f(A)$. Follows that $f(x) \in \gamma\theta_{v_1v_2}(f(A))$ and hence, $f(w_1C(A)) \subseteq \gamma\theta_{v_1v_2}(f(A))$.

2. Clear. □

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