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DISSERTATION

MORTALITY INVESTIGATION – DOES LIFE TABLE PA90 MODEL ANNUITANTS MORTALITY IN NIGERIA?

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SUPERVISION: ONOFRE ALVES SIMÕES

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Abstract

This study aims to investigate PA90 of the UK as a proxy for annuitant mortality table in Nigeria. Annuities seem to grow rapidly across the globe due to reformations and regulations in the public social security systems regarding post retirement plans. Nigerian annuity market is not left out in this global growth as annuity product now gains momentum by the day.

The primary focus of this dissertation is to compare PA90 of the UK with crude rates estimated from the national data available, an important topic nowadays in Nigeria. A literature review is provided – what life annuity means, mortality investigations in UK and Africa, and some of the reasons why mortality rates are being assessed. Data and methodology required to accomplish the objective of the work developed are also thoroughly discussed and used. Two smoothing techniques, natural basis spline (NCS) and penalised spline were applied on the training set, to obtain smoothed mortality rates.

The rates that have been estimated are then compared with the PA90 rates, to see whether this life table should continue to be used as a proxy for the mortality of Nigerian annuitants, or an independent study should be carried out.

Keywords: Annuity, Mortality, Standardised Mortality Ratio, Spline, Nigeria.

Resumo

Este estudo tem como objetivo investigar se a tábua PA90 do Reino Unido constitui um modelo aceitável para a experiência de mortalidade na Nigéria, no que diz respeito à população dos detentores de anuidades. A motivação para o trabalho provém do facto de o mercado nigeriano de anuidades se ter vindo a desenvolver nos últimos anos.

Nesta dissertação apresenta-se uma revisão de alguma da literatura relevante sobre o tópico, incluindo algumas noções de base - o que é uma renda vitalícia – e descrições necessariamente breves da investigação sobre questões de mortalidade desenvolvida no Reino Unido e em África, bem como de algumas das razões pelas quais as taxas de mortalidade estão a ser continuamente objeto de estudo. Os dados e as metodologias indispensáveis à prossecução do objetivo são de seguida discutidos e aplicados. Destaque deve ser dado aos dois métodos de suavização utilizados, *spline* com base natural (NCS) e *spline* penalizada, que foram usados no *training set data*, para a obtenção de taxas de mortalidade alisadas.

As taxas estimadas são posteriormente comparadas com a tábua PA90, para estudar se esta deve continuar a ser usada na Nigéria, ou se se impõe a realização de um estudo completo da mortalidade no país.

Palavras-chave: Anuidade, Mortalidade, Razão de Mortalidade Padrão, *Spline*, Nigéria.

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Abbreviations and Acronyms

AIC - Akaike's information Criterion

CMI - Continuous Mortality Investigation

CPFA – Closed Pension Fund Administrator

df – degree of freedom

NAICOM – National Insurance Commission

NCS - Natural Cubic Spline

PFA – Pension Funds Administrator

PFC – Pension Funds Custodian

PS - P Spline

SMR - Standardised Mortality Ratio

Chapter 1 – Introduction

1.1 Motivation

The birth of insurance in Nigeria could be traced to the year 1918 with about four (4) insurance agencies. Not until the 1960s, insurance was unable to pick up with first insurance company established in 1960. Despite this lengthy period of being in the market, insurers started writing annuity business in 2010 and PA90 UK Mortality table has been the proxy mortality table for Nigerian annuitants. This work is to statistically investigate the closeness to reality of using the UK Mortality table or better establish a locally estimated Mortality table for Nigerian market and to possibly project the mortality rate of annuitants.

1.1.1 The attitude of people towards annuity in Nigeria and probably elsewhere

Despite the economic benefits of annuities, it is a known fact that few people voluntarily annuitize their retirement savings. A risk averse consumer faces uncertainty about his/her remaining years on earth and is not wanting to place a value on annuities that guarantees a stable income till his/her death. In fact many papers have tried to find out the reason why a few individuals purchase annuities despite of the enormous benefits and unfortunately all these papers failed to establish a general explanation on why this is so. Luck befell this quest when Jeffrey R. Brown, Jeffrey R. Kling, Sendhil Mullainathan and Marian V. Wrobel (2008) in their paper concluded that "*the vast majority of individuals prefer annuity over alternative products when presented in a consumption frame whereas majority of individuals prefer non-annuitized products when presented in an investment frame*".

Not until recently, life insurance has not been embraced in Nigeria, attributable to religion and traditional beliefs and this could be pictured through its contribution to the nation's GDP at the rate of 0.1% while non-life business contributes 0.2% making a total contribution of 0.3% to the nation's GDP. These rates are as at 2014 research done and reported by ***datamarket***. In my personal opinion, the attitude of people towards the entirety of insurance is not helping the industry to grow due to the history of non-redemption of benefits as at when due. Large proportion of Nigerians with no exception of learned ones believe that insurance is a scam. Since the people of Nigerian state do not belief in insurance, hence selling its products to people has been difficult. The decision or idea of not wanting to insure against one's death is a societal norm, due to culture and religiosity, has eating deep in our nation - is like a taboo to insure against one's death since this could be likened to paying for one's death and this has made it difficult for life products to thrive.

On the other hand, insurers do not help matters by low penetration via marketing although some stakeholders believe the real situation is that the industry lost its integrity years ago. The industry should see marketing as a key driver to growing its business. History has it that benefits weren't redeemed as at when due in the 90's, when Nigerians started hoping it was going to work for them. Unfortunately, larger percentage of people who bought one or two policies were grossly disappointed. With the intervention of the regulator over the past few years, the industry has undoubtedly received a bit of life unlike the era of no regulation or almost self-regulated market.

1.1.2 The evolution of the contributory scheme in Nigeria

Not until the Pension Reform Act of 2004, Nigeria had been operating solely an unfunded defined benefit scheme which made payments of pensions reliant on the annual budgetary position. On the public sector, the scheme was flawed by inefficiencies and lack of good administrators which was due to financial malpractices whilst their peers in the private arena had situations whereby employees were not protected by the pension schemes placed by their employers or these neglected sufficiently funding the schemes. Additionally, problems were attributed to low coverage and non-compliance leaving most employees with no or inadequate retirement benefit arrangements. The government had to find a way to resolute this anomaly in the pension system, and introduced "the contributory scheme" in 2004 by the then government and later retracted by the Pension Reform Act of 2014, which now gives two pay-out options to employees at retirement - programmed withdrawal and life annuity. These two pay-out options are expanded in the subsequent paragraphs.

Under this act, both the employees and their employers are committed to contributing certain percentages monthly to the fund with licensed independent Pension Fund Administrators(PFAs) and Closed Pension Fund Administrators (CPFAs) and managed by Pension Fund Custodians (PFCs). The minimum statutory percentage of 7.5% (of employee's monthly emoluments) is expected from the employer and employee. The emoluments consist of basic salary, housing allowance, transport allowance and others depending on firm's structure or employee's grade level. The Nigerian Pension Commission (PENCOM). Established under the Nigeria Pension Reform Act No.2 of 2004, licenses, regulates and monitors the Pension Fund Administrators (PFAs) and Pension Fund Custodian (PFCs). It's worthy to note that an employee has the right to choose his own independent Pension Fund Administrator (PFA) with a unique but constant identification from inception. The Fund Administrators jointly invest the deposits with their respective Custodians (PFCs).

1.1.3 Why PA 90 Standard Life Table?

The implementation of PA 90 as a standard table in the Nigerian insurance space could be linked to the harmonisation regime that kicked off IFRS 4 in the market precisely 2011. Though IFRS 4 is an International Financial Reporting Standard issued in March 2004, it was not implemented in the region until 2011. Prior this time Generally Accepted Accounting Principles (GAAP) was the main standard.

To properly match with IFRS 4 standard, an unpublished study was carried out by a few active Life Actuaries in Nigeria and results at the time exhibited a close experience to that of PA 90 of the UK.

1.2 The need for mortality investigation

At the inception of rolling out the contributory scheme, more retirees settled for Programmed Withdrawal and this was attributed to lack of knowledge or misinformation. Even as at March 2016, 82% of retirees chose Programmed Withdrawal whilst 18% of retirees subscribed to Annuity. Of the total retirees (5630) in Programmed Withdrawal who subscribed during the last quarter, analysis by sector showed that 71.74% were from the public sector while the remainder were from the private sector.

As people are becoming more aware of annuity and its benefits, game has suddenly changed in the recent times as more people are leaving programmed withdrawal for annuity by the day. Also (Aladeloye Oluwaseyi and H. Hasim 2015) concluded that life annuity is better than programmed withdrawal. Now that annuity business is gaining momentum in the Nigerian market, stakeholders are much concerned about its pricing, reserving and the general management of the fund to mitigate against any future risks.

Since every stakeholder now shows interest in knowing what goes on in the background ranging from pricing to writing annuity business, adequate studies of mortality experience need to be done. In the quest of staying in the business using the best practices, concern on how to move away from using UK Mortality experience and come up with the real experience in Nigeria has been the focus and the exercise is termed "Mortality Investigation".

Investigating mortality rates is a very critical part of life insurance business and entails carrying out a research on mortality and morbidity experience of a defined population. The UK CMI has split this into four major areas namely: annuitant mortality, assurances (critical illness and mortality), income protection, self-administered pension scheme (SAPs) mortality and mortality projections considering future changes in mortality experience. This investigation in general examines and analyses the observed experience in the subsets of the total data and mortality and morbidity tables should be periodically produced.

For any life table constructed, it is scientifically assumed to reflect the mortality experience of a homogeneous group of lives and this table can then be used to model the experience of a homogeneous group of lives which is believed to have a similar experience. Otherwise if a life table is constructed for heterogeneous group then the mortality experience will depend on the exact mix of lives with different experiences and used to derive the table and such a life table is only useful for modeling mortality in a group with the same mixture.

This study aims to investigate PA90 of the UK as a proxy for annuitant mortality table in Nigeria. Pertinent to this objective, it is necessary to compare PA90 with crude rates estimated from the national data available. After the introductory chapter, a literature review is provided in Chapter 2 – what life annuity means, mortality investigations in some regions and some of the reasons why mortality rates are being assessed. Data and methodology required to accomplish the objective of the work developed are thoroughly discussed and used in Chapters 3 and 4. Particularly, two smoothing techniques, natural basis spline (NCS) and penalised spline, were applied on the training set, to obtain smoothed mortality rates. These rates are then compared with the PA90 rates, to see whether this life table should continue to be used as a proxy for the mortality of Nigerian annuitants, or an independent study should be carried put. Conclusions in Chapter 5 close the text.

Chapter 2 - Literature Review

2.1 Life annuities

Whenever any financial product comes up, the primary focus of every stakeholder is to know the impact on their wealth, socio-economic status of any subscriber and subsequently the overall impact on the nation's economy. Though there are different shades of annuities such as immediate versus differed, fixed versus variable, and whole life and temporary annuities, etc., but in general annuity is a contract between a buyer usually called a policyholder and the seller known as the insurance company, see Dickson, D., Hardy, M. and Waters, H. (2013). The two main perspectives, insurers and policyholders' perspectives, will be analysed. The main channel for delivering defined contribution (DC) pensions is via an annuity purchased from a life insurance company.

An annuity could be referred to as "reverse life insurance" since it plays out in opposite direction of a life insurance product. In the case of life insurance, the policyholder pays monthly or annually the insurer until he or she dies, after which the insurer pays a lump sum to the insured's estate(s). On the contrary in the case of annuities, the annuitant will make a lump sum payment to the insurer, and subsequently receives regular payouts from the latter.

Economists define the value of an annuity as what is called "Money's Worth", and this can be valued by juxtaposing the premium to the present value of lifelong future benefits to the policyholder, also known as the purchaser. Take for instance an immediate single life annuity that costs \$200,000 would pay an annual benefit of \$12,400 for life if the purchaser survives. Olivia S Mitchell (2004) thoroughly worked on valuing annuities.

2.2 Mortality investigation

Mortality needs to be investigated from time to time due to the differential in it and this was evident through the earliest work by Kitagawa and Hauser (1973). Mortality differs across racial lines through some major socioeconomic factors such as education, income, and gender. Despite living longer in sound health is a societal and governmental achievement, it could lead to enormous challenges in terms of policy making and annuity management for annuity providers, should the real pattern be unknown. This is related with carrying out a detailed statistical research on countries (or regions) mortality rates of a population by age. A few countries have dedicated institutions that frequently carry out and publish the continuous mortality investigations observed for their populations. In the next sections we will give a brief introduction to some of the mortality investigations work in the United Kingdom and Africa respectively.

2.2.1 Mortality investigation in the UK

In England, a body established by the Institute and Faculty of Actuaries known as the Continuous Mortality Investigation (CMI), is solely dedicated to carrying research on its nation's mortality and morbidity experience. The CMI's mortality projections working party was established to explore possible projection methodologies for use with "00" Series Tables. The Working Paper 15 completed its assessment of P-Spline model in early 2006 after setting out its initial consideration and discussion of projection methodologies. Then afterwards concluded to make use of P-Spline and Lee-Carter models.

Working Paper 20 was published in April 2006 which provided the summary of the study carried out hitherto by the Working Party and established practical advice on how to use the P-Spline model. The features of this model were set out in the Working Party and examples with results were provided, as well as implications.

2.2.2 Mortality investigation in Africa

Unfortunately, little or no work has been done in this region of the world due to lack of data or inadequate data, except in South Africa where insurance industry is fully developed. The Actuarial Society of South Africa in their own capacity has produced quite a few standard tables (SA56-62, SA 72-77 and SA85-90) for assured lives and (SAIML98 and SAIFL98) for annuitants. These are the first set of tables produced for South African immediate annuitants, using a parametric curve fitted to the data from normal retirement ages up to age 85. It was established that the increases in normal retirement ages, as expected implies higher mortality and this was due to ill-health retirement. Above age 85 the estimates were not reliable and the curve fit to the lower ages did not allow for expected drop in the increase rate with age, see R E Dorrington and S Tootla (2007).

2.3 Why assessing annuitant mortality rates?

It is very crucial to consider this since risk associated with mortality improvements and mortality risk are two of the major problems that annuity providers face. Among other problems such as interest risk, reinvestment and inflation risk, for further details see David Blake (1997).

2.3.1 Mortality improvements

Annuitant mortality has indisputably become one of the most discussed topics in the actuarial world, as mortality improvements over time tend to have huge impacts on the solvency, embedded value position and profitability of the insurer. There is a common error of about 20% over intervals as short as ten years, and some insurers have wrongly estimated the life expectancy in their annuity portfolio by up to two years, cf. David Blake (1997). Mortality improvement can result as either the outcome of removing some causes of death or by enabling people to live certain health conditions that would have resulted in death. In recent years, significant increase in life expectancy have been felt across the globe. "*Increase in male and female life expectancy at birth across North America, Europe, Australia and Asia averaged approximately 0.8% per year between 1900 and 1950, more recently dropping to levels around 0.3% per year"* (Purushotham, et al. (2007).

Sithole *et al*. (2000) analysed the differences in the present value of annuity payments after allowing for mortality improvements using log-link models. Another work by M. Khalaf-Allah, S. Haberman, R. Verrall (2006), demonstrated how to investigate the age ranges that contribute most to the extra cost related with permitting for future mortality improvements. They affirmed that if an annuitant starts at age 60, the critical age range is 73-80, which does not instantly follow the commencement of the annuity. It was shown that incorporating future mortality improvements for older ages (80 and above) is also essential reflecting the high level of the contributions made by these ages to the additional cost of annuity.

The recent mortality trends have led to the use of projected mortality tables when pricing and reserving for life annuities, as well as other living benefits. But mortality patterns evolve along time and so any projection might result weak when used for pricing new annuities and reserving for in-force business. Annamaria Olivieri and Ermanno Pitacco (2002).

2.3.2 Adverse selection and mortality risk

Adverse selection is directly linked to mortality risk. This occurs when there is information asymmetry between the insurance company selling the annuity and the prospective annuity purchaser. Also, this could be termed as a risk that only persons who believe they are likely to outlive the average age for the population voluntarily choose to buy annuities. The insurer is unable to distinguish between prospective policyholders who will experience lighter mortality and those who will experience heavier mortality. In the former the insurance company will make a loss while a profit will be made in the latter, although it is widely known that buyers of annuities have lower mortality rates than the general population.

Friedman and Warshawsky (1998; 1990). Warshawsky (1998) and Mitchell *et al*. (1999) shed light on the limited size of the US individual annuity market and the difference between premium and the expected present discounted value of annuity payments, for a typical individual; according to them this difference results from adverse selection. Olivia S. Mitchell & David McCarthy (2002) established that adverse selection associated with buying of individual annuities reduces the annuitants' mortality rates by 25% in the international setting.

Chapter 3 - Empirical Specifications

3.1 Data and Variables

3.1.1 Description of data included in the investigation

These data are expressed in numbers of lives, and therefore necessary to distinguish between the exposed lives and the already dead lives.

For the exposure data, the following information were made available:

- Record number;
- Policyholder names;
- Voluntary purchases of immediate annuity by individual;
- Sex;
- Entry date;
- Date of birth;
- Annual annuity.

Data on the different types of annuities were not included. Because of this, it was not possible to separate annuities with/guarantee periods from the ones without such periods, for instance.

For the death data, all the above were available, plus

- The year of death;
- The age at death.

3.1.2 Data excluded from the investigation

The following were removed from the available raw data;

Annuities with missing data (date of birth, sex, age, etc.) or incomplete information in general.

3.1.3 The amount of experience

These data used were collected from six (6) annuity providers, and three (3) closed pension fund administrators (CPFA), which will not be disclosed here. These data include exposure and death data from 2008 to 2017 calendar years.

3.1.4 Data limitations and problems

Missing data: The only important information that is missing across the ten (10) years is the gender classification and only an average of 5% of the information was missing. We assumed here that 80% were male while 20% were female and the distribution is presented accordingly in the next chapter. This assumption emanated from the pattern of the information available fact that the split of average annuity subscribers for the past five (5) years is 20% (female) to 80% (male).

3.2 Methodology

Our analyses entail three major steps, namely:

- Crude rates analysis;
- Usage of smoothing methods B-Spline and P-Spline (to smoothen the obtained rates);
- Estimation of standardised mortality ratios (SMRs) from the smoothing models (B-Spline and P-spline) and comparison of the estimates with the benchmark PA90 of the UK.

3.2.1 Crude Mortality Rate

This is the simplest single figure index and is just the ratio of total actual deaths (for all ages) to the total exposed to risk. It could also be expressed as the weighted average of the central rate of mortality using the central exposed to risk in the population as weights. The central rate of mortality could either be observed from a life table whilst the central exposed to risk could either be from a standard population or from a population under studied, see for instance The Actuarial Profession (2015). Another attractive feature of the crude rate is that the separate knowledge of central rate of mortality and central exposed to risk in the population is not needed. However, it is heavily influenced by the age and sex structure of the population.

$$CMR\ (Crude\ Mortality\ Rate) = \frac{Actual\ deaths\ (all\ ages)}{Total\ exposed\ to\ risk(all\ ages)} \qquad (1)$$

3.2.2 Spline Smoothing Methods

This is the mathematical method of smoothing in which a popular spline function is being considered. Spline is a word or term that originated from ancient draftsmen and spline tool was a thin flexible strip of wood, metal, or rubber used by a draftsman aid in drawing curved lines. Smoothing splines avoid the challenge of knot selection (as they just use the inputs as knots), and concurrently they control for overfitting by shrinking the coefficients of the estimated function (in its basis expansion)

Smoothing is creating an approximate function that attempts to capture important patterns in the data, leaving out noise or other fine-scale structures/rapid phenomena. Various algorithms are used in smoothing and the commonest algorithm is the “moving average” often used to capture important trends in repeated statistical surveys. A good reference for the remainder of this chapter is, for instance, Splines, knots, and Penalties by Eilers P.H.C & Marx B.D (2010).

More specifically, a P-spline is the spline function to be used in our work. About twenty-two years ago, precisely in 1996, Eilers & Marx imposed penalties on differences between adjacent coefficients. P-spline is simply the mix of B-splines (basis splines) and “penalization”, and the applications are generalized linear models or non-linear models. A P-spline is an alternative to non-parametric regression, see O’Sullivan (1986).

3.2.2.1 B - Spline

B or basis splines were developed way back 1970’s and were derived from Bezier splines. A curve can be parameterized by

$$P_0 f_0(t) + P_1 f_1(t) + P_2 f_2(t) + P_3 f_3(t), \qquad (2)$$

where $f_0, f_1, f_2,$ and f_3 are the combining/blending functions that indicate how the regulating points combine together to form the spline curve.

In the cases of Bezier curve, the combining functions are the Bernstein polynomials. To get a good fit, it is generally desirable that combining functions possess the following properties:

- Each function must be equal to zero almost all the time
- Smooth
- Permit interpolation of the regulating points.
- It must be rapid to compute

Basis splines are piece-wise polynomials. The degree of the polynomials is one less than their order, with smoothness being two less than the order. A simple case is the blending function of order five (5): will be piecewise polynomial of degree four (4) whose smoothness or continuity is C^3 and this results in low degree polynomials.

Considering a given case of $L + 1$ knot points and ($P_0, P_1, P_2,$ and P_3) that define a basis or blended function for each knot point of order m, $N_{0,m}(t), N_{1,m}(t), N_{2,m}(t), \dots N_{L,m}(t)$, we can write

$$P_0 N_{0,m}(t) + P_1\, N_{1,m}(t) + \, P_2 N_{2,m}(t) + \, \dots + P_L N_{L,m}(t) \qquad (3)$$

Basis function can therefore be parameterised for order $m > 1$ (degree m-1 and smoothness or continuityC^{m-2}).

$$N_{k,m}(t) = \frac{t-k}{m-1}\, N_{k,m-1}(t) + \, \frac{k+m-t}{m-1} N_{k+1,m-1}(t). \qquad (4)$$

3.2.2.2 P-Splines

The rightful handling of P-Splines is to assign the coefficients a central position as a skeleton with B-Splines merely putting “the flesh on the bones”, see (Eilers *et al.* (2015**)**). A smoother sequence coefficient leads to a soother curve. The number of splines and coefficients is immaterial, as long as the latter are smooth and this reflects the presence of penalty factor.

The model will be formulated for any mortality data.

Assume that Y_{ij} is a random variable representing death rate where I and j represent age and year of death respectively and following a Poisson distribution with parameter, $E_{ij} \times \, \mu_{ij}$, i.e., $Y_{ij} \sim P\big(E_{ij} \times \, \mu_{ij}\big)$. The expectation of this random variable is then

$$E(Y_{ij}\,) = \big(E_{ij} \times \, \mu_{ij}\big), \qquad (5)$$

where E_{ij} is the exposure and μ_{ij} is the force of mortality.

In a large population, the size of the exposure population is typically estimated by arranging the population size at the beginning and at the end of the year.

The non-parametric estimate of this hazard can be obtained by computing the actual death rates at the respective age and year using $\widehat{M}_{ij} = \, Y_{ij}/E_{ij}$

To simplify manipulations and presentations, data are usually prepared in a rectangular array whereby the rows are indexed by age and the columns are indexed by year.

In the next paragraphs, for the sake of a certain self-containment of the text, an approach to modelling with P-Splines is described.

If $y' = (y_1 \dots y_n)$ depends smoothly on a single variable $x' = (x_1 \dots x_n)$, then the non-parametric model for y can be written as, $y = f(x) + \epsilon$, where $f(\cdot)$ is a smoothly varying function and ϵ is a vector of independent errors with variance $\sigma^2 I$. (Eilers and Marx, 1996) set the following assumptions:

$E(y) = B_a$, where $B = (B_1(x),\ B_2(x) \dots\ B_k(x))$ is an $n \times k$ matrix of B-Splines (k depends on the number of knots and the degree of B-splines while a is the vector of regression coefficients)

It is assumed that the coefficients in adjacent B-Splines satisfy certain smoothness conditions that can be expressed in terms of finite differences of the a_is. Therefore, from the ordinary least-squares, the coefficients a are chosen to minimize

$$\delta(a) = (y - B_a)'(y - B_a)\ + \lambda a' D' D_a, \qquad (6)$$

where D is the difference matrix and λ is the penalty.

For any penaltyλ, the solution to this optimization problem satisfies

$$(B'B\ + \lambda\, D'D)\hat{a} = B'y \qquad (7)$$

$\hat{y} = B\hat{a} = Hy$, where H is the hat-matrix

$$H = B(B'B\ + \lambda\, D'D)^{-1} \qquad (8)$$

$(y - B_a)'(y - B_a)$, speaks of ordinary regression on columns of *B*.

$\lambda a' D' D_a$, caters for over-paramaterisation of the regression function by placing a penalty on the smoothness of the, a_j.

The key merit of using a P-Spline is that it solves the problem of selecting the number and position of knots.

3.2.2.3 P-Splines – one dimensional densities

Let (y_i, u_i) represent the Poisson count and bin median pairs from a (narrowly binned) histogram, i=1,…,n. The vector of counts is denoted y. The expected values of the counts can be modeled as

$$\mu_i = E(y_i) = \exp\left(\sum_{j=1}^{c} b_j(u_i)\theta_i\right), \tag{9}$$

Also in matrix terms, $\mu = B\boldsymbol{\theta}$ where B = $[b_j(u_i)]$ is a (n x c) B-spline basis built along the indexing axis u of the density. A rich basis is used (c sufficiently large) and knots for the basis are equally-spaced. Apart from a constant, the Poisson log-likelihood is

$$l = \sum_{i=1}^{n} \log\left(\mu_i^{y_i} e^{-\mu_i}\right) = \sum_{i=1}^{n}(y_i log\mu_i - \mu_i) \tag{10}$$

A penalty on the d-th differences of is deducted from the log-likelihood, to adjust smoothness. The number of basis functions in B is chosen large, to give full flexibility. The penalized log-likelihood, l^*, can then be written as

$$l^* = l - \frac{\lambda}{2}\sum_{j=d+1}^{c}(\Delta^d \boldsymbol{\theta}_j)^2 \tag{11}$$

Letting the gradient of equation (3) equal to zero gives

$$B'(y - \mu) = \lambda\, D'D\boldsymbol{\theta} \tag{12}$$

Such that D is a matrix of contrasts such that $D\boldsymbol{\theta} = \Delta^{\boldsymbol{d}}\boldsymbol{\theta}$. Therefore, linearization of (12) leads to

$$(B'\widetilde{W}B + \lambda\, D'D)\boldsymbol{\theta} = B'\widetilde{W}\tilde{z} \tag{13}$$

Such that $z = \eta + \widetilde{W}^{-1}(y - \mu)$ is the working variable and$\eta = B\boldsymbol{\theta}$. The matrix W = diagonal (μ) and $\tilde{\theta}$, $\tilde{\mu}$ are approximations to the solution of (13), iterated until convergence. One easily recognizes the familiar equations for fitting a GLM, modified by term $\lambda\, D'D$ that stems from the penalty. At convergence, one can interpret (13), as linear smoothing of the working variable z. We have

$$\hat{z} = B\hat{\theta} = \left(B'\widetilde{W}B + \lambda\, D'D\right)\boldsymbol{\theta} = B'\widetilde{W}\tilde{z} \;\; = \text{Hz}, \tag{14}$$

where is the smoothing or “hat” matrix. The effective dimension of smoother is approximated by race (H). We use it to define Akaike’s information criterion (AIC) as

$$AIC = 2\sum_{i=1}^{n} y_i \log(y_i/\hat{\mu}_i) + 2trace(H) \tag{15}$$

See Eilers & Marx (2006)

3.2.3 Standardised Mortality Ratio (SMR)

This is the ratio that can be employed to compare relative levels e.g. having applied the standard mortality the ratio of the actual number of deaths in a region to the expected number of deaths in that region. It's a useful index, since it requires only the age/sex specific rates for standard population, not the age/sex specific rates for the region/occupation etc.

SMR is usually expressed on a scale, and a value less than 1 indicates population with lighter mortality that that in the standard population and value greater than 1 indicates population with mortality heavier than that in the standard population. It can be expressed as below:

$$SMR\ (Standardised\ Mortality\ Ratio) = \frac{Actual\ deaths\ in\ the\ region}{Expected\ deaths\ in\ the\ region\ (standard\ mortality))}$$

Chapter 4 – Empirical Analysis and Results

4.1 General Data Summary

This is the summary of the data collected from the six (6) annuity providers and three (3) closed pension fund administrators (CPFAs), and all analyses are limited to the insights provided by these set of information.

Gender	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Male	7,411	7,573	7,592	8,128	8,995	11,290	16,616	23,750	34,297	41,914
Female	751	761	787	897	1,198	2,121	3,898	6,261	9,558	12,221
Total	**8,161**	**8,334**	**8,378**	**9,026**	**10,192**	**13,411**	**20,514**	**30,012**	**43,854**	**54,135**

Table 1- Summary of the exposure

Gender	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Male	56	60	81	95	123	198	142	332	513	464
Female	8	8	12	6	15	68	30	59	103	95
Total	**64**	**68**	**93**	**101**	**138**	**266**	**172**	**391**	**616**	**559**

Table 2 - Summary of the deaths

4.2 Crude Mortality Rates

The crude mortality rates (CMR) depend on the exposure and the number of deaths for each age band from the raw data, pooled for the period 2008 to 2017.

Age Band	Male Crude rates	Female Crude rates	Age Band	Male Crude rates	Female Crude rates
50-54	0.00338	0.00093	75-79	0.02136	0.00468
55-59	0.00519	0.00118	80-84	0.06812	0.01421
60-64	0.00625	0.00116	85-89	0.10473	0.03176
65-69	0.00705	0.00093	90-94	0.19252	0.06190
70-74	0.01745	0.00235	95-99	0.19067	0.04727

Table 3 – Average Crude Mortality Rates

It is clear from the average crude death rates table above, that the male lives experience much higher death rates than the female lives, as usual.

Also from the crude mortality rate estimates we can see that mortality is an increasing function of age as this will be obvious in the data visualizations following this section. It should however be noted that due to the data available for this work, female rates tend to show some volatility. On average, experience shows that male deaths are as five times as female deaths. This seems to be untrue but the data available shows this weird result and could possibly be because of class of annuity subscribers in Nigeria.

4.2.1 Relative proportion of female crude death rates

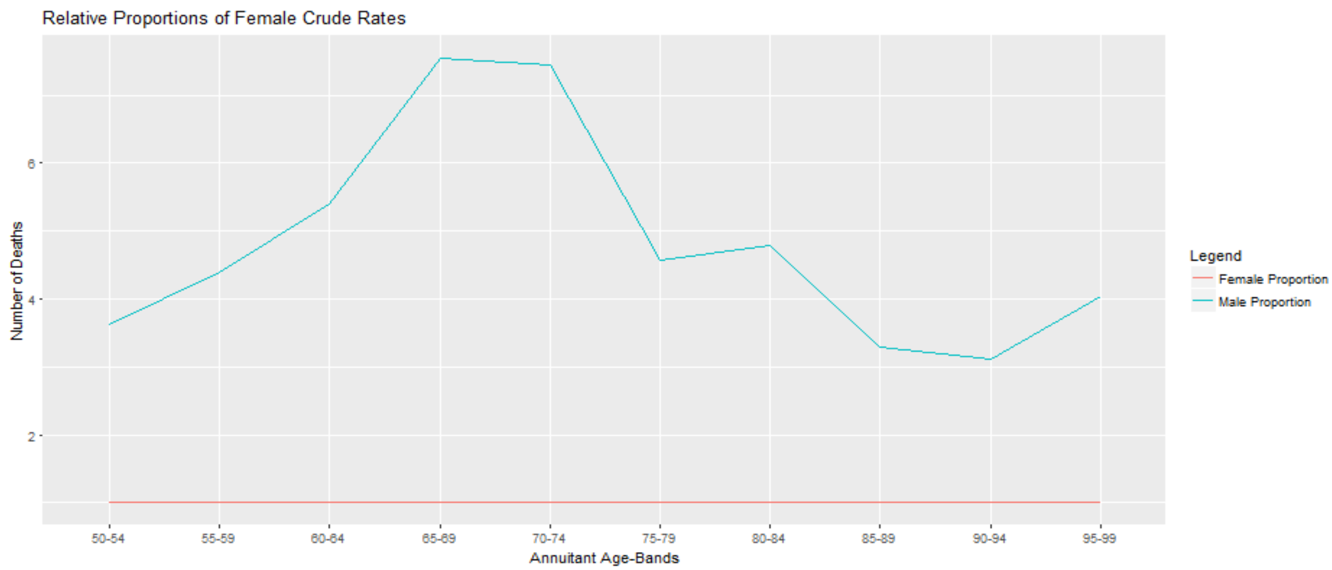

Figure 1 - Relative proportion of female crude death rates

Gender	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	95-99
Males	4	4	5	8	7	5	5	3	3	4
Females	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 4 – Relative proportion of female crude mortality rates

Analogously, from Table 4 above, the male annuitants experience much higher death rates than the female annuitants. On average, experience shows that male deaths are as five times as the female deaths.

4.2.2 Average Crude Rates

All data visualizations are presented with a log-normal transformation to make it easier to see and analyze patterns in the data.

4.2.2.1 Average Crude Mortality Rates: Age Effect

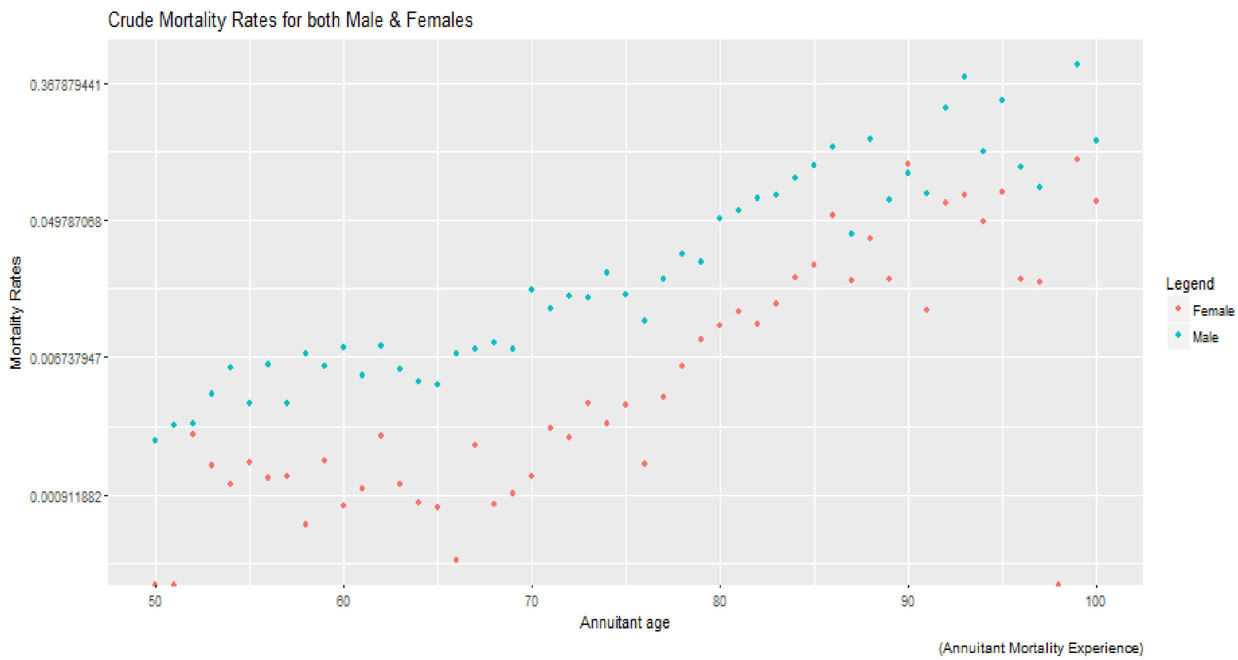

Figure 2 - Male & Female Average Crude Rates by Age

As expected mortality is an increasing function of age: As the annuitants age, the more likely they are exposed to death. This is common with all demographic data.

The population for female lives is approximately one fifth of the male population as seen in the previous section.

The smaller population size as also seen in the scatter plot above, usually shows unstable trends from year to year. This is very typical of the female experience as seen in the periodic plots below.

4.2.2.2 Average Crude Mortality Rates: Period Effect

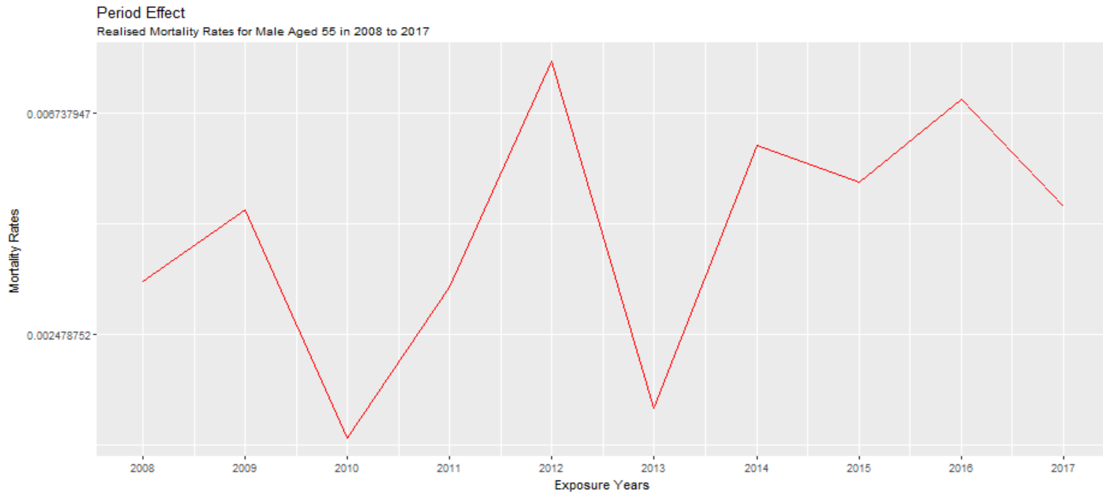

Figure 3 - Realized Death Rates for Male Aged 55 in 2008 to 2017

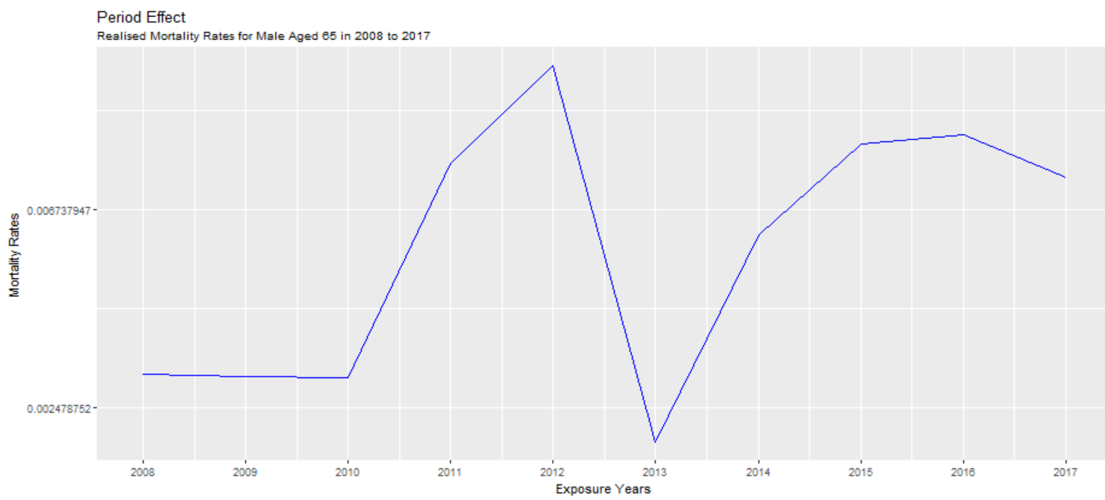

Figure 4 - Realized Death Rates for Male Aged 65 in 2008 to 2017

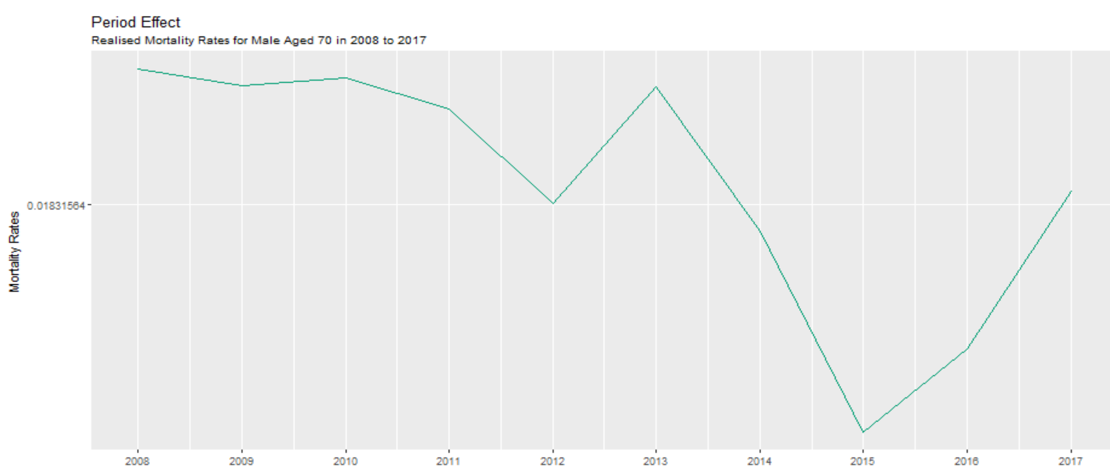

Figure 5 - Realized Death Rates for Male Aged 70 in 2008 to 2017

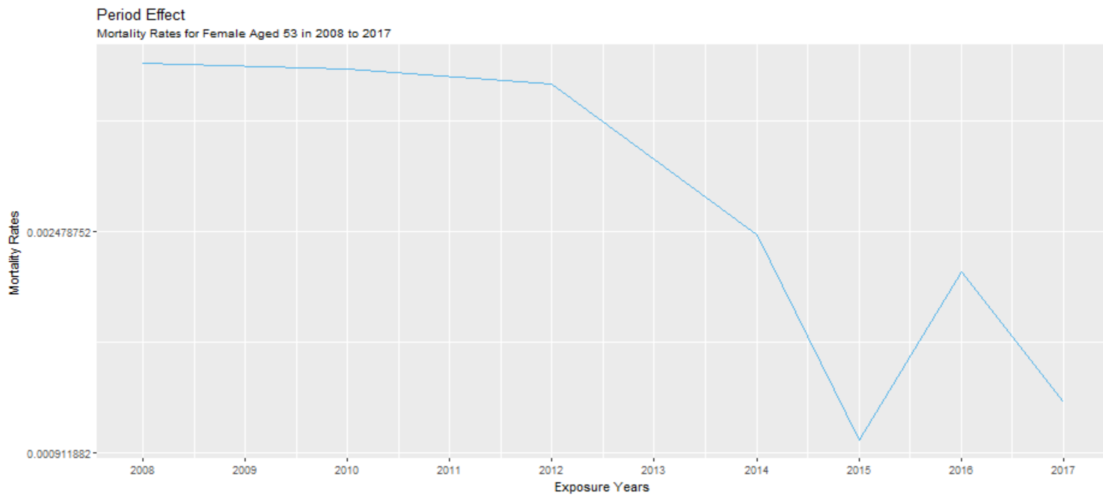

Figure 6 - Realized Death Rates for Female Aged 53 in 2008 to 2017

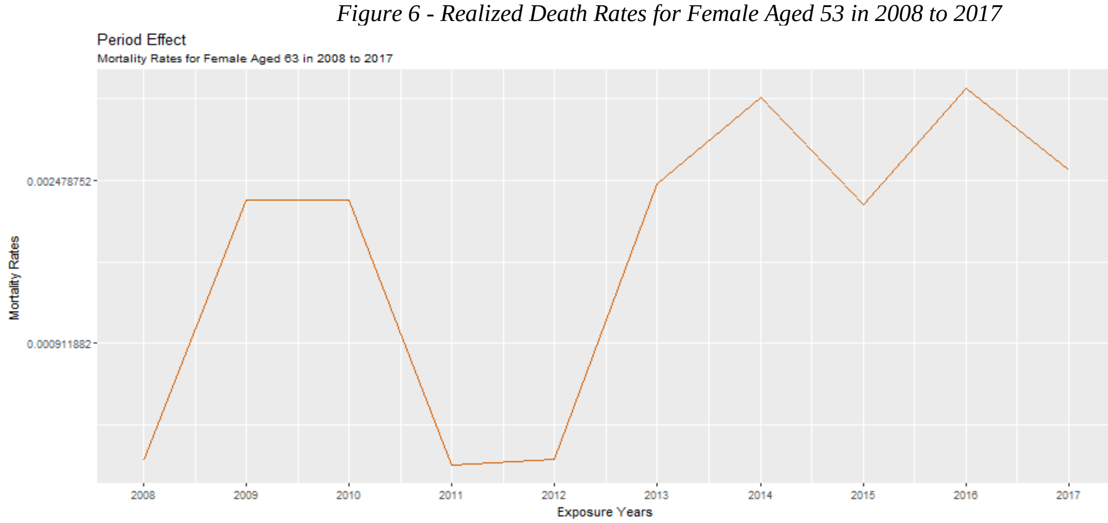

Figure 7 - Realized Death Rates for Female Aged 63 in 2008 to 2017

The figures above show an inconsistent trend in mortality rates during the years of investigation. However, there is a significant reduction in mortality between 2012 & 2013 in Figures 3 & 4 and subsequently in Figure 5. However, between 2013 & 2015, Figures 6 and 7 also show consistent trends in later years. This is because of a scanty data set for females in earlier exposure years.

It’s clear that the number of exposure within these years increased significantly, however not the same can be said for the number of deaths realized. Other factors such as improved economic conditions, technology and medical advancements may also contribute to this pattern.

4.3 The Graduation methodology

In this section we apply smoothing and parametric methods to the realized central mortality rates as estimated above.

The smoothing methods – The Natural Basis and Penalized Spline methods – produce, in principle, more adequate models to explain the variabilities in the mortality data.

The modeled rates derived from the regression model are then compared with PA90 (1967-70 UK Pensioners / Annuitants - Ultimate) and with SAI (MF) L98 (South African Annuitant Standard Mortality Table 1996-2000) using a ratio referred to as the Standardised Mortality Ratio (SMR). The SMR can be simply defined the SMR as the ratio of the actual to expected death - see for instance The Actuarial Profession (2015).

4.3.1 B Spline – Natural Cubic Spline

Using a sample (a training set) of the estimated average mortality rates for each age band across the exposure years, a natural basis spline is then constructed.

The tables below show the summary statistics derived under four different models:

Model	Number of model parameters	p-value	Adjusted R-squared	Model AIC*
Model -1	1	1.235e-08	56.74%	-175.17
Model -2	2	2.402e-12	75.18%	-196.46
Model -3	3	5.697e-12	76.31%	-197.42
Model-4	4	4.458e-11	75.52%	-195.28

Table 5 - Summary statistics for male using natural basis splines

Akaike's Information Criterion

Model	Number of model parameters	p-value	Adjusted R-squared	Model AIC*
Model -1	1	4.619e-07	49.74%	-253.3433
Model -2	2	2.282e-11	73.94%	-277.3694
Model -3	3	2.596e-11	76.17%	-279.8720
Model -4	4	2.082e-10	75.24%	-277.5569

Table 6 – Summary statistics for female using natural basis splines

From Table 5 above Model -3 has the lowest AIC score and shows that the model explains 76% of the variability in mortality rates, which is the highest as compared to the other three models. The scatter plot below shows the cubic spline constructed from this model:

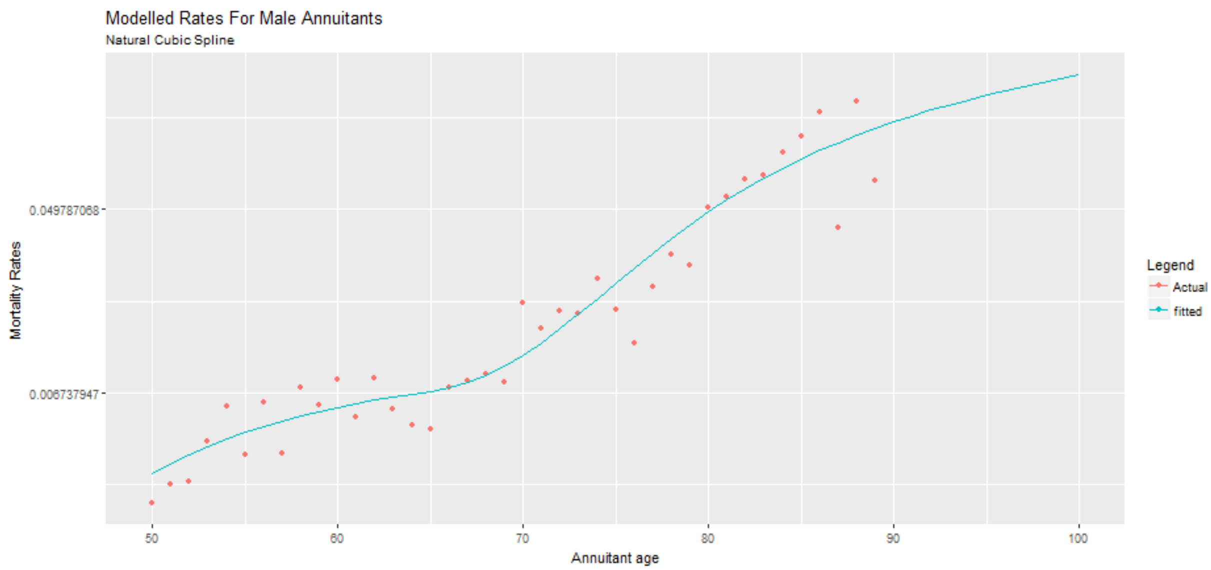

Figure 8 - Modelled Rates for Male Annuitants – Natural Cubic Spline

Similarly, for female, from Table 6 above, Model-3 also has the lowest AIC score and shows that the model explains 76% of the variability in mortality rates, which is the highest as compared to the other three models. The scatter plot below shows the cubic spline constructed from this model:

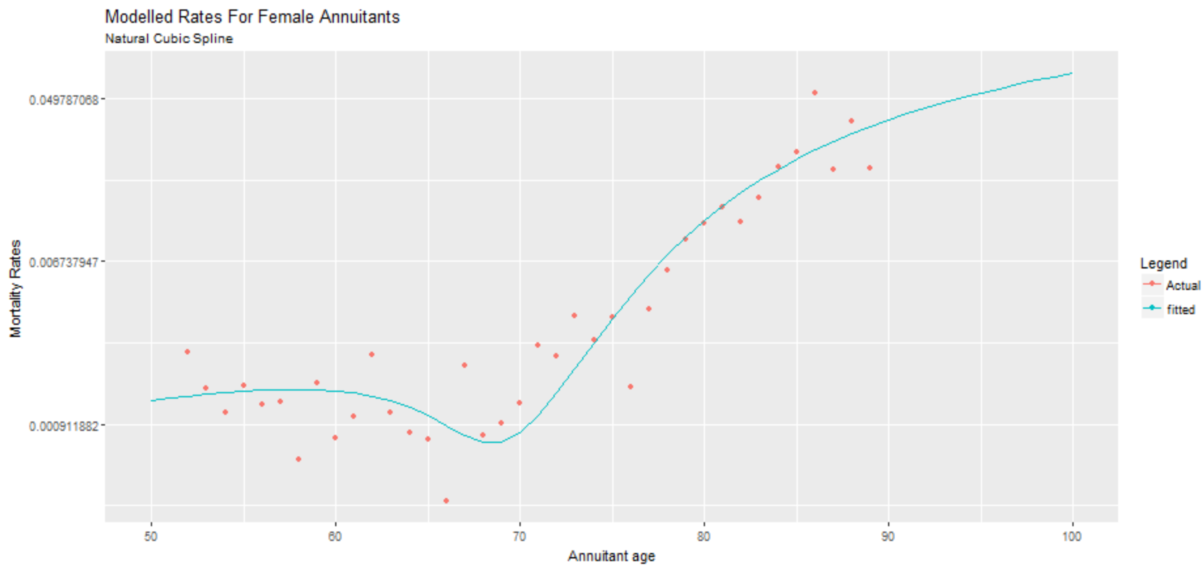

Figure 9 - Modelled Rates for Female Annuitants – Natural Cubic Spline

4.3.2 P Spline – Smoothing a one-dimensional mortality data using a P-Spline

In this section a Penalized basis spline is constructed from the training sets (refer to 2.1). The Actual and fitted death rates in addition to a 95% confidence level are shown in the figure below:

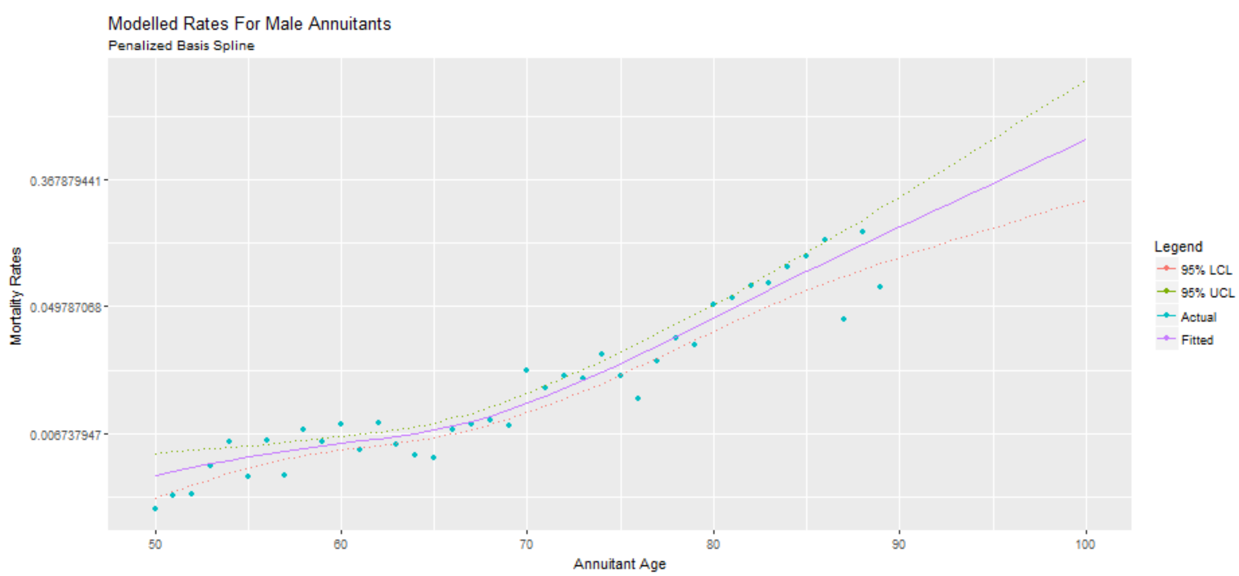

Figure 10 – P – Spline (Natural Cubic Spline) for Male Annuitants

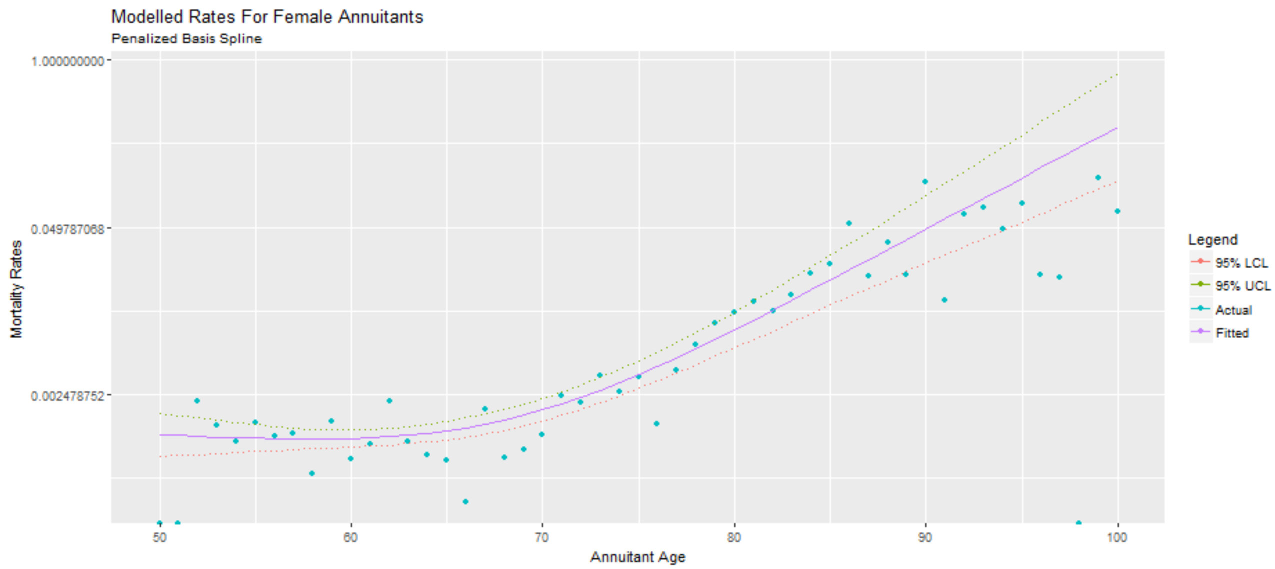

Figure 11 – P – Spline (Natural Cubic Spline) for Female Annuitants

This section presents at 95% confidence level, both the lower and upper bounds at which death rates could be for annuitants based on this data.

For Male annuitants in the age bracket (50 - 54), it is between (0.00298 and 0.00514), age (60 - 64), it's between (0.00556 and 0.00691) and age (80 - 84) death rate is between (0.04499 and 0.07272).

Similarly, in Figure 11, Female annuitants in age bracket (50 - 54) seem to experience between (0.00083 and 0.00161), death rate is between (0.00099 and 0.00136) for age (60 - 64) whilst death rate sits between (0.00791 and 0.01683) for age (80 - 84).

4.3.3 Benchmark Mortality Rates –Males & Females

Here are the pictorial representations for PA90 of the UK, and SAI (MF) L98 of South Africa rates:

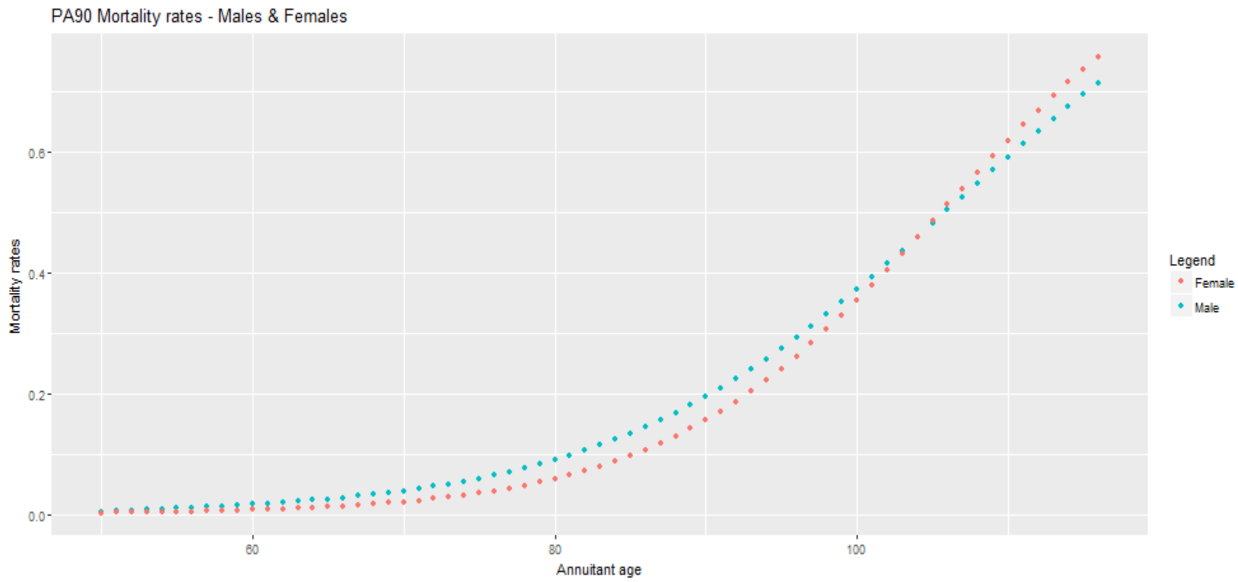

Figure 12 Benchmark Mortality Rates – PA90 for Males & Females

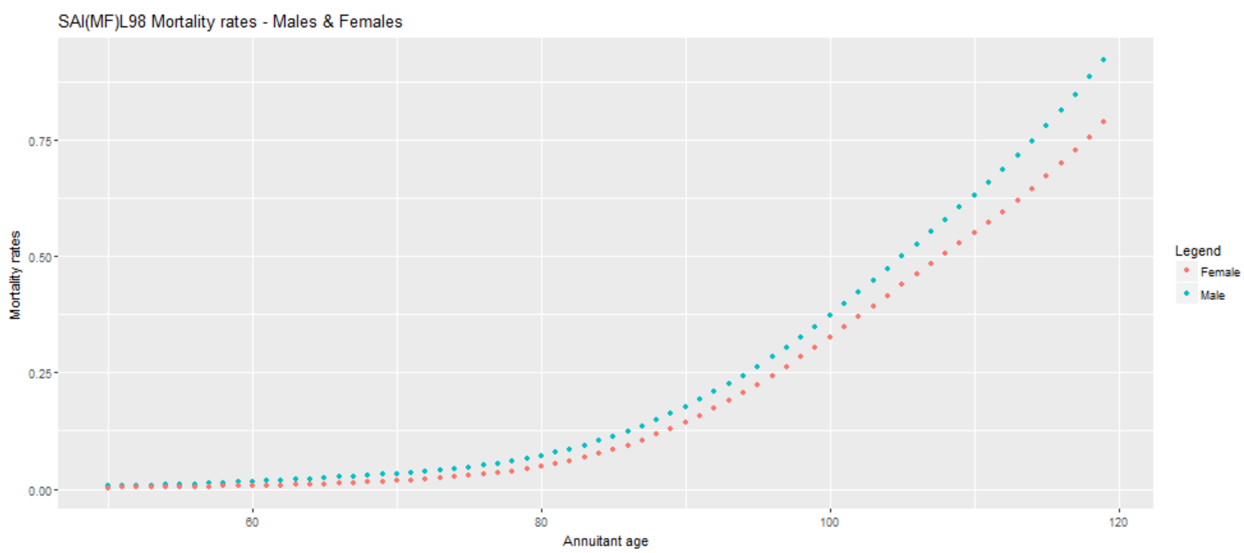

Figure 13 Benchmark Mortality Rates –SA L98 for Males & Females

From the Benchmark rates for both males and females, we observe the following that:

Overall the rate of death increases with increasing age as expected. In the UK, the male lives experience heavier mortality than females for ages less than 105. Male lives mortality improves for ages over 105 years.

4.4 Estimated crude rates and smoothened rates

4.4.1 Comparison of rates for male annuitants

Age Bands	Male Annuitants					Benchmark
	Crude Rates	NCS Best Estimate	PS Best Estimate	UCL Estimate	LCL Estimate	PA-90 M
50-54	0.00338	0.00463	0.00391	0.00514	0.00298	0.00627
55-59	0.00519	0.00358	0.00510	0.00586	0.00444	0.01216
60-64	0.00625	0.00579	0.00620	0.00691	0.00556	0.01938
65-69	0.00705	0.01206	0.00829	0.00945	0.00728	0.03000
70-74	0.01745	0.01999	0.01397	0.01651	0.01182	0.04616
75-79	0.02136	0.03020	0.02746	0.03335	0.02261	0.07038
80-84	0.06812	0.05788	0.05719	0.07272	0.04499	0.10590
85-89	0.10473	0.12120	0.11746	0.17061	0.08106	0.15629
90-94	0.19252	0.21644	0.23545	0.42057	0.13233	0.22458
95-99	0.19067	0.32452	0.46950	1.00000	0.20907	0.31162

Table 7 - Estimated rates for male under different smoothening methods and scenarios by age band

4.4.2 Comparison of rates for female annuitants

Age Bands	Female Annuitants					Benchmark
	Crude Rates	NCS Best Estimate	PS Best Estimate	UCL Estimate	LCL Estimate	PA-90 F
50-54	0.00093	0.00232	0.00116	0.00161	0.00083	0.00296
55-59	0.00118	0.00113	0.00111	0.00135	0.00091	0.00509
60-64	0.00116	0.00102	0.00116	0.00136	0.00099	0.00874
65-69	0.00093	0.00223	0.00147	0.00176	0.00122	0.01497
70-74	0.00235	0.00304	0.00240	0.00299	0.00192	0.02551
75-79	0.00468	0.00516	0.00491	0.00647	0.00373	0.04315
80-84	0.01421	0.01762	0.01154	0.01683	0.00791	0.07207
85-89	0.03176	0.04374	0.02860	0.04830	0.01697	0.11795
90-94	0.06190	0.07589	0.07163	0.14343	0.03586	0.18705
95-99	0.04727	0.10862	0.17952	0.43059	0.07504	0.28348

Table 8 - Estimated rates for female under different smoothening methods and scenarios by age band

In the above two tables (Tables 7 and 8), crude rates are placed alongside rates obtained from natural cubic spline (B Spline) and P Spline to see the discrepancies between each other. A further analysis was done where the upper confidence level (UCL) under P Spline (best case rates) and lower confidence level (LCL) under P Spline (worst case rates).

The upper confidence level which stands at 95% is being regarded as the highest death rate that could emerge should anything goes wrong with the annuitants. High death rates of annuitants mean a havoc to a layman whilst this means profit to an annuity writer (insurer). Also, the 95% lower confidence level gives the minimum death rates that could emerge from each age band within the portfolio.

Table 7 gives the comparisons of rates for male annuitants where benchmark rates are greater than other various rates except age (85 – 99) where benchmark (PA90) rates are lower than upper confidence level (UCL) rates whilst in Table 8, all benchmark (PA 90) rates are greater than other estimated rates except in age (95 – 99) where the benchmark rate is lower than UCL rates.

Differences between the benchmark rates and various estimates in absolute terms and percentages are also presented in (see Appendix 2).

4.5 Estimated number of deaths

In this section, we have assumed a sample (training set) for exposure across each age band and have estimated the number of deaths under each scenario.

For instance, from Table 9, under male annuitants for age band (55 – 59), 1216 deaths are expected under PA90, 358 deaths under NCS method, and 510 deaths under P Spline method. This result weird since this translates to annuitants' mortality being heavier in the UK than in Nigeria.

This result could be attributed to the fact that annuity business is new in Nigerian insurance market mostly dominated by retirees or retiring individuals of privately owned organisations. An equal argument could be that, the pattern of death rates in the market isn't fully developed to the level of establishing a standard mortality table.

4.5.1 Male deaths comparisons

Age Bands	Exposures	Number of Deaths				
		PA-90 M	NCS Deaths	PS Deaths	UCL Estimated Deaths	LCL Estimated Deaths
50-54	100,000	627	463	391	514	298
55-59	100,000	1216	358	510	586	444
60-64	100,000	1938	579	620	691	556
65-69	100,000	3000	1206	829	945	728
70-74	60,000	2769	1199	838	990	709
75-79	50,000	3519	1510	1373	1668	1131
80-84	10,000	1059	579	572	727	450
85-89	5,000	781	606	587	853	405
90-94	2,000	449	433	471	841	265
95-99	1,000	312	325	469	1000	209

Table 9 - Estimated deaths for male under different smoothening methods and scenarios by age band

4.5.2 Female deaths comparisons

Analogously, from Table 10, under female annuitants for age band (65 – 69), 1497 deaths are expected under PA90, 223 deaths under NCS method, and 147 deaths under P Spline method.

Age Bands	Exposures	Number of Deaths				
		PA-90 F	NCS Deaths	PS Deaths	UCL Estimated Deaths	LCL Estimated Deaths
50-54	100,000	296	232	116	161	83
55-59	100,000	509	113	111	135	91
60-64	100,000	874	102	116	136	99
65-69	100,000	1497	223	147	176	122
70-74	60,000	1531	182	144	179	115
75-79	50,000	2158	258	245	324	186
80-84	10,000	721	176	115	168	79
85-89	5,000	590	219	143	241	85
90-94	2,000	374	152	143	287	72
95-99	1,000	283	109	180	431	75

Table 10 - Estimated deaths for female under different smoothening methods and scenarios by age band

4.6 Standardised Mortality Ratio (SMR)

This section presents the SMRs under the smoothened rates using natural cubic spline (NCS) and P spline (PS).

4.6.1 Standardised Mortality Ratios (SMR) – Male Annuitants

Age Bands	Standardised Mortality Ratios (SMRs) – Male Annuitants			
	NCS	PS	UCL Estimate	LCL Estimate
50-54	74%	62%	82%	48%
55-59	29%	42%	48%	37%
60-64	30%	32%	36%	29%
65-69	40%	28%	32%	24%
70-74	43%	30%	36%	26%
75-79	43%	39%	47%	32%
80-84	55%	54%	69%	42%
85-89	78%	75%	109%	52%
90-94	96%	105%	187%	59%
95-99	104%	151%	321%	67%

Table 11 – Standardised Mortality Ratios for male annuitants under different smoothening methods and scenarios by age band

This section also buttresses the previous by showing the SMRs for all age bands under each method of estimation. Except under natural cubic spline (NCS) at age (95-99), P-spline (PS) at age (90-99) and at age (85-99) under UCL that show the contrary, mortality seems heavier in the UK than in Nigeria.

4.6.2 Standardised Mortality Ratios (SMR) – Female Annuitants

Age Bands	Standardised Mortality Ratios (SMRs) – Female Annuitants			
	NCS	PS	UCL Estimate	LCL Estimate
50-54	79%	39%	55%	28%
55-59	22%	22%	27%	18%
60-64	12%	13%	16%	11%
65-69	15%	10%	12%	8%
70-74	12%	9%	12%	8%
75-79	12%	11%	15%	9%
80-84	24%	16%	23%	11%
85-89	37%	24%	41%	14%
90-94	41%	38%	77%	19%
95-99	38%	63%	152%	26%

Table 12 – Standardised Mortality Ratios for female annuitants under different smoothening methods and scenarios by age band

Also in Table 12, this section also buttresses the previous by showing the SMRs for all age bands under each method of estimation. Except under upper confidence level (UCL) at age (95 -99) that shows the contrary, mortality seems heavier in the UK than in Nigeria.

Chapter 5 – Conclusions

The first analysis in this work was to calculate the crude mortality rates for male and female annuitants, using the data supplied from six (6) annuity providers in addition to a collection information gotten from three (3) closed pensions fund administrator. The discrepancies are quite huge for all age bands across both genders except for male annuitants in the age band (90-94).

A further analysis was done using two smoothing methods (NCS and PS) that give more useful models that explain variability in a data - in this case, it the annuitants' data. The first method is called natural cubic (basis) spline, NCS, and several versions were estimated, to choose the best fitted model to the male and female annuitants' data. This was examined by the AIC (Akaike's information criterion). The least AIC gives the best model and coincidentally the third models with three (3) parameters were selected in each case. The second method is the P-spline (PS) - in this method, the best estimates and confidence levels were estimated and pictorially represented. In addition to these analyses, worst case and best-case scenarios were also estimated. However, in this case, the worst-case scenario is when less people die in the portfolio whilst the best-case scenario is when more people die because of the nature of annuity business.

On the overall, establishing a standard mortality table for any region should be a continuous exercise. As such, annuitant mortality should be continuously investigated to see the true pattern of deaths amongst annuitants in Nigeria. Most importantly is that fact the potential policyholders could be overcharged or undercharged, and this consequently could lead to undue profits or losses, respectively. Specifically, for this work, the PA90 of the UK has highest estimated deaths for virtually all age bands, and this seems too optimistic if being continually considered as the standard table. This result could be associated to annuity business being new in the market with only corporate individuals or retirees from privately owned firms. Also, the pattern of death rates in the market isn't fully developed to the level of establishing a standard mortality table due to amount of information available, that is there was a considerable large amount of annuitants to the UK population compare to annuitants to total population in Nigeria.

Though this thesis work is limited due to the credibility of data available, it is advisable not to place much reliance on the estimates since this work is the first and foremost academic attempt to establishing a standard table for annuitants' mortality in Nigeria. Therefore, the primary purpose of this work is to demonstrate a possible way to answer the research question provided there is sufficient data. Further mortality studies on Nigerian annuity market are to be done, to accurately establish Nigerian own standard mortality table.

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The Actuarial Profession, Subject CT5 - Contingencies - Core Reading for the 2016 exams (2015).

Appendix 1

Below are tables showing absolute and ratio differences between various estimated rates and the benchmark (PA 90) rates.

4.4.3. (a) Death differences in absolute term - Male Annuitants

Age Bands	Male Annuitants				
	diff Crude rates	diff NCS	diff Best Estimate	diff UCL	diff LCL
50-54	0.00289	0.00164	0.00236	0.00113	0.00329
55-59	0.00697	0.00858	0.00706	0.00630	0.00772
60-64	0.01313	0.01359	0.01318	0.01247	0.01381
65-69	0.02295	0.01794	0.02170	0.02055	0.02272
70-74	0.02871	0.02617	0.03219	0.02965	0.03433
75-79	0.04903	0.04018	0.04292	0.03703	0.04777
80-84	0.03778	0.04802	0.04871	0.03317	0.06091
85-89	0.05156	0.03508	0.03883	-0.01433	0.07522
90-94	0.03206	0.00814	-0.01087	-0.19599	0.09225
95-99	0.12095	-0.01289	-0.15787	-0.68838	0.10255

Table 13 – Death differences in absolute term –Male Annuitants

4.4.3. (b.) Death differences in ratio term - Male Annuitants

Age Bands	Male Annuitants				
	PA90 as a % of Crude Rates	PA90 % of NCS	PA90 as a % of UCL	PA90 % of LCL	PA90 % of Crude Rates
50-54	186%	135%	160%	122%	210%
55-59	234%	339%	238%	208%	274%
60-64	310%	335%	313%	281%	348%
65-69	425%	249%	362%	317%	412%
70-74	265%	231%	330%	280%	390%
75-79	330%	233%	256%	211%	311%
80-84	155%	183%	185%	146%	235%
85-89	149%	129%	133%	92%	193%
90-94	117%	104%	95%	53%	170%
95-99	163%	96%	66%	31%	149%

Table 14 – Death differences in ratio term –Male Annuitants

4.4.4. (a) Death differences in absolute term - Female Annuitants

Age Bands	Female Annuitants				
	diff Crude rates	diff NCS	diff Best Estimate	diff UCL	diff LCL
50-54	0.00203	0.00064	0.00180	0.00135	0.00213
55-59	0.00391	0.00396	0.00398	0.00374	0.00418
60-64	0.00758	0.00772	0.00758	0.00738	0.00775
65-69	0.01403	0.01274	0.01350	0.01321	0.01374
70-74	0.02317	0.02247	0.02311	0.02252	0.02359
75-79	0.03847	0.03799	0.03824	0.03668	0.03943
80-84	0.05786	0.05446	0.06054	0.05524	0.06416
85-89	0.08619	0.07421	0.08935	0.06965	0.10098
90-94	0.12514	0.11116	0.11542	0.04362	0.15119
95-99	0.23621	0.17486	0.10396	-0.14711	0.20845

Table 15 – Death differences in absolute term –Female Annuitants

4.4.4. (b) Death differences in ratio term - Female Annuitants

Age Bands	Male Annuitants				
	PA90 as a % of Crude Rates	PA90 % of NCS	PA90 as a % of UCL	PA90 % of LCL	PA90 % of Crude Rates
50-54	319%	127%	255%	183%	355%
55-59	431%	451%	459%	376%	560%
60-64	754%	856%	753%	644%	879%
65-69	1603%	672%	1020%	851%	1224%
70-74	1088%	840%	1064%	854%	1326%
75-79	922%	836%	879%	667%	1158%
80-84	507%	409%	625%	428%	911%
85-89	371%	270%	412%	244%	695%
90-94	302%	246%	261%	130%	522%
95-99	600%	261%	158%	66%	378%

Table 16 – Death differences in ratio term –Female Annuitants

Appendix 2

These are the R codes for the whole analyses done in this master's final work.

```
#    File name: mortAnalysis.r
#
#    Nigeria Mortality Analysis - Annuitant Mortality
#
#   Load required packages & Read in data

library(tidyverse) #tidyverse: A collection of R packages designed for data science
library(splines) # Makes use of the Natural Cubic Spline Approach
library(MortalitySmooth) #Makes use of the P Spline Approach
library(modelr) # Data Grid function
library(scales) # for log transformation of data visualizations
library(StMoMo) #Loading R Package for Stochastic Mortality Modelling

source("dataPrep.R") #Read Datasets into Workspace

# ----Scatter Plots of Crude Rates---
ggplot(data = crudeRates) +
    geom_point(mapping = aes(Age, M, color = "Male")) +
    geom_point(mapping = aes(Age, F, color = "Female")) +
    labs(colour = "Legend") +
    labs(title = "Crude Mortality Rates for both Male & Females") +
    scale_y_continuous(name = "Mortality Rates", trans =log_trans())+
    xlab("Annuitant age") +
    labs(caption = "(Annuitant Mortality Experience)")

ggplot(data = bandedRates, aes(group = 1)) +
    geom_line(mapping = aes(x = AgeBand, y = M, color = "Male Proportion")) +
    geom_line(mapping = aes(x = AgeBand, y = F, color = "Female Proportion")) +
    labs(color = "Legend") +
    labs(title = "Relative Proportions of Female Crude Rates") +
    ylab("Number of Deaths") +
    xlab("Annuitant Age-Bands")


ggplot(data = mCRs) +
    geom_line(mapping = aes(x = mCRs[,1], y = mCRs[, 7]), color = "#009E73") +
    labs(title = "Period Effect ", subtitle = "Realised Mortality Rates for Male Aged 55
in 2008 to 2017") +
    scale_y_continuous(name = "Mortality Rates", trans = log_trans()) +
    scale_x_continuous(name = "Exposure Years", breaks = 2008:2017)

ggplot(data = fCRs) +
    geom_line(mapping = aes(x = fCRs[, 1], y = fCRs[, 14], color = "#56B4E9")) +
    labs(color = "Legend") +
    labs(title = "Period Effect ", subtitle = "Mortality Rates for Female Aged 63 in 2008
to 2017") +
    scale_y_continuous(name = "Mortality Rates", trans = log_trans()) +
    scale_x_continuous(name = "Exposure Years", breaks = 2008:2017)
```

```
# ----Smoothening Crude Rates---

# Natural Cubic Spline (Applying several DFs)
NCSM <- list()
NCSF <- list()

for (i in 1:4) {

    NCSM[[i]] <- lm(M ~ ns(Age, df = i), data = crudeRates[c(1:40),])
    print(summary(NCSM[[i]]))

    NCSF[[i]] <- lm(F ~ ns(Age, df = i), data = crudeRates[c(3:40),])
    print(summary(NCSF[[i]]))
}

#Predict mortality rates for Test Data using the constructed NCS

NCSMpredict <- predict(NCSM[[3]], newdata = Ages)
NCSFpredict <- predict(NCSF[[3]], newdata = Ages)
ncsPredictions <- as.tibble(data.frame(Age = Ages, M = NCSMpredict, F = NCSFpredict))

# ----Scatter Plots of NCS Predictions---
ggplot() +
    geom_point(data = crudeRates[c(1:40),], mapping = aes(Age, M, color = "Actual")) +
    geom_line(data = ncsPredictions, mapping = aes(Age, M, color = "fitted")) +
    labs(color = "Legend") +
    labs(title = "Modelled Rates For male Annuitants", subtitle = "Natural Cubic Spline")
+
    xlab("Annuitant age") +
    scale_y_continuous(name = "Mortality Rates", trans = log_trans())


ggplot() +
    geom_point(data = crudeRates[c(3:40),], mapping = aes(Age, F, color = "Actual")) +
    geom_line(data = ncsPredictions, mapping = aes(Age, F, color = "fitted")) +
    labs(color = "Legend") +
    labs(title = "Modelled Rates For Female Annuitants", subtitle = "Natural Cubic
Spline") +
    xlab("Annuitant age") +
    scale_y_continuous(name = "Mortality Rates", trans = log_trans())


#----Penalized Basis Spline under a 1D-Setting-----

PSM1D <- Mort1Dsmooth(x = psdata[c(1:40), 1], y = psdata[c(1:40), 3], offset =
log(psdata[c(1:40), 2]), overdispersion = TRUE)
PSF1D <- Mort1Dsmooth(x = psdata[c(3:38), 1], y = psdata[c(3:38), 4], offset =
log(psdata[c(3:38), 2]), overdispersion = TRUE)


#Predict mortality rates for Test Data using the constructed P-Spline

Ages.new <- 50:100
PSMpredict <- predict(PSM1D, newdata = Ages.new, se.fit = TRUE)
PSFpredict <- predict(PSF1D, newdata = Ages.new, se.fit = TRUE)

psBase <- as.tibble(data.frame(Age = Ages, M = exp(PSMpredict$fit), F =
exp(PSFpredict$fit)))
```

```
# Construct a 95% Confidence interval for Base P-Spline

psUCL <- as.tibble(data.frame(Age = Ages, M = exp(PSMpredict$fit + 2 *
PSMpredict$se.fit), F = exp(PSFpredict$fit + 2 * PSFpredict$se.fit)))
psLCL <- as.tibble(data.frame(Age = Ages, M = exp(PSMpredict$fit - 2 *
PSMpredict$se.fit), F = exp(PSFpredict$fit - 2 * PSFpredict$se.fit)))

# ----Scatter Plots of P-Spline Predictions---

ggplot() +
    geom_point(data = crudeRates, mapping = aes(Age, M, color = "Actual")) +
    geom_line(data = psBase, mapping = aes(x = Ages, M, color = "Fitted")) +
    geom_line(data = psUCL, mapping = aes(x = Ages, M, color = "95% UCL"), linetype =
"dotted") +
    geom_line(data = psLCL, mapping = aes(x = Ages, M, color = "95% LCL"), linetype =
"dotted") +
    labs(color = "Legend") +
    labs(title = "Modelled Rates For Male Annuitants", subtitle = "Penalized Basis
Spline") +
    scale_y_continuous(name = "Mortality Rates", trans = log_trans()) +
    scale_x_continuous(name = "Annuitant Age")

ggplot() +
    geom_point(data = crudeRates, mapping = aes(Age, F, color = "Actual")) +
    geom_line(data = psBase, mapping = aes(x = Ages, F, color = "Fitted")) +
    geom_line(data = psUCL, mapping = aes(x = Ages, F, color = "95% UCL"), linetype =
"dotted") +
    geom_line(data = psLCL, mapping = aes(x = Ages, F, color = "95% LCL"), linetype =
"dotted") +
    labs(color = "Legend") +
    labs(title = "Modelled Rates For Female Annuitants", subtitle = "Penalized Basis
Spline") +
    scale_y_continuous(name = "Mortality Rates", trans = log_trans()) +
    scale_x_continuous(name = "Annuitant Age")

# ----The End---
```