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Studies on open-channel cross-section shapes and their hydraulic exponents

II — Round-cornered trapezoid

by

Prof. P. DE VARENNES E MENDONÇA
Chair of General and Agricultural Hydraulics

RESUMO

Neste segundo estudo sobre formas de secção transversal de canais abertos e respectivos expoentes hidráulicos considera-se a secção trapezoidal de cantos arredondados, bem como os seus casos particulares: secções triangular de vértice arredondado e rectangular de cantos arredondados.

RESUME

En cette deuxième étude sur des formes de section droite de canaux découverts et les exposants hydrauliques respectifs, on considère la section trapézoïdale à coins arrondis et ses cas particuliers: section triangulaire à sommet arrondi et section rectangulaire à coins arrondis.

SYNOPSIS

In this second study the round-cornered trapezoidal section and its particular cases (round-bottomed triangle and round-cornered rectangle) are considered.

1. General geometric relations.

Consider the asymmetrical round-cornered trapezoid shown in Fig. 1.

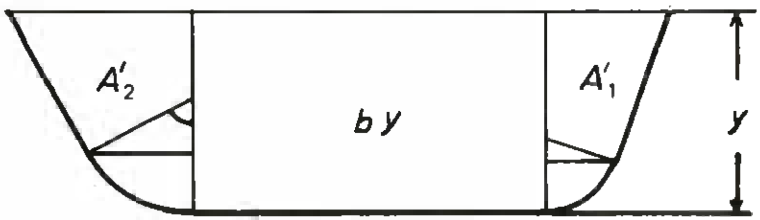

Fig. 1

The whole may be seen as composed by three portions: a mid rectangle b wide and y high, and two lateral parts. Each one of this is a right-angled triangle, A_i in area, of which the «hypotenuse» U_i is in general formed above by a straight line inclined an angle ξ_i to the horizon and below by an arc of circle of radius ρ_i and central angle ψ_i, the other sides being the water surface line and the height y,

Let h_i denote the elevation above the bottom of the point that separates the lateral straight line from the circular one.

If it is $y < h_i$, the following relations are found:

1) $$U_i = \rho_i \; \psi_i \quad ,$$

2) $$B_i = \rho_i \; \sin \; \psi_i$$

and

3) $$A_i = \frac{\rho_i^2}{2} \left(\psi_i - \frac{\sin \; 2 \; \psi_i}{2} \right) \quad .$$

If instead we have $y > h_i$, these formulas are replaced by

4) $$U'_i = y\sqrt{1 + c_i^2} + \rho_i \left(\text{arccot} \; c_i - \frac{1}{c_i + \sqrt{1 + c_i^2}} \right) \quad ,$$

5) $$B'_i = c_i \; y + \frac{\rho_i}{c_i + \sqrt{1 + c_i^2}}$$

and

6) $$A'_i = \frac{c_i\, y^2}{2} + \rho_i \left[\frac{y}{c_i + \sqrt{1+c_i^2}} + \right.$$
$$\left. + \frac{1 - (c_i + \sqrt{1+c_i^2})^2\, (1 - c_i \operatorname{arccot} c_i)}{2\, c_i\, (c_i + \sqrt{1+c_i^2})^2} \right] ,$$

where

7) $c_i = \cot \xi_i$.

When the section is symmetric ($\rho_1 = \rho_2 = \rho$ and $c_1 = c_2 = c$), it is $h_1 = h_2 = h$, and so we have to consider only the two cases $y < h$ and $y > h$.

In general however three different situations occur according to the relative values of y, h_1 and h_2 .

If y is smaller than both h_1 and h_2, then the total wetted perimeter U, the total top width B and the total sectional area A are

8) $U = U_1 + U_2 + b$,

9) $B = B_1 + B_2 + b$

10) $A = A_1 + A_2 + b\, y$,

where b denotes the bottom length.

If y lies in between h_1 and h_2 we have, assuming $h_1 < h_2$:

11) $U = U'_1 + U_2 + b$

12) $B = B'_1 + B_2 + b$

and

13) $A = A'_1 + A_2 + b\, y$.

Finally for a value of y larger then both h_1 and h_2 , as in Fig. 1, we get

14) $U = U'_1 + U'_2 + b$,

15) $B = B'_1 + B'_2 + b$

and

16) $A = A'_1 + A'_2 + b\ y$.

These formulas hold good for the particular cases of the round-bottomed triangle ($b = 0$) and also — with the obvious exception of 6) and consequently of 13) and 16) — for the round-cornered rectangle ($c_1 = c_2 = c$). Even the case of the ordinary trapezoid ($\rho_1 = \rho_2 = 0$) is not excluded.

2. *Round-bottomed triangle.*

In the case of the symmetrical round-bottomed triangle instead of taking ρ for standard length, as formulas 1) to 6) do, it might be preferred to take for such a purpose the distance a between the two vertices, that of the rounded bottom and that of the primitive triangle (Fig. 2).

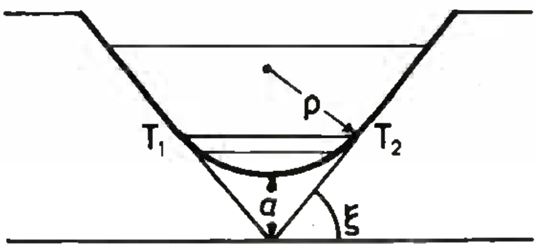

Fig. 2

The two lengths ρ and a are related by

17) $$\frac{\rho}{a} = c\,(c + \sqrt{1 + c^2}),$$

where, of course,

18) $c = \cot\ \xi$.

From 17) we get

19) $$h = \rho\,(1 - \cos\ \xi) = a\ c\,(c + \sqrt{1 + c^2}) \left[1 - \frac{c}{\sqrt{1 + c^2}} \right].$$

The pertinent formulas for U, B and A, written in dimensionless forms, are then those given below (Mendonça 1980).

For $y \geqslant h$ (surface line not under T_1T_2), we have

$$20)\quad \frac{U}{a} = 2\left[\left(\frac{y}{a}+1\right)\sqrt{1+c^2} - \left(c+\sqrt{1+c^2}\right)(1-c \operatorname{arccot} c)\right],$$

$$21)\quad \frac{B}{a} = 2\,c\left(\frac{y}{a}+1\right)$$

$$22)\quad \frac{A}{a^2} = c\left[\left(\frac{y}{a}+1\right)^2 - \left(c+\sqrt{1+c^2}\right)^2 (1-c \operatorname{arccot} c)\right],$$

but for $y \leqslant h$ these formulas must be replaced by

$$23)\quad \frac{U}{a} = 2\frac{\rho}{a}\psi\,,$$

$$24)\quad \frac{B}{a} = 2\frac{\rho}{a}\sin\psi$$

and

$$25)\quad \frac{A}{a^2} = \frac{\rho^2}{2a^2}(2\psi - \sin 2\psi)\,,$$

where

$$26)\quad \psi = \arccos\left(1-\frac{y}{\rho}\right) = \arccos\left[1-\frac{\frac{y}{a}}{c\left(c+\sqrt{1+c^2}\right)}\right].$$

3. Round-cornered rectangle.

As it has been remarked above, the formulas given in Section 1 for computing U and B still hold, but 6), containing a division by c_i (now equal to zero) cannot be employed.

In the symmetrical case (Fig. 3) the following formulas can be used.

For $y \geqslant h$ (surface line not below T_1T_2) (*) :

27) $$\frac{U}{\rho} = 2\left(\frac{y}{\rho} - 1\right) + \frac{b}{\rho} + \pi \quad ,$$

28) $$\frac{B}{\rho} = \frac{b}{\rho} + 2 \quad ,$$

29) $$\frac{A}{\rho^2} = \left(\frac{y}{\rho} - 1\right)\left(\frac{b}{\rho} + 2\right) + \frac{b}{\rho} + \frac{\pi}{2} \quad .$$

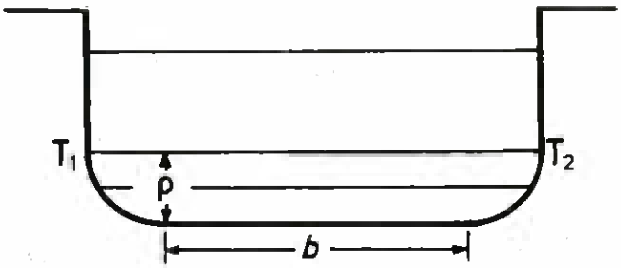

Fig. 3

For $y \leqslant h$:

30) $$\frac{U}{\rho} = \frac{b}{\rho} + 2\,\psi \, ,$$

31) $$\frac{B}{\rho} = \frac{b}{\rho} + 2 \sin \psi \quad ,$$

32) $$\frac{A}{\rho^2} = \frac{y}{\rho} \cdot \frac{b}{\rho} + \frac{1}{2}\,(2\,\psi - \sin 2\,\psi) \quad ,$$

with

33) $$\psi = \arccos\left(1 - \frac{y}{\rho}\right) \quad .$$

(*) Chow (1959), in his Table 2-1 (p. 21), gives formulas equivalent to 27), 28) and 29), but he forgot to point out that they apply only for $y \geqslant h$.

4. *General software.*

Using a pocket computer Casio FX-702P, we have found that a simple and sufficiently general algorithm for solving problems about the cross sections we are dealing with consists of making suitable combinations of Subroutines 1 to 5.

Subroutine 1

```
LIST #4
  10 PRT "R.C.TRAPEZ
     OID, SBR 1"
  20 INP "C",C
  30 S=SQR (1+C↑2)
  40 T=π/2-ATN C
  50 INP "G",G
  60 H=G*(1-COS T)
  65 INP "Y",Y
  70 IF Y>H THEN 130
  80 I=ACS (1-Y/G)
  90 U=G*I
 100 B=G*SIN I
 110 A=G↑2*(I-SIN (2
     *I)/2)/2
 120 GOTO 190
 130 U=Y*S+G*(T-1/(C
     +S))
 140 B=C*Y+G/(C+S)
 150 IF C=0 THEN 180
 160 A=C*Y↑2/2+G*Y/(
     C+S)+G↑2*(1-(C+
     S)↑2*(1-C*T))/2
     /C/(C+S)↑2
 170 GOTO 190
 180 A=G*Y-G↑2+π/4
 190 PRT "H=";H
```

Subroutine 2

```
LIST #5
  10 PRT "R.C.TRAPEZ
     OID, SBR 2"
  20 L=U
  30 M=B
  40 N=A
  50 PRT "END "
```

Subroutine 3

```
LIST #6
  10 PRT "R.C.TRAPEZ
     OID, SBR 3"
  20 INP "Z",Z
  25 K=Y
  30 V=L+U+Z
  40 P=M+B+Z
  50 E=N+A+Y*Z
  60 PRT "END "
```

Subroutine 4

```
LIST #7
  10 PRT "R.C.TRAPEZ
     OID, SBR 4"
  20 INP "Z",Z
  30 U=L+U+Z
  40 B=M+B+Z
  50 A=N+A+Y*Z
  60 SET F7
  70 PRT "U=";U
  80 PRT "B=";B
  90 SET E8
 100 PRT "A=";A
 110 SET N
 120 PRT "END "
```

Subroutine 5

```
LIST #8
  10 PRT "R.C.TRAPEZ
     OID, SBR 5"
  15 X=Y/K
  20 L=(V/U)↑(4/3)
  30 M=(A/E)↑(1/3)
  40 N=(A/E)↑3
  50 R=LN (L*M*N)/LN
     X
  60 Q=LN (L*M*B/P)/
     LN X
  70 W=LN (N*P/B)/LN
     X
  80 SET F2
  90 PRT "R=";R
 100 PRT "Q=";Q
 110 PRT "W=";W
 120 SET N
 130 PRT "END "
```

Subroutine 1 computes U_i, B_i and A_i, and ascribes their values to U, B and A respectively; it ends by printing $H = h_i$. The correspondence between the text symbols and those adopted in the program is as follows:

c_i	ρ_i	y_i	h_i	U_i	B_i	A_i	ψ_i	ξ_i
C	G	Y	H	U	B	A	I	T .

Subroutine 2 copies U into L, B into M and A into N.

Subroutine 3 inputs b as Z, transcribes Y into K, and computes U_{10}(V), B_{10}(P) and A_{10}(E), where the subscript zero denotes the normal values.

Subroutine 4 also inputs b as Z, computes U, B and A, and prints their values.

Subroutine 5 computes the hydraulic exponents r (R), q (Q) and w (W) — defined in Section 3.1 of the first part of this memoir (Mendonça 1984) — assuming that Manning or Gauckler-Strickler formula has been adopted (*), and prints their values.

These subroutines are employed in the manner related below.

The angles must be expressed in radians, so MODE 5 has to be used. The instructions in parentheses can be omitted when the section is symmetric.

For computing U, B and A, after calling SBR 1, input $C = c_1$, $G = \rho_1$ and $Y = y$, execute and thereafter perform SBR 2 (call SBR 1, input $C = c_2$ and $G = \rho_2$, execute), do SBR 4.

For computing not only those values but also the hydraulic exponents,

a) after calling SBR 1, input $C = c_1$, $G = \rho_1$ and $Y = y_0$, execute, do SBR 2 (call SBR 1, input $C = c_2$ and $G = \rho_2$, execute), do SBR 3;

b) continue by calling anew SBR 1, input $C = c_1$, $G = \rho_1$ and $Y = y$, execute, do SBR 2 (call SBR 1, input $C = c_2$ and $G = \rho_2$, execute), and thereafter perform SBR 4 and SBR 5 in succession;

c) for another value cf y, repeat b).

(*) For another uniform flow monomial formula, instructions 20 to 40 must be modified accordingly.

5. *Numerical examples.*

5.1. *Round-cornered trapezoid.*

5.1.1. *Asymmetric.* Data: $b=2$, $c_1=0.5$, $c_2=1$, $\rho_1=1$, $\rho_2=1.5$, $y_0=0.8$.

Using the algorithm of Section 4, we get: $h_1 = 0.5527864045$, $h_2 = 0.4393398282$, $U_0 = 5.0716897$, $B_0 = 4.4393543$ and $A_0 = 2.9586163$. Table 1, for instance, can then be written.

TABLE 1

y	U	B	A	r	q	w
0.3	3.7606503	3.6141428	0.93168817	3.52	0.20	3.32
0.5	4.3110812	3.9873457	1.6943450	3.49	0.16	3.33
0.792	5.0514317	4.4273543	2.9231494	3.47	0.14	3.33
0.808	5.0919477	4.4513543	2.9941791	3.47	0.14	3.33
1	5.5781392	4.4393543	3.8764871	3.47	0.13	3.34

5.1.2. *Symmetric.* Data: $b = 4$, $c = 1$, $\rho = 1$, $y_0 = 0.1$.

The said algorithm has been used. First, we got $h=0.2928932188$, $U_0 = 4.9020536$, $B_0 = 4.8717798$ and $A_0 = 0.45872591$. Then, Table 2 was obtained.

TABLE 2

y	U	B	A	r	q	w
0.101	4.9066311	4.8758973	0.46359975	3.42	0.31	3.10
0.2	5.2870022	5.2000000	0.96360111	3.42	0.31	3.12
0.293	5.5710983	5.4144271	1.4575492	3.43	0.30	3.13
0.4	5.8737401	5.6284271	2.0483419	3.42	0.29	3.13
0.8	7.0051109	6.4284271	4.4597127	3.42	0.27	3.15

5.2. Round-bottomed triangle.

5.2.1. Asymmetric. Data: $c_1 = 0.5$, $c_2 = 1$, $\rho_1 = 1$, $\rho_2 = 1.5$, $y_0 = 0.8$.

Using again the algorithm of Section 4, first we get $h_1 = 0.5527864045$ and $h_2 = 0.4393398282$ just as in Section 5.1.1. Thereafter, we obtain $U_0 = 3.0716897$, $B_0 = 2.4393543$ and $A_0 = 1.3586163$. Then, Table 3 can be constructed.

TABLE 3

y	U	B	A	r	q	w
0.3	1.7606505	1.6141428	0.33168817	4.04	0.14	3.89
05	2.3110812	1.9873457	0.69434501	3.91	0.10	3.87
0.792	3.0514317	2.4273543	1.3391494	3.91	0.09	3.82
0.808	3.0919477	2.4513543	1.3781791	3.91	0.09	3.82
1	3.5781392	2.7393543	1.8764871	3.91	0.09	3.82.

5.2.2. Symmetric. The algorithm of Section 4 can of course be used.

However it may be felt of interest to have an example of the employment of formulas 20) to 26). In order to make easier the computations for a great number of cases, we found pertinent to write Program 1. The correspondence between the text symbols and those used in the program is as follows:

c	$\sqrt{1+c^2}$	h/a	ρ/a	ψ_0	ψ	ξ	y_0/a	u	U_0/a	B_0/a	A_0/a^2	y/a	U/a	B/a	A/a^2	r	q	w
C	S	H	G	J	I	T	K	X	V	P	E	Y	U	B	A	R	Q	W.

It was by means of it that Tables 4 and 5, relative to $c = 1$ (right isosceles triangle), has been built.

5.3. Round-cornered rectangle.

To illustrate the symmetric situation dealt with in Section 3, i. e. the use of formulas 27) to 32), we have chosen the particular case in which the normal height y_0 equals the rounding radius ρ. For it we have written Program 2, where the correspondence between the text and the program symbols is the following:

b/ρ y/ρ ψ U_o/ρ B_o/ρ A_o/ρ^2 U/ρ B/ρ A/ρ^2 r q w
Z Y I V P E U B A R Q W .

It was by means of this program that Table 6 has been built assuming $b/\rho = 2$, a procedure much quicker than the use of the general algorithm described in Section 4.

Program 1

```
LIST #0
  10 PRT "R.-BOTTOME
     D TRIANGLE"
  20 INP "C",C
  30 S=SQR (1+C↑2):T
     =π/2-ATN C
  40 G=C*(C+S):H=G*(
     1-COS T)
  50 INP "K",K
  55 Y=K
  60 GSB #1
  70 V=U:P=B:E=A
  80 INP "Y",Y
  90 GSB #1
 100 L=(V/U)↑(4/3):M
     =(A/E)↑(1/3)
 110 N=(A/E)↑3:X=Y/K
 120 R=LN (L*M*N)/LN
     X
 130 Q=LN (L*M*B/P)/
     LN X
 140 W=LN (N*P/B)/LN
     X
 160 SET F7
 170 PRT "U=";U
 180 PRT "B=";B
 190 SET E8
 200 PRT "A=";A
 205 SET N:PRT "X=";
     X
 210 SET F2
 220 PRT "R=";R
 230 PRT "Q=";Q
 240 PRT "W=";W
 250 SET N
 260 INP "NEW K",D$
 270 IF D$="YES" THE
     N 50
 280 GOTO 80
```

```
LIST #1
  10 IF Y)H THEN 40
  20 I=ACS (1-Y/G):U
     =2*G*I:B=2*G*SI
     N I:A=G↑2*(I-SI
     N (2*I)/2)
  30 RET
  40 U=2*((Y+1)*S-(C
     +S)*(1-C*T):B=2
     *C*(Y+1)
  50 A=C*((Y+1)↑2-(C
     +S)↑2*(1-C*T))
  60 RET
```

Program 2

```
LIST #9
  10 PRT "R.C.SYM.RE
     CT.,W.H.=H=RHO"
  20 INP "Z",Z
  30 INP "Y",Y
  40 V=Z+π
  50 P=Z+2
  60 E=Z+π/2
  70 IF Y)1 THEN 130
  80 I=ACS (1-Y)
  90 U=2*I+Z
 100 B=2*SIN I+Z
 110 A=(2*I-SIN (2*I
     ))/2+Y*Z
 120 GOTO 160
 130 U=2*(Y-1)+V
 140 B=P
 150 A=(Y-1)*B+Z+π/2
 160 L=(V/U)↑(4/3)
 170 M=(A/E)↑(1/3)
 180 N=(A/E)↑3
 190 R=LN (L*M*N)/LN
     Y
 200 Q=LN (L*M*B/P)/
     LN Y
 210 W=LN (N*P/B)/LN
     Y
 220 SET F7
 230 PRT "U=";U
 240 PRT "B=";B
 250 SET E8
 260 PRT "A=";A
 270 SET F2
 280 PRT "R=";R
 290 PRT "Q=";Q
 300 PRT "W=";W
 310 SET N
 320 GOTO 30
```

TABLE 4

Round-bottomed triangle ($c = 1$)

y/a	U/a	B/a	A/a^2
0.004	0.2779859	0.2778324	7.4100920×10^{-4}
0.012	0.4816189	0.4808207	3.8484813×10^{-3}
0.016	0.5562025	0.5549733	5.9236494
0.020	0.6219395	0.6202211	8.2764894
0.040	0.8801654	0.8752990	2.3380256×10^{-2}
0.048	0.9644410	0.9580407	3.0718780×10^{-2}
0.060	1.0787282	1.0697769	4.2898567
0.080	1.2464775	1.2326787	6.5963855
0.10	1.3945800	1.3752712	9.2071385×10^{-2}
0.12	1.5287589	1.5033446	1.2087839×10^{-1}
0.16	1.7677509	1.7285235	1.8563353×10^{-1}
0.20	1.9792134	1.9242509	2.5877061
0.24	2.1712193	2.0987830	3.3929237
0.30	2.4327464	2.3311183	4.7234374
0.36	2.6707591	2.5366385	6.1849279×10^{-1}
0.40	2.8193574	2.6618571	7.2249102×10^{-1}
0.48	3.0976101	2.8894602	9.4472959×10^{-1}
0.60	3.4789116	3.1856279	1.3097131
$1/\sqrt{2}$	3.7922378	3.4142136	1.6634224
0.72	3.8287053	3.44	1.7076088
0.80	4.0549795	3.6	1.9892088
0.90	4.3378222	3.8	2.3592088
1.0	4.6206649	4.0	2.7492088
1.2	5.1863503	4.4	3.5892088
1.5	6.0348785	5.0	4.9992088
1.6	6.3177212	5.2	5.5092088
1.8	6.8834066	5.6	6.5892088
2.0	7.4490920	6.0	7.7492088
2.4	8.5804629	6.8	1.0309209×10
3.0	10.2776192	8.0	1.4749209×10
3.6	11.9745754	9.2	1.9909209×10
4.0	13.1059463	10	2.3749209
4.5	14.5201599	11	2.8999209
5.0	15.9343734	12	3.4749209
6.0	18.7628005	14	4.7749209×10
7.5	23.0054412	17	7.0999209×10
8	24.4196548	18	7.9749209×10
10	30.0765090	22	1.1974921×10^2
12	35.7333633	26	1.6774921
15	44.2186447	32	2.5474921×10^2
18	52.7039260	38	3.5974921×10^2
20	58.3607803	42	4.3974921
24	69.6744888	50	6.2374921
25	72.5029159	52	6.7474921
30	86.6450515	62	9.5974921×10^2
36	103.6156143	74	1.3677492×10^3
45	129.0714584	92	2.1147492
50	143.2135940	102	2.5997492
60	171.4978653	122	3.7197492×10^3
100	284.6349503	202	1.0199749×10^4
150	426.0563065	302	2.2799749×10^4
250	708.8990190	502	6.2999749
300	850.3203752	602	9.0599749×10^4
500	1416.005800	1002	2.5099975×10^5
1500	4244.432925	3002	2.2529997×10^6

TABLE 5

Round-bottomed triangle ($c = 1$) — Manning formula

Values of r

u \ y_o/a	0.2	0.6	1	2	3	5	10	30
0.02	4.32	4.29	4.26	4.22	4.21	4.23	4.30	4.53
0.08	4.31	4.28	4.23	4.17	4.16	4.20	4.35	4.76
0.2	4.31	4.25	4.19	4.11	4.12	4.23	4.51	4.95
0.4	4.30	4.23	4.14	4.06	4.15	4.35	4.68	5.06
0.6	4.29	4.21	4.10	4.08	4.21	4.46	4.77	5.11
0.8	4.29	4.19	4.05	4.11	4.27	4.52	4.83	5.13
0.99	4.28	4.17	4.03	4.15	4.32	4.57	4.87	5.15
1.01	4.28	4.17	4.03	4.15	4.33	4.57	4.87	5.16
1.2	4.28	4.15	4.03	4.18	4.37	4.62	4.90	5.17
1.5	4.27	4.11	4.04	4.23	4.42	4.67	4.94	5.18
2	4.26	4.08	4.06	4.30	4.49	4.73	4.98	5.20
5	4.19	4.12	4.23	4.51	4.69	4.89	5.08	5.24
10	4.15	4.23	4.38	4.65	4.81	4.98	5.13	5.26
50	4.30	4.53	4.67	4.88	4.99	5.10	5.20	5.29

Values of q

u \ y_o/a	0.2	0.6	1	2	3	5	10	30
0.02	0.32	0.31	0.29	0.26	0.24	0.23	0.22	0.23
0.08	0.32	0.29	0.27	0.23	0.21	0.19	0.20	0.25
0.2	0.32	0.28	0.24	0.19	0.18	0.18	0.22	0.28
0.4	0.31	0.26	0.21	0.17	0.17	0.20	0.24	0.29
0.6	0.31	0.25	0.19	0.16	0.18	0.21	0.25	0.30
0.8	0.30	0.23	0.18	0.17	0.18	0.22	0.26	0.31
0.99	0.30	0.22	0.17	0.17	0.19	0.23	0.27	0.31
1.01	0.30	0.22	0.17	0.17	0.19	0.23	0.27	0.31
1.2	0.29	0.21	0.17	0.17	0.20	0.23	0.27	0.31
1.5	0.29	0.19	0.16	0.18	0.20	0.24	0.28	0.31
2	0.28	0.18	0.16	0.19	0.21	0.25	0.28	0.31
5	0.24	0.18	0.18	0.22	0.24	0.27	0.30	0.32
10	0.22	0.19	0.20	0.24	0.26	0.28	0.30	0.32
50	0.22	0.23	0.24	0.27	0.28	0.30	0.31	0.33

Values of w

u \ y_o/a	0.2	0.6	1	2	3	5	10	30
0.02	4.00	3.99	3.98	3.96	3.96	4.00	4.08	4.30
0.08	3.99	3.98	3.96	3.94	3.95	4.00	4.15	4.51
0.2	3.99	3.97	3.95	3.92	3.94	4.05	4.30	4.67
0.4	3.99	3.97	3.93	3.89	3.97	4.16	4.44	4.76
0.6	3.99	3.96	3.91	3.91	4.03	4.24	4.51	4.80
0.8	3.99	3.95	3.88	3.95	4.09	4.30	4.57	4.83
0.99	3.98	3.95	3.87	3.98	4.13	4.34	4.60	4.85
1.01	3.98	3.95	3.86	3.98	4.13	4.35	4.60	4.85
1.2	3.98	3.94	3.86	4.01	4.17	4.38	4.63	4.86
1.5	3.98	3.92	3.87	4.05	4.22	4.43	4.66	4.87
2	3.98	3.90	3.90	4.11	4.28	4.48	4.70	4.89
5	3.95	3.94	4.05	4.30	4.45	4.62	4.78	4.92
10	3.94	4.04	4.18	4.42	4.55	4.69	4.83	4.94
50	4.08	4.30	4.43	4.61	4.70	4.80	4.89	4.96

TABLE 6

Round-cornered rectangle ($b = 2$) — Manning formula

y/ρ	U/ρ	B/ρ	A/ρ^2	r	q	w
0.01	2.2830789	2.2821347	2.1882787×10^{-2}	3.45	0.26	3.20
0.1	2.9020536	2.8717798	2.5872591×10^{-1}	3.47	0.19	3.28
0.2	3.2870022	3.2	5.6350111	3.45	0.15	3.30
0.3	3.5907977	3.4282857	8.9549884×10^{-1}	3.43	0.11	3.32
0.4	3.8545904	3.6	1.2472952	3.41	0.08	3.33
0.5	4.0943951	3.7320508	1.6141848	3.38	0.04	3.34
0.6	4.3185590	3.8330303	1.9926734	3.35	0.01	3.34
0.7	4.5322073	3.9078784	2.3799219	3.32	− 0.03	3.35
0.8	4.7388768	3.9595918	2.7734792	3.29	− 0.06	3.35
0.9	4.9412578	3.9899749	3.1711302	3.25	− 0.10	3.36
0.99	5.1215923	3.9999000	3.5307967	3.22	− 0.14	3.36
1	5.1415927	4	3.5707963	3.21	− 0.15	3.36
1.01	5.1615927	4	3.6107963	3.21	− 0.15	3.36
2	7.1415927	4	7.5707963	2.98	− 0.27	3.25
3	9.1415927	4	1.1570796×10	2.87	− 0.34	3.21
4	11.1415927	4	1.5570796×10	2.80	− 0.39	3.19
5	13.1415927	4	1.9570796	2.75	− 0.43	3.17
10	23.1415927	4	3.9570796	2.61	− 0.52	3.13
20	43.1415927	4	7.9570796×10	2.51	− 0.60	3.11
50	103.1415927	4	1.9957080×10^{2}	2.41	− 0.68	3.09

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