Received November the 9 th, 1984.

Studies on open-channel cross-section shapes and their hydraulic exponents

I – Trapezoid

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RESUMO

Neste primeiro estudo sobre formas de secção transversal de canais abertos e respectivos expoentes hidráulicos considera-se a secção trapezoidal, bem como os seus casos particulares: secções triangular e rectangular.

Indica-se a maneira correcta de determinar os expoentes hidráulicos e faz-se uma referência crítica aos métodos de Chugaev-Chow e de Curto.

Pela sua maior importância, dedica-se especial atenção ao caso da secção trapezoidal isósceles na hipótese de se adoptar como equação do movimento uniforme a fórmula de Manning ou a de Gauckler-Strickler.

RESUMÉ

En cette première étude sur des formes de section droite de canaux découverts et les exposants hydrauliques respectifs, on considère la section trapézoïdale et ses cas particuliers: sections triangulaire et rectangulaire.

On indique la façon correcte de déterminer les exposants hydrauliques et on fait une référence critique aux méthodes de Chugaev-Chow et de Curto.

Vue sa plus forte importance, on prête spéciale attention au cas de la section trapézoïdale isocèle dans l'hypothèse d'adoption, comme équation du mouvement uniforme, de la formule de Manning ou de celle de Gauckler-Strickler.

SYNOPSIS

In this first study the trapezoidal section and its particular cases (triangular and rectangular) are considered.

The correct manner of computing the hydraulic exponents is pointed out and a critical reference is made to the Chugaev-Chow and Curto methods.

Owing to its greatest importance, especial attention is paid to the case of the isosceles trapezoidal section, when the Manning or the Gauckler-Strickler formula is adopted as uniform flow equation.

1. *General geometric relations.*

Refer to Fig. 1, where ξ_1 is the left side slope angle, ξ_2 the right side slope angle, y the section height and b the bottom width.

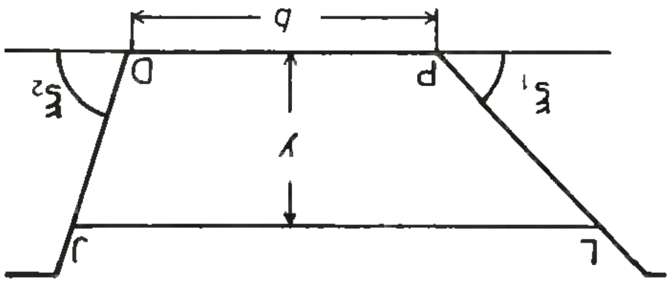

Fig. 1

Putting

1) $c_1 = \cot \xi_1$ left side slope

and

2) $c_2 = \cot \xi_2$ right side slope,

the following relations are easily obtained:

3) $a_1 = \overline{LP} = y / \sin \xi_1 = y\sqrt{1+c_1^2}$ length of left side,

4) $a_2 = \overline{JD} = y / \sin \xi_2 = y\sqrt{1+c_2^2}$ length of right side,

5) $B = \overline{LJ} = b + (c_1 + c_2)\ y$ top width,

6) $U = \overline{LP} + \overline{PD} + \overline{DJ} = b + y(\sqrt{1+c_1^2} + \sqrt{1+c_2^2})$ wetted perimeter

and

7) $A = by + \dfrac{c_1 + c_2}{2} y^2$ cross-sectional area.

When symmetric ($\xi_1 = \xi_2$, $c_1 = c_2 = c$) the trapezoid is called isosceles. As it is well known, this is the most common shape used in practice.

The formulas still hold good in the particular cases of the triangular ($b = 0$) and the rectangular ($c_1 = c_2 = 0$) cross sections.

2. *Best hydraulic sections.*

2.1. *Definition.*

The name best hydraulic section is given to any cross section that, for a known area A, has a minimum wetted perimeter U.

If no additional conditions are imposed, the best (absolutely optimal) hydraulic cross section is obviously the semicircular.

For the trapezoidal form there are several cases to be considered.

2.2. $b \neq 0$.

When the trapezoid does not reduce to a triangle, from 7) we get

$$b = \frac{A}{y} - \frac{c_1 + c_2}{2} y \ ,$$

a value that, introduced into 6), leads to

8) $$U = \frac{A}{y} + \left(\sqrt{1 + c_1^2} + \sqrt{1 + c_2^2} - \frac{c_1 + c_2}{2}\right) y \ .$$

2.2.1. c_1 and c_2 prescribed. In this case the condition U_{min} with A, c_1 and c_2 constants, i. e. $dU/dy = 0$, furnishes

9) $$A = \left(\sqrt{1 + c_1^2} + \sqrt{1 + c_2^2} - \frac{c_1 + c_2}{2}\right) y^2 \ ,$$

10) $$U = 2 \left(\sqrt{1 + c_1^2} + \sqrt{1 + c_2^2} - \frac{c_1 + c_2}{2}\right) y \ ,$$

11) $$b = \left(\sqrt{1 + c_1^2} + \sqrt{1 + c_2^2} - c_1 - c_2\right) y$$

and

12) $$B = (\sqrt{1 + c_1^2} + \sqrt{1 + c_2^2})\ y\ .$$

In Fig. 2 the point M was chosen so as to divide the surface line LJ in such a manner that we have $\overline{LM} = y\sqrt{1 + c_1^2}$ and $\overline{MJ} = y\sqrt{1 + c_2^2}$.

Then, paying attention to 3), we get $\overline{LM} = a_1 = \overline{LP}$ and see that the triangle LPM is isosceles, a fact that implies $\overline{MT_1} = \overline{PN_1} = y$. In the same way, it is seen to be $\overline{MT_2} = \overline{DN_2} = y$. Consequently the best hydraulic section is the trapezoid circumscribed to the semi-circle of radius y.

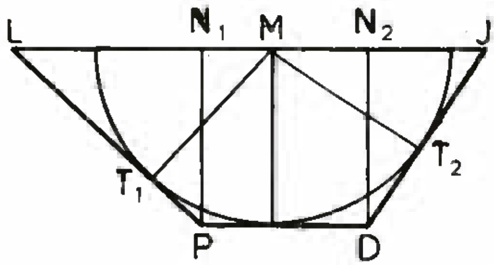

Fig. 2

2.2.2. Only c_1 or c_2 is prescribed. If for instance c_1 is prescribed, the additional condition $\partial U/\partial c_2 = 0$, which 10) shows to be equivalent to

$$\frac{\partial}{\partial c_2}\left[\ 2\ (2\sqrt{1 + c_2^2} - c_2)\ \right] = 0\ ,$$

yields

$$c_2 = \frac{1}{\sqrt{3}} \quad \text{or} \quad \xi_2 = 60^\circ\ .$$

Similarly, if only c_2 is prescribed, we get

$$c_1 = \frac{1}{\sqrt{3}} \quad \text{or} \quad \xi_1 = 60^\circ$$

2.2.3. c_1 and c_2 are not prescribed. In this case, from the above, we infer

$$c_1 = c_2 = c = \frac{1}{\sqrt{3}} \text{ or } \xi_1 = \xi_2 = 60^\circ ,$$

and the trapezoid is the regular semi-hexagon circumscribed to the semi-circle of radius y, the optimal trapezoidal cross section of area A.

2.3. $b = 0$.

In this case of the triangular section, the reasoning made to obtain 8) is not valid. Instead, from 6) and 7), we get

$$\text{13)} \qquad U = \sqrt{2A} \, . \, \frac{\sqrt{1 + c_1^2} + \sqrt{1 + c_2}}{\sqrt{c_1 + c_2}}$$

and then $\dfrac{\partial U}{\partial c_1} = 0$ is equivalent to

$$\frac{\partial}{\partial c_1} \left(\frac{\sqrt{1 + c_1^2} + \sqrt{1 + c_2^2}}{\sqrt{c_1 + c_2}} \right) = 0 ,$$

that gives

$$\text{14)} \qquad c_1^2 + 2 \, c_1 \, c_2 - \sqrt{(1 + c_1^2) \, (1 + c_2^2)} - 1 = 0 \, .$$

In the same manner, $\dfrac{\partial U}{\partial c_2} = 0$ furnishes

$$\text{15)} \qquad c_2^2 + 2 \, c_1 \, c_2 - \sqrt{(1 + c_1^2) \, (1 + c_2^2)} - 1 = 0 \, .$$

Then, by subtraction of 15) from 14), we get $c_1 = c_2 = c$ and so both these equations reduce to $c_1 = c_2 = 1$, which shows that the best triangular cross section is the symmetrical (isosceles) one with an angle of 90° at the vertex.

3. Hydraulic exponents.

3.1. Definition.

If we make the convention of adopting a monomial formula as the equation of uniform flow, it can be written (Mendonça 1945)

16) $$V = \sqrt{\chi \, R^{\varphi} \, S^{\vartheta}} \;,$$

where V is the mean velocity, $R = A/U$ the hydraulic radius and S the channel longitudinal bottom slope.

Labelling with the subscript zero the normal values and denoting by

17) $$u = y/y_o$$

the unit cross-sectional height, let us to consider the hydraulic exponents r, w and q defined by

18) $$u^r = \left(\frac{A}{A_o} \right)^{\frac{\varphi + 2}{\vartheta}} \left(\frac{U_o}{U} \right)^{\frac{\varphi}{\vartheta}} \;,$$

19) $$u^w = \left(\frac{A}{A_o} \right)^3 \frac{B_o}{B}$$

and

20) $$u^q = \left(\frac{A}{A_o} \right)^{\frac{\varphi + 2}{\vartheta} - 3} \left(\frac{U_o}{U} \right)^{\frac{\varphi}{\vartheta}} \frac{B}{B_o} \;.$$

These quantities are not independent for it is evident that the relation

21) $$q = r - w$$

exists.

The number r, so denoted after Mononobe (1936), is the classical hydraulic exponent n of Bakhmeteff (1932, p. 84); Chow (1959, p. 131) represented it by N and called it the hydraulic exponent for uniform-flow computation.

Chow (1959, p. 66) denoted by M and called hydraulic exponent for critical-flow computation the number w, the second hydraulic exponent of Von Seggern (1949).

Numbers q and r are the two parameters of the Dupuit function, that has been shown to introduce itself quite naturally in several hydraulic problems, namely the computation of backwater curves and volumes in uniform open channels (either in turbulent or laminar flow), the steady two-dimensional flow of groundwater, reservoir routing and the Italian method for computing drainage networks (Mendonça 1964a, 1964b, 1977, 1978, 1980).

From 18), 19) and 20), we see that for $u \neq 1$, i. e. $y \neq y_o$, the hydraulic exponents are given by

22) $$r = \left(\frac{\varphi + 2}{\vartheta} \log \frac{A}{A_o} - \frac{\varphi}{\vartheta} \log \frac{U}{U_o}\right) / \log u \ ,$$

23) $$w = \left(3 \log \frac{A}{A_o} - \log \frac{B}{B_o}\right) / \log u$$

and

24) $$q = \left[\left(\frac{\varphi + 2}{\vartheta} - 3\right) \log \frac{A}{A_o} - \frac{\varphi}{\vartheta} \log \frac{U}{U_o} + \log \frac{B}{B_o} \right] / \log u \ .$$

For $u = 1$, i. e. $y = y_o$, these equations lead to $r = w = q = \frac{0}{0}$,

and so Hospital rule must be used. It yields

25) $$r = \frac{\varphi + 2}{\vartheta} \cdot \frac{1}{A} \frac{dA}{du} - \frac{\varphi}{\vartheta} \frac{1}{U} \frac{dU}{du} \ ,$$

26) $$w = 3 \cdot \frac{1}{A} \frac{dA}{du} - \frac{1}{B} \frac{dB}{du}$$

and

27) $$q = \left(\frac{\varphi + 2}{\vartheta} - 3\right) \frac{1}{A} \frac{dA}{du} - \frac{\varphi}{\vartheta} \frac{1}{U} \frac{dU}{du} + \frac{1}{B} \frac{dB}{du} \ .$$

3.2. The case of the trapezoidal section.

3.2.1. In general. Let us apply the formulas found above to the case of the trapezoidal cross section.

Putting

28) $t = y / b$

and

29) $t_o = y_o / b$,

we have

30) $u = t / t_o$

and

31) $\mathrm{d}u = \mathrm{d}t / t_o$.

The substitution of $y = bt$ into 5), 6) and 7) gives

32) $$B = b \left[1 + (c_1 + c_2)\, t \right] ,$$

33) $$U = b \left[1 + \left(\sqrt{1 + c_1^2} + \sqrt{1 + c_2^2} \right) t \right]$$

and

34) $$A = \frac{b^2}{2} \left[2 + (c_1 + c_2)\, t \right] t .$$

Then, for $u \neq 1$, by combining 22), 23), 24), 30), 32), 33) and 34), we get

35) $$r = \left(\frac{\varphi + 2}{\vartheta} \log \frac{[2 + (c_1 + c_2)\, t]\, t}{[2 + (c_1 + c_2)\, t_o]\, t_o} - \right.$$
$$\left. - \frac{\varphi}{\vartheta} \log \frac{1 + (\sqrt{1 + c_1^2} + \sqrt{1 + c_2^2})\, t}{1 + (\sqrt{1 + c_1^2} + \sqrt{1 + c_2^2})\, t_o} \right) / \log \frac{t}{t_o} ,$$

$$36)\quad w = \left(3\ \log \frac{[2+(c_1+c_2)\ t]\ t}{[2+(c_1+c_2)\ t_o]\ t_o} - \log \frac{1+(c_1+c_2)t}{1+(c_1+c_2)t_o}\right) \Big/ \log \frac{t}{t_o}$$

and

$$37)\quad q = \left[\ (\frac{\varphi+2}{\vartheta} - 3)\ \log \frac{[2+(c_1+c_2)\ t]\ t}{[2+(c_1+c_2)\ t_o]\ t_o} - \right.$$

$$\left. - \frac{\varphi}{\vartheta} \log \frac{1+(\sqrt{1+c_1^2}+\sqrt{1+c_2^2})t}{1+(\sqrt{1+c_1^2}+\sqrt{1+c_2^2})t_o} + \log \frac{1+(c_1+c_2)t}{1+(c_1+c_2)t_o}\right] \Big/ \log \frac{t}{t_o}\ .$$

For $u = 1$, paying attention to 31), we deduce instead

$$38)\quad r = \frac{\varphi+2}{\vartheta} \cdot \frac{2[1+(c_1+c_2)\ t]}{2+(c_1+c_2)\ t} - \frac{\varphi}{\vartheta}\ \frac{(\sqrt{1+c_1^2}+\sqrt{1+c_2^2})t}{1+(\sqrt{1+c_1^2}+\sqrt{1+c_2^2})t}\ ,$$

$$39)\quad w = 3\ .\ \frac{2[1+(c_1+c_2)\ t]}{2+(c_1+c_2)\ t} - \frac{(c_1+c_2)\ t}{1+(c_1+c_2)\ t}$$

and

$$40)\quad q = (\frac{\varphi+2}{\vartheta} - 3)\ .\ \frac{2[1+(c_1+c_2)t]}{2+(c_1+c_2)t} -$$

$$- \frac{\varphi}{\vartheta}\ \frac{(\sqrt{1+c_1^2}+\sqrt{1+c_2^2})t}{1+(\sqrt{1+c_1^2}+\sqrt{1+c_2^2})t} + \frac{(c_1+c_2)t}{1+(c_1+c_2)t}\ .$$

3.2.2. The case of the symmetric trapezoid, adopting the Manning or the Gauckler-Strickler formula. For the Manning and the Gauckler-Strickler formulas we have $\varphi = 4/3$ and $\vartheta = 1$. Then, if the trapezoidal section is isosceles ($c_1 = c_2 = c$), formulas 35) to 40) take the following aspects:

$$35')\quad r = \left[\frac{10}{3} \log \frac{(1+ct)t}{(1+ct_o)t_o} - \frac{4}{3} \log \frac{1+2\sqrt{1+c^2}\ t}{1+2\sqrt{1+c^2}\ t_o}\right] \Big/ \log \frac{t}{t_o}\ ,$$

$$36')\quad w = \left[3 \log \frac{(1+ct)t}{(1+ct_0)t_0} - \log \frac{1+2\,ct}{1+2\,ct_0} \right] / \log \frac{t}{t_0} \,,$$

$$37')\quad q = \left[\frac{1}{3} \log \frac{(1+ct)t}{(1+ct_0)t_0} - \right.$$

$$\left. - \frac{4}{3} \log \frac{1+2\sqrt{1+c^2}\,t}{1+2\sqrt{1+c^2}\,t_0} + \log \frac{1+2\,ct}{1+2\,ct_0} \right] / \log \frac{t}{t_0} \,,$$

$$38')\quad r = \frac{10}{3} \cdot \frac{1+2ct}{1+ct} - \frac{4}{3} \cdot \frac{2\sqrt{1+c^2}\,t}{1+2\sqrt{1+c^2}\,t} \,,$$

$$39')\quad w = 3 \cdot \frac{1+2ct}{1+ct} - \frac{2ct}{1+2ct}$$

$$40')\quad q = \frac{1}{3} \cdot \frac{1+2ct}{1+ct} - \frac{4}{3} \cdot \frac{2\sqrt{1+c^2}\,t}{1+2\sqrt{1+c^2}\,t} + \frac{2ct}{1+2ct} \,.$$

For this particular case, the writer has long ago (1967/68) computed a detailed table containing 11960 values of r and the same number of q. It can be described thus:

r, q, 2 dec.
$c = 0(0.25)1(0.5)3(1)4$
$t_0 = 0.01(0.09)0.1(0.1)1(1)10(5)20(30)50$
$t = 0(0.01)0.1(0.02)0.2(0.05)0.5(0.1)1(0.2)4(0.5)5(1)10(5)20;\ \infty$.

Tables 1 and 2 give a sample of the above mentioned long table. Due to rounding errors, relation 21) brings sometimes for w, when computed after Tables 1 and 2, values differing $\pm$ 0.01 from the true ones. And so Table 3 has been especially computed for the present paper.

Table 4 gives the values of the hydraulic exponents under the hypothesis that the normal section is the best one. The value $c = 1/\sqrt{3}$, it is to be recalled, refers to the optimal trapezoidal isosceles cross section.

These and any other value can be computed by means of the programs shown in Section 3.4.

It is to be remarked that, owing to the symmetric role played by t and t_0 in 35'), 36') and 37'), Tables 1, 2 and 3 are made of symmetrical square matrices.

TABLE 1

Isosceles trapezoid, Manning formula
Values of r

t \ t_0	0.1	0.2	0.4	0.7	1	2	4	7	10	15
					$c = 0$					
0.1	3.11	3.04	2.94	2.86	2.80	2.70	2.61	2.54	2.50	2.47
0.2	3.04	2.95	2.85	2.76	2.70	2.60	2.51	2.44	2.41	2.38
0.4	2.94	2.85	2.74	2.65	2.59	2.49	2.40	2.35	2.32	2.29
0.7	2.86	2.76	2.65	2.56	2.50	2.40	2.32	2.27	2.25	2.22
1	2.80	2.70	2.59	2.50	2.44	2.35	2.28	2.23	2.21	2.18
2	2.70	2.60	2.49	2.40	2.35	2.27	2.20	2.16	2.14	2.13
4	2.61	2.51	2.40	2.32	2.28	2.20	2.15	2.12	2.10	2.09
7	2.54	2.44	2.35	2.27	2.23	2.16	2.12	2.09	2.08	2.06
10	2.50	2.41	2.32	2.25	2.21	2.14	2.10	2.08	2.06	2.05
15	2.47	2.38	2.29	2.22	2.18	2.13	2.09	2.06	2.05	2.04
					$c = 0.5$					
0.1	3.25	3.23	3.23	3.26	3.29	3.38	3.52	3.66	3.74	3.84
0.2	3.23	3.22	3.23	3.27	3.31	3.43	3.59	3.74	3.83	3.93
0.4	3.23	3.23	3.26	3.31	3.37	3.51	3.70	3.86	3.96	4.07
0.7	3.26	3.27	3.31	3.38	3.45	3.62	3.82	3.99	4.10	4.20
1	3.29	3.31	3.37	3.45	3.52	3.71	3.92	4.09	4.20	4.30
2	3.38	3.43	3.51	3.62	3.71	3.91	4.13	4.31	4.41	4.51
4	3.52	3.59	3.70	3.82	3.92	4.13	4.36	4.52	4.61	4.70
7	3.66	3.74	3.86	3.99	4.09	4.31	4.52	4.67	4.76	4.84
10	3.74	3.83	3.96	4.10	4.20	4.41	4.61	4.76	4.83	4.91
15	3.84	3.93	4.07	4.20	4.30	4.51	4.70	4.84	4.91	4.98
					$c = 1$					
0.1	3.34	3.37	3.42	3.50	3.57	3.72	3.88	4.02	4.09	4.18
0.2	3.37	3.41	3.48	3.58	3.65	3.82	4.00	4.14	4.22	4.31
0.4	3.42	3.48	3.58	3.69	3.78	3.97	4.16	4.30	4.38	4.46
0.7	3.50	3.58	3.69	3.82	3.92	4.12	4.31	4.45	4.53	4.61
1	3.57	3.65	3.78	3.92	4.01	4.22	4.41	4.55	4.62	4.70
2	3.72	3.82	3.97	4.12	4.22	4.42	4.61	4.73	4.80	4.86
4	3.88	4.00	4.16	4.31	4.41	4.61	4.77	4.88	4.94	5.00
7	4.02	4.14	4.30	4.45	4.55	4.73	4.88	4.98	5.03	5.08
10	4.09	4.22	4.38	4.53	4.62	4.80	4.94	5.03	5.08	5.12
15	4.18	4.31	4.46	4.61	4.70	4.86	5.00	5.08	5.12	5.16
					$c = 2$					
0.1	3.48	3.56	3.68	3.80	3.89	4.06	4.23	4.34	4.41	4.47
0.2	3.56	3.66	3.80	3.94	4.03	4.22	4.38	4.49	4.56	4.62
0.4	3.68	3.80	3.96	4.11	4.21	4.40	4.56	4.66	4.72	4.78
0.7	3.80	3.94	4.11	4.27	4.37	4.55	4.70	4.80	4.85	4.90
1	3.89	4.03	4.21	4.37	4.47	4.64	4.78	4.87	4.92	4.97
2	4.06	4.22	4.40	4.55	4.64	4.80	4.93	5.00	5.04	5.08
4	4.23	4.38	4.56	4.70	4.78	4.93	5.03	5.10	5.13	5.16
7	4.34	4.49	4.66	4.80	4.87	5.00	5.10	5.15	5.18	5.20
10	4.41	4.56	4.72	4.85	4.92	5.04	5.13	5.18	5.20	5.23
15	4.47	4.62	4.78	4.90	4.97	5.08	5.16	5.20	5.23	5.25
					$c = 3$					
0.1	3.59	3.70	3.86	4.00	4.09	4.26	4.41	4.51	4.57	4.62
0.2	3.70	3.84	4.01	4.16	4.26	4.43	4.57	4.67	4.72	4.77
0.4	3.86	4.01	4.20	4.35	4.45	4.61	4.74	4.83	4.87	4.91
0.7	4.00	4.16	4.35	4.50	4.59	4.75	4.87	4.94	4.98	5.02
1	4.09	4.26	4.45	4.59	4.68	4.83	4.94	5.01	5.04	5.07
2	4.26	4.43	4.61	4.75	4.83	4.95	5.05	5.10	5.13	5.16
4	4.41	4.57	4.74	4.87	4.94	5.05	5.13	5.17	5.20	5.22
7	4.51	4.67	4.83	4.94	5.01	5.10	5.17	5.21	5.23	5.25
10	4.57	4.72	4.87	4.98	5.04	5.13	5.20	5.23	5.25	5.26
15	4.62	4.77	4.91	5.02	5.07	5.16	5.22	5.25	5.26	5.27

TABLE 2

Isosceles trapezoid, Manning formula
Values of q

t \ t_o	0.1	0.2	0.4	0.7	1	2	4	7	10	15
					$c = 0$					
0.1	0.11	0.04	−0.06	−0.14	−0.20	−0.30	−0.39	−0.46	−0.50	−0.53
0.2	0.04	−0.05	−0.15	−0.24	−0.30	−0.40	−0.49	−0.56	−0.59	−0.62
0.4	−0.06	−0.15	−0.26	−0.35	−0.41	−0.51	−0.60	−0.65	−0.68	−0.71
0.7	−0.14	−0.24	−0.35	−0.44	−0.50	−0.60	−0.68	−0.73	−0.75	−0.78
1	−0.20	−0.30	−0.41	−0.50	−0.56	−0.65	−0.72	−0.77	−0.79	−0.82
2	−0.30	−0.40	−0.51	−0.60	−0.65	−0.73	−0.80	−0.84	−0.86	−0.87
4	−0.39	−0.49	−0.60	−0.68	−0.72	−0.80	−0.85	−0.88	−0.90	−0.91
7	−0.46	−0.56	−0.65	−0.73	−0.77	−0.84	−0.88	−0.91	−0.92	−0.94
10	−0.50	−0.59	−0.68	−0.75	−0.79	−0.86	−0.90	−0.92	−0.94	−0.95
15	−0.53	−0.62	−0.71	−0.78	−0.82	−0.87	−0.91	−0.94	−0.95	−0.96
					$c = 0.5$					
0.1	0.20	0.16	0.12	0.09	0.08	0.07	0.08	0.10	0.11	0.12
0.2	0.16	0.12	0.08	0.06	0.05	0.05	0.06	0.08	0.10	0.11
0.4	0.12	0.08	0.05	0.03	0.02	0.03	0.06	0.08	0.10	0.12
0.7	0.09	0.06	0.03	0.02	0.02	0.04	0.07	0.10	0.11	0.13
1	0.08	0.05	0.02	0.02	0.02	0.05	0.08	0.11	0.13	0.15
2	0.07	0.05	0.03	0.04	0.05	0.08	0.12	0.15	0.17	0.18
4	0.08	0.06	0.06	0.07	0.08	0.12	0.16	0.19	0.20	0.22
7	0.10	0.08	0.08	0.10	0.11	0.15	0.19	0.21	0.23	0.24
10	0.11	0.10	0.10	0.11	0.13	0.17	0.20	0.23	0.24	0.26
15	0.12	0.11	0.12	0.13	0.15	0.18	0.22	0.24	0.26	0.27
					$c = 1$					
0.1	0.24	0.21	0.20	0.19	0.18	0.19	0.20	0.21	0.22	0.22
0.2	0.21	0.19	0.18	0.17	0.17	0.18	0.20	0.21	0.22	0.22
0.4	0.20	0.18	0.17	0.16	0.17	0.18	0.20	0.22	0.22	0.23
0.7	0.19	0.17	0.16	0.17	0.17	0.19	0.21	0.23	0.24	0.25
1	0.18	0.17	0.17	0.17	0.18	0.20	0.22	0.24	0.25	0.26
2	0.19	0.18	0.18	0.19	0.20	0.22	0.24	0.26	0.27	0.27
4	0.20	0.20	0.20	0.21	0.22	0.24	0.26	0.28	0.28	0.29
7	0.21	0.21	0.22	0.23	0.24	0.26	0.28	0.29	0.30	0.30
10	0.22	0.22	0.22	0.24	0.25	0.27	0.28	0.30	0.30	0.31
15	0.22	0.22	0.23	0.25	0.26	0.27	0.29	0.30	0.31	0.31
					$c = 2$					
0.1	0.26	0.25	0.25	0.25	0.25	0.26	0.26	0.27	0.27	0.28
0.2	0.25	0.24	0.24	0.24	0.25	0.26	0.27	0.27	0.28	0.28
0.4	0.25	0.24	0.24	0.25	0.25	0.26	0.27	0.28	0.29	0.29
0.7	0.25	0.24	0.25	0.25	0.26	0.27	0.28	0.29	0.29	0.30
1	0.25	0.25	0.25	0.26	0.27	0.28	0.29	0.30	0.30	0.30
2	0.26	0.26	0.26	0.27	0.28	0.29	0.30	0.31	0.31	0.31
4	0.26	0.27	0.27	0.28	0.29	0.30	0.31	0.31	0.32	0.32
7	0.27	0.27	0.28	0.29	0.30	0.31	0.31	0.32	0.32	0.32
10	0.27	0.28	0.29	0.29	0.30	0.31	0.32	0.32	0.32	0.32
15	0.28	0.28	0.29	0.30	0.30	0.31	0.32	0.32	0.32	0.32
					$c = 3$					
0.1	0.27	0.26	0.26	0.26	0.27	0.27	0.28	0.29	0.29	0.29
0.2	0.26	0.26	0.26	0.27	0.27	0.28	0.29	0.29	0.30	0.30
0.4	0.26	0.26	0.27	0.27	0.28	0.29	0.29	0.30	0.30	0.31
0.7	0.26	0.27	0.27	0.28	0.28	0.29	0.30	0.31	0.31	0.31
1	0.27	0.27	0.28	0.28	0.29	0.30	0.31	0.31	0.31	0.31
2	0.27	0.28	0.29	0.29	0.30	0.31	0.31	0.32	0.32	0.32
4	0.28	0.29	0.29	0.30	0.31	0.31	0.32	0.32	0.32	0.32
7	0.29	0.29	0.30	0.31	0.31	0.32	0.32	0.32	0.33	0.33
10	0.29	0.30	0.30	0.31	0.31	0.32	0.32	0.33	0.33	0.33
15	0.29	0.30	0.31	0.31	0.31	0.32	0.32	0.33	0.33	0.33

TABLE 3

Isosceles trapezoid

Values of w

t \ t_0	0.1	0.2	0.4	0.7	1	2	4	7	10	15
					$c = 0$					
0.1	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
0.2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
0.4	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
0.7	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
1	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
4	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
7	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
10	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
15	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
					$c = 0.5$					
0.1	3.05	3.08	3.12	3.16	3.21	3.31	3.44	3.56	3.64	3.72
0.2	3.08	3.11	3.15	3.21	3.26	3.38	3.53	3.66	3.73	3.82
0.4	3.12	3.15	3.21	3.28	3.34	3.48	3.64	3.78	3.86	3.95
0.7	3.16	3.21	3.28	3.37	3.43	3.58	3.76	3.90	3.98	4.07
1	3.21	3.26	3.34	3.43	3.50	3.66	3.84	3.98	4.07	4.15
2	3.31	3.38	3.48	3.58	3.66	3.83	4.02	4.16	4.24	4.32
4	3.44	3.53	3.64	3.76	3.84	4.02	4.20	4.33	4.41	4.48
7	3.56	3.66	3.78	3.90	3.98	4.16	4.33	4.46	4.53	4.59
10	3.64	3.73	3.86	3.98	4.07	4.24	4.41	4.53	4.59	4.65
15	3.72	3.82	3.95	4.07	4.15	4.32	4.48	4.59	4.65	4.71
					$c = 1$					
0.1	3.11	3.15	3.23	3.31	3.38	3.53	3.69	3.81	3.88	3.95
0.2	3.15	3.21	3.30	3.40	3.48	3.64	3.81	3.93	4.01	4.08
0.4	3.23	3.30	3.41	3.53	3.61	3.79	3.96	4.09	4.16	4.23
0.7	3.31	3.40	3.53	3.65	3.74	3.92	4.10	4.22	4.29	4.36
1	3.38	3.48	3.61	3.74	3.83	4.02	4.19	4.31	4.38	4.44
2	3.53	3.64	3.79	3.92	4.02	4.20	4.36	4.47	4.53	4.59
4	3.69	3.81	3.96	4.10	4.19	4.36	4.51	4.61	4.66	4.70
7	3.81	3.93	4.09	4.22	4.31	4.47	4.61	4.69	4.74	4.78
10	3.88	4.01	4.16	4.29	4.38	4.53	4.66	4.74	4.77	4.81
15	3.95	4.08	4.23	4.36	4.44	4.59	4.70	4.78	4.81	4.84
					$c = 2$					
0.1	3.21	3.30	3.43	3.56	3.64	3.81	3.96	4.07	4.13	4.19
0.2	3.30	3.41	3.56	3.69	3.79	3.96	4.11	4.22	4.28	4.34
0.4	3.43	3.56	3.72	3.86	3.96	4.13	4.28	4.38	4.43	4.49
0.7	3.56	3.69	3.86	4.01	4.11	4.28	4.42	4.51	4.55	4.60
1	3.64	3.79	3.96	4.11	4.20	4.36	4.49	4.58	4.62	4.66
2	3.81	3.96	4.13	4.28	4.36	4.51	4.63	4.70	4.73	4.77
4	3.96	4.11	4.28	4.42	4.49	4.63	4.73	4.78	4.81	4.84
7	4.07	4.22	4.38	4.51	4.58	4.70	4.78	4.83	4.86	4.88
10	4.13	4.28	4.43	4.55	4.62	4.73	4.81	4.86	4.88	4.90
15	4.19	4.34	4.49	4.60	4.66	4.77	4.84	4.88	4.90	4.92
					$c = 3$					
0.1	3.32	3.44	3.59	3.73	3.82	3.99	4.13	4.22	4.28	4.33
0.2	3.44	3.58	3.75	3.90	3.99	4.15	4.29	4.38	4.42	4.47
0.4	3.59	3.75	3.93	4.08	4.17	4.32	4.45	4.53	4.57	4.61
0.7	3.73	3.90	4.08	4.22	4.31	4.45	4.57	4.64	4.67	4.71
1	3.82	3.99	4.17	4.31	4.39	4.53	4.63	4.70	4.73	4.76
2	3.99	4.15	4.32	4.45	4.53	4.65	4.74	4.79	4.81	4.84
4	4.13	4.29	4.45	4.57	4.63	4.74	4.81	4.85	4.87	4.89
7	4.22	4.38	4.53	4.64	4.70	4.79	4.85	4.89	4.90	4.92
10	4.28	4.42	4.57	4.67	4.73	4.81	4.87	4.90	4.92	4.93
15	4.33	4.47	4.61	4.71	4.76	4.84	4.89	4.92	4.93	4.95

TABLE 4

Best normal trapezoid, Manning formula

Values of r

u \ c	0	0.3	$1/\sqrt{3}$	1	1.5	2	2.5	3
0.02	3.10	3.20	3.31	3.50	3.73	3.93	4.11	4.25
0.08	3.01	3.15	3.32	3.60	3.92	4.19	4.41	4.58
0.2	2.91	3.11	3.34	3.73	4.12	4.43	4.64	4.80
0.4	2.81	3.09	3.40	3.87	4.31	4.61	4.81	4.94
0.6	2.75	3.09	3.46	3.98	4.43	4.71	4.89	5.01
0.8	2.70	3.10	3.51	4.06	4.51	4.78	4.94	5.05
1	2.67	3.11	3.56	4.13	4.57	4.82	4.98	5.07
1.2	2.64	3.13	3.60	4.18	4.61	4.86	5.00	5.09
1.5	2.60	3.15	3.65	4.25	4.67	4.90	5.03	5.12
2	2.55	3.19	3.73	4.33	4.73	4.94	5.07	5.14
3	2.49	3.27	3.85	4.44	4.81	5.00	5.11	5.17
5	2.42	3.40	4.01	4.57	4.89	5.06	5.14	5.20
10	2.35	3.60	4.21	4.71	4.98	5.11	5.18	5.23

Values of q

u \ c	0	0.3	$1/\sqrt{3}$	1	1.5	2	2.5	3
0.02	0.10	0.14	0.18	0.22	0.24	0.26	0.27	0.28
0.08	0.01	0.07	0.12	0.19	0.23	0.26	0.27	0.29
0.2	− 0.09	0.00	0.08	0.17	0.23	0.26	0.28	0.30
0.4	− 0.19	− 0.05	0.06	0.17	0.24	0.28	0.29	0.31
0.6	− 0.25	− 0.08	0.05	0.18	0.25	0.28	0.30	0.31
0.8	− 0.30	− 0.09	0.05	0.19	0.26	0.29	0.30	0.31
1	− 0.33	− 0.10	0.06	0.19	0.26	0.29	0.31	0.31
1.2	− 0.36	− 0.11	0.06	0.20	0.27	0.29	0.31	0.32
1.5	− 0.40	− 0.11	0.06	0.20	0.27	0.30	0.31	0.32
2	− 0.45	− 0.11	0.07	0.21	0.28	0.30	0.31	0.32
3	− 0.51	− 0.10	0.09	0.23	0.28	0.31	0.32	0.32
5	− 0.58	− 0.08	0.12	0.24	0.29	0.31	0.32	0.32
10	− 0.65	− 0.04	0.15	0.26	0.30	0.31	0.32	0.33

Values of w

u \ c	0	0.3	$1/\sqrt{3}$	1	1.5	2	2.5	3
0.02	3.00	3.05	3.13	3.29	3.49	3.67	3.84	3.98
0.08	3.00	3.08	3.19	3.41	3.69	3.93	4.13	4.29
0.2	3.00	3.11	3.26	3.55	3.89	4.16	4.36	4.50
0.4	3.00	3.14	3.34	3.70	4.07	4.34	4.51	4.63
0.6	3.00	3.17	3.40	3.80	4.18	4.43	4.59	4.70
0.8	3.00	3.19	3.46	3.87	4.25	4.49	4.64	4.73
1	3.00	3.22	3.50	3.93	4.31	4.53	4.67	4.76
1.2	3.00	3.24	3.54	3.98	4.35	4.57	4.70	4.78
1.5	3.00	3.26	3.59	4.04	4.40	4.60	4.72	4.80
2	3.00	3.31	3.66	4.12	4.45	4.64	4.75	4.82
3	3.00	3.38	3.76	4.22	4.53	4.69	4.79	4.85
5	3.00	3.48	3.90	4.33	4.60	4.75	4.83	4.87
10	3.00	3.64	4.07	4.45	4.68	4.80	4.86	4.90

3.3. Chugaev-Chow and Curto methods.

Using a method introduced by Chugaev (1931), Chow deduces formulas that, under the hypothesis of Section 3.2.2., reduce to 38′) and 39′) (Chow 1959, eq. 6-15, p. 131, and eq. 4-10, p. 66, respectively). This shows that such a method is only sound for uniform flow.

The use that Chow (1959, p. 256) makes of the values thus found in his Example 10-2 of a backwater curve problem is then equivalent to assume that the hydraulic exponents to be got are those pertaining to the normal section with a height equal to an average of the heights at the extremities of the reach.

Later, Curto (1966) suggested to replace in 18), 19) and 20) the normal values by those corresponding to a sectional height equal to 1. However, this method is not acceptable for it leads to formulas lacking dimensional homogeneity, which means that the hydraulic exponents would then be dependent on the unit of length adopted. Let us see a numerical example.

For the case of the isosceles trapezoidal cross section, Curto [1966, 3d eq. (26′)] obtained

$$41) \qquad w = \left[3 \log \frac{y(b+cy)}{b+c} - \log \frac{b+2cy}{b+2c} \right] / \log y.$$

Put $c = 1$, $b = 1\,\text{m} = 100\,\text{cm}$ and $y = 10\,\text{m} = 1000\,\text{cm}$.

If the meter is adopted as the unit of length, 41) gives, using decimal logarithms,

$$\begin{aligned} w &= (3 \log 55 - \log 7)/\log 10 \\ &= 5.22109 - 0.84510 \\ &\approx 4.38. \end{aligned}$$

If instead the centimeter is adopted, 41) yields

$$\begin{aligned} w &= (3 \log 10891.08911 - \log 20{,}58824)/\log 1000 \\ &= (12.11121 - 1.31362)/3 \\ &\approx 3.60. \end{aligned}$$

3.4. Software.

With a Casio FX-702P pocket computer, Program 1 can be employed to obtain the correct values of the hydraulic exponents for any isosceles trapezoidal cross section using Manning of Gauckler-Strickler formula. The correspondence between the text and the program symbols is as follows :

text	c	$\sqrt{1+c^2}$	t	t_o	A	A_o	B	B_o	U	U_o	r	q	w
program	C	S	Y	Z	A	E	B	P	U	V	R	Q	W .

Program 1

```
LIST #1
 10 PRT "TRAPEZ.ISOSCELES, MANNING"
 20 INP "C",C
 30 INP "Z",Z
 40 INP "Y",Y
 50 X=Y/Z
 60 B=1+2*C*Y
 70 S=SQR (1+C↑2)
 80 U=1+2*S*Y
 90 IF Y=Z THEN 210
100 A=(1+C*Y)*Y
110 E=(1+C*Z)*Z
120 V=1+2*S*Z
130 P=1+2*C*Z
140 L=(V/U)↑(4/3)
150 M=(A/E)↑(1/3)
160 N=(A/E)↑3
170 R=LN (L*M*N)/LN X
180 Q=LN (L*M*B/P)/LN X
190 W=LN (N*P/B)/LN X
200 GOTO 250
210 T=8*S*Y/3/U
220 R=10*B/3/(1+C*Y)-T
230 Q=8/3/(1+C*Y)-T+2*C*Y/B
240 W=3*B/(1+C*Y)+2*C*Y/B
250 SET F2
260 PRT "R=";R
270 PRT "Q=";Q
280 PRT "W=";W
290 SET N
300 IF Y=15 THEN 30
310 GOTO 40
```

In this manner all the values contained in Tables 1, 2 and 3 may be verified. And, of course, also computed those pertaining to any other combination of the data, c, t_o, t. Examples: $c = 1$, $t = t_o = 0.43 \rightarrow r = 3.60$, $q = 0.16$, $w = 3.44$; $c = 1$, $t = t_o = 0.28 \rightarrow r = 3.47$, $q = 0.18$, $w = 3.30$; $c = 0.25$, $t = 5$, $t_o = 3 \rightarrow r = 3.79$, $q = -0.03$, $w = 3.82$; $c = 1.5$, $t = 100$, $t_o = 0{,}5 \rightarrow r = 4.91$, $q = 0.30$, $w = 4.62$; $c = 0$, $t = t_o = 1000 \rightarrow r = 2.00$, $q = -1.00$, $w = 3.00$; $c = 5$, $t = 100$, $t_o = 200 \rightarrow r = 5.33$, $q = 0.33$, $w = 5.00$.

Program 2 has been used, with the same machine, to compute the values in Table 4. The correspondence between the text and the program symbols is the following:

Program 2

```
LIST #2
 10 PRT "TRAPEZOID, BEST NORMAL"
 20 INP "C",C
 30 INP "X",X
 40 S=SQR (1+C↑2)
 50 Z=1/2/(S-C)
 60 Y=X*Z
 70 V=2*(2*S-C)*Z
 80 P=2*S*Z
 85 E=(2*S-C)*Z↑2
 90 U=1+2*S*Y
100 B=1+2*C*Y
110 A=Y+C*Y↑2
120 L=(V/U)↑(4/3)
130 M=(A/E)↑(1/3)
140 N=(A/E)↑3
150 R=LN (L*M*N)/LN X
160 Q=LN (L*M*B/P)/LN X
170 W=LN (N*P/B)/LN X
180 SET F2
190 PRT "R=";R
200 PRT "Q=";Q
210 PRT "W=";W
220 SET N
230 IF X=10 THEN 20
240 GOTO 30
```

text	c	$\sqrt{1+c^2}$	t	t_o	u	A/b	A_o/b	B/b	B_o/b	U/b	U_o/b	r	q	w
program	C	S	Y	Z	X	A	E	B	P	U	V	R	Q	W .

Both programs are so arranged that the translation into any other BASIC dialect is quite easy to accomplish.

4. *About an example given in a previous paper.*

In an old paper (Mendonça 1964a), the writer tackled the following problem.

A uniform open channel, with the longitudinal slope $S_0 = 0.0036$, has an isosceles trapezoidal normal cross section characterized by $c = 1$, $b = 1$ m and $y_0 = 0.7$ m; from a bottom sluice comes out the constant discharge $Q = 3.605$ m^3/s, the height of the contracted section being $y_1 = 0.2$ m; the acceleration of gravity is $g = 9.81$ m/s^2. One wishes to determine the distance X downstream from the contracted section where the flow becomes approximately uniform ($y_2 = 0.99\ y_o = 0.693$ m).

We have $u_1 = y_1/y_o = 0.2857$, $u_2 = y_2/y_o = 0.99$, $B_o = 2.4$ m, $A_o = 1.19$ m^2 and then, putting $\alpha = 1$,

$$\omega = \frac{\alpha\, Q^2\ B_o}{g\ A_o^3} = 1.88674\ .$$

If the course illustrated by the Example 10-2 of Chow, cited in Section 3.3 were blindly followed, the values of r and q to be adopted should have been those corresponding to a normal height equal to the mean $\bar{y} = (0.2 + 0.693)/2 = 0.4465$ m, i. e. $r = 3.62$ and $q = 0.16$.

Instead — picking out from a table (Tableau 2) values computed by means of formulas 38′), 39′) and 40′) — the average values of the hydraulic exponents have been determined by (see the principal diagonal of the matrices of Tables 1 and 2)

$$r = (3.41 + 2 \times 3.58 + 3.82)\ /\ 4 \approx 3.60$$

and

$$q = (0.19 + 2 \times 0.17 + 0.17)\ /\ 4 \approx 0.18\ .$$

From the examples given in Section 3.4, we know that $r = 3.60$ corresponds to $y_o = 0.43$ m and $q = 0.18$ to $y_o = 0.28$ m (in uniform flow).

To allow for the S_3 type of the backwater curve (for which the Boudin and the Jaeger-Manzanares coefficients are both less than one), the value $y_0 \,/\, |S_0| = 0.7 \,/\, 0.0036 = 194.5$ was adopted in place of 194.44444 .

The answer thus obtained has been $X = 372$ m .

Let us now consider a somewhat different procedure, that utilizes the correct values of the hydraulic exponents, given by 35′) and 37′).

Taking from Tables 1 and 2 the values of r and q for t equal to 0.2, 0.4 and 0.7 corresponding to $t_0 = 0.7$, the following averages are determined:

$$r = (3.58 + 2 \times 3.69 + 3.82) \,/\, 4 \approx 3.70 \ ,$$

$$q = (0.17 + 2 \times 0.16 + 0.17) \,/\, 4 \approx 0.17 \ .$$

In Table 5 the numbers in italics were found by linear interpolation between the others there written (read in Mendonça unpubl.).

TABLE 5

u \ q	0.1	0.17	0.2	3.6	3.8
			$r = 3.6$		
0.280	2.1360	*2.1045*	2.0910	1.4568	
0.2857		*2.0995*		*1.4567*	
0.300	2.1141	*2.0869*	2.0752	1.4565	
0.990	0.6426	*0.6425*	0.6424	0.6339	
			$r = 3.8$		
0.280	2.0465	*2.0183*	2.0062		1.3705
0.2857		*2.0137*			*1.3704*
0.300	2.0287	*2.0020*	1.9905		1.3703
0.990	0.6090	*0.6089*	0.6088		0.6003

Then formula [Mendonça 1964a, eq. 34)]

$$X = \frac{y_0}{|S_0|} \left\{ \lambda \left[\overline{D}_r^{\,r}(u_2) - \overline{D}_r^{\,r}(u_1) \right] - \omega \left[\overline{D}_r^{\,q}(u_2) - \overline{D}_r^{\,q}(u_1) \right] \right\} ,$$

where $\overline{D}_r^{\,q}(u)$ means the minus branch of Dupuit function, gives for $r = 3.6$

$$X = 194.5[0.6339 - 1.4567 - 1.88674 \ (0.6425 - 2.0995)] \\ = 374.642$$

and for $r = 3.8$

$$X = 194.5 [0.6003 - 1.3704 - 1.88674 \ (0.6089 - 2.0137)]$$
$$= 365.736 \ .$$

In this way, the distance looked for may be estimated as

$$X = (374.642 + 365.736) \ / \ 2$$
$$= 370 \text{ m} \ .$$

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