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Backwater curves and volumes in uniform rectangular channels

by

Prof. P. DE VARENNES E MENDONÇA
of the Chair of General and Agricultural Hydraulics

RESUMO

Mostra-se como, mediante uma exposição coerente e unificada, se podem deduzir as fórmulas — estabelecidas inicialmente pelo autor em 1963 e 1964 — que, na hipótese da constância do coeficiente de Chézy, permitem determinar, de modo teoricamente exacto, as curvas e os volumes de regolfo em canais uniformes de secção rectangular, de qualquer largura, e com rasante descendente, ascendente ou horizontal.

Para os leitos inclinados, descrevem-se e indica-se onde se encontram tábuas numéricas pormenorizadas, que muito simplificam os cálculos, tornando prático o uso das fórmulas respectivas.

Dão-se exemplos numéricos de aplicação.

SYNOPSIS

It is shown how, by means of a coherent and unified exposition, one can deduce those formulas — originally established by the author in 1963 and 1964 — which, under the hypothesis of the constancy of the Chézy coefficient, allow to determine, in a theoretically exact manner, backwater curves and volumes in uniform rectangular channels, of any breadth, and with sustaining, adverse or horizontal slope.

For channels with sustaining and adverse slopes, detailed tables are available that much simplify the calculations, rendering practical the use of the pertinent formulas. These tables are described and indication is given where they can be found.

Numerical examples are given.

1. Introduction

Formulas and tables that, under the hypothesis of the constancy of the CHÉZY coefficient, allow to determine, in a theoretical exact manner, backwater curves and volumes in uniform rectangular channels have been established in previous papers [3, 4, 7, 8].

Here it is shown how this set of problems may be solved by means of a coherent and unified exposition.

2. Sloping channels

Taking the bottom line (its positive sense being that of the flow) as the axis of abscissas, let y be the height, S the rate of energy loss, λ the product of BOUDIN's coefficient ξ by that of JAEGER-MANZANARES β and $\mathbf{F}$ the FROUDE number at the cross section, perpendicular to the bottom, of abscissa x. Further, let $S_0 = \pm \sin \psi_0$, where ψ_0 is the angle with the horizon, be the bottom longitudinal slope, considered positive (+sign) when sustaining and negative (−sign) when adverse. It is recalled that we have $\xi = \cos \psi_0 = \sqrt{1 - S_0^2}$ and that β allows the curvature of the streamlines to be taken into consideration [2, 7].

We are going to start from the most compact form of the differential equation of backwater curves in open channels with cylindrical bed [5]:

$$1) \qquad \frac{dy}{dx} = \frac{S_0 - S}{\lambda\,(1 - \mathbf{F})}.$$

Identifying the rate of energy loss with that of the uniform flow tangent to the real one (steady, gradually varied, of discharge Q), denoting by A the water area, by U the wetted perimeter and by R the hydraulic radius of the cross section of abscissa x, calling V the mean velocity and labelling with zero as subscript the normal values. CHÉZY formula gives

$$2) \qquad S = \frac{V^2}{C^2 R} = \frac{Q^2 U}{C^2 A^3}$$

and

$$3) \qquad S_0 = \pm \frac{Q^2 U_0}{C^2 A_0^3},$$

where the upper sign refers to channels with sustaining slopes and the lower one to those with adverse slopes.

Hence, assuming a constant resistance factor C, from 2) and 3) we get

4) $$S_0 - S = S_0 \ (1 - \frac{S}{S_0}) = S_0 \ (1 \mp \frac{A_0^3 U}{A^3 U_0}) \ .$$

In the case of the rectangular section of width b, we have

5) $$A = by$$

and

6) $$U = b + 2y \ ,$$

Then it is

7) $$\frac{A_0^3 U}{A^3 U_0} = \left(\frac{y_0}{y}\right)^3 \frac{b + 2y}{b + 2y_0} = \left(\frac{y_0}{y}\right)^3 \left[p + (1 - p) \frac{y}{y_0}\right],$$

where the parameter

8) $$p = \frac{b}{U_0} = \frac{b}{b + 2y_0}$$

is a shape coefficient. And the Froude number

9) $$\mathbf{F} = \frac{1}{\lambda} \cdot \frac{\alpha Q^2 B}{g A^3} \ ,$$

expression in which α represents the Coriolis coefficient and B the top width, as it is also

10) $$B = b \ ,$$

putting (discharge per unit breadth)

11) $$q = \frac{Q}{b} \, ,$$

may be written

12) $$\mathbf{F} = \frac{\alpha\, q^2}{\lambda\, g\, y^3} = \frac{1}{\lambda} \cdot \frac{\alpha\, q^2}{g\, y_0^3} \left(\frac{y_0}{y}\right)^3 = \frac{\omega}{\lambda}\left(\frac{y_0}{y}\right)^3 ,$$

with

13) $$\omega = \frac{\alpha q^2}{g\, y_0^3} \, .$$

By means of 4), 7) and 12), equation 1) becomes

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{|S_0|} \cdot \frac{\mp\left[\omega - \lambda\left(\frac{y}{y_0}\right)^3\right]}{\left(\frac{y}{y_0}\right)^3 \mp\left[(1-p)\,\frac{y}{y_0} + p\right]} \, ,$$

that is, calling

14) $$u = \frac{y}{y_0}$$

the unit height,

15) $$\mathrm{d}x = \frac{y_0}{|S_0|} \cdot \frac{\pm\,(\lambda u^3 - \omega)}{u^3 \mp [(1-p)\; u + p]}\, \mathrm{d}u \, .$$

Supposing both λ and ω to be constant, the distance X between the sections of unit heights u_1 and u_2, the first upstream from the second one, has the value

$$X = x_2 - x_1 = \frac{y_0}{|S_0|}\left[\lambda \int_{u_1}^{u_2} \frac{\pm\, u^3\, \mathrm{d}u}{u^3 \mp [(1-p)\, u + p]} - \right.$$

$$\left. - \,\omega \int_{u_1}^{u_2} \frac{\pm\, \mathrm{d}u}{u^3 \mp [(1-p)\, u + p]}\right],$$

formula that, with

16) $$\Psi(u\,;p) = \int_a^u \frac{\pm u^3\,\mathrm{d}u}{u^3 \mp [(1-p)\,u + p]}$$

and

17) $$\Phi(u\,;p) = \int_a^u \frac{\pm\,\mathrm{d}u}{u^3 \mp [(1-p)\,u + p]}\,,$$

takes the form

18) $$X = \frac{y_0}{|S_0|}\left\{\lambda[\Psi(u_2\,;p) - \Psi(u_1\,;p)] - \omega[\Phi(u_2\,;p) - \Phi(u_1\,;p)]\right\}.$$

It is called backwater volume with respect to a given pair of cross sections the volume of liquid included in between these sections [6].

Bearing in mind that the height y is measured normally to the bottom line, it is evident that the infinitesimal backwater volume with respect to the sections of abscissas x and $x + \mathrm{d}x$ is

$$\mathrm{d}\tau = A\ \mathrm{d}x = by\ \mathrm{d}x = by_0 u\ \mathrm{d}x\ ,$$

or, by 15),

19) $$\mathrm{d}\tau = \frac{by_0^2}{|S_0|}\cdot\frac{\pm(\lambda u^4 - \omega u)}{u^3 \mp [(1-p)\ u + p]}\,\mathrm{d}u\ .$$

The backwater volume with respect to the sections of unit depths u_1 and u_2 is thus given by

20) $$\tau = \frac{by_0^2}{|S_0|}\left\{\lambda[\Gamma(u_2\,;p) - \Gamma(u_1\,;p)] - \omega\,[\Lambda(u_2\,;p) - \Lambda(u_1\,;p)]\right\}.$$

with

$$21) \qquad \Gamma\ (u\ ;\ p) = \int_a^u \frac{\pm\ u^4\ du}{u^3 \mp [(1-p)\ u + p]}$$

and

$$22) \qquad \Lambda\ (u\ ;\ p) = \int_a^u \frac{\pm\ u\ du}{u^3 \mp [(1-p)\ u + p]} .$$

The analytical expressions of these functions are the following ones.

For sustaining slopes ($S_0 > 0$), upper sign in equations 16), 17), 21) and 22):

$$16') \ \Psi\ (u\ ;\ p) = \left\{ u + \frac{1}{2\ (p+2)} \left[\log_e \frac{(u-1)^2}{u^2+u+p} - P \cdot F\ (u\ ;\ p) \right] \right\}_a^u$$

$$17') \ \Phi\ (u\ ;\ p) = \frac{1}{2\ (p+2)} \left[\log_e \frac{(u-1)^2}{u^2+u+p} - 3 \cdot F\ (u\ ;\ p) \right]_a^u ,$$

$$21') \ \Gamma\ (u\ ;\ p) = \frac{1}{2} \left\{ u^2 + \frac{1}{p+2} [\log_e (u-1)^2 + \right.$$

$$\left. + (1-p-p^2) \log_e (u^2+u+p) - (1-3\ p-p^2)\ F\ (u\ ;\ p)] \right\}_a^u ,$$

and

$$22') \ \Lambda\ (u\ ;\ p) = \frac{1}{2\ (p+2)} \left[\log_e \frac{(u-1)^2}{u^2+u+p} + (2p+1)\ F\ (u\ ;\ p) \right]_a^u .$$

For adverse slopes ($S_0 < 0$), with $p \neq 0$:

$$16'') \qquad \Psi\ (u\ ;\ p) = \Big\{ u - T\ [z\ G\ (L-M) - (3pz^2 + H)\ N] \Big\}_u^a ,$$

$$17'') \qquad \Phi\ (u\ ;\ p) = T \Big[z\ (L-M) - 3z^2 N \Big]_u^a ,$$

$$21'')\quad \Gamma(u\,;p) = \left| \frac{u^2}{2} + T\,\{ z^5\,L + [p\,z^2 - (1-p)\,(z^3-p)]\,M - \right.$$

$$\left. - [(1-p)^2\,z^2 + p\,(z^3 - 2p)]\,N\,\}\right|_u^a ,$$

and

$$22'')\quad \Lambda(u\,;p) = T\left[z^2\,(L-M) + (z^3-2p)\,N \right]_u^a .$$

In these equations, $z = z\,(p)$ denotes the real root of the trinomial $u^3 + (1-p)\,u + p$ and, for the sake of brevity, we put

$$P = 2p\,(p+1) - 1\,,$$

$$T = \frac{1}{2\,(2z^3 - p)}\,,$$

$$G = p + (1-p)\,z,$$

$$L = \log_e\,(u-z)^2\,,$$

$$M = \log_e\left(u^2 + zu - \frac{p}{z} \right)$$

$$N = \frac{1}{\sqrt{-\frac{p}{z} - \frac{z^2}{4}}}\quad \arctan \frac{2u+z}{2\sqrt{-\frac{p}{z} - \frac{z^2}{4}}}$$

and

$$F\,(u\,;p) = \int \frac{du}{u^2 + u + p}\,,$$

an integral that, according to the value of the discriminant of the trinomial $u^2 + u + p$, has the following three representations:

$$p < \frac{1}{4} \;\rightarrow\; F(u\,;p) = \frac{1}{\sqrt{1-4p}}\log_e \frac{2u + 1 - \sqrt{1-4p}}{2u + 1 + \sqrt{1-4p}} + \text{const.}\,,$$

$$p = \frac{1}{4} \rightarrow F(u\,;p) = -\frac{2}{2u+1} + \text{const.},$$

$$p > \frac{1}{4} \rightarrow F(u\,;p) = \frac{2}{\sqrt{4p-1}} \arctan \frac{2u+1}{\sqrt{4p-1}} + \text{const.}$$

Formulas 16''), 17''), 21'') and 22'') do not hold for $p = 0$, in which case they are to be replaced by

$$\Psi(u\,;0) = \Big[u - \arctan u \Big]_u^a ,$$

$$\Phi(u\,;0) = \frac{1}{2}\left[\log_e \left(1 + \frac{1}{u^2}\right)\right]_u^a ,$$

$$\Gamma(u\,;0) = \frac{1}{2}\Big[u^2 - \log_e (1 + u^2)\Big]_u^a ,$$

$$\Lambda(u\,;0) = \Big[\arctan u \Big]_u^a .$$

Since functions 16'), 17'), 21') and 22') diverge in the intervals that contain $u = 1$, in order to construct tables, we have taken $a = 0{,}999$ for $u < 1$ and $a = 1{,}001$ for $u > 1$, a procedure which has the advantage of getting positive and the least possible all the useful values. These tables [3, 8], obtained by electronic digital computation, can be described in the following way:

$\Psi(u;\,p)$, $\Phi(u;\,p)$, 4 dec.
$p =$ 0(0,1)0,2(0,05)0,3(0,1)1
$u =$ 0(0,02)0,64(0,01)0,85(0,005)0,95(0,002)0,996(0,001)0,999; 1,001(0,001)1,006(0,002)1,05(0,005)1,11(0,01)1,3(0,02)1,4 (0,05)1,7(0,1)2,2(0,2)4(0,5)6(1)15(5)20(10)30(20)50; ∞;

$\Gamma(u;\,p)$, $\Lambda(u;\,p)$, 3 dec.
$p =$ 0(0,1)0,2(0,05)0,3(0,1)1
$u =$ 0(0,04)0,64(0,02)0,86(0,01)0,95(0,004)0,994(0,003)0,997 (0,002)0,999; 1,001(0,002)1,003(0,003)1,006(0,004)1,038 (0,006)1,05(0,01)1,11(0,02)1,23(0,03)1,32(0,04)1,4(0,1)1,7 (0,2)2,1(0,3)2,4(0,4)4(1)10(5)20(10)30(20)50; ∞ .

For adverse sloping beds, z is calculated by means of

$$z = \sqrt[3]{\sqrt{\frac{p^2}{4} + \frac{(1-p)^3}{27}} - \frac{p}{2}} - \sqrt[3]{\sqrt{\frac{p^2}{4} + \frac{(1-p)^3}{27}} + \frac{p}{2}}$$

and, for dressing the tables, we have taken $a = 50$, a number that, in current practice, may be considered as an upper limit of the u values. These tables [4, 8] can be described thus:

$\Psi(u;\ p)$, $\Phi(u;\ p)$, 4 dec.
$p = 0(0{,}1)0{,}2(0{,}05)0{,}3(0{,}1)1$
$u = 0(0{,}02)1{,}4(0{,}05)1{,}7(0{,}1)2{,}2(0{,}2)3(0{,}5)6(1)10(5)20(10)30$
$(20)50;\ \infty$;

$\Gamma(u;\ p)$, $\Lambda(u;\ p)$, 3 dec.
$p = 0(0{,}1)0{,}2(0{,}05)0{,}3(0{,}1)1$
$u = 0(0{,}04)1{,}4(0{,}1)1{,}6(0{,}2)2{,}2(0{,}4)3(1)5(5)20(10)30(20)50;\ \infty$.

3. Horizontal channels

For $S_0 = 0$, we have $\xi = 1$ and then $\lambda = \beta$. Hence 1) and 12) can be written respectively

23) $$\frac{dx}{dy} = \frac{\beta(\mathbf{F} - 1)}{S}$$

and

24) $$\mathbf{F} = \frac{\alpha q^2}{\beta\, g\, y^3}\ .$$

Being, by 2), 5), 6) and 11),

25) $$S = \frac{q^2}{b\, C^2} \cdot \frac{b + 2y}{y^3}$$

we get, by means of 23), 24) and 25),

26) $$\mathrm{d}x = \frac{C^2\, b}{q^2}\left(\frac{\alpha q^2}{g} \cdot \frac{1}{b+2y} - \frac{\beta y^3}{b+2y}\right)\mathrm{d}y\,.$$

And, as it is

$$\int \frac{\mathrm{d}y}{b+2y} = \frac{1}{2}\log_e\,(b+2y) + \text{const.}$$

and

$$\int \frac{y^3\,\mathrm{d}y}{b+2y} = \frac{1}{2}\left[\frac{y^3}{3} - \frac{by^2}{4} + \frac{b^2y}{4} - \frac{b^3}{8}\log_e\,(b+2y)\right] + \text{const.}\,,$$

the integration of 26) yields

$$x = \frac{C^2 b}{2q^2}\left[\left(\frac{\alpha q^2}{g} + \frac{\beta b^3}{8}\right)\log_e\,(b+2y) - \right.$$

$$\left. - \beta\left(\frac{y^3}{3} - \frac{by^2}{4} + \frac{b^2y}{4}\right)\right] + \text{const.}$$

Then the distance between the sections of depths (heights) y_1 and y_2 is given by

27) $$X = \frac{C^2 b}{2q^2}\left\{\left(\frac{\alpha q^2}{g} + \frac{\beta b^3}{8}\right)\log_e\frac{b+2y_2}{b+2y_1} - \right.$$

$$\left. - \beta\left[\frac{y_2^3 - y_1^3}{3} - \frac{b\,(y_2^2 - y_1^2)}{4} + \frac{b^2\,(y_2 - y_1)}{4}\right]\right\}.$$

The infinitesimal backwater volume is, by 26),

$$\mathrm{d}\tau = A\,\mathrm{d}x = by\,\mathrm{d}x = C^2\,b^2\left(\frac{\alpha}{g}\cdot\frac{y}{b+2y} - \frac{\beta}{q^2}\cdot\frac{y^4}{b+2y}\right)\mathrm{d}y\,,$$

from which, since it is

$$\int \frac{y\,dy}{b+2y} = \frac{1}{2}\left[y - \frac{b}{2}\log_e (b+2y)\right] + \text{const.}$$

and

$$\int \frac{y^4\,dy}{b+2y} = \frac{1}{4}\left[\frac{y^4}{2} - \frac{by^3}{3} + \frac{b^2y^2}{4} - \frac{b^3y}{4} + \frac{b^4}{8}\log_e (b+2y)\right] + \text{const.}\,,$$

we get that the backwater volume with respect to the sections of depths y_1 and y_2 is

28) $$\tau = \frac{C^2b^2}{2}\left\{\left(\frac{\alpha}{g} + \frac{\beta b^3}{8q^2}\right)\left(y_2 - y_1 - \frac{b}{2}\log_e \frac{b+2y_2}{b+2y_1}\right) - \right.$$
$$\left. - \frac{\beta}{2q^2}\left[\frac{1}{2}(y_2^4 - y_1^4) - \frac{b}{3}(y_2^3 - y_1^3) + \frac{b^2}{4}(y_2^2 - y_1^2)\right]\right\}.$$

In the case of the infinitely wide section, since $2y$ is negligible in comparison to b, 26) is reduced to

$$dx = C^2\left(\frac{\alpha}{g} - \frac{\beta}{q^2}y^3\right) dy\;,$$

from which we get

29) $$X = C^2\left[\frac{\alpha}{g}(y_2 - y_1) - \frac{\beta}{4q^2}(y_2^4 - y_1^4)\right]$$

and the backwater volume by unit width, $\tau_1 = \tau/b$, is given by

30) $$\tau_1 = C^2\left[\frac{\alpha}{2\,g}(y_2^2 - y_1^2) - \frac{\beta}{5\,q^2}(y_2^5 - y_1^5)\right].$$

Formulas 27) to 30) where first found in 1964 [7, 8], but for $\beta = 1$ 27) and 29) become identical to those established by DUPUIT in 1848 [1].

4. Examples

A) Let us consider a uniform rectangular channel 7.00 m wide and ending by a free overfall. Let $Q = 12.75\ m^3/s$ be the discharge and $C = 60\ m^{1/2}/s$ the CHÉZY resistance factor. Supposing, in succession, that the bottom longitudinal slope has the value

a) $S_0 = 3.9\ ^{00}/_{00}$,

b) $S_0 = -\ 3.9\ ^{00}/_{00}$

and

c) $S_0 = 0$,

we wish to compute the distances between and the backwater volumes relative to the sections of heights $y_1 = 1.44$ m and $y_2 = 0.72$ m.

We have

$$q = \frac{Q}{b} = \frac{12.75}{7} = 1.821429, \qquad q^2 = 3.317604 \ ,$$

$$\frac{A_0}{U_0} = \frac{Q^2}{C^2 \,|\, S_0 \,|} = \frac{7^2 \times 3.317604}{60^2 \times 0.000\ 39} = 7^2 \times 2.363$$

or

$$\frac{7y_0^3}{7 + 2y_0} = 2.363$$

from which

$$y_0 = 1.50 \text{ m} \ ,$$

$$p = \frac{b}{b + 2\,y_0} = \frac{7}{7 + 2 \times 1.5} = 0.7 \ ,$$

$$\frac{y_0}{|\, S_0 \,|} = \frac{1.5}{0.000\ 39} = 3846$$

and

$$\frac{y_0^2}{|S_0|} = \frac{(1.5)^2}{0.000\ 39} = 5770\ .$$

Further, taking $\alpha = 1$ and $g = 9.81\ \mathrm{m/s^2}$, we get

$$\omega = \frac{\alpha q^2}{g\, y_0^3} = \frac{3.317604}{9.81 \times (1.5)^3} = 0.100\ .$$

The unit heights are $u_1 = 1.44/1.50 = 0.96$ and $u_2 = 0.72/1.50 =$ $= 0.48$.

a) *Sustaining slope. Backwater curve of type* M_2.

In the above mentioned tables it may be read:

$\Psi\ (0.48;\ 0.7) = 1.9857$
$\Phi\ (0.48;\ 0.7) = 2.5849$
$\Gamma\ (0.48;\ 0.7) = 1.876$
$\Lambda\ (0.48;\ 0.7) = 2.317$

$\Psi\ (0.96;\ 0.7) = 1.3391$
$\Phi\ (0.96;\ 0.7) = 1.3826$
$\Gamma\ (0.96;\ 0.7) = 1.325$
$\Lambda\ (0.96;\ 0.7) = 1.368\ .$

Then, putting $\lambda = 1$, formulas 18) and 20) give respectively

$$X = 3846\ [1.9857 - 1.3391 - 0.1\ (2.5849 - 1.3826)]$$
$$= 2025\ \mathrm{m}$$

and

$$\tau = 40\ 390\ [1.876 - 1.325 - 0.1\ (2.317 - 1.368)]$$
$$= 18.4 \times 10^3\ \mathrm{m}^3\ .$$

Thus the volume found corresponds, in uniform flow, to the height

$$\frac{\tau}{bX} = \frac{18\ 400}{7 \times 2025} = 1.30\ \mathrm{m}$$

which is very different from the arithmetic mean, 1.08 m, of y_1 and y_2.

b) *Adverse slope. Backwater curve of type* A_2.

Now the tables give

Ψ (0.48; 0.7) = 48.6639
Φ (0.48; 0.7) = 0.7603
Γ (0.48; 0.7) = 1247.973
Λ (0.48; 0.7) = 1.080

Ψ (0.96; 0.7) = 48.5242
Φ (0.96; 0.7) = 0.3854
Γ (0.96; 0.7) = 1247.866
Λ (0.96; 0.7) = 0.820

and then, with $\lambda = 1$, from formulas 18) and 20), we get

$$X = 3846 \; [48.6639 - 48.5242 - 0.1 \, (0.7603 - 0.3854)]$$

$$= 393 \text{ m}$$

and

$$\tau = 40\,390 \; [1247.973 - 1247.866 - 0.1 \, (1.080 - 0.820)]$$

$$= 3.27 \times 10^3 \text{ m}^3 .$$

This value of τ corresponds in uniform flow (made in the opposite direction) to the height 1.19 m.

c) *Horizontal bottom. Backwater curve of type* H_2.

We have $1/q^2 = 1/3.317604 = 0.301422$.

$$\frac{C^2 b}{2q^2} = \frac{60^2 \times 7 \times 0.301422}{2} = 3800 , \qquad \frac{C^2 b^2}{2} = \frac{(60 \times 7)^2}{2} = 88\,200 ,$$

$$\frac{b + 2y_2}{b + 2y_1} = \frac{7 + 2 \times 0.72}{7 + 2 \times 1.44} = 0.854251 , \qquad \log_e 0.854251 = -0.157530 .$$

Then formulas 27) and 28), with $\alpha = \beta = 1$, give

$$X = 3800 \{ (3.317604/9.81 + 7^3/8) \times -0.15753 - (1/3) [0.72)^3 - (1.44)^3] + (7/4) [(0.72)^2 - (1.44)^2] - (7^2/4)(0.72 - 1.44) \}$$

$$= 615 \text{ m}$$

and

$$\tau = 88\,200 \Big\{ [1/9.81 + (7^3/8) \times 0.301422] [0.72 - 1.44 - (7/2) \times -0.15753] - (0.301422/2) \{ (1/2) [(0.72)^4 - (1.44)^4] - (7/3) [(0.72)^3 - (1.44)^3] + (7^2/4) [(0.72)^2 - (1.44)^2] \} \Big\}$$

$$= 5.25 \times 10^3 \text{ m}^3 .$$

If the width could be assumed to be infinite, we would get, from 29) and 30),

$$X = 3600 \{ (1/9.81)(0.72 - 1.44) - (0.301422/4) [(0.72)^4 - (1.44)^4] \}$$

$$= 829 \text{ m}$$

and

$$\tau_1 = 3600 \{ (1/19.62) [(0.72)^2 - (1.44)^2] - (0.301422/5) [(0.72)^5 - (1.44)^5] \}$$

$$= 1016 \text{ m}^3 ,$$

whence

$$\tau = 7.11 \times 10^3 \text{ m}^3 ,$$

values much larger than those found above (1).

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(1) Both values of τ for this case correct those respectively given in reference [8].

B) Under the conditions of case a) of example *A*, supposing that the control section (overflow dam) has the height $y_2 = 3.00$ m, we wish to determine a few points of the backwater curve (of type M_1), including those which correspond to heights differing from the normal 1 % and 1 ‰ of this one.

We have $u_2 = 3.00/1.50 = 2$ and the tables yield

$$\Psi(2.00;\ 0.7) = 3.3291$$

and

$$\Phi(2.00;\ 0.7) = 2.2594\ (^2).$$

Hence formula 18) can be written, putting $\lambda = 1$,

$$X = 3846\ \{\ 3.3291 - \Psi(u_1) - 0.1\ [2.2594 - \Phi(u_1)]\}\ .$$

It is helpful to arrange the remaining computation in the following way, the values of columns (*3*) and (*5*) being also got out from the tables:

y_1 (m)	$u_1 = \frac{(1)}{1.5}$	$\Psi(u_1;\ 0.7)$	3.3291 — (3)	$\Phi(u_1)\ ;\ 0.7)$	2.2594 — (5)	0,1 × (6)	(4) — (7)	X 3846 × (8) (m)
(*1*)	(*2*)	(*3*)	(*4*)	(*5*)	(*6*)	(*7*)	(*8*)	(*9*)
2.400	1.600	2.8173	0.5118	2.1686	0.0908	0.0091	0.5027	1933
1.800	1.200	2.1052	1.2239	1.8863	0.3731	0.0373	1.1866	4564
1.650	1.100	1.7758	1.5533	1.6664	0.5930	0.0593	1.4940	5746
1.515	1.010	0.8591	2.4700	0.8491	1.4103	0.1410	2.3290	8957
1.5015	1.001	0.0000	3.3291	0.0000	2.2594	0.2259	3.1032	11935

(2) In the example of reference [3] this number has been wrongly written, so that the values of X there arrived at are slightly inaccurate.

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