UNIVERSIDADE DE LISBOA
INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

PORTFOLIO INSURANCE STRATEGIES: FRIEND OR FOE?

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A Thesis submitted to Universidade de Lisboa in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Management at Instituto Superior de Economia e Gestão.

2018

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2018

Preface

There are always challenges and new environments to explore. A thesis on management is certainly an adventure for me. I am a professional in financial services and not an academic thus adaptation has been permanent over last years to cope with the PhD requirements and to write this document. Nonetheless, I have been fortune in taking along my professional career several courses and certifications that kept me in touch with academia and very inspiring minds.

The need to be updated in some areas of finance to be able to bridge between the cutting edge on theory and the industry, drove me to the challenge of pursuing this PhD.

The selection of a topic for a deep dive into finance and build some innovative feature and contribute to the field is something that is difficult, takes time and is very demanding. In my case the idea of protective strategies was growing since the Master's degree in finance, in 2011, and the missing pieces were combined at the lectures during the PhD course along with the discussions with my Supervisor, Prof. Raquel M. Gaspar. From the practitioners' point of view, the implementation of protective strategies or downside protection mechanisms brings always a question: when individual investors are seeking return from risky assets and still demand a protection in bear markets are they being rational or just relying on asset managers capabilities? Institutional investors, in particularly pension funds, insurance companies, endowments and foundations are in the asset management business to define the best strategies to face their liabilities on the short and long run and thus their objectives are set into a statement of investment policy. The term liability driven investments and asset and liability management are concepts and strategies that are very common and effective, once investors are aware of the pitfalls and limitations.

However, this awareness of the pitfalls and limitations for individual investors regarding protective strategies is not effective, even when investors are protected with regulation on sales and risk profiling by the sell side. Therefore this general idea for the thesis has evolved and became the central point for a question: Are portfolio insurance strategies a good option for investment?.

After the classes of the PhD course, in 2012, the objective of finalizing a thesis become almost an utopia due to the competing tasks and commitments in a daily basis. Managing all these factors and not giving up was only possible because of my wife and our daughter support. Critical was also my supervisor motivation that thrive me to keep on the topic and to challenge the research. She was enthusiastic and capable of creating all conditions for keeping me involved in this process.

During the PhD programme I also had the privilege of taking graduation course for a Master in Science in Risk Management at Stern in New York University where I met challenging colleagues in my cohort with whom I worked closely in a strategic capstone on robo-advising investments. This work allowed a different approach into digital economy and to all financial aspects on a very sophisticated market.

During this journey, there were a lot of people that either on my daily job or on my “second life” environment contributed to this personal project. For them all I am truly grateful for their prompt and unconditional support. Finally, from a pure personal and professional perspective, I do believe that people in the business of professional services and corporations or public service should be focus on delivering the best to the entities they work for and their final beneficiaries. In some cases there is no need for external scrutiny from third parties, but regulatory environments made almost compulsory a portfolio of certifications. Nevertheless, being eager about assessments towards a continuous learning process, this work made my awareness on how important it is to bridge between academia and industry in a very different level.

The most important is not the destination, but the journey and how we embrace it. Regardless of the outcome and the level of contribution to the financial field, the relevance of this work is about using all the techniques that are available and respond to the question: what strives individual investors into strategies that are intended to protect their wealth, but in extreme scenarios are the ones that will hurt them directly? The answers, however, are not complete, which is something that will keep me focus on this particular topic in the future.

Thank you all for this amazing opportunity to keep on learning.

Statement of Originality

I hereby declare that except where specific reference is made, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This thesis is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text.

Acknowledgements

First and foremost I would like to acknowledge my deepest thank to my supervisor, Prof. Raquel M. Gaspar, for her continuous efforts to thrive me through the research and for enhancing the capabilities to learn more. It has been a long but stimulating journey. I am also grateful for her trust in inviting me to lecture the course of Risk Management at IDEFE.

I'm also grateful to my colleagues at the Master in Science in Risk Management at Stern Business School in New York University, specially Aized Gill, Amit Sinha, Faisal Azim, and Juan Bernal. I am indebted for their support during the class of 2017.

In the journey that led to this project there are two references that are really special and to whom I owe some of the conditions that led me in this way: Rui Lopes, my boss at the bank, back in 2009-2010, and today my friend and mentor, that believed in my capabilities and taught some of the requirements to excel and focus on the important things; and Rui Guerra, my dear friend (over 30 years!), that challenged me to embrace professional services, became my mentor in the consultancy business and created all conditions for my return to school.

Special word for anonymous reviewers of two papers accepted in the Portuguese Finance Network conferences in 2016 and 2018.

Finally, for all the people with whom I work on a daily basis at my job and the colleagues at the PhD course, I'm grateful for their support and friendship.

Dedication

I dedicate this work to my wife Lucília and to our daughter Ana for their unconditional and endless support and motivation.

Abstract

This work focus on a specific protective investment strategy developed in the foundations of options theory.
Although individual investors' risk profile has evolved to accommodate remuneration on risks taken, still averse investors tend to appreciate the rallies of risky markets when relatively protected from downward movements.
One of the strategies addressing this conundrum was developed in the 1980s and evolved from inclusion of options. In fact portfolio insurance strategies are important financial solutions sold to institutional and individual investors, that protect against downside risk while maintaining some upside valuation potential. The way these strategies are engineered has been criticized, and some analysts point them as one of the causes for increasing market volatility in depressed markets. In spite of the negative opinion, and the difficulties to explain their solid market share, investors keep on buying portfolio insurance.
As these strategies are reactive to risky assets price movements we review the impact of portfolio insurance strategies on stability of financial markets. In particular, we go from the crisis of October 1987 to some of the current resurgence of protective views on recent equity market rallies.
The objective of this thesis is three-fold: have a transversal approach to portfolio insurance strategies using current tools and assess the fitting of these financial solutions to individual investors; contribute to the literature on portfolio insurance, specially, on the discussion on the values derived from protective strategies; finally, taking account new business platforms, discuss how new digital tools for investments may enhance capabilities for profiling individual risks and set strategies that are proper per each investor. The work points to some features that may define the characteristics of individual investors' risk profile with the product definition for portfolio strategies. In particular we set the common approach for different utility functions and evaluate how these strategies respond to investors' risk and return requirements. We find no relevant results under the Expected Utility Theory (EUT) to explain why individuals invest in portfolio insurance.
In this thesis we support the use of behavioural finance to explain the popularity of portfolio insurance investments. In order to clarify their popularity, we compare investors' decision using two distinct frameworks: the EUT and the behavioural approach based on the prospect and cumulative prospect theory. We rely on Monte Carlo simulation techniques to compare portfolio insurance investment strategies against uninsured basic benchmark strategies. Our compara-

tive analysis allows us to conclude that cumulative prospect theory may be a viable framework to explain the popularity of (at least some) portfolio insurance investments. The results point the best choices to be the naïve portfolio insurance strategies instead of the complex products. Ultimately we take a view on the digital marketplace for portfolio management in particular when using robo-advisors. There is a growing number of automatic platforms that define investors risk profile using a set of questions on psychological and behaviour features. Based on these characteristics, robo-advisors propose asset allocation into portfolios that tend to address investors aspirations within their risk profile. However, we found that even using some questions on downside risks - which tend to be responded by portfolio insurance strategies - there is a biased approach on the sample of robo-advisors in our study that may hide future mismatching from individual investors aspirations and deliverables from these platforms. A cross sectional analysis for the same risk type investor end up with different risk reward patterns from the sample of robo-advisors. There is, therefore, a potential long term mismatch between risks and risk tolerance levels that investors think they are bearing. This opens the space to review how the regulation is actually addressing mis-selling and effective risk profiling on individuals. In this case we point out the need for guidelines on policy issues regarding robo-advisor.

Keywords: portfolio management, portfolio insurance, expected utility, behavioural finance and robo-advisor.

Resumo

Este trabalho centra-se nas estratégias de proteção baseadas nos fundamentos da teoria das opções. Apesar da avaliação do perfil de risco dos investidores individuais ter vindo a evoluir no sentido de incorporar o conceito de remuneração pelo risco tomado, continuam a existir evidências que apontam para os investidores avessos ao risco apreciarem o movimentos de valorização dos ativos com risco, desde que beneficiem de uma proteção relativamente a situações de perda.

Uma das estratégias de investimento que permite responder a este desafio foi desenvolvida nos anos 80 a partir da evolução da teoria das opções. De facto, as estratégias de "portfolio insurance" são soluções relevantes, tanto para investidores institucionais como individuais, para a proteção de carteiras em situção de perdas nos mercados de ativos de risco mas que permitem, simultaneamente, beneficiar do potencial de valorização nos movimento de subida de preços. A forma como estas estratégias têm sido desenvolvidas é alvo de críticas, tendo alguns analistas apontado estas estratégias como uma das causas de maior volatilidade nos mercados em situações de quedas nos preços. No entanto, os investidores continuam a alocar os seus recursos a soluções de "portfolio insurance" apesar destas avaliações negativas.

Dado que estas estratégias são reativas quando ocorrem movimentos nos preços, neste trabalho efetuamos uma revisão dos impactos das estratégias de portfolio insurance na estabilidade dos mercados financeiros. Em particular, revimos a crise de Outubro de 1987 e damos nota sobre alguns dos movimentos mais recentes relativamente ao ressurgimento das estratégias de proteção face aos últimos movimentos de subida dos preços do mercado acionista.

São três os objetivos desta Tese: estabelecer uma abordagem transversal às estratégias "portfolio insurance" utilizando as técnicas disponíveis para avaliar a adequação destas soluções aos investidores individuais; contribuir para a literatura, em particular para a avaliação do valor obtido pelos investidores em "portfolio insurance"; finalmente, tendo em conta a evolução das novas plataformas digitais de gestão de ativos com base em "robo-advisors", discutir como se pode incrementar a capacidade de desenhar perfis de risco e níveis de tolerância adequados a cada investidor. No âmbito do trabalho avaliamos algumas das características que permitem, de alguma forma, avaliar a adequação das soluções de "portfolio insurance" ao perfil de risco dos investidores individuais. A metodologia que utilizamos baseia-se na abordagem da Teoria da Utilidade Esperada, com diversas funções e parâmetros, de forma a analisar a resposta em termos de utilidade face a diversos cenários de risco e retorno. Os resultados obtidos nesta abordagem não permitem uma explicação suficientemente robusta para justificar o investimento dos

investidores individuais nestas soluções.

Na Tese é desenvolvida uma análise que suporta a abordagem das finanças comportamentais para a explicação da popularidade dos investimentos em estratégias de "portfolio insurance". Com efeito, baseamos a análise na comparação das decisões dos investidores em enquadramentos distintos, i.e. teoria da utilidade e finanças comportamentais baseadas em "prospect theory" e "cumulative prospect theory". A partir de simulações de Monte Carlo comparamos o valor atríbuido pelos investidores num conjunto de estratégias "portfolio insurance" frequentemente disponibilizados no mercado com estratégias sem mecanismos de proteção. Os resultados desta análise comparativa permitem concluir que a "cumulative prospect theory" pode ser um enquadramento viável para explicar a popularidade de, pelo menos, algumas das estratégias de "portfolio insurance". Os resultados apontam para que de entre as estratégias de "portfolio insurance" as mais simples - do tipo naive - sejam preferidas, face a estratégias assentes em soluções mais complexas.

Por último, tendo presente a evolução digital que tem vindo a ser introduzida na gestão de investimentos para particulares, com especial incidência nos designados "robo-advisors", abordamos a forma como estas plataformas determinam as estratégias de investimento. Existe um número cada vez maior de plataformas que define o perfil dos investidores através de um questionário com recolha de elementos psicológicos e comportamentais. Com base nas características recolhidas a partir dessas questões, as plataformas de "robo-advisors" propõem uma alocação a diversas classes de ativos que pretendem endereçar as aspirações dos investidores individuais de acordo com o seu perfil de risco. No entanto, na nossa análise, verificamos que apesar da utilização de perguntas sobre situações de perda de valor nos ativos - cenários a que as estratégias de "portfolio insurance" pretendem dar resposta - regista-se uma abordagem enviesada na amostra de plataformas "robo-advisors" que podem, no futuro, potenciar divergências entre as aspirações dos investidores e os resultados estimados a partir das alocações definidas pelas plataformas. Isto significa que podemos estar perante um "mismatch" no longo prazo entre os riscos e níveis de tolerância ao risco em que os investidores julgam que podem incorrer, face a questionários recolhidos no processo de definição da sua estratégia de investimentos. Este facto abre espaço para a discussão sobre a forma como a regulamentação pretende gerir, em termos de orientações, os procedimentos de "mis-selling" e efetiva definição dos perfis de risco dos investidores individuais. Face a estas situações, identificamos alguns aspetos que podem vir a ser relevantes para orientações sobre políticas quanto a risco, retorno e comportamento dos investidores individuais.

Palavras-Chave: gestão de portfolio, “portfolio insurance”, teoria da utilidade esperada, finanças comportamentais e “robo-advisor”.

"The first thing you have to know is yourself. A man who knows himself can step outside himself and watch his own reactions like an observer."

Adam Smith, The Money Game

"Never complain of that of which it is at all times in your power to rid yourself."

Adam Smith, The Theory Of Moral Sentiments

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Abbreviations

- ARA: Absolute Risk Aversion
- AT: Algorithm Trading
- BFT: Behavioural Finance Theory
- B&H: Buy-and-Hold
- CE: Certainty Equivalent
- CARA: Constant Absolute Risk Aversion
- CBT: Computer-Based Trading
- CPPI: Constant Proportion Portfolio Insurance
- CPT: Cumulative Prospect Theory
- CRRA: Constant Relative Risk Aversion
- DARA: Decreasing Absolute Risk Aversion
- DRRA: Decreasing Relative Risk Aversion
- EMH: Efficient Market Hypothesis
- EUT: Expected Utility Theory
- ETF: Exchange Tradable Fund
- FINRA: Financial Industry Regulatory Authority
- GBM: Geometric Brownian Motion
- HFT: High Frequency Trading
- HARA: Hyperbolic Absolute Risk Aversion
- IARA: Increasing Absolute Risk Aversion
- IRRA: Increasing Relative Risk Aversion
- MVA: Mean-Variance Analysis
- MPT: Modern Portfolio Theory
- OBPI: Option Based Portfolio Insurance
- PT: Prospect Theory
- RRA: Relative Risk Aversion
- SEC: U.S. Securities and Exchange Commission
- SLPI: Stop Loss Portfolio Insurance
- TIPP: Time Invariant Portfolio Protection

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Chapter 1

Introduction

The paradigm of risk vs returns evolved through times as individuals' capability to understand the impact of losses changed under a framework that specifies the rationality of their decisions based on a set of assumptions. In fact, knowing that some of the decisions by individuals may differ from the prescriptive results from Modern Portfolio Theory (MPT) (Markowitz, 1952, 1959) rises questions on the drivers of individual investment decisions.

Portfolio insurance is an asset allocation or hedging strategy that gives an investor the capability to decide the amount of risk he or she is willing to accept through a trade-off between risk and expected return. In these strategies the objective is to limit the downside risk while maintaining the possibility of benefiting partially from the upside potential from risky assets (Leland and Rubinstein, 1976).

Portfolio insurance are dynamic hedging strategies that have been pointed as contributors to relevant financial crisis, specially the October 1987 market crash (Schiller, 1988), although they were not the sole causes of such crisis. Like other investment strategies, portfolio insurance are based on models, which are always a simplification of reality, yet the more complexity to be included into the model and the level of sophistication for estimations the more accuracy is reached in order to adjust the results with investors expectations. However, the different market players - insures, hedgers and speculators - and interdependencies between markets (cash and futures) in moments of instability along with incomplete information creates imbalances that may impact portfolio insurance strategies and, simultaneously, enhances feedback loops with effects on liquidity. All these effects are stressed with computer based trading and, under specific circumstances, vicious cycle of forced selling conducts to violations on portfolio insurance targets (Leland, 2011).

In spite of these risks, the specificity of the rational on loss preventing investment strategies can be the trigger to understand the reason for individual investors to keep on investing in portfolio insurance strategies. Thus, in this thesis we want to identify the reasons that lead individual investors to invest in portfolio insurance strategies and to set the conditions that may find these investments as the choices of specific segment of investors.

It is common practice in the literature to set a framework for analysis of investment decisions under uncertainty based on different theories. Therefore, our work comprehends the normative Expected Utility Theory (EUT) and Prospect Theory (PT) and Cumulative Prospect Theory (CPT) to test different attitudes towards risk. Investors are not all alike, their decisions are focus either on utility derived from a certain amount of estimated wealth, an expected monetary value or gains and losses against a pre-established reference. The valuation or utility is driven by a set of factors that may be collected and perceived using a formulation. The way portfolio insurance strategies fit the rationality in each of the explanatory theories enhances a possible segmentation on investors appetite for this type of products. Our work supports the use of behavioural finance to explain the popularity of portfolio insurance investments.

Due to the emergency of modern platforms to develop asset management based on advanced algorithms that match individual aspirations and investors' risk profiles and risk tolerance — robo-advisors — we dive into a sample of platforms that are growing in relevance. This is a new space for defining risk profiles based on questionnaires that aim to set risk tolerances and some guidelines on how individual behave before scenarios of downside and how much they can accommodate on the loss side. We investigate how these mechanisms are mimicking human decision in order to bridge between risk aversion and value seeking behaviours. The risks involved in such process may not be perceived by investors using robo-advisors and that can trigger an unsustainable confidence on expected future returns that may not be protected.

The critical point is how risk profiling is being adequately defined with the algorithms on robo-advisors. We raise some issues that may be addressed with new policy and guidelines.

During the research we presented our work at the Portuguese Finance Network conferences on June 2016 and July 2018. A version of a joint work developed on robo-advising and risk profiling was also presented at the International Conference on Computational Finance in September 2017 in Lisbon.

The thesis combines the findings of individual research papers, but was organized in seven chapters, being the Chapter 2 a general approach to portfolio insurance strategies and their characteristics. The remaining four chapters are dedicated to the topics that were the basis for research: A Review on the Impact of Portfolio Insurance Investments on Market Stability; Who Can Portfolio Insurance Strategies Attract? Mapping Solutions to Investors; Investor's Perspective on Portfolio Insurance - Expected Utility vs Prospect Theories; and Market Innovation, Robo-Advising and Protection. The final chapter presents the conclusions of this work and highlights areas for future research.

Chapter 2

Fundamentals of Portfolio Insurance

Portfolio insurance is an asset allocation or hedging strategy that gives an investor the capability to decide the amount of risk he or she is willing to accept through a trade-off between risk and expected return. Although it is not a formal insurance policy it may resemble the insurance concept as it is possible to limit the downside risk while maintaining the possibility of benefiting partially from the upside potential of the investment strategy.

Portfolio insurance is related with any investment strategy that protects the amount of the portfolio, which can be made of bonds, equity or real assets. If the value of the assets increases, the increase of the insured portfolio will be lower, but even so it will increase. These strategies allow the investor to participate in the appreciation of value of a risky portfolio while limiting the potential of losses. Thus can also be considered similar to the features of investment in options.

There are four main mechanisms to implement a portfolio insurance investment (Ho et al., 2013, Meucci, 2010, Lee et al., 2013):

1 Stop loss orders

2 Purchase of exchange-trade put options

3 Synthetic options

4 Dynamic hedging using futures contracts

Derived from these mechanisms, the most common portfolio insurance strategies are Option

Based Portfolio Insurance (OBPI), Constant Proportion Portfolio Insurance (CPPI) and the Stop Loss Portfolio Insurance (SLPI).

The most simple strategy is the SLPI and is based on stop limit orders as its underlying order type. These are placed at a specific price, and if the market price reaches the order price, the order will be executed as a limit order. In this case all the risky assets under this condition will be sold and revert into non-risky assets. The SLPI strategy was used by Rubinstein (1985) and is also a dynamic portfolio strategy, but the portfolio is fully invested in risky assets and only considers a single transaction that may occur when the minimum amount for a target on wealth is breached.

The first approach to more complex portfolio insurance strategies was developed by Leland and Rubinstein (1976), who analysed the portfolio insurance techniques based on the options pricing formula of Black and Scholes (1973). The original OBPI concept consists in an investment based on financial options. The investment is allocated between a risk-free investment and a call option on the underlying portfolio[1]. Nevertheless, to build this strategy is necessary to find listed options with specific strike prices and maturities for each investment portfolio, which is often not possible. Thus, Leland and Rubinstein (1981) based on the pricing formula of Black and Scholes (1973) and Merton (1973) developed a replication of the payoff of the OBPI strategy which means a replication of a call and a bond or a put and the underlying asset. This strategy is also known in literature by Dynamic OBPI and consists in a dynamic portfolio strategy that allocates the capital between risk-free assets and risky asset. The proportion in-vested between these two assets is defined through delta hedging according to the Black-Scholes model. The underlying idea of these strategies is to provide protection against potential market losses, while preserving the upward potential (see e.g. Grossman and Villa (1989) and Basak (1995)), allowing participation in market rallies. Due to a dynamic allocation strategy the portfolio is protected against market falls by a guaranteed floor, which preserves a minimum level of wealth at a specific time horizon. The investor has the ability to limit downside risk, particularly in falling markets, while allowing some participation in upside markets (Bertrand and Prigent, 2016).

The CPPI strategy was introduced by Perold (1986) and later by Black and Jones (1987) for

[1] Due to the put-call parity theorem the investment can also be combined between the underlying portfolio and a put option on the underlying portfolio.

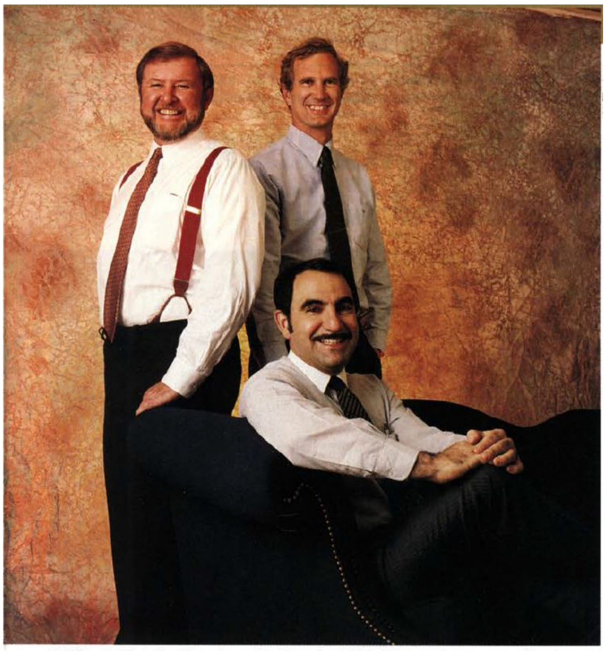
equity instruments and by Perold and Sharpe (1988) for fixed-income instruments. This strategy is also a dynamic portfolio strategy which divides the portfolio between risk-free and risky investments. The proportion invested is defined by setting a floor and a multiplier. This strategy ensures a predefined floor by dynamically rebalancing allocations between a risky asset and a risk-free asset. A constant proportion or multiplier, m, of the excess value of the investment above the floor (the buffer) is allocated to the risky asset, the rest is invested in the risk-free asset. The floor and the multiplier are exogenous variables to the model and are determined by the investor's risk attitude and his or her views on the evolution of the risky asset. The lower the floor and the higher the multiplier, the greater the allocation to the risky asset. The investor then has a higher upside potential but the floor is reached more quickly if the risky asset price falls.

2.1 Portfolio Insurance designers

Hayne Leland had the idea of portfolio insurance in the mid 1970s when his brother mentioned that institutional investors had been away from the market after the 1974 slump and missed the recovery period after. After the initial idea, he discussed it with Mark Rubinstein and set out a product for pension funds. The marketing and business actions took off after John O'Brian, a former executive at A. G. Becker for pension fund investments analysis, joined them at the Leland O'Brian & Rubinstein (LOR) as chief executive.

After the crisis of October 19th, 1987, portfolio insurance was referred as a major factor for the plunge of the Dow Jones of 508 points. At the time LOR clients also noted that the product did not delivered what was expected. The technique has pitfalls, as pointed out decades after the crisis by their designers, but is still running and being loaded with funding and also enlarging the scope for more asset classes. Innovations come with a learning curve and portfolio insurance has made that path.

Figure 2.1: Portfolio Insurance - The inventors and missionaries Hayne Leland, John O'Brian and Mark Rubinstein, from Business People of the Year, The Guys Who Gave Us Portfolio Insurance, Fortune, January 4, 1988



O'Brien (left), Leland (rear), and Rubinstein of LOR: Why are these men smiling?

2.2 Portfolio Insurance Strategies - Mechanics

Despite the misleading designation, portfolio insurance is not an insurance contract where an investor pays a premium for a risk transference to an insurance company to limit losses from adverse market conditions. Instead, is a strategic asset allocation that may limit the risk taken by the investor through techniques that actually change return distributions.

These strategies are not recent, they had their origins on Black-Scholes-Merton's work on option theory in the early 1970s (Black and Scholes, 1973, and Merton, 1973). To some extent, portfolio insurance can be seen as an investment on a financial option: an investment on a risky underlying, plus a put option written on the asset, or, by the put-call option parity theorem (Cox and Rubinstein, 1985), as an investment in the bank account and call options on the risky underlying. With this strategy a minimum is set for the value of the portfolio (floor), regardless of the price movement of the risky underlying.

Due to falling costs on trading and exuberant product innovation (e.g. structured financial products, index notes, warrants, etc.), portfolio insurance managers have extended portfolio insurance solutions from institutional to their retail segments.

Financial markets, specially since de 1980s, have suffered several crises impacting severely on investors' wealth. The intensity of those crises varied across markets and asset classes, but the uncertainty became a permanent factor in investors' decisions. This new environment of uncertain outcomes coupled with global integration, and the increasing complexity of investment alternatives, drove investors' decisions into a more demanding risk management process, which lead the market players to provide both hedging and leverage solutions for a large array of investors (Pain and Rand, 2008).

In spite of continuous concerns from different parties, and the increasing complexity of risk management decisions (Brady Report, 1988; Rubinstein, 1999; Tucker, 2005), portfolio insurance popularity has not decreased. If some strategies turned to be polemic and lost popularity, such as CPPIs (see Costa and Gaspar, 2014, and Carvalho et al., 2016), other have emerged in their place as is the case of Time Invariant Portfolio Protection (TIPP). Nowadays investors are presented with a wide range of strategies, offered from very active distribution channels on retail banking and institutional segments.

The implementation of portfolio insurance is possible using naïve, or advanced strategies. Therefore an investor can choose from a simple SLPI to a complex Risk-Based portfolio insurance. The most common portfolio insurance strategies on retail and institutional markets amongst the array of possibilities are the synthetic OBPI and the CPPI - with a multiplier superior to 1. Along with these strategies, investors and portfolio managers also utilize the naïve insurance strategies such as SLPI or CPPI with a multiplier equal to 1. Table 2.1 presents major strategies that can be developed under the portfolio insurance framework.

Table 2.1: List of major portfolio insurance strategies

<table>
<tr><th colspan="6">Portfolio Insurance Strategies</th></tr>
<tr><th colspan="2">Naïve strategies</th><th colspan="4">Complex strategies</th></tr>
<tr><td rowspan="2">SLPI</td><td rowspan="2">CPPI
$0 < m \leq 1$</td><td rowspan="2">CPPI
$m > 1$</td><td>TIPP</td><td colspan="2">OBPI</td></tr>
<tr><td>Dynamic Floor</td><td>Listed Options</td><td>Synthetic Options</td></tr>
</table>

2.2.1 Stop Loss Portfolio Insurance

In a SLPI strategy, the investor sets the equivalent to a stop loss order, which is a conditional instruction to sell the risky underlying if it's value falls below a given level. In this case, an investor, who allocates the total initial wealth (V_0) in the underlying risky asset, cuts the loss to a predetermined level, should the market fall, and, simultaneously, allows gains in an upward market. A major problem with this strategy is the path dependency (Rubinstein, 1985): when market falls below the predefined floor portfolio, stock is sold and converted into cash/bonds, which are held until maturity. Unless the investor decides to re-enter the risky underlying market there is independence on future market movements until maturity. The investor position in the risky underlying is held as long as the present value (PV) of the floor (F_T), which is the minimum acceptable wealth value at maturity defined by investor, is smaller than current wealth (portfolio value at time t), i.e. the investor will be 100% invested in the risky underlying as long as $V_t > PV_t(F_T)$ and it will be 100% invested in the risk free asset otherwise. Thus the exposure to risky underlying (ES_t):

$$ES_t = \begin{cases} W_t, & \text{if } V_t > F_t \\ 0, & \text{if } V_t \leq F_t, \end{cases} \tag{2.1}$$

where the value of the floor at maturity $F_T = K$. During the investment period at any time t, $F_t = PV_t(F_T) = Ke^{-r(T-t)}$, r is the risk-free rate and $(T-t)$ is the time to maturity.

If the market value of the risky underlying falls below the discounted floor (F_t), portfolio is sold and converted into the risk-free asset (EB_t), and held until maturity.

Therefore, at maturity, if we consider the final wealth of the investor as the value of a stop loss portfolio (V^{SLPI}), we have the following:

$$V_T^{SLPI} = ES_T + EB_T \begin{cases} ES_T = 0 \wedge EB_T = ES_t e^{r(T-t)}, & \text{if } V_t < F_t, \\ ES_T = W_T \wedge EB_T = 0, & \text{if } V_t \geq F_t. \end{cases} \tag{2.2}$$

2.2.2 Option-Based Portfolio Insurance

A simple OBPI strategy consists of an investment in a risky asset (usually a financial index) plus an option written on that asset - i.e. a contract that gives the holder the right to sell a certain quantity of the underlying asset to the writer of the option at a specified price, up to a specified date. This strategy permits the investor to set a floor equal or below the value of the portfolio should the value of the risky asset fall.

Figure 2.2: Example of a OBPI profit at expiration - Pain and Rand (2008)

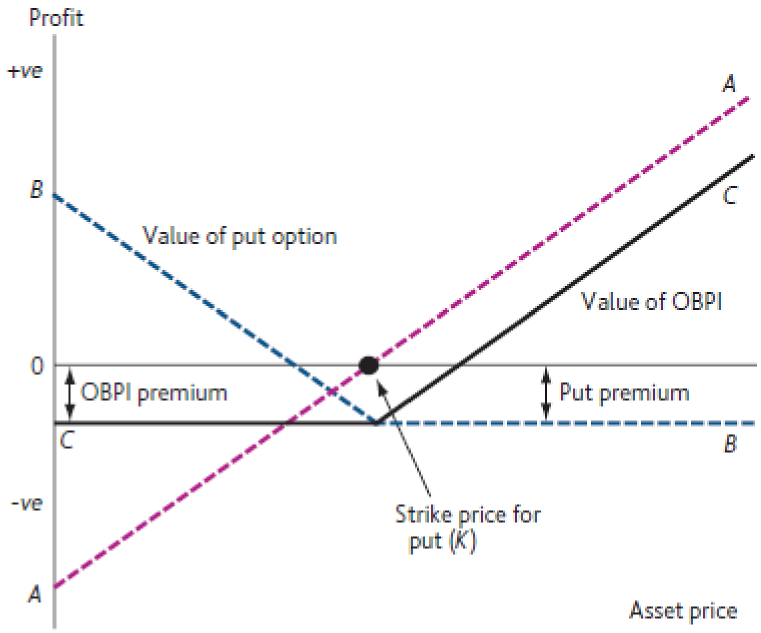


Figure 2.2 describes the net pay-off profile for an investor in an OBPI position at the maturity. The dashed magenta line (AA) shows the pay-off to the investor, at different levels of the price of the underlying asset, from simply owning that asset. If the value of the risky asset is below the cost of purchase (K) the investor would be facing a loss. The dashed blue line (BB) shows the net pay-off from simply owning a put option on the underlying asset with the strike price for the option set at the initial capital investment, K. If at expiration of the option the value of the risky asset is below the strike price, the investor can profit by buying the asset in the open market and selling it to the writer of the option (i.e., a financial institution) at the agreed price (less the premium paid for the option itself). In contrast, if the price of the asset is above the strike price at expiration, the investor does not exercise the put option and it expires

with no value. As with all derivatives, an option transaction is a zero-sum game - for every person who gains on a contract, there is a counter-party that loses. Thus in this case, if the put option expires with positive value the investor gains but the writer of the option (i.e., the counter-party of the contract) loses. By combining the two investments (the underlying risky asset and a put option) in a single strategy, the OBPI enables the investor to obtain the pay-off line CC which limits the potential downside risk - the pay-off on the option offsets any loss on holding the risky asset, thereby providing the capital protection. In principle, the pay-off from an OBPI is identical to the pay-off from a call option on the underlying asset (a contract that gives the holder the right to buy the underlying asset at a specified price) and investing the remnant of the funds in a risk-free asset such as a government security. The maximum loss for the investor is the cost of the premium for the OBPI.
Due to this similarity, it is normal to create an analogy with traditional types of insurance. The investor seeks an assured value for his investment in return for paying a premium for the option while the option writer hopes to make profits from these deals by charging premiums (typically across a range of options that he may have sold) that compensate for the risk taken.

Some investors seeking portfolio insurance, for example retail investors, may not have direct access to options markets. And for some asset classes, an options market may not exist at all. However, in theory at least, it is possible to achieve the pay-off on an option without using options directly. Using the insights of Black-Scholes (1973), Leland and Rubinstein (1981) showed that it was possible to replicate the pay-off of an option by creating a dynamic portfolio of the underlying asset and a risk-free asset. By adjusting the holding of the underlying asset in response to changes in the underlying asset price over time (dynamic hedging), the returns to the portfolio replicate those of a call option.

Investors, pursuing an OBPI strategy, hold a portfolio of a risky asset, and an at-the-money option on that asset. The investor defines a floor on the value of the portfolio, and protects wealth if the risky asset market price falls. The proportions of the risk-free and risky assets depend on the price of at-the-money options at the time of initial investment. OBPI is, usually, a static approach if it happens that the option can be bought, but in practice the option often needs to be replicated using a dynamic, discretely monitored investment strategy. Additionally, as perfect hedging is often rare, due to the possible correlation of the market index between the strategy's risky underlying and the used option, there may be tracking problems (Lee et al., 2013), which lead asset managers to avoid options. A solution to the tracking problem is using dynamic rebalancing (synthetic option) to hedge the portfolio of risk-free and risky assets against downside risk, according to the delta of an option. Therefore, synthetic option, is used as an alternative in the OBPI (Rubinstein and Leland, 1981).

In fact, a static OBPI strategy can be implemented either using put or call options (Leland, 1980). With the put option, investor holds the underlying risky portfolio and buys a put option with striking price equal to floor. When an investor insures the portfolio with call options, there is a call on the underlying risky asset with striking price equal to the floor and holds risk free (cash/bonds) asset discounted by the risk-free interest rate until maturity.
As in Costa and Gaspar (2014), an OBPI using call option has the following pay-off:

$$V_t^{OBPI} = ES_t^{OBPI} + EB_t^{OBPI}, \tag{2.3}$$

where:

$$\begin{cases} ES_t^{OBPI} = qS_tN(d_1), \\ EB_t^{OBPI} = qKe^{-r(T-t)}N(-d_2) + EB_{t=0}^{OBPI}, \end{cases}$$

and C_0 is the price of a call at the money in inception, S_t is the price of the underlying risky asset, K is the strike price (floor), the number of call options is given by $q = \frac{V_{t=0}^{OBPI} - Ke^{-rT}}{C_0}$, and the initial amount in risk free asset is $EB_{t=0}^{OBPI} = (1-q)Ke^{-rT}$. The factor $N(x)$ is the cumulative probability distribution function for a standardized normal distribution, and d_1 and d_2, which are defined by:

$$d_1 = \frac{(ln(\frac{S}{K}) + (\frac{1}{2}r\sigma^2)T)}{(\sigma\sqrt{T})}, \tag{2.4}$$

$$d_2 = d_1 - \sigma\sqrt{T}, \tag{2.5}$$

where σ is the standard deviation of the underlying risky asset returns.

As portfolio insurance strategies must be self-financed, the option prices are incorporated into portfolio weights to accommodate the insurance price.

2.2.3 Constant Proportion Portfolio Insurance

The basic structure of a CPPI is a portfolio that switches the investment mix dynamically between a risk-free and a risky asset according to a discrete trading rule. Each period the investor calculates how much needs to be invested in the risk-free asset in order to guarantee a given percentage of the initial investment - this is known as the cost of the guarantee or the 'floor' - as well as the value of the portfolio in excess of that floor (the 'cushion' or 'reserve'). A constant 'multiple' is then applied to the cushion to determine the amount to be invested in the risky asset in each period. The multiple is typically chosen to reflect the expected performance of the risky asset as well as the risk preferences of the investor. The multiple determines the potential leverage of the investment. A multiple of one implies no leverage; a multiple of zero is equivalent to a purely risk-free investment.

Table 2.2 provides an illustrative worked example of a simple CPPI strategy for a 100 monetary units investment over ten years where the price of the underlying asset is assumed to first rise and then fall over the investment period. At time zero, the guarantee of 100% of principal costs 78.1 (the present value of 100 received in ten years' time at a risk-free rate of 2.5%) so the initial cushion is 21.9 (100 - 78.1). With a multiple of 3 this implies an investment in the risky asset of 65.6 and 34.4 in the risk-free asset. Over time, if the growth in the value of the risky asset exceeds the risk-free rate of interest, the cushion will rise and more of the portfolio should be switched into the risky and away from the risk-free asset. In the second panel a a multiplier of $m = 5$ is considered instead. By period 1 the CPPI strategy involves a negative position in the risk-free asset (i.e., borrowing funds to invest more in the risky asset than the value of the portfolio). As the risky asset price falls, more of the portfolio is reallocated away from the risky asset. In the second half of the investment period, the portfolio is

Table 2.2: Example of a CPPI evolution - adapted from Pain and Rand (2008)

CPPI - Example of strategy evolution of a 100 monetary units (m = 3)

Period	Cost of guarantee	Risky asset price	Cushion	Risky asset exposure	Risk-free asset exposure	Portfolio value
	A	B	C = F - A	D = C x multiple	E = F - D	F = E_{t-1} (A_t/A_{t-1}) + D_{t-1} (B_t/B_{t-1})
0	78.12	100.00	21.88	65.64	34.36	100.00
1	80.07	110.00	27.35	82.05	25.37	107.42
2	82.07	130.00	40.90	122.70	0.27	122.98
3	84.13	110.00	19.98	59.94	44.17	104.11
4	86.23	102.00	14.62	43.86	56.99	100.85
5	88.39	100.00	13.03	39.09	62.33	101.42
6	90.60	95.00	10.42	31.27	69.75	101.02
7	92.86	90.00	8.26	24.77	76.35	101.12
8	95.18	80.00	5.09	15.27	85.00	100.27
9	97.56	85.00	5.79	17.38	85.98	103.35
10	100.00	85.00	5.50	16.51	89.00	105.50
Multiple	3					
risk free rate	2.50%					

CPPI - Example of strategy evolution of a 100 monetary units (m = 5)

Period	Cost of guarantee	Risky asset price	Cushion	Risky asset exposure	Risk-free asset exposure	Portfolio value
	A	B	C = F - A	D = C x multiple	E = F - D	F = E_{t-1} (A_t/A_{t-1}) + D_{t-1} (B_t/B_{t-1})
0.00	78.12	100.00	21.88	109.40	-9.40	100.00
1.00	80.07	110.00	30.63	153.16	-42.46	110.71
2.00	82.07	130.00	55.42	277.08	-139.59	137.49
3.00	84.13	110.00	7.25	36.23	55.14	91.37
4.00	86.23	102.00	3.89	19.43	70.68	90.12
5.00	88.39	100.00	3.12	15.59	75.92	91.50
6.00	90.60	95.00	2.03	10.13	82.49	92.62
7.00	92.86	90.00	1.29	6.45	87.70	94.15
8.00	95.18	80.00	0.44	2.22	93.40	95.63
9.00	97.56	85.00	0.54	2.69	95.41	98.10
10.00	100.00	85.00	0.49	2.43	98.06	100.49
Multiple	5					
risk free rate	2.50%					

Note: Results in table are rounded, thus values may present differences from calculations.

switched mostly into the risk-free asset. If developments in the risky asset require that the CPPI portfolio is entirely reallocated to the risk-free asset we have a situation known as "cash-locked".

CPPI strategy was first presented by Black and Jones (1987). It is a portfolio of risky and risk-free assets, where holdings are dynamically rebalanced according to a discrete trading rule in order to achieve at maturity a minimum amount of the initial portfolio. In each period, the investor sets the weight on risk-free asset to guarantee the predefined minimum amount—the floor ($F_T = K_T$). The difference between this discounted floor and the initial portfolio value ($V_{t=0}$) is called the cushion (Cu). The amount invested in the risky asset is defined by applying a factor—multiple (m)—to the cushion. The result is the exposure ($ES_t^{CPPI} = m \times Cu_t$). In this strategy, the investor must decide both the floor and the multiplier.

The outcome of the CPPI strategy is the following:

$$V_t^{CPPI} = ES_t^{CPPI} + EB_t^{CPPI}, \tag{2.6}$$

where:

$$\begin{cases} ES_t^{CPPI} = m \times Cu_t, \\ EB_t^{CPPI} = V_t^{CPPI} - ES_t^{CPPI}, \end{cases}$$

and the floor at any time $t \in [0; T]$ is $K_t = K_T e^{-r(T-t)}$, the cushion is calculated at rebalancing moments $Cu_t = V_t^{CPPI} - K_t$ and m remains fixed until maturity. A simple approach to CPPI determines daily adjustments, between the fraction of risky and risk-free assets, in order to assure the floor. When $m > 1$, there is a a potential leverage of the investment.

If the multiplier is set to $m = 1$ it is equivalent to put aside the present value of the floor in the risk free asset and investing the remaining (i.e., the cushion) in the risky underlying, we have what is sometimes called a naïve strategy (see Costa and Gaspar, 2014).

2.2.4 Time Invariant Portfolio Protection

The TIPP is a variant of CPPI proposed by Estep and Kritzman (1988), and incorporates a dynamic absolute floor, which is ratchet up whenever the portfolio value increases. In this way, the investor not only protects a percentage of the initial wealth, but can incorporate intra-period gains into the protection floor.

$$V_t^{TIPP} = ES_t^{TIPP} + EB_t^{TIPP}, \tag{2.7}$$

where:

$$\begin{cases} ES_t^{TIPP} = m \times Cu_t, \\ EB_t^{TIPP} = V_t^{TIPP} - ES_t^{TIPP}, \end{cases}$$

and the floor at any time $t[0;T]$ is:

$$K_t^{TIPP} = \begin{cases} V_t^{TIPP} e^{-r(T-t)}, & \text{if } V_t^{TIPP} > K_t, \\ K_T e^{-r(T-t)}, & \text{if} V_t^{TIPP} \leq K_t, \end{cases}$$

The cushion is calculated at rebalancing moments $Cu_t = V_t^{TIPP} - K_t^{TIPP}$ and m remains fixed until maturity. Due to this mechanism, at the end of period (T), the TIPP strategy is expected to have a greater percentage of risk-free assets than the CPPI.

2.3 Literature Review

There have been extensive theoretical work on the optimality of portfolio insurance strategies, as well as empirical studies comparing different portfolio strategies against alternative basic investment strategies (i.e., Buy-and-Hold (B&H), mix B&H and risk-free allocation). There are also comparative studies using simulation methods that focus on the dominance of portfolio insurance strategies in specific market conditions.

The first studies on the evaluation of the performance of CPPI and synthetic OBPI were made by Zhu and Kavee (1988). They find that both strategies capture partial upside, and protect on the downside risk. However, this market protection has two costs: explicit (transactions costs), and implicit (returns that are forgone when using the protection). Perold and Sharp (1988) simulate the performance of four dynamic asset allocation strategies in bull, bear, trendless and volatile markets—namely, the B&H, constant mix, CPPI, and OBPI; they find that there is no dominance of a particular strategy in all situations.

Cesari and Cremonini (2003) use measures for risk, returns, and risk adjusted performance to compare portfolio insurance strategies. This study compares plain portfolio strategies (B&H,

constant mix,) and portfolio insurance strategies (CPPI, OBPI, and SLPI). Some features are included in the strategies and the comparison is made between several alternatives. Cesari and Cremonini (2003) find that there are no dominant strategies in any of the simulated market conditions.

In Ho et al. (2013), several portfolio strategies are tested, and the authors find that depending on the assessment perspective, different strategies are dominant in specific market conditions. The performances are evaluated from six perspectives: in terms of the Sharpe ratio and the volatility of portfolio returns, the CPPI is the best performer; whereas the VaR based upon the normal distribution is the worst; concerning the average and the cumulative portfolio returns across years, the Expected Shortfall (ES)-based strategy, using the historical distribution, ranks first; moreover, the ES-based strategy results in a lower turnover within the investment horizon, thereby saving transaction costs [2].

Anaert et al. (2009) evaluate the performance of the stop-loss, synthetic OBPI, and constant proportion portfolio insurance techniques, based on a block-bootstrap simulation; they consider traditional performance measures, along with some measures that capture the non-normality of the return distribution (value-at-risk, expected shortfall, and the Omega measure). Anaert et al. (2009) compare them to the more comprehensive stochastic dominance criteria, and find that, even though a B&H strategy generates higher average excess returns, it does not stochastically dominate the portfolio insurance strategies, or vice versa. They indicate that a 100% floor value should be preferred to lower floor values, and that daily-rebalanced synthetic OBPI, and CPPI strategies, dominate their counterparts with less frequent rebalancing.

Zagst and Kraus (2011) also analyse, and compare, two standard portfolio insurance strategies:

[2] The complexity of portfolio insurance strategies has increased and some features related with downside risk were added. The Risk Based Portfolio Insurance used the VaR and Expected Shortfall concepts. The Expected Shortfall is defined as the expected value of the loss of a portfolio in a certain percentage of worst cases within a given holding period. The allocation to the risky asset is adjusted each day so that the expected shortfall of the portfolio does not exceed a target value. The VaR-based portfolio insurance is a strategy that permanently controls the shortfall risk of the portfolio. The allocation to the risky asset is adjusted each day so that the shortfall probability does not exceed a target value. The first authors to analyse these features in portfolio insurance were Zhao and Ziemba (2000)

OBPI and CPPI. Various stochastic dominance criteria, up to third order are considered. Zagst and Kraus (2011) derive parameter conditions implying the second and third order stochastic dominance of the CPPI strategy. In particular, restrictions on the CPPI multiplier resulting from the spread between implied and empirical volatilities are analysed. As they consider risk-aversion in stochastic dominance analysis, it is possible to derive specific conditions for the market parameters, as well as the CPPI multiplier, m, implying the second and third order stochastic dominance of the CPPI strategy.

Almeida and Gaspar (2012) analyse the performance of most common portfolio insurance strategies based on a block-moving bootstrap simulation and find that unleveraged CPPI strategy should be preferred in terms of stochastic dominance. Recently, Costa and Gaspar (2014) compare naïve strategies of portfolio insurance with the OBPI, and CPPI classical strategies, and find that the ones that seem to be the best, in general, are the unleveraged CPPI ($m = 1$), and the SLPI strategy. As these two strategies can be implemented by any investor, the more complex investment strategies—whether based in options, or in CPPI strategies with multipliers used in real life—seem to make little sense, as they lead to worse stochastic performances.

Dichtl et. al. (2017) test for statistical significance of the differences in downside performance risk measures between pairs of portfolio insurance strategies using a bootstrap-based hypothesis test. They find that the classical portfolio insurance strategies (synthetic put and CPPI) provide superior downside protection compared to a simple stop-loss trading rule and that more recently developed strategies, as the TIPP strategy or the dynamic VaR-strategy, do not provide significant improvements over the more traditional portfolio insurance strategies.

Carvalho et al. (2016) also study the design problem of CPPI due to the cash-lock effect that is caused by the path dependency of these strategies.

From the studies referred above there is evidence that, for some investors, in specific market conditions, downside risk can drive the investment decision. In behavioural finance theory there is evidence that investors, contrary to the expected utility axioms, are more sensitive to losses than to gains. As portfolio insurance strategies target a protection on the downside that characteristic may explain the popularity of such strategies in the marketplace. In the field of behavioural finance prospect theory became a relevant framework to explain investors' selection of protection strategies.

Vrecko and Branger (2009) are among the first to consider prospect theory to explain portfolio insurance popularity. Their study analyses the two most popular portfolio insurance strategies, OBPI and CPPI. The analysis is done both for an investor with Constant Relative Risk Aversion (CRRA), and for a Cumulative Prospect Theory (CPT) investor. They find that a CRRA investor does not profit from portfolio insurance, and chooses rather low protection levels. A CPT investor, on the other hand, strongly prefers portfolio insurance to constant proportion strategies. Both loss aversion, and probability weighting of CPT, turn out to be critical to explain the attractiveness of portfolio insurance, as utility gains drop sharply if one of these two elements of CPT is eliminated.

The study of Dichtl and Drobetz (2011) analyses portfolio insurance strategies in a behavioural finance context, and find that most portfolio insurance strategies are the preferred investment strategy for a prospect theory investor.

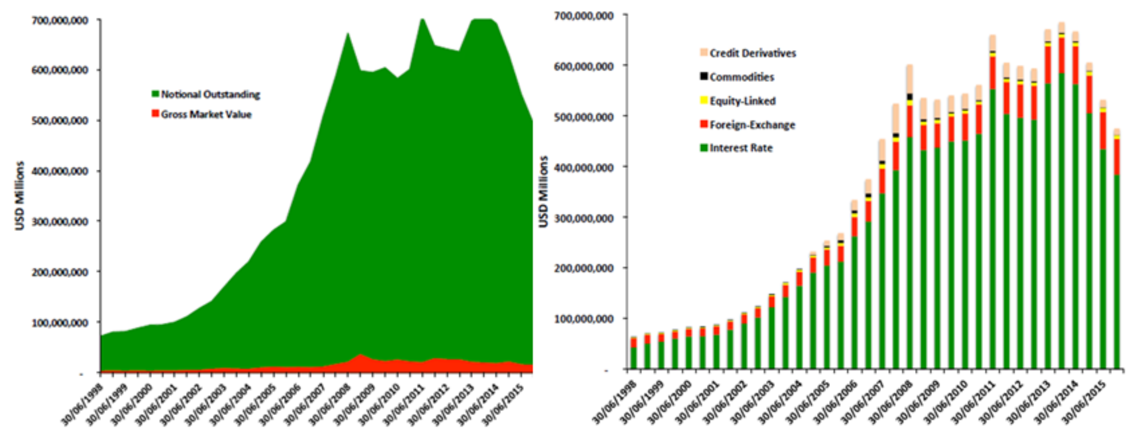
The paper of Tawill (2017) differentiates between investors' risk preferences and their choice of either OBPI or CPPI using partial-moments-based risk-adjusted performance measures. The analysis covers EUT and PT investors and the results show investors' risk preferences in the gain domain are the key determinants of the choice between OBPI and CPPI. The author finds that OBPI is the preferred strategy for expected utility and prospect utility investors who are risk averse in the gain domain and CPPI provides a higher risk-adjusted performance for investors who are risk seeking in gains and risk averse in losses.

Our work on finding the reasons for individual investors to allocate assets to portfolio insurance strategies differs from Dichtl and Drobetz (2011) approach, as we include a different perspective to compare investors' preferences on both traditional and naïve portfolio insurance strategies. The comparative approach between the EUT and CPT thrives to define a reasonable explanation for investors' decision regarding the popularity of portfolio insurance strategies based on a CPT framework.

2.4 Market Developments

From an incumbent market in the late 1970s, portfolio insurance investments gained momentum, and became an important strategy in asset management industry. According to Pain and Rand (2008), it is difficult to precise the size, and rate of growth, of portfolio insurance, since isolated data are not available. Data on portfolio insurance is still very disperse and not categorized in the different strategies. An indicative way of monitoring the amount of assets under the umbrella of portfolio protective strategies is obtained from the statistics of equity linked structured notes that comprises Structured Notes with Principal Protection and some variants of "principal protection", "capital guarantee", "absolute return", "minimum return" or similar terms. The volume of assets depicted in the graphic reveals a consistent evolution:

Figure 2.3: Market estimates on structured products - Source: BIS and Sundaram, Rangarajan K. (2016, Notes on Derivatives, Stern NYU)



Pain and Rain (2008) report that traditional OBPI investments have not been particularly common. In part, this reflects the difficulty in explaining options to investors. But CPPI products

have become much more prevalent and over time have been designed with additional features in hedge funds and funds of hedge funds (i.e., investment funds that consist of a portfolio of other investment funds rather than a direct investment in shares, bonds or other securities). CPPI has also been written on corporate bonds and credit derivatives such as credit default swaps (so-called credit CPPI), property and private equity.
As referred by Pain and Rand (2008), CPPI investments evolved since their inception, and incorporated different features, particularly the constraints on leverage and investment level; the variable floors and multipliers, and, also, the inclusion of caps. The authors reinforce the role of portfolio insurance as a distributor of financial risk among agents willing to absorb it. In fact, issuers of portfolio insurance solutions can be exposed to relevant downside risk (unexpected high losses), enhancing the conditions to more volatile financial markets.
Specially on the US market there have been several issues and alerts to individual investors either by U.S. Securities and Exchange Commission and Financial Industry Regulatory Authority related with the complexity of the protective strategies and the risks involved [3].

Popular features in CPPI investments have evolved to incorporate various different features. Of particular note are the following:

- Constraints on the investment level. In the event that the underlying asset price falls, the allocation to the risky asset can potentially fall to zero. Once that happens there is no chance for the strategy to recover. To counter this, some products have been developed to incorporate a minimum level of investment in the risky asset. Equally, to avoid unbounded investment in the risky asset as its price rises, a maximum investment level is sometimes imposed.
- Constraints on leverage. Exposure to the risky asset of more than the initial available funds can be achieved by allowing borrowing. But often there will be limits on how much can be borrowed, depending on collateral or margin limits. In relation to variable and 'straight-line' floors, when the price of the underlying asset increases, any gains made by the CPPI strategy can still be lost if prices subsequently fall. To address this, products with so-called 'ratcheting' are available which allow the investor to lock-in gains made

[3]For a comprehensive listing of investors alerts please see https://www.sec.gov/investor/alerts and http://www.finra.org/investors/alerts.

from upward movements in the risky asset price. More specifically, the floor is increased if the cushion exceeds some agreed threshold, with the trigger typically set as a percentage of the highest portfolio value or as a percentage of any gains achieved. The floor in a conventional CPPI is sensitive to the level of interest rates (since it affects the present value of the pay-off on the risk-free asset). As interest rates fall, the floor would rise and the investment switches away from the risky asset.This in turn would limit the potential upside from the CPPI, which could be significant (if interest rates and the risky asset are negatively correlated for example).

- Time constraints. The floor can be allowed to vary linearly with time, a feature sometimes known as a 'straight-line' floor. Concerning variable multiples, rather than having a fixed multiple, some product structures allow for the multiple to vary over time in relation to the volatility of the risky asset and reflecting investors' appetite for risk. This is sometimes referred to as dynamic portfolio insurance (DPI). There is often a maximum level for the multiple, which is often based on the results of stress tests performed on the risky asset.
- Volatility caps. Some CPPI products include mechanisms that allow the percentage exposure to the risky asset to be reduced if its realised volatility exceeds a certain level.

Although portfolio insurance has been around for some time, it has experienced something of a re-emergence over the past few years. This appears to stem from lower structuring and trading costs and a broadening in, and growth of, asset classes on which investors find the idea of principal protection attractive. Many of the developments in principal-protected products are common to structured products more generally.

In the next chapter we go through major crisis associated with portfolio insurance and highlight some of the recent resurgence signs of capital protection solutions in the market. In spite of the criticism and vicious cycle that are attributable to portfolio insurance strategies, there are individual investors allocating resources into this structured products and banks and assets managers still offer these solutions to their clients. Although nowadays there is awareness of the risks involved in the mismatch between investors risk profile and selling procedures by the sell-side, the past events in some crisis have confirmed feedback loop effects that can be exacerbated in depressed markets (Leland, 2011).

Chapter 3

The Impact of Portfolio Insurance on Market Stability

An Overview

Portfolio insurance are dynamic hedging strategies with special characteristics that led, amongst other factors, to increase downward pressure on prices during bear markets (Grossman and Zhou, 1996). For that reason they are thought to have contributed to relevant financial crisis (Schiller, 1988, Leland, 2011). In fact, in the last decades, several crisis on financial markets occurred with impacts on investors' wealth. The severity of those crises enhanced uncertainty as a determinant factor on investors' decision process. Financial innovation made the asset management industry a laboratory of new techniques, either for hedging, or for leveraging positions on a variety of investors. The several types of investors demanded different solutions, so they could achieve their goals. A response for investors, who prefer protection on the downside, and simultaneously take advantage from the market upside, was developed based on the work of Black, Jones and Merton on early 1970s. In this chapter we review the impact of portfolio insurance strategies on asset price and individual investor perspective, specially under a decision making process for non sophisticated individuals. There has been recent protection environment for individual investors either from a stronger scrutiny for asset management and distribution channels of investment products or from strict requirements imposed to selling side on the investment products and adequacy of investors (e.g. The European Markets in Financial Instruments Directive - MiFID), but the evolution on computer based trading and the high frequency trading and algorithm trading can impact the market quality. Portfolio insurance strategies have a distinctive characteristic: they tend to sell assets after market prices have declined, and to buy after market prices rise. In equities and hedge funds this method has been used extensively, and it has been applied also to the credit market, since 2006. Due to such particular characteristic of portfolio insurance strategies, the 1987 and 2007 crises emerged as real case scenarios for the analysis of the impact caused by portfolio insurance strategies, or algorithm trading, which were developed by asset managers. Generally, it has been accepted by official institutions (Brady Report, 1988) that massive selling orders on a depressed market contributed to the increased volatility and losses on investment portfolios. Both the industry and academic literature have also pointed out portfolio insurance strategies as a major driver for the increased volatility in financial markets, mainly on the crash of 1987. The "illusion of liquidity" that is kept under stress market conditions was especially relevant on the crash of October 1987, but also on the extreme conditions of the 2007 financial crisis (Carlson, 2006).

In portfolio protection strategies the basic design is to capture the upside markets, and protect portfolios on the event of downside market movements. Some portfolio managers define minimum rate of return, or a guarantee (a floor) on the initial investment, and thus a permanent and dynamic re-balancing between risky assets and low risk asset classes (bonds and cash) encapsulate a very simple strategy, which is against common sense: buy on the upside and sell on the downside. This biased view may be supported by the fact that buyers of portfolio insurance are investors with very low tolerance to downside risk. The 2007 credit crisis became an interesting area for discussion between those who think portfolio insurance strategies were to be blame again, and those that point different reasons for the crisis. In fact, the evolution on the portfolio insurance strategies, which extended the dynamic hedging techniques to credit portfolios (i.e., constant portfolio debt obligations - CPDO), was referred as a relevant factor for the credit crisis (Pain and Rand, 2008). This "new generation" of portfolio insurance - including credit Constant Proportion Portfolio Insurance (CPPI) and CPDO - benefited from the ratings assigned from major agencies, and thus some opinions tend to highlight the role of portfolio insurers on the depressed credit market, due to the effects of the complex credit structures used by these dynamic hedging techniques.

The management of portfolio insurance strategies based on the concepts and mechanisms, highlighted in previous chapter, relies on models which are always a simplification of reality. The more complexity to be included into the model and the level of sophistication for estimations, the more accuracy is reached in order to adjust the results of insured portfolios with investors expectations (Rubinstein, 1999). However, the different market players - insures, hedgers and speculators - and interdependencies between markets (cash and futures) in moments of instability along with incomplete information creates imbalances that may impact portfolio insurance strategies and, simultaneously, enhances feedback loops with effects on liquidity. All these effects are stressed with computer based trading and, under specific circumstances, vicious cycle of forced selling conducts to violations on portfolio insurance floors, which, in turn push more selling orders into the market.

3.1 An Integrated Approach On Computer Based Trading

Almost native to the design of portfolio insurance is the Computer-Based Trading (CBT) that relies on High Frequency Trading and Algorithm Trading. To address this ecosystem, the Government Office for Science in the United Kingdom ended in 2012 the Foresight Project on Computer Trading in Financial Markets to assess the impact of CBT on market quality: liquidity, price efficiency/discovery and transaction costs. A relevant message from this study is that "despite commonly held negative perceptions, the available evidence indicates that high frequency trading (HFT) and algorithmic trading (AT) may have several beneficial effects on markets. However, HFT/AT may cause instabilities in financial markets in specific circumstances".

Although the effect of CBT on market quality is controversial, the studies developed in this Project suggest that CBT has several beneficial effects on markets:

- Improvement on liquidity, as measured by bid-ask spreads and other metrics;
- Decrease on transaction costs for both retail and institutional traders, due to changes in trading market structure, which are related closely to the development of HFT in particular;

- Increased efficiency on market prices, consistent with the hypothesis that CBT links markets and thereby facilitates price discovery.

In spite of the improvements, there are concerns relating to market quality which are relevant. In particular, periodic iliquidity has a greater potential. The nature of market making has changed, with high frequency traders now providing the bulk of such activity in both futures and equities. However, high frequency traders typically operate with little capital and have no obligations to provide liquidity during periods of market stress. These factors, together with the ultra-fast speed of trading, create the potential for periodic illiquidity. One of the recent events from this market characteristics is the US Flash Crash. The Foresight Project also refers that there is no direct evidence that HFT has increased volatility in financial markets. But under specific circumstances, the authors point that CBT can lead to significant instability. In fact, self-reinforcing feedback loops, as well as a variety of informational features inherent in computer-based markets, can amplify internal risks and lead to undesired interactions and outcomes. It is highlighted that this can happen even in the presence of well-intentioned management and control processes. There are three mechanisms that within CBT may enhance instability:

- Non-linear sensitivities to change, where small changes can have very large effects, not least through feedback loops;
- Incomplete information in CBT environments where some agents in the market have more, or more accurate, knowledge than others and where few events are common knowledge;
- Internal "endogenous" risks based on feedback loops within the system.

Although the feedback loops can be worsened by incomplete information and a lack of common knowledge, a behaviour factor designated by "normalisation of deviance", where unexpected and risky events come to be seen as increasingly normal, until a disastrous failure occurs, is a relevant contributor for the negative effects of feedback loops.

The analysis on the market impacts has come a long way since the Brady Report (1988). Several studies addressed the price and volatility effect under equilibrium models. The conclusions are contrary and based on a diversity of assumptions. Basak (1995) states that at a general equilibrium level the comparison between economies with no portfolio insurance investors and economies where these investors are active indicates a decrease on market volatility and risk premium. The findings of Basak oppose those from Brennan and Schwartz (1989) and Grossman and Zhou (1996), but follow the views of Donaldson and Uhlig (1993). The major difference between Brennan and Schwartz (1989) and Grossman and Zhou (1996) is the type of maximization technique as in the former paper insurers are managing risk in an automaton way. The insurers' trading is based on the rules defined for the risk exposure and floor affecting the portfolio strategy instead of maximizing a utility function. The difference between Basak (1995) and Grossman and Zhou (1996) is the timeframe of the insurers' actions as their horizon ends before maturity, thus allowing the security price model as a diffusion process relative to a Brownian movement to include jumps before the payoffs. This assumption creates a price jump at the insurance maturity, anticipating a price discontinuity that is required for equilibrium. This technique although enhancing the equilibrium seems to be difficult to incorporate in an economy with anticipation of price jumps, as referred by Grossman and Zhou (1996).

This component of volatility was also a particular area of the aforementioned Foresight Project: "Leverage, Forced Asset Sales, and Market Stability: Lessons from Past Market Crises and the Flash Crash". It includes a clear description on potential factors to explain the crisis, and one of the contributors was Leland (2011), who is one of the designers of portfolio insurance strategies.

Leland (2011) refers that "While most market participants base their trading on their view of asset fundamentals relative to price, an important subset of investors must sell even if they believe market fundamentals don't warrant selling. Such forced selling may be idiosyncratic, i.e. uncorrelated across market participants. But in other cases, forced selling may be related to general declines in asset prices and thus correlated across investors. This type of forced selling can lead to price declines which in turn force further selling and further price declines, a positive feedback situation that can lead to extreme market volatility and crashes.". The elements towards this view are based on the Brady Report (1988), a previous work by Gennotte and Leland (1990) and a post-mortem analysis of the Crash of 20th October 1987 by Carlson (2006).

Portfolio insurance grew rapidly between 1982 and 1987. Estimates suggested \$70-\$100 billion in funds were following formal portfolio insurance programs by mid-1987 (Gennotte and Leland, 1990), but there were also other protection strategies such as stop loss orders. The Brady Report (1988) discusses the causes of the crash of October 19th, 1987, when the market fell more than 20% in one day. There were no significant news of events of sufficient importance to explain the magnitude of the price fall. The Brady Report therefore focused on internal market causes rather than external events. In particular, the Brady Report centred attention on a number of large institutions following "price insensitive strategies" such as portfolio insurance. It describes enormous waves of portfolio insurance selling forcing down the equity prices. This led to a "vicious circle", as the selling enhanced further price declines which in turn led to additional portfolio insurance selling. Thus portfolio insurance selling played a role in 1987 that was similar to forced margin sales in 1929. While forced margin selling was not initially highlighted in 1987, there were margin calls in both futures and options markets, where margins were as low as 8% of underlying values. This view is stressed by Carlson (2006): "Failure of retail investors to meet margin calls spurred liquidations in options markets. Brokers placed emergency margin calls to their retail investors with exposed options positions. In the absence of additional margin, these positions were supposed to be liquidated. The Brady Report indicates that this happened frequently and these liquidations likely added to the selling pressure in financial markets." Some observers have noted that portfolio insurers were just one group of sellers amongst many on October 19th, 1987. They did about 15% of the total volume of trading on that day. However, these trades were all in one direction, and trading was at a record level that day. Carlson (2006) refers to three factors, amongst several, explaining the severity of the crash: margin calls, program trading and misinformation. He notes that different investors with incomplete information operating in cash and futures markets, sometimes simultaneously, led to relevant imbalances that eventually were settled. Portfolio insurance trading was an early example of Algorithmic Trading (AT). The trading strategy was dictated by computer algorithms (determining Black-Scholes or related hedges and defining necessary sales/purchases) and implemented primarily in the S&P 500 futures markets. The liquidity of these markets had increased dramatically in the preceding five years of the crisis.

The major conclusions of the 2011 study by Leland for the Project on The Future of Computer

Trading in Financial Markets on the section about recent crisis are the following:

"

1. The speed with which insurers, hedgers, and other forced sellers needed to trade was far greater than the speed at which natural counter-parties could place orders. Market makers were overwhelmed by the imbalance of orders from such sellers during the crises.

2. The rapid fall in prices resulting from forced selling and order imbalances was mistaken by many investors for some terrible but unobserved event affecting fundamentals. Thus, the market did not quickly recover. This is in contrast with theories based solely on limited liquidity of market makers, in which case (with unchanged fundamentals) the decline in values should be temporary.

3. Theory (and common sense) predict that the forced selling by portfolio insurers and others would have had much less price impact if it were known to be forced (rather than informed) selling.

4. Market makers who normally would have provided liquidity saw their equity rapidly disappear. They withdrew from the market, and there were no financial entities prepared to take their place on short notice.

"

Previous to this study, in 1999, Mark Rubinstein wrote the book "Rubinstein on Derivatives" and some comments on the risks of implementing portfolio insurance were discussed on the DerivativesStrategy.com, on September 1999: The Real-World Pitfalls of Portfolio Insurance. Rubinstein notes the attractiveness of portfolio insurance for investors who feel strongly, relative to other investors, that cannot tolerate losses at all — but nonetheless would like to invest in risky assets because of their high expected returns. This topic was addressed before by Rubinstein (1988) in relation to the 1987 crisis. The insured portfolio by design should perform better than the underlying portfolio on the downside, in this way it must perform worse on the upside. The degree of attractiveness is managed by the shortfall relative to the performance of the underlying portfolio on the upside.

In reality, the insured portfolio is not typically literally insured (guaranteed by a second party). Although portfolio insurance can sometimes be implemented with listed options, taking into account the costs and the gap risks, most portfolio insurance are implemented by dynamic hedging using only the underlying portfolio (or, more commonly, a highly correlated portfolio of futures) and cash. Because the common way to determine exactly how much to be selling or buying over time is to use a modified version of the Black-Scholes formula and its derived delta, there are problems with portfolio insurance implementations that can arise which are not anticipated by the Black-Scholes formula. If Treasury bills are used as the proxy for cash, their interest rate is known with certainty over their life. However, the rate of change of the T-bill price over each day is still uncertain. For the most part, this can be handled by using as cash a zero-coupon bond maturing near the payoff date of the strategy. In addition, the upside capture (and delta) can be recalculated each day based on the remaining riskless return through the payoff date. With this feature the floor is preserved even in the presence of uncertain future spot returns. Due to the need of increasing accuracy on more precise portfolio insurance models, some strategies require a generalization of the Black-Scholes formula that allows for uncertain future spot returns. However a major issue is the volatility of the underlying portfolio that is unknown. There are several techniques to reduce the uncertainty by using the realized volatility after periods of low or high volatility and thus predict the upside capture. The estimates are done on a daily basis to preserve the floor of the insured portfolio. Then, the rebalancing calculation should be performed joining the realized volatility and the estimation, but with a decreasing weight on the estimation as maturity decreases. If there is a high degree of confidence of how the volatility is about to evolve, some specific models of option pricing can be used, instead of the Black-Scholes formula.

Rubinstein (1999) addresses all these problems and notes that "can be reduced by combining static positions in options with dynamic replication. Indeed, if the replicating strategy could be achieved using only buy-and-hold positions in European exchange-traded options (static replication), then uncertain riskless returns and uncertain volatility would not be a problem at all. Jumps in underlying portfolio prices would also not pose a problem. In fact, any dependence of the results on the special assumptions behind the Black-Scholes formula would be removed." However there are still major issues on monitoring and rebalancing the insured portfolio. One unexpected factor is price jumps that can derail the insured portfolio and violate the floor. But for rebalancing when static hedging is not enough, transaction costs must be factor in into the rebalancing calculations. Rubinstein (1999) refers that these pitfalls can be managed and thus improve the results of portfolio insurance strategies.

Although the first designers of portfolio insurance are aware of the pitfalls of it, sophistication on the protective solutions on financial markets is a mean to satisfy the needs of loss aversion investors. The development of these solutions is based on risk management techniques in order to deliver what is agreed between investors and asset managers. Estimations are not flawless but are the backbone of dynamic hedging.

Because models are always a simplification of reality, moments of instability along with potential incomplete information create imbalances that may impact portfolio insurance strategies and, simultaneously, enhance feedback loops with effects on liquidity. All these effects are stressed with computer based trading and, under specific circumstances, vicious cycle of forced selling enhances violations on portfolio insurance floors. General equilibrium models are relevant to assess the price effects and the volatility impacts, but microstructure models are determinant to assess information and liquidity that feeds into feedback loops.

The resurge of portfolio insurance strategies incorporated into products that are distributed to individual investors is a way to satisfy the needs of investors that are loss averse but, simultaneously, tend to be attracted to the up scale of risky markets as the equity asset class. Individual investors are protected with by-laws of transparency on distribution rules, transparency and information on product characteristics, with rules on selling procedures about investments, definition of scope for investment consultants, but are not protected against crisis that can be exacerbated by the investments they hold.

The risks that were not mitigated in last financial crisis persist. Although there are new approaches to portfolio insurance, and even more sophisticated trading algorithms and robots, there is a wrong sense of availability of data that can be masked as complete information instead of a misinterpretation of data or information asymmetry. Individual investors should be aware of it, because regulators and asset managers as well as institutional investors are all part of a system that always reach moments of equilibrium even after major imbalances. Portfolio insurance strategies aggregate all of these entities within the same system, however is just a component and although it may accelerate imbalances should not be viewed as their sole cause.

3.2 The Industry Perspective

Portfolio insurance strategies represent a standard practitioners "buy high and sell low" rule for the risky asset. When the underlying asset markets move in one particular direction, either trending up or down, these actions may enhance feedback effects in markets that amplify price movements. Before markets that are deep and liquid, these feedbacks effects are expected to be limited. However, if the underlying asset markets do not have these characteristics, the feedback effects may reinforce market prices movements. Additionally, in stress situations the various loss preventing strategies that may present low correlations became more correlated with the underlying assets when all strategies are set to reduce risky exposure in the same time-frame. This was the situation described about the 1987 stock market crash in the Brady Report (1988) .

A major risk of implementing a portfolio insurance strategy is that the payoffs are particularly sensitive to rapid losses in the risky asset prices before the portfolio can be rebalanced. In these situations, the value of the insured portfolio could fall below the floor and, for example, the issuers of CPPI products would suffer a gap risk. In particular, if the issuers send out several CPPI products written on different underlying assets, and those seemingly uncorrelated assets suddenly become much more correlated in stressed conditions (such as the credit crunch led by the subprime mortgage crisis since September 2007), then the scale of the gap risk may be very much underestimated.

Pain and Rain (2008) state that "overall, it seems unlikely that portfolio insurance-related investments contributed significantly to the latest bout of financial market volatility that began in Summer 2007. And, in all but a handful of cases, market contacts observe that the gap risk in CPPI products has not crystallised. Nonetheless, financial markets currently remain fragile and vulnerable to further shocks. It is therefore important that market participants and policymakers alike are alert to situations when portfolio insurance could potentially work to amplify financial market instability".

Under severe conditions, unexpected big losses may jeopardize the issuers and the stability of financial markets. In theory, the issuer of CPPI products can hedge such exposure to gap risk by options. The issuer needs to model the likely worst-case move in the risky asset price before the next rebalancing opportunity and build the cost of this implicit option into the premiums and fees charged to the investors (or in the case of limited possibilities to transfer costs to clients, the issuer needs to calculate the amount of capital that needs to be provided). But, in practice, the pricing of such options can be quite complex, because the issuer does not exactly know the underlying asset price process and correlations in stressed markets. Furthermore, it is difficult for the issuers to find available options through which to hedge their exposures. Pain and Rand (2008) report that some issuers of CPPI products create securities that package up the gap risk and sell these to investors, including private banks and funds.

Although it is technically clear that portfolio insurance strategies could distribute the risks amongst different risk profiles of individual and institutional investors, markets are not perfect and in some conditions portfolio insurance can induce market instability. The conditions to factor in the instability may be the following:

1. Impact of dynamic hedging on illiquid markets.
2. Imperfect information and the gap risk.
3. Limited available instruments to hedge the exposure to gap risk.

Pain and Rand (2008) reinforce the role of portfolio insurance as a distributor of financial risk among agents that are willing to absorb it. In fact, issuers of portfolio insurance solutions can be exposed to relevant downside risk (unexpected high losses) enhancing conditions to a more fragile stability of financial markets with increasing volatility. More generally, portfolio insurance is an example of how financial innovations, which in most circumstances enable risk to be better managed, can also potentially accentuate market instability. Specifically, it is relevant to investigate the impact of these portfolio management techniques on price, liquidity and volatility. Also of importance is the level of agency costs by portfolio managers: are they managing accordingly to a target rate of return for a client, or are they protecting their balance sheets from underperformance results on minimum rate of return contracts with retail clients? As an announcement of all the ingredients for the crisis of 2007, Tucker (2005) made a speech about risk dispersion and risk management and highlighted the need to understand risk in order effectively managed it. The innovation enhanced by the dynamic strategies is refereed as a positive contribution, but financial intermediaries should identify structural imbalanced options positions in markets that could face illiquidity after some stressed conditions and major price downfalls.

After the crisis, portfolio insurance resurged and retail investors poured into protection of wealth but keeping in mind the upside potential of markets. The newspaper The New York Times, November 11th 2015, Wealth Special Section,“For Investors, ’Portfolio Insurance’ Against Market Declines.”, describes some of the portfolio insurance techniques available to individual investor - directly or through their investment advisors - that can be employed to protect wealth over time. The short term volatility protection is referred as being different from shielding against future loss in retirement income. For the short term protection or betting views on volatility the fear index Vix provides investment on Exchange Tradable Fund (ETF) and the entry timing and exit are the drivers for investors’ returns. As an alternative protection the options are also highlighted as an adequate investment. The sense of protection and the adoption of such shields against market declines always emerges after some losses are pocketed by investors.

Several market opinions started to highlight, again, the benefits of portfolio insurance - Forbes, March 4th 2016, “Add Portfolio Insurance Just in Case of Market Crash In 2016.” - after the strong equity returns on 2016 and the rising fear of loses on the increasing wealth from market returns. But one year after, on the Financial Times, March 21 2017, “Rise In New Form of ’Portfolio Insurance’ Sparks Fears.”, the rise of new forms of portfolio insurance is said to spark fears. It is referred that institutional investors were allocating considerable amounts of money into risk mitigation or crisis offset programmes - long maturity government bonds and trend-following hedge funds, like "Commodity Trading Advisor" (CTA) - that act as a counterweight when markets are in turmoil. The CTA are computer-driven vehicles that take advantage of financial markets’ tendency towards momentum. These instruments often bet against an already falling market, shorting it to profit from further declines and thrive when the other strategies are unravelling. Therefore, the fear of losses has driven institutional investor towards this strategy. But, in practice this idea is similar to portfolio insurance which is fuelling some concerns over re-incidence on feedback loops and mostly on a generalized approach where short-term market behaviour can be destabilising.

On addition to these strategies, the fear index, VIX, has been a new trend for portfolio insurance. In this case volatility ETF's track the Vix volatility index, which tends to move in the opposite direction of stock prices in general. Bloomberg, October 25th 2017, "Sometimes You Beat the Bear. Lately, the Bear Beats You.", has referred the emergent impact of such funds due to its structure which is design to amplify short-term moves. Although some providers market these vehicles for traders instead of long term investors, the selling point is still protection on downside markets. Thus, not being aware of some of the technicalities of protection vehicles can be as bad as the worst scenario they tend to prevent.

The Fear index was also pointed on Financial Times, February 6th 2018, "Shorting Volatility: Its Role in the Stocks Sell-Off.", as the corner of a complex and expanding volatility ecosystem that has evolved over the past decade and is being compared with the portfolio insurance strategies. The Vix Exchange-Traded Products are tradable asset but volatility is a relevant input into investment strategies. When asset managers short volatility and others use it as a proxy for risk on turmoil markets, volatility can be-get more volatility causing a selling vicious cycle from a feedback loop.

From The Wall Street Journal of 19th October 2017, "A Stock Market Panic Like 1987 Could Happen Again", the Nobel prize winner Prof. Robert Shilller notes that there is a resurgent idea about portfolio insurance as a solution that was designed to protect investors from falling markets but, instead, it helped to exacerbate a breakdown on stock market. However nothing happened solely on the 19th October. Some previous trigger events caused the selling cycle: the drop of 9% on the S&P 500 index, one week before the crash, signalled a selling action for existing portfolio insurance models, then mutual funds were forced to alleviate their equity holdings to meet redemptions and finally, the impact on liquidity from margin calls that shrunk liquidity and enhance lack of reliable information for trading (Carlson M, 2006). The time-line of the crash presented by Carlson (2006) is an impressive sequence of events for which portfolio insurance contributed but is hardly the sole explanation for those events.

More than 30 year after the October stock market crash of 87 the market views are now envisaging new forms of risks similar to the causes of Black Monday. Larry Summers, former US Treasury Secretary, said on a comment to Prospect Magazine of April 2018, "Back to School: Top Economists On What Their Subject Needs to Learn Next.", that modern economies are not self-equilibrating systems and are dominated by positive feedback effects that destabilize. He refers that margin calls, bank runs, portfolio insurance, option hedging, all cause more selling of assets as their values go down. In this way, when selling causes lower prices, which cause more selling, the market mechanism is in trouble. The challenge Mr Summers refers is to prevent vicious cycles from developing and to contain them when they start, which means more smarter government policy and not a retreat into market fundamentalism.

Recently, after the 2016 and 2017 rallies on equities, the beginning of 2018 presented some signs of correction on equity prices. Amongst several reasons pointed out by practitioners, the dominance of machines, algorithms and passive investment instruments and strategies were referred as the causes for the market instability, much like portfolio insurance disrupted markets in October, 1987 (The Street, April 2018, "Why the Stock Market Collapsed Monday and What's Next.").

Specially related with the individual investor perspective, in the next chapter we focus on the attractiveness of portfolio insurance strategies to identify which rational investors, despite potential crisis, would select these protection strategies in distinctive market conditions.

Chapter 4

Who Can Portfolio Insurance Strategies Attract?

Mapping Solutions to Investors

The analysis of investment decisions under uncertainty in the literature is based on different theories. From the normative Expected Utility Theory (EUT) to Behavioural Finance Theory (BFT) one can test different attitudes towards risk. Investors are not all alike, their decisions are focused either on utility derived from a certain amount of estimated wealth, an aspiration, an expected monetary value or gains and losses against a certain reference. The valuation or utility is driven by a set of factors that may be collected and perceived using a formulation. The way portfolio insurance strategies fit the rationality in each of the explanatory theories enhances a possible segmentation on investors appetite for this type of products.

The specificity of the rational on loss preventing investment strategies can be the trigger to understand the reason for individual investors to keep on investing in portfolio insurance strategies. The aim of this chapter is to identify the conditions that drive individual investors into portfolio insurance strategies and also to set the descriptive characteristics of segments of individual investors that choose protective strategies. We address firstly a prescriptive framework for rational investors and in the next chapter we change the framework towards BFT.

The paradigm of risk *vs* returns evolved through times as individuals' capability to understand the impact of losses changed under a framework that specifies the rationality of their decisions based on a set of assumptions. In fact, knowing that some of the decisions by individuals may differ from the prescriptive results of Modern Portfolio Theory (MPT) raises questions on the drivers of individual investment decisions. Although Mean-Variance Analysis (MVA) and EUT are foundational pillars in any advance course in asset management, the recognition that behaviour factors must be taken into account for individual decisions is gaining grounds (Hens and Bachmann, 2011).

To test the drivers of investors' choice we based our analysis on simulation of a risky asset and set several portfolio insurance strategies and simple investments on the risky asset, a risk-free asset and a 50:50 portfolio of risky and risk-free assets. The ultimate wealth level derived from portfolio strategies and the latter, is valued using MVA and some utility functions on different risk aversion levels. The results of the valuation are compared in high to low volatility market and high to low expected returns using a market conditions matrix.

The remaining of the chapter is organized as follows: Section 4.1 defines the framework for portfolio insurance. Section 4.2 provides a short review on the investment decision under uncertainty. The methodology and results are set in Sections 4.3 and 4.4. The conclusions are presented in Section 4.5.

4.1 Portfolio Insurance

The way portfolio insurance strategies set the rationality in each of the explanatory theories enhances a possible segmentation on investors' appetite for this type of products. Investors that under EUT are characterized by having average expectations, but with marginal increasing risk tolerance with wealth, may prefer to be exposed to portfolio insurance. Investors who have average risk tolerance, but whose expectations of returns are more optimistic than average, may buy portfolio insurance strategies (Leland, 1980). However, not all portfolio insurance strategies are able to perform according with investors objectives, as is the case of Constant Proportion Portfolio Insurance (CPPI). In fact, a simple Stop Loss Portfolio Insurance (SLPI) strategy may perform better than the more elaborate portfolio insurance strategies (Costa and Gaspar, 2014). As Option Based Portfolio Insurance (OBPI) and CPPI strategies offer alternative protected pay-offs, it is relevant to examine under what circumstances an investor should prefer one type of protection over the other. Zhu and Kavee (1988) use Monte Carlo simulation to compare various statistic samples of replicated OBPI and CPPI pay-offs. El Karoui et al. (2006) investigate which strategy over a finite horizon maximizes a utility criterion and prove the optimality of OBPI strategies when a position in the risky portfolio requires a given level of guarantee. Bertrand and Prigent (2011), Annaert et al. (2009), and Zagst and Kraus (2011) compare OBPI and CPPI using stochastic dominance criteria.

When the risky asset price follows a geometric Brownian diffusion the portfolio value can, theoretically, never reach the floor, but in reality markets experience jumps. Benninga and Blume (1985) show that portfolio insurance strategies are still desired by investors in incomplete markets. In the presence of market jumps Bertrand and Prigent (2016) study how extreme moves in asset returns may impact portfolio insurance and Cont and Tankov (2009) examine the gap risk - the risk of falling below the floor - and derive the gap loss distribution and various associated risk measures in the context of a jump-diffusion model. Zhu and Kavee (1988) and Bertrand and Prigent (2011) compare OBPI and CPPI strategies with an underlying following a compound Poisson process. With this perspective of protection, investors demand different solutions so that they could achieve their goals on expected wealth, or on potential gains and losses.

Leland (1980) stated the following:

> "Since much of the demand for options is attributed to investors who are either more bullish or more bearish on the expected return of the underlying stocks, it seems important to include differing expectations as a possible source of demand for options or for portfolio insurance. Our principal conclusions are:
> 1) Investors who have average expectations, but whose risk tolerance increases with wealth more rapidly than average, will wish to obtain portfolio insurance.

2) Investors who have average risk tolerance, but whose expectations of returns are more optimistic than average, will wish to obtain portfolio insurance.
Institutional investors falling in class (1) might include pension or endowment funds which at all costs must exceed a minimum value, but thereafter can accept reasonable risks. "Safety-first" investors would find portfolio insurance attractive on this basis. Institutional investors falling in class (2) would include well-diversified funds which believe themselves to have positive "alphas"—i.e., funds which expect on average to achieve excess returns by superior stock selection. In order to exploit these excess returns to equities, but at the same time keep risk within tolerable levels, insured-type strategies are optimal."

The search for a segmentation of portfolio insurance investors is not novel. In fact, the way these solutions are presented to investors in order to fit their needs is still a major issue, as theoretical frameworks defines specific conditions to assess investors' expectations. In fact, even theorists of PT (Kaheman and Tversky, 1979) have stated that reasonable people could, in most of the time, follow EUT axioms having thus a rational choices.

Portfolio insurance investors may have different perspectives on risk and return on reference values for their gains and losses. However, investors' decisions under uncertainty may be non-rational according to prevailing normative theoretical frameworks, like the EUT, but may follow guidelines on a descriptive framework as Prospect Theory (PT) and Cumulative Prospect Theory (CPT). In this twofold environment it is necessary to identify de drivers for selecting portfolio investment strategies and understand the mapping between protective strategies and investors preferences. We are addressing the first environment.

4.2 Investors' Decision Under Risk

The decision process deals with two definitions, risk and uncertainty, that sometimes are not sufficiently clarified leading to some misunderstanding on the analysis of investors' financial

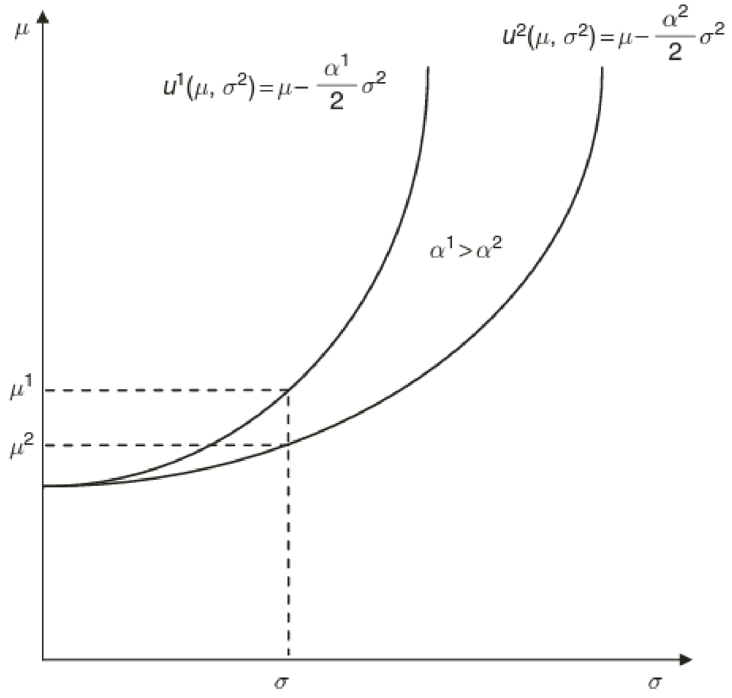
decisions results. The definition of risk commonly accepted in the literature which was presented by Knight (1921), refers to situations in which the probabilities of different outcomes are known, while uncertainty refers to situations in which the probabilities are unknown. Based on these definitions it is possible to distinguish uncertainty attitudes from risk attitudes.
We are using a definition of decision theory that deals with how investors make decisions and with how investors should make decisions. Therefore, the heartland of decision theory is centred in choice under uncertainty. From the idea in the 17th century of Pascal of expected value and the paper of Bernoulli (1954) where he defines a utility function and computes expected utility instead of expected financial value, there have been different approaches. In 1947, Von Neuman and Morgenstern presented an axiomatic framework of EUT that defines a normative theory. Based on this economic theory people behave as rational agents setting the grounds to a decision-making behaviour under risk. Against this normative theory some authors (Allais, 1953; Ellsberg, 1961) show that describing investors' behaviour leads to violations of the EUT axioms. Incorporating actual human behaviour in decision-making demonstrates that losses loom larger than gains and that people are more focused on changes in their utility states then the states themselves and estimation of subjective probabilities is very biased (Tversky and Kahneman, 1979, 1992).

The decision theory can be detailed through the Expected Utility Theory (EUT) and Prospect Theory (PT). Although the abnormal behaviours in financial markets may be explained using different models, in this chapter we focus our attention in EUT because it is still a dominant framework to assess investment strategies.

4.2.1 Mean-Variance Analysis

The most used framework to support investment decision is based on objective criteria: choices between alternatives are defined using mean and variance (Markowitz, 1959). However, the two fund theorem of Tobin (1958) is supported on two restrictive conditions: Constant Rela-tive Risk Aversion and normally distributed log-returns (Merton, 1973).
Under MVA the best portfolio is set by two variables: the risk-return opportunities and in-

Figure 4.1: Example of a risk aversion comparison - adapted from Hens and Bachmann (2011)



vestor's preferences. The risk-return combinations of assets defines the set of efficient portfolios from which we derive the efficient frontier. Investor's preferences are given by an utility function:

$$u^i(\mu, \sigma^2) = \mu - \frac{\alpha^i}{2}\sigma^2, \tag{4.1}$$

where $\alpha^i > 0$ is a parameter describing the risk aversion of investor i (the higher is α^i the more risk averse is the investor). In this case, the higher the risk aversion, the higher is the required expected return for a unit increase in risk[1]. Although the two fund theorem limits the heterogeneity of investors to a single portfolio of risk-free asset and the tangent portfolio, market solutions rarely rely on this theoretical approach (conservative, moderate or aggressive type investors with different mix of cash, bonds or stocks). When investors present a high level of risk aversion they tend to select investment strategies with low expected returns and risk, moving down the efficient frontier. Investors with a low level of risk aversion select portfolios that move up along the efficient frontier.

[1]See p. 154-155 in T. Sargent, Macroeconomic Theory, 2nd. edition for a detailed analysis on the "Mean Variance Utility Function".

4.2.2 Investors' Decision in Expected Utility Theory

The Expected Utility Theory (EUT) is based on the assumption that investors maximize their final expected wealth when making investment decisions. Utility is analysed based on two approaches: the cardinal and the ordinal.
Cardinal utility, a quantitative approach, measures the satisfaction of individuals by utils and the marginal utility derived from consumption or amount of wealth. This has been promoted by classical and neo-classical economists. The ordinal utility, promoted by modern economists (Hicks and Allen, 1934), measures ranks and the qualitative approach using indifference curves states a comparative level of satisfaction between differences of consumption or amount of wealth. The ordinal utility is used mainly on consumer theory under certainty and represents preferences before certain outcomes (Debreau, 1954). The analysis of consumer theory under uncertainty is based on the work of Von Neumann-Morgenstern (1947) that designs the cardinal framework for utility that represents random outcomes. Cardinal utility function is an utility index that maintains preference orderings uniquely up to positive affine transformations.

The analysis we perform relies on the cardinal utility approach, which states that choice over lotteries, satisfying certain axioms (presented hereunder), implies maximization of the expectation of a utility function, defined over outcomes. Our work is based on tractable utility functions of two kinds: the class of linear risk tolerance or Hyperbolic Absolute Risk Aversion (HARA). The special cases are the quadratic, the exponential (or Constant Absolute Risk Aversion (CARA)) and the power utility function (or Constant Relative Risk Aversion (CRRA)).
The expected utility model, initially designed by Bernoulli (1954), was developed by Von Neumann and Morgenstern (1947). The theory is based on preference axioms that define decision making of a rational investor over uncertain prospects: completeness, transitivity, continuity and independence. The EUT assumes investors verify this set of rational axioms:

- **Completeness.** For every A and B either$A \succeq B$ or $A \preceq B$. This means that the individual either prefers A to B, or is indifferent between A and B, or prefers B to A.
- **Transitivity.** For every A, B and C with $A \succeq B$ and $B \succeq C$ then $A \succeq C$. If an investor decides according to the completeness axiom then also decides consistently.

- **Continuity.** Let A, B, and C be three lotteries with $A \succeq B$, and $\alpha \in (0,1]$; then $\alpha A + (1-\alpha)C \succeq \alpha B + (1-\alpha)C$. When two gambles are mixed with a third one, they maintain the same preference order as when the two are presented independently of the third one.

- **Independence.** Let A, B and C be lotteries with $A \succeq B \succeq C$; then there exists a probability p such that B is equally good as $pA+(1-p)C$. When there are three lotteries (A, B and C) and the investor prefers A to B and B to C, then there should be a possible combination of A and C in which the individual is then indifferent between this mix and the lottery B.

As individuals do not care directly about monetary values of the outcomes, but care about utility that money provides, their goal is to maximize expected utility. The expected utility function measures the expected utility of a set of possible outcomes as the sum of the products of the utility received from each outcome, multiplied by its respective probability of occurrence[2]:

$$E[U(X_i)] = p_1U(X_1) + p_2(X_2) + \cdots + p_nU(X_n) = \sum_{i=1}^{n} p_iU(X_i). \tag{4.2}$$

Based on this formulation, it is possible to define alternative investor attitudes toward risk [3]. An individual is risk averse if the expected utility from the outcome associated with a risky choice is less than the utility from a certain outcome—which is equal to the expected, or mean, outcome associated with the risky choice:

$$E[U(X)] < U[E(X)]. \tag{4.3}$$

In general, an investor is considered risk loving if the expected utility from the outcome associated with a risky choice is greater than the utility from one outcome with certainty—which is equal to the expected, or mean, outcome associated with the risky choice:

$$E[U(X)] > U[E(X)]. \tag{4.4}$$

[2]The formula applies only to discrete time model, which is the one we select to perform the Monte Carlo simulation on the risky asset price.

[3]Investor attitude towards risk is evaluated using Jensen's inequality, and the utility function derivative.

An individual can be characterized as risk neutral if the expected utility obtained, from the outcome associated with a risky choice, is precisely equal to the utility he gets from an outcome with certainty—which is equal to the expected, or mean, outcome associated with the risky choice:

$$E[U(X)] = U[E(X)]. \tag{4.5}$$

In order to map between the physical measure of money and the perceived value of money it is necessary to have functions, which are called utility functions. The most common utility functions used in finance are the exponential, logarithmic, power and iso-elastic, due to its mathematical tractability.

There were experimental works in the decades after Von Neumann and Morgenstern (1947), that showed individuals systematically violate EUT when choosing among risky gambles. There have been alternative theories to explain the experimental evidence. Some are better known such as the weighted-utility theory [Chew and MacCrimmon (1979), Chew (1983)], the implicit expected utility [Chew (1989), Dekel (1986)], the disappointment aversion [Gul (1991)], the regret theory [Bell (1982), Loomes and Sugden (1982)], the rank-dependent utility theory [Quiggin (1982), Segal (1987, 1989), Yaari (1987)], and the prospect theory [Kahneman and Tversky (1979), Tversky and Kahneman (1991, 1992)].

Regarding the measures of risk aversion, there are two possibilities to measure it: Absolute Risk Aversion (ARA), and Relative Risk Aversion (RRA). The ARA measures aversion to a loss in absolute terms, and the RRA measures aversion to a loss relative to investor's wealth.

From Figure 4.2 it is possible to verify the difference of the curvature of utility function. The evolution of marginal utility determines the way an investor reacts before uncertainty. Arrow-Pratt measures risk aversion for different utility characterizations:

- absolute risk aversion

$$R_a(X) = -\frac{U''(X)}{U'(X)} \tag{4.6}$$

- and relative risk aversion

$$R_r(X) = -X\frac{U''(X)}{U'(X)}. \tag{4.7}$$

An investor's allocation of wealth to risky assets depends on the risk aversion characteristics of his utility function. If he has Increasing Absolute Risk Aversion (IARA), then, as wealth increases, he will hold less money in risky assets. If an investor has Constant Absolute Risk Aversion (CARA), he will have the same amount of money in risky assets, as wealth increases. If an investor has Decreasing Absolute Risk Aversion (DARA), then, as wealth increases, he will hold more money in risky assets. If an investor has Increasing Relative Risk Aversion (IRRA), then, as wealth increases, he will hold a lower percentage of wealth in risky assets. If an investor has Constant Relative Risk Aversion (CRRA), he will have the same percentage of wealth in risky assets, as wealth increases. If an investor has Decreasing Relative Risk Aversion (DRRA), then, as wealth increases, he will hold a higher percentage of his wealth in risky assets.

If a decision maker accepts the EUT axioms it is possible to evaluate the consequences of uncertainty based on an utility function, which lead him/her to maximize the expected utility. In fact the expected monetary value of the pay-offs $X - i$ with probabilities $p_i, i = 1, 2, 3, ..., n$ can be compared with the utility derived from a Certainty Equivalent (CE):

$$U(CE) = \sum_{i=1}^{n} p_i U(X_i). \tag{4.8}$$

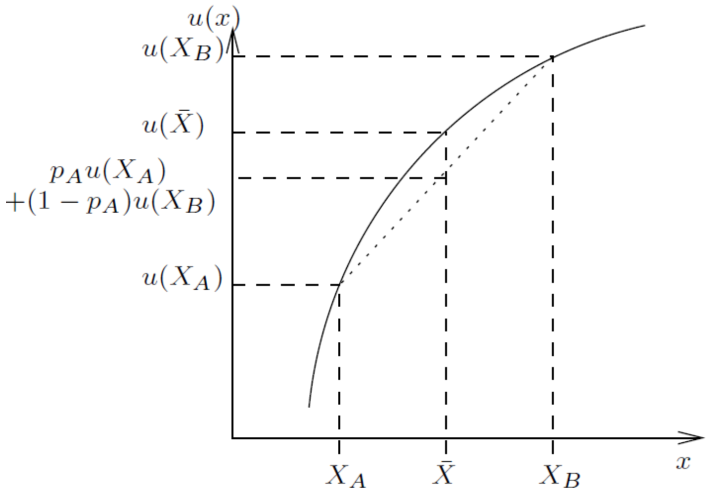
The choice between different investment strategies depends on how the decision maker assesses the certainty equivalent with the expected monetary value. If CE is less than the expected monetary value investor shows a risk aversion attitude. On the contrary if CE is larger than the expected monetary value, investors are keen on risk and attractiveness of lotteries/uncertainty.

Concerning the shape of the utility function, a risk averse investor shows a concave curve, a risk neutral investors shows linear utility function and a risk seeker presents a convex curve. In our analysis we follow the more simple utility functions of the CARA, CRRA, or HARA,

instead of more complex like the ones proposed by Markowitz (1959), that have concave as well as convex segments along the curve. For a detailed revision of utility curves please see Friedman and Sunder (2011).

The most tractable utility functions used in literature are the following:

Figure 4.2: Example of a risk aversion investor - adapted from Jonhson (2007)



- linear

$$U(X) = a + bX, b > 0 \tag{4.9}$$

- quadratic

$$U(X) = X - \frac{1}{2}bX^2, b > 0 \tag{4.10}$$

- exponential

$$U(X) = \frac{1}{a}(1 - e^{-aX}), a > 0 \tag{4.11}$$

- logarithmic

$$\begin{cases} U(X) = \frac{x^{1-c}-1}{1-c}, c \neq 1 \\ U(X) = ln(X), c = 1 \end{cases} \tag{4.12}$$

- power

$$\begin{cases} U(X) = \frac{s^{c+1}-(s-X)^{c+1}}{(c+1)s^c}, s > 0, c > 0, X < s \\ U(X) = \frac{s}{c+1}, X \geq s, \text{ where s can be considered a level of saturation.} \end{cases} \tag{4.13}$$

We also stated the power utility function, but we will not use it due to the unknown level of saturation that is set per individual. As utility functions are the base of a "moral" value, as stated by Bernoulli (1873), the cardinal analysis of the utility is not changed by positive affine transformations..

4.3 Methodology

We use Monte Carlo method, which is widely use in the literature[4], to simulate a risky asset pattern—see Table 4.1—, in order to evaluate the performance of portfolio insurance strategies, and benchmark investments. After the outcomes of the different strategies we use several approaches to assess the level of satisfaction of each strategy. Based on the different utility functions we identify the segments of investors that may be attracted to portfolio insurance strategies and then we group by homogeneous risk aversion investors in order to map their risk attitude to the expected returns from portfolio insurance.

We run numerical solutions on Monte Carlo simulations to build stock market scenarios with normally distributed price returns. The continuous compound stock market returns are generated 100.000 times, using a Geometric Brownian Motion (GBM), for a one year time horizon, with 252 trading days. We assume there is a risk-free asset paying a constant rate of return (r), and a risky asset with dynamics:

$$dS_t = \mu S_t d_t + \sigma S_t W_t, \tag{4.14}$$

where μ is the mean, σ is the volatility and W is a Wiener process. In order to define a stochastic process for stock returns consistent with the assumptions of Black and Scholes (1973), which support the engineering of portfolio insurance strategies, we set the dynamics of log returns of the risky asset as:

$$d(lnS) = (\mu - \frac{\sigma^2}{2})dt + \sigma dW_t, \tag{4.15}$$

where dW_t is a Wiener process describing the development of a normally distributed variable. We define 9 stock market scenarios: high, medium and low volatility; high, medium and equivalent risk-free equity returns, in order to capture a large span of possible bear and bull market conditions. We set the nominal risk-free interest rate at 5% in order to allow a larger portion of investment in risky assets and comparison with previous studies (e.g., Dichtl and Drobetz, 2011, Costa and Gaspar, 2014).

[4]Several research papers use this numerical method: e.g. Pézier and Scheller, 2011, Dichtl and Drobetz, 2011, Costa and Gaspar, 2014

Table 4.1: Stock market scenarios

	Volatility		
	High: $\sigma = 30\%$	Medium: $\sigma = 20\%$	Low: $\sigma = 10\%$
Expected return μ			
15%	Scenario 1	Scenario 4	Scenario 7
10%	Scenario 2	Scenario 5	Scenario 8
5%	Scenario 3	Scenario 6	Scenario 9

In each of the scenarios of Table 4.1 we have 100.000 paths on stock market returns to compare the results amongst strategies[5]. The stochastic process for the stock returns is consistent with the assumption of Black and Scholes (1973) option pricing model, which we use to depict the features of the synthetic options strategy for portfolio insurance performance as in Bird et al. (1990).

Following the conditions used in most empirical studies of portfolio insurance, simulations do not consider tax, bid and offer spreads, nor transaction costs on the stock market. However, we are aware that tax and transaction costs are relevant facts that impact the performance and may affect the final wealth.[6]

We do not apply any filter on portfolio shifts (single or cumulative), in order to avoid biased comparisons. The time horizon of the investment period is one year, which is the standard maturity in retail and institutional markets before a regular feview on investment strategy, hence we consider the 252 trading days of the stock market[7]. Regarding the trading days they were also used by Dichtl and Drobetz (2011), and Branger and Vrecko (2009). With simulated data, we compare the final wealth from portfolio insurance, and benchmark strategies, using the

[5]We simulate one hundred thousand paths for the Monte Carlo simulation, and run the exercise several times to test robustness. We find the results to be coherent. The calculations were made on Matlab programming and partially on Excel with VB programming.

[6]The transaction costs factor was addressed by Leland (1985) using an adjustment to volatility in order to accommodate the upward and downward final price of the transactions:

$$\sigma_A^2 = \sigma^2 (1 + \frac{\sqrt{2/\pi k}}{\sigma\sqrt{t}}),$$

where σ is the annualized standard deviation of the stock returns, k is the round-trip transaction costs as a proportion of the volume of transaction, and t is the revision interval as a proportion of a year. In our study we do not include this adjustment as the gained precision does not have a relevant impact of strategies' returns distributions.

[7]We use a one year investment period in order to compare several strategies. However, for CPPI and TIPP, it is necessary to perform additional studies for longer maturities, specially, if the focus of the investor is the long term objective for specific levels of wealth (i.e. pension funds, target funds, life-cycle funds).

first two moments of the distributions (following the standards in portfolio insurance literature) and use different classes of utility function to calculate the investor's expected utility.

The portfolio insurance strategies considered are:

1. Stop Loss Portfolio Insurance (SLPI)
2. Option Based Portfolio Insurance (OBPI) with synthetic option
3. Constant Proportion Portfolio Insurance with $m = 1$ (CPPI m=1)
4. Constant Proportion Portfolio Insurance with $m = 3$ (CPPI m=3)
5. Time Invariant Portfolio Protection (TIPP) .

The portfolio insurance strategies are benchmarked against:

1. Passive stock market strategy (Risky Asset)
2. Portfolio with risky and risk free assets (50:50)
3. Risk-free cash market deposit (Risk Free).

In SLPI, the investor sets a stop loss order, which is a conditional order to sell portfolio stock if the value of the stock falls below a given level (i.e., the floor). Once the market value of the risky asset portfolio falls below the discounted floor, the portfolio is sold, and converted into risk-free assets, then held until maturity.

Regarding the simulation of the OBPI strategy, we use Bird et al.(1990) approach on synthetic put portfolio insurance strategy, because perfect hedging in portfolio insurance with listed put options is frequently not possible. The synthetic put strategy was suggested by Boyle and Schwartz (1977), Brennan and Schwartz (1979), Rubinstein and Leland (1981), and uses the Black and Scholes (1973) option pricing model to create a synthetic European put option on the risky asset. The synthetic put strategy continuously adjusts a portfolio of a protective put $(S + P)$ and using a dynamic replica protects an equity portfolio using a basket of equity and debt investments.

Using the Black and Scholes (1973) option pricing model and the call-put parity theorem (Cox and Rubinstein, 1985), the portfolio is defined by:

$$S + P = S - SN(-d_1) + Ke^{-rT}N(-d_2) \tag{4.16}$$
$$S + P = S[1 - N(-d_1)] + Ke^{-rT}N(-d_2)$$
$$S + P = SN(d_1) + Ke^{-rT}N(-d_2),$$

where S is the underlying risky asset price, P is the price of the put option, K is the strike price, r is the risk-free rate, and T is the time to maturity. The functions $N(x)$ are standard normal cumulative distribution functions with d_1 and d_2.
The insured portfolio is implemented by investing $SN(d_1)$ in the underlying risky asset and $Ke^{-rT}N(-d_2)$ in the risk-free asset. However, to ensure a self-financed strategy, Bird et al. (1990) define two conditions in relation to the fraction of the insured portfolio and the risk free asset (B):

$$g(S + P) + B = S, 0 \leq g \leq \ 1 \tag{4.17}$$

$$gK + Be^{rT} = K \tag{4.18}$$

With these conditions the strategy keeps the target to achieve the floor and is self-financed. Solving equations 4.17 and 4.18 and based on the put-call option parity theorem:

$$g = \frac{S - Ke^{-rT}}{S + P - Ke^{-rT}} \tag{4.19}$$

and

$$B = \frac{PKe^{rT}}{C} \tag{4.20}$$

As the call option (C), using the call-put option parity theorem, is:

$$C = S + P - Ke^{rT} \tag{4.21}$$

the g fraction can be written as:

$$g = \frac{C-P}{C} \tag{4.22}$$

From equation 4.22 the value invested in the underlying risky asset is:

$$gSN(d_1) = SN(d_1)\frac{C-P}{C} = ES_t^{OBPI} \tag{4.23}$$

As the total initial value to invest is the value of the equity S, the fraction to invest in the strategy is:

$$w = N(d_1)\frac{C-P}{C} \tag{4.24}$$

The investment in the risk-free asset is:

$$gN(-d_2)Ke^{-rT} + B = Ke^{-rT}\frac{CN(-d_2) + PN(d-2)}{C} = EB_t^{OBPI} \tag{4.25}$$

The fraction of the strategy investment in the risk free asset is:

$$1 - w = 1 - N(d_1)\frac{C-P}{C} \tag{4.26}$$

Using the market value of the strategy and recalculating d_1 and d_2 it is possible to determine the relative holdings of ES_t^{OBPI} and EB_t^{OBPI}. However, during the investment period, the synthetic approach involves a dynamic management of the proportion of both asset classes.

The CPPI strategy is simulated with two multipliers ($m = 1; m = 3$). Under the scope of naïve strategies, we simulate a CPPI with a multiplier of one, hence we include a static downside protection strategy. In relation to CPPI with $m = 3$ the simulation eliminates short positions by using a constraint on the risk-free asset:$w_{risk-free} = \frac{EB_t^{CPPI}}{V_t^{CPPI}} \in [0;1]$, as in Benninga (1990), Annaert et al. (2009), and Dichtl and Drobetz (2011):

$$ES_t^{CPPI} = max[min(mCu_t, V_t^{CPPI}), 0]. \tag{4.27}$$

The constraint is included in each simulated path at the end of each trading day, when risky and risk-free asset allocations are recalculated.

We also include in the simulation the TIPP, which is a variant of CPPI, that allow us to compare a solution where an investor incorporates the intra-period gains into the protection floor. The protection level is ratchet up when the portfolio value increases.

We use a passive stock market strategy in order to have a static strategy on the stock market (ES_t). In the portfolio of risky and risk-free assets (50:50), we use a passive approach on both markets: in stock market we simulate the risky asset (S_t), and for the risk-free bond/cash market we set a riskless asset (B): strategy is implemented with 50% allocated in each market ($0.5ES_t + 0.5B_t$). As a fixed return risk-free product is a frequent benchmark, for one year period risky investments, we include an interest rate (r) of 5% for a riskless asset (B) (i.e., a 1 year deposit).

4.4 Results

The results are presented in two perspectives: a performance measurement in a statistical approach; and under the frameworks of valuation for the MVA environment and also for EUT. On each of these two valuation frameworks we define different risk profiles for investors in order to create a wider scope. The results are compared per strategy for specific market conditions. Simulation results are presented using a floor of 100% for the portfolio insurance strategies per scenario.

4.4.1 Performance Measures

Portfolio insurance and benchmark strategies are compared using descriptive statistics (1 to 4 moments). Additionally, we also include two of the most common performance ratios: Sharpe and Sortino. Table 4.2 contains four moments of the distributions of returns at maturity, and Table 4.3 shows the performance ratios. Although the descriptive statistics, and some of the performance ratios, are not sufficient to conclude on the best portfolio insurance strategy, when the focus is the wealth protection, or potential wealth increase (Annaert et al., 2009), it is interesting to point results under several market conditions.

Table 4.2: Distributions of returns (Floor = 100%)

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Mean return p.a. (%)								
S1-$\mu: 15\%; \sigma: 30\%$	9.09	7.70	5.67	6.91	5.87	15.08	10.04	5.00
S2-$\mu: 10\%; \sigma: 30\%$	6.98	5.40	5.39	5.97	5.51	10.03	7.49	5.00
S3-$\mu: 5\%; \sigma: 30\%$	5.11	3.43	5.13	5.13	5.02	5.01	5.00	5.00
S4-$\mu: 15\%; \sigma: 20\%$	10.49	9.56	5.66	6.89	5.89	15.06	10.02	5.00
S5-$\mu: 10\%; \sigma: 20\%$	7.51	6.61	5.39	5.96	5.52	10.01	7.50	5.00
S6-$\mu: 5\%; \sigma: 20\%$	5.09	4.24	5.12	5.11	5.12	4.99	5.00	5.00
S7-$\mu: 15\%; \sigma: 10\%$	13.96	12.77	5.66	6.91	5.90	15.00	10.0	5.00
S8-$\mu: 10\%; \sigma: 10\%$	9.00	8.28	5.39	5.96	5.51	10.00	7.50	5.00
S9-$\mu: 5\%; \sigma: 10\%$	5.08	4.84	5.12	5.12	5.12	5.00	5.00	5.00
Panel B: Volatility p.a. (%)								
S1-$\mu: 15\%; \sigma: 30\%$	18.37	17.16	1.63	6.40	2.47	29.97	15.07	0.00
S2-$\mu: 10\%; \sigma: 30\%$	16.32	15.55	1.56	5.65	2.46	30.09	15.06	0.00
S3-$\mu: 5\%; \sigma: 30\%$	14.26	13.91	1.48	4.95	2.42	29.97	14.94	0.00
S4-$\mu: 15\%; \sigma: 20\%$	15.88	13.58	1.08	4.05	1.69	20.02	10.02	0.00
S5-$\mu: 10\%; \sigma: 20\%$	13.64	11.97	1.03	3.56	1.68	19.98	10.06	0.00
S6-$\mu: 5\%; \sigma: 20\%$	11.50	10.25	0.98	3.10	1.66	19.95	9.95	0.00
S7-$\mu: 15\%; \sigma: 10\%$	10.92	8.97	0.53	1.95	0.85	9.98	5.03	0.00
S8-$\mu: 10\%; \sigma: 10\%$	9.51	7.90	0.51	1.71	0.85	10.03	5.03	0.00
S9-$\mu: 5\%; \sigma: 10\%$	7.59	6.31	0.49	1.48	0.85	10.02	5.00	0.00
Panel C: Skewness								
S1-$\mu: 15\%; \sigma: 30\%$	2.46	1.40	0.89	3.04	0.20	0.00	0.43	-
S2-$\mu: 10\%; \sigma: 30\%$	2.80	1.55	0.88	3.16	0.24	0.00	0.43	-
S3-$\mu: 5\%; \sigma: 30\%$	3.20	1.69	0.90	3.32	0.29	0.01	0.45	-
S4-$\mu: 15\%; \sigma: 20\%$	1.60	1.11	0.57	1.84	0.07	-0.01	0.28	-
S5-$\mu: 10\%; \sigma: 20\%$	1.98	1.36	0.58	1.95	0.09	-0.01	0.28	-
S6-$\mu: 5\%; \sigma: 20\%$	2.47	1.62	0.60	2.01	0.15	0.01	0.31	-
S7-$\mu: 15\%; \sigma: 10\%$	0.31	0.45	0.27	0.86	-0.01	-0.01	0.13	-
S8-$\mu: 10\%; \sigma: 10\%$	0.79	0.83	0.29	0.89	0.01	0.00	0.15	-
S9-$\mu: 5\%; \sigma: 10\%$	1.40	1.31	0.28	0.88	0.02	-0.01	0.14	-
Panel D: Excess Kurtosis								
S1-$\mu: 15\%; \sigma: 30\%$	5.56	2.29	1.41	15.27	-0.29	0.00	0.24	-
S2-$\mu: 10\%; \sigma: 30\%$	7.73	3.09	1.39	17.27	-0.31	-0.01	0.23	-
S3-$\mu: 5\%; \sigma: 30\%$	10.44	4.18	1.44	19.26	-0.32	0.00	0.27	-
S4-$\mu: 15\%; \sigma: 20\%$	1.62	0.99	0.53	5.78	-0.12	0.00	0.09	-
S5-$\mu: 10\%; \sigma: 20\%$	3.23	1.98	0.62	6.93	-0.16	0.01	0.13	-
S6-$\mu: 5\%; \sigma: 20\%$	5.77	3.39	0.64	7.61	-0.18	0.01	0.14	-
S7-$\mu: 15\%; \sigma: 10\%$	-0.87	-0.30	0.14	1.32	-0.02	0.00	0.02	-
S8-$\mu: 10\%; \sigma: 10\%$	-0.36	0.25	0.18	1.46	-0.02	0.02	0.05	-
S9-$\mu: 5\%; \sigma: 10\%$	1.13	1.64	0.14	1.40	-0.03	0.00	0.03	-

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1.

The annual expected return is presented in Panel A of Table 4.2. In relation to expected returns in three bullish market conditions (Scenarios $1-2$, $4-5$ and $7-8$), holding the risky asset delivers the highest expected return. Amongst the portfolio insurance strategies, the SLPI is the option with the highest expected return. The synthetic OBPI is better than the unleveraged CPPI($m=1$), when market presents high returns, which is explained by the cost of protection that is accommodated in markets with high expected returns. The TIPP, with the ability to protect intermediary gains, presents the second lowest expected returns, amongst the portfolio insurance strategies, due to the cost of "cashing in" the intermediary upward movements of the risky asset. However, in depressed markets it tends to present better performance, excluding the unleveraged CPPI. The reason for the dominant results of the passive stock market strategy in bullish markets, compared with any of the portfolio insurance strategies, lies on the protection cost of downside risk, which is implicit on the latter. In a neutral market (Scenarios 3, 6 and 9), expected returns from the simulated strategies are all very similar, except for the synthetic OBPI, which penalizes returns due to volatility.

The annual volatilities are presented in Panel B of Table 4.2. The results are dependent on volatility parameter we set for the scenarios. TIPP and unleveraged CPPI are the portfolio insurance strategies with the lowest volatility in all scenarios. Regarding the benchmark strategies, naturally, the $50:50$ strategy delivers the lowest volatility (except for the case of risk-free asset). The characteristic of moving up the floor of TIPP limits the dispersion of the returns at maturity, which gives the investor the lowest volatile strategy in all scenarios,except in the case of unleveraged CPPI. A particular feature of the SLPI strategy is that volatility decreases as scenarios change from more unstable market conditions to less volatile market (in relation to expected returns), due to the lower probability of activating stop loss orders.

The skewness measure is presented in Panel C of Table 4.2. Return distributions with positive skewness have frequent small losses, and some large gains; whereas, those with negative skewness have frequent small gains, and some large negative returns. Due to the characteristics of portfolio insurance strategies, we expected the results to show positive skewness, which was confirmed by the results. The SLPI return distributions have the highest skewness, as the loss cutting features make these distributions very right tailed. However, this does not occur in all scenarios, as the more bullish and volatile the market, the bigger the probability of the stop loss order being activated, and in these situations the leveraged CPPI ($m = 3$) strategy has the most right tailed returns distribution.

The excess kurtosis results are presented in Panel D of Table 4.2. Higher kurtosis means that variance results from infrequent extreme deviations, rather than frequent modest deviations (Gujarati, 1992). As expected, return distribution of the benchmark strategies are mesokurtic, because the returns follow a lognormal distribution. The TIPP return distributions, are near mesokurtic in all scenarios, due to the fact that a dynamic floor limits deviations. In scenarios 7 to 9, in particular, the SLPI strategy is near mesokurtic, since low volatility and high returns make it less probable to execute the stop loss order, thus becoming a proxy of the lognormal return distribution. A leptokurtic distribution has more returns around the mean, but with large deviations. The return distributions of portfolio insurance strategies, in general, have leptokurtic behaviour and in the scenarios with the highest volatility we observe higher excess kurtosis.

Table 4.3 presents the results for the performance ratios: Sharpe and Sortino. The Sharpe ratio (Sharpe, 1994) is a measure of reward to volatility, which indicates the performance of an investment adjusted by the incurred risk. The ratio measures the excess return per unit of deviation in an investment or a strategy, which is generally referred to as risk[8]. The higher the ratio, the better the performance adjusted by the risk. The ratio is given by:

$$SharpeRatio = \frac{\bar{r_p} - r_f}{\sigma_p}, \tag{4.28}$$

where $\bar{r_p}$ is the expected return of the portfolio, σ_p is the volatility of portfolio and r_f is the risk-free interest rate of the market.

The Sortino ratio measures the risk-adjusted return of an investment (Sortino and Price, 1994), and is a modification of the Sharpe ratio, penalizing only those returns falling below an investor's

[8]The use of Sharpe ratio for a highly non-normal investment outcome like the distribution obtained from portfolio insurance is not adequate - thus the monitoring by risk-based indicators - but is still very common by practitioners.

required rate of return, while the Sharpe ratio penalizes both upside and downside volatility equally. Though both ratios measure an investment's risk-adjusted returns, they do so in significantly different ways, which will frequently lead to differing conclusions, regarding to the true nature of the investment's return-generating efficiency. The Sortino ratio is commonly used to compare the risk adjusted performance of strategies with differing risk and return profiles. As most investors consider risk as the probability of not attaining the target return, this is an important measure for the downside risk. The Sortino ratio is given by:

$$SortinoRatio = \frac{\bar{r_p} - MAR}{\sigma_d}, \tag{4.29}$$

where $\bar{r_p}$ is the expected return of the portfolio, MAR is the target return defined by the investor for the portfolio (in some cases it is named the minimum acceptable return), and σ_d is the downside deviation volatility of portfolio, that can be interpreted as the annualized standard deviation of returns below the target. The formula for the σ_d is:

$$\sigma_d = \sqrt{\sum_{i=1}^{n} \frac{min[(r_i - MAR), 0]^2}{n}}. \tag{4.30}$$

When results from Sharpe and Sortino ratios are negative, conclusions are difficult to reach and additional performance ratios such Omega and Upside potential are necessary to complement the analysis. The study by Costa and Gaspar (2014) is an example of a complete performance ratio study, but in our work we use only a brief analysis of performance, as our focus is on the expected utility and prospect theory perspectives.

The Sharpe ratio results are presented in Table 4.3, Panel A. In all market scenarios the best reward to risk is given by the unleveraged CPPI ($m = 1$) strategy, which dominates all the other strategies.

The results of the Sortino ratio are presented in Panel B of Table 4.3. In bullish and volatile markets, because the ratio penalizes the downside, the Stop Loss strategy delivers the best risk-adjusted return. However, in the majority of market conditions, the unleveraged CPPI delivers the highest Sortino ratio, as the returns are right tailed.

Table 4.3: Performance ratios (Floor = 100%)

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Sharpe								
S1-$\mu: 15\%; \sigma: 30\%$	0.49	0.45	3.45	1.07	2.35	0.54	0.68	-
S2-$\mu: 10\%; \sigma: 30\%$	0.42	0.34	3.42	1.05	2.21	0.35	0.51	-
S3-$\mu: 5\%; \sigma: 30\%$	0.36	0.24	3.42	1.03	2.10	0.17	0.34	-
S4-$\mu: 15\%; \sigma: 20\%$	0.66	0.70	5.21	1.69	3.46	0.80	1.01	-
S5-$\mu: 10\%; \sigma: 20\%$	0.55	0.55	5.20	1.66	3.26	0.53	0.76	-
S6-$\mu: 5\%; \sigma: 20\%$	0.44	0.41	5.18	1.63	3.05	0.25	0.50	-
S7-$\mu: 15\%; \sigma: 10\%$	1.27	1.42	10.50	3.51	6.90	1.61	2.01	-
S8-$\mu: 10\%; \sigma: 10\%$	0.94	1.04	10.41	3.45	6.43	1.04	1.50	-
S9-$\mu: 5\%; \sigma: 10\%$	0.66	0.76	10.39	3.42	5.96	0.50	1.00	-
Panel B: Sortino (MAR = 5%)								
S1-$\mu: 15\%; \sigma: 30\%$	9.55	1.29	9.05	5.69	4.90	1.21	1.59	-
S2-$\mu: 10\%; \sigma: 30\%$	7.25	0.91	8.27	4.85	4.43	0.76	1.13	-
S3-$\mu: 5\%; \sigma: 30\%$	5.33	0.58	7.62	4.16	4.04	0.36	0.71	-
S4-$\mu: 15\%; \sigma: 20\%$	14.09	2.37	13.09	6.97	6.86	1.71	2.27	-
S5-$\mu: 10\%; \sigma: 20\%$	10.01	1.64	11.75	5.84	6.07	1.06	1.58	-
S6-$\mu: 5\%; \sigma: 20\%$	6.82	1.06	10.62	4.88	5.38	0.48	0.97	-
S7-$\mu: 15\%; \sigma: 10\%$	12.88	5.59	28.95	12.79	15.06	3.69	4.88	-
S8-$\mu: 10\%; \sigma: 10\%$	8.35	3.69	23.99	9.81	12.36	2.09	3.12	-
S9-$\mu: 5\%; \sigma: 10\%$	5.00	2.26	19.91	7.56	10.07	0.88	1.77	-

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1.

We elaborate a complete rerun of the model using a floor of 80% on the portfolio insurance strategies. The results confirm the overall findings when floor is at 100%.

Table 4.4 presents the performance indicators. As expected, the portfolio insurance strategies yield wider range of returns and high volatility. The new conditions result in positive skewness for portfolio insurance strategies, but bigger exposure to risky assets leads to a less positive skew distribution.

Table 4.4: Distribution of returns (Floor = 80%)

Floor = 80%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Mean return p.a. (%)								
S1-$\mu:15\%;\sigma:30\%$	15.04	9.93	7.77	13.04	11.56	15.08	10.04	5.00
S2-$\mu:10\%;\sigma:30\%$	9.86	5.50	6.41	8.84	8.22	10.03	7.49	5.00
S3-$\mu:5\%;\sigma:30\%$	5.12	5.12	5.12	5.13	5.12	5.01	5.00	5.00
S4-$\mu:15\%;\sigma:20\%$	16.02	16.02	7.79	13.56	12.00	15.06	10.02	5.00
S5-$\mu:10\%;\sigma:20\%$	10.37	8.08	6.42	9.07	8.44	10.01	7.50	5.00
S6-$\mu:5\%;\sigma:20\%$	5.11	3.26	5.12	5.11	5.11	4.99	5.00	5.00
S7-$\mu:15\%;\sigma:10\%$	16.20	15.60	7.77	13.83	12.23	15.00	10.00	5.00
S8-$\mu:10\%;\sigma:10\%$	10.49	9.93	6.41	9.16	8.58	10.00	7.50	5.00
S9-$\mu:5\%;\sigma:10\%$	5.16	4.63	5.13	5.13	5.14	5.00	5.00	5.00
Panel B: Volatility p.a. (%)								
S1-$\mu:15\%;\sigma:30\%$	28.70	24.97	7.69	22.53	18.31	30.02	15.49	0.00
S2-$\mu:10\%;\sigma:30\%$	27.46	23.68	7.41	21.07	17.66	29.96	15.11	0.00
S3-$\mu:5\%;\sigma:30\%$	26.09	26.09	7.15	19.58	16.96	30.03	14.77	0.00
S4-$\mu:15\%;\sigma:20\%$	20.16	19.08	5.15	16.19	12.88	20.05	10.44	0.00
S5-$\mu:10\%;\sigma:20\%$	19.72	18.43	4.95	14.99	12.50	19.97	10.16	0.00
S6-$\mu:5\%;\sigma:20\%$	19.10	17.55	4.77	13.77	12.03	19.99	9.92	0.00
S7-$\mu:15\%;\sigma:10\%$	10.03	10.02	2.58	8.66	6.63	10.03	5.26	0.00
S8-$\mu:10\%;\sigma:10\%$	10.03	10.02	2.49	7.92	6.57	10.03	5.13	0.00
S9-$\mu:5\%;\sigma:10\%$	10.03	9.99	2.40	2.40	6.42	10.03	5.01	0.00
Panel C: Skewness								
S1-$\mu:15\%;\sigma:30\%$	0.47	0.68	0.67	1.09	0.51	0.00	0.42	-
S2-$\mu:10\%;\sigma:30\%$	0.62	0.83	0.70	1.25	0.63	0.01	0.45	-
S3-$\mu:5\%;\sigma:30\%$	0.78	0.78	0.69	1.40	0.75	0.00	0.45	-
S4-$\mu:15\%;\sigma:20\%$	0.07	0.25	0.43	0.78	0.24	-0.01	0.27	-
S5-$\mu:10\%;\sigma:20\%$	0.19	0.38	0.45	0.93	0.35	0.00	0.29	-
S6-$\mu:5\%;\sigma:20\%$	0.33	0.53	0.45	1.07	0.47	0.00	0.29	-
S7-$\mu:15\%;\sigma:10\%$	0.00	0.00	0.22	0.46	0.02	0.00	0.14	-
S8-$\mu:10\%;\sigma:10\%$	0.01	0.01	0.24	0.59	0.09	0.01	0.16	-
S9-$\mu:5\%;\sigma:10\%$	0.01	0.03	0.22	0.22	0.18	0.00	0.14	-
Panel D: Excess Kurtosis								
S1-$\mu:15\%;\sigma:30\%$	-0.57	-0.05	0.75	0.81	-0.35	0.00	0.24	-
S2-$\mu:10\%;\sigma:30\%$	-0.41	0.22	0.82	1.35	-0.20	0.01	0.29	-
S3-$\mu:5\%;\sigma:30\%$	-0.20	-0.20	0.81	1.90	-0.04	0.02	0.29	-
S4-$\mu:15\%;\sigma:20\%$	-0.36	-0.37	0.29	0.26	-0.39	0.00	0.08	-
S5-$\mu:10\%;\sigma:20\%$	-0.45	-0.34	0.34	0.69	-0.35	0.00	0.12	-
S6-$\mu:5\%;\sigma:20\%$	-0.50	-0.26	0.33	1.14	-0.29	-0.01	0.11	-
S7-$\mu:15\%;\sigma:10\%$	-0.01	-0.02	0.07	0.03	-0.11	-0.01	0.01	-
S8-$\mu:10\%;\sigma:10\%$	0.03	0.00	0.12	0.36	-0.16	0.02	0.06	-
S9-$\mu:5\%;\sigma:10\%$	-0.06	-0.12	0.06	0.06	-0.27	-0.01	0.01	-

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1.

The Sharpe ratio results are presented in Table 4.5. Overall, the unleveraged CPPI with a lower floor, continues to yield the best reward to risk. However, a greater exposure to risk, with increasing probability of losses, results on a lower Sharpe ratio for portfolio insurance strategies. The results of the Sortino ratio show high dispersion, as the chances of negative outcomes are bigger. Comparing these results with the Sortino ratio for portfolio insurance strategies with a floor of 100%, we observe lower values, as expected.

Table 4.5: Performance ratios (Floor = 80%)

Floor = 80%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Sharpe								
S1-μ : 15%; σ : 30%	0.52	0.40	1.00	0.58	0.63	0.54	0.68	-
S2-μ : 10%; σ : 30%	0.36	0.23	0.86	0.42	0.46	0.35	0.51	-
S3-μ : 5%; σ : 30%	0.19	0.19	0.71	0.26	0.30	0.17	0.34	-
S4-μ : 15%; σ : 20%	0.79	0.84	1.50	0.83	0.93	0.81	1.01	-
S5-μ : 10%; σ : 20%	0.52	0.44	1.29	0.60	0.67	0.53	0.76	-
S6-μ : 5%; σ : 20%	0.26	0.18	1.06	0.37	0.42	0.25	0.50	-
S7-μ : 15%; σ : 10%	1.61	1.55	2.99	1.59	1.84	1.61	2.01	-
S8-μ : 10%; σ : 10%	1.04	0.99	2.56	1.15	1.30	1.04	1.50	-
S9-μ : 5%; σ : 10%	0.51	0.46	2.12	2.12	0.79	0.51	1.00	-
Panel B: Sortino (MAR = 5%)								
S1-μ : 15%; σ : 30%	1.71	1.16	2.43	2.14	1.86	1.20	1.59	-
S2-μ : 10%; σ : 30%	1.14	0.66	1.94	1.45	1.31	0.75	1.13	-
S3-μ : 5%; σ : 30%	0.61	0.61	1.50	0.83	0.82	0.35	0.71	-
S4-μ : 15%; σ : 20%	1.79	1.79	3.52	2.73	2.35	1.74	2.30	-
S5-μ : 10%; σ : 20%	1.15	1.02	2.74	1.77	1.60	1.07	1.59	-
S6-μ : 5%; σ : 20%	0.57	0.41	2.05	0.97	0.94	0.48	0.98	-
S7-μ : 15%; σ : 10%	3.71	3.53	7.56	5.05	4.38	3.71	4.89	-
S8-μ : 10%; σ : 10%	2.08	1.96	5.41	2.97	2.72	2.09	3.12	-
S9-μ : 5%; σ : 10%	0.90	0.80	3.79	3.79	1.47	0.90	1.79	-

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1.

4.4.2 Investors' Decision

We set the investor's risk profile based on the parametric utility function which give us a set of points from neutral to high level of risk aversion (α). Based on the return distributions per each market scenario, presented on previous section, the utility values derived on each strategy are calculated and compared amongst insurance portfolio strategies and with the benchmark portfolio strategies.

Table 4.6: Mean variance analysis (Floor = 100%)

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: MVA $\alpha^2 = 0$ (risk neutral)								
S1-$\mu: 15\%; \sigma: 30\%$	0.091	0.077	0.057	0.069	0.059	0.151	0.100	0.050
S2-$\mu: 10\%; \sigma: 30\%$	0.070	0.054	0.054	0.060	0.055	0.100	0.075	0.050
S3-$\mu: 5\%; \sigma: 30\%$	0.051	0.034	0.051	0.051	0.051	0.050	0.050	0.050
S4-$\mu: 15\%; \sigma: 20\%$	0.105	0.096	0.057	0.069	0.059	0.151	0.100	0.050
S5-$\mu: 10\%; \sigma: 20\%$	0.075	0.066	0.054	0.060	0.055	0.100	0.075	0.050
S6-$\mu: 5\%; \sigma: 20\%$	0.051	0.042	0.051	0.051	0.051	0.050	0.050	0.050
S7-$\mu: 15\%; \sigma: 10\%$	0.140	0.128	0.057	0.069	0.059	0.150	0.100	0.050
S8-$\mu: 10\%; \sigma: 10\%$	0.090	0.083	0.054	0.060	0.055	0.100	0.075	0.050
S9-$\mu: 5\%; \sigma: 10\%$	0.051	0.048	0.051	0.051	0.051	0.050	0.050	0.050
Panel B: MVA $\alpha = 2$ (low risk aversion)								
S1-$\mu: 15\%; \sigma: 30\%$	0.023	0.018	0.056	0.061	0.058	-0.029	0.053	0.050
S2-$\mu: 10\%; \sigma: 30\%$	0.017	0.006	0.053	0.053	0.054	-0.081	0.029	0.050
S3-$\mu: 5\%; \sigma: 30\%$	0.010	-0.004	0.051	0.046	0.050	-0.130	0.007	0.050
S4-$\mu: 15\%; \sigma: 20\%$	0.054	0.059	0.056	0.066	0.058	0.070	0.078	0.050
S5-$\mu: 10\%; \sigma: 20\%$	0.038	0.037	0.054	0.057	0.055	0.020	0.054	0.050
S6-$\mu: 5\%; \sigma: 20\%$	0.024	0.021	0.051	0.049	0.051	-0.030	0.031	0.050
S7-$\mu: 15\%; \sigma: 10\%$	0.116	0.112	0.057	0.068	0.059	0.130	0.095	0.050
S8-$\mu: 10\%; \sigma: 10\%$	0.072	0.070	0.054	0.059	0.055	0.080	0.045	0.050
S9-$\mu: 5\%; \sigma: 10\%$	0.039	0.040	0.051	0.051	0.051	0.030	0.046	0.050
Panel C: MVA $\alpha = 5$ (medium risk aversion)								
S1-$\mu: 15\%; \sigma: 30\%$	-0.331	-0.291	0.053	0.018	0.051	-0.961	-0.193	0.050
S2-$\mu: 10\%; \sigma: 30\%$	-0.263	-0.248	0.051	0.020	0.047	-1.026	-0.209	0.050
S3-$\mu: 5\%; \sigma: 30\%$	-0.203	-0.207	0.049	0.021	0.044	-1.071	-0.220	0.050
S4-$\mu: 15\%; \sigma: 20\%$	-0.210	-0.135	0.055	0.048	0.055	-0.340	-0.030	0.050
S5-$\mu: 10\%; \sigma: 20\%$	-0.158	-0.113	0.053	0.044	0.052	-0.393	-0.051	0.050
S6-$\mu: 5\%; \sigma: 20\%$	-0.114	-0.089	0.050	0.039	0.048	-0.448	-0.072	0.050
S7-$\mu: 15\%; \sigma: 10\%$	-0.010	0.027	0.056	0.064	0.058	0.037	0.072	0.050
S8-$\mu: 10\%; \sigma: 10\%$	-0.023	0.005	0.054	0.056	0.054	-0.021	0.045	0.050
S9-$\mu: 5\%; \sigma: 10\%$	-0.021	-0.001	0.051	0.048	0.050	-0.075	0.020	0.050
Panel D: MVA $\alpha = 10$ (high risk aversion)								
S1-$\mu: 15\%; \sigma: 30\%$	-1.596	-1.395	0.043	-0.135	0.028	-4.329	-1.090	0.050
S2-$\mu: 10\%; \sigma: 30\%$	-1.262	-1.154	0.042	-0.100	0.025	-4.421	-1.071	0.050
S3-$\mu: 5\%; \sigma: 30\%$	-0.966	-0.933	0.040	-0.071	0.022	-4.439	-1.035	0.050
S4-$\mu: 15\%; \sigma: 20\%$	-1.156	-0.826	0.051	-0.013	0.045	-1.843	-0.437	0.050
S5-$\mu: 10\%; \sigma: 20\%$	-0.856	-0.650	0.049	-0.004	0.041	-1.891	-0.438	0.050
S6-$\mu: 5\%; \sigma: 20\%$	-0.610	-0.483	0.046	0.003	0.037	-1.941	-0.440	0.050
S7-$\mu: 15\%; \sigma: 10\%$	-0.457	-0.275	0.055	0.050	0.055	-0.336	-0.030	0.050
S8-$\mu: 10\%; \sigma: 10\%$	-0.362	-0.229	0.053	0.045	0.052	-0.398	-0.053	0.050
S9-$\mu: 5\%; \sigma: 10\%$	-0.237	-0.151	0.050	0.040	0.048	-0.452	-0.074	0.050

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The utility is calculated per each stylized aversion level increasing α factor.

A risk neutral investor (Panel A) in general values the risky strategy against the other strategies in most market conditions. The expected value of its wealth is, in most scenarios, higher when investing in the risky asset. In market conditions where the expected returns are lower, regardless of the volatility conditions, the choices are almost indifferent amongst all strategies except for the OBPI which is not selected in any circumstances.

As the level of risk aversion increases (Panel B), as expected under MVA, the required return per unit of risk increases. In the low level of risk aversion the strategy selection shift occurs from the risky asset towards protective strategies as CPPI (m=3) and TIPP. In high volatility conditions with low expected returns the preferred strategy is the CPPI(m=1). In low volatile market conditions and high and medium returns the preferred strategy is the risky asset. In low return environment investors shift from risky asset exposure to protective strategies like CPPI(m=1), CPPI(m=3) and TIPP. In medium volatile conditions the shift from risky asset exposure to protective strategies (CPPI(m=1) and TIPP occurs at the lower end of returns.

A medium risk aversion investor (Panel C) in low volatile conditions selects a 50:50 risky exposure and in low return environment changes towards a protective strategy (CPPI(m=1) or TIPP).

The higher the volatility the more protective strategies are chosen by medium risk version investors.

For investors with high levels of risk aversion (Panel D) this trend is reinforced, and there is a generalized exposure to the risk free asset.

General results point to a risk exposure towards the risk free - fly to safety - on investors with medium to high level of risk aversion. As the level of risk aversion decreases the return seeking investments tend to become more attractive as the return per unit of risk is perceived with a higher utility. Investors with decreasing level of risk aversion to medium levels, depending on market conditions, tend to exhibit a safety attitude benefiting protective strategies (the more simple portfolio strategies like CPPI and TIPP) in more volatile conditions. On less volatile conditions investors seek return and show more exposure to risky assets.

The graphics on return and risk deploys strategy results based on different scenarios for market conditions. The portfolio insurance strategies are set with a floor of 100%.

Figure 4.3: Return and risk space - Floor set at 100% for portfolio insurance strategies

To test these results we also change the floor from 100% to 80%. The distribution of returns is presented in Table 4.4.

Table 4.7: Mean variance analysis(Floor = 80%)

Floor = 80%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: MVA $\alpha^2 = 0$ (risk neutral)								
S1-μ : 15%; σ : 30%	0.150	0.099	0.078	0.130	0.116	0.151	0.100	0.050
S2-μ : 10%; σ : 30%	0.099	0.055	0.064	0.088	0.082	0.100	0.075	0.050
S3-μ : 5%; σ : 30%	0.051	0.051	0.051	0.051	0.051	0.050	0.050	0.050
S4-μ : 15%; σ : 20%	0.160	0.160	0.078	0.136	0.120	0.151	0.100	0.050
S5-μ : 10%; σ : 20%	0.104	0.081	0.064	0.091	0.084	0.100	0.075	0.050
S6-μ : 5%; σ : 20%	0.051	0.033	0.051	0.051	0.051	0.050	0.050	0.050
S7-μ : 15%; σ : 10%	0.162	0.156	0.078	0.138	0.122	0.150	0.100	0.050
S8-μ : 10%; σ : 10%	0.105	0.099	0.064	0.092	0.086	0.100	0.075	0.050
S9-μ : 5%; σ : 10%	0.052	0.046	0.051	0.051	0.051	0.050	0.050	0.050
Panel B: MVA $\alpha = 2$ (low risk aversion)								
S1-μ : 15%; σ : 30%	-0.014	-0.025	0.066	0.029	0.049	-0.019	0.059	0.050
S2-μ : 10%; σ : 30%	-0.052	-0.057	0.053	0.000	0.020	-0.075	0.032	0.050
S3-μ : 5%; σ : 30%	-0.085	-0.085	0.041	-0.025	-0.006	-0.129	0.008	0.050
S4-μ : 15%; σ : 20%	0.079	0.087	0.073	0.083	0.087	0.082	0.085	0.050
S5-μ : 10%; σ : 20%	0.026	0.013	0.059	0.046	0.053	0.026	0.058	0.050
S6-μ : 5%; σ : 20%	-0.022	-0.029	0.047	0.013	0.022	-0.029	0.031	0.050
S7-μ : 15%; σ : 10%	0.142	0.136	0.076	0.123	0.114	0.142	0.101	0.050
S8-μ : 10%; σ : 10%	0.085	0.079	0.063	0.079	0.077	0.085	0.073	0.050
S9-μ : 5%; σ : 10%	0.031	0.026	0.050	0.050	0.043	0.031	0.046	0.050
Panel C: MVA $\alpha = 5$ (medium risk aversion)								
S1-μ : 15%; σ : 30%	-0.879	-0.680	0.004	-0.504	-0.303	-0.965	-0.193	0.050
S2-μ : 10%; σ : 30%	-0.844	-0.646	-0.005	-0.466	-0.308	-1.017	-0.207	0.050
S3-μ : 5%; σ : 30%	-0.800	-0.800	-0.013	-0.428	-0.308	-1.076	-0.221	0.050
S4-μ : 15%; σ : 20%	-0.348	-0.295	0.045	-0.192	-0.087	-0.340	-0.029	0.050
S5-μ : 10%; σ : 20%	-0.382	-0.344	0.034	-0.190	-0.111	-0.393	-0.051	0.050
S6-μ : 5%; σ : 20%	-0.405	-0.353	0.023	-0.186	-0.130	-0.449	-0.072	0.050
S7-μ : 15%; σ : 10%	0.036	0.030	0.069	0.045	0.067	0.036	0.072	0.050
S8-μ : 10%; σ : 10%	-0.021	-0.026	0.056	0.013	0.032	-0.021	0.045	0.050
S9-μ : 5%; σ : 10%	-0.074	-0.078	0.044	0.044	0.000	-0.074	0.020	0.050
Panel D: MVA $\alpha = 10$ (high risk aversion)								
S1-μ : 15%; σ : 30%	-3.968	-3.018	-0.218	-2.407	-1.560	-4.345	-1.094	0.050
S2-μ : 10%; σ : 30%	-3.673	-2.750	-0.211	-2.131	-1.478	-4.383	-1.063	0.050
S3-μ : 5%; σ : 30%	-3.353	-3.353	-0.204	-1.866	-1.387	-4.459	-1.039	0.050
S4-μ : 15%; σ : 20%	-1.872	-1.659	-0.055	-1.176	-0.709	-1.847	-0.438	0.050
S5-μ : 10%; σ : 20%	-1.840	-1.617	-0.058	-1.033	-0.697	-1.889	-0.438	0.050
S6-μ : 5%; σ : 20%	-1.773	-1.508	-0.062	-0.896	-0.673	-1.947	-0.441	0.050
S7-μ : 15%; σ : 10%	-0.341	-0.346	0.044	-0.236	-0.098	-0.341	-0.031	0.050
S8-μ : 10%; σ : 10%	-0.398	-0.402	0.033	-0.222	-0.130	-0.398	-0.053	0.050
S9-μ : 5%; σ : 10%	-0.451	-0.452	0.023	0.023	-0.155	-0.452	-0.074	0.050

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The utility is calculated per each stylized aversion level increasing α factor.

The change of the floor from 100% to 80% increases the potential for the returns from portfolio insurance strategies, although in the case of the SLPI strategy the leverage tends to increase the number of sell-offs of the risky asset. For a risk neutral investor (Panel A) the most simple portfolio strategy — Stop Loss — is almost indifferent compared with a risky asset portfolio in every scenario.

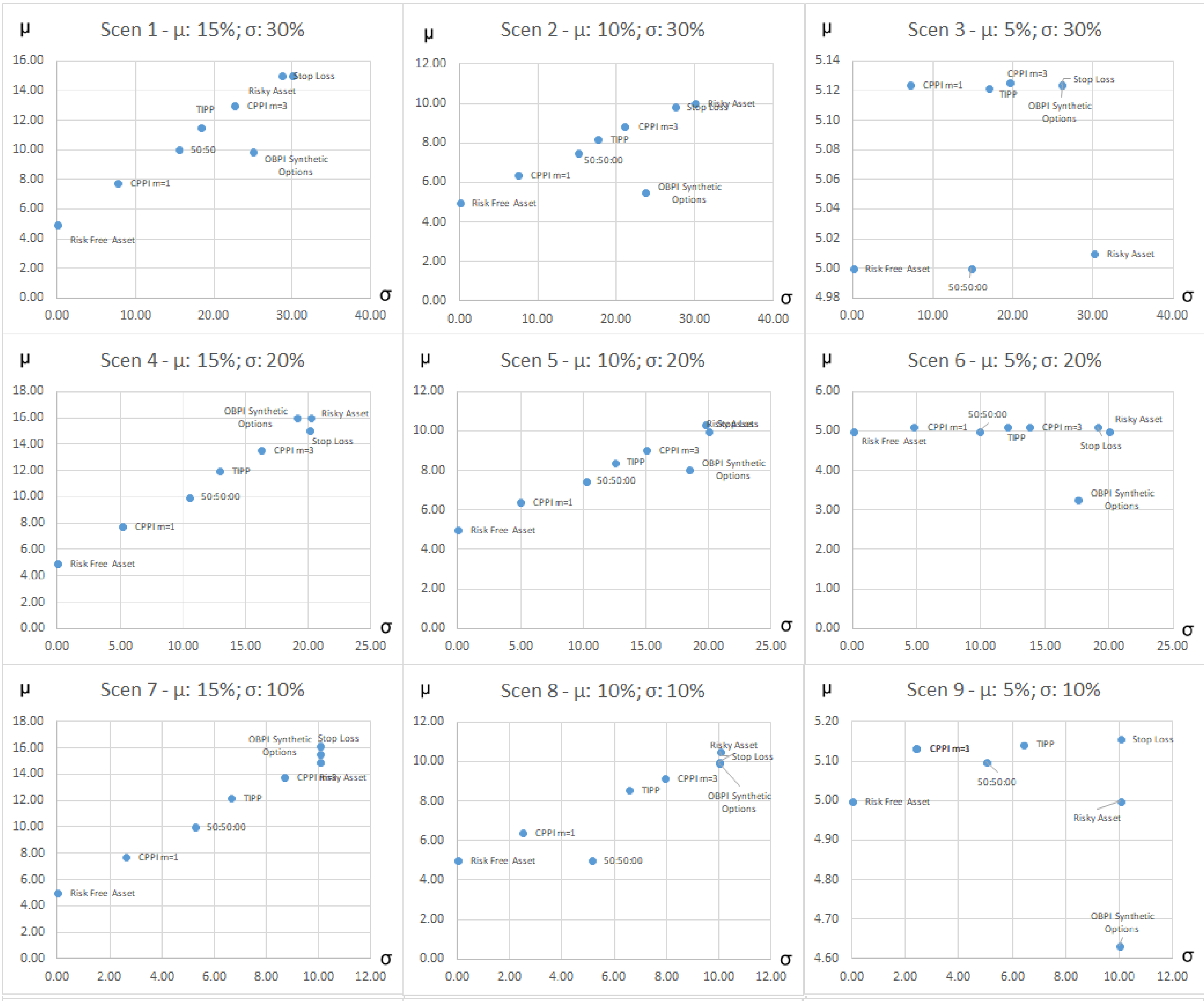
In volatile market conditions a low risk aversion investor values simple protective strategies (CPPI (m=1)) as the potential for upward movements is more frequent if the floor is decreased, thus allowing more risk appetite. However, in low return scenarios with high and medium volatility, a low risk investor tends to favor a risk-free exposure.

In low volatility and low return there is indifference between the risk-free and CPPI strategies. In medium return scenario with medium volatility a CPPI (m=1) is the most valuable strategy, which is the same for high volatility conditions and medium returns. The reason is based on the lower floor for protective strategies. In low volatility conditions with medium to high returns the choices between exposure to risky asset or SLPI strategy is indifferent.

For medium to high risk aversion investors the strategy decision relays generally on the risk-free exposure, reflecting a fly to safety approach with no valuation on the more flexible definition of minimum guaranty at the maturity.

The graphics on return and risk deploys strategy results based on different scenarios for market conditions. The portfolio insurance strategies are set with a floor of 80%.

Figure 4.4: Return and risk space-Floor set at 80% for portfolio insurance strategies



4.4.3 Expected Utility Analysis

The analysis is based on the assessment of different risk profiles per each scenario. The returns' distribution simulated on 9 scenarios depicts a different approach to risk that lead to a possible aggregation of homogeneous behaviours. We use four different utility functions with a wide level of risk aversion in order to test numerically in what market conditions investors are selecting portfolio insurance strategies.

Table 4.8: Scenario 1 - Utility (Floor = 100%)

Floor 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Linear utility: $U(X) = a + bX$								
$a = 2, b = 5$	223.18	220.39	216.33	218.83	216.75	237.36	226.31	215.25
Quadratic utility: $U(X) = X - \frac{b}{2}X^2$								
$b = 0.025$	-47.75	-42.86	-33.94	-36.69	-34.33	-68.42	-46.37	-33.02
$b = 1$	-6,164	-5,915	-5,478	-5,637	-5,502	-7,268	-6,170	-5,421
$b = 5$	-31,257	-30,004	-27,815	-28,615	-27,934	-36,804	-31,294	-27,524
Exponential utility: $U(X) = \frac{1}{a}(1 - e^{-aX})$								
$a = 0.025$	0.067	0.058	0.055	0.065	0.057	0.104	0.089	0.050
$a = 1$	0.051	0.044	0.053	0.061	0.055	0.058	0.074	0.049
$a = 5$	0.021	0.006	0.048	0.049	0.048	-0.162	0.032	0.044
Logarithmic utility: $U(X) = \frac{X^{1-c}}{1-c}$								
$c = 0.5$	20.779	20.669	20.558	20.668	20.577	21.317	20.973	20.506
$c = 1, U(X) = \ln(X)$	4.672	4.663	4.660	4.670	4.662	4.710	4.694	4.655
$c = 1.25$	-1.245	-1.248	-1.248	-1.245	-1.247	-1.236	-1.238	-1.249
$c = 1.5$	-0.194	-0.195	-0.195	-0.194	-0.194	-0.192	-0.192	-0.195

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The utility is calculated per each utility function.

Investors in a high volatile market with high level of expected return on the risky asset whose utility is represented by a linear function will have higher value by exposure to the risky asset. Linear transformations do not change the order amongst the selected strategies. Under a quadratic utility valuation, which implies globally increasing absolute risk aversion, an investor with low b factor tends to select the CPPI(m=1) strategy. As b increases the risk aversion also increases thus leading investors into risk-free assets. In this cases, the possibility of benefiting from a protective strategy and gain from the upside of the risky asset is not valued. An investor with a negative exponential utility function has a constant attitude towards risk expressed in absolute dollar terms. In this case an increasing coefficient of risk aversion changes the selection from the risky asset exposure to a 50:50 strategy and for higher coefficient a change towards CPPI and TIPP strategies. An investor with a low risk aversion, represented with the logarithmic utility function, will select the risky asset in any circumstances.

Table 4.9: Scenario 2 - Utility (Floor = 100%)

Floor 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Linear utility: $U(X) = a + bX$								
$a = 2, b = 5$	218.96	215.79	215.79	216.94	216.01	226.15	220.70	215.25
Quadratic utility: $U(X) = X - \frac{b}{2}X^2$								
$b = 0.025$	-42.28	-37.85	-33.49	-34.95	-33.72	-56.73	-41.16	-33.02
$b = 1$	-5,864	-5,625	-5,450	-5,531	-5,464	-6,582	-5,853	-5,421
$b = 5$	-29,746	-28,544	-27,670	-28,078	-27,740	-33,350	-29,695	-27,524
Exponential utility: $U(X) = \frac{1}{a}(1 - e^{-aX})$								
$a = 0.025$	0.052	0.039	0.052	0.056	-0.451	0.054	0.064	0.050
$a = 1$	0.040	0.028	0.051	0.053	-0.567	0.010	0.051	0.049
$a = 5$	0.016	-0.004	0.046	0.044	-1.699	-0.269	0.012	0.044
Logarithmic utility: $U(X) = \frac{X^{1-c}}{1-c}$								
$c = 0.5$	20.600	20.463	20.532	20.579	20.542	20.795	20.709	20.506
$c = 1, U(X) = \ln(X)$	4.657	4.645	4.658	4.661	4.658	4.660	4.669	4.655
$c = 1.25$	-1.250	-1.253	-1.248	-1.247	-1.248	-1.251	-1.246	-1.249
$c = 1.5$	-0.195	-0.197	-0.195	-0.195	-0.195	-0.197	-0.194	-0.195

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The utility is calculated per each utility function.

Investors in a high volatile market with medium level of expected return on the risky asset, whose utility is represented by a linear function, will have higher value by exposure to the risky asset. Under a quadratic utility valuation an investor with low b factor tends to select the risk-free exposure. As b increases the risk aversion also increases thus keeping investors in risk-free assets. In this cases, the possibility of benefiting from a protective strategy and gain from the upside of the risky asset is not valued. Investors with a negative exponential utility function change the selection from the 50:50 strategy towards CPPI strategies as coefficient of risk aversion increases. An investor with a low risk aversion, represented with the logarithmic utility function, will select the risky asset and as risk coefficient increases there is a drift from risky asset to CPPIm=1 and TIPP and also towards 50:50 strategy.

Table 4.10: (continued)

Floor 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free

Table 4.10: Scenario 3 - Utility (Floor = 100%)

Floor 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Linear utility: $U(X) = a + bX$								
$a = 2, b = 5$	215.23	211.86	215.25	215.25	215.25	215.24	215.25	215.25
Quadratic utility: $U(X) = X - \frac{b}{2}X^2$								
$b = 0.025$	-37.57	-33.66	-33.05	-33.42	-33.10	-46.01	-36.26	-33.02
$b = 1$	-5,602	-5,380	-5,422	-5,437	-5,424	-5,940	-5,550	-5,421
$b = 5$	-28,432	-27,315	-27,530	-27,605	-27,539	-30,121	-28,173	-27,524
Exponential utility: $U(X) = \frac{1}{a}(1 - e^{-aX})$								
$a = 0.025$	0.038	0.023	0.050	0.049	0.050	0.004	0.039	0.050
$a = 1$	0.029	0.014	0.049	0.046	0.048	-0.041	0.028	0.049
$a = 5$	0.010	-0.013	0.044	0.039	0.043	-0.396	-0.010	0.044
Logarithmic utility: $U(X) = \frac{X^{1-c}}{1-c}$								
$c = 0.5$	20.439	20.285	20.506	20.499	20.504	20.277	20.449	20.506
$c = 1, U(X) = \ln(X)$	4.643	4.628	4.655	4.654	4.655	4.610	4.644	4.655
$c = 1.25$	-1.254	-1.258	-1.249	-1.250	-1.249	-1.267	-1.254	-1.249
$c = 1.5$	-0.197	-0.198	-0.195	-0.195	-0.195	-0.202	-0.197	-0.195

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The utility is calculated per each utility function.

Investors in a high volatile market with low level of expected return on the risky asset, whose utility is represented by a linear function, present no relevant preference except for OBPI strategy that is not a choice, compared with the other strategies. Under a quadratic utility valuation an investor with low b factor tends to select the risk-free exposure. As b increases the risk aversion also increases thus keeping investors in risk-free assets. However, due to the low expected return, the CPPI(m=1) presents a value very close to the risk-free. Investors with a negative exponential utility function are indifferent between risk-free asset and the CPPI(m=1). Except when $a = 1$, where the TPPI present the highest value. An investor with a low risk aversion, represented with the logarithmic utility function, will select either the risky asset or the CPPI(m=1) and as risk coefficient increases there is a wider choice between the risk-free, CPPI(m=1) and TIPP.

Table 4.11: Scenario 4 - Utility (Floor = 100%)

Floor 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Linear utility: $U(X) = a + bX$								
$a = 2, b = 5$	225.99	224.12	216.32	218.79	216.78	237.11	226.18	215.25
Quadratic utility: $U(X) = X - \frac{b}{2}X^2$								
$b = 0.025$	-47.10	-43.73	-33.91	-36.19	-34.31	-59.18	-44.01	-33.02
$b = 1$	-6,193	-6,022	-5,477	-5,616	-5,502	-6,893	-6,073	-5,421
$b = 5$	-31,408	-30,549	-27,807	-28,509	-27,933	-34,931	-30,809	-27,524
Exponential utility: $U(X) = \frac{1}{a}(1 - e^{-aX})$								
$a = 0.025$	0.086	0.081	0.055	0.066	0.057	0.128	0.095	0.050
$a = 1$	0.072	0.070	0.053	0.063	0.055	0.103	0.086	0.049
$a = 5$	0.039	0.041	0.048	0.053	0.049	0.026	0.059	0.044
Logarithmic utility: $U(X) = \frac{X^{1-c}}{1-c}$								
$c = 0.5$	20.948	20.882	20.558	20.673	20.580	21.438	21.004	20.506
$c = 1, U(X) = \ln(X)$	4.691	4.687	4.660	4.671	4.662	4.734	4.700	4.655
$c = 1.25$	-1.239	-1.240	-1.248	-1.244	-1.247	-1.226	-1.236	-1.249
$c = 1.5$	-0.192	-0.192	-0.195	-0.194	-0.194	-0.188	-0.191	-0.195

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The utility is calculated per each utility function.

Investors in a medium volatile market with high level of expected return on the risky asset, whose utility is represented by a linear function, select the risk asset exposure. Under a quadratic utility valuation an investor with low b factor tends to select the risk-free exposure. As b increases the risk aversion also increases thus keeping investors in risk-free assets

Investors with a negative exponential utility function value the risky asset exposure, but as the risk aversion increases they seek security on 50:50 strategy and also CPPI(m=3). An investor with a low risk aversion, represented with the logarithmic utility function, will select the risky asset. As risk coefficient increases there is no change on the strategy, due to the high expected return.

Table 4.12: Scenario 5 - Utility (Floor = 100%)

Floor 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Linear utility: $U(X) = a + bX$								
$a = 2, b = 5$	220.02	218.22	215.79	216.93	216.04	226.17	220.71	215.25
Quadratic utility: $U(X) = X - \frac{b}{2}X^2$								
$b = 0.025$	-40.52	-37.88	-33.47	-34.58	-33.70	-48.50	-39.11	-33.02
$b = 1$	-5,814	-5,673	-5,449	-5,516	-5,463	-6,253	-5,771	-5,421
$b = 5$	-29,499	-28,791	-27,667	-28,003	-27,737	-31,707	-29,286	-27,524
Exponential utility: $U(X) = \frac{1}{a}(1 - e^{-aX})$								
$a = 0.025$	0.062	0.054	0.052	0.057	0.054	0.080	0.070	0.050
$a = 1$	0.052	0.048	0.051	0.055	0.052	0.059	0.063	0.049
$a = 5$	0.029	0.027	0.046	0.048	0.046	-0.021	0.041	0.044
Logarithmic utility: $U(X) = \frac{X^{1-c}}{1-c}$								
$c = 0.5$	20.682	20.610	20.532	20.584	20.544	20.927	20.744	20.506
$c = 1, U(X) = \ln(X)$	4.667	4.662	4.658	4.662	4.659	4.686	4.676	4.655
$c = 1.25$	-1.246	-1.248	-1.248	-1.247	-1.248	-1.241	-1.243	-1.249
$c = 1.5$	-0.194	-0.195	-0.195	-0.194	-0.195	-0.193	-0.193	-0.195

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The utility is calculated per each utility function.

Investors in a medium volatile market with medium level of expected return on the risky asset, whose utility is represented by a linear function, select the risk asset exposure. Under a quadratic utility valuation an investor selects the risk-free exposure regardless of the b factor.

Investors with a negative exponential utility function value the risky asset exposure, but as the risk aversion increases they seek security on 50:50 strategy and also CPPI(m=3). An investor with a low risk aversion, represented with the logarithmic utility function, will select the risky asset. As risk coefficient increases there is no change on the strategy, due to the high expected return. Results, on general, are very similar with the outcomes of scenario 4.

Table 4.13: Scenario 6 - Utility (Floor = 100%)

Floor 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Linear utility: $U(X) = a + bX$								
$a = 2, b = 5$	215.17	213.48	215.24	215.22	215.23	214.99	215.12	215.25
Quadratic utility: $U(X) = X - \frac{b}{2}X^2$								
$b = 0.025$	-35.40	-33.29	-33.02	-33.13	-33.04	-38.42	-34.31	-33.02
$b = 1$	-5,514	-5,397	-5,421	-5,424	-5,421	-5,631	-5,470	-5,421
$b = 5$	-27,992	-27,401	-27,523	-27,542	-27,526	-28,577	-27,770	-27,524
Exponential utility: $U(X) = \frac{1}{a}(1 - e^{-aX})$								
$a = 0.025$	0.042	0.036	0.050	0.049	0.050	0.028	0.044	0.050
$a = 1$	0.036	0.030	0.049	0.048	0.048	0.009	0.039	0.049
$a = 5$	0.020	0.015	0.044	0.042	0.044	-0.084	0.020	0.044
Logarithmic utility: $U(X) = \frac{X^{1-c}}{1-c}$								
$c = 0.5$	20.463	20.390	20.505	20.502	20.505	20.391	20.474	20.506
$c = 1, U(X) = \ln(X)$	4.647	4.641	4.655	4.654	4.655	4.634	4.650	4.655
$a = 1.25$	-1.252	-1.254	-1.249	-1.249	-1.249	-1.257	-1.251	-1.249
$a = 1.5$	-0.196	-0.197	-0.195	-0.195	-0.195	-0.198	-0.196	-0.195

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The utility is calculated per each utility function.

Investors in a medium volatile market with low level of expected return on the risky asset, whose utility is represented by a linear function, select the risky asset exposure. Under a quadratic utility valuation an investor with low b factor tends to select the risk-free exposure. As b increases the risk aversion also increases thus keeping investors in risk-free assets.

Investors with a negative exponential utility function value the risk-free asset exposure and CPPI and TIPP, and as the risk aversion increases they continue to seek security. An investor with a low risk aversion, represented with the logarithmic utility function, will select the risk-free asset. As risk coefficient increases there is a wider scope os choices between CPPI, TIPP and risk-free strategies. Results from market conditions as in scenario 6 determine similar choices as the scenario 3. There is a shift towards safety, except for the linear utility function.

Table 4.14: Scenario 7 - Utility (Floor = 100%)

Floor 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Linear utility: $U(X) = a + bX$								
$a = 2, b = 5$	232.91	230.55	216.33	218.82	216.81	237.31	226.28	215.25
Quadratic utility: $U(X) = X - \frac{b}{2}X^2$								
$b = 0.025$	-50.37	-47.54	-33.90	-36.02	-34.30	-54.18	-42.80	-33.02
$b = 1$	-6,459	-6,300	-5,477	-5,610	-5,502	-6,697	-6,027	-5,421
$b = 5$	-32,751	-31,950	-27,807	-28,479	-27,934	-33,951	-30,577	-27,524
Exponential utility: $U(X) = \frac{1}{a}(1 - e^{-aX})$								
$a = 0.025$	0.124	0.116	0.055	0.067	0.057	0.144	0.100	0.050
$a = 1$	0.112	0.106	0.054	0.064	0.056	0.130	0.094	0.049
$a = 5$	0.077	0.077	0.048	0.056	0.050	0.090	0.074	0.044
Logarithmic utility: $U(X) = \frac{X^{1-c}}{1-c}$								
$c = 0.5$	21.318	21.217	20.559	20.678	20.582	21.528	21.030	20.506
$c = 1, U(X) = \ln(X)$	4.730	4.721	4.660	4.672	4.662	4.750	4.705	4.655
$c = 1.25$	-1.227	-1.229	-1.248	-1.244	-1.247	-1.220	-1.234	-1.249
$c = 1.5$	-0.188	-0.189	-0.195	-0.193	-0.194	-0.186	-0.190	-0.195

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The utility is calculated per each utility function.

Investors in a low volatile market with high level of expected return on the risky asset, whose utility is represented by a linear function, select the risky asset exposure. Investor with quadratic utility values a strategy based on risk-free assets.

Investors with a negative exponential utility function value the risky asset strategy in any circumstances. An investor with a low risk aversion, represented with the logarithmic utility function, will select the risky asset. As risk coefficient increases there is no change of strategy. These market conditions are relatively rare and may not occur for longer periods of time, but if they persist less risk averse investor favour expected return.

Table 4.15: Scenario 8 - Utility (Floor = 100%)

Floor 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Linear utility: $U(X) = a + bX$								
$a = 2, b = 5$	223.00	221.56	215.78	216.92	216.03	226.05	220.53	215.00
Quadratic utility: $U(X) = X - \frac{b}{2}X^2$								
$b = 0.025$	-40.96	-39.25	-33.45	-34.42	-33.66	-43.72	-37.78	-32.81
$b = 1$	-5,890	-5,793	-5,448	-5,509	-5,462	-6,059	-5,714	-5,408
$b = 5$	-29,884	-29,397	-27,663	-27,970	-27,730	-30,738	-29,002	-27,458
Exponential utility: $U(X) = \frac{1}{a}(1 - e^{-aX})$								
$a = 0.025$	0.081	0.076	0.052	0.058	0.054	0.095	0.074	0.050
$a = 1$	0.074	0.071	0.051	0.056	0.52	0.086	0.070	0.049
$a = 5$	0.053	0.054	0.046	0.050	0.047	0.059	0.057	0.044
Logarithmic utility: $U(X) = \frac{X^{1-c}}{1-c}$								
$c = 0.5$	20.856	20.795	20.532	20.586	20.544	21.000	20.755	20.494
$c = 1, U(X) = \ln(X)$	4.687	4.682	4.658	4.663	4.659	4.700	4.679	4.654
$c = 1.25$	-1.240	-1.241	-1.248	-1.247	-1.248	-1.236	-1.242	-1.250
$c = 1.5$	-0.192	-0.193	-0.195	-0.194	-0.195	-0.191	-0.193	-0.195

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The utility is calculated per each utility function.

Investors in a low volatile market with medium level of expected return on the risky asset, whose utility is represented by a linear function, select the risky asset exposure. Under a quadratic utility valuation an investor selects the risk-free strategy.

Investors with a negative exponential utility function value the risky asset. An investor represented with the logarithmic utility function will select the risky asset.

Table 4.16: Scenario 9 - Utility (Floor = 100%)

Floor 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Linear utility: $U(X) = a + bX$								
$a = 2, b = 5$	215.17	214.67	215.25	215.24	215.24	215.15	215.20	215.25
Quadratic utility: $U(X) = X - \frac{b}{2}X^2$								
$b = 0.025$	-33.83	-33.14	-33.02	-33.04	-33.02	-34.32	-33.32	-33.02
$b = 1$	-5,452	-5,414	-5,421	-5,421	-5,420	-5,471	-5,432	-5,421
$b = 5$	-27,678	-27,491	-27,523	-27,526	-27,523	-27,774	-27,579	-27,524
Exponential utility: $U(X) = \frac{1}{a}(1 - e^{-aX})$								
$a = 0.025$	0.047	0.045	0.050	0.050	0.050	0.044	0.048	0.050
$a = 1$	0.043	0.042	0.049	0.048	0.049	0.039	0.046	0.049
$a = 5$	0.032	0.033	0.044	0.044	0.044	0.018	0.038	0.044
Logarithmic utility: $U(X) = \frac{X^{1-c}}{1-c}$								
$c = 0.5$	20.487	20.467	20.506	20.505	20.505	20.475	20.497	20.506
$c = 1, U(X) = \ln(X)$	4.652	4.650	4.655	4.655	4.655	4.650	4.654	4.655
$c = 1.25$	-1.250	-1.251	-1.249	-1.249	-1.249	-1.251	-1.250	-1.249
$c = 1.5$	-0.196	-0.196	-0.195	-0.195	-0.195	-0.196	-0.195	-0.195

This table shows descriptive statistics of the distributions of returns at maturity for the portfolio insurance and the benchmark strategies. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The utility is calculated per each utility function.

Investors in a low volatile market with low level of expected return on the risky asset, whose utility is represented by a linear function, select the CPPI(m=1). Under a quadratic utility valuation an investor with low b factor tends to select the risk-free exposure, CPPI(m=1) and TIPP. As b increases the risk aversion also increases but exposure remains the same. Investors with a negative exponential utility function value the risk-free asset exposure and CPPI and TIPP, and as the risk aversion increases they continue to seek security either from risk-free of portfolio insurance.

An investor with a low risk aversion, represented with the logarithmic utility function, will select the risk-free asset, CPPI and TIPP. As risk coefficient increases the strategy remains stable.

4.5 Conclusions

In our work we set different risk aversion levels for several descriptive utility functions. The numerical examples, using a stochastic simulation of the returns for portfolio insurance strategies and benchmark investments, create conditions to define homogeneous groups of individual investors for whom the selection of protective strategies can be relevant. In order to identify portfolio insurance strategies that are attractive for investors we defined market conditions. Based on this procedure it is possible to check if categories of investors with certain type of utility functions value protection and, simultaneously, are keen on the upside potential of risky asset.

The determinants of choices are set by market conditions used for the simulation. When markets present high level of volatility and low returns, in absolute or relative terms, except for the investors whose utility is characterized by a quadratic function, where risk free exposure is always more value, investors somehow tend to have a wider scope of choices by using protective strategies but also with a twist into the upside potential. Results from scenarios 3, 6 and 9 represent a group of investors that tend to define some security on the expected return, but value the possibility of benefiting from market upside. However, as the level of expected return increases there is a shift towards risky asset exposure and also CPPI with lower multiplier.

Investors with utility represented by a quadratic utility function value security. When expected return is low and volatility is within medium to low levels, there is a change from absolute security to some level of security given by protective strategies that set a floor, but permits some benefit from the upside potential of risky assets.

Remaining groups of investors, either with exponential or logarithmic utility functions, do not value protective strategies, because they are very focused on expected return. However, as the risk aversion level increases, there is a reduction from total exposure to risky markets to some level of protection, by 50:50 strategies or CPPI with multiplier of 1 or 3, depending on the volatility level and expected returns.

Table 4.17: Summary of results - Investors' strategies choices

Floor =100%	S 1 μ=15%; σ=30%	S 2 μ=10%; σ=30%	S 3 μ=5%; σ=30%	S 4 μ=15%; σ=20%	S 5 μ=10%; σ=20%	S 6 μ=5%; σ=20%	S 7 μ=15%; σ=10%	S 8 μ=10%; σ=10%	S 9 μ=5%; σ=10%
Linear utility	Risky asset	Risky asset	CPPI m=1 CPPI m=3 TIPP 50:50 Risk free	Risky asset	Risky asset	Risk free	Risky asset	Risky asset	CPPI m=1 Risk free
Quadratic utility									
b = 0.025	Risk free	Risk free	Risk free	Risk free	Risk free	CPPI m=1 Risk free	Risk free	Risk free	CPPI m=1 TIPP Risk free
b = 1	Risk free	Risk free	Risk free	Risk free	Risk free	CPPI m=1 TIPP Risk free	Risk free	Risk free	TIPP
b = 5	Risk free	Risk free	Risk free	Risk free	Risk free	CPPI m=1	Risk free	Risk free	CPPI m=1 TIPP
Exponential utility									
a = 0.025	Risky asset	50:50	CPPI m=1 TIPP Risk free	Risky asset	Risky asset	CPPI m=1 TIPP Risk free	Risky asset	Risky asset	CPPI m=1 CPPI m=3 TIPP Risk free
a = 1	50:50	CPPI m=3	CPPI m=1 Risk free	Risky asset	50:50	OBPI	Risky asset	Risky asset	CPPI m=1 TIPP Risk free
a = 5	CPPI m=3	CPPI m=1	CPPI m=1 Risk free	50:50	CPPI m=3	OBPI	Risky asset	Risky asset	CPPI m=1 CPPI m=3 TIPP Risk free
Logarithmic utility									
c = 0.5	Risky asset	Risky asset	CPPI m=1 Risk free	Risky asset	Risky asset	Risk free	Risky asset	Risky asset	CPPI m=1 Risk free
c = 1	Risky asset	50:50	CPPI m=1 TIPP Risk free	Risky asset	Risky asset	CPPI m=1 TIPP Risk free	Risky asset	Risky asset	CPPI m=1 CPPI m=3 TIPP Risk free
c = 1.25	Risky asset	50:50	CPPI m=1 TIPP Risk free	Risky asset	Risky asset	CPPI m=1 CPPI m=3 TIPP Risk free	Risky asset	Risky asset	CPPI m=1 CPPI m=3 TIPP Risk free
c = 1.5	Risky asset 50:50	50:50	CPPI m=1 CPPI m=3 TIPP Risk free	Risky asset	Risky asset 50:50	CPPI m=1 CPPI m=3 TIPP Risk free	Risky asset	Risky asset	CPPI m=1 CPPI m=3 TIPP 50:50 Risk free

Under MVA framework, general results reveal a risk exposure towards the risk free - fly to safety - on investors with medium to high level of risk aversion. As the level of risk aversion decreases, the return seeking investments tend to become more attractive as the return per unit of risk is perceived with higher utility. Investors with decreasing level of risk aversion, depending on market conditions, tend to exhibit a safety attitude benefiting protective strategies (the more simple portfolio strategies like CPPI and TIPP) in more volatile conditions. On less volatile conditions investors seek return and show more exposure to risky assets. Additionally, a general result from the analysis is that the higher the expected return the lower is the level of protection required by investors, and the higher the level of volatility the higher is the protection required.

These results on investors with descriptive utility functions support the idea that there are not potential buyers of portfolio insurance strategies, except in very specific market conditions: scenarios of low expected returns and increasing volatility. That situation is also visible under MVA, where a flight to safety also depicts some valuation on protective strategies with upside potential.

This study does not take into account the level of wealth on each group of investors, but having tractable utility functions this point is not relevant, except for different wealth segments of investors, which are out of scope on our work as there is no impact on simulation results. We expect to include in future work some of the utility functions that define a bridge between expected utility and prospect theory.

The absence of clear circumstances where portfolio insurance strategies are demanded by investors and the changing market conditions may raise the question of adequacy of investment solutions to individual investors. Portfolio insurance solutions are complex investments that need to be matched with individual investors risk profile. When financial institutions sell these protective solutions they must, under current regulation, avoid mis-selling. The most common financial frameworks as are the MVA and EUT may not support entirely the definition of risk profile of individual investors to whom a portfolio insurance would match.

The results do not support a clear explanation for individual investors' choices into portfolio insurance. In the next chapter we introduce behavioural finance factors in order to evaluate the adequacy of a non-prescriptive utility valuation framework to evaluate the selection of protective investment strategies. Based on the same distribution results from the simulation performed we compare standard utility functions with cumulative prospect value functions.

Chapter 5

Investor's Perspective on Portfolio Insurance

Expected Utility *vs* Prospect Theories

In spite of controversial opinions surrounding the benefits for investors, along with analysis that portfolio insurance strategies may have contributed to exacerbate financial crises (Shiller, 1988; Dybvig, 1988; Brennan and Schwartz, 1989; Rubinstein, 1999; Tucker, 2005; Leland, 2011; Vandenbroucke, 2015), the fact is that the idea of insurance of risky investments remains attractive for a large segment of investors (Pain and Rand, 2008; Vrecko and Branger, 2009).

Some theoretical studies, such as those of Benninga and Blume (1985), have questioned the optimality of portfolio insurance strategies under investors' expected utility functions. Black and Perold (1992) documented that a CPPI strategy, with unconstrained borrowing, only maximizes expected utility when investors show Hyperbolic Absolute Risk Aversion (HARA) utility functions.

The classical Expected Utility Theory (EUT) establishes the foundation to explain investors' choices under uncertainty. However, it has faced difficulties in explaining individual's investment decisions.

The traditional finance paradigm pursues an explanation for financial markets using models in which agents are rational, based on two assumptions: first, when agents receive new information, they update their beliefs correctly; second, given their beliefs, agents make choices that are normatively acceptable. This traditional framework is appealingly simple, and it would be very satisfying if its predictions were confirmed in the data. However, after years of effort, it has become clear that basic facts about the aggregate stock market, the cross-section of average returns and individual trading behaviour are not easily understood in this framework (Barberis and Thaler, 2003).

Behavioural finance is a new approach to financial markets that emerged in response to the difficulties faced by the traditional paradigm. In broad terms, it argues that some financial phenomena can be better understood using models in which some agents are not fully rational. Portfolio insurance allows downside protection of wealth, which is an objective aligned with the experimental results of several studies on behavioural finance (see for example Tversky and Kahneman, 1974, 1979, 1992, and Barberis and Thaler, 2003). Investors tend to be more averse to losses, i.e., they are more reactive to losses than gains, which implies that it may exist a match between this behaviour with the characteristics of portfolio insurance. In fact, prospect and cumulative prospect theories (Tversky and Kahneman, 1974, 1992) can support investors' decision for downside protection mechanisms, as investment decisions in this framework are not assessed by the amount of expected final wealth, but on the valuation of outcomes, taking into consideration the prospects of both gains and losses (Barberis and Tahler, 2003).

In this chapter we use portfolio insurance strategies and plain investments on stock market and risk-free assets to find how capable are these alternative theories — EUT, PT, CPT — to explain the persistent popularity of portfolio insurance. Concretely we use a set of popular portfolio insurance strategies benchmarked against plain strategies: a risk-free cash investment, a passive stock market investment and a buy and hold investment between risk-free cash and equities. For all these investment strategies we compare the return distributions and compute standard performance measures. Nonetheless our focus is on understanding investors' investment decisions under alternative theoretical settings.

The results of the comparison between the level of utility for risk averse, risk neutral and risk lover investors under expected utility present no evidence that portfolio insurance strategies dominate the investment on stocks or risk-free assets. In order to explain portfolio insurance popularity, we use prospect and cumulative prospect theories, and our findings point to cumulative prospect theory as a strong framework to explain the attractiveness of portfolio insurance strategies. The results are also very interesting from the perspective of product design as they point the best choices to be the very simple portfolio insurance strategies instead of the complex products. Results are in line with previous work developed by Dichtl and Drobetz (2011), and Vrecko and Branger (2009).

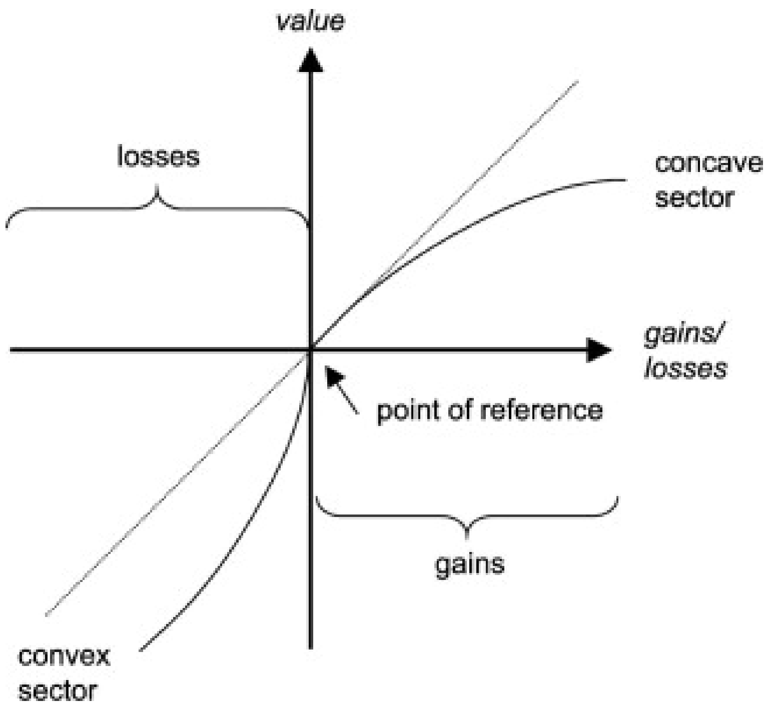
5.1 From Expected Utility Theory to Cumulative Prospect Theory

The expected utility theory (EUT) is based on the assumption that investors maximize their final expected wealth when making investment decisions. Prospect theory characterizes choices made by individuals between risky alternatives, based on empirical evidence. In this theory, investors assess their decisions, against a reference target, in terms of gains and losses. This formulation means that utility is defined over gains and losses rather than over final wealth positions, which was an idea proposed by Markowitz (1952). This fits naturally with the way gambles are often presented and discussed in everyday life. According to experiments by Kahneman and Tversky (1979), there is consistency with the way people perceive attributes such as brightness, loudness, or temperature relative to earlier levels, rather than in absolute terms. They show that for two gambling problems where, individually, bets achieve the same level of expected wealth, people decide differently on each of the gamble, because they focus on gains and losses.

Barberis and Thaler (2003) refer that "prospect theory may be the most promising for financial applications". In fact it is seen as "the most successful at capturing the experimental results", which exhibit most of the anomalies in financial market behavior. As prospect theory is not a normative theory, like the EUT, the authors Kahneman and Tversky intend to get people's attitudes to risky gambles as parsimoniously as possible. Although risk aversion investors are considered to be always rational under expected utility theory, they showed that when investors face losses, a risk seeking behaviour would be rational. There are now cognitive explanations for increasing and decreasing marginal utility of wealth, which lead to risk tolerant or risk seeking behaviours[1].

Prospect theory investors' value function has a sigmoid curve—with the curve being concave for gains, and convex for losses; the degree of response is defined by a factor λ—, representing risk-aversion in the domain of gains, and risk-seeking in the domain of losses. This means that, for the same referential absolute variation, the impact of losses is bigger than the impact of gains. Investors weight the probabilities of outcomes, and evidence highlights the fact that events with low probabilities are overweighted.

Figure 5.1: Example of a value function - adapted from Tversky and Kahneman (1992)



In its first formulation prospect theory was concerned with simple prospects of the form $(x, p; y, q)$, which had at most two non-zero outcomes. In this prospect an individual receives x with probability p, y with probability q, and nothing with probability $(1 - p - q)$, where $p + q \leq 1$. The decision between the prospects depends on its value (V). The overall value of a prospect is defined in two scales π and v, where π associates with each probability p a decision weight $\pi(p)$, reflecting the impact of p on the value of the prospect, and v assigns each outcome x a number $v(x)$ which reflects a subjective value for the outcome. The outcomes are defined

[1] There are several well known examples in the literature that question the normative behaviour of individuals (e.g. Allais (1953) and St Petersburg (Bernoulli, 1954) paradoxes).

relative to a reference point, which is the zero point of the value scale (i.e. measures losses and gains). That means that V is defined on prospects and v is defined in outcomes. Therefore:

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y), \tag{5.1}$$

where $v(0) = 0$, $\pi(0) = 0$, $\pi(1) = 1$ and when prospects are sure $V(x, 1) = V(x) = v(x)$.

The evaluation of strictly positive or negative prospects follows a different formula. First, all prospects are edited into two components: one where there is no risk (the minimum gain or loss that is certain), and a risky component (the additional gain or loss that derives from the game). When $p + q = 1$ and $x > y > 0$ or $x < y < 0$:

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)] \tag{5.2}$$

With this value function, the risk-less component has no decision weight, which is applied to the value difference between the outcomes. Kahneman and Tversky (1979) propose a value function that is steeper for losses than the value function for gains, generally concave for gains and convex for losses, and is defined on deviations from a reference point. The weighting function defined by the authors is set to capture the impact of events on the desirability of prospects and not the likelihood of the events. Thus the weights should not be stated as probabilities.

The theory originally introduced by Tversky and Kahneman in 1979 has been modified since then. Based on additional evidence, Tversky and Kahneman (1992) propose a generalization of prospect theory which can be applied to gambles with more than two outcomes, leading to the cumulative prospect theory (CPT).

The cumulative prospect value function is defined as strictly increasing and assess the negative and positive outcomes of each prospect (f):

$$V(f) = V(f^+) + V(f^-) = \sum_{i=-m}^{n} \pi_i v(x_i), \tag{5.3}$$

where $V(f^+) = \sum_{i=-m}^{0} \pi_i^- v(x_i)$, $V(f^-) = \sum_{i=0}^{n} \pi_i^+ v(x_i)$ and $-m \leq i \leq n$.

With this value function it is possible to evaluate investment strategy prospects (from negative to positive outcomes in relation to a reference point—initial wealth) for a cumulative prospect theory investor, as in the case of expected utility framework. In this theory, a two component power valuation function is defined to characterize the curve pattern describing decision behaviour:

$$v(\Delta x) = \begin{cases} (\Delta x)^{\alpha} & (\Delta x) \geq 0 \\ -\lambda(-\Delta x)^{\beta} & (\Delta x) < 0, \end{cases} \tag{5.4}$$

where i denotes outcomes, Δx_i and $i = (1, \cdots, N)$, which are ranked in ascending order. The parameters $\alpha \approx \beta \approx 0.88$ for the exponent reflect the diminishing sensitivity. The parameter $\lambda \approx 2.25$ is the coefficient of loss aversion, a measure of the relative sensitivity to gains and losses. These parameters were set based on experiences by Tversky and Kahneman (1992).

Investors assign the value to all outcomes by applying single probabilities in prospect theory, while in cumulative prospect theory probabilities are weighted:

$$\pi_i = \begin{cases} \pi_i^- = w^-(p_{-m} + \cdots + p_i) - w^-(p_{-m} + \cdots + p_{i-1}), 1 - m \leq i \leq 0, \\ \pi_i^+ = w^+(p_i + \cdots + p_n) - w^+(p_{i+1} + \cdots + p_n), 0 \leq i \leq n - 1. \end{cases} \tag{5.5}$$

Assuming that the value function is linear, the smoothing of curves are weighting functions. They are fitted using:

$$w^+(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{(1/\gamma)}} \text{ and } w^-(p) = \frac{p^{\delta}}{(p^{\delta} + (1-p)^{\delta})^{(1/\delta)}}, \tag{5.6}$$

where the parameter weighting for gains $\gamma = 0.61$, and for losses $\delta = 0.69$, were set based on empirical results (Tversky and Kahneman, 1992). They show the curvature for the value function, and indicate that risk aversion for gains is more pronounced than risk seeking for losses (for moderate and high probabilities). These weight functions determine the shape of the indifference curves for CPT for non-negative and non-positive prospects, while the value of outcomes control their slope.

Although EUT is still a dominant framework to assess investment strategies, the abnormal behaviours in financial markets may be explained using different models. In particular, it is interesting to test the features of portfolio insurance strategies in order to explore an explanation for its attractiveness and popularity, as not all investors follow the EUT axioms.

5.2 Methodology

We use the same Monte Carlo method as presented in previous chapter to simulate a risky asset pattern—see Table 4.1—, in order to evaluate the performance of portfolio insurance strategies, and benchmark investments. We run Monte Carlo simulations to build stock market scenarios with normally distributed price returns. The continuous compound stock market returns are generated 100.000 times, using a geometric Brownian motion (GBM), for a one year time horizon, with 252 trading days.

We keep the 9 stock market scenarios: high, medium and low volatility; high, medium and equivalent risk-free equity returns, in order to capture a large span of possible bear and bull market conditions. In each of the scenarios of Table 4.1 we have 100.000 paths on stock market returns to compare the results amongst strategies. With simulated data we compare the final utility from portfolio insurance and benchmark strategies in the case of investor's expected utility and also the cumulative prospect values.

We continue to use the same portfolio insurance strategies we used for the simulation in previous chapter:

1 Stop Loss Portfolio Insurance (SLPI);

2 Option Based Portfolio Insurance (OBPI) with synthetic option;

3 Constant Proportion Portfolio Insurance (CPPI1 and CPPI3); and

4 Time Invariant Portfolio Protection (TIPP).

These portfolio insurance strategies are benchmarked against:

1 Passive stock market strategy;

2 Portfolio with risky and risk free assets (50:50); and

3 Risk-free cash market deposit.

In the case of the utility function selected for the calculation of the expected utility under EUT we use three utility functions ($U_1; U_2; U_3$) representative of investors' attitude towards risk: aversion, neutrality and risk seeking. Utility functions are calculated using the final wealth distribution at maturity of the portfolio insurance and benchmark strategies. We use simple utility functions in order to compare the strategies with three different type of investors.

5.3 Results

The results provide a valuation for an investor that maximizes expected utility; and valuation of outcomes by investors under prospect and cumulative prospect theories. Simulation results are presented using a floor of 100% for the portfolio insurance strategies per scenario—robustness tests in section 5.4 present results for a 80% floor.

5.3.1 Expected Utility Theory

The results of the utility for investors are depicted in Table 5.1. The results for a risk averse investor are presented in Panel A of Table 5.1. In bullish markets (Scenarios 1, 4 and 7), for a risk averse investor, the highest utility is obtained from a passive stock market portfolio. The reason for this outcome is that the passive stock market delivers the highest expected wealth at maturity. Regarding portfolio insurance strategies, SLPI is the strategy that delivers the highest utility. When the market is neutral (Scenarios 3, 6 and 9), the results are inconclusive,

as there is no dominant strategy between portfolio insurance and benchmark strategies.
The results for the risk neutral investor are presented in Panel B of Table 5.1. In market neutral conditions (Scenarios 3, 6 and 9), utility value is very similar amongst all strategies, except for the synthetic OBPI, where the utility values are the lowest. In this strategy, due to the costs of protection, the lower the volatility, the higher the utility value. Although very similar, the passive stock market strategy delivers the highest utility. When market conditions are very positive (Scenarios 1-2, 4-5 and 7-8) investors should choose the passive stock market investment in order to maximize utility.
The results for a risk seeking investor are presented in Panel C of Table 5.1. The results are very interesting, because, as expected, investor should choose a passive stock market strategy for all market conditions. In any scenario, for a risk loving investor, portfolio insurance strategies should not be appealing options, as the upward movement is only partial. In this framework, investing in the equity market to collect a risk premium provides the best utility choice.

The results from the expected utility perspective show no strong evidence that portfolio insurance strategies are appealing to risk averse investors. In fact, averse investors are expected to prefer strategies that avoid downside risk, and not waiving an upward potential. Portfolio insurance, which meet these conditions, do not appear to be a clear choice, and these results are in line with the work of Benninga and Blume (1985), and Black and Perold (1992). The authors find that utility maximization of CPPI, under expected utility theory is attainable under restrictive conditions for some utility functions (i.e., only if the investor has Hyperbolic Absolute Risk Aversion (HARA)).

The results under the expected utility framework cannot explain the popularity of the portfolio insurance strategies. Though portfolio insurance strategies have the necessary characteristics to attract risk averse investors, we find no evidence from the simulations we performed that these strategies are in the top rank for investors choices.

Table 5.1: Results - Expected Utility Theory (Floor = 100%)

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Risk averse $[U_1(w)] = ln(w)$								
S1-$\mu: 15\%; \sigma: 30\%$	**4.672***	4.663*	4.660*	4.670*	4.662*	4.710	4.694	4.654
S2-$\mu: 10\%; \sigma: 30\%$	4.657*	4.645*	4.658*	**4.661***	4.658*	4.660	4.668	4.654
S3-$\mu: 5\%; \sigma: 30\%$	4.643*	4.628*	**4.655****	4.654*	**4.655***	4.610	4.643	4.654
S4-$\mu: 15\%; \sigma: 20\%$	**4.691***	4.687*	4.660*	4.671*	4.662*	4.734	4.700	4.654
S5-$\mu: 10\%; \sigma: 20\%$	**4.667***	4.662*	4.658*	4.662*	4.659*	4.686	4.675	4.654
S6-$\mu: 5\%; \sigma: 20\%$	4.647*	4.641*	**4.655***	4.654*	**4.655***	4.634	4.649	4.654
S7-$\mu: 15\%; \sigma: 10\%$	**4.730***	4.721*	4.660*	4.672*	4.662*	4.750	4.704	4.654
S8-$\mu: 10\%; \sigma: 10\%$	**4.687***	4.682*	4.658*	4.663*	4.659*	4.700	4.679	4.654
S9-$\mu: 5\%; \sigma: 10\%$	4.652*	4.650*	**4.655****	**4.655***	**4.655***	4.650	4.653	4.654
Panel B: Risk neutral $E[U_2(w)] = w_T$								
S1-$\mu: 15\%; \sigma: 30\%$	**109.09***	107.70*	105.67*	106.91*	105.87*	115.18	110.09	105.00
S2-$\mu: 10\%; \sigma: 30\%$	**106.98***	105.40*	105.39*	105.97*	105.51*	110.08	107.59	105.00
S3-$\mu: 5\%; \sigma: 30\%$	105.11	103.43*	**105.13**	**105.13**	105.12	105.02	105.01	105.00
S4-$\mu: 15\%; \sigma: 20\%$	**110.49***	109.56*	105.66*	106.89*	105.89*	115.06	110.03	105.00
S5-$\mu: 10\%; \sigma: 20\%$	**107.51***	106.61*	105.39*	105.96*	105.52*	110.09	107.55	105.00
S6-$\mu: 5\%; \sigma: 20\%$	105.09	104.24*	**105.12****	105.11***	**105.12*****	104.99	105.00	105.00
S7-$\mu: 15\%; \sigma: 10\%$	**113.96***	112.78*	105.66*	106.91*	105.90*	115.15	110.58	105.00
S8-$\mu: 10\%; \sigma: 10\%$	**109.00***	108.28*	105.39*	105.96*	105.52*	110.53	107.56	105.00
S9-$\mu: 5\%; \sigma: 10\%$	105.08	104.84*	**105.12**	**105.12**	**105.12****	105.07	105.04	105.00
Panel C: Risk lover $E[U_3(w)] = {w_T}^2$								
S1-$\mu: 15\%; \sigma: 30\%$	**12,547***	12,045*	11,168*	11,489*	11,216*	14,768	12,548	11,025
S2-$\mu: 10\%; \sigma: 30\%$	**11,941***	11,460*	11,110*	11,273*	11,138*	13,384	11,908	11,025
S3-$\mu: 5\%; \sigma: 30\%$	**11,415***	10,967*	11,054*	11,084*	11,058*	12,090	11,298	11,025
S4-$\mu: 15\%; \sigma: 20\%$	**12,607***	12,263*	11,165*	11,447*	11,216*	14,019	12,354	11,025
S5-$\mu: 10\%; \sigma: 20\%$	**11,843***	11,559*	11,109*	11,244*	11,137*	12,727	11,744	11,025
S6-$\mu: 5\%; \sigma: 20\%$	**11,239***	11,002*	11,051*	11,059*	11,053*	11,473	11,137	11,025
S7-$\mu: 15\%; \sigma: 10\%$	**13,146***	12,825*	11,165*	11,434*	11,216*	13,627	12,261	11,025
S8-$\mu: 10\%; \sigma: 10\%$	**11,997***	11,802*	11,107*	11,230*	11,134*	12,340	11,644	11,025
S9-$\mu: 5\%; \sigma: 10\%$	**11,113***	11,038*	11,051*	11,053*	11,051*	11,152	11,060	11,025

This table shows the results of three utility functions ($U_1; U_2; U_3$) representative of investors' attitude towards risk: aversion, neutrality and risk seeking. Utility functions are calculated using the portfolio value at maturity of the portfolio insurance and benchmark strategies. Wealth at maturity is the result of accumulated daily returns of each strategy. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year, with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The null hypothesis in the paired t-test is that the utility value of a portfolio insurance strategy is equal to that of the benchmark strategy with the highest utility value. * The test statistic is significant at the 1% level. ** The test statistic is significant at the 5% level. *** The test statistic is significant at the 10% level.

The results lead us to change the framework towards prospect theory, in order to evaluate if behavioural theory explains why these strategies are still popular in retail and institutional markets.

5.3.2 Prospect and Cumulative Prospect Theories

The results for prospect value are presented in Table 5.2. We use the valuation function defined by Tversky and Kahneman (1979, 1992), and simulate the different approaches: prospect theory, and cumulative prospect theory. All the parameters are from the studies of the referred authors. In Panel A of Table 5.2 we use mean prospect with $\lambda = 1$, which means that reaction to losses is not different from the reaction to gains. In Panel B, we incorporate a different loss reaction, using $\lambda = 2.25$. In both situations, there is no distinct weighting of the gains, and losses; therefore, we calculate the simple mean of the simulated prospect values. In Panel C of Table 5.2, we incorporate the cumulative probability weightings.

The results of Panel A of Table 5.2 present the prospect values for an investor with similar reactions to gains and losses. The prospect values show that, for bullish market conditions (scenarios 1-2, 4-5, and 7-8), a passive stock market strategy yields the best global results. In neutral conditions (scenarios 3, 6 and 9), the unleveraged CPPI strategy is the best choice. The leveraged CPPI and TIPP, as well as the risk-free investment have prospect values very similar to unleveraged CPPI.

In Panel B of Table 5.2, we incorporate the loss reaction for investors in valuing outcomes, and find that results become more favourable to portfolio insurance strategies. In some bullish market conditions with medium to low volatility, a passive stock market investment delivers high prospect value, as high negative returns are not frequent. In market conditions with high and medium expected returns and volatility, portfolio insurance strategies yields high prospect values for investors. In most cases, leveraged CPPI is the best strategy. When market conditions are relatively stable, and with high expected returns (scenarios 7 and 8), a risky investment continues to deliver the highest prospect value. Except for the bearish market conditions, where risk-free investment has the highest value, and the low to medium volatile markets with high expected returns, where a passive stock market strategy yields the best result, in all other scenarios portfolio insurance strategies are the best choices.

Table 5.2: Results - Prospect and Cumulative Prospect Theories (Floor = 100%)

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Mean prospect value ($\lambda = 1.0$)								
S1-μ : 15%; σ : 30%	6.93*	6.42*	7.75*	**8.65***	7.92*	11.61	10.50	7.16
S2-μ : 10%; σ : 30%	5.43*	4.58*	7.45***	**7.73***	7.51	6.43	7.70	7.16
S3-μ : 5%; σ : 30%	3.63*	2.47*	**7.12***	6.73*	7.02*	0.73	4.62	7.16
S5-μ : 15%; σ : 20%	9.68*	**9.85***	7.79*	9.02*	8.03*	15.22	11.95	7.16
S6-μ : 10%; σ : 20%	6.82*	6.59*	7.45*	**7.92***	7.56*	9.14	8.65	7.16
S7-μ : 5%; σ : 20%	4.67*	4.27*	**7.15***	6.99*	7.11*	3.52	5.64	7.16
S9-μ : 15%; σ : 10%	**14.96***	14.36*	7.79*	9.15*	8.06*	17.53	12.84	7.16
S10-μ : 10%; σ : 10%	**9.98***	9.69*	7.47*	8.08*	7.60*	11.76	9.77	7.16
S11-μ : 5%; σ : 10%	5.74*	5.92*	**7.16***	7.12*	7.15*	5.68	6.58	7.16
Panel B: Mean prospect value ($\lambda = 2.25$)								
S1-μ : 15%; σ : 30%	5.10*	1.11*	7.75*	**8.65***	7.92*	0.74	6.45	7.16
S2-μ : 10%; σ : 30%	3.52*	-0.87*	7.45*	**7.73***	7.51*	-7.33	2.48	7.16
S3-μ : 5%; σ : 30%	1.65*	-3.28	**7.12***	6.73*	7.02*	-15.96	-1.77	7.16
S4-μ : 15%; σ : 20%	8.60*	7.43*	7.79*	**9.02***	8.03*	10.44	10.43	7.16
S5-μ : 10%; σ : 20%	5.64*	3.65*	7.45*	**7.92***	7.56*	1.96	6.27	7.16
S6-μ : 5%; σ : 20%	3.38*	1.09*	**7.15***	6.99*	7.11*	-6.44	2.23	7.16
S7-μ : 15%; σ : 10%	**14.69***	14.08*	7.79*	9.15*	8.06*	16.91	12.75	7.16
S8-μ : 10%; σ : 10%	**9.61***	9.16*	7.47*	8.08*	7.60*	10.19	9.51	7.16
S9-μ : 5%; σ : 10%	5.21*	5.15*	**7.16***	7.12*	7.15*	2.07	5.81	7.16
Panel C: Mean cumulative prospect value ($\lambda = 2.25$)								
S1-μ : 15%; σ : 30%	**17.04***	7.00*	7.86*	11.61*	7.20**	6.96	7.65	7.16
S2-μ : 10%; σ : 30%	**14.34***	3.65*	7.52*	10.25*	6.88*	-0.37	3.82	7.16
S3-μ : 5%; σ : 30%	**11.30***	0.14*	5.21*	8.70*	6.68*	-8.06	-0.18	7.16
S4-μ : 15%; σ : 20%	**15.65***	9.08*	4.13*	9.13*	7.71*	10.48	9.44	7.16
S5-μ : 10%; σ : 20%	**12.53***	5.31*	7.49*	8.84*	7.26*	3.42	5.82	7.16
S6-μ : 5%; σ : 20%	**9.65***	1.37*	3.70*	7.06**	6.80*	-4.12	1.97	7.16
S7-μ : 15%; σ : 10%	**15.50****	12.98*	2.52*	6.58*	8.01*	15.83	11.98	7.16
S8-μ : 10%; σ : 10%	**11.25***	8.36*	7.55*	8.21*	7.53*	8.67	8.42	7.16
S9-μ : 5%; σ : 10%	**7.56***	3.60*	7.22*	7.19**	4.63*	1.04	4.70	7.16

This table shows the results of (mean or cumulative) prospect value. Prospect values are calculated using the portfolio gains and losses relative to a reference point (100 or 0% return) at maturity of the portfolio insurance and benchmark strategies. Gains and losses at maturity are the result of the accumulated daily returns of each strategy. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year, with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The null hypothesis in the paired t-test is that the (mean or cumulative) prospect value of a portfolio insurance strategy is equal to that of the benchmark strategy with the highest prospect value. * The test statistic is significant at the 1% level. ** The test statistic is significant at the 5% level. *** The test statistic is significant at the 10% level.

The results of Panel C in Table 5.2 present the cumulative prospect values. There is only one scenario where benchmark strategies dominate portfolio insurance strategies: scenario 7 where the best choice is a passive stock market strategy. The SLPI strategy dominates all other strategies in the remaining scenarios. The results support a possible explanation for the popularity of portfolio insurance on a behavioural finance context, and especially regarding the preference of naïve strategies as the SLPI. Amongst portfolio insurance strategies, the SLPI is always the best choice.

The way investors react against prospects with losses or gains, and also on the different valuation of low or high probabilities, is not captured in an EUT. Hence, this is not a complete framework to explain the attractiveness of portfolio insurance. On the contrary, a prospect theory approach may support the longevity of strategies that offer protection on the downside and, at the same time, keep a potential to benefit from the upside.

5.4 Robustness Tests

We perform several tests in order to assess the adherence of the results. We begin by changing the floor from 100% to 80% and rerun the model. On a second analysis, we test the parameters of cumulative prospect theory used by Tversky and Kanhneman (1992): we change these assumptions in order to increase the overweight of small probability events in negative outcomes (we change the δ from 0.69 to 0.77, and the γ from 0.61 to 0.44, while keeping the α constant at 0.88), in line with parameters defined in the experiment by Wu and Gonzalez (1996) regarding the weighting process. The final tests are related with the changes of risk-free rates (we stress the risk-free rate by 150 basis points, from 5% to 3.5% and from 5% to 6.5%) in order to check the impact on investor choices.

5.4.1 Setting the Floor at 80%

We elaborate a complete rerun of the model using a floor of 80% on the portfolio insurance strategies. The results confirm the overall findings when floor is at 100%.

The results under the expected utility framework are presented in Table 5.3. For a risk averse investor, indicated in Panel A of Table 9, when the floor is changed from 100% to 80%, the results confirm, overall, the utility derived from the passive stock market strategy, and, also, the risk-free investment. However, with the new floor, in market conditions with low volatility, SLPI strategy and unleveraged CPPI yield a higher utility value than benchmark strategies.

The results for a risk neutral investor, presented in Panel B of Table 5.3, point that portfolio insurance strategies do not yield higher utility to a neutral investor. In the case of a 80% floor, a SLPI becomes a possible choice for a risk neutral investor, in high expected market returns with low volatility, as utility is very similar to a passive stock market strategy—as expected, because the probability of executing the stop loss order is lower.
For a risk loving investor, the reduction of the floor, does not change the results. They are very similar to results from Panel B of Table 5.1, and the passive stock market strategy yields the highest utility for all bullish or neutral scenarios.

Overall results by portfolio insurance strategies, when the floor is reduced, and, consequently, an increasing exposure to risky assets is allowed, present higher utility values. However, this increase is not enough to change investors choices. The explanation for the popularity of portfolio insurance strategies is, according with our simulation, not explained under the framework of expected utility theory, even if we increase the upside potential.

Table 5.3: Results - Expected Utility Theory (Floor = 80%)

Floor = 80%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Risk averse $[U_1(w)] = ln(w)$								
S1-μ : 15%; σ : 30%	**4.702***	4.667*	4.677*	4.700*	4.697*	4.710	4.694	4.654
S2-μ : 10%; σ : 30%	4.659*	4.629*	4.665*	4.666*	**4.668**	4.660	4.668	4.654
S3-μ : 5%; σ : 30%	4.619*	4.595*	**4.653***	4.634*	4.640*	4.610	4.643	4.654
S4-μ : 15%; σ : 20%	**4.733***	4.712*	4.679*	4.719*	4.710*	4.736	4.701	4.654
S5-μ : 10%; σ : 20%	**4.684***	4.665*	4.666*	4.680*	4.678*	4.686	4.675	4.654
S6-μ : 5%; σ : 20%	4.636*	4.621*	**4.654**	4.645*	4.648*	4.635	4.649	4.654
S7-μ : 15%; σ : 10%	**4.750*****	4.745*	4.680*	4.731*	4.718*	4.750	4.705	4.654
S8-μ : 10%; σ : 10%	**4.700***	4.695*	4.667*	4.690*	4.685*	4.700	4.678	4.654
S9-μ : 5%; σ : 10%	4.650*	4.645*	**4.655***	4.653*	4.653*	4.650	4.653	4.654
Panel B: Risk neutral $E[U_2(w)] = w_T$								
S1-μ : 15%; σ : 30%	**115.04***	109.93*	107.77*	113.04*	111.56*	116.18	110.59	105.00
S2-μ : 10%; σ : 30%	**109.86***	105.50*	106.41*	108.84*	108.22*	110.49	107.75	105.00
S3-μ : 5%; σ : 30%	105.12	101.61*	105.12*	**105.13*****	105.12**	105.11	105.06	105.00
S4-μ : 15%; σ : 20%	**116.02***	113.38*	107.79*	113.56*	112.00*	116.29	110.64	105.00
S5-μ : 10%; σ : 20%	**110.37***	108.08*	106.42*	109.07*	108.44*	110.55	107.78	105.00
S6-μ : 5%; σ : 20%	105.11***	103.26*	**105.12***	105.11**	105.11*	105.10	105.05	105.00
S7-μ : 15%; σ : 10%	**116.20*****	115.60*	107.77*	113.83*	112.23*	116.20	110.60	105.00
S8-μ : 10%; σ : 10%	**110.49***	109.93*	106.41*	109.16*	108.58*	110.50	107.75	105.00
S9-μ : 5%; σ : 10%	**105.16**	104.63*	105.13	105.15	105.14	105.16	105.08	105.00
Panel C: Risk lover $E[U_3(w)] = {w_T}^2$								
S1-μ : 15%; σ : 30%	**14,494***	12,994*	11,687*	13,624*	12,907*	14,771	12,548	11,025
S2-μ : 10%; σ : 30%	**13,158***	11,905*	11,389*	12,546*	12,122*	13,361	11,897	11,025
S3-μ : 5%; σ : 30%	**11,982***	10,973*	11,111*	11,624*	11,414*	12,093	11,298	11,025
S4-μ : 15%; σ : 20%	**14,024***	13,350*	11,651*	13,283*	12,759*	14,075	12,380	11,025
S5-μ : 10%; σ : 20%	**12,678***	12,109*	11,354*	12,206*	11,952*	12,720	11,740	11,025
S6-μ : 5%; σ : 20%	**11,481***	11,026*	11,076*	11,293*	11,218*	11,497	11,148	11,025
S7-μ : 15%; σ : 10%	**13,638*****	13,499*	11,623*	13,058*	12,651 *	13,638	12,266	11,025
S8-μ : 10%; σ : 10%	**12,332***	12,207*	11,330*	11,994*	11,840*	12,333	11,640	11,025
S9-μ : 5%; σ : 10%	**11,170**	11,058	11,060*	11,117*	11,101*	11,170	11,069	11,025

This table shows the results of three utility functions ($U_1; U_2; U_3$) representative of investors' attitude towards risk: aversion, neutrality and risk seeking. Utility functions are calculated using the portfolio value at maturity of the portfolio insurance and benchmark strategies. Wealth at maturity is the result of accumulated daily returns of each strategy. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year, with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The null hypothesis in the paired t-test is that the utility value of a portfolio insurance strategy is equal to that of the benchmark strategy with the highest utility value. * The test statistic is significant at the 1% level. ** The test statistic is significant at the 5% level. *** The test statistic is significant at the 10% level.

The results of prospect values are presented in Table 5.4. The outcomes of the simulation for portfolio insurance strategies with a floor set at 80% confirms the possibility of explaining the choices by investors for these type of strategies under the cumulative prospect valuation. The comparison is between investors with $\lambda = 2.25$, which represent different reactions to gains and losses.

The mean prospect values of Panel B in Tables 5.2 and 5.4 denote a consistent option of the investor for strategies with less chances of negative outcomes. There is evidence that, when a bigger exposure to risky assets in low volatile markets (scenarios 7 and 8) is allowed, the choice between a passive stock market and a SLPI strategy is almost indifferent. Since the return distributions of these two strategies tend to coincide, and present more chances of losses, the risk-free investments becomes more attractive. We also observe a shift from leveraged to unleveraged CPPI strategies for high and medium volatile markets(scenarios 1, 2, 4 and 5). In general, amongst portfolio insurance strategies, the more protective the strategies are, the more valuable to prospect investors.

The results for cumulative investors point to a possible explanation for some flight to safety when markets are neutral (scenarios 3, 6 and 9). In these market conditions, we can expect a change from SLPI to risk-free investments. In positive market expectations, independently of volatility, the cumulative value of leveraged CPPI strategies is higher than SLPI (scenarios 1-2, 4-5, and 8). The portfolio insurance strategies, under these market conditions, are still providing better results than benchmarking strategies.

Results from Panel C of Tables 5.2 and 5.4, for positive market conditions (i.e. expected positive risk premium), strengthens the possibility that cumulative prospect theory explains investors' choices. The specific decisions of portfolio insurance are, nonetheless, dependent on the market conditions and characteristics of each portfolio insurance strategy—e.g. the reduction of the percentage floor from 100% to 80% depicts a shift from SLPI to CPPI strategies.

Table 5.4: Results - Prospect and Cumulative Prospect Theories (Floor = 80%)

Floor = 80%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Mean prospect value ($\lambda = 1.0$)								
S1-$\mu : 15\%; \sigma : 30\%$	**15.64***	10.02*	9.87*	13.65*	12.90*	17.17	12.37	7.16
S2-$\mu : 10\%; \sigma : 30\%$	**10.00***	5.78*	8.21*	9.03*	9.08*	11.05	9.08	7.16
S3-$\mu : 5\%; \sigma : 30\%$	5.38*	2.10*	**6.68***	5.28*	5.81*	5.55	6.09	7.16
S4-$\mu : 15\%; \sigma : 20\%$	**18.13***	13.56*	10.30*	15.63*	14.31*	18.46	13.14	7.16
S5-$\mu : 10\%; \sigma : 20\%$	**11.06***	8.22*	8.40*	9.79*	9.63*	11.30	9.32	7.16
S6-$\mu : 5\%; \sigma : 20\%$	5.59*	4.19*	**6.92***	5.70*	6.04*	5.76	6.35	7.16
S7-$\mu : 15\%; \sigma : 10\%$	**19.58*****	16.52*	10.51*	17.06*	15.43*	19.58	13.74	7.16
S8-$\mu : 10\%; \sigma : 10\%$	**12.95***	11.23*	8.85*	11.60*	11.05*	12.95	10.31	7.16
S9-$\mu : 5\%; \sigma : 10\%$	6.10*	5.84*	**7.15***	6.35*	6.50*	6.10	6.74	7.16
Panel B: Mean prospect value ($\lambda = 2.25$)								
S1-$\mu : 15\%; \sigma : 30\%$	5.23*	2.72*	**8.98***	7.49*	7.56*	7.78	8.57	7.16
S2-$\mu : 10\%; \sigma : 30\%$	-2.19*	-3.15*	**7.01***	1.59*	2.54*	-0.67	4.26	7.16
S3-$\mu : 5\%; \sigma : 30\%$	-8.62*	-8.68*	**5.08***	-3.55*	-2.06*	-9.00	-0.03	7.16
S4-$\mu : 15\%; \sigma : 20\%$	**13.27***	10.28*	10.10*	12.87	11.90*	13.94	11.63	7.16
S5-$\mu : 10\%; \sigma : 20\%$	4.23*	3.49*	**8.07***	5.86*	6.14*	4.71	7.03	7.16
S6-$\mu : 5\%; \sigma : 20\%$	-3.49*	-2.19*	**6.38***	0.47*	1.32*	-3.21	3.10	7.16
S7-$\mu : 15\%; \sigma : 10\%$	**19.03*****	16.19*	10.51*	16.84*	15.23*	19.03	13.66	7.16
S8-$\mu : 10\%; \sigma : 10\%$	**11.44***	10.28*	8.85*	10.94*	10.45*	11.44	10.04	7.16
S9-$\mu : 5\%; \sigma : 10\%$	2.63*	3.58*	**7.14***	4.75*	5.02*	2.63	6.00	7.16
Panel C: Cumulative prospect value ($\lambda = 2.25$)								
S1-$\mu : 15\%; \sigma : 30\%$	9.53*	-0.88*	8.15*	**12.55***	8.10*	6.97	7.66	7.16
S2-$\mu : 10\%; \sigma : 30\%$	3.88*	-0.88*	6.35*	**7.57***	4.18*	-0.27	3.88	7.16
S3-$\mu : 5\%; \sigma : 30\%$	-1.66*	-1.66*	**4.51***	2.74*	0.30*	-7.68	0.02	7.16
S4-$\mu : 15\%; \sigma : 20\%$	11.21*	11.21*	8.91*	**13.16***	9.93	10.75	9.58	7.16
S5-$\mu : 10\%; \sigma : 20\%$	4.66*	2.53*	7.17*	**7.89***	5.65*	3.42	5.82	7.16
S6-$\mu : 5\%; \sigma : 20\%$	-1.82*	-3.62*	**5.38***	2.73*	1.32*	-4.20	1.93	7.16
S7-$\mu : 15\%; \sigma : 10\%$	**15.85*****	15.15*	9.76*	15.20*	13.00*	15.86	12.00	7.16
S8-$\mu : 10\%; \sigma : 10\%$	8.69***	7.98*	8.10*	**9.76***	8.50*	8.69	8.43	7.16
S9-$\mu : 5\%; \sigma : 10\%$	1.18*	0.48*	**6.47***	4.43*	3.87*	1.09	4.73	7.16

This table shows the results of (mean or cumulative) prospect value. Prospect values are calculated using the portfolio gains and losses relative to a reference point (100 or 0% return) at maturity of the portfolio insurance and benchmark strategies. Gains and losses at maturity are the result of the accumulated daily returns of each strategy. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year, with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1. The null hypothesis in the paired t-test is that the (mean or cumulative) prospect value of a portfolio insurance strategy is equal to that of the benchmark strategy with the highest prospect value. * The test statistic is significant at the 1% level. ** The test statistic is significant at the 5% level. *** The test statistic is significant at the 10% level.

5.4.2 Changes of Cumulative Prospect Theory Parameters

The Table 5.5 presents the cumulative prospect value with changes on the parameters for the curvature and the elevation of the smoothing of curves. In this sensitivity analysis we modify the assumptions in order to increase the overweight of small probability events in negative outcomes (we change the δ from 0.69 to 0.77, and the γ from 0.61 to 0.44, while keeping the α constant at 0.88), in line with parameters defined in the experiment by Gonzalez and Wu (1999) regarding the weighting process. The results on cumulative prospect values indicate portfolio insurance dominates benchmark strategies in all market conditions. The portfolio insurance strategy with higher value is the SLPI, since there is an overreaction to losses. Comparing the results with scenarios of Panel C in Table 5.2, we find that cumulative values are higher, and the downside risk aversion is managed using the same strategy—SLPI–, hence confirming our results that prospect theory is a viable framework to explain portfolio insurance popularity.

Table 5.5: Results - Cumulative Prospect Theories - Changing Parameters

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Cumulative prospect value ($\lambda = 2.25$)								
S1-μ : 15%; σ : 30%	**21.80**	10.30	7.31	13.29	6.29	8.94	8.30	7.23
S2-μ : 10%; σ : 30%	**19.59**	7.86	7.00	11.92	5.71	2.19	4.88	7.23
S3-μ : 5%; σ : 30%	**16.77**	5.43	6.70	10.47	5.59	-4.87	1.30	7.23
S4-μ : 15%; σ : 20%	**16.64**	10.61	3.63	8.96	6.66	10.14	8.66	7.23
S5-μ : 10%; σ : 20%	**14.36**	8.14	7.02	8.20	6.23	4.10	5.70	7.23
S6-μ : 5%; σ : 20%	**12.13**	5.84	6.73	7.17	5.84	-2.93	2.29	7.23
S7-μ : 15%; σ : 10%	**13.25**	12.17	7.57	8.68	7.41	13.36	9.90	7.23
S8-μ : 10%; σ : 10%	**10.28**	8.88	7.26	7.64	6.94	7.68	7.12	7.23
S9-μ : 5%; σ : 10%	**7.63**	6.11	6.93	6.67	6.44	1.08	4.12	7.23

This table shows the results of (mean or cumulative) prospect value. Prospect values are calculated using the portfolio gains and losses relative to a reference point (100 or 0% return) at maturity of the portfolio insurance and benchmark strategies. Gains and losses at maturity are the result of the accumulated daily returns of each strategy. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 5%. Stock returns were simulated daily for a period of one year, with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1.

5.4.3 Using Risk-Free Rates at 3.5 % and 6.5 %

The Table 5.6 presents the cumulative prospect value with changes on the risk-free rate. We modify the assumptions for the scenario generation for the stock price considering a lower risk-free than the central macro-scenario and a higher risk-free rate (we change the risk-free rate from 5.0% to 3.5% and from 5.0% to 6.5%).

The results on cumulative prospect values - Panel A with risk-free rate set at 3.5% - indicate portfolio insurance dominates benchmark strategies in all market conditions, except in scenario 7, where benchmark investment yields higher. The portfolio insurance strategy with higher value is the SLPI. Results of Panel B also present portfolio insurance strategies with higher cumulative value in all scenarios. These results confirm general findings that point for preferences on portfolio insurance investments, in particular for SLPI strategies.

Table 5.6: Results - Cumulative Prospect Theory - Risk Free at 3.5% and 6.5%

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Cumulative prospect value ($\lambda = 2.25$) **and Risk Free = 3.5%**								
S1-$\mu: 15\%; \sigma: 30\%$	**14.73**	5.41	4.19	8.67	5.55	6.55	6.22	5.30
S2-$\mu: 10\%; \sigma: 30\%$	**12.53**	2.95	3.80	7.56	5.24	-0.47	2.52	5.30
S3-$\mu: 5\%; \sigma: 30\%$	**9.83**	0.19	3.63	6.60	4.94	-8.07	-1.49	5.30
S4-$\mu: 15\%; \sigma: 20\%$	**14.07**	8.61	3.14	7.60	5.02	10.66	8.38	5.30
S5-$\mu: 10\%; \sigma: 20\%$	**11.25**	5.43	5.52	6.70	4.68	3.32	4.54	5.30
S6-$\mu: 5\%; \sigma: 20\%$	**8.33**	2.40	5.28	5.80	5.03	-4.37	0.54	5.30
S7-$\mu: 15\%; \sigma: 10\%$	**14.41**	12.98	1.71	7.08	3.16	15.89	10.97	5.30
S8-$\mu: 10\%; \sigma: 10\%$	**10.41**	8.42	5.56	6.21	3.31	8.66	7.29	5.30
S9-$\mu: 5\%; \sigma: 10\%$	**7.00**	4.82	5.32	3.63	5.23	1.08	3.47	5.30
Panel B: Cumulative prospect value ($\lambda = 2.25$) **and Risk Free = 6.5%**								
S1-$\mu: 15\%; \sigma: 30\%$	**18.48**	8.70	9.81	13.61	9.04	6.75	8.70	9.08
S2-$\mu: 10\%; \sigma: 30\%$	**15.73**	6.19	9.41	12.47	8.53	-0.31	5.06	9.08
S3-$\mu: 5\%; \sigma: 30\%$	**12.77**	3.52	8.99	10.88	8.30	-7.94	1.15	9.08
S4-$\mu: 15\%; \sigma: 20\%$	**16.91**	11.75	9.79	12.28	9.61	10.75	10.69	9.08
S5-$\mu: 10\%; \sigma: 20\%$	**13.59**	8.32	9.37	10.01	9.03	3.42	7.01	9.08
S6-$\mu: 5\%; \sigma: 20\%$	**10.59**	5.57	8.97	9.45	8.44	-4.16	3.21	9.08
S7-$\mu: 15\%; \sigma: 10\%$	**16.31**	15.15	9.86	11.40	9.97	15.86	12.99	9.08
S8-$\mu: 10\%; \sigma: 10\%$	**11.79**	10.64	9.44	6.73	5.40	8.72	9.52	9.08
S9-$\mu: 5\%; \sigma: 10\%$	7.97	6.99	**9.03**	8.74	8.73	0.96	5.85	9.08

This table shows the results of cumulative prospect value. Prospect values are calculated using the portfolio gains and losses relative to a reference point (100 or 0% return) at maturity of the portfolio insurance and benchmark strategies. Gains and losses at maturity are the result of the accumulated daily returns of each strategy. Returns of the stock market were generated using a GBM. Portfolio insurance and benchmark strategies' returns were simulated using Monte Carlo, and the risk-free rate was set at 3.5% and 6.5%. Stock returns were simulated daily for a period of one year with 252 trading days. The returns were simulated 100.000 for each of the 9 scenarios presented in Table 4.1.

5.5 Conclusions

The results we computed from the expected utility perspective show no strong evidence that portfolio insurance strategies are appealing to risk averse investors. In fact, by nature, investors that are averse to losses are expected to prefer strategies that limit downside risk, and, simultaneously, not giving away an upward potential. Although portfolio insurance strategies have the necessary characteristics to attract risk averse investors, we find no evidence from the simulations we performed under the expected utility framework that these strategies are in the top rank for investors choices. Therefore, results cannot explain the popularity of the portfolio insurance strategies.

As we change the framework towards prospect theory by Kahneman and Tversky (1979, 1992), results from Panel C of Tables 5.2 and 5.4, for positive market conditions (i.e. expected positive risk premium), strengthens the possibility that cumulative prospect theory may explain investors' choices. The specific decisions of portfolio insurance are, nonetheless, dependent on the market conditions and characteristics of each portfolio insurance strategy—e.g., the reduction of the percentage floor from 100% to 80% depicts a shift from SLPI to CPPI strategies.

To the best of our knowledge, based on innovative approach that compares the value perceived by investors between expected utility and prospect theories, we find that cumulative prospect theory is a viable framework to explain the popularity of portfolio insurance investments. Our findings also support the findings of Dichtl and Drobetz (2011), and Vrecko and Branger (2009).

In spite of these results, we understand that it is necessary further analysis regarding the higher cumulative prospect values for SLPI strategies against CPPI. In future research we will analyse determinant factors for investment decision such as time horizon, portfolio management costs and tax.

The framework of behavioural finance may have the relevant factor to explain the selection of portfolio insurance strategies by individual investors. In the market there is a trend towards automatic platforms to help investors on the definition of their risk profiles to set investment strategies that may accomplish either their aspirations or objectives. As individual choices are being integrated into a digital world, in the next chapter we address the integration that is being made by robo-advisors and focus on the risks and on the limitations that these new tools may have on protective strategies.

Chapter 6

Robo-Advising and Investors' Protection

In this chapter we review the factors affecting wealth management and highlight the risks of portfolio management in robot-advising as well as the pitfalls of these platforms towards individual investors' risk profile[1].

The role of technology in finance is constantly evolving. The newest idea being the concept of robo-advising, which consists of using an algorithm based asset allocation model via an on-line platform. The history of asset management is embedded within the concepts of Modern Portfolio Theory (MPT), Mean-Variance Analysis (MVA) and principles of asset allocation which are the building blocks of the algorithms used by robo-advising platforms.

The first introduction of technology in the stock market occurred in the early 1970s when the world's first on-line stock market, NASDAQ, was created and launched (FINRA, 2016). By the 1980s to 1990s, program trading had taken off and computers were being used to trade and perform analysis. By the early 2000s the National Association of Securities Deals allowed investment analysis tools to become available on-line to investors which was the birth of robo-advising (FINRA, 2016). The vast rise of today's robo-advisors tool place after the financial 2007 crisis when consumers lost trust in large financial institutions. With the advancements in technology, firms were able to offer low cost investment planning via the internet while still

[1]The findings presented in this chapter are an extension of a joint work with colleagues from the MSc Risk Management Class of 2017 at Stern NYU, Aized Gill, Amit Sinha, Faisal Azim and Juan Bernal. We were involved in research for a Strategic Capstone and found grounds for a business case, specially from the drawbacks of investor profiling on a sample of on-line advisory platforms. A perspective on consumer guide to robo-advising based on this joint work was presented on September 2017 on the 2nd International Conference on Computational Finance in Lisbon, Portugal.

allowing customers some sense of control over their decisions (Lam, 2016).

One obvious observation of robo-advising is the lack of human touch throughout the investment process. The involvement of humans largely depends on the service being provided and the brokers model. Some brokers add robo-advising to their current financial advisory services offer, while others hire advisors to reaffirm or amend the suggestions offered by the algorithm. For traditional robo-advisory firms, the human component is very limited. The entire portfolios rebalancing, monitoring and assessing are done via the algorithm and then reviewed by humans to ensure their compliance with securities regulations. On the customer side, the human element revolves around how much time and attention he/her dedicates to answering the questionnaire as this determines the accuracy of the algorithm to match his/her needs (FINRA, 2016, Lam, 2016, QPLUM, 2016). It also places a burden on the customer to notify the software of any changes to his/her risk strategy or changes in the investment goals (Fish and Turner, 2017, Fish et al., 2017).

An important aspect of robo-advising and questionnaires are the behavioural elements associated with an investment decision making process. Most platforms agree that understanding investor psychology and investors expectations is important to create a sound financial strategy (Fish and Turner, 2017). A relevant argument is the distinction between clients' decisions, which are driven by their preferences and those that are driven by psychological biases. This is also the reason why it is important for advisors to understand how their clients make financial decisions. Not understanding behaviour biases may result in inappropriate financial advising. This perspective is important for establishing a link between the clients needs and certain financial objectives because the typical perception is that traditional finance defines a rational benchmark, and all behaviourally motivated decisions of the clients are irrational. This is the frontier between a framework for rational and irrational investors. Advising is helping clients to take rational decisions, which are consistent with their needs and preferences. Although some theories define a strict framework for rationality, this may not fit clients needs (Tversky and Kanheman, 1976, 1992).

In the industry the challenge is how advisors calibrate the theoretical framework with a sound and clear classification on clients needs and perceptions (Kofman and Payne, 2018). Another frontier is how algorithms used by robot-advising incorporate these features on their questionnaires. In fact, there are clear distinctions between the results from the robo-advisors sample we tested, either from a client risk profiling or from an asset allocation perspective. These outcomes are a major concern for individual investors that have no financial literacy to perform backtesting of each proposed strategy by the robot. In spite of individual investors being alerted either by financial market regulators or trading regulators (e.g., FINRA, SEC, PRA, ...), and regardless the fact that some markets, as the European, being under the umbrella of strict directives (i.e., MIFID), there are relevant risks of mismatching between investors goals and solutions provided by robo-advisors, due to some large scope of understanding within the strategies put forward by the algorithms (OECD, 2017).

Over the last several years, with an ageing population transferring assets to millennial, these individuals are more susceptible to utilizing internet based platforms for most of their financial needs. Large investment institutions are mindful of these trends and have started investing in the infrastructure of robo-platforms (McKinsey, 2015). As artificial intelligence evolves, the software will be designed to handle more complex, tougher strategies, whereas the current robo-platforms are not equipped to handle these scenarios. The strategies are offered based on Exchange Tradable Fund (ETF), and there are questions on how based investing performs would survive during a financial crisis as it would present itself the next time. Nonetheless, the levels of trust humans are willing to have on a fully automated investment platform will determine how fast robo-advising is integrated, specially on the event of financial crises and the way conflicts of interests are managed (OECD, 2017, Baker and Dellaert, 2018).

6.1 The Robot and the Individual Investor

Robo-advising is a type of financial advisor that provides investment advice and creates portfolio recommendations based on broad characteristics of principles of asset allocation, mean variance analysis and modern portfolio theory (QPLUM, 2016). There is little to no human interaction as robo-advisers use an on-line platform created with an algorithm to provide the recommendations. These characteristics highlight important elements such as an ETF based strategies, rebalancing and tax loss harvesting (Lam, 2016). The behavioural components that drive investor's expectations are associated with risk questionnaires at platforms that provides consumers a high level overview of various robo-advisors. We work an in-depth assessment of a sample of five on-line platforms: Charles Schwab, SigFig, Wealthfront, ToleRisk and RiskAlyze. In this analysis, back-testing is conducted to assess performance, volatility, specific risk exposures, value at risk and sharpe ratios.

6.1.1 Environment

Sixty years have passed since the first publication of Markowitz's article on portfolio-selection (1952), which has set the foundation for the ground-breaking concept commonly known today as the Modern Portfolio Theory (MPT). Markowitz research primarily focused on effects of correlation, diversification, asset-risk and investment returns on portfolios. The MPT aims for a better understanding of the relationship between investment risk and return and is independent of asset pricing theories (Elton and Gruber, 1997). Furthermore, Portfolio Theory provides a method to analyse mean and variance of returns based on the assets contained in the portfolio. The efficient frontier consists of a set of portfolios that gives the highest level of expected-return for each level of risk. Moreover, many aspects of the MPT and robo-advisory platform utilize these principles as a framework to select portfolios based on expected return and investors' appetite for risk (Fabozzi et al., 2002).

Although, the Markowitz's model laid the foundation for the theory, in historical practice it had its limitations since it required a large amount of data (Marling and Emanuelsson, 2012). In robo-advising, this limitation is managed since these platforms can be designed to efficiently utilize large quantities of historical data and optimize it in a way that can still use the MPT.

The term robo-advising is broader than most understand. Robo-advisors are not literal robots who are conducting investment decisions. Robo-advisors are automated investment platforms that use software to do the same duties a typical financial advisor would do (D'Acunt et al., 2017). In the United States, as of today, almost all robo-advisors are registered as "RIA"s (Registered Investment Advisers) with the U.S. Securities and Exchange Commission (SEC), which serve to provide investment advice through the use of a website (Strzelczyk, 2018). The process of using a robo-advising would begin with the customer answering a set of questions. The profile begins with income, age, family demographics such as spouse and children, expected retirement, etc. Then the customer would be asked a series of questions that would determine his/her tolerance and acceptability to risk. This is done either through selecting goals or determining a particular investing style. An algorithm used by the software would then determine a recommended portfolio of ETF with a variety of asset classes (such as stock, bonds, etc.). The algorithm's recommendation generally follows ten fundamental principles of asset allocation (Lam, 2016) presented in Table 6.1.

6.1.2 Characteristics

Robo-advising platforms use an ETF based strategy to follow the principles noted above. ETFs are a passive investment vehicle which allows its participants to take market exposure via index based strategies[2]. In passive indexing strategies follow the Efficient Market Hypothesis (EMH) which in essence believes that all material, public and non-public information, is priced in the current market valuation. The proponents of indexing strategies use the EMH to claim that active management is unable to outperform a broad market based index as presented by Malkiel (2003). Since indexing eliminates the need for excess portfolio analysis, rebalancing and trading, it keeps internal costs much lower compared to an actively managed fund.

Liquidity is an important element to many sound financial strategies. Customers using robo-advisors have the ability to retract their assets at any time. Given this limitation, robo-advisors must select asset classes that are fairly liquid and stay away from classes such as private equity as those funds are generally tied for certain time-frames. They also tend to limit their exposure to actively managed mutual funds in order to keep internal costs low. As customers have

[2]For a detailed definition of indexing strategy see https://www.etf.com/publications/journalofindexes/joi-articles/1791.html.

Table 6.1: Fundamental Principles of Asset Allocation

Fundamental Principles	
Market Efficiency	Market-efficiency is the one of the most important factors associated with asset allocation. In the absence of market efficiency asset allocation would not be employed and the aim would be to focus on security-selection. Asset allocation focuses on two well-known investment concepts which are Efficient Market Hypothesis (EMH) and Modern Portfolio Theory (MPT).
Investor Risk Profile	An investor's optimal-portfolio is selected and designed to cater for the needs and risk-tolerance of investors. Once the risk-tolerance is established the next goal is to maximize the overall portfolio's return by establishing the risk-return trade-off profiles.
Identifiable Financial Goals	When identifying financial goals, asset allocation enables the division of assets among different asset classes and securities to achieve the highest level of total-return for the level of risk investors are willing to accept and take. As a result, investors should be primarily focused on their short-term, medium-term and long-term financial goals.
Time Horizon	An investor's time horizon plays a significant role in determining the overall risk levels, asset class returns and price correlations. Accurate forecasts are required to build an optimal portfolio for an investor, thereby determining the balance of portfolio between fixed income, equity, cash, alternative investments and equivalents. As a rule, the longer the time horizon of investment, the more investors should allocate towards equity investments and the shorter the time horizon of investments, the more should be allocated towards income producing investments.
Expected Total Return	During the interment holding period; the expected total return is the total-return for each asset-class and subclass. Once the investor's risk tolerance levels are established the next step is to maximize the overall expected total return. To build the optimal portfolio, there is a need to clearly understand the overall expected total return for each asset class and sub-class.
Risk-and-Return Trade-off Profile	The relationship between risk and return and investment specific-risk is central to asset-allocation. Risk and return are both interlinked, and one cannot generate or earn a higher return by taking low levels of risk. Asset allocation theory suggests that the higher the investors risk-tolerance levels, the higher the potential returns.
Correlation	Correlation in the case of asset allocation refers to the co-movement of two assets in relation with each other. Asset allocation theory aims to have assets selected in a portfolio that do not have perfect positive correlation. The greater the difference in correlation and lower the correlation levels, the more attractive the portfolio's return will be for the investor. Therefore, this enhances the risk return profile for the overall portfolio.
Diversification	Principles of diversification should be universally applied to minimize the risk of a portfolio. Diversification in portfolio management is the idea of spreading your assets amongst various asset classes. By holding diversified portfolios, investors overall risk return can be enhanced. The process of diversification involves holding a significant number of different investments in the overall portfolio. However, it is also necessary to note that diversification does not take account off or protect against systematic risk (market risk).
Optimal Asset Mix	The optimal asset mix refers to the mix of assets classes and sub-asset classes in a portfolio. By allocating assets and sub asset classes the desired risk-return trade-off defines the optimal asset-mix. Creating a portfolio with sub-optimal mix will assume more risk than the investor's tolerance levels or is likely to assume less-risk than the investor's tolerance levels. Furthermore, in order to manage an optimum portfolio there is a constant need to rebalance the portfolio which requires constant monitoring and rebalancing.
Re-optimization	Overtime, there is a constant need to re-optimize the portfolio resulting from price-level and economic-level fluctuations in the market. These price level and economic level changes will result in the risk-return profiles to change and therefore to manage investors overall risk-tolerance there is a constant need to re-optimize the portfolio. Re-optimization of a portfolio consists of the following tasks: i.e., re-evaluating, re-balancing, and re-allocating.

different needs, risk profiles and time horizons, robo-advisors will also select numerous asset classes to satisfy their customer's situation (Lam, 2016).

Another characteristic of robo-advisors is adjusting portfolio drifts while still maintaining the asset allocation. Volatility may cause an overweighting or under-weighting in an asset class which may cause the portfolio to deviate from its benchmark. Due to the use of index funds, robo-advisors are able to correct this drift without affecting returns. In comparison, actively managed mutual funds would rebalance its holdings which could lead to a combination of issues such as increased portfolio turnover, higher tax drag and possibility of lower net of fee returns. On the other hand, robo-advisors in their strategies factor in the ETF costs along with the spreads of bid-ask, which typically are lower for assets that are liquid and tradable. Lastly, robo-advisors are able to minimize their tracking error risk which arises from the difference between the target index and the index fund (Lam, 2016).

In robo-advising, trades are typically cleared through the robo-advisor's broker deal (i.e., a traditional advisor may clear their trades through a clearing house, and then a trade is passed onto the market maker or an exchange). Some robo-advisors have further advanced the trading algorithms by incorporating a high frequency component to ensure efficient pricing and timing of the order. Once the client strategy is implemented, the portfolio is systematically realigned to keep the asset allocation in line with his/her risk return profile (Lam, 2016).

In the US Tax-loss harvesting, is another value added feature that robo-advisors have been able to capitalize on[3].

[3] Tax-loss harvesting is achieved when a losing position is used to offset gains while still maintaining a portfolio's variance/covariance mix. While tax loss harvesting, a portfolio must remain sensitive to SEC's Wash Sale Rule which does not allow to trade a security at a loss within 30 days before or after the sale of an identical security. Algorithms claim they are designed in a way to work around the wash sale rule by selling something at a taxable loss and then repurchasing comparable assets which would yield a similar risk return profile.

6.2 The Human Component and Influence of Behavioural Finance

One obvious observation of robo-advising is the lack of human touch throughout the investment process. The entire portfolio's rebalancing, monitoring and assessing are done via the algorithm and then reviewed by humans to ensure their compliance with securities regulations. On the customer side, the human element revolves around how much time and attention the customer dedicates to answering the questionnaire as this determines the accuracy of the algorithm to match a customer's needs. It also places a burden on the customer to notify the software of any changes to their risk strategy or changes in the investment goals.
An important aspect of robo-advising and questionnaires are behavioural elements associated with an investment decision making process. Most platforms agree that understanding investor psychology and investors' expectations is important for creating a sound financial strategy (FINRA, 2016).

The most important factor determining the client retention is the service quality from financial advisors, besides the effective accomplishment of the goals set at portfolio level. Investors tend to rely more on personal experience than marketing campaigns from financial services. This evidence tends to focus advisors actions towards clients' experience when designing their strategy approach on wealth (Lam, 2016).

Although clients' needs are per nature individual and specific, financial advisors try to reach some level of standardization. They use segmentation by aggregating homogeneous groups by needs and expectations. The most common factors for criteria are:

- Geographic
- Demographic;
- Psychographic;
- Client's profitability;
- Client's wealth.

The least important in knowing clients' needs and expectations is wealth. When clients do not differ with respect to their aversion toward uncertainty, which is the main feature of their preferences, they should receive the same advice independently on their wealth. The absolute measure of wealth may have some constrains but when dealing with ETF there are no limitations when implementing investment strategies (Fish et al., 2017).

A relevant argument is the distinction between clients' decisions which are driven by their preferences and those that are driven by psychological biases. This is also the reason why it is important for advisors to understand how their clients make financial decisions. Not understanding behaviour biases may result in inappropriate financial advising. In fact, clients' loss aversion may enhance protective strategies, like portfolio insurance, but the key point is to derive from the questionnaires levels of loss aversion and degrees of "greed"/aspirations, making the complete process adherent under a valuation framework.

Table 6.2: Bias irrational

Biases leading to irrational choice	How to fight against biases
Availability bias/Attention bias	Show long term results before going into recent news. Examples of good stories that failed.
Representativeness/Gamblers' fallacy	When clients see patterns from the data it is necessary to present statistical data.
Anchoring/Conservatism	Clients' expectations must respond to new changes in fundamentals and do not stick to last estimates.
Framing	Confirm that clients are coherent by changing the representation of the problem.
Overconfidence	Clients must be aware that there are other risk profiles that may obtain better results by using different skills and styles.
Mental accounting	Point out the benefits of diversification.
Home bias	Point out the gains on country and sector risk diversification.
Self attribution bias	Present success/failure is most likely good/bad luck.
Emotions	Help client to avoid spontaneous trades.
Hindsight bias	Present the risks and chances when a decision is taken and recall them after the returns have realized.

Source: A resume from "Behavioral Finance for Private Banking", 2008, Thorsten Hens and Kremena Bachman, Wiley Publications.

This perspective is important for establishing a link between the clients' needs and certain financial objectives because the typical perception is that traditional finance defines a rational benchmark, and all behaviourally motivated decisions of the clients are irrational. This is the frontier between a framework for rational and irrational investors.

Advising is helping clients to rational decisions, which are consistent with their needs and preferences. Although some theories define a strict framework for rationality, as discussed in previous chapters, this may not fit clients' needs. The challenge is how advisors calibrate the theoretical framework with a sound and clear classification on clients' needs and perceptions. Another frontier is how algorithms used by robot-advising incorporate these features on their questionnaires. Trust still is a very important factor when selecting a financial advisor, therefore using algorithms is a new approach on the market that is making a steady and consolidating path towards matching clients' needs and financial outcomes.

6.3 Analysis of On-line Platforms and Risks

The following exhibit compares standard features of the sample of five robo-advisors selected for analysis. The selection was limited due to access conditions to account opening process and data availability on risk profiling, regardless of robo-advising capabilities to provide an end-to-end process from advising to investing.

Table 6.3: Assessment of on-line platforms

..	charles SCHWAB INTELLIGENT PORTFOLIOS	SIGFIG	wealthfront	TOLERI$K THE SCIENCE OF RISK TOLERANCE	riskalyze
Assets Under Management	$12.3B	$114M	$5.0B	N/A*	N/A*
Minimum Deposit	$5,000	$2,000	$500	--	--
Fees	0.28% of assets[5]	None first $10k, 0.25%/yr for more	None first $10k, 0.25%/yr for more	$49-$59 Per User, Per Month[6]	$145-$245 Per Month[7]
Automatic Rebalancing	Yes	Yes	Yes	Yes	Yes
Tax Loss Harvesting	Yes	Yes	Yes	Yes	Yes
Advice	Automated with Human Advice Available	Automated	Automated	Automated	Automated

Source: Robo-Advisor Comparison – Detailed Breakdown (2017)

Both ToleRIsk and Riskalyze do not have assets under management since they are advisory software's available to consumers.

The tables in the appendix include a compilation of questions presented by the 5 robot-advising companies. In our analysis we aggregated them in four behaviour vectors: expectations, risk ability, risk preference and risk awareness. The more complete questionnaires into behaviour approach are marked with colour, the less matching ones are identified with grey.
In our opinion the reason for these differences is not an incomplete algorithm, but rather a strategy defined by these financial providers where subjective biases may be subject to a lighter or deeper analysis.

Robo-advisors rebalance funds using threshold-based rebalancing. Threshold rebalancing will rebalance a portfolio once any asset class deviates from a specified percentage or target volatility. Robo-advising on-line platforms' algorithm will automatically trade assets to bring the portfolio back to its optimal asset mix relevant to its investor's objectives. Another important item to consider is an investor's target, which changes with time. As time passes, the investors risk tolerance and time horizon will also change (QPLUM, 2016).

To create the portfolio on each robo-advising platform we took the following steps:

1. Each question asked by the robo-advising platform was evaluated
2. Based on the answers to the risk evaluating questionnaires, each robo-advisory recommendation was noted.
3. The risk level was checked due to different options of each question.
4. Based on the risk appetite of the user three types of profiles were created.
   (a) Conservative: This profile was created for a consumer who has a very low level of risk exposure. These assets are mostly used for retirement funds that are expected to have low volatility and high liquidity. Asset allocation of these portfolios generally tend to have low percentage allocations to speculative asset classes.
   (b) Moderate: This profile was created for a consumer who has a medium level of risk exposure and targets to attain returns at market level. Consumers with this risk appetite are mostly seeking to invest for saving purposes and funds may not be needed for immediate needs. This portfolio is expected to have a moderate amount

of investments in speculative asset classes with some volatility and medium term liquidity.

(c) Aggressive: This profile was created for a consumer who has a high level of risk exposure and is seeking to earn returns higher than the market. This consumer is seeking to invest generally for wealth creation that is long term in nature while utilizing speculative and long term growth strategies. This portfolio is expected to have a high degree of allocation in speculative asset classes with a high volatility and low liquidity.

Investors face many risks in the market, however the following ones are those that are specific to the robo-advisory platforms compared in this work.

Table 6.4: Market Risks - Adapted from GARP: Risk Concepts

Market Risks	
Interest Rate Risk	Interest rate risk is the risk of changes in interest rates which may reduce (or increase) the market value of a bond portfolio.
Equity Risk	Equity risk arises when an investment is exposed to systematic risk drivers that may cause a portfolio to depreciate or incur in capital losses due to broad based market shocks. Equity risk can also be specific to a company. In that case it is an idiosyncratic risk. Most fund managers diversify the company specific risk while using assorted strategies to manage market risk.
Currency Risk	Currency risk, sometimes referred to as exchange rate risk, is the negative impact of currency depreciation on the value of one's assets, investments and their related interest and dividend payment streams. Investments that hold a long position on assets which are denominated in foreign currency are most susceptible to currency risk.
Commodity Risk	Commodity risk refers to the uncertainties of future market values and to the amount of future income, caused by fluctuation in the prices of commodities. Examples of commodities are grains, metals, gas, electricity etc. Investments have exposure to commodity risk when the underlying commodities in the ETF based portfolio perform adversely.
Inflation Risk	Inflation risk, also known as purchasing power risk arises when changes in the real return (due to inflation) are negative. Interest rates tend to directly impact an inflation exposure of an investor's portfolio. Investments such as TIPS or other inflation protected bonds mitigate exposure to inflation risk.
Real Estate Risk	Real estate risk arises in a portfolio due to devaluation of the underlying real estate, investment properties, corporate real estate or residential home values. Typical investments that are exposed to this risk are REITS, MBS, ABS and CMBS, etc. These assets also are exposed to secondary risks such as interest rate risk and currency risk (if denominated in foreign currency).

After the selection of stylized investors' profiles the next step in the analysis focused on portfolio recommendations provided by each robo-advisory platform.

In most platforms a portfolio is created by the algorithm which constructs an asset allocation and provides the associated weighted percentages to those asset classes relevant to each profile. In some robo-advising platforms the underlying assets were not clearly defined, in which case a comparable ETF or an index was used to map the risk.

The indexes are outlined in Table 6.5 and are mapped according to their primary and secondary risk. The assets used for portfolio construction were assessed. The primary purpose and the assets underlying those ETFs and indexes are considered and mapped to those relevant risks mentioned above. The next step assessed each portfolio's asset classes and determined its primary and secondary risks relevant to each of the Index and ETF. It should be noted that some of the ETFs and indexes may have multiple risk exposures, but on this work only primary risk drivers were mapped.

As all robo-advising platforms use ETFs or indexes to create their asset allocation plans, we back tested each portfolio using 10 years of price data of various indexes. In some instances, the advice only mentioned a broad asset class without a specific investment recommendation. In those instances was selected a liquid ETF or an index which aligned with the risk return profile of the asset class. In some cases, when historical prices were not available for indices, the prices from the secondary indices were used to complement the price history. Secondary indices that are chosen have a similar asset class profile as the primary index they are trying to complement. In this way we can obtain a more sound price information and capture more market trends [4].

All the return calculations were done at the monthly frequency of the underlying ETFs and indexes. Returns of each ETF and index were aggregated at the portfolio level to get the monthly return of the portfolio. The return aggregation of the portfolio was done based on the portfolio weight given by the robo-advisory platform. After this, log returns were computed on a monthly frequency and final aggregated returns were done for 1 year and 5 years. We

[4] For example, in one of the portfolios, VOO-Vanguard S&P 500, with an inception date of September 30th 2010 was substituted with SPY- SPDR S&P 500 ETF Trust which has inception date in 1993. It should be noted that the sources for all price information are Thomson Reuters or Yahoo Finance's database.

Table 6.5: Mapping risks and indexes

Index Name	Index Type	Risk Exposure	Secondary Risk Exposure
PRF	US Large Company Stocks - Fundamental	Equity Risk	
PXF	International Developed Large Company Stocks - Fundamental	Equity Risk	
VOO	US Large Company Stocks	Equity Risk	
PRFZ	US Small Company Stocks - Fundamental	Equity Risk	
VEA	International Developed Large Company Stocks	Equity Risk	Currency Risk
VNQ	US Exchange-Traded REITs	Real Estate Risk	
VB	US Small Company Stocks	Equity Risk	
PDN	International Developed Small Company Stocks - Fundamental	Equity Risk	
IEMG	International Emerging Market Stocks	Currency Risk	Equity Risk
PXH	International Emerging Market Stocks - Fundamental	Currency Risk	Equity Risk
VSS	International Developed Small Company Stocks	Equity Risk	Currency Risk
MBG	US Securitized Bonds	Interest rate risk	
VGIT	US Treasuries	Interest rate risk	
VYM	US Corporate High Yield Bonds	Equity Risk	
VCIT	US Investment Grade Corporate Bonds	Interest rate risk	
STIP	TIPS	Inflation risk	Interest rate risk
IGOV	International Developed Country Bonds	Interest rate risk	Currency Risk
VWOB	International Emerging Market Bonds	Interest rate risk	Currency Risk
DGL	Gold and Other Precious metals	Commodity Risk	
VMMXX	Cash	Interest rate risk	
TFI	Municipal Bonds	Interest rate risk	
BND	BND · Vanguard Total Bond Market ETF	Interest rate risk	
SHY	SHY · iShares 1-3 Year Treasury Bond	Interest rate risk	
SPY	US Equities	Equity Risk	
EFA	EFA · iShares MSCI EAFE	Equity Risk	Currency Risk
HYG	HYG · iShares iBoxx $ High Yield Corporate Bd	Interest rate risk	
FLOT	FLOT · iShares Floating Rate Bond	Interest rate risk	
VNQ	US Exchange-Traded REITs	Real Estate Risk	
QQQ	QQQ · PowerShares QQQ ETF	Equity Risk	
DBC	DBC · PowerShares DB Commodity Tracking ETF	Commodity Risk	
DBL	DBL · Doubleline Opportunistic Credit Fund	Interest rate risk	
EFR	EFR · Eaton Vance Senior Floating-Rate Fund	Interest rate risk	
XLU	XLU · Utilities Select Sector SPDR® ETF	Equity Risk	
EEM	EEM · iShares MSCI Emerging Markets	Currency Risk	Equity Risk
FPX	FPX · First Trust US IPO ETF	Equity Risk	
FXI	FXI · iShares China Large-Cap	Equity Risk	Currency Risk
VNRSQ	Natural Resources	Commodity Risk	

assumed that dividend and interim cash flows amongst all indexes are reinvested.

Portfolio volatility was computed using the standard deviation of the log returns previously computed. The volatility was annualized to get the final value. The final volatility figures attributed to various underlying risks such as equity risk, currency risk, commodity risk, inflation risk and interest rate risk were computed from the weights assigned by the robo-advising platforms and the volatility of that ETF and Index. The assumption was made that there is one primary market risk to each index or ETF. Additionally, we assumed that there is zero correlation between the various market risks mentioned above. The calculations consist of one year, five year and ten year volatility for the given portfolios. One year volatility risk was further segmented to reflect equity risk, currency risk, commodity risk, inflation risk and interest rate risk of the portfolio.

The following performance measures were used to evaluate portfolio recommendations.

- Worst return in 10 year period
- 5% historical Value at Risk
- 10% historical Value at Risk
- 1 year Sharpe ratio
- 5 year Sharpe ratio
- Risk profile of the return

Table 6.6: Assessment of on-line platforms - Wealthfront

	Wealthfront		
risk/return/performance	**Conservative**	**Moderate**	**Aggressive**
1 Year return	0.4%	4.5%	6.7%
5 Year return	0.0%	1.4%	2.0%
5 Year Volatility	9.0%	10.1%	11.6%
10 year Volatility	11.3%	14.4%	17.0%
1 year Volatility	13.3%	11.3%	11.5%
Interest rate risk	2.0%	1.2%	0.3%
Equity risk	2.1%	2.7%	3.1%
Currency risk	0.6%	1.6%	2.0%
Commodity risk	13.0%	10.8%	10.8%
Inflation risk	0.1%	0.0%	0.0%
Worst return	-12.5%	-17.3%	-20.6%
5% VaR	5.9%	7.7%	9.3%
10% VaR	4.8%	6.5%	7.7%
1 year sharpe ratio	0.04	0.45	0.58
5 year sharpe ratio	-0.23	-0.06	0.00

Table 6.7: Assessment of on-line platforms - Charles Schwab

Schwab			
risk/return/performance	**Conservative**	**Moderate**	**Aggressive**
1 Year return	5.3%	6.4%	10.7%
5 Year return	2.9%	3.4%	4.7%
5 Year Volatility	5.4%	6.2%	8.6%
10 year Volatility	9.5%	11.0%	15.2%
1 year Volatility	1.5%	1.7%	2.4%
Interest rate risk	0.7%	0.6%	0.5%
Equity risk	1.2%	1.4%	2.0%
Currency risk	0.4%	0.4%	0.9%
Commodity risk	0.3%	0.7%	0.8%
Inflation risk	0.1%	0.0%	0.0%
Worst return	-13.1%	-15.2%	-20.4%
5% VaR	5.4%	6.1%	8.3%
10% VaR	3.0%	3.4%	5.1%
1 year sharpe ratio	0.99	1.03	1.25
5 year sharpe ratio	0.17	0.23	0.31

Table 6.8: Assessment of on-line platforms - SigFig

SigFig			
risk/return/performance	**Conservative**	**Moderate**	**Aggressive**
1 Year return	2.0%	8.7%	12.2%
5 Year return	1.6%	4.0%	5.0%
5 Year Volatility	3.5%	7.3%	10.6%
10 year Volatility	5.1%	12.8%	17.7%
1 year Volatility	1.8%	2.9%	4.3%
Interest rate risk	1.5%	0.6%	0.4%
Equity risk	0.9%	2.5%	3.4%
Currency risk	0.4%	1.3%	2.6%
Commodity risk	0.0%	0.0%	0.0%
Inflation risk	0.3%	0.4%	0.0%
Worst return	-6.5%	-16.7%	-22.6%
5% VaR	2.4%	6.9%	9.4%
10% VaR	1.7%	4.5%	7.4%
1 year sharpe ratio	1.10	3.00	2.86
5 year sharpe ratio	-0.12	0.27	0.29

Results from the risk profile derived from platform questionnaires are not consistent with the expected results, neither on a 1 year nor on a 5 year period. Thus it is clear the possible inconsistencies between the life style aspirations with the strategies delivered from this automatic risk profiling techniques. It is unclear how these mismatchs are impacting the wealth growth of

Table 6.9: Assessment of on-line platforms - Tolerisk

Tolerisk			
risk/return/performance	**Conservative**	**Moderate**	**Aggressive**
1 Year return	0.9%	11.6%	13.9%
5 Year return	1.6%	8.8%	10.3%
5 Year Volatility	4.2%	8.3%	9.7%
10 year Volatility	4.7%	12.8%	15.1%
1 year Volatility	4.1%	5.2%	6.1%
Interest rate risk	4.1%	0.9%	0.2%
Equity risk	0.6%	5.2%	6.1%
Currency risk	0.0%	0.0%	0.0%
Commodity risk	0.0%	0.0%	0.0%
Inflation risk	0.0%	0.0%	0.0%
Worst return	-4.2%	-15.2%	-17.6%
5% VaR	2.1%	6.8%	7.9%
10% VaR	1.6%	4.6%	5.8%
1 year sharpe ratio	0.22	2.22	2.27
5 year sharpe ratio	-0.08	0.82	0.86

Table 6.10: Assessment of on-line platforms - Riskalyze

Riskalyze			
risk/return/performance	**Conservative**	**Moderate**	**Aggressive**
1 Year return	1.8%	5.6%	12.6%
5 Year return	1.5%	2.6%	6.4%
5 Year Volatility	2.7%	5.9%	10.5%
10 year Volatility	4.6%	10.5%	17.8%
1 year Volatility	1.7%	2.6%	3.8%
Interest rate risk	1.3%	1.0%	0.0%
Equity risk	1.0%	2.3%	3.6%
Currency risk	0.0%	0.0%	0.8%
Commodity risk	0.0%	0.7%	1.0%
Inflation risk	0.0%	0.0%	0.0%
Worst return	-7.6%	-16.8%	-23.6%
5% VaR	10.4%	4.6%	5.3%
10% VaR	7.7%	3.4%	0.9%
1 year sharpe ratio	1.10	2.15	3.32
5 year sharpe ratio	0.40	1.50	2.24

investors' portfolios. They may be piling up implicit losses when opting by theses "automatic" strategies.

The cross-sectional analysis sets the grounds to quantify potential mismatches between specific individual aspirations and strategies that are not delivering those objectives. The probable

Figure 6.1: Results - Conservative portfolios

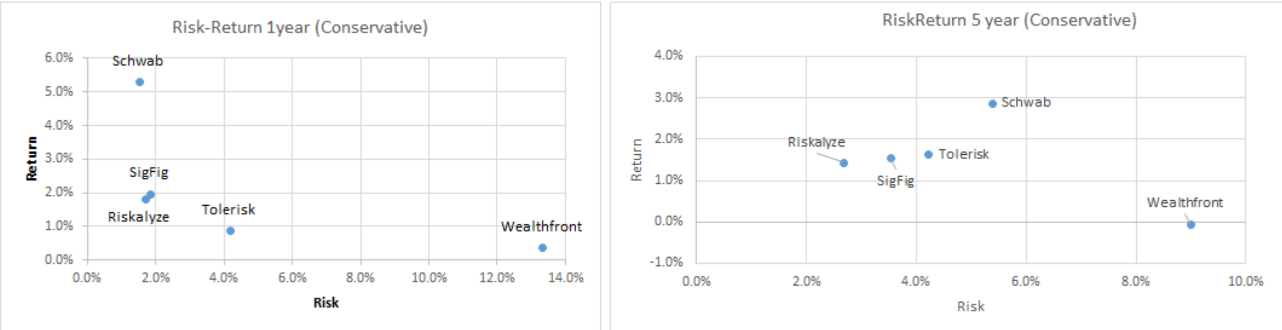


Figure 6.2: Results - Moderate portfolios

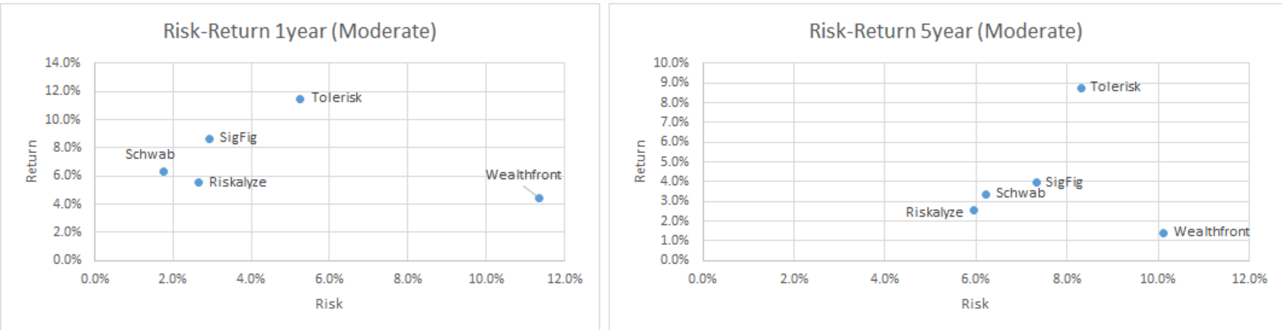
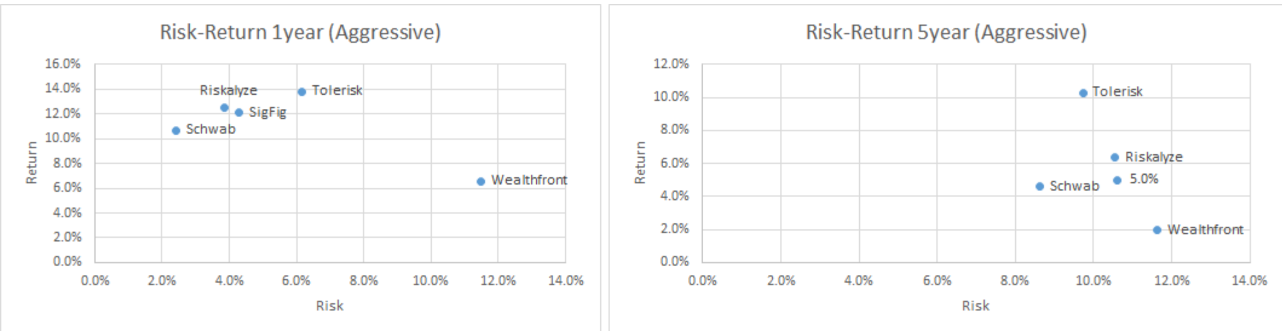


Figure 6.3: Results - Aggressive portfolios



cause of potential mismatches may be the fact that platforms are not capable of capturing the individual risk profile on on-line questionnaire. One of the reasons is that there may be some incomplete behaviour factor missing on questionnaires.

The three risk profiles used for all robo-platforms come up with different asset allocations. The divergence between this approach on a risk-return selection has some pitfalls. The possibility of introducing some dynamic approach for securing some level of protection is something that needs to be thoroughly analysed as the same stylized investor may end up with severe differences after being profiled in several platforms.

Table 6.11: Summary results

	Wealthfront			Schwab			SigFig			Tolerisk			Riskalyze		
	Conservative	Moderate	Aggressive	Conservative	Moderate	Aggressive	Conservative	Moderate	Aggressive	Conservative	Moderate	Aggressive	Conservative	Moderate	Aggressive
1 Year return	0.4%	4.5%	6.7%	5.3%	6.4%	10.7%	2.0%	8.7%	12.2%	0.9%	11.6%	13.9%	1.8%	5.6%	12.6%
5 Year return	0.0%	1.4%	2.0%	2.9%	3.4%	4.7%	1.6%	4.0%	5.0%	1.6%	8.8%	10.3%	1.5%	2.6%	6.4%
5 Year Volitility	9.0%	10.1%	11.6%	5.4%	6.2%	8.6%	3.5%	7.3%	10.6%	4.2%	8.3%	9.7%	2.7%	5.9%	10.5%
1 year Volatility	13.3%	11.3%	11.5%	1.5%	1.7%	2.4%	1.8%	2.9%	4.3%	4.1%	5.2%	6.1%	1.7%	2.6%	3.8%
Worst return	-12.5%	-17.3%	-20.6%	-13.1%	-15.2%	-20.4%	-6.5%	-16.7%	-22.6%	-6.5%	-16.7%	-22.6%	-7.6%	-16.8%	-23.6%
5% VaR	5.9%	7.7%	9.3%	5.4%	6.1%	8.3%	2.4%	6.9%	9.4%	2.4%	6.9%	9.4%	1.9%	4.9%	9.0%
10% VaR	4.8%	6.5%	7.7%	3.0%	3.4%	5.1%	1.7%	4.5%	7.4%	1.7%	4.5%	7.4%	1.4%	3.3%	6.8%
1 year sharpe ratio	0.04	0.45	0.58	0.99	1.03	1.25	1.10	3.00	2.86	1.10	3.00	2.86	1.10	2.15	3.32
5 year sharpe ratio	-0.23	-0.06	0.00	0.17	0.23	0.31	-0.12	0.27	0.29	-0.12	0.27	0.29	0.40	1.50	2.24

6.4 Limitation of Robo-Advising

While automation is one of the largest strengths of robo-advising, it also has its shortfalls. The shortfall arises when a complex scenario such as estate, trust or endowment planning is needed. For example, an estate plan will typically involve an understanding of wills, estate taxes, state death taxes (if applicable), along with current regulations. The idea behind robo-advising is lowering fees, but if investors have to supplement their robo-advisor with advising they may end up spending more than if they would have used a traditional advisor.

Another limitation of robo-advising is that the portfolio is created based on static information that is relevant at a given point in time. Traditional financial advisors adjust portfolios in order to match a client's ever-changing financial situations. Although robo-advisors do not have face-to-face interaction, their algorithms do not lack an ability to tailor a portfolio strategy. For example, if an investor already has an income based rental property, the portfolio recommendation would minimize its exposure to income driven asset classes such as Real Estate Investment Trust (REIT) or corporate bonds.

The robo-advisor platforms consider factors that are significant to an investor's financial needs. These recommendations are completely determined by the depth of questionnaires used by each platform. A second factor is how thorough the answers to the questions are and if the pre-selected fields used to answer the questions are 100% matching the client's individual situation. The robo-platforms also consider the type of account an investor is seeking advice for. In the United States, for tax sheltered accounts, robo-advisors may recommend corporate debt as opposed to a taxable account, they may recommend municipalities which have similar tax equivalent yields. This element of the strategy would allow an investor to optimize his/her tax burden. Some platforms are focused on goal-based investing, in which a strategy is designed to manage his/her investment objectives while keeping costs and expenses low. Therefore, it is evident that robo-platforms have the capability to constantly adjust and address the changing needs of an investor (Lam, 2016).

In relation to investors' protection, either on the capability of the financial services firms providing robo-advising, or at the wealth investment security, there are limitations that individual investors should be aware of (FINRA, 2016). These are related with safety of investments and also with the necessary due diligences on financial firms. Although financial literacy is a relevant topic on financial decisions, there is potential for the inability to correctly describe the relationship between investors and robo-advisor firms (Strzelczyk, 2018) specially on fiduciary duties. The potential to better alignment between investor targets and robo-advisors is being tested, as its popularity is growing, but there are signs that portfolio management improvements are possible and some of the pitfalls derived from pervasive behavioral biases can be reduced (D'Acunto et al., 2017, Shefrin, 2016).

6.5 Conclusions

Humans tend to execute decisions based on emotions which can end up costing their portfolios. These emotions can be triggered by biases or swift decisions due to changes in the market. With robo-advising, the emotional component is removed because all decisions are based on objective data that an algorithm utilizes to determine the most effective asset allocation.

Over the last several years, quasi-robo advising models have begun to show at the traditional investment houses. A hybrid model has been implemented, which contains asset management advice from robo-platforms along with over-the-phone human advisory support. With an ageing population transferring assets to millennial, these individuals are more susceptible to use internet based platforms for most of their financial needs.

For larger institutions, complete replacement of human advisors seems unlikely but they will be influenced by a few items. First, as artificial intelligence evolves, the software will be designed to handle more complex, tougher strategies, whereas the current robo-platforms are not equipped to handle these scenarios. Second, how ETF-based investing will perform during a financial crisis. Lastly, the levels of trust humans are willing to have on a fully automated investment platform will determine how fast robo-advising is integrated.

These large financial institutions face both a challenge and an opportunity, especially given the costs associated with marketing and acquiring new clients. In a human advisory model, the burden of acquiring new assets is placed on the financial advisor and their team. As firms shift towards cost cutting models, they have started to incorporate Business to Business to Consumer (B2B2C) models in which robo-advisory firms partner with traditional wealth management institutions to gain access to their customer base (particularly those sensitive to cost and not in need of a particularly sophisticated investment strategy).

Robo-advising platforms although still within their early phases of development, have gained quite a bit of traction within the wealth management sector. From our analysis we expect robo-advisory platforms will continue to evolve over the years. As technology evolves, the robo-advisory platforms may continue to experience an up-tick. This could be affected given the millennial preference for on-line based services. If the focus remains on cost cutting elements then passive indexing strategies could remain a viable option. Another important component is the regulatory landscape and its evolution. In this sense, the focus that regulatory frameworks take into the role of asset managers, depositories, custodians, exchanges, must go beyond the nature of the supervisory of markets to the conduct of the firms managing the robo-advisory platforms. The value chain of these emergent financial services is unclear regarding the fiduciary duties and is necessary a clarification due to the velocity of technological evolution. However, risk profiling is still a very grey area when financial literacy is not robust enough for investors to comprehend the pitfalls of a wider class of protective strategies, that include portfolio insurance.

The setting of protective strategies by robo-advisers is still under analysis and remains yet a concern. As an example, in our study the risk/return outputs obtained from the platforms for the same set of investors' characteristics were very different. The level of complexity necessary for defining floors and risk leverage (multipliers) can be developed but the density of the questions and cross-checks could discourage investors to go through questionnaires, opting to interact with their investment advisor.

The existence of stylized investors with different risk profiles allowing the industrialization of the asset allocation process, may limit the capital protection strategies that are still attracting a large number of investors, as is the case of portfolio insurance. However we are seeing innovation using artificial intelligence and machine learning that can trigger the offer of downside protective strategies. These areas are evolving and, along with regulation, there is a need to deeper research on such dynamic topics.

Chapter 7

Conclusions

Summary of Thesis Achievements

The wide use of protective strategies like portfolio insurance reflects the wish of safe-harbours and simultaneously the eagerness of capturing market gains by investors. The conciliation of this two objectives comes with a cost which is relevant whenever there are imbalances in financial markets. There are records of such moments but investors still allocate assets into these portfolio insurance strategies.

Under EUT framework, general results reveal a risk exposure towards the risk free - fly to safety - on investors with medium to high level of risk aversion. As the level of risk aversion decreases, the return seeking investments tend to become more attractive as the return per unit of risk is perceived with higher utility. Investors with decreasing level of risk aversion, depending on market conditions, tend to exhibit a safety attitude benefiting protective strategies (the more simple portfolio strategies like CPPI and TIPP) in more volatile conditions. On less volatile conditions investors seek return and show more exposure to risky assets.
These results on investors with descriptive utility functions support the idea that there are not potential buyers of portfolio insurance strategies, except in very specific market conditions: scenarios of low expected returns and increasing volatility. That situation is also visible under MVA, where linear utility functions points to a flight to safety and some valuation on protective strategies with upside potential.

As some of the portfolio insurance solutions are complex investments that need to match with individual investors risk profile, the absence of clear circumstances where these strategies are demanded by investors should encourage financial institutions to avoid mis-selling.
However, as we change the framework for decision making towards prospect theory by Kahneman and Tversky (1979, 1992), results in positive market conditions (i.e. expected positive risk premium), strengthens the possibility that cumulative prospect theory may explain investors' choices.
The specific decisions of portfolio insurance are, nonetheless, highly dependent on the market conditions and characteristics of each portfolio insurance strategy—e.g., the reduction of the percentage floor from 100% to 80% depicts a shift from SLPI to CPPI strategies.

To the best of our knowledge, based on such innovative approach that compares the value perceived by investors between expected utility and prospect theories, we find that cumulative prospect theory is a viable framework to explain the popularity of portfolio insurance investments.
In spite of these results, we understand that it is necessary further analysis regarding the higher cumulative prospect values for SLPI strategies against CPPI. In future research we will review determinant factors for investment decision such as time horizon, portfolio management costs and tax.

The framework of behavioural finance may be the relevant factor to explain the selection of portfolio insurance strategies by individual investors. This rational behaviour fuels a market where there is a trend towards automatic platforms to help investors on the definition of their risk profiles and the selection of strategies to accomplish either their aspirations or objectives.

Future Work

The evolution of portfolio management for individual investments has been speeding towards algorithms and artificial intelligence, but we are still anchored into personal and human behaviour. The loss aversion is striving new investment allocations and dynamic management, however investors must be aware of the crisis that wiped out personal savings allocated to financial assets. Due to this situation there are three areas for future work that, in our opinion, need to be addressed:

1 Analysis in continuous time framework to develop analytical models, specially to integrate life cycle and protective solutions;

2 Augment the numerical analysis towards structured products in the area of capital protection solutions;

3 Development of risk profiling methodologies that mitigate future gaps between strategies suggested by robo-advisors algorithms and long term savings objectives.

Based on the findings of protective strategies and the high level of digitalization in the wealth management industry these topics are becoming extremely relevant for financial policies. This is specially true on the oversight of markets and institutions, with the view to promote financial stability, market efficiency, and client-asset and consumer protection.

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Appendix

Table 7.1: Risk Questionnaire from Robot-Advisors - Mapping the behaviour approach

Client - Profiling	**Schwab**	**SigFig**	**Riskalyse**	**Tolerisk**	**WealthFront**
Basic description	- Initial account settings.	- Initial account settings.	- Initial account settings.	- Initial account settings.	Current Age
					Which of the following best describes your household? - Single income, no dependents - Single income, at least one dependent - Dual income, no dependents - Dual income, at least one dependent - Retired or financially independent
Wealth		How much of your household income are you able to save (including retirement savings)?			Annual pre-tax income?
					What is the total value of your cash and liquid investments?
Expectations	My goal for this account is to" - prepare for retirement. - save for major upcoming expenses - save for something special - build a rainy day fund for emergencies. generate income for expenses.	How long do you expect to hold your portfolio (i.e., your investment horizon)?	What are your financial goals? - Retirement - College savings - Wealth accumulation - Income - Paying down debt - Other	When it comes to my investments, my primary goal is preservation of what I have.	What is your primary reason for investing? - General Investing - Retirement - College Savings - Other

Table 7.1 - (Continued)

Client - Profiling	Schwab	SigFig	Riskalyse	Tolerisk	WealthFront
	- build long term wealth.				
				When you think of the word "risk" in a financial context, which of the following words comes to mind first? Danger Uncertainty Opportunity Thrill	What are you looking for in a financial advisor? - I'd like to create a diversified investment portfolio - I'd like to save money on my taxes - I'd like someone to completely manage my investments, so that I don't have to - I'd like to match or beat the performance of the markets
Risk Ability	I have __ understanding of stocks, bonds and ETFs. (no, some, good, extensive)	Compared to your peers, do you think you are more or less tolerant of investment risk?	Type of Investment - Capital Preservation - Conservative - Growth and Income - Growth - Aggressive	Compared to others, how do you rate your willingness to take financial risks?	When deciding how to invest your money, which do you care about more? - Maximizing gains - Minimizing losses - Both equally
	When it comes to making important financial decisions: - I try to avoid making decisions. - I reluctantly make decisions. - I confidently make decisions and don't look back.		How much risk you can handle in 6 months -5% - +5% -10% - +10% -15% - +15% -20% - +20%	How do you usually feel about your major financial decisions shortly after you make them?	
			What if you could	"Would you prefer a fixed	

Table 7.1 - (Continued)

Client - Profiling	Schwab	SigFig	Riskalyse	Tolerisk	WealthFront
			improve that? - Less risk Less reward - Less risk Same reward - Same risk Same reward - Same risk More reward	rate mortgage on your home or a floating rate mortgage that is currently HALF of the fixed rate? If you could choose one option or a mix of both, what would you choose? Note that the floating rate loan is riskier."	
	Have you ever experienced a 20% or more decline in the value of your investments in one year? (yes, no)	"Imagine a choice between a stable job with limited opportunity for pay increases, or a less secure job with the option of a big payday in the future. Which job would you be more likely to take today?"	What is your preference? Certain gain? 10%, 4%, 3%, 2%, 1%, 0% Certain Loss? -10%, -4%, -3%, -2%, -1% 50% Loss and 50% Gain	If the stock market is up 30% this year, I would expect my account to be up: <5% 5%-10% 10%-15% 15%-20% 20%-25% 25%-30% 30%+	"The global stock market is often volatile. If your entire investment portfolio lost 10% of its value in a month during a market decline, what would you do?" - Sell all of your investments - Sell some - Keep all - Buy more
Risk Preference	If I ever were to experience a 20% or more decline in the value of my investments in one year, I would - sell everything. - sell some. - do nothing. - reallocate my investments. - buy more.	The value of investments can decline in market downturns. How much could your portfolio fall before you felt uncomfortable?		Have you ever invested a large sum in a risky investment mainly for the "thrill" of seeing whether it went up or down in value? No Yes, very rarely Yes, somewhat rarely Yes, somewhat frequently Yes, very frequently	
				If you had to choose between more job security	

Table 7.1 - (Continued)

Client - Profiling	Schwab	SigFig	Riskalyse	Tolerisk	WealthFront
				with a smaller paycheck and less job security with a larger paycheck, which would you choose?	
				If the stock market were to DECLINE 20% next year, I would expect my account to be DOWN: <5% 5%-8% 8%-11% 11%-14% 14%-17% 17%-20% 20%+	
				. Insurance can cover a wide variety of life's major risks - theft, fire, accident, illness, death etc. How much insurance coverage do you have?	
				"Given that risk and return generally go hand in hand, what portion of your investments are you willing to place in investments where BOTH returns and risks are expected to be above average?"	

Table 7.1 - (Continued)

Client - Profiling	Schwab	SigFig	Riskalyse	Tolerisk	WealthFront
				Imagine you were in a job where you could choose whether to be paid salary, commission, or a mix of both. Which would you pick? All salary Mainly salary Equal mix of salary and commission Mainly commission All commission	
Risk Awareness	When I hear "risk" related to my finances: - I worry I could be left with nothing. - I understand that it's an inherent part of the investing process. - I see opportunity for great returns. - I think of the thrill of investing.	Investment decisions often come down to a tradeoff between potential gains and the risk of loss. Do you tend to focus on the potential upside or downside?	Define a Devastating loss? -50% -40% -30% -20% -10%	How easily do you adapt when things go wrong financially?	
				When faced with a major financial decision, are you more focused on the possible losses or the possible gains?	
				What degree of risk have you taken with your financial decisions in the	

Table 7.1 - (Continued)

Client - Profiling	Schwab	SigFig	Riskalyse	Tolerisk	WealthFront
				past?	
				What degree of risk are you currently prepared to take with your financial decisions?	
				I'm okay if my investments drop a lot in the short-term because they will go up a lot in the long run too.	
				Investments can go up or down in value, and experts often say you should be prepared to weather a downturn. By how much could the total value of ALL your investments go DOWN before you would begin to feel uncomfortable?	
				Market volatility does NOT scare me.	
				I'm very afraid of the stock market.	