INTELLIGENT MULTI-ATTRIBUTE DECISION MAKING APPLICATIONS: DECISION SUPPORT SYSTEMS FOR PERFORMANCE MEASUREMENT, EVALUATION AND BENCHMARKING

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Under the Supervision of Dr. Elif Kongar

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Dedicated to my parents, Leyla and Hüseyin Duman

ABSTRACT

Efficiency has been and continues to be an important attribute of competitive business environments where limited resources exist. Owing to growing complexity of organizations and more broadly, to global economic growth, efficiency considerations are expected to remain a top priority for organizations. Continuous performance evaluations play a significant role in sustaining efficient and effective business processes. Consequently, the literature offers a wide range of performance evaluation methodologies to assess the operational efficiency of various industries. Majority of these models focus solely on quantitative criteria omitting qualitative data. However, a thorough performance measurement and benchmarking require consideration of all available information since accurately describing and defining complex systems require utilization of both data types. Most evaluation models also function under the unrealistic assumption of evaluation criteria being dependent on one another. Furthermore, majority of these methodologies tend to utilize discrete and contemporary information eliminating historical performance data from the model environment. These shortcomings hinder the reliability of evaluation outcomes leading to inadequate performance evaluations for many businesses. This problem gains more significance for business where performance evaluations are tied in to important decisions relating to business expansion, investment, promotion and compensation.

The primary purpose of this research is to present a thorough, equitable and accurate evaluation framework for operations management while filling the existing gaps in the literature. Service industry offers a more suitable platform for this study since the

industry tend to accommodate both qualitative and quantitative performance evaluation factors relatively with more ease compared to manufacturing due to the intensity of customer (consumer) interaction. Accordingly, a U.S. based food franchise company is utilized for data acquisition and as a case study to demonstrate the applications of the proposed models.

Compatible with their multiple criteria nature, performance measurement, evaluation and benchmarking systems require heavy utilization of Multi-Attribute Decision Making (MADM) approaches which constitute the core of this research. In order to be able to accommodate the vagueness in decision making, fuzzy values are also utilized in all proposed models.

In the first phase of the study, the main and sub-criteria in the evaluation are considered independently in a hierarchical order and contemporary data is utilized in a holistic approach combining three different multi-criteria decision making methods. The cross-efficiency approach is also introduced in this phase.

Building on this approach, the second phase considered the influence of the main and sub-criteria over one another. That is, in the proposed models, the main and sub-criteria form a network with dependencies rather than having a hierarchical relationship. The decision making model is built to extract the influential weights for the evaluation criteria. Furthermore, Group Decision Making (GDM) is introduced to integrate different perspectives and preferences of multiple decision makers who are responsible for different functions in the organization with varying levels of impact on decisions. Finally, an artificial intelligence method is applied to utilize the historical data and to obtain the final performance ranking.

Owing to large volumes of data emanating from digital sources, current literature offers a variety of artificial intelligence and machine learning methods for big data analytics applications. Comparing the results generated by the ANNs, three additional well-established methods, viz., Adaptive Neuro Fuzzy Inference System (ANFIS), Least Squares Support Vector Machine (LSSVM) and Extreme Learning Machine (ELM), are also employed for the same problem. In order to test the prediction capability of these methods, the most influencing criteria are obtained from the data set via Pearson Correlation Analysis and grey relational analysis. Subsequently, the corresponding parameters in each method are optimized via Particle Swarm Optimization to improve the prediction accuracy.

The accuracy of artificial intelligence and machine learning methods are heavily reliant on large volumes of data. Despite the fact that several businesses, especially business that utilize social media data or on-line real-time operational data, there are organizations which lack adequate amount of data required for their performance evaluations simply due to the nature of their business. Grey Modeling (GM) technique addresses this issue and provides higher forecasting accuracy in presence of uncertain and limited data. With this motivation, a traditional multi-variate grey model is applied to predict the performance scores. Improved grey models are also applied to compare the results. Finally, the integration of the fractional order accumulation along with the background value coefficient optimization are proposed to improve accuracy.

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I believe that every living creature has a purpose in the universe. I also would like to believe that the one and only purpose of individuals is to explore and understand the beautiful harmony in it. This role is given to us as the most intelligent life forms in the known universe. As Albert Einstein once said, "the time is not the same for all of us but different for each one us". Similarly, I consider this work to be the result of 35 years of learning and exploring, not as the outcome of my studies for the past four years. This dissertation, however small its contribution might be, belongs to all humankind.

Pursuing a Ph.D. degree is a difficult journey requiring a long-term commitment and high motivation. I would not have been able to complete this journey without the support of my family and my wife Fazila. I would like to thank them for their patience, faith and trust.

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CHAPTER 1: INTRODUCTION

1.1. Research Problem and Focus

Productivity and efficiency are two important factors which are expected to remain as top priorities for especially competitive business environments where limited resources exists. Varying in their intensity, existing market conditions and growing competition force companies to utilize available resources in the most effective manner. Performance and efficiency analyses are significant management measurement tools that help companies determine the relationships between the outcomes and the inputs used to obtain these outcomes.

Today, globalization has led to an increase in competitiveness among companies and organizations at all levels and in all sectors [1]. In this highly competitive business environment measuring efficiency and performance of the processes in any organization has gained significant importance [2]. An accurate evaluation of an organization's ability to transform its resources into its corresponding outputs efficiently requires careful selection of appropriate performance measurement and evaluation tools. In addition, as also stated by Samoilenko and Osei-Bryson [3], the dynamic nature of competition requires these tools to be adaptable ensuring the sustainability of such mechanisms.

Related literature survey results indicate that continuous process and productivity measurement and evaluations have always been critical components of sustainability in operations, and even more so in today's highly saturated and competitive business environment. Due to the multi-criteria nature of these efforts, various Multiple Criteria

Decision Making (MCDM) methodologies have been published to address similar issues. However, these studies mostly focus on quantitative measurements and fall short in including qualitative perspective in the evaluation process [4]. Furthermore, the inter-relationships and influences among the evaluating criteria and the past performance data are not considered in previously published work. Instead, proposed methodologies tend to capture the contemporary performance data and weigh the considered criteria independently. These drawbacks naturally lead to imprecise and biased evaluations which constitute the basis for the proposed methodologies in this research.

1.2. Research Motivation

Measuring productivity accurately is crucial for increasing the overall efficiency of operations. In line with its significance the topic has been the subject of several studies in the literature. Many methodologies have been applied to assess the productivity of individual retail stores, groups of stores, and retail industry as a whole [5]. Compared to relatively conventional manufacturing industries, establishing appropriate measures for service industries is considered to be a more challenging task, particularly for restaurants [6]. While earlier studies focus on labor productivity, recent studies examine additional factors which have potential effect on store productivity such as merchandise assortment, location, pricing, and promotion [7]. These studies mostly focus on quantitative measurements lacking in qualitative perspective in their evaluations. Excluding qualitative factors in performance evaluation hinders the ability to gain meaningful insight regarding the retail store performance. For instance, store image, an important measure in demonstrating how the company is performing from its customers' perspective has been omitted from early studies prohibiting the companies from understanding and hence meeting the customer needs.

Furthermore, the inter-relationships and influences among the evaluating criteria and the past performance data are not considered in previously published work. Instead, proposed methodologies tend to capture the contemporary performance data and weigh the considered criteria independently. These drawbacks naturally lead to imprecise and biased evaluations which constitute the basis for the proposed methodology.

In order to establish a reliable standardized performance measurement and evaluation system, various franchise businesses developed key performance indicators (KPIs) to be used in periodical performance evaluation, continuous monitoring and management of quality across the entire company. However, these performance evaluation systems are mostly based on discrete numerical data excluding any information that relates to the historical performance of the business. Furthermore, the criteria considered for evaluation tend to be independent of one another failing to incorporate the inter-relations among and influences on each other. For instance, current performance evaluation models exclude factors such as the land area of the restaurant territory, the neighborhood demographics or their impact on delivery times and/or sales. Moreover, the influences of qualitative criteria in the overall performance evaluation are not fully represented either. As a result of these omissions, current measurement systems often lead to erroneous promotion and retention decisions. The purpose of this study is to propose an equitable and accurate evaluation model in a food retail and delivery franchise operating in the U.S. The stores in this particular franchise are periodically evaluated via scheduled and unscheduled audits by the corporate auditors. The data collected through these evaluations are then used to initiate a stream of evaluations and constitute the starting point for internal and external benchmarking. Findings are also tied to an incentive system providing store managers and employees with varying levels of rewards if the target values are achieved and/or exceeded. The process however, falls short in including the relationships among the evaluation criteria while also neglecting to account for historical performance data. Furthermore, the influenced weights of each criterion are not included in the current evaluation model.

This study addresses these shortcomings and proposes several approaches for performance evaluation. To utilize the significant benefits artificial intelligence and machine learning applications bring to formal performance evaluation systems, various artificial intelligent methods utilizing historical data are also tested using the same problem domain.

1.3. Contributions

This research builds on a previously proposed multi-criteria decision making approach for performance evaluation and benchmarking. The aim of this research is to present a thorough, equitable and accurate evaluation framework for operations management applicable in every industry which has both qualitative and quantitative measures in the performance evaluation.

In the first phase of the study, the main and sub criteria in the evaluation process are considered independently in a hierarchical order and contemporary data is utilized in a holistic approach combining Fuzzy Analytic Hierarchy Process (Fuzzy AHP), Data Envelopment Analysis (DEA) with the extension of Cross-Efficiency measurement and

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). This approach is used to rank the efficient and inefficient retail stores according to the evaluation criteria determined by a decision maker in the company.

In the second phase, the influence of the main and sub-criteria on each other is considered forming a network with dependencies rather than establishing a hierarchical relationship matrix. This part of the study proposes an integrated approach combining Fuzzy Decision-Making Trial and Evaluation Laboratory (DEMATEL), Analytic Network Process and Artificial Neural Network (ANN) methodologies for performance evaluation. In the proposed model, DEMATEL and ANP methodologies are utilized to obtain priorities of the evaluation criteria. Following this, an ANN model is designed and trained with historical performance data collected from the company and the results of the Fuzzy DEMATEL-ANP model. The outcomes include the relational data among the criteria and alternatives used in the model in addition to their relative rankings. Furthermore, Group Decision Making is introduced to integrate different perspectives and preferences from multiple decision makers from different levels in the company management.

Although very widely employed, ANN is not the only artificial intelligence technique utilized in multi-criteria decision making problems. Several other artificial intelligence and machine learning methods are also applied for performance predictions. To reflect this variety, three additional well-established methods, viz., Adaptive Neuro Fuzzy Inference System (ANFIS), Least Squares Support Vector Machine (LSSVM) and Extreme Learning Machine (ELM), are also employed for the same problem. In order to test the prediction capability of these methods, the most influencing criteria are obtained from the data set via Pearson Correlation Analysis and grey relational analysis. Subsequently, the corresponding parameters in each method are optimized via Particle Swarm Optimization to improve the prediction accuracy.

The accuracy of artificial intelligence and machine learning methods are heavily reliant on large volumes of data. However, there are organizations, especially newly opened businesses without adequate amount of data required for performance evaluations. Grey Modeling (GM) technique addresses this issue and provides higher forecasting accuracy in presence of uncertain and limited data. Considering a business with similar data scarcity concerns, this study applied a traditional multi-variate grey model to predict performance scores. Improved grey models are also applied to compare the results. The integration of the fractional order accumulation along with the background value coefficient optimization are proposed to improve accuracy.

CHAPTER 2: BACKGROUND AND LITERATURE REVIEW

This section summarizes the findings of the literature survey while highlighting the gaps in the related literature.

2.1. Performance Evaluation and Benchmarking in Food Industry

In their paper, Ismail, et al. [8] provided the historical background of the modern efficiency measurement which began with Farrell [9]. The author defined measurement as a simple degree of a firm's efficiency which could account for multiple inputs. Proposing that the efficiency of a firm consisted of two components, viz., technical efficiency and allocative efficiency, the author defined technical efficiency as the ability to produce the maximum number of outputs with a fixed number of inputs, and allocative efficiency as the ability to use the inputs in the most optimal proportion, given their respective prices. These measures together are considered to constitute economic efficiency [8]. Müller [10] also provided historical information regarding efficiency mentioning that the term "*efficiency"* has dated back to Pareto [11] and the extensions of Koopmans [12]. Yadav, et al. [13] explained that organizations involved in similar activities could quantify their relative performance by comparing by the results with one another and then could develop strategic plans for improvements in their performance taking into consideration the best in the class as a benchmark.

Retail productivity is an important issue and hence has been the subject of an extensive amount of research. A review of related literature indicates that multiple methodologies have been applied to assess the productivity of individual retail stores, groups of stores, and the retail industry as a whole [5]. Compared to manufacturing industries, establishing an appropriate measure of production efficiency is more difficult in service industries, particularly in restaurants since customer interactions play an important in such businesses. Also, early studies tend to focus on labor productivity because labor expenditures were and continue to be of great importance. More recent studies however, have examined additional factors that may influence store productivity, such as merchandise assortment, location, pricing, and promotion [14]. It also became a common practice to conduct and heavily rely on "within chain" comparisons for franchise businesses when making important strategic management decisions. . The evaluation, promotion, and development store management personnel relies on factors that affect store financial performance. In addition, strategic resource-allocation decisions—such as advertising budgets, store expansions and store closings—are based on company management's understanding of what drives store performance. For instance, if the factors that contribute to low performance are deemed to be unalterable or prohibitively expensive to modify, management may choose to close the store. By adopting a `best practices' approach to continuous improvement and corporate learning, company management continues its ongoing monitoring of overall store operational management procedures and their influence on store performance.[14]. Keh and Chu [15] acknowledged that various constructs of output such as sales revenue, physical units, value added, and gross margin, etc., have been proposed and utilized.

The food industry has been a major contributor to the United States economy. In 2016, the food and related industries ranked third (12.6%) following housing (33%) and transportation (15.8%) in a typical American household's expenditures [16]. The food industry is composed of numerous segments responsible for producing, processing, manufacturing and selling a large variety of products such as food, beverage and supplements. Food and beverage retail sector is highly sensitive to economic fluctuations since point-of-sale locations are responsible for the delivery of end products to consumers. According to the U.S. Bureau of Economic Analysis, the contribution of the food services and drinking places to the Gross Domestic Product (GDP) has increased from 272.2 to 400.7 billion dollars between the years 2008 and 2016 [17]. With a 227.3 billion dollar revenue in 2016, the fast food restaurants industry has the highest share in this market with an expect annual growth rate of 1.8% during the next five years [18]. The food industry accounts for more than nine percent of the total employment in the country. It is estimated that there are a total of 31 thousand food and beverage industries in the United States employing about 1.5 million in the manufacturing of these raw materials to products ready for consumption [19]. On a micro scale, food takes up the third largest share of household incomes after housing and transportation [20].

For a country as urbanized as the United States, transportation represents a significant cost item for any food-related company. Therefore, efficient and cost-effective product delivery rises as an important issue [21]. Increasing the utilization of e-commerce that allows customers to place orders remotely also reduces the overall transportation cost. As e-commerce develops, it is expected that demand for food deliveries will also grow [22]. However, there has not been much development or change in the logistics surrounding the food industry [23].

Franchising strategy has been widely used as a restaurant management tool since mid-1940s. Several fast food companies such as McDonald’s, Dunkin Donuts and

Domino's Pizza successfully established franchising operations and obtained relatively fast and significant business growth throughout the years in both domestic and global markets [24].

Recently, significant advances in technology and changes in consumer behavior have compelled companies to invest in technology-driven services to provide value-added options to their customers such as online ordering, order tracking platforms and smartphone applications. Today, this introduction of digitization is investigated under a new concept, Industry 4.0, which refers to the transformation of current markets due to various advances in data, communication and computation related fields [25]. This transformation requires businesses to reevaluate and restructure their operations to be compatible with changing dynamics of industry.

These inevitable changes in the standard operations made franchise business management more complex and challenging. However, improvements in data related technologies have also provided these companies with the ability to monitor and collect online real-time operational performance data. Collected operational data which involve most recent performance measures, if utilized properly, would lead to more accurate and reliable evaluations as well.

2.2. Data Envelopment Analysis (DEA) Analytic Hierarchy Process (AHP) and The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) Literature Survey

This section provides a literature review on the concepts considered in this section of the research. First, a review on Data Envelopment Analysis (DEA) is presented. This is followed by the Analytic Hierarchy Process [26] and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) literature with a focus on performance evaluation. An overview of the previous research is also presented in this section.

Over the past two decades, DEA has been used as a popular benchmarking technique for performance measurement [27]. Mishra [28] mentioned that benchmarking is used by many leading companies such as Xerox Corporation and American Express in order to excel in their respective industries on a global scale as well as for small and medium size firms [28]. Paradi and Zhu [29] explained that the limitations of ratio and regression analysis have led to the development of more advanced tools for assessing corporate performance [29]. Supporting Paradi and Zhu's conclusions [29], Lau [27] discussed that based on DEA's simplicity of use and flexibility in data requirement, the technique has become a popular tool [27]. Mostafa [30] further emphasized DEA as an adequate tool for benchmarking due to its ability to identify a group of efficient DMUs for each non-efficient one [30]. Furthermore, Lee and Kim [31] reported DEA was highly useful in providing benchmarking guidelines for inefficient DMUs. That is, for each inefficient DMU, DEA identifies a set of efficiency units called the reference set, which constitutes DEA's benchmark, containing information on the percentage of the efficiency improvement [31]. Donthu et al.[32] aimed at filling the gap in marketing productivity benchmarking using DEA. Gonzales-Padrone et al. [33], also proposed a DEA model as a benchmarking tool in order to conduct an efficiency assessment of the sales staff in dealerships. Mishra [28] utilized DEA to assess the relative efficiency of 25 different retail stores in India. Takouda and Dia [34] performed aDEA-based internal and external benchmarking study comparing the technical, pure technical and scale efficiency of three main hardware retail stores in Canada. Similarly, Erdumlu and Saricam [35] applied a DEA model to evaluate the technical and scale efficiencies of 30 different apparel retailers in Turkey. Mostafa used a non-parametric DEA approach to measure the relative efficiency of 45 retailers in the U.S. [30].

In food industry, Donthu and Yoo [36] utilized a traditional DEA approach to assess the retail productivity of a restaurant chain. Roh and Choi [37] evaluated three different brand efficiencies using DEA. Similarly, Duman and Kongar [38] used DEA to obtain the efficiency scores of food franchise stores by focusing their quantitative inputs and outputs for service performance.

The Analytic Hierarchy Process [26] was first proposed by Prof. Thomas L. Saaty [39] in the early 1970s. AHP is interposed between operational research and decision analysis and is considered as a Multi-Criteria Decision Making (MCDM) method, based on the relative measurement theory. AHP, using linguistic expressions, derives the ratio scales from pairwise comparisons and is designed to help decision makers to make a choice among a set of alternatives [40]. The problem description in AHP consists of goal, criteria, and alternatives.

AHP and Fuzzy AHP methods are well studied in complex decision modeling. Both methods are either applied alone or, in majority of the cases, in a hybrid sense in conjunction with other decision support techniques. Ho [41] argues that hybrid approaches perform better due to their wide applicability, ease of use, and flexibility, and, hence, are also more commonly applied [42] compared to stand-alone AHP applications.

In this regard, Vaidya and Kumar [43], analyzed the hybrid approaches combining the AHP and other decision support techniques and listed the application areas of these methods. Kubler, et al. [44] studied the methods combining Fuzzy AHP and others elaborating on their application areas. There are several studies reporting the advantages and shortcomings of each method while agreeing on the fact that combining AHP or fuzzy AHP and other models help to avoid certain limitations and to improve the performance of the overall approach [45-47]. For instance, as far as the agility in the decision process is concerned, Fuzzy TOPSIS is proven to perform better than Fuzzy AHP in most cases except when there are insufficient number of criteria and decision making units (DMUs) [46]. However, while there is no pair-wise comparison in fuzzy TOPSIS [48], fuzzy AHP considers pair-wise comparisons for both the decision criteria and the alternatives. For further information regarding the comparison of these methods please see [46, 48].

While TOPSIS and DEA are two approaches which are frequently integrated with Fuzzy AHP, there are only a couple of studies combining these three methodologies. Zeydan, et al. [47] combined fuzzy AHP, fuzzy TOPSIS and DEA methods for the selection and evaluation of suppliers in a car manufacturing factory. Çelen and Yalçın [45] applied a combined Fuzzy AHP, TOPSIS and DEA methods to assess the performance of Turkish electricity distribution utilities.

Multi-Criteria Decision Making (MCDM) is well-studied in the literature. An extensive literature review was conducted by Ilgin, et al. [49] including over 190 applications of various MCDM techniques focusing on environmentally conscious manufacturing principles and product recovery activities. However, the literature offers relatively limited studies where combined MCDM techniques are used in food and retail industry related studies. Among these, Chen, et al. [50] built a fuzzy MCDM method to rank the service providers in the Taiwan fast food industry.Hu and Chen [51] utilized a Choquet integral-based hierarchical network approach to evaluate customer service perceptions on fast food stores in Taiwan. Pi-Fang and Bi-Yu [52] proposed a combined Delphi Technique, AHP, entropy and compromised weighting method to select the most appropriate franchisee in durable goods industry. Joshi, et al. [53] employed a combined Delphi-AHP-TOPSIS approach as a benchmarking technique to evaluate the cold chain performance of a food retailer. Dogan, et al. [54] utilized AHP, simple additive weighting (SAW), TOPSIS and elimination and choice translating reality (ELECTRE) techniques to evaluate the sensory properties of instant hot chocolate beverage with different fat contents.

2.3. Group Decision Making (GDM), Decision-Making Trial and Evaluation Laboratory (DEMATEL), Analytic Network Process (ANP) and Artificial Neural Network (ANN) Literature Survey

Group decision making has been utilized by many organizations to obtain a satisfactory solution in decision-making problems [55]. The concept aims at forming a consensus by considering various perspectives of decision makers. Based on the level of experience and managerial position, some decision makers are more influential on the outcome of the decision process than the others [56]. Hence, assessments of decision makers can be better reflected if they are prioritized by assigning a unique weight factor to each decision maker. Several studies with various group decision making approaches in multi-criteria environment have been published in the literature. For instance, Lai, et al.

[57] employed AHP and GDM for software selection. Büyüközkan, et al. [56] incorporated fuzzy Choquet integral approach via GDM to rank the sustainable urban transportation alternatives. Lin and Wu [58] developed a fuzzy DEMATEL method for group decision-making to gather group ideas and to analyze the relationship among R&D project selection criteria. Xia, et al. [59] integrated GDM and Grey-DEMATEL to analyze the internal barriers for automotive parts remanufacturers in China.

Decision-Making Trial and Evaluation Laboratory (DEMATEL) [60, 61] and Analytic Network Process [62] are two widely-studied MCDM approaches in order to determine the influences and interdependence between criteria [63]. Various combinations of these two approaches have also been proposed in the literature [64]. The DEMATEL based ANP (DANP) is essentially the general form of cluster-weighted ANP. In traditional ANP method, the unweighted super-matrix is formed according to pairwise comparisons, and the criteria weights which correspond to eigenvalues are obtained by limiting the super-matrix. In order to lessen the burden of the pairwise comparison questions for the decision makers, DANP forms a comprehensive unweighted super-matrix by building a direct influence matrix where pairwise comparisons are not only conducted within clusters but the influences between clusters are also obtained. Once the unweighted super-matrix is constructed, the total relation matrices between clusters are applied to weigh the appropriate portions of the super-matrix to obtain the weighted super-matrix. For further information regarding the DANP method, please see [64]. Several studies utilizing DANP in various areas can also be found in the literature [65-72].

Artificial neural network (ANN) applications in multi-criteria decision making environment is very promising [73, 74]. However, the literature indicates that ANN is generally combined with other MCDM methods as an integrated approach and the motivation behind its utilization varies widely by study. For instance, Kuo, et al. [75] applied Fuzzy AHP and ANN for convenience store location selection. Fuzzy AHP is utilized to obtain the factor weights and ANN is employed to study the relationship between the factors and the store performance. Wu, et al. [76] proposed an integrated DEA–ANN model to examine the relative bank branch efficiency. The findings are then compared with the traditional DEA. Çelebi and Bayraktar [77] applied DEA and ANN for supplier evaluation in automotive industry. In their study, Ha and Krishnan [78] utilized AHP to rank the suppliers based on qualitative criteria and ANN and DEA together to calculate the combined supplier score in auto parts manufacturing industry. Efendigil, et al. [79] employed Fuzzy AHP to obtain the criteria weights and then applied to determine the appropriate third party reverse logistics provider. Yazgan, et al. [80] combined ANP and ANN in ERP software selection. A study carried out by Kuo, et al. [81] presented the comparisons of integrated ANN–DEA, ANP–DEA and ANN-ANP-DEA approaches in green supplier selection. In his paper, Golmohammadi [73] utilized Fuzzy AHP to calculate the weights of the suppliers in the automotive industry and then ANN is trained with historical performance data to obtain the rankings of the suppliers. In another study, Kar [82] used Fuzzy AHP for group decision making to estimate group preferences and then applied ANN to map suppliers in steel industry to suitable classes. In their study, Shabanpour, et al. [83] combined ANN and Dynamic DEA for green supplier selection in manufacturing industry. ANN is trained to forecast inputs and outputs for green suppliers and then the forecasts derived from ANN is fed into Dynamic DEA to obtain the efficiency scores.

2.4. Adaptive Neuro Fuzzy Inference System (ANFIS), Least Squares Support Vector Machine (LSSVM), Extreme Learning Machine (ELM) and Particle Swarm Optimization (PSO) Literature Survey

ANN is often compared with Adaptive Fuzzy Inference System (ANFIS) and several comparative studies have been published in the decision making domain [84-87]. Furthermore, ANFIS is utilized solely or with other decision making methods for prediction and forecasting as well. For instance, Tan, et al. [88] applied ANFIS for measuring country sustainability performance. Asgari, et al. [89] conducted a comparative study between ANFIS and Fuzzy AHP- Fuzzy Goal Programming (FGP) methods for supplier selection and concluded that ANFIS outperforms in predicting performance scores. Similarly, Khalili-Damghani, et al. [90] proposed a hybrid approach based on ANFIS and FGP to deal with the supplier selection problem. Güneri, et al. [91] and Tavana, et al. [26] utilized ANFIS for determining the input criteria and supplier selection.

Least Squares Support Vector Machine (LSSVM) [92] and Extreme Learning Machine (ELM [93] are other fairly new techniques for prediction and forecasting in multi-attribute environment. For instance, Guosheng and Guohong [94] compared ANN and SVM performances in supplier selection problem. In their study, Kaytez, et al. [95] presented a comparative study via applying ANN, multiple linear regression and LSSVM to forecast electricity consumption in Turkey. Vahdani, et al. [96] applied LSSVM for project selection problem in construction industry. Chen and Ou [97] proposed a sales forecasting system based on Grey Relational Analysis (GRA) and ELM. They utilized

GRA to extract the most influential factors from raw data and then transform them as the input data for ELM. Wong, et al. [98] utilized ELM for modeling and optimization of biodiesel engine performance. Several other applications of ELM in various areas for system modeling and prediction can be found in the literature [99].

Particle Swarm Optimization (PSO), first proposed by Eberhart and Kennedy [100], has been widely used to optimize the parameters and configurations in ANN, ANFIS, LSSVM and ELM methods to improve the accuracy in predictions. Various PSO integrated artificial intelligence and machine learning studies can be found in the literature as well [101-108]. Therefore, this study also utilizes PSO to obtain the optimal/near optimal values of the corresponding parameters in each method.

2.5. Grey Relational Analysis and Multi-Variate Grey Modeling Literature Survey

Grey Relational Analysis (GRA) and Multi-Variate Grey Modeling (GM), approaches are part of the Grey System Theory pioneered by Deng [109, 110]. Both methods have been utilized in multi-attribute or multi-criteria decision making [111], and have been extensively applied on various fields, such as supplier selection [112, 113], evaluation of energy systems [114-116], analyzing financial performance [117], end-of-life product strategy selection [118], and integrated circuit output forecasting [119].

Over the years, traditional GM(1,n) method has been extended and new models with higher accuracy in prediction have been built on the previous techniques. Out of these, Tien [120, 121] claimed that the original GM(1,n) model could be corrected via integrating it with convolution integral and proposed the GMC(1,n) model. In their paper, Ma and Liu

[122] developed a multi-variate discrete grey model, DGM(1,n), to predict the oil field production in China.

The traditional GM(1,n) models generally utilize an integer order, mostly the first order, accumulation generating operator. However, recent studies indicate that employing fractional order accumulation could lead to higher prediction accuracy [123, 124]. Another issue addressed in the grey modeling literature is the optimization of background value coefficient. PSO has been widely used in several applications to obtain the optimal/near optimal value of the background value coefficient [125-128]. Hence, this study also utilizes PSO to obtain the optimal/near optimal both fractional order value and background value coefficient.

CHAPTER 3: RESEARCH PLAN

This section presents the proposed approaches for performance evaluations and benchmarking to obtain final ranking of Decision Making Units (DMUs). In the first part, the evaluation criteria are considered to be independent and in a hierarchical order with contemporary discrete data.

In the second section, the dependency of the evaluation criteria is considered to form a network. In this part, multiple DMs are involved in the process and historical data is utilized via Artificial Neural Network.

The third section deals with the extraction, a.k.a., dimension reduction, of the most influential evaluation criteria via Pearson Correlation Analysis and grey relational analysis. Subsequently, the prediction capability of Artificial Neural Network is investigated via comparing its produced error rate with other artificial intelligence and machine learning methods, viz., Adaptive Neuro Fuzzy Inference System, Least Squares Support Vector Machine and Extreme Learning Machine. The corresponding parameters in each method are optimized by Particle Swarm Optimization.

In the last part, the traditional multi-variate Grey Modeling approach along with Discrete Grey Modeling and Grey Model with Convolutional Integral are implemented to predict the performance scores in presence of uncertain and limited data. Fractional order accumulation and background value coefficient optimization are also integrated to improve the prediction accuracy.

3.1. Mathematical Foundation of Fuzzy AHP, DEA with Cross-Efficiency and TOPSIS

In this part of the study, Fuzzy Analytic Hierarchy Process (Fuzzy AHP), Data Envelopment Analysis (DEA) with the extension of Cross-Efficiency measurement and The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) are utilized in a holistic approach to rank the efficient and inefficient retail stores according to the evaluation criteria determined by a decision maker in the company. The following steps are used for this process.

Step 1: Fuzzy AHP is employed to determine the relative weights of the main criteria by utilizing linguistic ratings obtained from the decision makers.

Step 2: DEA with Cross-Efficiency measurement is utilized to evaluate each Decision Making Unit [32], according to their service performance. The quantitative input and output data for the DEA model is obtained from the USA Census 2010 and the corporate store management system.

Step 3: TOPSIS approach is applied to obtain the overall ranking of each retail store. This is done by utilizing the DEA results and other qualitative ratings of each store which are obtained through the operational excellence audits.

Figure 1 provides a schematic representation of the methodology. The details of the methodology are as follows.

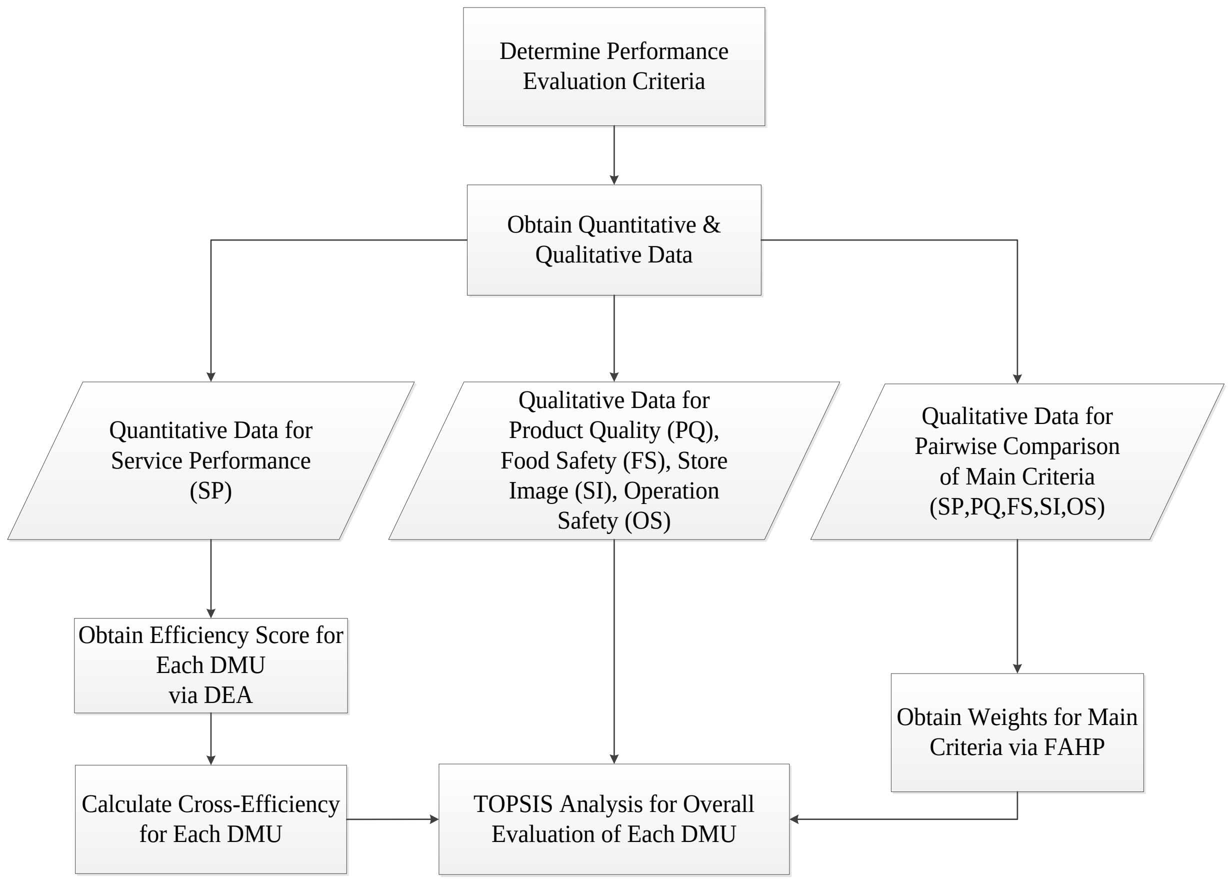

Figure 1. Schematic Representation of the Proposed MCDM Methodology

3.1.1. Determining the Main Criteria Weights via Fuzzy AHP

Integrating fuzzy set theory into decision analysis has received some criticism despite the fact that it has been also argued that Dubois [129] fuzzy set theory offered a bridge between numerical and qualitative approaches in decision analysis [90]. For instance, Zhü [130] stated that the AHP with crisp numbers was more effective than the fuzzy judgments in complex and uncertain environments. The author also noted that the utilization of the fuzzy AHP should be justified by the complexity of the decision behavior. Wang, et al. [131] stated that Chang's extent analysis [132] in Fuzzy AHP might result in obtaining a zero weight in the pairwise comparison leading to their exclusion from the decision analysis. Furthermore, Wang, et al. [131] proposed a revised version of the extent analysis and utilized it on several numerical examples. Their findings indicated that the priority vectors determined by the extent analysis method fell short in representing the relative importance of the decision criteria or alternatives. The authors stated that the method was more suitable for showing to what degree the priority of one decision criterion or alternative was bigger than those of others in a fuzzy comparison matrix [131].

Despite the criticism it has received, Chang's extent analysis [132] still remains as the most popular method for deriving weights of each criterion from the fuzzy pairwise comparison weight matrix [44]. Kubler, et al. [44] conducted an extensive literature survey on Fuzzy AHP application and reviewed 190 academic journal articles published between 2004 and 2016. The authors stated that the majority of the papers, *i.e.*, 109 out of the 190, utilized this method and that they expected this trend to continue despite the criticism it has received [44].

In order to overcome these limitations, several authors [133-137] combined the revised version of the extent analysis [131] and the total integral value method proposed by Liou and Wang [138] to obtain the importance weight of the criteria. This study followed the same framework. The steps of this approach are provided in the following.

(i) Let X = $\{x_1, x_2, x_{3,} \ldots, x_n\}$ be an object set, whereas U = $\{u_1, u_2, u_{3,} \ldots, u_m\}$ is a goal set. According to the method of extent analysis, each object is taken and the extent analysis for each goal is performed respectively. Therefore, m extent analysis values for each u can be obtained with the following signs:

$M_{gi}^{1}, M_{gi}^{2}, M_{gi}^{3}, \ldots\ldots, M_{gi}^{m}$, *i=1, 2,….., n.* Here, all M_{gi}^{j} , *j=1, 2,….., m* are triangular fuzzy numbers. The value of fuzzy synthetic extent with respect to the *i*th object can then be defined as:

$$S_i = \sum_{j=1}^{m} M_{gi}^{j} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} \tag{1}$$

The symbol $\otimes$ denotes the fuzzy arithmetic multiplication operation and $\sum_{j=1}^{m} M_{gi}^{j}$ is obtained by performing the fuzzy addition operation of *m* extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right). \tag{2}$$

To obtain $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1}$, the fuzzy addition operation of M_{gi}^{j} , *j=1, 2,….., m* values is performed as in Eq (3).

$$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{i=1}^{n} l_j, \sum_{i=1}^{n} m_j, \sum_{i=1}^{n} u_j\right). \tag{3}$$

The inverse vector is computed as in Eq (4),

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right). \tag{4}$$

According to Wang, et al. [131], Eq (4) can be corrected as Eq (5):

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m}M_{gi}^{j}\right]^{-1} = \left(\frac{\sum_{j=1}^{m} l_{ij}}{\sum_{j=1}^{m} l_{ij} + \sum_{k=1,k\neq 1}^{n}\sum_{j=1}^{n} u_{kj}}, \frac{\sum_{j=1}^{m} m_{ij}}{\sum_{k=1}^{n}\sum_{j=1}^{n} m_{kj}}, \frac{\sum_{j=1}^{m} u_{ij}}{\sum_{j=1}^{m} u_{ij} + \sum_{k=1,k\neq 1}^{n}\sum_{j=1}^{n} l_{kj}}\right) \quad (5)$$

(ii) Chang's extent analysis methodology computes the degree of possibility of $S_2(l_2, m_2, u_2 \geq S_1(l_1, m_1, u_1)$, where S_2 and S_1 are obtained by Eq (1 . The degree of possibility between two fuzzy synthetic extents is defined as follows:

$$V(S_2 \geq S_1) = sup_{y \geq x}\left[min\left(\mu_{S_2}(y), \mu_{S_1}(x)\right)\right] \quad (6)$$

which can be expressed as in Eq (1.7):

$$V(S_2 \geq S_1) = hgt(S_1 \cap S_2) = \mu_{S_2}(d), \quad (7)$$

$$where, \mu_{S_2}(d) \begin{cases} 1, & if\ m_2 \geq\ m_1 \\ 0, & if\ l_1 \geq\ u_2 \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & otherwise \end{cases},$$

and *d* is the ordinate of the highest intersection point *D* between μ_{S_1} and μ_{S_2}. In order to compare S_1 and S_2, the values of both $V(S_2 \geq S_1)$ and $V(S_1 \geq S_2)$ are required. However, as mentioned above, Wang, et al. [131] indicate that the degree of possibility defined by the extent analysis method is an index for comparing two triangular fuzzy numbers rather than calculating their relative importance. This problem can be solved by utilizing the total integral value with index of optimism proposed by Liou and Wang [138]. Therefore, equations 6 and 7 were not utilized.

(iii) Obtain the importance weight of the criteria the total integral value with index of optimism to prioritize the synthetic extent values by employing Eq (8):

$$I_T^{\alpha} = \frac{1}{2}\alpha(m_i + u_i) + \frac{1}{2}(1-\alpha)(l_i + m_i)$$

$$= \frac{1}{2}[\alpha u_i + m_i + (1 - \alpha)m_i], \tag{8}$$

where α is index of optimism which represents degree of optimism for decision makers. If α approaches 0 in [0, 1], the decision makers are more pessimistic and otherwise they are more optimistic. In this study, the index of optimism α value is taken as 0.5.

(iv) Compute the normalized importance weight vector $W = (w_1, w_2, \dots, w_n)^T$ which is given in Eq (9):

$$w_i = \frac{I_T^{\alpha}(S_i)}{\sum_{i=1}^{n} I_T^{\alpha}(S_i)}, \tag{9}$$

where, *W* is a non-fuzzy number calculated for each comparison matrix.

3.1.2. Determining the Efficiency Scores via DEA and Cross-Efficiency DEA as a Quantitative Evaluation

Data Envelopment Analysis (DEA), first proposed by Charnes, et al. [139], is a non-parametric approach that compares similar entities, *i.e.*, decision making units (DMUs), against the "best virtual decision making unit". Usually modeled as a linear programming (LP) model, the method provides relative efficiency score for each decision making unit under consideration. DEA method allows the introduction of multiple inputs and multiple outputs and obtains an "efficiency score" of each DMU with the conventional output/input ratio analysis. Defining basic efficiency as the ratio of weighted sum of outputs to the weighted sum of inputs, the relative efficiency score of a test DMU *i* can be obtained by solving the following DEA ratio model proposed by [139]:

Let E_i is the efficiency score of DMU i,

$$E_i = \sum_{k=1}^{K} u_k y_{ki} \Big/ \sum_{h=1}^{H} v_h x_{hi}$$

s.t.

$$\sum_{k=1}^{K} u_k y_{kp} \Big/ \sum_{h=1}^{H} v_h x_{hp} \leq 1 \qquad \forall\, p$$

$$u_k, v_h \geq 0 \qquad \forall\;\; k, h \tag{10}$$

where,

i = index of DMU being compared in the DEA,

y_{ki} = amount of output k produced by DMU i,

x_{hi} = amount of input h produced by DMU i,

y_{kp} = amount of output k produced by DMU p,

x_{hp} = amount of input h produced by DMU p,

v_h = weight assigned to the h-th type input,

u_k = weight assigned to the k-th type output.

Eq (10) can be easily converted into a linear program as in Eq (11)

$$Max\ E_i = \sum_{k=1}^{K} u_k y_{ki}$$

s.t.

$\sum_{h=1}^{H} v_h x_{hi} = 1,$

$$\sum_{k=1}^{K} u_k y_{kp} - \sum_{h=1}^{H} v_h x_{hp} \leq 0 \qquad \forall\, p,$$

$$u_k, v_h \geq 0 \qquad \forall\ k, h. \tag{11}$$

Equation (11) provides the relative efficiency score of DMU *i* by comparing it with other DMUs. DMU *i* is efficient if its calculated score E_i is equal to 1. Otherwise it is considered as not efficient.

The classic DEA is known to perform well in distinguishing the efficient and inefficient DMUs when there is sufficient number of DMUs compared to the total number of inputs and outputs [140-142]. The conventional method, however, does not rank the efficient DMUs. In order to overcome this shortcoming, DEA with Cross-Efficiency Measurement was first proposed by Sexton, et al. [143]. The main idea of Cross-Efficiency Measurement is to utilize DEA in a peer evaluation instead of a self-evaluation mode [144]. Using the determined weights, this approach allows each DMU to be evaluated with not only by itself but also by the other DMUs. These results are then used to obtain a cross-efficiency matrix, in which the diagonal members show the DEA efficiency scores of the DMUs while the off-diagonal cells provide the cross-efficiency scores as provided in Table 1.

Table 1. Cross-efficiency matrix for DMUs

Target DMU	**DMU**			
	1	2	j	*F*
1	E_{11}	E_{12}	…	E_{1F}
2	E_{21}	E_{22}	…	E_{2F}
i	…	…	E_{ij}	…
F	E_{F1}	E_{F2}	…	E_{FF}

Cross-Efficiency CE_j	$\bar{E}_1$	$\bar{E}_2$	$\bar{E}_j$	$\bar{E}_F$

Cross-Efficiency Measurement has two essential advantages [145]: (i) it provides a unique ordering of the DMUs, and (ii) it eliminates unrealistic weight schemes without requiring the elicitation of weight restrictions from application area experts [144, 146, 147]. Doyle and Green [148] formulated the method as follows.

$$\min \sum_{k=1}^{K} u_k \left(\sum_{j=1, j\neq i}^{F} y_{kj} \right) \tag{12}$$

or

$$\max \sum_{k=1}^{K} u_k \left(\sum_{j=1, j\neq i}^{F} y_{kj} \right) \tag{13}$$

s.t.

$$\sum_{h=1}^{H} v_h \left(\sum_{j=1, j\neq i}^{F} x_{hj} \right) = 1,$$

$$\sum_{k=1}^{K} u_k \, y_{ki} - E_i \sum_{h=1}^{H} v_h \, x_{hi} = 0,$$

$$\sum_{k=1}^{K} u_k \, y_{ki} - \sum_{h=1}^{H} v_h \, x_{hi} \leq 0, \quad j = 1,2, \ldots, F \text{ } and \text{ } j \neq i \,, \quad v_h \,, u_k \geq 0 \,, \quad \forall k, h.$$

Since the model minimizes the sum of weighted output of other DMUs, Eq (12) is determined as the aggressive formulation. In contrast, Eq (13) is accepted as the benevolent formulation since it aims at maximizing the sum of the weighted outputs of other DMUs. Both approaches present a set of optimal input and output weights for the *i*th DMU $(u_1^i, u_2^i, \dots, u_K^i, v_1^i, v_2^i, \dots, v_H^i)$. Target DMU *i* can be considered as a pivot whose weights are used to determine its own efficiency and also the efficiency values of all other DMUs which are calculated according to *i*. These values can be observed in the *i*th row of the matrix provided in Table 3. Here, the generic element E_{ij} represents the efficiency value of the *j*th DMU with respect to the optimal weights for the *i*th target DMU [149].

This study used benevolent formulation to obtain the set of the optimal weights. By applying the cross-efficiency DEA, all the DMUs are assessed by the weights of target DMU *i*. Following this the average value is calculated. Therefore, the cross-efficiency score of the *j*th DMU is calculated as follows:

$$CE_j = \frac{\sum_{i=1}^{F} E_{ij}}{F} = \frac{\sum_{i=1}^{F}\left(\frac{u_1^i y_{1j} + u_2^i y_{2j} + \cdots + u_K^i y_{Kj}}{v_1^i x_{1j} + v_2^i x_{2j} + \cdots + v_H^i x_{Hj}}\right)}{F} \qquad i = 1,2,\dots,F. \tag{14}$$

The ranking of each DMU can be obtained using its cross-efficiency (CE_j) score where the higher CE_j values indicate more efficient DMUs.

3.1.3. Overall Evaluation via TOPSIS

The Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) is a multi –criteria decision making technique which was initially proposed by Hwang and Yoon [150]. TOPSIS method can be formulated as follows:

(i) Let A_{ij} represent the decision matrix where a_{ij} is the score of each DMU according to the evaluation criteria.

The normalized matrix $R = (r_{ij})_{mxn}$ includes the normalized values of each DMU where r_{ij} can be obtained as:

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}} \quad i = 1, \dots, m; j = 1, \dots, n \tag{15}$$

(ii) The weighted normalized decision matrix values (v_{ij}) are calculated as in Eq (16).

$$v_{ij} = w_i r_{ij} \qquad j = 1, \dots, m\ ; i = 1, \dots, n, \tag{16}$$

where, w_j represents the weight of criterion j, and $\sum_{j=1}^{n} w_j = 1$.

The w_j values are derived from the Fuzzy AHP provided in Step 1.

(iii) Positive ideal (A^+) and negative ideal (A^-) solutions are obtained via Eq(17) and Eq (18). The positive ideal solution (A^+) represents the solution with the highest benefit and with the lowest cost out of all alternatives. Similarly, the negative ideal (A^-) solution represents the one with the least benefits and the highest costs. The solutions are formulated as follows:

$$A^+ = \{v_1^+, \dots \dots, v_n^+\} = \left\{(\max_j v_{ij} \mid j \in J'), (\min_j v_{ij} \mid j \in J'')\right\}, \tag{17}$$

$$A^- = \{v_1^-, \dots \dots, v_n^-\} = \left\{(\min_j v_{ij} \mid j \in J'), (\max_j v_{ij} \mid j \in J'')\right\}, \tag{18}$$

where J' is associated with the criteria having positive impact and J'' is associated with the criteria having negative impact.

(iv) The separation measures are calculated via Euclidean distance. The separation of each DMU from the positive ideal solution and negative ideal solution are provided as follows:

$$d_i^+ = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_i^+)^2} \;, \qquad i = 1, \dots, m, \tag{19}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_i^-)^2} \;, \qquad i = 1, \dots, m. \qquad (20$$

(v) The closeness coefficient (CC_i) of each alternative is computed as in Eq(21).

$$CC_i = \frac{d_i^-}{d_i^- + d_i^+}. \tag{21}$$

(vi) The alternatives are ranked according to the closeness coefficient, CC_i, in decreasing order.

In the TOPSIS method, the criteria are divided into two groups, namely, benefit and cost. The benefit criterion has a positive impact on alternatives while the cost criteria has a negative one. For the positive ideal solution (A^+), the alternative (here, DMU) with the highest score is selected for each benefit criterion whereas the alternative with the lowest score is chosen for each cost criterion. For the negative ideal solution (A^-), the alternative with the lowest score is chosen for each benefit criterion whereas the alternative with the highest score is chosen for each cost criterion. In our study, every criterion has a positive impact on the alternatives.

3.2. Mathematical Foundation of Fuzzy DEMATEL, ANP and ANN

In this part of the study, Group Decision Making via Fuzzy DEMATEL based ANP (DANP) approaches are utilized in an integrated approach to obtain the influenced weights of the evaluation criteria determined by multiple decision makers in the company. Following this, ANN is employed to obtain the final ranking of the DMUs via utilizing historical data.

Figure 2 provides a schematic representation of the methodology. The details of the methodology are provided in the following text.

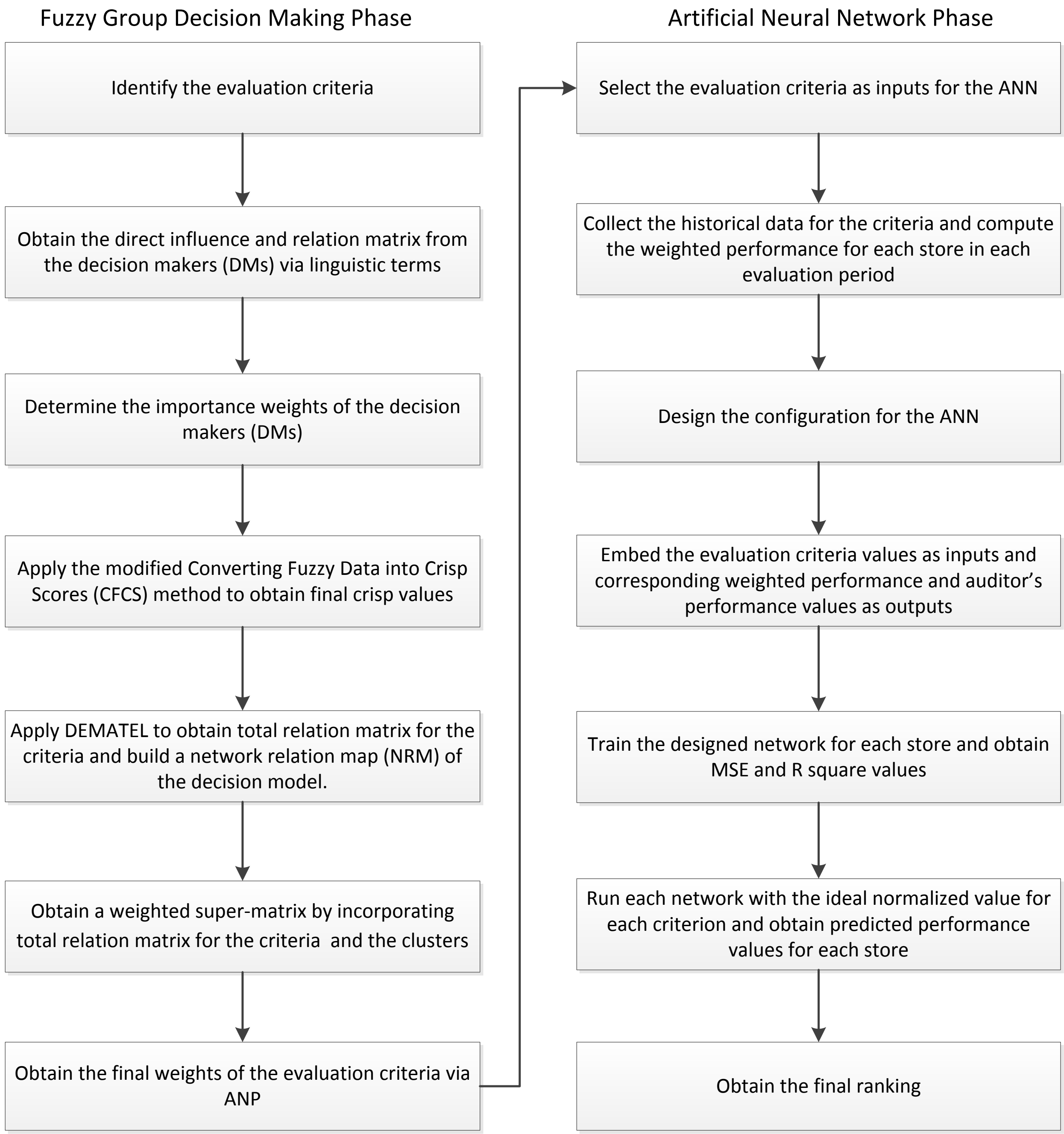

Figure 2.Flow diagram of the DANP and ANN approach

3.2.1. Converting Fuzzy Data into Crisp Scores (CFCS)

With its ability to integrate diverse and different perspectives into decision making process, Group Decision Making (GDM) becomes a viable option for situations where decision-making is open to all/multiple levels of the organization requiring the involvement of various experts or decision makers (DMs) from different organizational levels and/or functions of the business. This integration allows a consensus to be built so that a more favorable solution can be achieved. However, the existence of multiple fuzzy assessments obtained from various DMs require an effective fuzzy aggregation and defuzzification method.

There are several fuzzy aggregation and defuzzification methods that are proposed in the literature [151]. Out of these, Opricovic and Tzeng [152] proposed Converting Fuzzy Data into Crisp Scores (CFCS) method to deal with the fuzziness in multi-criteria decision models. CFCS is designed to distinguish between two fuzzy numbers which have the same crisp value obtained by the Centroid (center of gravity) method regardless of the shape of fuzzy numbers [72]. Therefore, this study utilizes CFCS to obtain the final crisp value of the fuzzy assessments from the DMs. The steps of the CFCS method are provided below.

Let $\widetilde{a_{ij}}^k = (l_{ij}{}^k, m_{ij}{}^k, u_{ij}{}^k)$ indicate the fuzzy assessment of evaluator *k*, (*k=1,2,..,p*), that will evaluate the influence of criterion *i* on criterion *j*. Then the following steps are followed.

Step 1: Normalization.

$$xl_{ij}{}^{k} = \left[l_{ij}{}^{k} - min_j l_{ij}{}^{k}\right] / \Delta_{min}^{max}, \tag{22}$$

$$xm_{ij}{}^{k} = \left[m_{ij}{}^{k} - min_j m_{ij}{}^{k}\right] / \Delta_{min}^{max}, \tag{23}$$

$$xu_{ij}{}^{k} = \left[u_{ij}{}^{k} - min_j u_{ij}{}^{k}\right] / \Delta_{min}^{max}, \tag{24}$$

where $\Delta_{min}^{max} = max_j u_{ij}{}^{k} - min_j l_{ij}{}^{k}$. (25)

Step 2: Compute the upper and lower bound normalized values.

$$xlb_{ij}{}^{k} = xm_{ij}{}^{k} / (1 + xm_{ij}{}^{k} - xl_{ij}{}^{k}). \tag{26}$$

$$xub_{ij}{}^{k} = xu_{ij}{}^{k} / (1 + xu_{ij}{}^{k} - xm_{ij}{}^{k}). \tag{27}$$

Step 3: Compute total normalized crisp value.

$$x_{ij}{}^{k} = [\, xlb_{ij}{}^{k} * (1 - xlb_{ij}{}^{k}) + xub_{ij}{}^{k} * xub_{ij}{}^{k} \,] / (1 - xlb_{ij}{}^{k} + xub_{ij}{}^{k}). \tag{28}$$

Step 4: Compute the crisp value

$$a_{ij}{}^{k} = min_j l_{ij}{}^{k} + \Delta_{min}^{max} , \tag{29}$$

Step 5: Calculate the final crisp values from the DMs.

$$a_{ij} = (a_{ij}{}^{1} * \omega^{1} + a_{ij}{}^{2} * \omega^{2} + \cdots + a_{ij}{}^{p} \omega^{p}) , \tag{30}$$

where ω is the weight of the DM and $\sum_{1}^{p} \omega = 1$.

3.2.2. The DEMATEL-Based ANP (DANP) Method

The DEMATEL-Based ANP (DANP) is a novel method that combines the DEMATEL and ANP methods to utilize total relation matrix for the criteria and the clusters, viz., the qualitative and quantitative perspectives in this study, and to establish a

Network Relation Map (NRM) of the decision model. Based on the NRM, the influential relationships are then obtained [64].

The basic steps of the DANP method are given below:

1. *Generate the direct relation matrix*

The first step in this process is to obtain the decision maker assessments regarding the direct influence between each pair of elements. The pairwise comparison is designated by five levels: “no influence”, “low influence”, “medium influence”, “high influence” and “very high influence” which are represented as grey numbers. The initial direct-relation matrix $\boldsymbol{A}$ is a $n \times n$ matrix in which $\boldsymbol{a_{ij}}$ indicates the degree that the criterion i affects the criterion j; where, $A=\left[a_{ij}\right]_{nxn}$.

2. *Normalize the direct relation matrix*

The normalized direct-relation matrix $X=\left[x_{ij}\right]_{nxn}$ can be obtained through

$$X = A/s \tag{31}$$

$$\text{where } s = max\left[max \sum_{i=1}^{n} a_{ij}\, , max \sum_{j=1}^{n} a_{ij}\right]. \tag{32}$$

Here, Equation (31) represents the normalized initial direct-relation matrix while Equation (32) represents the maximum values of the sums of all the rows and the sums of all the columns.

3. *Obtain the total relation matrix*

Once the normalized direct relation matrix $\boldsymbol{X}$ is obtained, the total relation matrix $\boldsymbol{T}=\left[t_{ij}\right]_{nxn}$ can be derived using the equation below where $\boldsymbol{I}$ is the identity matrix:

$$T= X+X^2 + X^3 + \cdots + X^k=\sum_{k=1}^{\infty} X^k=X(I - X)^{-1}. \quad (33)$$

In addition, the method uses each row and column sums of the matrix *T* to build the NRM.

$$d_i = (r_i)_{nx1}=\left[\sum_{j=1}^{n} t_{ij}\right]_{nx1}. \quad (34)$$

$$r_j = (c_j)_{nx1}=\left[\sum_{i=1}^{n} t_{ij}\right]_{1xn}. \quad (35)$$

Here, d_i denotes the row sum of the *i*th row of matrix *T* and shows the sum of direct and indirect effects of criterion *i* on the other criteria. Similarly, r_j denotes the column sum of the *j*th column of matrix *T* and shows the sum of direct and indirect effects that criterion *j* has received from the other criteria. Furthermore, the sum, (d+r) shows the effects among criteria whereas (d - r) is the causal relations among criteria. In other words, (d+r) reveals the importance of the criterion and if (d - r) is positive, it is implied that the criterion has an effect on others. Similarly, when (d - r) is a negative value then the criterion is affected by the others.

3. *Formation of an unweighted super-matrix*

The first step in the ANP approach is to build an unweighted super-matrix by pair-wise comparisons of the criteria. The weighted super-matrix is obtained by dividing each element in a column by the number of clusters where each cluster is assumed to have equal weight. However, the equal weight assumption for each cluster to obtain the weighted super-matrix seems irrational because of the different degrees of influence among the criteria [153]. Therefore, two different total influence matrices are then applied. The first one, $T_c = \left[t_c^{ij}\right]_{nxn}$ pertains to *m* criteria, while the second one $T_D = \left[t_D^{ij}\right]_{nxn}$ is devoted to *n* dimensions, *i.e.*, clusters, as presented below.

$$T_C = \begin{array}{cc} & \begin{array}{cccc} D_1 & D_2 & \cdots & D_n \\ c_{11} \ldots c_{1m1} & c_{21} \ldots c_{2m2} & \cdots & c_{n1} \ldots c_{nmn} \end{array} \\ \begin{array}{cc} D_1 & \begin{array}{c} c_{11} \\ c_{12} \\ \vdots \\ c_{1m1} \end{array} \\ D_2 & \begin{array}{c} c_{21} \\ c_{22} \\ \vdots \\ c_{2m2} \end{array} \\ \vdots & \vdots \\ D_n & \begin{array}{c} c_{n1} \\ c_{n2} \\ \vdots \\ c_{nmn} \end{array} \end{array} & \begin{bmatrix} T_c^{11} & T_c^{12} & \cdots & T_c^{1n} \\ T_c^{21} & T_c^{22} & \cdots & T_c^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ T_c^{n1} & T_c^{n2} & \cdots & T_c^{nn} \end{bmatrix} \end{array} \tag{36}$$

$$T_D = \begin{bmatrix} t_D^{11} & \cdots & t_D^{1j} & \cdots & t_D^{1n} \\ \vdots & & \vdots & & \vdots \\ t_D^{i1} & \cdots & t_D^{ij} & \cdots & t_D^{in} \\ \vdots & & \vdots & & \vdots \\ t_D^{n1} & \cdots & t_D^{nj} & \cdots & t_D^{nn} \end{bmatrix}. \tag{37}$$

4. Normalize the total relation and total influence matrices

The normalized total relation matrix of criteria T_C^{nor} is calculated by dividing the sum of each row in each sub-matrix. For instance, the normalized sub-matrix T_C^{nor12} indicating the relationship between Clusters 1 and 2 is calculated as follows.

$$T_C^{12} = \begin{array}{cc} & \begin{matrix} c_{21} & c_{2j} & & c_{2m2} \end{matrix} \\ \begin{matrix} c_{11} \\ \vdots \\ c_{1j} \\ \vdots \\ c_{1m1} \end{matrix} & \begin{bmatrix} t_{11}^{12} & t_{1j}^{12} & \cdots & t_{1m2}^{12} \\ t_{i1}^{12} & t_{ij}^{12} & \cdots & t_{im2}^{12} \\ \vdots & \vdots & \ddots & \vdots \\ t_{mi1}^{12} & t_{mij}^{12} & \cdots & t_{m1m2}^{12} \end{bmatrix} \begin{matrix} \rightarrow & r_1^{12} = \sum_{j=1}^{m2} t_{1j}^{12} \\ \rightarrow & r_i^{12} = \sum_{j=1}^{m2} t_{1j}^{12} \\ \vdots & \vdots \\ \rightarrow & r_{m1}^{12} = \sum_{j=1}^{m2} t_{mij}^{12} \end{matrix} \end{array}, \qquad (38)$$

where r_i^{12} represents the sum of each row in the sub matrix T_C^{12}. Then T_C^{nor12} is obtained as shown below.

$$T_C^{nor12} = \begin{bmatrix} t_{11}^{12}/r_1^{12} & t_{1j}^{12}/r_1^{12} & \cdots & t_{1m2}^{12}/r_1^{12} \\ t_{i1}^{12}/r_i^{12} & t_{ij}^{12}/r_i^{12} & \cdots & t_{im2}^{12}/r_i^{12} \\ \vdots & \vdots & \ddots & \vdots \\ t_{mi1}^{12}/r_{m1}^{12} & t_{mij}^{12}/r_{m1}^{12} & \cdots & t_{m1m2}^{12}/r_{m1}^{12} \end{bmatrix}. \qquad (39)$$

Similar to T_C^{nor}, the normalized total influential matrix for clusters T_D^{nor} is formed as shown below.

$$T_D^{nor} = \begin{bmatrix} t_D^{11}/t_D^1 & \cdots & t_D^{1j}/t_D^1 & \cdots & t_D^{1n}/t_D^1 \\ \vdots & & \vdots & & \vdots \\ t_D^{i1}/t_D^i & \cdots & t_D^{ij}/t_D^i & \cdots & t_D^{in}/t_D^i \\ \vdots & & \vdots & & \vdots \\ t_D^{n1}/t_D^i & \cdots & t_D^{nj}//t_D^i & \cdots & t_D^{nn}/t_D^i \end{bmatrix} = \begin{bmatrix} t_D^{nor11} & \cdots & t_D^{nor1j} & \cdots & t_D^{nor1n} \\ \vdots & & \vdots & & \vdots \\ t_D^{nori1} & \cdots & t_D^{norij} & \cdots & t_D^{norin} \\ \vdots & & \vdots & & \vdots \\ t_D^{norn1} & \cdots & t_D^{spornj} & \cdots & t_D^{nornn} \end{bmatrix}, \qquad (40)$$

where the sum of each cluster is defined as $t_D^i = \sum_{j=1}^{n} t_D^{ij}$.

5. Build a weighted super-matrix

The unweighted super-matrix U_C is the matrix transposed from the normalized total relation matrix for the criteria T_C^{nor} as shown below.

$$U_C = (T_C^{nor})' = \begin{array}{cc} & \begin{array}{cccc} \overset{D_1}{c_{11} \cdots c_{1m1}} & \overset{D_2}{c_{21} \cdots c_{2m2}} & \overset{\cdots}{\cdots} & \overset{D_n}{c_{n1} \cdots c_{nmn}} \end{array} \\ \begin{array}{c} D_1 \begin{array}{c} c_{11} \\ c_{12} \\ \vdots \\ c_{1m1} \end{array} \\ D_2 \begin{array}{c} c_{21} \\ c_{22} \\ \vdots \\ c_{2m2} \end{array} \\ \vdots \quad \vdots \\ D_n \begin{array}{c} c_{n1} \\ c_{n2} \\ \vdots \\ c_{nmn} \end{array} \end{array} & \begin{bmatrix} U^{11} & U^{i1} & \cdots & U^{n1} \\ U^{1j} & U^{ij} & \cdots & U^{nj} \\ \vdots & \vdots & \ddots & \vdots \\ U^{11} & U^{in} & \cdots & U^{nn} \end{bmatrix} \end{array}. \tag{41}$$

The weighted super-matrix W is obtained by incorporating the unweighted super-matrix U_C and the normalized total influential matrix for clusters T_D^{nor} is shown below.

$$W = \begin{bmatrix} t_D^{nor11} x U^{11} & \cdots & t_D^{nori1} x U^{i1} & \cdots & t_D^{nor1n} x U^{n1} \\ \vdots & & \vdots & & \vdots \\ t_D^{nor1j} x U^{1j} & \cdots & t_D^{norij} x U^{ij} & \cdots & t_D^{norinj} x U^{nj} \\ \vdots & & \vdots & & \vdots \\ t_D^{nor1n} x U^{1n} & \cdots & t_D^{nornj} x U^{in} & \cdots & t_D^{nornn} x U^{nn} \end{bmatrix}. \tag{42}$$

6. Limit the weighted super-matrix to obtain criteria weights

In order to obtain the influential criteria weights, the weighted super-matrix W needs to be limited by raising it to a sufficiently large power s until it converges and becomes a long-term stable super-matrix, $\lim_{s \to \infty} (W)^s$.

3.2.3. Artificial Neural Network (ANN)

Artificial Neural Networks (ANNs) are developed to emulate the cognitive learning process of a human brain which is a very complex and non-linear system with parallel processing. ANN is a powerful tool to amass knowledge by detecting the patterns and relationships in data. The fundamental processing component of an ANN is a neuron. In a neuron, the input signals multiplied by the connection weights are first summed and then passed through an activation function to produce the output for that neuron [154]. A single neuron can perform simple data processing functions whereas the computational power of an ANN comes from the connections between neurons in the network. The neurons in a neural network are organized into a sequence of layers.

A widely used model called the multi-layered perceptron (MLP) ANN consists of one input layer, one or more hidden layers and one output layer. The input layer receives the input from the external environment. The output layer delivers the output of the system to the user. The hidden layers are the black box of the system which help to link the relationship between input and output when it is nonlinear. The number of hidden layers and the number of nodes in each layer are usually heuristically set by determining the number of intermediate steps to translate the input variables into an output value [79, 155]. Various studies addressing this issue can be found in the literature. Figure 3 depicts the general structure of a MLP neural network.

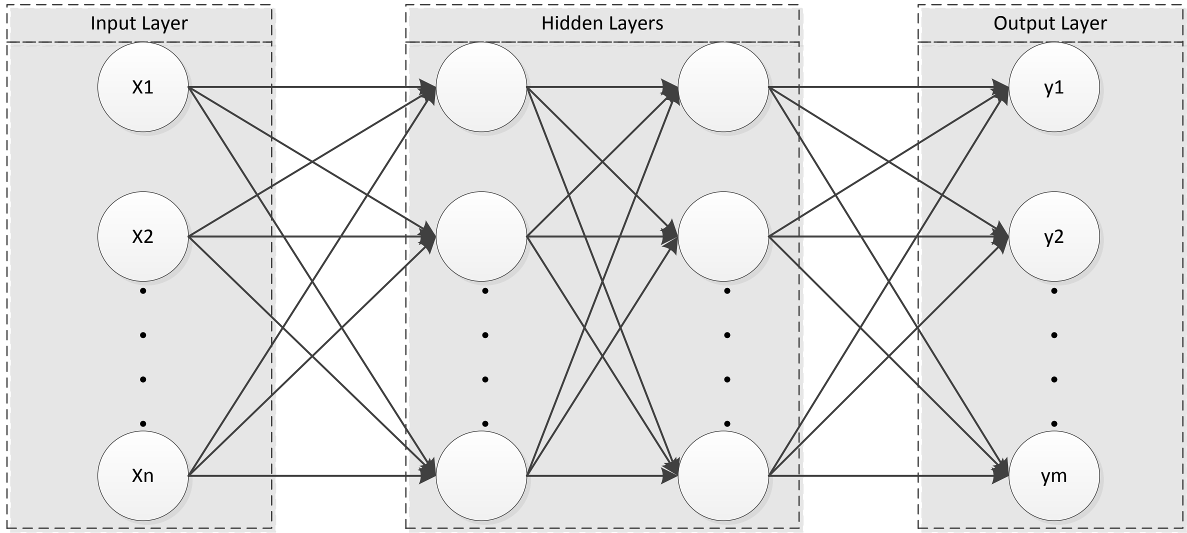

Figure 3.General structure of an MLP neural network

A feed-forward backpropagation MLP network is utilized in this study. MLPs require a learning algorithm and a desired response to be trained. The error between the expected output and the calculated output is computed. Following this, a minimization procedure is used to adjust the weights between two connection layers starting backwards from the output layer to input layer [79]. There are a number of variations of minimization procedures that are based on different optimization methods, such as gradient descent, conjugate gradient, Quasi-Newton, and Levenberg–Marquardt methods. Levenberg–Marquardt method is commonly used in the literature due to its success rate [156-160] and it is utilized in this study as well. For detailed explanations of these methods, please see [161-165].

A transfer function is also required to introduce the non-linearity characteristics into the network. The sigmoid transfer function is utilized due to its ability to help the generalization of learning characteristics to yield models with improved accuracy [155].

The hyperbolic tangent function is a rescaling of the logistic sigmoid function, such that its outputs range from -1 to 1 as shown in Equation (43).

$$f(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \tag{43}$$

A proper normalization method for the data is required to put all the inputs at a comparable range in the neural network. The normalization method used in this study to convert the data between 0 and 1 range is provided in Equation (44).

$$X_i^{norm} = \frac{X_i - X_{min}}{X_{max} - X_{min}} \tag{44}$$

The statistical indicators Mean Square Error (MSE) and the coefficient determinations (R^2) are calculated to illustrate the performance of the model as shown in Equations (45) and (46).

$$MSE = \sum_{i=1}^{n} (Y_i - Y_i')^2 / n \,, \tag{45}$$

$$R^2 = 1 - \left(\frac{\sum_{i=1}^{n} \left(Y_i - Y_i'\right)^2}{\sum_{i=1}^{n} \left(Y_i'\right)^2} \right), \tag{46}$$

where Y_i is the observed value and Y_i' is the predicted value.

3.3. Comparative Analysis of ANN, ANFIS, LSSVM and ELM

This section investigates the prediction capability of ANN, ANFIS, LSSVM and ELM models on the same problem via the data set utilized in the previous section. In order to avoid multicollinearity and to reduce the computation complexity, the four most influential criteria on the final performance score are extracted from the original data set via Pearson Correlation Analysis and grey relational analysis. Subsequently, ANN, ANFIS, LSSVM and ELM methods are employed to obtain Root Mean Square Error (RMSE) values. In order to improve the prediction accuracy, the related parameters in each model are optimized by Particle Swarm Optimization.

Figure 4 provides a schematic representation of the methodology. The details of the methodology are provided in the following text.

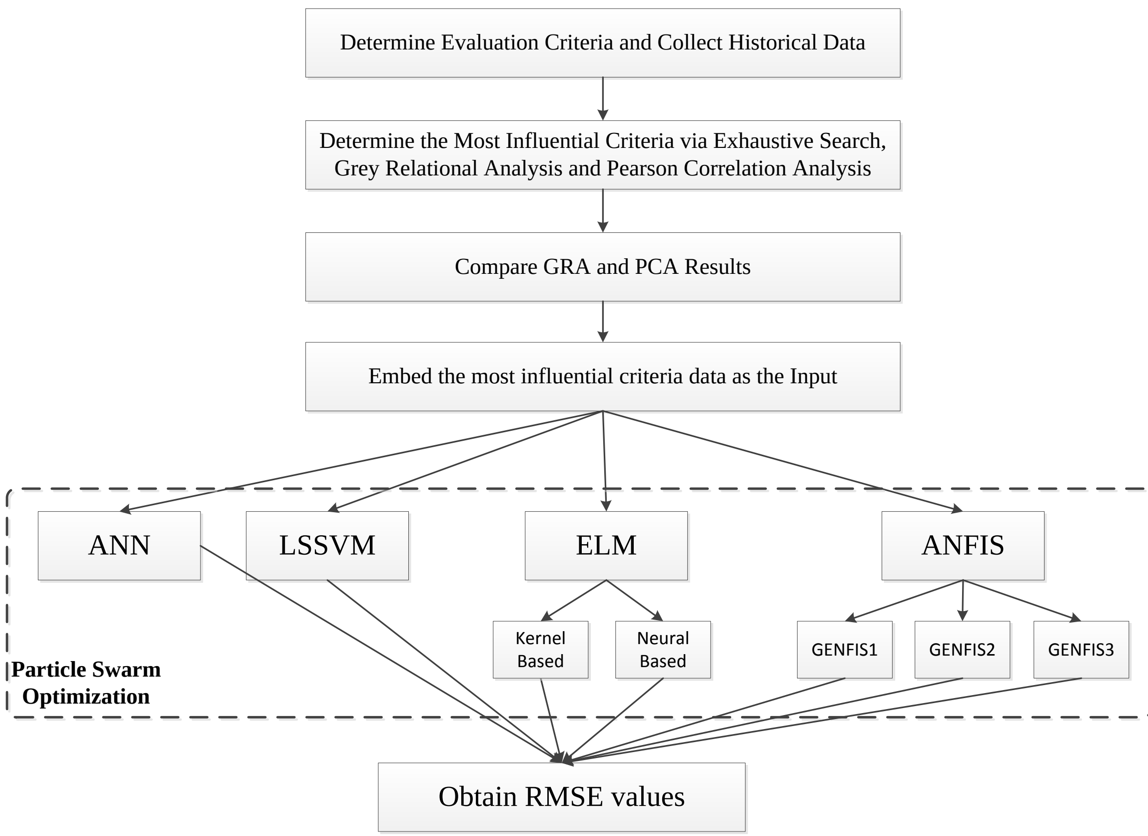

Figure 4.Flow diagram of the ANN, ANFIS, LSSVM and ELM comparison

3.3.1. Pearson Correlation Analysis and Grey Relational Analysis

Pearson Correlation Analysis is a well-known statistical method and widely used to determine the relationship between two data sets. Let X and Y be two zero-mean real-valued random variables. The Pearson correlation coefficient is defined as:

$$\rho(X,Y) = \sum_{i=1}^{n}(X_i - \bar{X})\,(Y_i - \bar{Y})/\sqrt{(X_i - \bar{X})^2}\sqrt{(Y_i - \bar{Y})^2} \tag{47}$$

The formula basically represents dividing the covariance by the product of the standard deviations. The higher ρ indicates the higher correlation between the variables.

Grey relational analysis is utilized to determine the relationship between reference series and compared series, which are denoted as $x_0 = (x_0(1), x_0(2), x_0(3), \dots, x_0(k))$ and $x_i = (x_i(1), x_i(2), x_i(3), \dots, x_i(k))$, *(i=1,2,…,n)* respectively. The grey relational coefficient can be obtained via Eq(48) while Eq (49) provides the grey relational grade.

$$\gamma\big(x_0(k), x_i(k)\big) = \frac{min_i min_k|x_0(k)-x_i(k)|+p(max_i max_k|x_0(k)-x_i(k)|)}{|x_0(k)-x_i(k)|+p(max_i max_k|x_0(k)-x_i(k)|)} \tag{48}$$

$$\gamma(x_0, x_i) = \frac{1}{m}\sum_{k=1}^{m}\gamma\big(x_0(k), x_i(k)\big)\ , i = 1,2,\dots,n\ and\ k = 1,2,\dots,m \tag{49}$$

where, $p\,\epsilon(0,1)$ is the resolution coefficient and generally taken as 0.5.

3.3.2. Adaptive Neuro Fuzzy Inference System (ANFIS)

Adaptive neuro-fuzzy inference systems (ANFIS) first proposed by Jang [166]is a well-studied data driven modelling technique that combines ANN and fuzzy logic. The structure of ANFIS consists of if-then rules and input-output data processing where the learning algorithm of a neural network is used for training. ANFIS is a methodology employed to simulate complex nonlinear mappings using neural network learning and fuzzy inference methodologies. It adjusts membership functions and the related parameters towards the target data sets [26]. An ANFIS structure includes five layers: a fuzzified layer, product layer, normalized layer, defuzzified layer, and a total output layer. A simple ANFIS structure is the one associated with the Sugeno Fuzzy model. This model is known for allowing to generate fuzzy rules from an input-output data set, a typical fuzzy rule being:

Rule 1: If x is A_1 and y is B_1; then $f_1 = p_1x + q_1y + r_1$

Rule 2: If x is A_2 and y is B_2; then $f_2 = p_2x + q_2y + r_2$

where A_i and B_i are the fuzzy sets, fi is the output set within the fuzzy region specified by the fuzzy rule p_i and q_i and r_i are the design parameters that are determined during the training process [167]. Figure 5 illustrates the ANFIS structure.

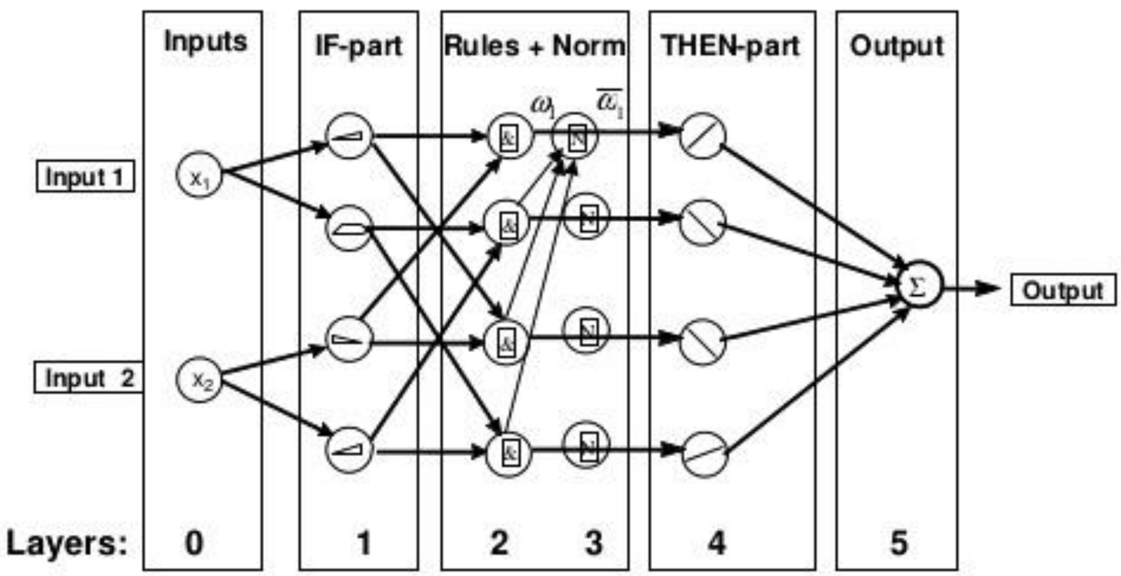

Figure 5.ANFIS structure

<u>Layer 1:</u> Each node *i* in this layer has a node function as

$$O_i^1 = \mu_{Ai}(x), \quad i = 1,2. \tag{50}$$

where, *x* is the input of node *i*, and A_i is the linguistic label (low, high, etc.) associated with this node function. In other words, O_i^1 is the membership function of A_i and it specifies the degree to which the given x satisfies the quantifier A_i. Usually $\mu_{Ai}(x)$ is chosen as a generalized bell shaped function within the range of [0,1] and can be expressed as:

$$\mu_{Ai}(x) = \frac{1}{1 + \left[(\frac{x - c_i}{a_i})^2 \right]^{b_i}} \tag{51}$$

where $\{a_i, b_i, c_i\}$ is the membership function parameter set.

<u>Layer 2</u>: Every node in this layer multiplies the incoming signals and sends the product out. It represents the firing strength of a rule.

$$O_i^2 = \omega_i = \mu_{Ai}(x).\mu_{Bi}(y), \ i=1,2. \tag{52}$$

<u>Layer 3</u>: Every node in this layer calculates the ratio of the *i*th rule's firing strength:

$$O_i^3 = \overline{\omega_i} = \frac{\omega_i}{\omega_1 + \omega_2}, i = 1,2. \tag{53}$$

<u>Layer 4:</u> Every node in this layer has a node function as:

$$O_i^4 = \overline{\omega_i} f_i = \overline{\omega_i}(p_i x + q_i y + r_i) \tag{54}$$

<u>Layer 5:</u> The single node in this layer computes the overall output as the summation of all incoming signals. The last layer has a single fixed node and its outputs have crisp characteristics.

$$O_i^5 = \sum_{i=1}^{2} \overline{\omega_i} f_i = \frac{\sum_{i=1}^{2} \omega_i f_i}{\sum_{i=1}^{2} \omega_i} \tag{55}$$

The hybrid algorithm utilized in ANFIS combines the backpropagation method with least squares method. The backpropagation is used for the parameters in Layer 1 and the least squares method is employed for training the parameters. For a given dataset, different ANFIS models can be constructed using different identification methods. Grid partitioning, subtractive clustering and fuzzy c means clustering methods are utilized in this study and denoted as *Genfis1*, *Genfis2*, *Genfis3*, respectively.

Grid Partitioning (Genfis1): The grid partitioning method proposes independent partitions of each antecedent variable. The data space is divided into rectangular sub-spaces using axis-paralleled partitions based on a predefined number of MFs and their types in each dimension. The limitation of this method is that the number of rules rises rapidly as the number of inputs increases. For example, if the number of input is n and the partitioned fuzzy subset for each input is m, then the number of possible fuzzy rules is m^n. While the number of variables raises, the number of fuzzy rules increases exponentially, which requires a large computer memory[168].

Subtractive Clustering (Genfis2): The subtractive clustering method assumes each data point is a potential cluster center and based on the density of the surrounding data points, calculates a measure of the possibility that each data point would define the cluster center. The potential cluster center P_i at a data point x_i is described as:

$$P_i = \sum_{j=1}^{m} exp\left(-\frac{||x_i - x_j||^2}{(r_a/2)^2}\right) \tag{56}$$

where m is the total number of data points in the N-Dimensional space. X_i and X_j are data points and r_a is a positive constant defining neighborhood radius and ||-|| denotes the Euclidean distance. The first cluster center is selected as the c_1 data point, which has the highest potential value, P_{c1}. For the second cluster center, the effect of the first cluster center is subtracted to obtain the new density values.

$$P_i = P_i - P_{c1} exp\left(-\frac{||x_i - x_{c1}||^2}{(r_b/2)^2}\right) \tag{57}$$

$$r_b = \delta r_a \tag{58}$$

where δ is a constant greater than 1 to avoid cluster centers being in too close proximity. Generally, after determining the kth cluster center c_k, the potential is revised as follows:

$$P_i = P_i - P_k exp\left(-\frac{||x_i - x_k||^2}{(r_b/2)^2}\right) \tag{59}$$

where c_k is the location of the kth cluster center and P_k is the largest potential density value. Cluster centers are chosen iteratively until the stopping criteria are met.

Fuzzy C Means Clustering (Genfis3): Fuzzy c-means is a soft clustering method in which each data point belongs to a cluster, with a degree specified by a membership grade and does not consider sharp boundaries between the clusters. The centers of each cluster c_i is selected randomly from n data patterns. The membership matrix (μ) is computed via Eq(60).

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{c}(\frac{d_{ij}}{d_{kj}})^{\frac{2}{m}-1}} , \tag{60}$$

where, μ_{ij} is the degree of membership of object j in cluster i, m is the fuzziness index and d_{ij} is the Euclidian distance between c_i and x_j. The objective function is calculated via Eq(61).

$$J(U, c_1, c_2, \dots, c_c) = \sum_{i=1}^{c} J_i = \sum_{i=1}^{c} \sum_{i=1}^{c} \mu_{ij}{}^{m} d_{ij}^2 , \tag{61}$$

The new c fuzzy cluster centers c_i are calculated as follows:

$$c_i = \frac{\sum_{j=1}^{n} \mu_{ij}{}^{m} x_{ij}}{\sum_{j=1}^{n} \mu_{ij}{}^{m}} . \tag{62}$$

3.3.3. Least Squares Support Vector Machine (LSSVM)

LSSVM is an alteration of the standard SVM and was improved by Suykens et al. [92] and has been successfully applied in optimal control, classification and regression problems. The LSSVM uses the least squares loss function to construct the optimization problem based on equality constraints.

In an LSSVM model, the training dataset is assumed to be $\{x_k, y_k\} k=1,2,\ldots,l$, where $x_k \in R^n$ is an input vector and $y_k \in R$ is its corresponding target vector. The regression function can be formulated as feature space representation.

$$y = f(x) = w^T \varphi(x) + b \ . \tag{63}$$

The regression can be transformed into the following optimization problem [169].

$$min J_1(w, b, e) = \frac{1}{2} w^T w + \frac{1}{2} C \sum_{i=1}^{l} e_i^2 \tag{64}$$

Subject to

$$y_i = w^T \varphi(x_i) + b + e_i \ , \ \text{i=1,2…,l} \tag{65}$$

where w is the weight vector, C is the regularization parameter, $\varphi(x_i): R^n \rightarrow R^f$ is a function used to map the input space to a higher dimensional space and e_i is the error between the prediction value and true value of the system which introduces a Lagrangian function. Hence, the LSSVM model is formulated as follows:

$$f(x) = \sum_{i=1}^{l} \alpha_i K(x, x_i) + b \ , \tag{66}$$

where $K(x, x_i)$ indicates a kernel function and α_i and b are the solutions to the linear system. This study utilizes the radial basis function (RBF) kernel was selected as the kernel function because the RBF kernel manages the nonlinear relationship well and has a high overall performance. The RBF kernel maps samples to high dimensional space in a nonlinear fashion and has fewer required parameters. Therefore, it is the most commonly used kernel function type among others [169]. The RBF kernel function is formulated as follows:

$$K(x_i, x_j) = \exp\left(-\frac{\left||x_i - x_j|\right|^2}{\sigma^2}\right), \tag{67}$$

Therefore, the LSSVM model can be reformulated as in the following:

$$f(x) = \sum_{i=1}^{l} \alpha_i \exp\left(-\frac{\left||x_i - x_j|\right|^2}{\sigma^2}\right) + b \;. \tag{68}$$

3.3.4. Extreme Learning Machine (ELM)

Extreme learning machine is first proposed as a single layer hidden feedforward network by Huang et al. [93] using randomly assigned input weights and bias. Unlike the back-propagation method, it does not require adjustment of input weights. However, the randomly assigned weights could cause a large variation in the accuracy in different trials with the same number of hidden nodes [170]. In order to overcome this issue, Huang et al. [171] proposed replacing the hidden layer of the ELM with a kernel function which does not include randomness in assigning connection weights between input and hidden layers. Therefore, this study utilizes both neural based and kernel based ELM for comparison purposes. The brief explanations of each method is provided in the following.

<u>Neural Based ELM:</u> Let the number of nodes in the input, hidden and output layers of the network denoted as U, R and O, respectively. Given the input vector $= [x_1, x_2, \dots, x_U]^T$, the output vector y= $[y_1, y_2, \dots, y_O]^T$ of a single-layer feedforward neural network can be expressed as:

$$y_o = \sum_{j=1}^{R} \beta_{io} G_i(\,\omega_i, b_i, x), \;\; (o=1,2,\dots,O) \tag{69}$$

where, ω_i is the connection weight between the input layer and *i*th node in the hidden layer, b_i is the bias of the *i*th hidden node, β_i is the connection weight between the *i*th node in the hidden layer and the output layer and $G_i(\omega_i, b_i, x) = g(\omega_i x + b_i)$ is the output of the *i*th hidden node. Here, $g(.)$ represents the activation function and RBF is utilized as the activation function in this study. The objective of training ELM is to minimize the empirical error and structural error by assigning the best input and output weights (ω_i, β_i). Therefore, given the training dataset (X_j, Y_j) the training of ELM is a nonlinear optimization problem and the objective function is formulated as in the following.

$$minE(\omega_i, \beta_i) = \sum_{j=1}^{W} ||y_j - Y_j||, \tag{70}$$

where $y_j = [y_{j1}, y_{j2}, \dots, y_{jO}]^T$ are the outputs of the given inputs $X_j = [X_{j1}, X_{j2}, \dots, X_{jU}]^T$and $Y_j = [Y_{j1}, Y_{j2}, \dots, Y_{jO}]^T$ represents the real values of the corresponding dependent variables. When the weights and biases are randomly assigned solving the optimization problem in Eq(70) is equivalent to solving $G.\beta = Y$ for its least squares solution β. Based on Moore-Penrose's generalized inverse matrix theory, the weights can be obtained analytically as $\beta = G^+.Y$, where G^+ is the generalized inverse matrix of the hidden layer output matrix *G* in the ELM network [172].

Kernel Based ELM: In order to have an ELM with better generalization capabilities in comparison with the least square solution-based ELM, which requires randomly assigned input weights, Huang et al. [171] proposed adding a positive value I/ρ (where ρ is a user-defined parameter) for the calculation of the output weights *G* as provided in the following.

$$G = \beta^T (\mathrm{I}/\rho + \beta\beta^T)^{-1} Y, \tag{71}$$

If the hidden layer feature mapping $g(.)$ is unknown using a kernel function is suggested. Hence, A kernel matrix Ω for ELM can be represented as in Eq (72) and the output function can be formulated as in Eq(73) where $K(x_i, x_j)$ is a kernel function.

$$\Omega_{ELM} = \beta\beta^T : \Omega_{ELMi,j} = g(x_i).\, g(x_j) = K(x_i, x_j), \tag{72}$$

$$G = \begin{bmatrix} K(x, x_j) \\ \vdots \\ K(x, x_n) \end{bmatrix}^T (\mathrm{I}/\rho + \Omega_{ELM})^{-1} Y\,. \tag{73}$$

3.3.5. Particle Swarm Optimization

Particle Swarm Optimization, developed by Eberhart and Kennedy [100] is a population based heuristic computation technique that simulates the social behavior metaphor of the birds. In the algorithm, the population is considered as the swarm and the individuals are called particles. PSO performs iterative searches to obtain the optimal or near optimal solution where each particle changes its searching direction according to its own best previous experience and the best experience of the entire swarm. In this study, PSO is utilized to obtain the optimal or near optimal values of the corresponding parameters in each method while minimizing the calculated Root Mean Square Error (RMSE):

$$\text{RMSE}=\sqrt{\textstyle\sum_{i=1}^{n} (X_i - \hat{X}_i)^2 / n}\ . \tag{74}$$

Five basic steps of the PSO are as follows:

Step 1. Initialize randomly the position (pBest) and speed for each particle.

Step 2. Set pBest as the current position and gBest as the optimal particle position in initial swarm.

Step 3. Compute the RMSE of the GM when the value of the variable is pBest.

Step 4. Compute the velocity and the position for each particle using:

$$V = \omega * V + c_1 * rand * (pBest - Present) + c_2 * rand * (gBest - Present), \quad (75)$$

and

$$Present = Present + V, \quad (76)$$

where, V is the velocity, $rand$ is the random number generator in the range [0,1], ω is the inertia factor, and c_1 and c_2 are the learning factors. If the fitness of this particle is superior to pBest then pBest becomes the new position. If the fitness of this particle is superior to gBest then gBest is accepted as the new position.

Step 5. Go back to Step 3 until one of the two termination criteria is met: i. obtaining sufficiently good fitness value, or, ii. reaching the maximum number of iterations.

3.4. Mathematical Foundation of Multi-variate Grey Model GM(1,n)

This section investigates the prediction capability of traditional multi-variate grey model GM(1,n), grey model with convolutional integral, GMC(1,n) and discrete grey model, DGM(1,n) using the same problem via the data set provided in the previous section.

The grey model, proposed by Deng [109], is usually represented as GM(r,n) for the *r*th order of the differential equation and with *n* variables. First order grey model, GM(1,n), is commonly used in a variety of applications. However, recent studies indicate that fractional order could lead to higher accuracy levels. With this motivation, the fractional order accumulation is integrated into each model. In order to improve the prediction accuracy, the associated parameters in each model are optimized by Particle Swarm Optimization. Figure 6 provides a schematic representation of the methodology. The details of the methodology are provided in the following text.

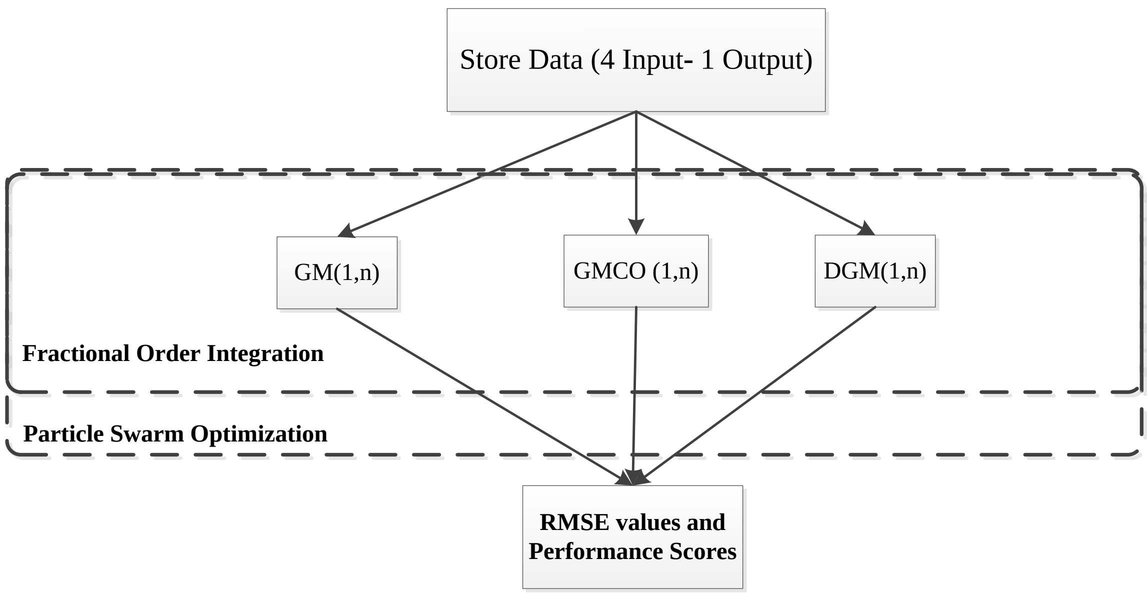

Figure 6.Flow diagram of the GM(r,n), DGM(r,n) and GMC(r,n) comparison

The traditional GM(1,n) method is the foundation of other improved grey models. The modeling procedures of GM(1,n) can be carried out as follows. Let an initial series be

$$x_1^{(0)} = \left\{ x_1^{(0)}(1),\ x_1^{(0)}(2), \ldots,\ x_1^{(0)}(k) \right\},$$
$$x_2^{(0)} = \left\{ x_2^{(0)}(1),\ x_2^{(0)}(2), \ldots,\ x_2^{(0)}(k) \right\},$$
$$\ldots$$

$$x_N^{(0)} = \left\{ x_N^{(0)}(1),\ x_N^{(0)}(2), \dots,\ x_N^{(0)}(k) \right\},\ k=1,2,\dots,\ m. \tag{77}$$

Based on the initial series, the first order accumulation generating operation is defined as:

$$x_1^{(1)} = \left\{ x_1^{(1)}(1),\ x_1^{(1)}(2), \dots,\ x_1^{(1)}(k) \right\},$$
$$x_2^{(1)} = \left\{ x_2^{(1)}(1),\ x_2^{(1)}(2), \dots,\ x_2^{(1)}(k) \right\},$$
$$\dots$$
$$x_N^{(1)} = \left\{ x_N^{(1)}(1),\ x_N^{(1)}(2), \dots,\ x_N^{(1)}(k) \right\},\quad k=1,2,\dots,\ m, \text{ and } x_i^1(k) = \sum_{j=1}^{k} x_i^{(0)}(j) \tag{78}$$

The discrete equation of GM(1,n) can be formulated as follows:

$$x_1^{(0)}(k) + a z_1^{(1)}(k) = \sum_{i=2}^{N} b_i x_i^{(1)}(k)\ . \tag{79}$$

where, k≥2 and the background value *z* can be obtained via Eq (80).

$$z = p.x(k) + (1-p).x(k+1)\ , \tag{80}$$

where, where the background value coefficient $p \epsilon R^{+}$ and between [0,1] and generally taken as 0.5. The parameters, *a* the developing coefficient and b_i the control variable can be estimated by ordinary least squares method.

$$[a, b_2, \dots, b_N] = (B^T B)^{-1} B^T Y\ , \tag{81}$$

where,

$$Y=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix},\ B=\begin{bmatrix} -z_1^{(1)}(2) & x_2^{(1)}(2) \dots & x_N^{(1)}(2) \\ -z_1^{(1)}(3) & x_2^{(1)}(3) \dots & x_N^{(1)}(3) \\ \vdots & \vdots \dots & \vdots \\ -z_1^{(1)}(n) & x_2^{(1)}(n) \dots. & x_N^{(1)}(n) \end{bmatrix}. \tag{82}$$

The forecasting equation of GM(1,n) is denoted via Eq(83).

$$\hat{x}_1^{(1)}(k+1) = \left[x_1^{(1)}(0) - \frac{1}{a}\sum_{i=2}^{N} b_i x_i^{(1)}(k+1) \right] e^{-ak} + \frac{1}{a}\sum_{i=2}^{N} b_i x_i^{(1)}(k+1) \tag{83}$$

where, $x_1^{(1)}(0)$ is taken to be $x_1^{(0)}(0)$ which is the initial value of GM(1,n) model. Hence, the restoration of the inverse accumulation of GM(1,n) can be computed via Eq(84).

$$\hat{x}_1^{(0)}(k+1) = \hat{x}_1^{(1)}(k+1) - \hat{x}_1^{(1)}(k) \; . \tag{84}$$

3.4.1. Grey Model with Convolutional Integral GMC(1,n)

The structure of GMC(1,n) is very similar to GM(1,n) while the grey control parameter u is introduced to GMC(1,n)[120]. Thus, GMC(1,n) can be represented as follows:

$$x_1^{(0)}(k) + a z_1^{(1)}(k) = \sum_{i=2}^{N} b_i x_i^{(1)}(k) + u \tag{85}$$

The parameters, the developing coefficient a, the associated variables b_i and the grey control parameter u can be estimated by ordinary least squares method as in the following.

$$[a, b_2, \dots, b_N, u] = (B^T B)^{-1} B^T Y \tag{86}$$

where,

$$\mathrm{Y}=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \ \mathrm{B}=\begin{bmatrix} -z_1^{(1)}(2) & z_2^{(1)}(2) \dots & z_N^{(1)}(2) & 1 \\ -z_1^{(1)}(3) & z_2^{(1)}(3) \dots & z_N^{(1)}(3) & 1 \\ \vdots & \vdots \quad \dots & \vdots & \vdots \\ -z_1^{(1)}(n) & z_2^{(1)}(n) \dots . & z_N^{(1)}(n) & 1 \end{bmatrix}. \tag{87}$$

$$f(k) = \sum_{i=2}^{N} b_i x_i^{(1)}(k) + u \ , \quad k=1,2,..,n \tag{88}$$

The estimated value can be obtained via Eqs.(89) and (90).

$$\hat{x}_1^{(1)}(k) = x_1^{(0)}(1)e^{-a(k-1)} + \frac{1}{2}e^{-a(k-1)} f(1) + \sum_{\tau}^{k-1}\left[e^{-a(k-\tau)}f(\tau)\right] + \frac{1}{2}f(k) \quad (89)$$

$$\hat{x}_1^{(0)}(k) = \hat{x}_1^{(1)}(k) - \hat{x}_1^{(1)}(k-1) \quad (90)$$

3.4.2. Discrete Grey Model DGM(1,n)

The basic equation of DGM(1,n) can be defined as in the following [122].

$$x_1^{(1)}(k) = \beta_i x_i^{(1)}(k-1) + \sum_{i=2}^{N} \beta_i x_i^{(1)}(k) + u \quad (91)$$

The parameters β_i can be obtained via least squares estimation method

$$[\beta_1, \beta_2, \dots, \beta_N, u] = (B^T B)^{-1} B^T Y \quad (92)$$

where,

$$Y=\begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{bmatrix}, B=\begin{bmatrix} x_1^{(1)}(1) & x_2^{(1)}(2) \dots & x_N^{(1)}(2) & 1 \\ x_1^{(1)}(2) & x_2^{(1)}(3) \dots & x_N^{(1)}(3) & 1 \\ \vdots & \vdots \quad \dots & \vdots & \vdots \\ x_1^{(1)}(n-1) & x_2^{(1)}(n) \dots & x_N^{(1)}(n) & 1 \end{bmatrix}. \quad (93)$$

Thus, the response function and the restored values of DGM(1,n) model can be obtained via Eqs.(94) and (95).

$$\hat{x}_1^{(1)}(k+1) = \beta_1^k x_1^{(0)}(1) + \sum_{m=0}^{k-1} \beta_1^m \left(\sum_{i=2}^{N} \beta_i x_i^{(1)}(k+1-m)\right) + \frac{1-\beta_1^k}{1-\beta_1} u \,, \quad (94)$$

$$\hat{x}_1^{(0)}(k+1) = \hat{x}_1^{(1)}(k+1) - \hat{x}_1^{(1)}(k) \,. \quad (95)$$

3.4.3. Fractional Order Accumulation

Let *r=p/q* and $x_1^{(0)} = \left\{ x_1^{(0)}(1),\, x_1^{(0)}(2), \dots,\, x_1^{(0)}(n) \right\}$ is the original data sequence as in Eq(77). Then, $x_1^{(r)} = \left\{ x_1^{(r)}(1),\, x_1^{(r)}(2), \dots,\, x_1^{(r)}(n) \right\}$ is called the *r.th* order cumulative generation sequence. $x^{(r)}(k)$ can be expressed as follows:

$$x^{(r)}(k) = \sum_{i=1}^{k} \frac{(k-i-1)(k-i+2)\dots(k-i+r-1)}{(r-1)!} x^{(0)}(i)\ ,\ \ r \in R^{+}\ , k = 1,2,..,n \tag{96}$$

In order to express the *r.th* order cumulative Gamma function Γ is utilized. Therefore,

$$x^{(r)}(k) = \sum_{i=1}^{k} \frac{\Gamma(k-i-1)}{\Gamma(\mathrm{r})\Gamma(\mathrm{k-i-1})} x^{(0)}(i) \quad r \in R^{+}\ \ , k = 1,2,..,n \tag{97}$$

The grey reducing generation $x^{(-r)} =\ x^{(-r)}(1),\, x^{(-r)}(2), \dots,\, x^{(-r)}(n)$ corresponds to the grey accumulating generation implying that these two operators meet the reciprocity condition.

$$x^{(-r)}(k) = \sum_{i=0}^{k-1} \frac{\Gamma(r+1)}{\Gamma(\mathrm{r+1})\Gamma(\mathrm{r-i-1})} x^{(0)}(k-i)\ r \in R^{+}\ \ , k = 1,2,..,n \tag{98}$$

As in the original GM(1,n), the background sequence values of $x^{(r)}$ is defined as $z^{(r)}$, where,

$$z^{(r)}= z^{(-r)}(1),\, z^{(-r)}(2), \dots,\, z^{(-r)}(n)\ \text{, and} \tag{99}$$

$$z(k)^{(r)} = px(k)^{(r)} + (1-p)x^{(r)}(k-1) \quad k = 2,3,..,n\ and\ p\ \epsilon\ [0,1] \quad . \tag{100}$$

The associated parameters are obtained by least squares method where the elements in the *B* and *Y* matrices need to be in the *r*.th order. For instance, for the original GM(1,n) model:

$$Y=\begin{bmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{bmatrix}, B=\begin{bmatrix} -z_1^{(r)}(2) & x_2^{(r)}(2) & \dots & x_N^{(r)}(2) \\ -z_1^{(r)}(3) & x_2^{(r)}(3) & \dots & x_N^{(r)}(3) \\ \vdots & \vdots & \dots & \vdots \\ -z_1^{(r)}(n) & x_2^{(r)}(n) & \dots. & x_N^{(r)}(n) \end{bmatrix}. \tag{101}$$

Then the predicted *r.th* cumulative data sequence $\hat{x}^{(r)}(k)$ can be obtained using the response function in the associated model. Following that, the restored value of the original data sequence is computed using Eq(102).

$$\hat{x}^{(0)}(k) = x^{(r)(-r)}(k) = \sum_{i=0}^{k-1}(-1)^i \frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i-1)} \hat{x}^{(r)}(k-i) \ , k = 2,3 \dots, n \tag{102}$$

CHAPTER 4: IMPLEMENTATION AND RESULTS

4.1. Implementation of the Fuzzy AHP, DEA with Cross-Efficiency and TOPSIS

Previously, Duman and Kongar [38] employed DEA to evaluate the service performance of twenty franchise retail stores of one of the leading pizza restaurant chain stores in the US. The evaluation of these stores, a.k.a. Decision Making Units (DMUs), was solely based on quantitative data obtained from each DMU (Table 2). The quantitative data set was sufficient for the evaluation of the Service Performance (SP), a measure commonly used by corporate auditors, but was inadequate in evaluating other significant aspects of the business which often require utilization of qualitative criteria. Among these, Product Quality (PQ), Food Safety (FS), Operational Safety (OS), and Store Image (SI) can be considered as the most significant criteria since they are frequently included in the overall performance assessment. With this motivation, to provide an accurate, precise and comprehensive evaluation, this study provides a holistic approach utilizing both quantitative and qualitative data which are detailed in the following sub-sections.

4.1.1. Criteria Definition

In order to evaluate the overall efficiency of 20 franchise retail stores, the data was collected from the franchise stores and the demographic data, population density and territory data, are retrieved from USA Census 2010 results. The analysis was conducted in a homogenous sample. In order to comply with assured confidentiality agreements with the franchisee, the identity of the name of the business and the restaurant chain is undisclosed.

Qualitative Criteria:

1. Product Quality (PQ): A measure related to quality of the products produced in each store, the auditors evaluate the properties of the product and the process such as proper make, portion and make.
2. Food Safety (FS): A measure related to product safety factors such as hygiene, temperature control, proper sanitation etc.
3. Operational Safety (OS) : A measure related to indoor and outdoor operational safety factors in each store such as, traffic violations, proper cash drop in the store, secure cashier till and safe.
4. Store Image (SI): A measure related to the overall image from a customer view in each store. The factors affecting this criterion are proper greeting, proper uniform, attire and grooming of the employees, organization of the signs, banners and advertisements, cleanness of the service internal and external service environment and compliance with customer expectations.

Quantitative Criteria:

Table 2 presents the input and output data used in DEA to evaluate the Service Performance (SP).

Table 2. Input and Output Criteria in the DEA Model

Input Criteria (per week)	Unit
Store Territory	Sq. Mile
Population Density	Population/Sq Mile
Weekly Expenses	Dollars/week
Total hours worked by in-store personnel	Hours/week
Total hours worked by delivery personnel	Hours/week
Output Criteria (per week)	**Unit**
Total Number of Carry-out Orders	Number of Orders/week
Total Number of Delivery Orders	Number of Orders/week
Sales	Dollars/week
% Delivery On Time	Percentage
% Out to Door Time	Percentage

4.1.2. Criteria Weights via Fuzzy AHP

As mentioned previously, Fuzzy AHP is applied to determine the weights of the main criteria, Service Performance (SP), Product Quality (PQ), Food Safety (FS), Operational Safety (OS) and Store Image (SI). Criteria weights and decision matrix are formed according to franchisee and supervisors point of view. The decision makers used linguistic terms to assess the criteria weights. The evaluation scale used in Fuzzy AHP is provided in Table 3. In order to be able to obtain the consistency ratio, the model is also solved via traditional AHP. The consistency ratio is calculated as (CR=0.08) which validates the model consistency. The results were shared with the franchise management and based on the feedback received, Fuzzy AHP results were considered to be more meaningful and suitable for future decision making.

Table 3. Comparative linguistic scale for ratings of alternatives and weights of criteria

Linguistic Terms	Triangular Fuzzy Number (TFN)
Just equal (EQ)	(1,1,1)
Weak importance of one over another (WI)	(1,1,3)
Fairly Preferable (FP)	(1,3,5)
Essential importance of one over another (EI)	(3,5,7)
Strongly Preferable (SP)	(5,7,9)
Absolutely Preferable (AP)	(7,9,9)
In the pairwise comparison matrix, the value of each criterion *j* compared to criterion *i* is calculated as the reciprocal of the value of criterion *i* compared to criterion *j*	

The data set obtained from the decision makers for pairwise comparison is provided in Table 4.

Table 4. Pairwise comparison matrix

Criteria	Service Performance (SP)	Product Quality (PQ)	Food Safety (FS)	Store Image (SI)	Operational Safety (OS)
Service Performance (SP)	EQ	WI	1/WI	SP	EI
Product Quality (PQ)	1/WI	EQ	1/FP	FP	FP
Food Safety (FS)	WI	FP	EQ	SP	FP
Store Image (SI)	1/SP	1/FP	1/SP	EQ	WI
Operational Safety (OS)	1/EI	1/FP	1/FP	1/WI	EQ

The weights are then fed into the Fuzzy AHP algorithm detailed in section 3.1.1. Resulting weights are given in Table 5.

Table 5. Weights of the main criteria

Service Performance (SP)	0.3527
Product Quality (PQ)	0.1753
Food Safety (FS)	0.3234
Store Image (SI)	0.0780
Operational Safety (OS)	0.0706

4.1.3. Service Performance Evaluation via DEA

The input and output data utilized in the DEA is obtained from Duman and Kongar's study [38]. According to the criteria determined in Table 2, the data shown in Table 6 and Table 7 reflect the total weekly average for 3 months of operations from 20 stores. Since there is a major difference between weekday and weekend operations, only weekly operation data is considered using the total weekly averages of 3 months obtained from 20 DMUs. The demographic data, population density and territory data, are retrieved from USA Census 2010 and the franchise management. For further information about the quantitative measurements please see [38].

Table 6. Input Data used in the DEA Model

DMU	Store territory	Population density	Total hours worked by in-store personnel	Total hours worked by delivery personnel	Weekly expenses
Store 1	18.797	1147.0	203.7	248.15	3321
Store 2	2.684	115685.1	577.34	828.33	14934.75
Store 3	1.315	182169.5	214.29	548.7	8312.75
Store 4	1.326	16216.4	259.77	500.07	7857.75
Store 5	2.635	16235.3	204.15	303.44	4962.75
Store 6	14.094	35240.7	135.71	244.86	3901.25
Store 7	14.489	2730.0	124.81	193.03	2993.5
Store 8	24.725	1163.8	198.2	261.96	4245.5
Store 9	7.007	1533.5	212.2	331.28	4319.5
Store 10	8.971	6137.0	265.72	518.56	7332
Store 11	8.89	3682.9	218.32	364.48	4667.25
Store 12	9.936	2411.0	132.51	203.96	4253
Store 13	35.986	924.5	212.6	291.83	4390.5
Store 14	5.06	79371.2	358.42	498.96	10035
Store 15	12.434	4907.9	223.14	378.47	4585.5

Store 16	1.352	12159.0	128.34	236.72	4225.75
Store 17	17.209	801.6	215.6	324.58	4918
Store 18	4.535	16587.0	238.56	387.62	5890.5
Store 19	4.561	5670.0	139.42	267.89	4036.25
Store 20	2.592	11506.9	179.87	265.72	3746

Table 7. Output Data used in the DEA Model

DMU	Total Number of Delivery Orders	Total Number of Carry-out Orders	Sales	% Out to Door Time	% Delivery On Time
Store 1	371	406	16132	0.7345	0.712
Store 2	2378	522	56359	0.6257	0.7605
Store 3	1202	260	32771	0.8294	0.8654
Store 4	758	252	31576	0.6543	0.7306
Store 5	577	212	17540	0.8455	0.7981
Store 6	376	240	14470	0.8891	0.8253
Store 7	293	270	9577	0.8759	0.8688
Store 8	387	291	11206	0.7698	0.5923
Store 9	406	297	16024	0.8735	0.8448
Store 10	682	235	15104	0.7124	0.6903
Store 11	513	243	16984	0.8788	0.8953
Store 12	354	248	8517	0.8634	0.8453
Store 13	487	247	15939	0.7698	0.6321
Store 14	1087	906	34131	50.23	0.5869
Store 15	482	254	10959	0.8345	0.7942
Store 16	331	256	11742	0.7881	0.9156
Store 17	302	501	13748	0.8247	0.8843
Store 18	546	208	8793	0.7163	0.7362
Store 19	369	205	8336	0.8176	0.8247
Store 20	318	188	7964	0.775	0.7906

The output-oriented DEA, Eq (11), is executed and the results are presented in Figure 7.

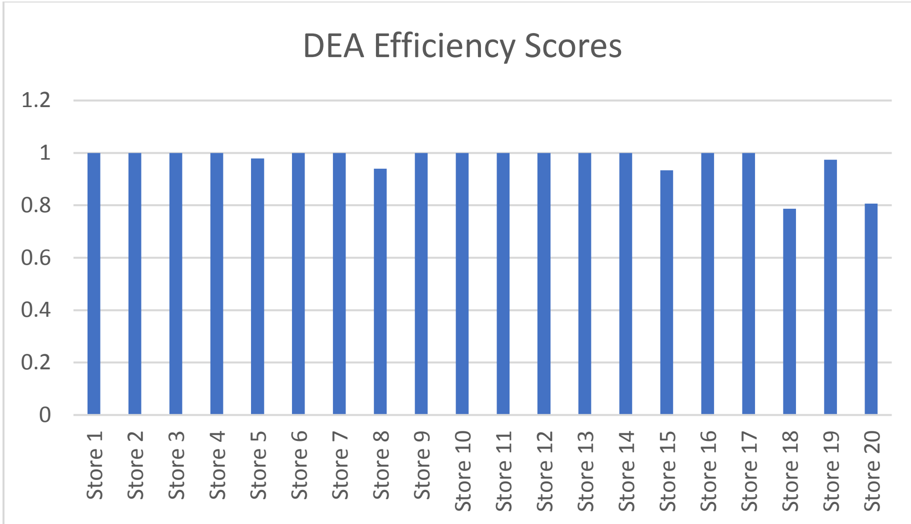

Figure 7. Service Performance Efficiencies of 20 Stores

As seen in Figure 4, the classic output oriented DEA method only differentiates inefficient and efficient DMUs and does not allow the ranking of DMUs. Since the initial DEA aims to maximize the output values with each DMU being differentiated as much as possible, cross-efficiency evaluation with benevolent formulation is chosen as the most appropriate tool for this case study. As also stated by Wang and Wang [173], the aggressive formulation leads to too many zero weights causing significant input and output information loss in the efficiency assessment. To overcome this setback, this study utilizes the benevolent formulation. Eq (13) as detailed in section 3.1.2 was applied and the results are demonstrated in Table 8.

Table 8. DEA with Cross-Efficiency Results

DMU	Score	DMU	Score
Store 1	0.944098	Store 11	0.920784
Store 2	0.902979	Store 12	0.889298
Store 3	0.775104	Store 13	0.919067
Store 4	0.890248	Store 14	0.944138
Store 5	0.874323	Store 15	0.76502

Store 6	0.796506	Store 16	0.816641
Store 7	0.991819	Store 17	0.730699
Store 8	0.773516	Store 18	0.613586
Store 9	0.869382	Store 19	0.823149
Store 10	0.739424	Store 20	0.631316

According to the results above Store 7 has the highest efficiency score.

4.1.4. Overall Evaluation of the Stores via TOPSIS

In this step, overall evaluation of the stores are obtained as a result of TOPSIS analysis. The qualitative measures for Product Quality, Food Safety, Operational Safety, and Store Image are collected for each store from the previous operational evaluation reports conducted by the franchise auditors. The comparative scale used in the evaluation of the ratings of these qualitative variables is provided in Table 9. Notice that the bottom line for these qualitative measures is 1 implying that if a store is not even good enough to receive a value of 1, it will be issues a warning stating that if the problem is not resolved in timely manner, this could result in that particular store’s closing.

Table 9. Comparative Scale for Qualitative Variables

Evaluation Scale	**Rating**
Excellent	5
Very Good	4
Good	3
Bad	2
Very Bad	1

These qualitative measures are grouped with the Cross-Efficiency DEA results and the main criteria weights coming from Fuzzy AHP analysis to form a decision matrix for the overall evaluation. This grouped data is provided in Table 10.

Table 10. Input Data for TOPSIS Analysis

	Service Performance (SP)	**Product Quality (PQ)**	**Food Safety (FS)**	**Store Image (SI)**	**Operational Safety (OS)**
Weights	0.352683329	0.175298751	0.323413994	0.078024057	0.070579869
DMU	**Service Performance (SP)**	**Product Quality (PQ)**	**Food Safety (FS)**	**Store Image (SI)**	**Operational Safety (OS)**
Store 1	0.94409762	4	4	2	2
Store 2	0.90297909	2	3	1	1
Store 3	0.77510395	5	4	3	1
Store 4	0.89024831	2	2	5	3
Store 5	0.874322603	2	3	4	4
Store 6	0.796505673	3	2	2	3
Store 7	0.991819373	4	2	5	5
Store 8	0.773515568	3	2	1	2
Store 9	0.869382228	5	2	3	2
Store 10	0.739424292	2	4	2	1
Store 11	0.920783949	1	2	5	2
Store 12	0.889297738	4	5	5	1
Store 13	0.919067457	4	5	4	4
Store 14	0.944137614	4	2	2	3
Store 15	0.765020033	4	5	1	3
Store 16	0.81664117	5	3	2	5
Store 17	0.730699164	2	2	2	3
Store 18	0.61358567	5	5	4	3
Store 19	0.823149083	5	5	2	4
Store 20	0.63131626	5	5	3	4

The closeness coefficient (CC_i) is calculated for each DMU by employing Eq (15) through Eq (21). The results are presented in Figure 8.

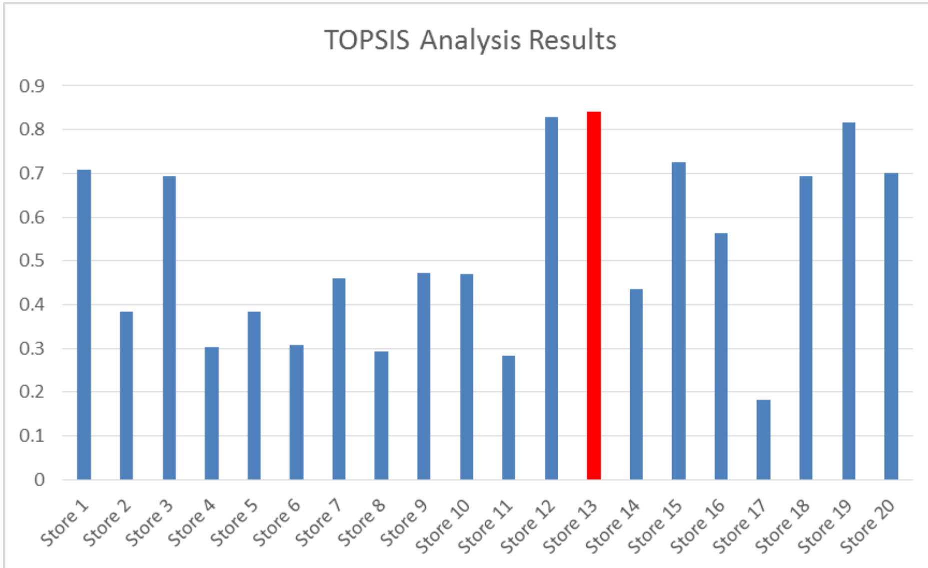

Figure 8. The Overall Evaluation Scores of Each Store

As seen in Figure 8, store 13 obtained the highest rank in the overall evaluation. The findings have been communicated with the head of the franchise operation. The administration validated the results of the proposed method. Since the findings were in line with the problems the management were also primarily concerned about, the franchise management decided to use the methodology in their periodic store audits.

In order to compare the aggressive and benevolent formulations results, overall evaluation was run using the aggressive formulation. The comparison of the results is provided in Figure 9.

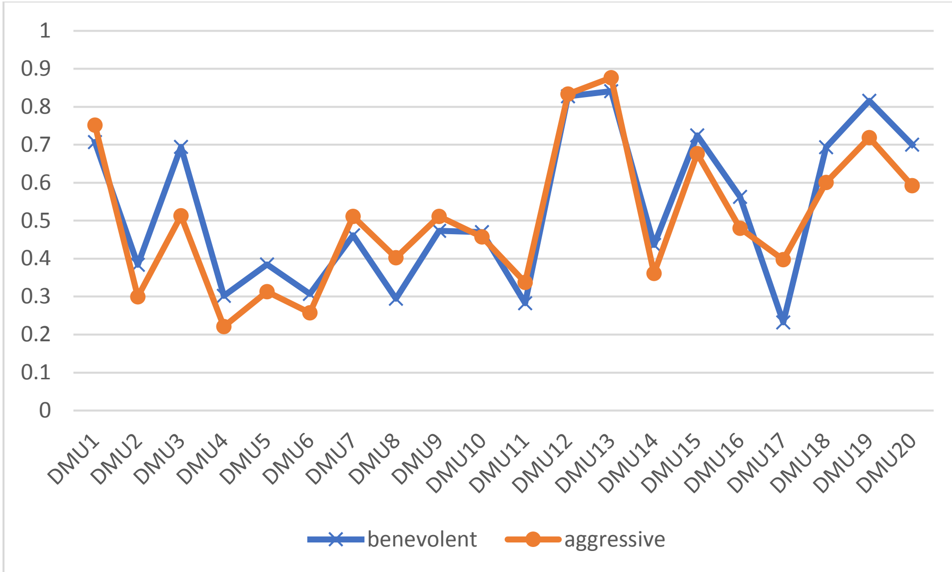

Figure 9. The Comparison of Benevolent and Aggressive Approaches in the Overall Evaluation

As it can be seen from Figure 9, the difference between the two approaches is minimal. The major difference is caused by DMU 17 since the benevolent approach created a more significant difference in the efficiency values compared to the aggressive approach.

4.1.5. Sensitivity Analysis

A sensitivity analysis is conducted to obtain the store ranking under different criteria weights. Sensitivity analysis requires exchanging the weight of each criterion produced by fuzzy AHP technique with another criterion weight as [174-178]. For instance, for scenario C_{12}, the weight of the first criterion C_1 is exchanged with the second criterion C_2 while the weights of other criteria remained constant. Following this, TOPSIS analysis is operated to obtain the new results via ten different calculations. The results are provided in Table 11.

Table 11. Sensitivity Analysis Results

DMU	C_0	C_{12}	C_{13}	C_{14}	C_{15}	C_{23}	C_{24}	C_{25}	C_{34}	C_{35}	C_{45}
DMU1	0.651	0.675	0.648	0.421	0.413	0.686	0.547	0.538	0.419	0.409	0.651
DMU2	0.373	0.285	0.360	0.172	0.166	0.337	0.333	0.327	0.231	0.224	0.372
DMU3	0.622	0.743	0.629	0.571	0.359	0.714	0.552	0.443	0.543	0.323	0.615
DMU4	0.347	0.281	0.323	0.591	0.401	0.373	0.470	0.387	0.719	0.501	0.341
DMU5	0.433	0.340	0.420	0.581	0.589	0.388	0.526	0.534	0.642	0.649	0.434
DMU6	0.298	0.366	0.279	0.257	0.400	0.433	0.253	0.314	0.328	0.482	0.301
DMU7	0.477	0.528	0.448	0.632	0.642	0.661	0.519	0.526	0.853	0.865	0.477
DMU8	0.266	0.351	0.250	0.155	0.244	0.406	0.183	0.216	0.212	0.302	0.267
DMU9	0.434	0.573	0.411	0.449	0.338	0.672	0.366	0.315	0.564	0.426	0.433
DMU10	0.460	0.391	0.478	0.362	0.275	0.318	0.449	0.402	0.265	0.154	0.457
DMU11	0.327	0.203	0.303	0.568	0.243	0.301	0.465	0.342	0.668	0.341	0.319
DMU12	0.753	0.753	0.764	0.838	0.409	0.714	0.782	0.574	0.800	0.334	0.738
DMU13	0.842	0.802	0.850	0.789	0.787	0.777	0.835	0.833	0.759	0.758	0.842
DMU14	0.414	0.495	0.386	0.301	0.426	0.608	0.344	0.385	0.416	0.551	0.416
DMU15	0.675	0.724	0.696	0.409	0.600	0.646	0.542	0.631	0.292	0.511	0.682
DMU16	0.539	0.668	0.525	0.401	0.721	0.721	0.419	0.566	0.440	0.809	0.545
DMU17	0.203	0.214	0.191	0.221	0.378	0.255	0.206	0.282	0.265	0.434	0.208
DMU18	0.669	0.832	0.698	0.790	0.636	0.695	0.654	0.612	0.652	0.527	0.667
DMU19	0.769	0.848	0.784	0.517	0.777	0.789	0.632	0.745	0.443	0.723	0.777
DMU20	0.678	0.835	0.706	0.642	0.788	0.703	0.623	0.662	0.534	0.662	0.680

Individual comparisons of each scenario with the initial results (C_0) are depicted in Figure 10. For instance, Figure 10 (a) represents the comparison of the initial result (C_0) with the scenario where the first criterion is exchanged with the second criterion (C_{12}).

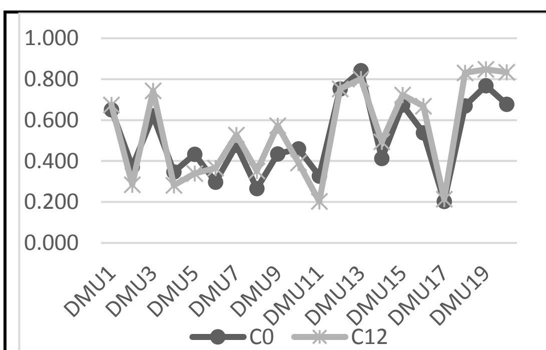

Fig. 10(a)

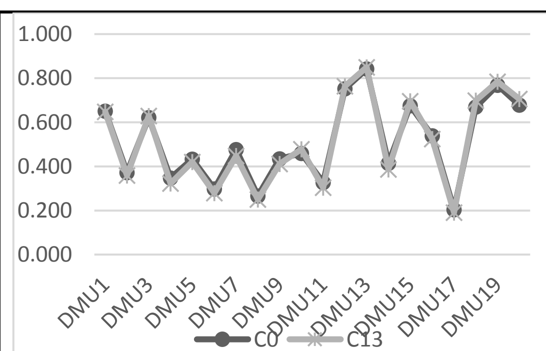

Fig.10(b)

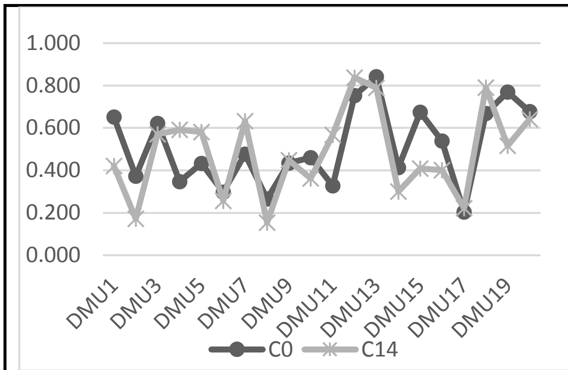

Fig.10(c)

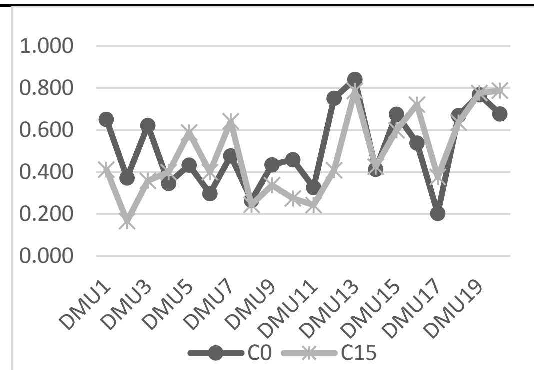

Fig.10(d)

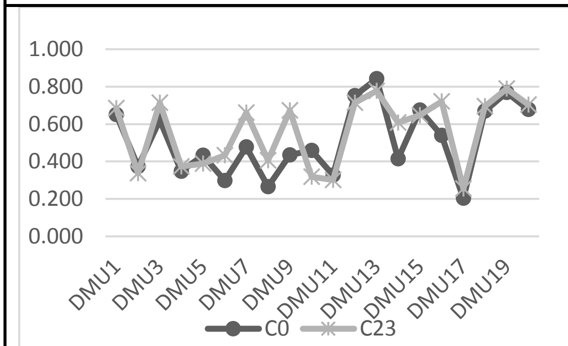

Fig.10(e)

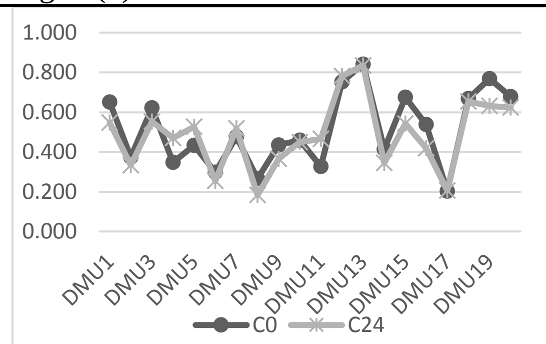

Fig.10(f)

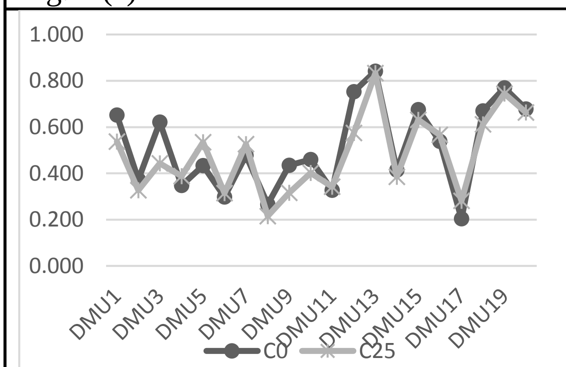

Fig.10(g)

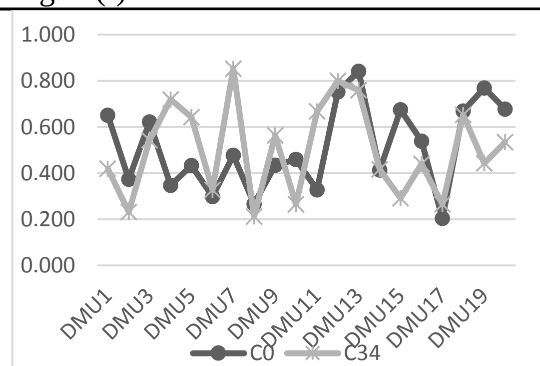

Fig.10(h)

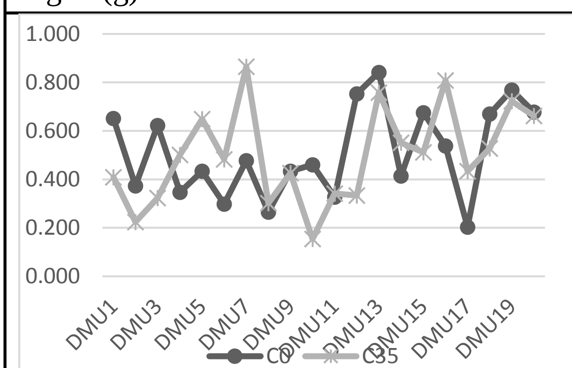

Fig.10(i)

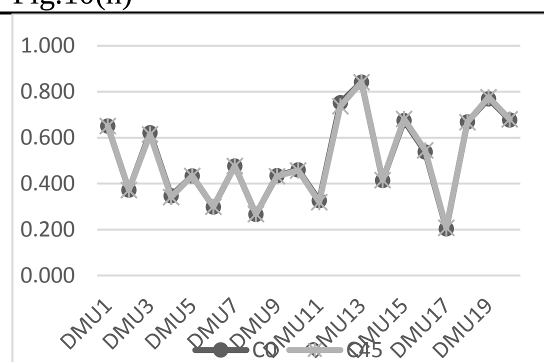

Fig.10(j)

Figure 10. Comparisons of initial results with each scenario

As it can be observed from both Table 11 and Figure 10, Store 13 holds the first rank in scenarios C_0, C_{13}, C_{25} and C_{45}. Sensitivity analysis shows that the ranking among the alternatives is sensitive to the changes in the weights of the evaluation criteria. Due to the fact that each store has different evaluation grade for each evaluation criterion, the rankings are subject to change under varying weights of the evaluation criteria. That is, for instance, Store 13 out performs all the other stores when Service Performance criterion has the highest weight, whereas Store 20 obtains the highest when Operational Safety criterion has the highest weight.

4.2. Implementation of the Group Decision Making via Fuzzy DEMATEL based ANP and ANN

This section details the findings from the case study conducted in a U.S.-based fast food restaurant company with seven franchise retail stores located in the northeast region via utilizing GDM, Fuzzy DANP and ANN. These stores are periodically and randomly audited and their business processes and performance metrics are benchmarked against each other at least four times a year. The key evaluation criteria are divided into two groups as qualitative and quantitative measures. However, these stores operate in various designated areas with different demographics and population sizes which directly influence their overall performances. The company management aims to create a more reliable performance evaluation system that would take these influential factors into account. To this end, the proposed evaluation system takes the relative weight of each factor into account reflecting their true impact on the performance criteria while also considering the relationship among them. Definitions of each utilized criterion as obtained from the company is provided in Table 12.

Table 12. List of performance criteria utilized in the case study

Dimensions	Criteria		Definition
Influencing factors (D_1)	C_{11}	Store territory	Size of the territory designated for each store in sq/mile.
	C_{12}	Population density	Population density in the designated territory for each store (population/sq. mile).
	C_{13}	Weekly expenses	Total weekly expenses in each store.
	C_{14}	Total hours worked by in-store personnel	Total weekly hours worked by in-store personnel in each store.
	C_{15}	Total hours by delivery personnel	Total weekly hours worked by delivery personnel in each store.
Quantitative Criteria (D_2)	C_{21}	Increase in sales	The ratio of the increase in weekly sales in each store.
	C_{22}	Increase rate in the number of carry-out orders	The ratio of the increase in weekly carry-out orders in each store.
	C_{23}	Increase rate in total number of delivery orders	The ratio of the increase in weekly delivery orders in each store.
	C_{24}	Resource utilization ratio	The ratio of the utilization of in-store personnel, delivery personnel, materials and other resources in the store.
	C_{25}	On-time delivery ratio	The ratio of the amount of orders delivered no later than the estimated time.
	C_{26}	Out to door time ratio	The ratio of the amount of orders completed in the store no later than the estimated time.
Qualitative Criteria (D_3)	C_{31}	Store Image	A measure related to the overall image from a customer view in each store.
	C_{32}	Service quality	A measure related to quality of the service quality in each store, the auditors evaluate the entire service starting from the proper greeting to farewell.
	C_{33}	Product quality	A measure related to quality of the products produced in each store, the auditors evaluate the properties of the product and the process such as proper make and portion.

	C_{34}	Food safety	A measure related to product safety factors such as hygiene, temperature control, proper sanitation etc.
	C_{35}	Operational safety	A measure related to indoor and outdoor operational safety factors in each store such as, traffic violations, proper cash drop in the store, secure cashier till and safe.

4.2.1. Obtaining Criteria Weights

In order to obtain the weights of the evaluating criteria, three decision makers, namely, the franchisee (DM_1), the supervisor (DM_2) and the store manager (DM_3), from the company were consulted. The DMs were asked to determine the degrees of influence of the relationships among the criteria and the assessment conducted based on the linguistic judgements. The assessment scale and the corresponding fuzzy numbers are provided in Table 13.

Table 13. The linguistic scale for the assessments

Linguistic terms	**Fuzzy Numbers**
No influence (N)	(0.00, 0.00, 0.25)
Low influence (L)	(0.00, 0.25, 0.50)
Medium influence (M)	(0.25, 0.50, 0.75)
High influence (H)	(0.50, 0.75, 1.00)
Very high influence (V)	(0.75, 1.00, 1.00)

The direct influence diagram among the criteria obtained from DM_1 is partially shown in Figure 11.

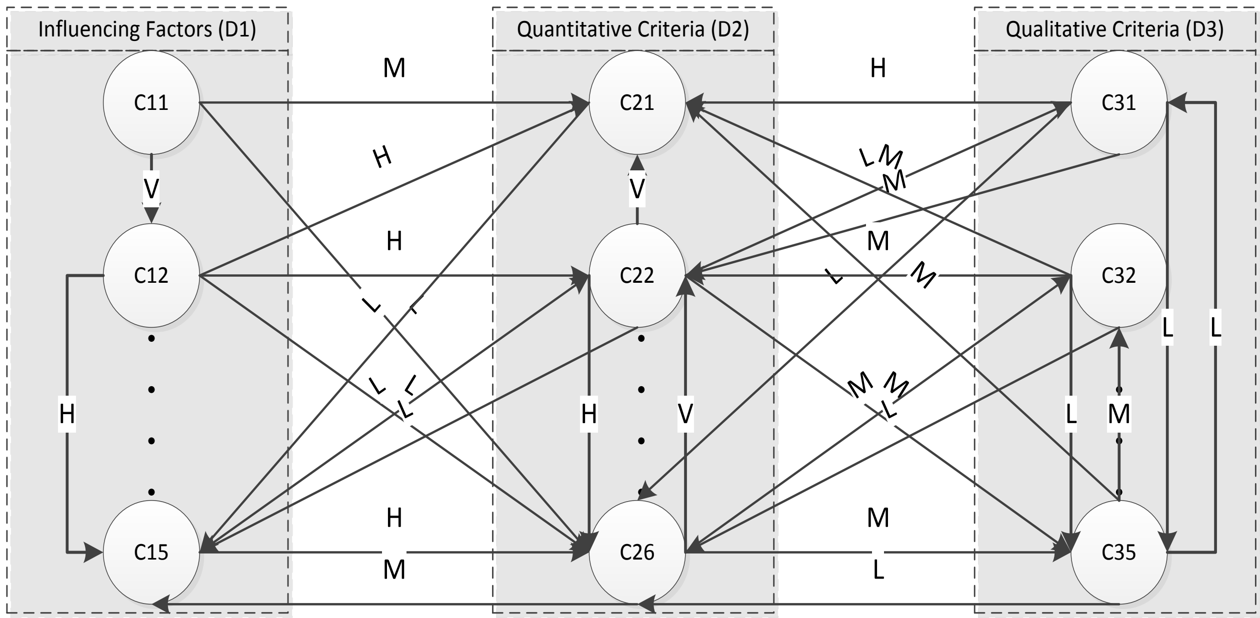

Figure 11. Partial representation of DM_1's assessments

The direct influence matrix obtained from all the DMs are provided in Tables 14, 15 and 16, respectively.

Table 14. Assessments of DM_1

	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}
C_{11}	N	V	N	M	N	M	N	N	V	H	L	N	N	N	N	N
C_{12}	N	N	N	H	H	H	H	M	V	H	L	N	N	N	N	N
C_{13}	N	N	N	N	N	L	N	N	N	N	N	L	M	M	L	L
C_{14}	N	N	H	N	N	N	N	M	V	V	H	N	N	N	M	L
C_{15}	N	N	V	N	N	N	L	L	L	H	H	L	N	L	N	M
C_{21}	N	N	V	L	L	N	N	N	V	V	N	N	N	N	N	N
C_{22}	N	N	N	N	L	V	N	N	N	N	H	L	N	N	N	M
C_{23}	N	N	N	L	N	V	N	N	V	V	H	L	N	N	M	N
C_{24}	N	N	N	L	L	N	N	V	N	V	H	V	N	N	M	M
C_{25}	N	N	N	L	L	L	N	V	V	N	H	V	N	N	M	M
C_{26}	N	N	M	M	M	N	V	V	V	V	N	N	M	M	M	M
C_{31}	N	N	N	N	N	H	M	M	N	N	L	N	M	L	M	L
C_{32}	N	N	N	N	N	M	M	N	N	N	L	H	N	V	L	L
C_{33}	N	N	N	N	N	L	M	N	N	N	L	H	M	N	L	N
C_{34}	N	N	N	N	N	L	L	N	N	N	L	N	M	L	N	L
C_{35}	N	N	N	N	N	M	H	H	L	N	L	L	M	M	L	N

Table 15. Assessments of DM_2

	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}
C_{11}	N	H	L	M	H	M	L	L	H	V	M	L	N	N	N	L
C_{12}	N	N	L	M	M	V	H	H	H	H	M	N	N	N	N	L
C_{13}	N	N	N	L	L	L	L	L	L	N	N	M	M	M	N	N
C_{14}	N	N	H	N	N	L	L	M	M	H	H	L	M	L	H	N
C_{15}	N	N	M	N	N	L	N	M	M	V	V	M	L	L	L	H
C_{21}	N	N	L	H	H	N	N	N	M	H	M	L	L	N	N	N
C_{22}	N	N	L	L	L	V	N	N	M	N	H	M	M	N	N	M
C_{23}	N	N	L	L	H	V	N	N	M	V	V	M	L	N	N	L
C_{24}	N	N	L	M	M	N	L	H	N	V	H	H	L	L	L	M
C_{25}	N	N	L	L	M	M	N	V	H	N	V	H	L	N	L	L
C_{26}	N	N	L	M	H	L	H	V	H	V	N	L	M	L	L	M
C_{31}	N	N	N	N	N	H	H	H	M	L	L	N	M	M	M	M
C_{32}	N	N	L	L	N	H	H	L	N	N	L	H	N	H	L	L
C_{33}	N	N	N	L	N	M	H	L	N	N	N	H	M	N	M	N
C_{34}	N	N	N	N	N	N	L	L	N	N	N	L	M	V	N	M
C_{35}	N	N	N	N	N	L	M	M	L	N	L	M	M	M	M	N

Table 16. Assessments of DM_3

	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}
C_{11}	N	V	M	L	V	H	M	M	M	V	H	N	N	N	N	M
C_{12}	N	N	L	H	H	H	H	H	M	H	H	L	L	N	N	L
C_{13}	N	N	N	M	M	L	N	N	N	N	N	M	H	M	L	N
C_{14}	N	N	M	N	N	M	M	M	M	H	V	M	H	L	M	N
C_{15}	N	N	M	N	N	N	N	H	L	V	H	L	M	N	N	H
C_{21}	N	N	M	M	M	N	M	M	H	H	H	M	M	L	N	N
C_{22}	N	N	M	M	L	V	N	N	H	N	M	H	H	L	N	M
C_{23}	N	N	M	L	V	V	N	N	H	V	V	L	M	L	L	L
C_{24}	N	N	N	L	L	N	L	L	N	H	H	M	M	L	L	M
C_{25}	N	N	M	M	H	H	N	H	H	N	V	V	H	L	N	L
C_{26}	N	N	M	M	H	M	N	V	H	V	N	H	H	L	L	M
C_{31}	N	N	N	N	N	M	H	M	H	M	M	N	M	M	H	M
C_{32}	N	N	N	N	N	V	H	M	L	M	L	H	N	M	M	L
C_{33}	N	N	N	N	N	M	V	V	L	N	N	H	M	N	V	N
C_{34}	N	N	N	N	N	L	L	N	N	N	N	H	M	V	N	H
C_{35}	N	N	N	N	N	L	L	L	N	L	L	M	M	M	V	N

Linguistic data are transformed to TFNs using the relevant scale provided in Table 13. For each criterion given in Tables 14, 15 and 16 DMs' judgments are aggregated using the CFCS method. Based on the role of each DM in the company, different importance weights are used in the overall assessment process which are 0.5, 0.3 and 0.2, respectively. The initial crisp direct relation matrix for the criteria is obtained using Equations from (22) to (30). Table 17 presents the initial crisp direct relation matrix for all criteria.

Table 17. The initial direct relation matrix

Criteria	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}
C_{11}	0.00	0.80	0.15	0.40	0.40	0.50	0.15	0.15	0.78	0.83	0.38	0.06	0.00	0.00	0.00	0.15
C_{12}	0.00	0.00	0.10	0.62	0.62	0.78	0.70	0.57	0.78	0.70	0.38	0.04	0.04	0.00	0.00	0.11
C_{13}	0.00	0.00	0.00	0.15	0.15	0.21	0.06	0.06	0.06	0.00	0.00	0.33	0.50	0.45	0.15	0.10
C_{14}	0.00	0.00	0.65	0.00	0.00	0.15	0.15	0.45	0.71	0.83	0.75	0.15	0.27	0.11	0.52	0.10
C_{15}	0.00	0.00	0.71	0.00	0.00	0.06	0.11	0.38	0.28	0.83	0.78	0.28	0.15	0.17	0.06	0.57
C_{21}	0.00	0.00	0.64	0.40	0.40	0.00	0.09	0.09	0.76	0.83	0.27	0.15	0.15	0.04	0.00	0.00
C_{22}	0.00	0.00	0.15	0.15	0.21	0.97	0.00	0.00	0.27	0.00	0.65	0.38	0.27	0.04	0.00	0.45
C_{23}	0.00	0.00	0.15	0.21	0.40	0.97	0.00	0.00	0.76	0.97	0.83	0.28	0.15	0.04	0.27	0.11
C_{24}	0.00	0.00	0.06	0.28	0.28	0.00	0.11	0.74	0.00	0.91	0.70	0.78	0.15	0.11	0.33	0.45
C_{25}	0.00	0.00	0.15	0.26	0.38	0.38	0.00	0.91	0.83	0.00	0.83	0.89	0.20	0.04	0.29	0.33
C_{26}	0.00	0.00	0.38	0.45	0.57	0.15	0.69	0.97	0.83	0.97	0.00	0.20	0.50	0.33	0.33	0.45
C_{31}	0.00	0.00	0.00	0.00	0.00	0.65	0.57	0.52	0.27	0.15	0.26	0.00	0.45	0.33	0.50	0.33
C_{32}	0.00	0.00	0.06	0.06	0.00	0.63	0.57	0.15	0.04	0.09	0.21	0.70	0.00	0.78	0.26	0.21
C_{33}	0.00	0.00	0.00	0.06	0.00	0.33	0.63	0.26	0.04	0.00	0.11	0.70	0.45	0.00	0.43	0.00
C_{34}	0.00	0.00	0.00	0.00	0.00	0.15	0.21	0.06	0.00	0.00	0.11	0.20	0.45	0.59	0.00	0.38
C_{35}	0.00	0.00	0.00	0.00	0.00	0.33	0.52	0.52	0.17	0.04	0.21	0.33	0.45	0.45	0.43	0.00

Following this, the total relation matrix is obtained by utilizing Equations (31) through (33). The results are provided in Table 18.

Table 18. The total relation matrix

Criteria	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}
C_{11}	0.00	0.11	0.08	0.11	0.12	0.16	0.09	0.14	0.23	0.25	0.18	0.11	0.07	0.05	0.06	0.09
C_{12}	0.00	0.00	0.09	0.14	0.15	0.21	0.16	0.20	0.24	0.24	0.19	0.12	0.08	0.05	0.07	0.09
C_{13}	0.00	0.00	0.02	0.03	0.04	0.07	0.04	0.04	0.04	0.03	0.04	0.09	0.10	0.09	0.05	0.04
C_{14}	0.00	0.00	0.13	0.05	0.06	0.11	0.08	0.17	0.20	0.22	0.21	0.12	0.11	0.07	0.13	0.08
C_{15}	0.00	0.00	0.14	0.04	0.05	0.09	0.08	0.15	0.13	0.21	0.20	0.13	0.09	0.08	0.07	0.14
C_{21}	0.00	0.00	0.13	0.09	0.10	0.06	0.06	0.10	0.18	0.20	0.13	0.11	0.08	0.05	0.05	0.06
C_{22}	0.00	0.00	0.06	0.06	0.07	0.19	0.05	0.07	0.11	0.09	0.16	0.12	0.09	0.05	0.05	0.10
C_{23}	0.00	0.00	0.09	0.08	0.12	0.22	0.07	0.13	0.23	0.27	0.23	0.15	0.10	0.07	0.11	0.09
C_{24}	0.00	0.00	0.06	0.08	0.09	0.10	0.09	0.22	0.11	0.24	0.21	0.21	0.10	0.07	0.12	0.13
C_{25}	0.00	0.00	0.08	0.09	0.12	0.16	0.08	0.25	0.23	0.14	0.24	0.23	0.11	0.07	0.12	0.12
C_{26}	0.00	0.00	0.12	0.12	0.15	0.15	0.17	0.27	0.25	0.28	0.16	0.17	0.16	0.12	0.13	0.15
C_{31}	0.00	0.00	0.04	0.03	0.04	0.16	0.13	0.14	0.11	0.10	0.11	0.07	0.11	0.09	0.11	0.09
C_{32}	0.00	0.00	0.04	0.04	0.03	0.15	0.13	0.08	0.07	0.07	0.09	0.16	0.05	0.14	0.08	0.07
C_{33}	0.00	0.00	0.02	0.03	0.02	0.11	0.13	0.08	0.06	0.05	0.07	0.14	0.10	0.04	0.09	0.04
C_{34}	0.00	0.00	0.01	0.01	0.01	0.06	0.06	0.04	0.03	0.03	0.05	0.07	0.09	0.11	0.03	0.07
C_{35}	0.00	0.00	0.03	0.03	0.03	0.11	0.12	0.13	0.08	0.07	0.09	0.11	0.11	0.10	0.10	0.04

Furthermore, the total influences given and received on the criteria and dimensions can be calculated using Equations (34) and (35) as shown in Table 19.

Table 19. The sum of influences provided and received on the criteria and dimensions

Dimensions/ Criteria		di	rj	di+rj	di-rj
D_1	Influencing Factors	0.29	0.13	0.42	0.16
C_{11}	Store territory	0.43	0.00	0.43	0.43
C_{12}	Population Density	0.38	0.11	0.49	0.27
C_{13}	Weekly expenses	0.09	0.46	0.55	-0.37
C_{14}	Total hours worked by in-store personnel	0.24	0.37	0.61	-0.14
C_{15}	Total hours worked by delivery personnel	0.23	0.42	0.65	-0.19
D_2	Quantitative Criteria	0.33	0.40	0.73	-0.07
C_{21}	Increase in sales	0.74	0.88	1.62	-0.14
C_{22}	Increase rate in total number of carry-out orders	0.67	0.52	1.19	0.16
C_{23}	Increase rate in total number of delivery orders	1.14	1.03	2.17	0.10
C_{24}	Resource utilization ratio	0.96	1.12	2.08	-0.16
C_{25}	On-time delivery ratio	1.10	1.22	2.31	-0.12
C_{26}	Out to door time ratio	1.28	1.12	2.40	0.15
D_3	Qualitative Criteria	0.20	0.28	0.48	-0.09
C_{31}	Store Image	0.48	0.55	1.04	-0.07
C_{32}	Service Quality	0.50	0.47	0.97	0.04
C_{33}	Product Quality	0.41	0.48	0.89	-0.07
C_{34}	Food safety	0.36	0.41	0.77	-0.04
C_{35}	Operational safety	0.45	0.31	0.76	0.15

Thus, the influence diagram, a.k.a. the network relation map (NRM) from the DEMATEL method can be obtained as illustrated in Figure 12.

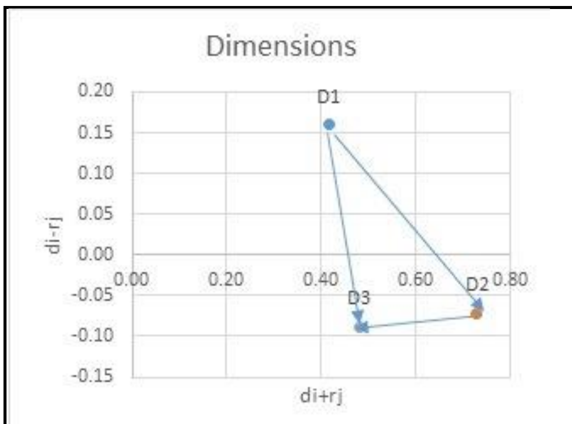

Fig.12(a)

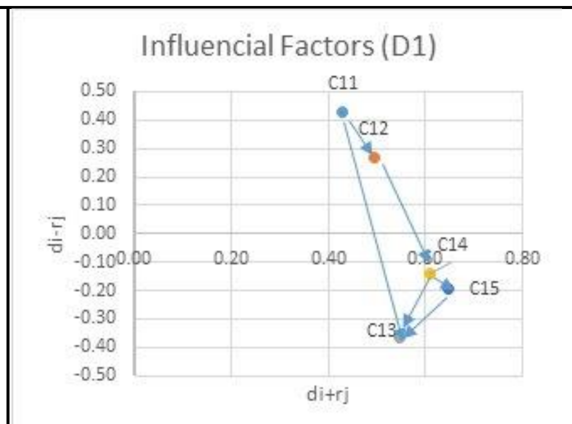

Fig.12(b)

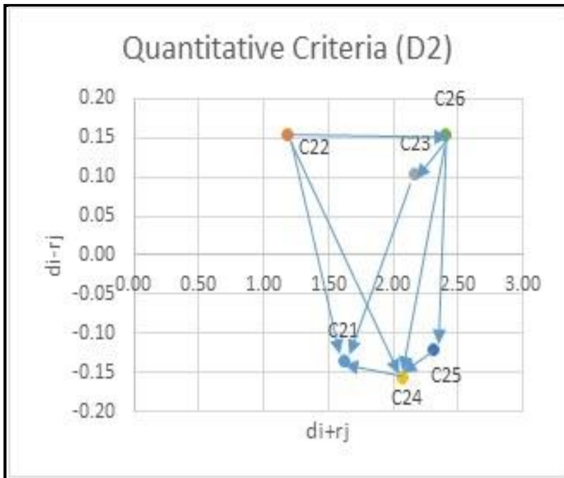

Fig.12(c)

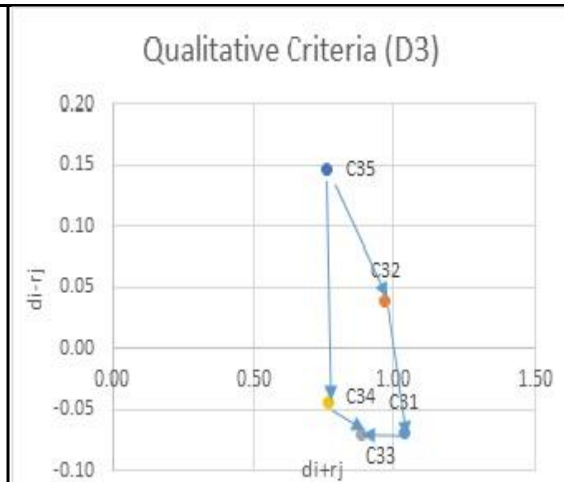

Fig.12(d)

Figure 12. The network relation map

Based on the results of the total relation matrix provided in Table 18, the ANP method is utilized to obtain the influential weights of each criterion. Initially, the influence of the relationship between the criteria was compared based on the NRM. The normalized super-matrix T_C^{nor} is formed by using Equations from (36) to (39). An unweighted super-matrix U_C can be obtained by transposing the normalized matrix as shown in Equation (41). Although the weights of the dimensions (clusters) can be obtained from additional pairwise comparisons amongst dimensions, this information can be extracted from the improved DANP method without using additional surveys as shown in Equation (40) [153]. Hence, the weighted super-matrix W can be obtained by using Equation (42). Finally, the influential weights can be obtained by limiting the power of the weighted super-matrix until it converges and reaches a steady state, $\lim_{s \to \infty} (W)^s$. The limited super-matrix is provided in Table 20.

Table 20. The limit super-matrix

	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}
C_{11}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C_{12}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C_{13}	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
C_{14}	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
C_{15}	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
C_{21}	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
C_{22}	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
C_{23}	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
C_{24}	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
C_{25}	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
C_{26}	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
C_{31}	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
C_{32}	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
C_{33}	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
C_{34}	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
C_{35}	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06

The DANP method derives the global weights of the criteria which are represented as the rows in Table 20. This is done to understand the absolute weight of each criterion in the overall perspectives and also local weights of the criteria and dimensions at their respective hierarchical levels. The global and local weights along with the rankings of the criteria and the dimensions are provided in Table 21.

Table 21. The weights of the dimensions and the criteria

Dimension	Local Weight	Ranking	Criteria	Local Weight	Global Weight	Ranking
D_1	0.13	3	C_{11}	0.0000	0.0000	15
			C_{12}	0.0000	0.0000	16
			C_{13}	0.3429	0.0457	13
			C_{14}	0.3000	0.0400	14
			C_{15}	0.3570	0.0475	12
D_2	0.53	1	C_{21}	0.1737	0.0924	3
			C_{22}	0.1296	0.0689	7
			C_{23}	0.1709	0.0909	4
			C_{24}	0.1673	0.0890	6
			C_{25}	0.1786	0.0950	2
			C_{26}	0.1799	0.0957	1
D_3	0.33	2	C_{31}	0.2711	0.0908	5
			C_{32}	0.2057	0.0689	8
			C_{33}	0.1726	0.0578	11
			C_{34}	0.1774	0.0594	9
			C_{35}	0.1732	0.0580	10

As it can be seen in Figure 12 and Table 21, the quantitative criteria have the highest ranks followed by qualitative criteria. Both are highly affected by the influential factors. These qualitative and quantitative measures and their corresponding weights are utilized to obtain the final ranking in the following section.

4.2.2. Ranking via ANN

In order to obtain the store rankings, historical data from 2011 to 2017 for seven stores have been retrieved from the company's store management system. This historical data contains the numerical values for the evaluation criteria and the final performance score provided by the external auditor provided annually for each period. Furthermore, these evaluation criteria values are weighted with the corresponding value obtained from the consensus via DANP. A final weighted performance score is then computed. For simplicity, partial representation for the data set is provided in Table 22.

Table 22. The partial representation of the ANN data set

Store	**Year**	**Period**	$\boldsymbol{C_{21}}$	$\boldsymbol{C_{22}}$	**……….**	$\boldsymbol{C_{34}}$	$\boldsymbol{C_{35}}$	**Auditor's Score**	**Weighted Score**
Store1	2011	1	-0.08	1.86	……….	4	5	0.78	0.687
Store1	2011	2	7.82	5.94	……….	3	4	0.75	0.696
Store1	2011	3	2.17	2.03	……….	3	5	0.82	0.726
Store1	2011	4	-0.02	-3.23	……….	4	4	0.69	0.634
⋮ ⋮	⋮ ⋮	⋮ ⋮	⋮ ⋮	⋮ ⋮	……….	⋮ ⋮	⋮ ⋮ ⋮	⋮ ⋮	⋮ ⋮
Store7	2017	1	-3.46	-0.59	……….	2	4	0.57	0.526
Store7	2017	2	4.18	3.36	……….	5	5	0.89	0.793
Store7	2017	3	7.64	6.18	……….	2	5	0.69	0.621
Store7	2017	4	-3.23	3.36	……….	3	5	0.7	0.637

In the artificial neural network model, the evaluation criteria and both weighted scores and auditors' scores were embedded as inputs and outputs, respectively. The configuration of the neural network is given in Table 23.

Table 23. The neural network configuration

Architecture:	Multi-layer perceptron feedforward backpropagation neural network
Input Neurons:	11
Output Neurons:	2
Hidden Layers:	2
Hidden Neurons	11 and 2
Learning Algorithm:	Levenberg-Marquardt Optimization
Transfer Function:	Hyperbolic Tangent Sigmoid function in hidden and output layers
Learning Rate:	0.5
Normalization:	All input data are normalized via Equation (44)

The model was executed in Matlab 2017b with seven runs based on the values of each store. Hence, a total of seven networks with identical structures were created. Resulting MSE and R^2 values are provided in Table 24.

Table 24. Calculated MSE and R^2

Network	MSE	R^2
1	0.0004	0.967
2	0.0011	0.929
3	0.0008	0.929
4	0.0002	0.988
5	0.0004	0.957
6	0.0008	0.905
7	0.0006	0.928

Normalized ideal value for each criterion was determined as “1”. Running the networks with this ideal value provided the highest predicted performance scores for each store. The simulated results and the final rankings are presented in Table 25.

Table 25. The predicted performance scores and the rankings

DMU	Output 1 Scores and Rank		Output 2 Scores And Rank		Average Scores and Final Rank	
Store1	0.9052	3	0.8346	1	0.8699	2
Store2	0.8988	4	0.7999	3	0.8494	3
Store3	0.9708	1	0.8341	2	0.9025	1
Store4	0.9296	2	0.7637	5	0.8467	4
Store5	0.8251	6	0.6749	7	0.7500	7
Store6	0.8024	7	0.7273	6	0.7649	6
Store7	0.8903	5	0.7783	4	0.8343	5

As it can be seen in Table 25, the rankings vary according to the auditors' perspective (Output 1) and the weighted internal perspective (Output 2). In order to reach a consensus, the averages of these scores are calculated and the final ranking is obtained.

4.3. Comparative Analysis of ANN, ANFIS, LSSVM and ELM

As mentioned in Section 3.3, the purpose of this comparative analysis is to investigate the prediction capability of other artificial intelligence and machine learning methods, viz., ANFIS, LSSVM and ELM via using the same data set. However, predicting the external auditor's score is determined as the main goal of this problem.

Theoretically, there might be many factors affecting the performance score. In practice however, only a few criteria are truly influential on the evaluation process. An unnecessarily large number of input data set not only weakens the clarity of the underlying model, but also increases the computational complexity [26]. Furthermore, the issue of multicollinearity may arise when two or more independent variables in the model are highly correlated [127]. In order to reduce the computation complexity, and negative effect of multi multicollinearity, Pearson Correlation Analysis followed by grey relational analysis are applied via utilizing Eqs(47), (48) and (49), respectively. Rankings of the evaluation criteria obtained from each method are provided in Table 26.

Table 26. The ranking of the evaluation criteria

Criteria	Ranks via Pearson Correlation Analysis	Ranks via Grey Relational Analysis
C_{21}	**2**	**3**
C_{22}	11	6
C_{23}	7	5
C_{24}	6	10
C_{25}	**3**	**2**
C_{26}	**1**	**1**
C_{31}	**4**	**4**
C_{32}	9	8
C_{33}	11	9
C_{34}	8	7
C_{35}	10	11

As it can be observed from Table 26, although their rankings are slightly different, the most influential four criteria (marked in bold) affecting the external auditor's score are determined as C_{21}, C_{25}, C_{26} and C_{31}. The normalized values of these criteria are extracted from the original dataset (Table 22) and then embedded as the inputs and external auditor's score as the output into new ANN, ANFIS, LSSVM and ELM methods. The architectures of each method and obtained results are provided in the following sections.

4.3.1. ANN and ANN Integrated with PSO

After obtaining the most influential criteria, the architecture of the original ANN model (Table 23) is restructured and the new architecture of the ANN model is provided in Table 27.

Table 27. New architecture of the ANN model

Architecture:	Multi-layer perceptron feedforward backpropagation neural network
Input Neurons:	4 (C_{21}, C_{25}, C_{26} and C_{31})
Output Neurons:	1 (Auditor's score)
Hidden Layers:	1
Hidden Neurons	4 and 1
Learning Algorithm:	Levenberg-Marquardt Optimization
Transfer Function:	Hyperbolic Tangent Sigmoid function in hidden and output layers
Learning Rate:	0.5
Normalization:	All input data are normalized via Equation (44)

70% of the data set is randomly allocated for training and the rest is utilized for testing. The mode is executed in Matlab 2017b and RMSE values are computed via Eq(74). The RMSE values for train and test data sets are obtained as 0.23400 and 0.20743, respectively which are unexpectedly high.

Due to high inaccuracy in prediction, Particle Swarm Optimization is introduced to optimize the weights and the biases in the ANN model. According to the new architecture, there are 27 weights and biases in total. The schematic representation of this procedure is provided in Figure 13.

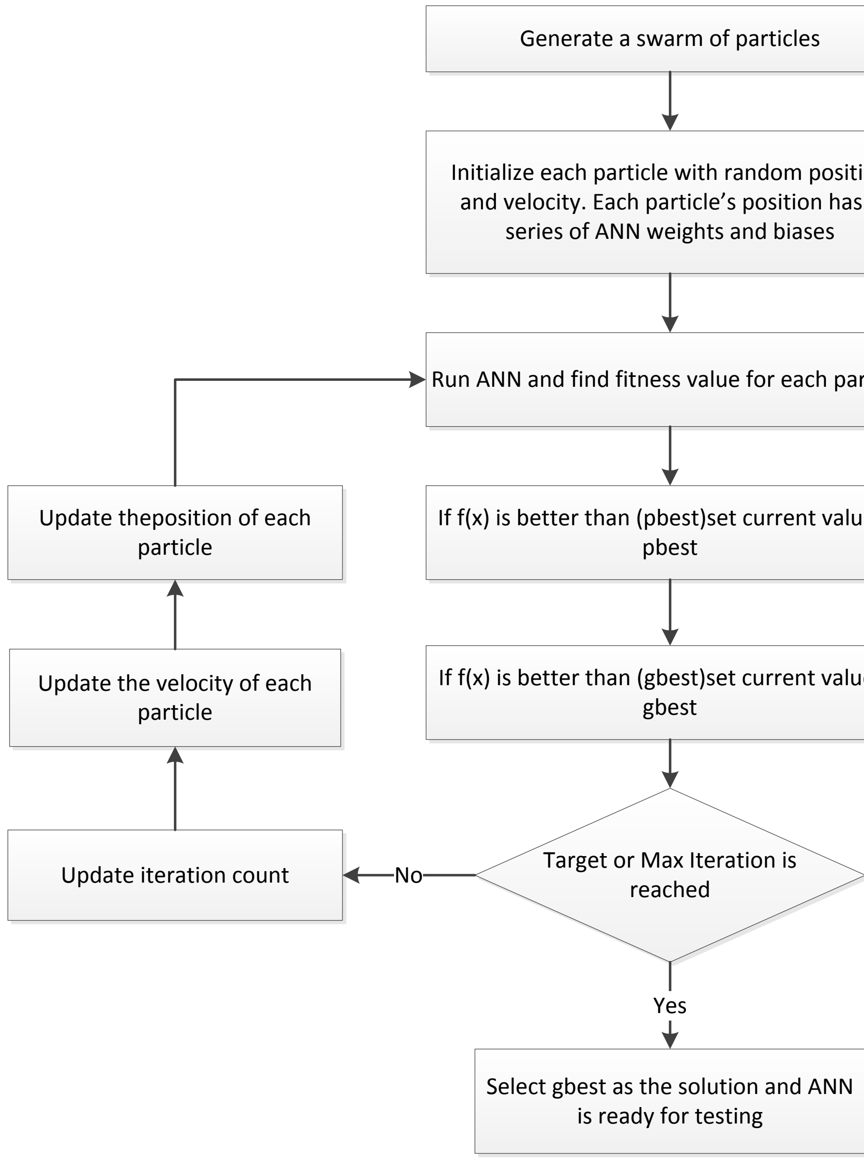

Figure 13. The proposed ANN-PSO algorithm

The ANN-PSO parameters are provided in Table 28.

Table 28. The parameters in ANN-PSO model

Parameters	**Value**
Learning factors c1 and c2	2.5 and 1.5
Inertia weight	0.4<w<0.8
Population Size	50
Max Iteration	300
Upper and Lower Bounds of weights and biases	-2.5<x<2.5

The combined model is executed in Matlab2017b and the RMSEs for training and testing are obtained as 0.061114 and 0.073515.

As it can be observed, PSO integration results in lower RMSE values. Based on these results, the following models are also executed with PSO integration.

4.3.2. ANFIS Integrated with PSO

As explained in section 3.3.2, this study utilizes three different ANFIS models, namely Genfis1, Genfis2 and Genfis3. The membership types and functions and the parameters to be optimized in each model are briefly provided in the following.

- Grid Partitioning (Genfis1): 2 mf and 'gbellmf‛ type, 16 rules and 104 parameters,
- Subtractive Clustering (Genfis2):0.5 influence range and 'gbellmf‛ type, 12 rules and 143 parameters,
- Fuzzy C Means (Genfis3): 10 clusters 2 fuzzy partition matrix and 'gbellmf type ,10 rules and 130 parameters.

The corresponding parameters in each model are optimized via PSO and RMSEs for training and testing are obtained. The results are provided in Table 29.

Table 29. The RMSE values obtained via ANFIS-PSO model

RMSE	Genfis1	Genfis2	Genfis3
RMSE (Train)	0.03189	0.057383	0.049841
RMSE (Test)	0.12395	0.075427	0.065929

4.3.3. LSSVM Integrated with PSO

As mentioned in section 3.3.3, the hyper parameters in the LSSVM methods, namely the regularization parameter C (or gamma) and the RBF kernel parameter σ (sigma), must to be optimized. The schematic representation of the proposed LSSVM-PSO method is provided in Figure 14.

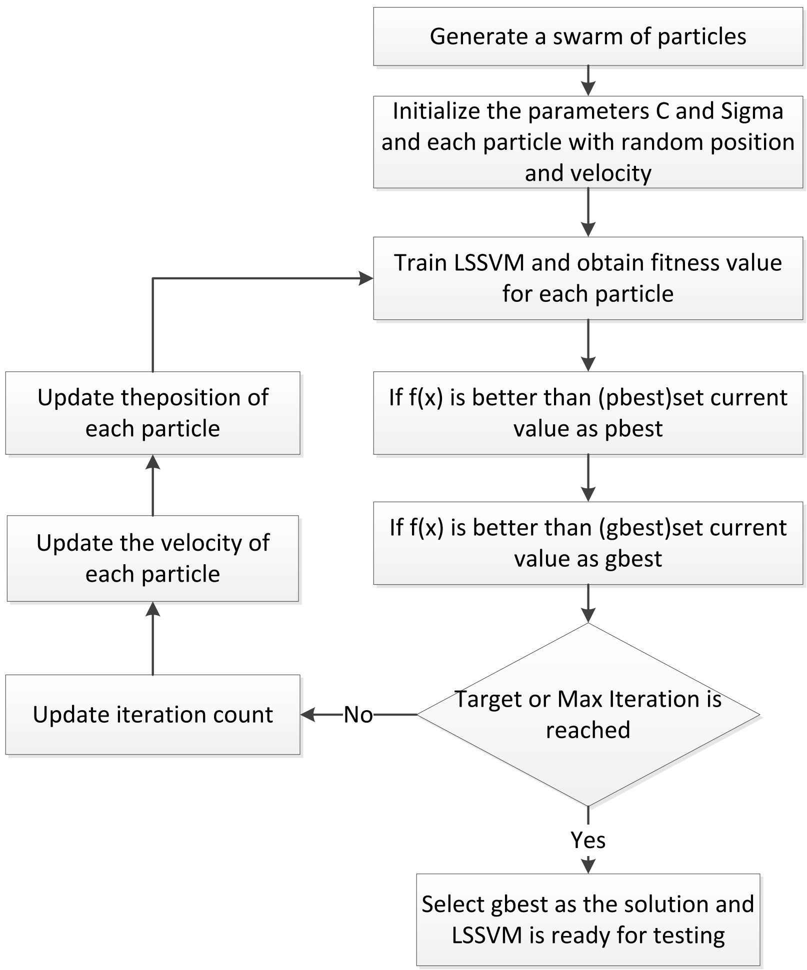

Figure 14. The proposed LSSVM-PSO algorithm

The LSSVM-PSO parameters are provided in Table 30.

Table 30. The parameters in LSSVM-PSO model

PSO Parameters	**Value**
Learning factors c1 and c2	2.5 and 1.5
Inertia weight	0.4<w<0.8
Population Size	50
Max Iteration	300
C or gamma range	100<C<20000
Sigma range	1<Sig<200

The combined model is executed in Matlab2017b and the RMSEs for training and testing are obtained as 0.05849 and 0.06787.

4.3.4. ELM Integrated with PSO

As explained in section 3.3.4, both neural and kernel based ELM models are utilized in this study. The architecture in the ANN-PSO model is also utilized in neural based ELM. The only difference is that the ELM model uses single hidden layer so that there are total of 25 weights and biases to be optimized. The rest of the parameters are preserved as in Tables 27 and 28 for neural-based ELM. For kernel-based ELM, the structure and the parameter range provided in Table 30 are utilized. The resulting RMSE values are provided in Table 31.

Table 31. The RMSE values obtained via ELM-PSO model

RMSE	Neural based ELM	Kernel based ELM
RMSE(Train)	0.099451	0.06120
RMSE (Test)	0.059942	0.07038

4.3.5. Overall Comparison of the Obtained Results

The findings of the comparative analysis are listed below. All RMSE values calculated in the previous sections are also included in Table 32.

Table 32. The calculated RMSE values

RMSE	**ANN**	**Genfis1**	**Genfis2**	**Genfis3**	**LSSVM**	**ELM - Neural**	**ELM-Kernel**
RMSE(Train)	0.061114	0.03189	0.057383	**0.049841**	0.05849	0.099451	0.06120
RMSE (Test)	0.073515	0.12395	0.075427	**0.065929**	0.06787	0.059942	0.07038

- ELM has the shortest overall computation time.
- As literature survey also indicated that there is no consensus on how to determine the superior method in terms of prediction capability. (Case-sensitive; data-dependent).
- The lowest RMSE is obtained via Genfis3-PSO model. Hence, ANN could be replaced with Genfis3 optimized w/PSO model for better prediction accuracy in the original model.

4.4. Comparative Analysis of GM(r,n), GMC(r,n) and DGM(r,n)

As stated in section 3.4, multivariate grey modeling approaches could be applied in presence of limited and uncertain data. Each store (DMU) data, 28 observations in each, could be utilized to predict the performance score via grey models. Therefore, traditional multivariate grey model GM(r,n), grey model with convolutional integral GMC(r,n) and discrete grey model DGM(r,n) are utilized in this part of the study. In order to improve the prediction capability of each model, fractional order accumulation are integrated and background value coefficient is optimized via PSO as detailed in section 3.4. The schematic representation of the procedure is provided in Figure 6.The original grey model utilizes the background value coefficient p as 0.5, and uses first order accumulation (r=1). For comparison purpose,s the RMSE values for each store are computed via both original and optimized models. The PSO parameters utilized in the algorithms and the ranges for *p* and *r* values are demonstrated in Table 33.*Table 33. The parameters in GM-PSO models*

PSO Parameters	Value
Learning factors c1 and c2	2 and 2
Inertia weight	0.4<w<0.8
Population Size	50
Max Iteration	500
p value range	0<p<1
r value range	0<r<2

All models are built and executed in Matlab 2017b and the obtained RMSE and the corresponding *p* and *r* values are provided in Table 34.

Table 34. The RMSE, *p* and *r* values obtained from grey models

Store	**RMSE-GM(1,n) p=0.5**	**RMSE GMFO-PSO**	**p**	**r**
Store 1	0.0524	0.0458	0.471	0.393
Store 2	0.9606	0.0834	0.246	0.427
Store 3	0.1161	0.0677	0.431	0.154
Store 4	0.0909	0.0799	0.507	0.582
Store 5	0.0805	0.0531	0.131	0.002
Store 6	0.0721	0.0699	0.354	0.214
Store 7	0.0875	0.0870	0.503	0.969
Store	**RMSE-DGM(1,n)**	**RMSE-DGMFO-PSO**	**r**	
Store 1	0.0601	0.0488	0.374	
Store 2	0.0718	0.0451	0.257	
Store 3	0.0669	0.0604	0.058	
Store 4	0.0703	0.0633	0.560	
Store 5	0.0666	0.0485	0.142	
Store 6	0.0677	0.0608	0.231	
Store 7	0.0829	0.0744	0.417	
Store	**RMSE-GMC(1,n) p=0.5**	**RMSE-GMCFO-PSO**	**p**	**r**
Store 1	0.9378	0.0916	0.498	0.558
Store 2	0.0979	0.0947	0.535	0.522
Store 3	0.1001	0.0893	0.505	1.261
Store 4	0.1174	0.1058	0.513	0.062
Store 5	0.0961	0.0955	0.504	1.096
Store 6	0.0837	0.0777	0.502	0.494
Store 7	0.5601	0.2516	0.327	0.902

As it can be seen in Table 34, except the first store's (DMU's) RMSE, the lowest RMSE values (shaded in grey) are obtained via fractional order discrete grey model integrated with particle swarm optimization (DGMFO-PSO).Therefore, the first 27 observations from each store and the corresponding values of the associated parameters are embedded into improved grey models with the lowest RMSE value, and the predicted performance score of the 28th observation for each store is obtained. The original values and the predicted values are provided in Table 35.

Table 35. The comparison of actual and predicted performance values

Store	**Original Value**	**Predicted Value**
Store 1	0.55	0.5597
Store 2	0.75	0.7846
Store 3	0.66	0.7200
Store 4	0.73	0.7206
Store 5	0.64	0.6446
Store 6	0.65	0.7460
Store 7	0.70	0.7298

CHAPTER 5: CONCLUSIONS AND DISCUSSION

The current literature focusing on food service industry performance comparisons is solely based on quantitative Key Performance Indicators (KPIs). Aiming at filling this gap, this study measured the performances of franchised food retail stores using both qualitative and quantitative data. Introduction of qualitative measures allows the decision makers to conduct a more comprehensive and accurate performance evaluation. This research also integrated the preferences of decision makers regarding the criteria used allowing a dynamic performance evaluation that is flexible and responsive to the potential changes in the market dynamics. The evaluation process composed of two stages and three multiple criteria decision making methods, namely, Fuzzy AHP, Data Envelopment Analysis (DEA) cross efficiency and TOPSIS. In the first stage, the weights of both qualitative and quantitative KPIs are obtained through the Fuzzy AHP model allowing the imprecision of experts' opinions. Following this, DEA cross efficiency is utilized to evaluate the performances of franchise stores using only quantitative KPIs. The findings are then fed into TOPSIS combining both quantitative and qualitative data for better assessment of the service providers. In the second stage, to be able to account for both types of KPI data, i.e., quantitative and qualitative, TOPSIS method is utilized to rank the stores according to their performances. The findings indicate that, as with many hybrid approaches, the proposed method provided a holistic approach for a more thorough evaluation of the food delivery network. Furthermore, the performance measures obtained from the franchise are used for both self- and peer-review evaluation which is one of the most appealing advantages of cross efficiency approach in DEA.

The quality of performance appraisal and evaluation systems is heavily reliant on their consistency, the appropriateness of information and analysis they employ in addition to their systematic implementations. In this regard, utilization of criteria, both quantitative and qualitative in nature, plays an important role in ensuring the accuracy and reliability of these systems requiring a holistic approach. However, qualitative assessments and the weight of each criterion are usually determined by the preferences of the evaluators which could potentially result in deviations from purported objectivity. Forming a consensus via group decision making (GDM) techniques is one viable way to lessen the impact of subjectivity in the decision making process. The method proposed in this study explicitly consider different perspectives from multiple decision makers (DMs) by varying the weights of each DM to be used in conjunction with hybrid multi-criteria decision making and artificial intelligence methodologies. That is, each decision maker is assigned an individual weight depending on their actual influence on the decision outcome. Moreover, fuzzy values were utilized to integrate the effect of vagueness in the decision making process. A case study in the food industry was also presented to illustrate the applicability of the proposed model.

A combined DEMATEL-ANP (DANP) approach was employed to obtain the weights of the evaluation criteria. This novel approach integrated the influences and inter-relationships among the clusters and the evaluation criteria. The algorithm then considered these as the nodes of a network rather than hierarchical elements. DANP is employed to extract the information regarding the weight of each cluster without requiring an additional step for pairwise comparison. In the second phase, the obtained weights and the corresponding criteria weights were used to compute a series of historical weighted performance values for each food store. Past performance evaluations conducted by an external auditor were also obtained from the store management system. Developed artificial neural network utilized evaluation criteria values as inputs and both internal and external performance scores as outputs. Resulting network was then run for each store to obtain the predicted performance values. This approach has several advantages. First and foremost, since linearity assumption is not required in artificial neural networks (ANNs), the model was able to produce more realistic outcomes. In addition, ANN brought the learning ability by accommodating historical data rather than employing only present-day data which was one of the major reasons why artificial neural networks were used to obtain the final ranking of the stores.

The literature offers a variety of artificial intelligence and machine learning methods. Each method has several advantages and shortcomings in terms of computation speed and prediction capability. Therefore, a comparative analysis could result in providing a better results for performance prediction. With this motivation, ANN, ANFIS, LSSVM and ELM models improved by PSO are compared with each other in the same problem domain. It is observed that the RMSE values obtained from each model slightly differ from one another and as the literature survey indicated that there is no consensus on how to determine the superior method in terms of prediction capability. The success rate in a model is most likely data dependent.

In recent years, Grey Modeling has drawn attention among researchers to run predictive analysis in presence of uncertain and limited data. Since each store's historical performance evaluation data could be considered limited, the original grey model, grey model with convolutional integral and discrete grey model are also utilized. In order to improve the prediction accuracy, fractional order accumulation is introduced into each model and the parameters are optimized by PSO. Although the obtained RMSE values are satisfying, grey modeling would not be an effective method in the long run due to its ability to solely handle limited data.

In other words, the artificial intelligence and machine learning methods which are also considered as the foundations of Big Data Analytics should be the focus of the future research. With this motivation, in the future, the data set will be expanded by including the daily operational data. The predictive analysis will be conducted using artificial intelligence and machine learning methods to gain insight regarding the future performance of the operations.

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