

Discrete LPV Modeling of Diabetes Mellitus for Control Purposes

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Abstract—The utilization of modern and advanced control engineering related methods for the control, estimation and assessment of physiological applications is widespread. It is also well-known that this engineering apparatus is executed on digital computers. The current insufficiency of available and accurate discretized models, especially in case of Diabetes Mellitus (DM), provides incentive for this research. The researchers typically approximate the continuous solutions which may not be the best alternative in many cases, in particular considering numerical stability and cost-effectiveness. In this paper we performed an analysis of the available discretization options in order to develop discrete models with a special focus on the Linear Parameter Varying (LPV) systems. LPV techniques are very useful frameworks which allow the application of linear controller, observer and estimator design. In this study, three LPV discretization and two Jacobian based discretization methods are introduced and analyzed to provide a basis for our further investigations in the topic.

Index Terms—Linear Parameter Varying techniques, Discretization, Discrete LPV Modeling, Diabetes Mellitus

I. INTRODUCTION

Control Engineering (CE) approaches have inevitable role regarding physiological, biological and chemical applications. In the recent years many practical application shown that by involving the latest results of CE into physiological applications better performance and better management of the treatments can be achieved [1]–[7].

On the field of biomedical engineering DM receives an ever growing scientific attention due to the continuously increasing number of people who are affected. By the newest approximations, there are 425 million people worldwide who live with DM in 2018, this estimation involves undiagnosed cases also beside the diagnosed ones [8]. Furthermore, the current predictions imply that this number will reach 629 million, the 6.65% of the expected world population by 2045 [8].

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This paper focuses on Type 1 Diabetes Mellitus (T1DM). This is a disorder of the natural blood glucose level regulatory system. Insulin hormone plays one of the most important part in the glucose homeostasis. T1DM involves an acute autoimmune reaction, where the result is the perishing of insulin producer β -cells [9]. Insulin is the indicator for the body cells, that the blood glucose level is elevated, and it ought to facilitate the glucose molecules by making the cell membrane permeable for them. Patients in such a diabetic condition are in need of external insulin injection, because due to the osmotic pressure cells dehydrate, on the long run energetic collapse can occur. [10], [11]. The control of the blood glucose level of a diabetic patient is crucial, because the uncontrolled disease can lead to several negative effect [8]. Furthermore, the characteristics and quality of the control is also essential [12].

In case of T1DM the tight glycemic control (TGC) is the key of good glycemic status. TGC requires frequent BG measurements via finger pricks (manual measurement) or Continuous Glucose Monitoring System (CGMS) [13]. There are still many challenges regarding CGMS technology, however. For example, the sampling frequency is around 5 minutes on basis, noise and disturbance sensitivity, inaccuracies of available mathematical models, etc. [2], [5], [14]. State estimation for control applications is also an important aspect which is not trivial and the requirements of it determined by the type of control to be applied [15], [16]. In our previous work, we have presented the use of LPV methodology regarding the control of DM [17]–[19].

Both for LPV based state estimation and control the investigation of possible discretization techniques is necessary. Through the scaled and discretized models it is possible to taking into account all requirements coming from the estimators (e.g. Kalman filter), LPV controller and available sensor technology (sampling time, disturbances, etc.).

The paper is structured as follows. First, we introduce the models to be investigated. After, the discretization possibilities regarding the presented models are shown. Finally, our findings are described and we conclude our work.

II. APPLIED MODELS

A. The Minimal Model

We have investigated the well-known T1DM model of Bergman in the form presented by [20]. The model has three state variables: $G(t)$ mg/dL is the blood glucose (BG) concentration, $X(t)$ 1/min insulin-excitabile tissue glucose uptake activity, $I(t)$ U/mL blood insulin concentration. The $w(t)$ mg/dL/min is the disturbance input (the glucose intake) and $u(t)$ U/mL/min is the control input (insulin intake). The output of the system is the BG concentration $G(t)$ which is measurable. The p_1, p_2, p_3 and n , are model parameters. In this study we have applied the following data set: $[p_1, p_2, p_3, n] = [0.028, 0.025, 0.00013, 0.23]$ based on [20].

$$\dot{G}(t) = -(p_1 + X(t))G(t) + p_1 G_B + w(t). \quad (1a)$$

$$\dot{X}(t) = -p_2 X(t) + p_3 [I(t) - I_B]. \quad (1b)$$

$$\dot{I}(t) = -n [I(t) - I_B] + u(t). \quad (1c)$$

B. LPV Modeling in General

In the following the continuous and discrete state-space LPV (LPV-SS) model description – in a general manner – are presented.

Definition 1. Continuous-Time LPV-SS Model [21], [22].

The continuous time LPV-SS model (CT-LPV) can be described by the following differential equations (without considering separated disturbance input):

$$\dot{\mathbf{x}}_c(t) = \mathbf{A}_c(\mathbf{p}_c(t))\mathbf{x}_c(t) + \mathbf{B}_c(\mathbf{p}_c(t))\mathbf{u}_c(t) + \mathbf{E}_c(\mathbf{p}_c(t))\mathbf{d}_c(t), \quad (2a)$$

$$\mathbf{y}_c(t) = \mathbf{C}_c(\mathbf{p}_c(t))\mathbf{x}_c(t) + \mathbf{D}_c(\mathbf{p}_c(t))\mathbf{u}_c(t) + \mathbf{D}_{2,c}(\mathbf{p}_c(t))\mathbf{d}_c(t), \quad (2b)$$

where $\mathbf{A}_c(\mathbf{p}_c(t)) \in \mathbb{R}^{n \times n}$, $\mathbf{B}_c(\mathbf{p}_c(t)) \in \mathbb{R}^{n \times m}$, $\mathbf{E}_c(\mathbf{p}_c(t)) \in \mathbb{R}^{n \times z}$, $\mathbf{C}_c(\mathbf{p}_c(t)) \in \mathbb{R}^{k \times n}$, $\mathbf{D}_c(\mathbf{p}_c(t)) \in \mathbb{R}^{k \times m}$ and $\mathbf{D}_{2,c}(\mathbf{p}_c(t)) \in \mathbb{R}^{k \times z}$ are the $\mathbf{p}_c(t)$ dependent system-, input-, disturbance-, output-, input-coupling- and disturbance-coupling matrices, respectively and $\mathbf{x}_c(t) \in \mathbb{R}^n$ is the state vector. The $\mathbf{u}_c(t) \in \mathbb{R}^m$ is the input vector, while $\mathbf{d}_c(t) \in \mathbb{R}^z$ is the disturbance vector. The $\mathbf{p}_c(t) = [p_{1,c}(t) \dots p_{R,c}(t)]$ parameter vector consists of the so-called scheduling parameters $p_{i,c}(t)$. $\mathbf{p}_c(t) \in \Omega^R \in \mathbb{R}^R$ is an R dimensional real vector within the $\Omega = [p_{1,c,min}, p_{1,c,max}] \times [p_{2,c,min}, p_{2,c,max}] \times \dots \times [p_{R,c,min}, p_{R,c,max}] \in \mathbb{R}^R$ hyperplane inward the \mathbb{R}^R real vector space. If any of the states is selected as scheduling variable, the given LPV model becomes a quasi-LPV (qLPV) model [21], [22].

By assuming an ideal zero-order hold (ZOH) device, $\mathbf{p}_c(t)$, $\mathbf{u}_c(t)$ and $\mathbf{d}_c(t)$ are constant within each sampling interval. In this case the continuous system can be written as:

$$\dot{\mathbf{x}}_c(t) = \mathbf{A}_c(\mathbf{p}_c(kT_d))\mathbf{x}_c(t) + \mathbf{B}_c(\mathbf{p}_c(kT_d))\mathbf{u}_c(kT_d) + \mathbf{E}_c(\mathbf{p}_c(kT_d))\mathbf{d}_c(kT_d), \quad (3a)$$

$$\mathbf{y}_c(t) = \mathbf{C}_c(\mathbf{p}_c(kT_d))\mathbf{x}_c(t) + \mathbf{D}_c(\mathbf{p}_c(kT_d))\mathbf{u}_c(kT_d) + \mathbf{D}_{2,c}(\mathbf{p}_c(kT_d))\mathbf{d}_c(kT_d), \quad (3b)$$

where k is the discrete step and T_d is the sampling time.

Definition 2. Discrete-Time LPV-SS Model [21], [22].

The discrete time LPV-SS model (CT-LPV) can be described by the following difference equations:

$$\mathbf{x}_d(k+1) = \mathbf{A}_d(\mathbf{p}_d(k))\mathbf{x}_d(k) + \mathbf{B}_d(\mathbf{p}_d(k))\mathbf{u}_d(k) + \mathbf{E}_d(\mathbf{p}_d(k))\mathbf{d}_d(k), \quad (4a)$$

$$\mathbf{y}_d(k) = \mathbf{C}_d(\mathbf{p}_d(k))\mathbf{x}_d(k) + \mathbf{D}_d(\mathbf{p}_d(k))\mathbf{u}_d(k) + \mathbf{D}_{2,d}(\mathbf{p}_d(k))\mathbf{d}_d(k), \quad (4b)$$

where the matrices and vectors are the discrete time equivalents of the continuous time counterparts and k is the sampling time instance.

C. Investigated LPV Model

Due to our aim is to provide appropriate discrete time LPV models for our further research, we have developed a qLPV model form of (1a)-(1c).

Because the $G(t)$ variable is the nonlinearity causing term, we have selected it as scheduling parameter, namely, $\mathbf{p}(t) = p(t) = G(t)$ from (1a). This is the most straightforward way to represent the system in qLPV form.

The (1a)-(1c) contain the $p_1 G_B$, $-p_3 I_B$ and $n I_B$ constant terms. These are needed to be handled as "input signals" from the qLPV system point of view. Since these are constant terms, it does not matter whether we represent them in the \mathbf{B} or in the \mathbf{E} matrix. However, our aim is to apply them for controller and estimator design in the future. Hence, it is more suitable if these terms are part of the \mathbf{E} matrix – beside the disturbance input. Thus, the $\mathbf{d}(t) = [1 \ 1 \ 1 \ w(t)]^T$.

By considering the aforementioned conditions and using the LPV principle the following qLPV state-space representation can be written:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(p(t))\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{d}(t), \quad (5a)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) + \mathbf{D}_2\mathbf{d}(t), \quad (5b)$$

$$\mathbf{A}(p(t)) = \begin{bmatrix} -p_1 & -G(t) & 0 \\ 0 & -p_2 & p_3 \\ 0 & 0 & -n \end{bmatrix}, \quad (5c)$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} p_1 G_B & 0 & 0 & 1 \\ 0 & -p_3 I_B & 0 & 0 \\ 0 & 0 & n I_B & 0 \end{bmatrix},$$

$$\mathbf{C} = [1 \ 0 \ 0], \quad \mathbf{D} = [0 \ 0 \ 0],$$

where the constant terms are represented in \mathbf{E} .

III. DISCRETIZATION PROCEDURES

We intended to provide a full picture about the available discretization techniques, how they can be applied regarding the given cases. Hence, we investigated the "classical" methods as well (Jacobian based discretization), not just the LPV discretization opportunities. In this way, the comparison between them became possible.

A. Complete LPV-SS discretization

In this case, the CT-LPV system as it is described by (3) is transformed by using the LTI assumption of complete signal evolution approach sampling with ZOH [21]. The method results an approximating DT-LPV system in the form of (4) with given approximation error. The transformation is step-by-step described by (6a)–(6d) in accordance with [23], [24].

$$\mathbf{A}_d(\mathbf{p}_d(k)) = e^{(\mathbf{A}_c(\mathbf{p}_c(kT_d))T_d)} , \quad (6a)$$

$$\mathbf{A}_c^{-1}(\mathbf{p}_c(kT_d))(e^{(\mathbf{A}_c(\mathbf{p}_c(kT_d))T_d)} - \mathbf{I})\mathbf{B}_c(\mathbf{p}_c(kT_d)) , \quad (6b)$$

$$\mathbf{A}_c^{-1}(\mathbf{p}_c(kT_d))(e^{(\mathbf{A}_c(\mathbf{p}_c(kT_d))T_d)} - \mathbf{I})\mathbf{E}_c(\mathbf{p}_c(kT_d)) , \quad (6c)$$

$$\mathbf{C}_d(\mathbf{p}_d(k)) = \mathbf{C}_c(\mathbf{p}_c(kT_d)) , \quad (6d)$$

$$\mathbf{D}_d(\mathbf{p}_d(k)) = \mathbf{D}_c(\mathbf{p}_c(kT_d)) , \quad (6e)$$

$$\mathbf{D}_{2,d}(\mathbf{p}_d(k)) = \mathbf{D}_{2,c}(\mathbf{p}_c(kT_d)) . \quad (6f)$$

B. Rectangular LPV-SS discretization

The rectangular method can be derived from the first order forward Euler's method [23]–[25].

$$\mathbf{A}_d(\mathbf{p}_d(k)) = \mathbf{I} + T_d\mathbf{A}_c(\mathbf{p}_c(kT_d)) , \quad (7a)$$

$$\mathbf{B}_d(\mathbf{p}_d(k)) = T_d\mathbf{B}_c(\mathbf{p}_c(kT_d)) , \quad (7b)$$

$$\mathbf{E}_d(\mathbf{p}_d(k)) = T_d\mathbf{E}_c(\mathbf{p}_c(kT_d)) , \quad (7c)$$

$$\mathbf{C}_d(\mathbf{p}_d(k)) = \mathbf{C}_c(\mathbf{p}_c(kT_d)) , \quad (7d)$$

$$\mathbf{D}_d(\mathbf{p}_d(k)) = \mathbf{D}_c(\mathbf{p}_c(kT_d)) , \quad (7e)$$

$$\mathbf{D}_{2,d}(\mathbf{p}_d(k)) = \mathbf{D}_{2,c}(\mathbf{p}_c(kT_d)) , \quad (7f)$$

C. Adams-Bashforth LPV-SS discretization

From the family of multi-step methods, the Adams-Bashforth discretization can be implemented due to the necessity of fixed step size. Below, the general form of the three step method given, which results in a 9th order system [23], [24], [26]:

$$\begin{bmatrix} \mathbf{I} + \frac{23T_d}{12}\mathbf{A}_c(\mathbf{p}_c(kT_d)) & -\frac{16T_d}{12}\mathbf{I} & \frac{5T_d}{12}\mathbf{I} \\ \mathbf{A}_c(\mathbf{p}_c(kT_d)) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix} , \quad (8a)$$

$$\begin{bmatrix} \mathbf{B}_d(\mathbf{p}_d(k)) = \\ \left[\frac{23T_d}{12}\mathbf{B}_c^\top(\mathbf{p}_c(kT_d)) \quad \mathbf{B}_c^\top(\mathbf{p}_c(kT_d)) \quad \mathbf{0} \right]^\top , \end{bmatrix} \quad (8b)$$

$$\mathbf{E}_d(\mathbf{p}_d(k)) = \begin{bmatrix} \frac{23T_d}{12}\mathbf{E}_c^\top(\mathbf{p}_c(kT_d)) & \mathbf{E}_c^\top(\mathbf{p}_c(kT_d)) & \mathbf{0} \end{bmatrix}^\top , \quad (8c)$$

$$\mathbf{C}_d(\mathbf{p}_d(k)) = [\mathbf{C}_c(\mathbf{p}_c(kT_d)) \quad \mathbf{0} \quad \mathbf{0}] , \quad (8d)$$

$$\mathbf{D}_d(\mathbf{p}_d(k)) = \mathbf{D}_c(\mathbf{p}_c(kT_d)) , \quad (8e)$$

$$\mathbf{D}_{2,d}(\mathbf{p}_d(k)) = \mathbf{D}_{2,c}(\mathbf{p}_c(kT_d)) , \quad (8f)$$

$$\mathbf{x}'_d(k) = \begin{bmatrix} \mathbf{x}_d^\top & \mathbf{f}|_{(k-1)T_d}^\top & \mathbf{f}|_{(k-2)T_d}^\top \end{bmatrix}^\top . \quad (8g)$$

where \mathbf{x}'_d is the altered state matrix, in which the \mathbf{f} vectors corresponds to the original continuous system equations.

D. Complete Jacobian Linearized-SS Discretization

With the Jacobian-matrix, a linear approximation has been acquired around the current step, and the system discretized by the complete method mentioned before (6a)–(6d) [23], [24].

$$\mathbf{A}_d(k) = e^{(\mathbf{A}_j(kT_d)T_d)} , \quad (9a)$$

$$\mathbf{B}_d(k) = \mathbf{A}_j^{-1}(kT_d)(e^{(\mathbf{A}_j(kT_d)T_d)} - \mathbf{I})\mathbf{B}_j(kT_d) , \quad (9b)$$

$$\mathbf{E}_d(k) = \mathbf{A}_j^{-1}(kT_d)(e^{(\mathbf{A}_j(kT_d)T_d)} - \mathbf{I})\mathbf{E}_j(kT_d) , \quad (9c)$$

$$\mathbf{C}_d(k) = \mathbf{C}_j(kT_d) , \quad (9d)$$

$$\mathbf{D}_d(k) = \mathbf{D}_j(kT_d) , \quad (9e)$$

$$\mathbf{D}_{2,d}(k) = \mathbf{D}_{2,j}(kT_d) , \quad (9f)$$

where $\mathbf{A}_j(kT_d)$, $\mathbf{B}_j(kT_d)$, $\mathbf{E}_j(kT_d)$, $\mathbf{C}_j(kT_d)$, $\mathbf{D}_j(kT_d)$ and $\mathbf{D}_{2,j}(kT_d)$ are the Jacobian form of the above defined matrices.

E. Jacobian Linearized Recursive Discretization

In this method the state matrices are not specified, because they were calculated according to the complete method (6a)–(6d) given in [23], [24]. The current values are calculated in each time step from the initial values, by calculating the total effect of the inputs up to the given point.

$$\mathbf{x}_d(k) = \mathbf{A}_d(k)^k \mathbf{x}(0) + \sum_{i=0}^{k-1} \left(\mathbf{A}_d(k)^{k-i-1} \left(\mathbf{B}_d(k)\mathbf{u}_d(i) + \mathbf{E}_d(k)\mathbf{d}_d(i) \right) \right) , \quad (10a)$$

$$\mathbf{y}_d(k) = \mathbf{C}_d\mathbf{x}_d(k) + \mathbf{D}_d\mathbf{u}_d(k) + \mathbf{D}_{2,d}\mathbf{d}_d(k) . \quad (10b)$$

IV. RESULTS

In this paper we do not aim to characterize the discretization error caused by the LPV discretization procedures in Sec. III. We have investigated only the applicability of them in case of the given highly non-linear and rapidly varying glucose-insulin model. The further investigation in this regard will be the part of our future work.

It should be noted that we have used the MATLAB 2017b environment for the developments.

In order to validate the discretized systems we have applied the inbuilt `ode45()` function of the MATLAB [27] in which we realized the original model described by Eqs. 1a-1c. This system was applied as reference system during the investigations to which all of the developed discretized systems have been compared. The `ode45()` is a suitable nonstiff solver solving nonlinear ordinary differential equations (ODEs) on a variable basis [27].

Since the investigated DM model has impulse like inputs, the functions had to be solved for every minutes independently. This results a quasi discrete reference model. The quantitative comparisons have been made by applying 2-norm error between the state variables of the system in all cases, namely, $\|\mathbf{x}_{ODE_{45}}(t) - \mathbf{x}_{Discretized\ Systems}(kT)\|$, where the $\mathbf{x}_{Discretized\ Systems}(kT)$ were the realized DT-LPV systems (LPV_{Complete}, LPV_{Rectangular}, LPV_{Adams-Bashforth}, LPV_{Complete}) and the DT-SS systems based on Jacobians (Jacobian Linearized-SS and Jacobian Linearized Recursive Discretization). The states of the discretized systems were hold for the applied T in each step in order to approximate the original continuous system. As we compared all of the discretized systems to the reference system we got a picture about how the discretized systems related to each other indirectly.

The following initial values have been applied: $\mathbf{x}(t_0) = [G(0), X(0), I(0)]^T = [115, 0, 15]^T$.

For testing purposes a realistic manual glucose and insulin administration scheme have been developed [28]. The glucose input consists of 500 g carbohydrate, divided into five smaller boluses. The insulin injections introduced with a five minute delay. Therefore, the inputs defined as follows: $w(t_i) = [65, 25, 75, 25, 60]$ g in each $t_i = [30, 250, 500, 800, 1200]$ min and $u(t_j) = [15, 5, 17, 5, 14]$ mU/min in each $t_j = [35, 255, 505, 805, 1205]$ min.

The basal levels of the three state variable have been chosen to be: $\mathbf{x}_B = [G_B, X_B, I_B]^T = [85, 0, 15]^T$. From physiological point of view basal levels correspond to the fasting conditions, from mathematical point of view the basal levels are the equilibrium points of the isolated system [29], [30].

Sampling time T was chosen to be $T = 5$ min due to the current blood glucose concentration measuring devices are capable of providing data at similar frequencies [5], [13] – with one exception. During our investigations it became clear that the Adams-Bashforth method was not able to provide acceptable outcome with $T = 5$. In accordance with our examinations we applied $T = 2$ min as the highest sampling

time in this case. Thus, it's practical applicability strongly depends on the available sensor technology.

Figure 1 shows the result of the simulation of the reference and discretized systems by using the complete LPV-SS discretization in accordance with III-A. It is clearly visible that the Complete LPV-SS model is able to approach the reference system with good accuracy. The norm based error shows that higher difference occurs when the systems changing due to the intakes. However, the state errors decay fast.

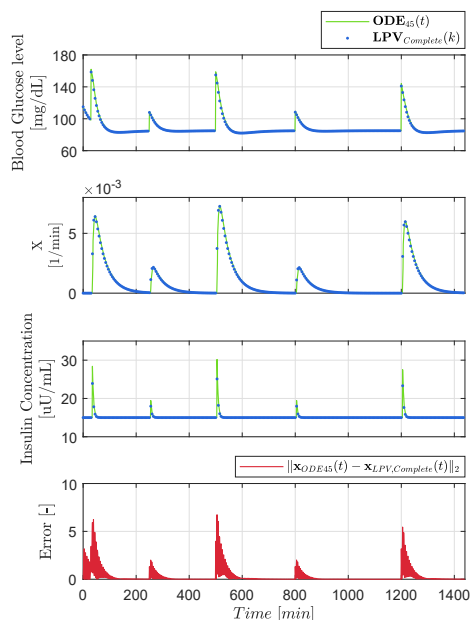


Figure 1. Comparison of the reference and complete DT-LPV systems.

Figure 2 presents the comparison of the rectangular DT-LPV system (III-B) and the original system during operation. The results are similar to the previous case. The amplitude of the state errors are bigger, however, they decay faster compared to the complete DT-LPV model. We have found that the state error – in general – increased rapidly by using $T > 5$ minutes, thus the use of higher sampling time is not recommended. The potential instability can be seen on Fig. 2 as well. After the injection of each bolus, the insulin concentration deviated in negative direction.

Figure 3 shows the comparison of the DT-LPV system provided by the Adams-Bashforth method (III-C) and the original system during operation. That was the only method in which we have not been able to apply the determined $T = 5$ sampling time (instead $T = 2$ was used) – as we mentioned above. It is clearly visible that this method provided the highest state errors and its inaccuracy was at the border of acceptability. As we found, the method is not able to handle the fast changes (around the intakes) at all. However, in case of "slower" systems it may be applicable. We could not apply $T > 5$ due to instability.

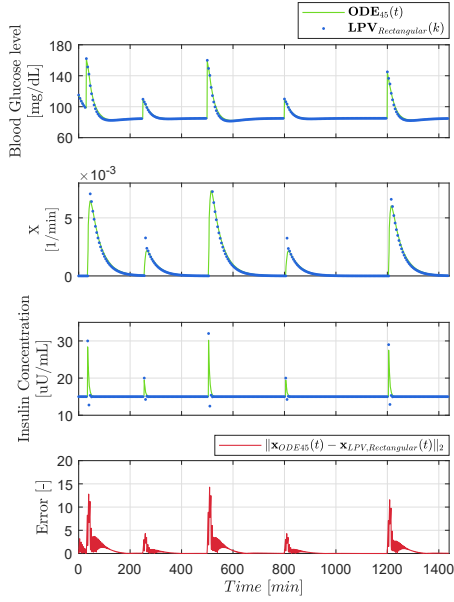


Figure 2. Comparison of the reference and the rectangular DT-LPV models.

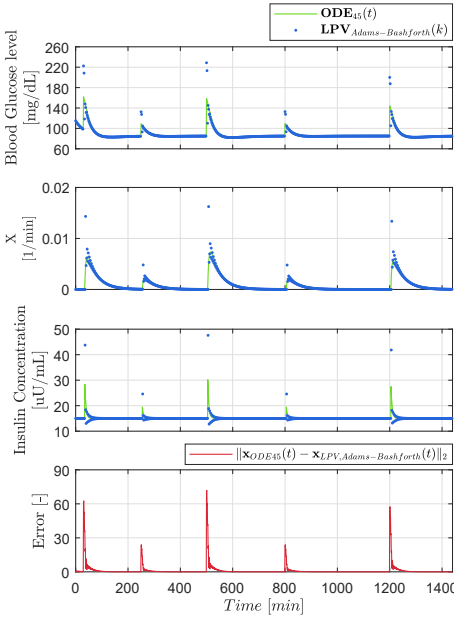


Figure 3. Comparison of the reference model and the DT-LPV model provided by the Adams-Basforth LPV-SS discretization.

Figure 4 shows the comparison of the discrete system (according to III-D) and the original system during operation. It can be seen that the highest state error is at the $G(t)$ due to the nonlinearity – other states of the reference system are approached by the discrete system with good accuracy. We have found, that the stability of this opportunity is good and even $T = 10$ can be applied without critical issues in accuracy

and numerical stability.

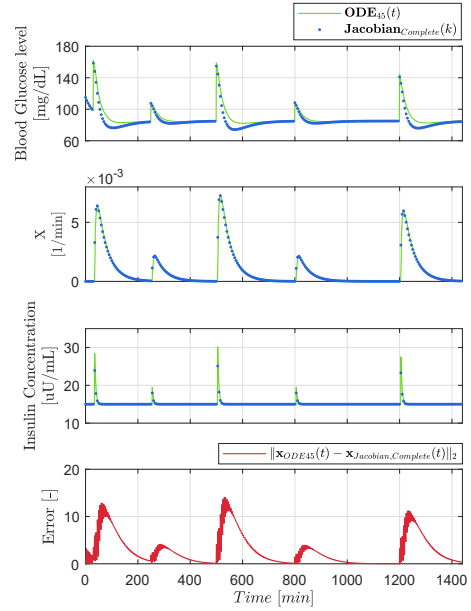


Figure 4. Comparison of the reference model and the discretized model (by using Jacobian linearization and complete discretization).

Figure 5 shows the comparison of the discrete system provided by the Jacobian recursive method (III-E) and the original system during operation. The key difference between the recursive and the complete method is that in case of the recursive method the states are calculated from the initial values in each step. The accuracy of the discrete model is similar as in the previous case – the reasons are similar as well. Although, the state errors are bigger in general and the discrete model provided unexpected behavior at the beginning of the simulations (initial deviations).

The numerical assessment can be found in Table I, where the Root Mean Square Errors (RMSE) based metric [31] has been applied to assess the quantitative differences between the systems during operation.

Table I
RMSE ERRORS

Systems	$RMSE_G$	$RMSE_X$	$RMSE_I$
ODE₄₅/Complete	0.739	1.722	0.327
ODE₄₅/Rectangular	1.321	5.676	0.921
ODE₄₅/Adams-Basforth	6.167	7.141	1.560
ODE₄₅/Jacobian, Complete	4.292	1.722	0.327
ODE₄₅/Jacobian, Recursive	5.065	2.042	0.384

As it can be seen both in the Figs 1–5 and in Table I the complete method provided the most accurate results both in case of the LPV and the Jacobian based cases. Hence, the use of this method is recommended in the further researches as a basis to design discrete LPV controllers or discrete estimators (eg. Kalman filter).

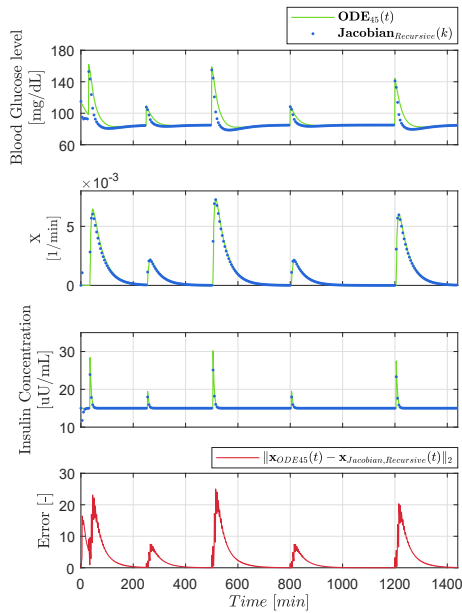


Figure 5. Comparison of the reference model and the discretized model (by using Jacobian linearization and recursive discretization).

V. CONCLUSION AND FURTHER WORK

In this paper we have analyzed the discretization opportunities of a well-known DM model with a special focus on the LPV (qLPV) techniques.

We have found that the best DT-LPV model and discrete Jacobian based models have been provided by the complete method – which were proven by the RMSE based error assessment as well.

In our future work we will use the developed DT-LPV model (provided by the complete LPV-SS discretization algorithm) to develop DT-LPV controllers and advanced Kalman-filters (e.g. extended or unscented Kalman-filter on LPV basis).

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