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Critical exponents in quantum Einstein gravity

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The quantum Einstein gravity is treated by the functional renormalization group method using the Einstein-Hilbert action. The ultraviolet non-Gaussian fixed point is determined and its corresponding exponent of the correlation length is calculated for a wide range of regulators. It is shown that the exponent provides a minimal sensitivity to the parameters of the regulator which correspond to the Litim's regulator.

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I. INTRODUCTION

The quantum Einstein gravity (QEG) can be a good candidate to describe the gravitational interactions in the framework of quantum field theory [1–3]. The action of the model is integrated out for all the possible paths of the field variables, whose role is now played by the metrics. The Einstein-Hilbert action of QEG contains only the cosmological and the Newton couplings [4]. The functional renormalization group (RG) method allows us to perform the path integration systematically for the degrees of freedom, and gives us the scaling behavior of the couplings starting from the high energy, UV region down to the low energy IR regime [5–7]. The method provides an integro-differential equation for the effective action, the Wetterich equation. Besides QEG the RG technique is widely used recently in many areas of gravitational issues starting from black holes [8,9] to cosmological problems [10,11]. Renormalization group calculations could show that QEG is asymptotically safe [1,12,13]. This result can be considered as one of the greatest achievements of the RG method. Asymptotic safety means that there is a UV non-Gaussian fixed point (NGFP) in the phase space of the model with a finite number of relevant couplings [14,15].

The Wetterich equation contains a regulator function to remove the divergences of the momentum integrals. In the IR limit the effective action should not depend on the regulator because it is an artificial term that is put by hand into the action. However, during the solution of the Wetterich equation we use several approximations and truncations which may introduce some regulator-dependence. In the RG method mostly the optimized Litim's regulator is used [16,17], especially due to its analytic form. The sensitivity of the physical quantities on the regulator parameters should be minimal [18], since the regulator itself is an artificial element of the RG method and the results should be independent on it ideally.

There are several types of regulators which are widely used, e.g., the power law, the exponential and the Litim ones, which can be get by the limiting cases of the so-called compactly supported smooth (css) regulator [19,20]. This regulator provides us with such a wide class of regulators that contains all the important types of regulator functions and enables us to investigate the sensitivity of the physical quantities on a broader set range of regulators and their parameters.

Using the css regulator, first we calculate the value of the exponent ν for the 3-dimensional $O(1)$ model. We investigate how the exponents depend on the css regulator parameters and search for those parameter regions where the exponents are practically parameter independent or have at least a minimal sensitivity of them. Then we calculate the critical exponent ν of the correlation length and the anomalous dimension η around the fixed points of QEG. We obtain that $\eta = -2$ independently on the regulators at the UV NGFP, but the value of the other exponent ν can be arbitrary there. The strong truncation of the action might cause such a nonuniversal behavior. We also determine how the exponents scale around the IR fixed point.

The paper is organized as follows. In Sec. II the investigated $O(1)$ model, the RG method, and the regulators are introduced and the critical exponent ν is calculated. In Sec. III the UV and IR critical behavior is discussed for the QEG. Finally, in Sec. IV the conclusions are drawn up.

II. 3-DIMENSIONAL $O(1)$ MODEL

The Wetterich equation for the effective average action Γ_k is [5]

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma''_k + R_k} \tag{1}$$

with the " RG time" $t = \ln k$, Furthermore, the prime is the differentiation with respect to the field variable and the trace Tr denotes the integration over all momenta and the summation over the internal indices. The function R_k plays the role of the IR regulator.

As it is well known, the 3-dimensional $O(1)$ model plays the role of the testing ground for any new improvement of renormalization. Therefore, in order to test the scaling criticality first we treat this model around the Wilson-Fisher (WF) fixed point and the corresponding critical exponent ν. We start with the effective potential

$$\tilde{V} = \sum_{i=1}^{N} \frac{\tilde{g}_i}{(2i)!}\phi^{2i}, \tag{2}$$

with the dimensionless couplings $\tilde{g}_i$ and restrict ourselves to the truncations $N = 2$ and $N = 4$. In local potential approximation the evolution equations for the first two couplings read as

$$\dot{\tilde{g}}_1 = -2\tilde{g}_1 + \tilde{g}_2\bar{\Phi}^2_{3/2}(\tilde{g}_1), \qquad \dot{\tilde{g}}_2 = -\tilde{g}_2 + 6\tilde{g}_2^2\bar{\Phi}^3_{3/2}(\tilde{g}_1), \tag{3}$$

where the threshold function is introduced according to

$$\bar{\Phi}^p_n(\omega) = \frac{1}{(4\pi)^n\Gamma(n)}\int_0^\infty dy y^{n+1}\frac{r'}{(y(1+r)+\omega)^p}, \tag{4}$$

where $y = p^2/k^2$ and $r = r(y)$ is the dimensionless regulator $r = R/p^2$. The dimensionless css regulator has the form

$$r_{\rm css} = \frac{s_1}{\exp[s_1 y^b/(1-s_2 y^b)] - 1}\theta(1 - s_2 y^b), \tag{5}$$

where $b \geq 1$ and s_1, s_2 are positive parameters. Although this regulator cannot provide an analytic form of the evolution equations, it gives a broader range of regulators. The limiting cases of the css regulator provide us the following commonly used regulator functions

$$\lim_{s_1\to 0} r_{\rm css} = \left(\frac{1}{y^b} - s_2\right)\theta(1 - s_2 y^b), \qquad \lim_{s_1\to 0, s_2\to 0} r_{\rm css} = \frac{1}{y^b},$$
$$\lim_{s_2\to 0} r_{\rm css} = \frac{s_1}{\exp[s_1 y^b] - 1}, \tag{6}$$

where the first limit gives the Litim's optimized regulator when $s_2 = 1$, the second gives the power-law regulator, and the third gives the exponential one, if $s_1 = 1$. It provides us with a possibility of simultaneous optimization of some physical quantities among the Litim's, the power-law and the exponential regulators, which can be continuously deformed from one to the other by only two parameters s_1 and s_2.

The exponent at the WF fixed point is well known, and the optimization of the RG regulator was originally performed for the WF fixed point giving the Litim's regulator. Furthermore, the value of ν was calculated for every available type of regulator in the literature. However, by using the css regulator we can get an optimized exponent among fundamental regulators which was not investigated so far, so now we look for the extremum of ν on the parameter space s_1, s_2, and b. We considered the RG evolution for 2 and 4 couplings. The truncation of the scalar potential to only two terms in the Taylor expansion is very strong, but we also have only two couplings in QEG with the same level of truncation. The values of the exponents are plotted in Fig. 1.

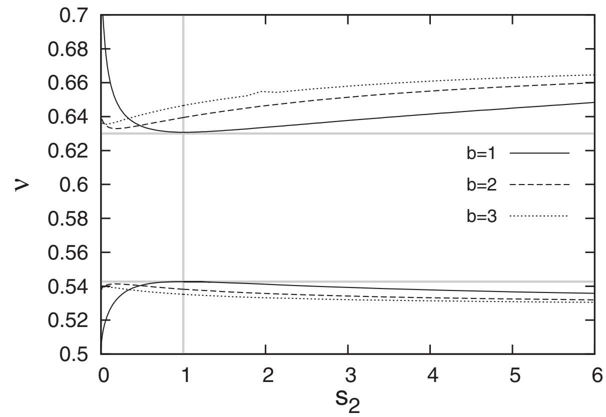

FIG. 1. The exponent ν is calculated for 2 and 4 couplings and shown by the curves at the bottom and at the top, respectively. The Litim's regulator gives the extremal exponent in both cases at the point of the crossing of the grey lines.

For 2 couplings we obtained that we have a maximum of the exponent ν at $b = s_2 = 1$ and $s_1 \to 0$ which corresponds to the Litim's regulator in Eq. (6). There the sensitivity of ν on the regulator parameters is minimal. Fortunately even the truncation with $N = 2$ can provide us a real physical exponent although it is smaller than the proper value of ν. We repeated the calculations for $N = 4$ couplings. We got another extremum but now it is a minimum and it corresponds to the Litim's regulator, too. Further increase of the number of the couplings and the inclusion of the evolving anomalous dimension η may give the Litim's regulator, too.

III. QUANTUM EINSTEIN GRAVITY

The Einstein-Hilbert effective action is

$$\Gamma_k = \frac{1}{16\pi G_k}\int d^d x\sqrt{g}(-R + 2\Lambda_k), \tag{7}$$

with the dimensionful Newton constant G_k and the cosmological constant Λ_k. The RG equations are formulated by the dimensionless couplings, i.e., $\lambda = \Lambda_k k^{-2}$ and $g = G_k k^{d-2}$. Since the determinant of the metric occurs only in Eq. (7) and nowhere does it occur anymore in our paper, the usage of g for the Newton coupling below shall not be confusing. The action contains the first two terms in the Taylor expansion of the curvature R. If one inserts Eq. (7) into the Wetterich equation then one obtains the evolution equations for the couplings, which are derived and given in [4] and have the form

$$\begin{aligned}\dot{\lambda} &= 2(2-\eta)\lambda + \frac{1}{2}(4\pi)^{1-d/2}g \\ &\quad\times [2d(d+1)\Phi^1_{d/2}(-2\lambda) - 8d\Phi^1_{d/2}(0)] \\ &\quad - d(d+1)\eta\tilde{\Phi}^1_{d/2}(-2\lambda)], \\ \dot{g} &= (d-2+\eta)g,\end{aligned} \tag{8}$$

with the anomalous dimension

$$\eta = \frac{gB_1(\lambda)}{1-gB_2(\lambda)}. \tag{9}$$

The functions $B_1(\lambda)$ and $B_2(\lambda)$ are

$$\begin{aligned}B_1(\lambda) &= \frac{1}{3}(4\pi)^{1-d/2}[d(d+1)\Phi^1_{d/2-1}(-2\lambda) \\ &\quad - 6d(d-1)\Phi^2_{d/2}(-2\lambda) \\ &\quad - 4d\Phi^1_{d/2-1}(0)24\Phi^2_{d/2}(0)], \\ B_2(\lambda) &= -\frac{1}{6}(4\pi)^{1-d/2}[d(d+1)\tilde{\Phi}^1_{d/2-1}(-2\lambda) \\ &\quad - 6d(d-1)\tilde{\Phi}^2_{d/2}(-2\lambda)],\end{aligned} \tag{10}$$

with the threshold functions

$$\begin{aligned}\Phi^p_n(\omega) &= \frac{1}{\Gamma(n)}\int_0^\infty dy y^n \frac{r-yr'}{(y(1+r)+\omega)^p}, \\ \tilde{\Phi}^p_n(\omega) &= \frac{1}{\Gamma(n)}\int_0^\infty dy y^n \frac{r}{(y(1+r)+\omega)^p}.\end{aligned} \tag{11}$$

A. Ultraviolet criticality

The phase structure of QEG is quite involved [4]. In the positive Newton coupling region the model has two phases, as is shown in Fig. 2.

The phase space contains 3 fixed points. The UV NGFP is said to be UV attractive, however it repels the trajectories from the viewpoint of the RG flow. The trajectories start there and flow toward the hyperbolic Gaussian fixed point (GFP). The trajectories with positive λ in the IR limit constitute the weak coupling phase of the model which contains an attractive IR fixed point [21–23]. This phase gives degenerate geometry as $\langle g_{\mu\nu}\rangle = 0$. The cosmological constant appears formally as the coupling $-\tilde{g}_1$ in the $O(1)$ model thus the weak coupling phase with positive λ shows similarity to the (symmetry-) broken phase of the $O(1)$ model [24]. The other trajectories belong to the strong coupling phase, and it gives a smooth geometry, i.e., $\langle g_{\mu\nu}\rangle \sim \delta_{\mu\nu}$ and a negative curvature $R = 4\Lambda$. This corresponds to the symmetric phase of the $O(1)$ model. One can linearize the RG equations around the fixed points. In the case of the UV NGFP the eigenvalues of the corresponding stability matrix can be written as

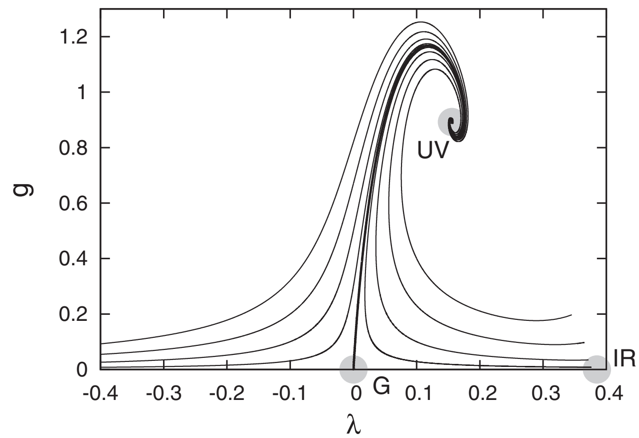

FIG. 2. The phase structure of QEG is shown for css regulator with the parameters $s_1 = s_2 = b = 1$. The grey points denote the critical points. The thick line represents the separatrix.

$$\theta^{\rm UV}_{1,2} = \theta' \pm i\theta'', \tag{12}$$

where the real part of the exponent can be related to the critical exponent ν of the correlation length, i.e., $\nu = 1/\theta'$. We calculated the position of the fixed point and the corresponding exponent in QEG with the Einstein-Hilbert action in $d = 4$. In Fig. 3 we plotted how the reciprocal of the exponent $1/\nu$ depends on the parameters of the css regulator. The figure demonstrates the appearing limiting regulators in Eq. (6).

We look for the extremum of $1/\nu$ as the function of s_1, s_2, and b. The increase of the parameters s_1 and s_2 gives monotonically increasing exponents, for large values they

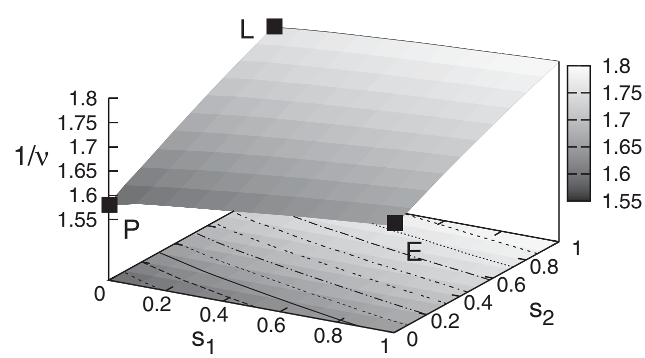

FIG. 3. The reciprocal of the exponent $1/\nu$ is calculated for various parameters of the css regulator, with $b = 3$. The value of $1/\nu$ at the corner $s_1 = s_2 = 0$ corresponds to Power-law regulator result, the corner $s_1 = 0$, $s_2 = 1$ gives the Litim's regulator result, and the corner $s_1 = 1$, $s_2 = 0$ results in the Exponential regulator value.

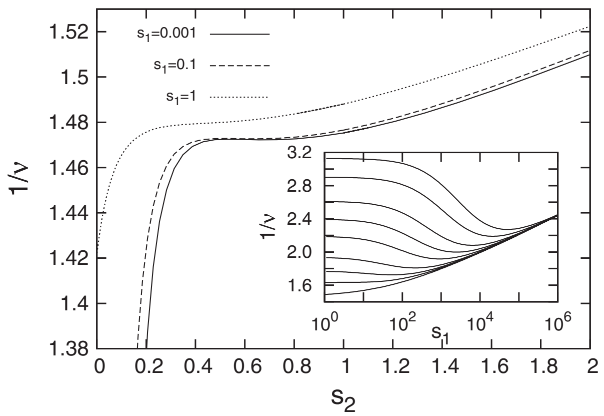

FIG. 4. The exponent $1/\nu$ is calculated for various values of parameters of the css regulator. In the inset the different curves correspond to different values of $s_2 = 1$, 5, 10, 20, 50, 100, 200, 500, 1000, increasing from below. There we chose $b = 1$.

logarithmically grow. The limit of the power-law regulator $s_1 = s_2 = 0$ gives the minimal value for $1/\nu$, when $b = 3$. However, the exponent grows with increasing b, therefore one might expect another local extremum at $b = 1$. We calculated $1/\nu$ at the Litim's limit ($s_1 \to 0$) for different values of s_2 as shown in Fig. 4 for $b = 1$.

Although the optimization of the exponent ν was performed earlier [25,26], the figure shows that the css regulator can mimic such a great set of regulators (including the Litim's, the power-law, and the exponential ones) that it can generate arbitrary values of $1/\nu$ and enables us to perform the optimization program among these fundamental regulators simultaneously. Renormalization group calculations give $\nu \approx 0.4 - 0.67$ [25,27,28,29–31] and the UV exponent is $\nu = 1/3$ according to lattice analysis [32]. By a proper parameter choice of the css regulator any of these values can be easily obtained and can even be exceeded up to practically ∞. For example, one can tune the regulator parameters to get the lattice result of the exponent, e.g., according to Fig. 4 the choice $b = 1$, $s_1 = 251$, and $s_2 = 1000$ does the trick. However the exponent shows high sensitivity on the parameters, which implies that it cannot be considered as the physical exponent, since we usually look for the parameter regime where the physical quantities show minimal sensitivity on the regulator [18,33]. We cannot find any absolute extremum in the parameter space of b, s_1, and s_2. However we have an inflection point at $b = 1$, $s_1 \to 0$, and $s_2 \approx 1/2$, where we have minimal sensitivity to the regulator parameters. It corresponds to a rescaled Litim's regulator of the form

$$r_{\rm opt} = \left(\frac{1}{y} - \frac{1}{2}\right)\theta(1 - y/2). \tag{13}$$

The value of the exponent there should be considered as the physical exponent, that is $\nu \approx 0.679$. Other RG equations provide different values for ν even in the framework of the Einstein-Hilbert action [25,34], however in this model if we consider this large set of regulator functions the principle of minimal sensitivity gives this result. The power-law limit of the css regulator does not exist for small b in the case of $d = 4$ because the RG equation gives UV divergent integrals, however the finite small value of s_2 serves as a UV cutoff of the loop integral. As s_2 gets smaller and smaller the value of $1/\nu$ decreases quickly and tends to zero and negative values. When $1/\nu = 0$ then the eigenvalue θ' in Eq. (12) is purely imaginary, then the UV NGFP is not attractive anymore and the trajectories evolve toward a limit cycle as was obtained in [35,36]. Further decrease of s_2 changes the sign of θ' to negative making the UV NGFP a UV repulsive one.

We also calculated the position of the fixed point in the phase space. If $s_1 \to \infty$ the values of λ^* and g^* become s_2 independent and scale according to $\lambda^* \sim s_1^{-0.89}$ and $g^* \sim s_1^{0.89}$ implying that the product $\lambda^* g^*$ tends to a constant, $\lim_{s_1\to\infty}\lambda^* g^* \approx 0.133$. The regulator in Eq. (13) gives $\lambda^* g^* = 0.136$, for which the value is practically the same. In the limit $s_1 \to \infty$ the UV NGFP disappears, since $g^* \to \infty$ making the QEG nonrenormalizable.

There is a crossover (CO) fixed point in QEG, namely the hyperbolic GFP. However its critical behavior is trivial since the eigenvalues corresponding to the linearized RG flows equal the negative of the canonical dimension of the couplings, and the inclusion of further couplings or taking into account corrections from the gradient expansion beyond the local potential approximations do not change their values. The reciprocal of the negative exponent $s_1 = -2$ provides $\nu = 1/2$ in the crossover regime.

B. Infrared criticality

In the broken phase of field theoretical models we usually find an IR fixed point. This is the case in QEG, too [21–23]. We calculated the exponent ν by the dynamically induced correlation [23,37,38] around the IR fixed point. The correlation length scales as

$$\xi \sim (\kappa - \kappa^*)^{-\nu}, \tag{14}$$

with the exponent ν. Furthermore $\kappa = g\lambda$, and $\kappa^* = g^*\lambda^*$ equals the value which is taken at the fixed point. The value of κ is calculated from those values of the couplings where the RG evolutions stop. In the IR limit $\kappa^* = 0$. In Fig. 5 the scaling of ξ is shown. We obtained that the exponent ν around the IR fixed point equals that value, which is calculated around the GFP and it is $\nu = 1/2$.

We also determined the scaling of the anomalous dimension η, and the results are plotted in Fig. 6. From the evolution of the Newton constant g it is trivial that in the UV NGFP we have $\eta = -2$. This corresponds to marginal scaling of η in Fig. 6. Then, as the flow approaches the GFP it tends to zero as a power-law function $\eta \sim k^2$, where the exponent 2 is universal. Going further, the η starts to

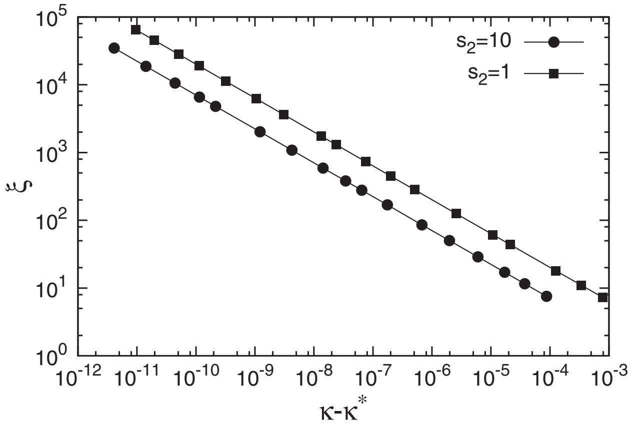

FIG. 5. The power-law scaling of the correlation length ξ is shown for various parameters of the css regulator, we chose $s_1 = b = 1$. From the negative slope of the logarithmic plot we obtained that $\nu = 1/2$.

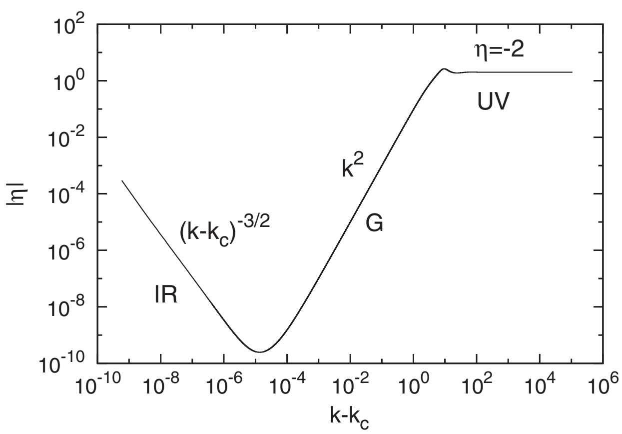

FIG. 6. The anomalous dimension η is plotted as the function of the scale. In the vicinity of the fixed points the η shows different scaling behaviors. We chose $s_1 = s_2 = b = 1$ for the css regulator.

diverge as $(k - k_c)^{-\alpha}$, where α is around $3/2$ and shows a slight parameter dependence, e.g., we have $\alpha = 7/4$ for the Litim's regulator. The value k_c denotes the critical scale where the evolution stops due to the appearing singularity in the threshold functions in Eq. (11). For example, the singularity appears there at $\lambda^* = 1/2$ in the case of the Litim's regulator. However the β functions for the couplings in Eq. (8) are finite at the singular point. The exponents are summarized in Table I.

IV. SUMMARY

By using the functional RG method the critical exponent ν of the correlation length and the anomalous dimension η are calculated for the QEG with the css regulator. We considered the Einstein-Hilbert action with the Newton constant g and the cosmological constant λ. The value of ν is calculated around the UV NGFP, the GFP, and the IR fixed point. We obtained that ν can be arbitrary around the UV NGFP. There can be several reasons for this result. On the one hand although the regulator is a masslike term, it possesses a complicated momentum dependence which might introduce nonlocal interactions into the action. Furthermore, the deep IR physics cannot be altered by the regulator, but the UV limit can show strong regulator dependence. On the other hand, the Einstein-Hilbert action contains only two couplings. The inclusion of further couplings in the Taylor expansion in the curvature [39–41] may restrict the value of ν to a certain interval. Several possible extensions [42,43] can also give some restriction to the value of ν. We showed that the value of ν has a minimal sensitivity on the regulator parameters around a rescaled Litim's regulator, and there we have $\nu = 0.679$.

TABLE I. The summary of the critical exponent ν and the anomalous dimensions η at the various fixed points of QEG.

Exponent	UV	G	IR
ν	0.679	0.5	0.5
η	-2	k^2	$(k - k_c)^{-3/2}$

In QEG the CO scaling around the GFP gives $\nu = 1/2$ independently of the approximations that were used. We demonstrated that the CO scaling in the 3-dimensional $O(1)$ model can give a wide range of the value of ν, however an extremum appears in the parameter space, which also corresponds to the Litim's regulator. Around the IR fixed point we obtained that $\nu = 1/2$, and it is independent of the regulator parameters. The scaling came from the dynamically induced correlation near the singularity of the RG flows. The value of ν is inherited from the hyperbolic CO fixed point, whose role is now played by the GFP as in the case of the d-dimensional $O(N)$ model [37].

We also showed that the anomalous dimension takes the value $\eta = -2$ at the UV NGFP. It scales in an irrelevant manner by a universal exponent toward the evolution as the trajectory approaches the GFP in the CO regime. As the flow reaches the IR region the scaling turns to a relevant one with such an exponent that shows moderate dependence on the regulator parameters.

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