INCREASING EFFICIENCY OF ON-LINE SHOPPING BY OPTIMIZING THE STAFF SCHEDULE

136th ESGI was jointly organised by
Università degli Studi dell'Aquila
Gran Sasso Science Institute

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May 18, 2018

Description of the problem

Problem proposed by: COOP Drive (online food shopping service provided by COOP Liguria),
`http://www.e-COOP.it/virtualShop/`.

The customers are making an online purchase and choose a time range for pick up.

Liguria

Description of the problem

Making optimal staff scheduling such that each employee has 'constant' working hours, satisfying customers and employees in the same time.

Constraints:

- Each employee can work at most 6 days a week.
- Each employee can work maximum 8 hours a day.
- Each employee needs to work continuous hours.

Additional questions:

- All the orders have to be processed.
- Can the schedule change if customers order in advance?

Introduction

Staff scheduling can be divided into three groups:

- days off scheduling,
- shift scheduling
- tour scheduling.

Mathematical models and algorithms should be based on historical data in order to provide convenient schedules.

The characteristics of mathematical models and algorithms depends on the area of application.

Introduction

Scheduling processes are classified into several modules:

- demand modeling
- days off scheduling
- line of work construction
- task assignment
- staff assignment.

Introduction

There are many different approaches in the reviewed literature:

- Integer linear programing (ILP)
- Mixed Integer Linear Programing (MILP)
- Column generation
- Constructive heuristic
- Tabu search
- Genetic Algorithm
- Simulation
- Agent-Based Solution
- Queuing

Preliminary data analysis

Two data sets were available:

a) Daily percentages of orders in 2017

b) The following variables for February and March 2018

   - date and time of orders
   - pick up date and time
   - number of ordered items
   - total amounts
   - type of payment.

Preliminary data analysis

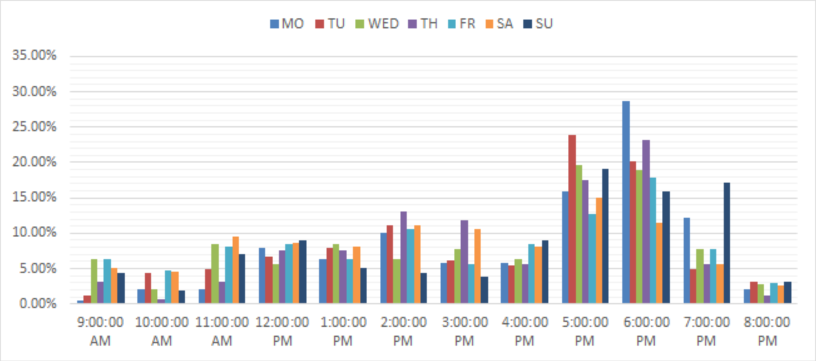


Figure: Distribution of pick up times by weekdays

Preliminary data analysis

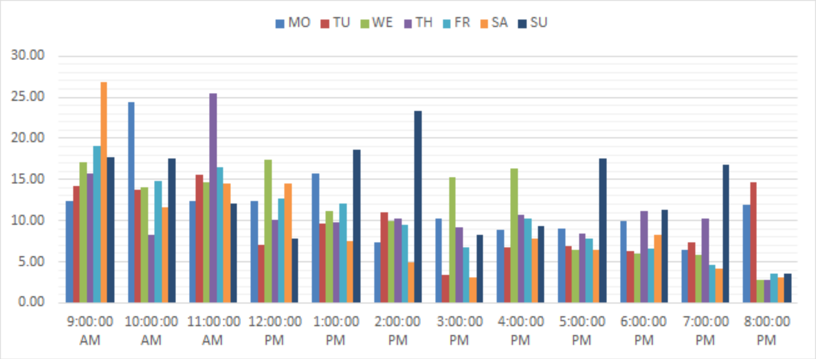


Figure: Average difference between order and pick up times (in hours) by weekdays

Preliminary data analysis

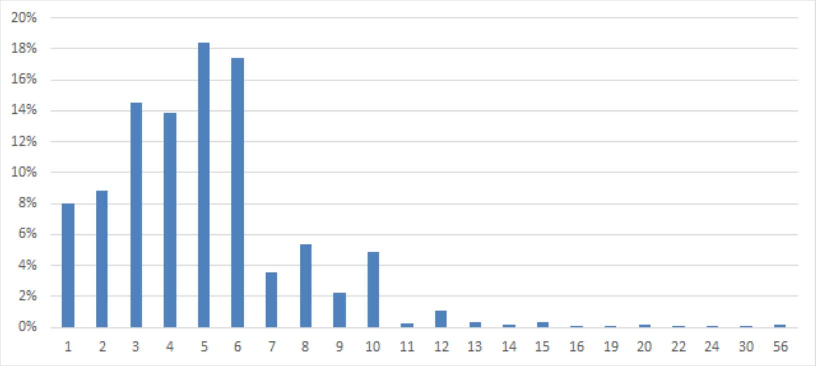


Figure: Percentage of number of ordered items

Models

Models

- Three models for staff scheduling.
- Agent-based model for the store operations.

Model Assumptions

- Weekly schedule: objective function is the weekly variance of working hours.
- We only require that the number of staff working hours is at least the number of hours needed to fulfill all the orders.
- The fulfillment of each order takes the same number of time.

Model 1 – with a Penalty Function

$$\min_{x_{ij},\delta_{ij}} \sum_{j=1}^{n}\sum_{i=1}^{7}\left(\delta_{ij}x_{ij} - \frac{1}{\sum_k \delta_{kj}}\sum_{k=1}^{7}\delta_{ij}x_{kj}\right)^2 + \frac{1}{2\mu}\sum_i\prod_j x_{ij}$$

subject to $\sum_i x_{ij} \geq C_j \quad \text{for } \forall j = \overline{1,n}$

$$\sum_j x_{ij} \geq D_i \quad \text{for } \forall i = \overline{1,7}$$

$$0 \leq x_{ij} \leq 8 \quad \text{for } \forall j = \overline{1,n}$$

$$x_{ij} \in \mathbb{R},$$

where

x_{ij} is the number of working hours by employee j on the day i;

n was taken to be 12.

Model 2 - Extension of Model 1

Bilevel nonlinear optimization problem.

- The upper-level problem: minimize the squared error of number of employees at each working hour each day compared to the estimated number of employees at each working hour each day
- The inner-level problem: minimize the discrepancy between the amount of hours each worker works during the day and the mean of hours the same worker works during the week

Model 2 - Decision variables

- $x_{ik}-$ starting time for $i-$th worker on $k-$th day (integer), $i = 1, \dots, 12$, $k = 1, \dots, 7$
- $y_{ik}-$ number of working hours of $i-$th worker on $k-$th day (integer), $i = 1, \dots, 12$, $k = 1, \dots, 7$
- $\delta_{ijk}-$ indicator of presence of $i-$th worker in the $j-$th working hour on $k-$th day (binary), $i = 1, \dots, 12$, $j = 1, \dots, 14$, $k = 1, \dots, 7$

$$\delta_{ijk} = \begin{cases} 1, & \text{if } x_{ik} \leq t_j < x_{ik} + y_{ik}, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

where $t_j = j + 6$ are beginnings of the working hours, $j = 1, \dots, 14$

Model 2 - Notation

- $n_{jk}-$ number of workers on $j-$th interval on $k-$th day, where for all j, k

$$n_{jk} = \sum_{i=1}^{12} \delta_{ijk}$$

- $\tilde{n}_{jk}$ is estimated number of needed workers on $j-$th interval on $k-$th day,
- $\bar{y}_i = \frac{1}{7} \sum_{k=1}^{7} y_{ik}, \ \forall i$
- index $i = 1, \ldots, 12$, denotes number of workers,
  index $j = 1, \ldots, 14$ denotes workings intervals,
  index $k = 1, \ldots, 7$ denotes days in the week.

Model 2 - Constraints

The upper-level problem:

- $7 \le x_{ik} \le 20, \quad \forall i, k$ bounds the starting working time for $i-$th employee on $k-$th day.
- rewrite δ_{ijk} as $\max\left(0, \frac{a_{ijk}}{M}\right) \le \delta_{ijk} \le \min\left(a_{ijk}, 1\right) \quad \forall i, j, k,$
  - $a_{ijk} = \max\left(0, t_j - x_{ik} + 1\right) \cdot \max\left(0, x_{ik} + y_{ik} - t_j\right)$
  - $M > 0$ large constant

The inner-level problem:

- $0 \le y_{ik} \le 8, \quad \forall i, k$ bounds the total working hours per day for each employee on each day, according to the labor law
- $\prod_{k=1}^{7} y_{ik} = 0, \quad \forall i$ provides at least one day off per week for each employee, according to the state labor law
- $20 \le \sum_{k=1}^{7} y_{ik} \le 38, \quad \forall i$ provides at least 20, and at most 38 total working hours per week for each employee, since they are all part time employees

Model 2 - Formulation

$$\min_{x_{ik}, y_{ik}, \delta_{ijk}} \quad \sum_{j=1}^{14} \sum_{k=1}^{7} (n_{jk} - \tilde{n}_{jk})^2$$

$$\text{subject to} \quad 7 \leq x_{ik} \leq 20, \quad \forall i, k$$

$$\max\left(0, \frac{a_{ijk}}{M}\right) \leq \delta_{ijk} \leq \min\left(a_{ijk}, 1\right) \quad \forall i, j, k$$

$$x_{i,k}, y_{ik}, \delta_{i,j,k} - \text{integers}, \quad \forall i, j, k$$

$$y_{ik} \quad \text{solves} \quad \min_{y_{ik}} \sum_{i=1}^{12} \sum_{k=1}^{7} (y_{ik} - \bar{y}_i)^2$$

$$0 \leq y_{ik} \leq 8, \quad \forall i, k$$

$$\prod_{k=1}^{7} y_{ik} = 0, \quad \forall i$$

$$20 \leq \sum_{k=1}^{7} y_{ik} \leq 38, \quad \forall i$$

Model 2 - Optimal schedule (solution)

- (x^*, y^*, δ^*) - the solution of the Model 2.
- The optimal schedule, according to Model 2, will be such that the $i-$th employee should work on the $k-$th day starting from x^*_{ik} hour, y^*_{ik} hours continuously.
- The contract for $i-$th worker should be for $\sum_{k=1}^{7} y^*_{ik}$ hours per week.

Model 3 – INP – Shifts & Starting Times

$$\min_{x_{des}, s_{de}} \sum_{d=1}^{7} \sum_{e=1}^{n} \left(s_{de} - x_{des} \frac{\sum_d s_{de}}{\sum_d x_{des}} \right)^2$$

$$\text{subject to} \sum_{d=1}^{7} s_{de} = C_e \quad \forall e = \overline{1, n}$$

$$\sum_{s=1}^{s_{de}} \sum_{e=1}^{n} x_{des} = 2 \quad \forall d = \overline{1, 7}$$

$$0 \leq s_{de} \leq 8 \quad \forall (d, e)$$

$$\sum_{d=1}^{7} x_{des} \leq 6 \quad \forall e = \overline{1, n}$$

Model 3 – INP - Shifts & Starting Times

$$t_{de} + s_{de} \leq 21 \quad \forall (d, e)$$
$$x_{des} \in \mathbb{B}$$
$$t_{de}, s_{de} \in \mathbb{R}_0^+,$$

where

$$x_{des} = \begin{cases} 1 & \text{if employee } e \text{ works shift } s \text{ as a starting one on day } d, \\ 0 & \text{otherwise;} \end{cases}$$

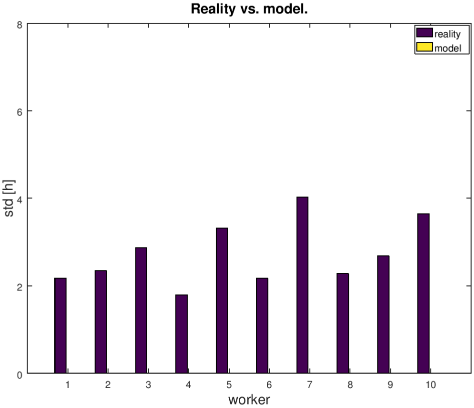
s_{de} is # shifts that e works on day d;
t_{de} is the starting time for e on day d;
$n = 12$.

Results from Model 1 (0 out of 4)

- Solved with the `Matlab's` built-in function `fmincon`.
- The results were rounded to the next 0.5 (except for the zeros).

Results from Model 1 (1 out of 4)

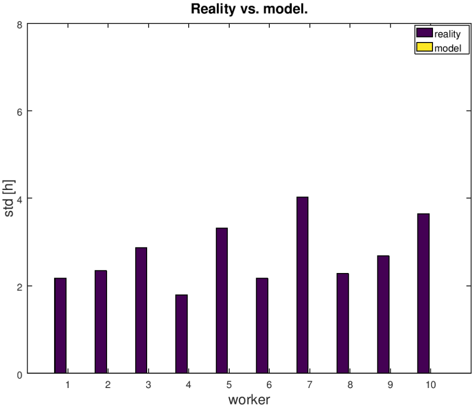
- Discrepancy from the mean weekly working hours (demand according to the daily working hours):



(!) Ours is zero. ;)

Results from Model 1 (2 out of 4)

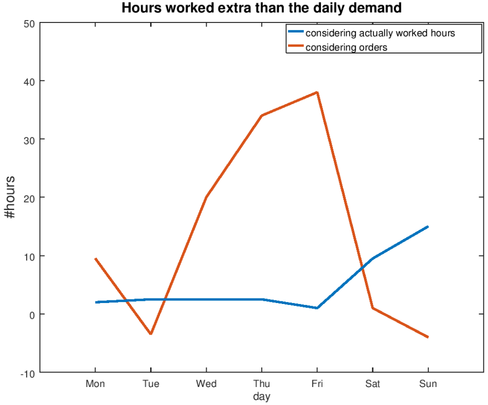
- Discrepancy from the mean weekly working hours (demand according to the daily orders):



(!) Ours is zero. ;)

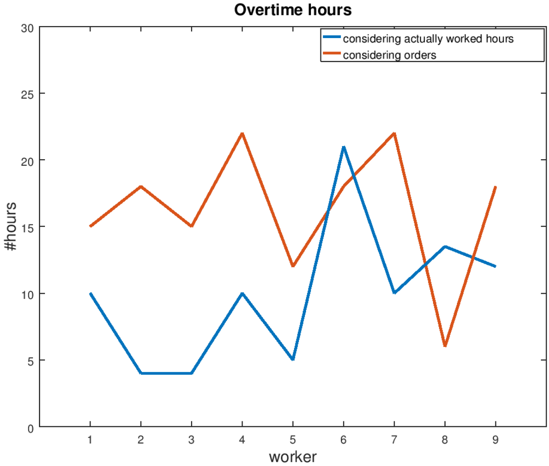
Results from Model 1 (3 out of 4)

- When the demand D is based on the number of orders, the amount of extra working hours is bigger than the one from the data.



Results from Model 1 (4 out of 4)

- When the demand D is based on the number of orders, the overtime is bigger than the one from the data.

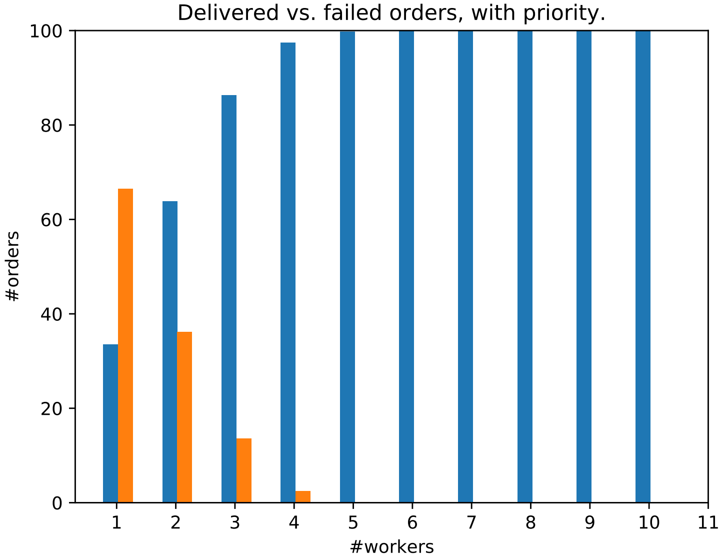


Agent-based model

Agent-based model

- Simulate store with fixed number n of employees and orders coming in (use historical data).
- Each order is processed/fulfilled by an employee if available. Otherwise, wait.
- Order is successful if processed.
- Order is failed if order is not processed by collection time.
- We wish to see how the percentage of failed order depends on the number n of employees.

Agent-based model



Conclusion

- Formulated staff scheduling problem as MINLP
- Set up agent-based model of store that can be used to analyze the dependence of performance of the COOP Drive with the number of employees present.

Future work

Scheduling

- Solve the scheduling models

Agent based modeling

- Analyze how making the order time earlier change the maximum number of workers required
- Incorporate splitting fulfillment of orders by perishable and non-perishable goods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